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## HEAVY ION PION PRODUCTION: SPECTRAL IRREGULARITIES



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# HEAVY ION
# PION PRODUCTION: SPECTRAL IRREGULARITIES

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Abstract: Data on $\pi^-/\pi^+$ ratios and on hills and valleys in spectra from heavy ion collisions are reviewed. Theoretical studies to handle Coulomb effects on pion spectra are examined. The possible role of strongly-bound pion orbitals of nuclear size is discussed.

## 1. Introduction

With the conversion of proton synchrotrons to heavy-ion acceleration capability came the opportunity to study pions produced by heavy ions. Here I am thinking of nuclei heavier than alpha particles as constituting heavy ions. Indeed, we may recall that the first artificially detected pions were made with alpha particles, since the internal Fermi motion made production cross sections with alpha particles more favorable than with protons of about the same energy then available in the 300-400 MeV range.[1]) At the Princeton-Penn accelerator Schimmerling *et al.*[2]) studied pion production by $^{14}$N ions at 520A MeV. There have followed numerous studies of pions at the Berkeley Bevalac, which I shall refer to at various places in this paper.

Since presumably others at this conference will be covering threshold pion production, pion thermometry, etc., I shall concentrate on other special features. First, we shall look at the data and implications of $\pi^-/\pi^+$ ratios, with special attention to the region near rest in the c.m. system for nearly symmetric collision systems. A nice feature of the pion probe is that the pions come in both positive and negative charge states. By studying spectra of both charges it may be possible to dissociate Coulomb effects from other effects on the spectra. In contrast to the protons from the heavy ion reactions, the pions will be emitted only from the hot firecloud region and not from the relatively cold spectator fragments.

## 2. The ratio $\pi^-/\pi^+$ near rest velocity in the center-of-mass

### 2.1. MONTE CARLO TRAJECTORY STUDIES

Theory for the $\pi^-$ to $\pi^+$ ratio at center-of-mass rest velocity for near-symmetric systems (hereafter called RC) preceded experiment. That is, Cugnon and Koonin ran sophisticated Monte Carlo trajectory studies for the system $^{40}$Ar on $^{40}$Ca at E/A of 1.05 GeV, as well as other systems.[3]) They used their cascade code with $\Delta$-resonance formation and decay to give the average nuclear charge distributions. They then ran relativistic trajectories for the pions with thermal and direct initial distributions of pion energies. Pions were allowed to feel the electromagnetic force field only, and not to be scattered or absorbed by the strong interaction with nucleons. Fig. 1 is from their paper and shows the predicted ratio R(y) for various center-of-mass rapidities y along 0°.

Subsequent to this theoretical work our MSU-Tokyo-Berkeley collaboration made 0° to 30° measurements on this collision system for both $\pi^-$ and $\pi^+$. The results are reported in two papers by Frankel *et al.*[4] The experimental data show a ratio R(y) that is quite flat with a value $1.5 \pm 0.2$, in marked contrast to the RC value in Fig. 1 of 5.5 and the peaked ratio around y=0. Fig. 2 shows a cut at $\theta_{lab}=16°$ of pion production cross sections with three different targets. The lowest pion energies on this figure are close to the rest velocity in the center-of-mass, and our data for Ca and U targets do go in to the center-of-mass (nucleon-nucleon).

Radi and I with collaborators decided to carry out a new Monte Carlo trajectory study on the problem.[5] There are several minor differences we introduced, but probably the most significant is that we introduced pion absorption by the spectator fragments and implicitly by the firecloud, since pions were started from the surface and firecloud charge was neglected at first.

This absorption effect was simulated by specifying that a pion orbit that passed within the arbitrary distance of 0.8 of a spectator fragment radius was absorbed. The effects of the hot, expanding, participant charge cloud were omitted in the initial Monte Carlo trajectory study, since they are difficult to treat and will be model-dependent on poorly known details of the heavy-ion collision process. Our calculations were non-relativistic so were confined to the lowest beam energy for which data were available, namely, 670 A MeV $^{20}$Ne on NaF. Later a specialized formula for the participant effect on the ratio RC was derived and applied to the

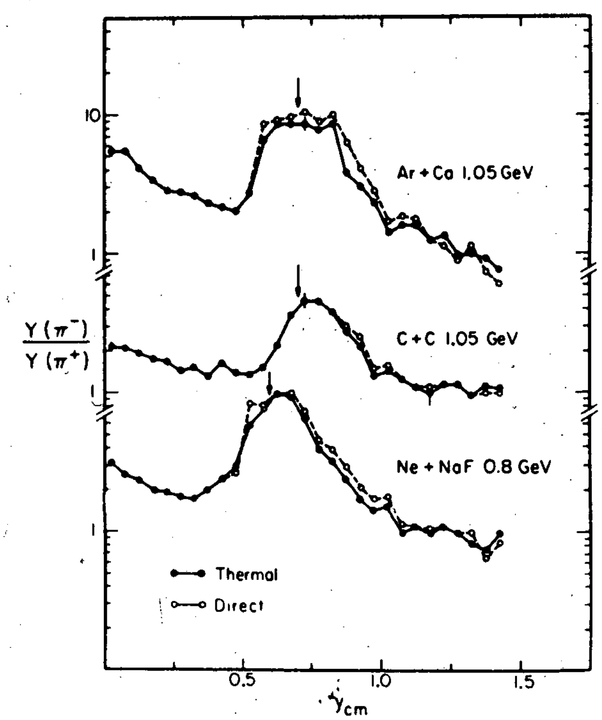


Fig. 1. The $\pi^-/\pi^+$ ratio at 0° as a function of the c.m. rapidity. The solid dots correspond to a thermal source, and the open dots to a direct source. The error bars indicate typical statistical uncertainties in the Monte Carlo calculation. The arrows point to the projectile rapidity. (From ref. 3)

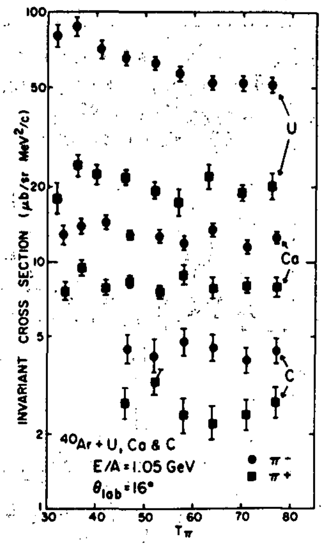


Fig. 2 Pion production cross sections at 16° (lab) for $^{40}$Ar on various targets at E/A of 1.05 GeV. The circles are for $\pi^-$ and the squares for $\pi^+$. The error bars indicate statistical errors. (From ref. 4)

calculation. There are also differences in the pion source velocity distribution, but these are probably insignificant so far as the central ratio RC is concerned.

The results of our new Monte Carlo calculations can be seen in Figs. 3-5.

Let us return later to examine the agreement or lack thereof in the projectile-velocity region. For the moment we continue to focus on the $\pi^-$ to $\pi^+$ ratio at the center, RC. The

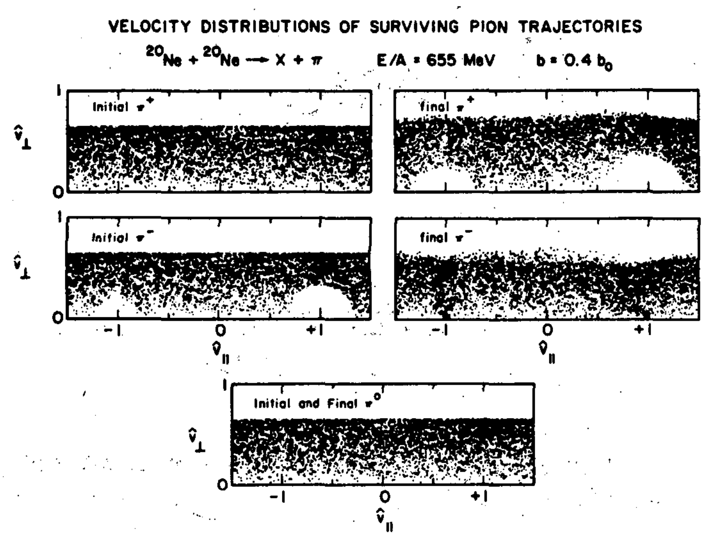


Fig. 3 Scatter plot in velocity space showing distributions of initial velocities giving rise to trajectories that are not absorbed and also distributions of final velocities. This scatter plot is based on an initial flat distribution in velocity space, and the weighting according to a 2-fireball thermal source is put into the later binning of events, along with impact-parameter averaging. (From ref. 5)

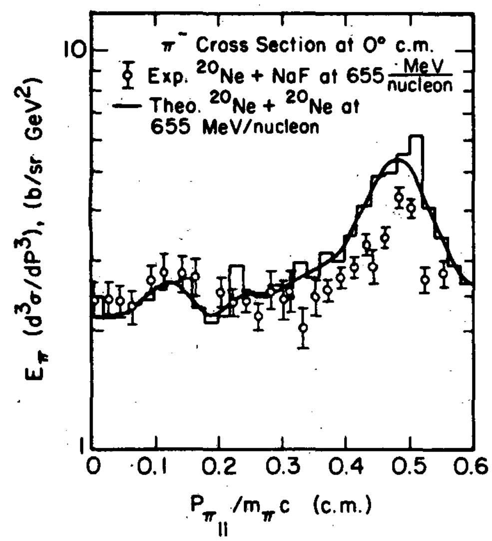


Fig. 4 Cut at zero degrees for $\pi^-$ with histogram giving the Monte Carlo calculation and the data from ref. 4 shown as points with error flags. The theory is arbitrarily normalized to the data in the low-velocity (c.m.) region. The width of the cut is 0.1c in the perpendicular velocity. (From ref. 5)

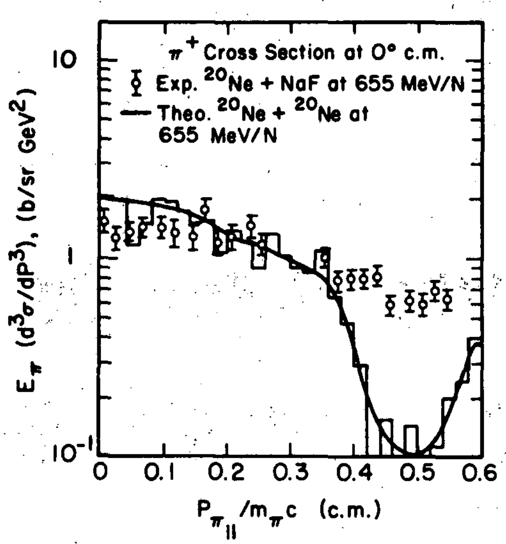


Fig. 5 Same as Fig. 4 except for $\pi^+$. The arbitrary normalization constant is the same as for $\pi^-$ above. (From ref. 5)

factor in the $\pi^-$ to $\pi^+$ ratio due to the spectator charges, as calculated by the Monte Carlo work, is 1.40.

Besides this we must examine the factors arising from the neutron-excess in the target (NaF has a five percent excess of neutrons over protons) and from the participant charge. Details of calculating these factors are given in ref. 5, but it is worth reproducing here the derivation of the expression for the effect of participant charge, specialized to the case of pions with final velocity zero in the center of mass. The more general case of non-zero velocity does not appear to have an analytical solution.

We represent the fireball protons as expanding in a spherical shell initially of some mean radius $R_c$ with a speed $v_c$. The pion starts from the nuclear surface $R_r$. For the first brief interval of time before the expanding charge shell overtakes the pion, the pion will experience the regular inverse square Coulomb force, as if the charge were concentrated in a point at the center. After the cloud passes the pion, the pion will feel no further Coulomb force. Thus, to have a final velocity of zero a $\pi^-$ must have a small initial velocity $v_i$, outwardly directed. If we assume the pion moves a distance much less than the nuclear radius before being overtaken by the protons, we can by simple Newtonian physics write the equation relating the final to the initial velocity as follows:

$$v_{xf} = v_c - \left[ (v_c - v_i)^2 + \frac{2Ze^2(R_r - R_c)}{m_\pi R_r^2} \right]^{1/2} , \tag{1}$$

where Z is the total participant charge. Assuming the velocities are directed along the x-axis, the classical Jacobian matrix with elements the derivatives of final velocity components with respect to initial velocity components appears as follows:

$$J_- = \begin{vmatrix} \frac{\partial v_{xf}}{\partial v_{xi}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}^{-1} = \left[ \frac{\partial v_{xf}}{\partial v_{xi}} \right]^{-1} \tag{2}$$

The Jacobian is evaluated by differentiating eq. (1) and substituting it into eq. (2):

$$J_\mp = \left[ 1 \mp \frac{2Ze^2(R_r - R_c)}{m_\pi R_r^2 (v_c - v_i)^2} \right]^{-1/2} \approx 1 \pm \frac{Ze^2(R_r - R_c)}{R_r m_\pi (v_c - v_i)^2 R_r} , \tag{3}$$

where we have indicated also the result for $\pi^+$ by the lower signs. In the numerical evaluation of eq. (3) for the cases listed in Table 1 we have taken the impact parameter averaged collision as having half the total charge as participants and half as spectators.

**Table 1. Zero-velocity $\pi^-/\pi^+$ Ratio Factors**

| Projectile-target<br>E/A (lab) | $^{20}$Ne+NaF<br>655 MeV | $^{40}$Ar+Ca<br>1050 MeV |
|---|---|---|
| Spectator Coulomb factor | 1.40 | 1.36* |
| Neutron-excess factor | 1.083 | 1.17 |
| Participant Coulomb factor | 1.11 | 1.10 |
| Final ratio (theo.) | 1.68 | 1.75 |
| Final ratio (expt.)(ref. 4) | 1.76±0.1 | 1.5±0.2 |

* Calculated by scaling from $^{20}$Ne Monte Carlo results.

The agreement with experimental ratios seems satisfactory, although our treatment of the participant charge effects is specialized to zero-velocity pions, and refined treatments are certainly called for. My guess is that the high RC ratio and the peaking at center of mass in

Fig. 1 of ref. 3 comes from a kind of Coulomb focusing effect within the hot matter region, analogous to that observed in the projectile velocity region. In our new Monte Carlo calculations that effect does not arise because the charge in the hot matter region is neglected in the trajectory part. However, such $\pi^-$ as would tend to focus within the firecloud would undergo nuclear absorption according to the prescription we applied explicitly to the spectators.

## 2.2. FUTURE WORK

It is certainly important to get more data on pion production cross sections around c.m. velocities. It will be helpful to get data on strictly symmetric isospin-zero systems. Harris, Schroeder, Wolf, and collaborators are currently working up data for 1.05A GeV $^{40}$Ca on natural calcium, and these data should be of great interest. The value of such data will be enhanced to the extent that some tagging to separate central and peripheral events can be done. Next year should see experiments by the Japanese collaboration at the Bevalac HISS spectrometer in which pions at 0° and small angles will be measured concurrently with heavy-ion fragment identification in downstream drift chambers.

There is need for further theoretical trajectory studies as well, building on the foundations of previous work but including pion-nucleon scattering and absorption effects carefully. There is not sufficient space here to cite and review the various papers that have derived and applied Coulomb corrections to heavy-ion pion spectra. Almost any treatment will reproduce qualitatively the beam velocity $\pi^-$ peak and $\pi^+$ depression, but I believe that a great deal of caution is in order regarding the lowest energy pions (c.m.). This region in momentum space is not dominated by any single charge center, and there appears to be a delicate cancellation of Coulomb forces that may occur. For example, in ref. 5 we show that for pions formed at the point of tangency in a barely grazing collision the Coulomb effect goes away, and all charges of pions have unity values for their classical Jacobians. This result is contrary to the analytical expressions of Gyulassy and Kauffman[6]), where each charge center contributes a scalar term to the total Coulomb effect. The clever approach that allows them to get analytical expressions is to assume in perturbative limit proton and pion coordinates that always maintain the same vector relations as if expanding from a common center, though when the Coulomb interaction is turned on, the charges are at some average nuclear radial distance. Now the pion formed at the point of tangency in a grazing collision obviously is not moving from a common center with the two nuclear centers. Contrasting with this counter-example, though, we find that eq. (3) above bears a very close similarity to eq. (2.15) of ref. 6. If $v_i$ is set to zero, eq. (3) is the same as their expression except for a factor of $(R_r - R_c)/R_r$, which is of the order of 0.5.

Despite these differences for special cases, the predictions of the Gyulassy-Kauffman expressions and our Monte Carlo work, when impact-parameter averaged, are not too different. It is probably relatively safe to use their analytical expressions for situations where the Coulomb effect is dominated by a single near-lying charge cloud in velocity space and perhaps where there are multiple sources removed in the same direction from the pion in velocity space. Otherwise, considerable caution should be exercised.

Another point of doubt raised about classical trajectory calculations is that quantum effects may not be negligible here, where the deBroglie wave length of the pions is comparable to nuclear dimensions. Of course, quantal effects could be introduced by calculating the classical action for trajectories and using the semiclassical methods of classical limit S-matrix studies. I rather believe that the quantum interference effects so calculated would not survive averaging over the large number of final states at these high energies. One place where quantum effects may be important is for $\pi^+$ close to beam velocity. As can be seen in Fig. 5 the theory vastly exaggerates the depression of the $\pi^+$ cross section, compared to experiment.

## 3. The $\pi^-/\pi^+$ ratio near beam velocity

Benenson *et al.*[7]) were the first to report the dramatic spectral effects occurring near beam velocity. These effects have been studied in more detail and reported by Sullivan *et al.*[8])

### 3.1. DEDUCTIONS ABOUT PROJECTILE FRAGMENTATION

The $\pi^-$ peak seems to be centered just slightly lower than beam velocity, consistent with the slight slowing of heavy-ion projectile fragments, so prominently produced at these high energies. In like manner the principal contribution to the width of the peak is thought to be the velocity dispersion of the projectile fragments.[9]) (In refs. 3 and 6 the width was governed by a spectator temperature, with the models assuming an expansion of free charges. Expressions from these models can be used provided one reinterprets the spectator "temperature" as being a measure of the fragment velocity dispersion, as is discussed in ref. 9.)

We were unable to fit the $\pi^-$ peak using the experimentally measured final yields of projectile fragments, but had to use a calculated primary distribution of fragments, before particle evaporation cools them. Thus, the pion peak probes the velocity dispersion of the primary fragments, a quantity not otherwise measurable in the laboratory. Fig. 6 shows the $\pi^-$ data for one case and the theoretical curves for three different assumed values of the velocity dispersion parameter.

The middle value, 86 MeV/c, is the value that fits most final fragment velocity distributions, but evidently the lower value, 60 MeV/c, gives the best agreement with pion data.

### 3.2. FUTURE STUDIES OF BEAM-VELOCITY PIONS

Whether this seeming difference in velocity dispersions between primary and final fragments is significant can best be determined in the type of experiment mentioned above for the

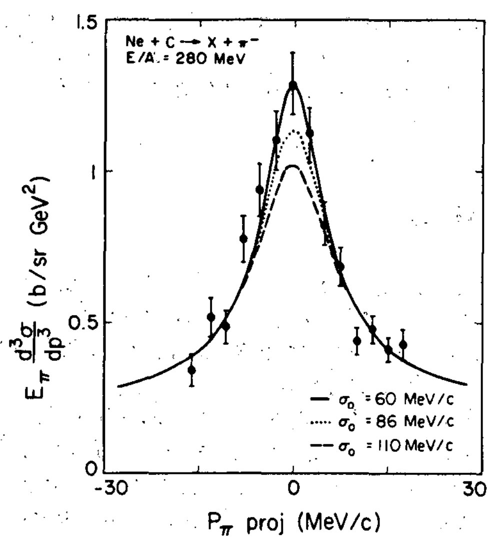


Fig. 6 Data points and theoretical curves for $\pi^-$ from the Ne + C reaction at E/A of 280 MeV. The theoretical curves have had the experimental resolution folded into them, and they are shown for three values of the velocity dispersion parameter, $\sigma_0$, namely, 60 MeV/c (solid), 86 MeV/c (dotted), and 110 MeV/c (dashed). (From ref. 9)

Japanese collaboration at the Bevalac. If the momentum and Z of a projectile fragment is determined in each event along with the pion momentum, then the dispersion introduced by de-exciting particle evaporation should be dominant in establishing the width of the $\pi^-$ peak about the fragment velocity.

For the inclusive data now available in the beam velocity region another feature of interest is the general height of the peak relative to the smooth general background. This ratio is affected by the way in which pion production depends on the impact parameter. In the row-on-row model employed in ref. 5 pion production in central collisions is more enhanced relative to peripheral collisions than in the geometrical fireball model of Swiatecki. Again we note that the next generation of experiments, where beam velocity pions are measured along with some kinds of tags distinguishing central from peripheral collisions, can give us much more direct information on this "profile" function for dependence on impact parameter. This, in turn, relates to central questions of heavy-ion pion production: (1) to what extent is there chemical equilibrium for the pion concentration in hot nuclear matter, (2) to what extent is there charge-exchange equilibrium among pions and nucleons, and (3) to what extent is there thermal equilibration of kinetic energies and momenta. The heavier ion beams newly available should help in finding answers to these questions.

## 4. Bumps in pion spectra in the center-of-mass region

### 4.1. EARLY OBSERVATIONS AND EXPLANATIONS

One of the most fascinating current puzzles in the pion data from heavy ions is the bump or ridge of extra cross section at very low energy (c.m.), around 15 MeV. This was reported in range telescope work on $\pi^+$ for the $^{20}$Ne + NaF system by Nakai *et al.*[10]) and for the $^{40}$Ar + Ca system by Wolf *et al.*[11])

The low-energy pion bumps have been observed at E/A of 0.8 and 1.05 GeV but not at 0.4 GeV, and they are not seen in the reaction $p + p = \pi^+ + X$.

There have been several possible explanations advanced, among them (1) the decay of strongly bound pairs of $\Delta$ particles, (2) an imaging of some collective hydrodynamic flow patterns, (3) decay of $\Delta$ particles rescattered to spectator velocities after formation, or (4) a simple Coulomb effect.[12])

### 4.2. RECENT OBSERVATIONS

More recent experimental work rules out the simple Coulomb explanation, since the bump is also observed for $\pi^-$ as well as $\pi^+$. Fig. 7 illustrates this with data from work of Frankel *et al.*[13]). These data and a discussion of the phenomenon appear in a report to be published in the 1982 Banff Winter School Proceedings.[13])

The matching with Nagamiya data where they touch adds confidence to the measurements. Our data clearly indicate a bump that attains a maximum around $p_\perp/m_\pi c$ of 0.4. Unfortunately we ran out of beam time before extending the $\pi^+$ measurements out far enough along 90° (c.m.). However, since the $\pi^+$ bumps in this vicinity of $p_\perp$ were seen in the early range telescope work referred to above, we presume there should be a $\pi^+$ bump also at this slightly lower beam energy.

Other data also make unlikely any simple Coulomb explanation of the low-energy bump. Wolf *et al.*[15]) present and analyze data from many target-projectile systems. They observe a persistence of the low energy bump at roughly the same $p_\perp$ even for considerably heavier targets.

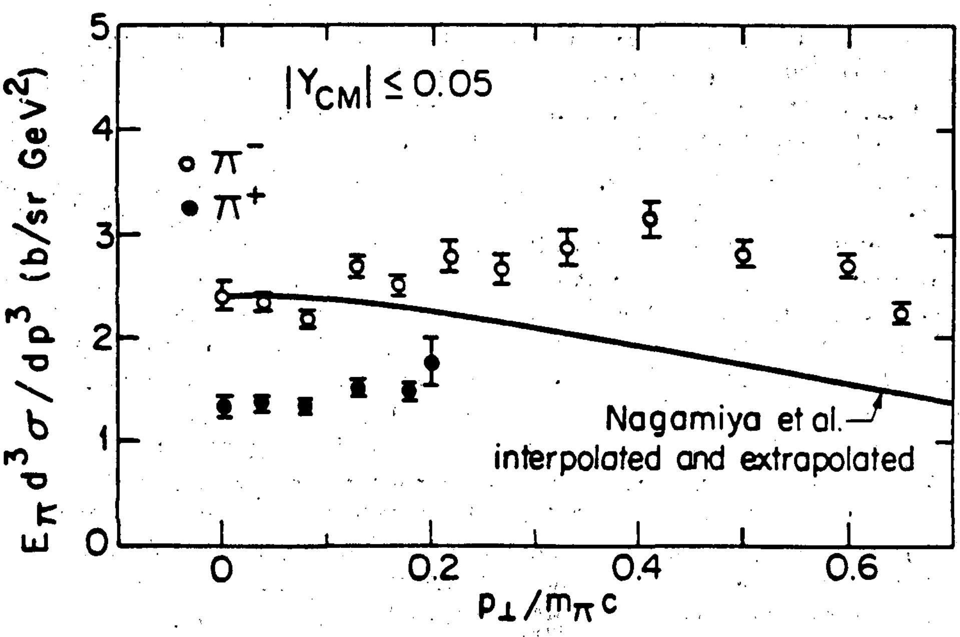


Fig. 7 Pion invariant production cross sections for Ne + NaF at E/A=655 MeV at 90° (c.m.) Data for $\pi^-$ are shown as open circles and for $\pi^+$ as solid dots. The solid line is taken from data of Nagamiya *et al.*[14]) The line represents an interpolation in beam energy between their 400 and 800 MeV data and a parabolic (thermal) extrapolation down in pion energy from their lowest points, which would be just on the right-hand margin of our figure. To insure adequate statistics we took the cross section as an average not over a fixed angle (c.m.) but rather over a fixed interval ±0.05 in rapidity $y_{cm}$. (From ref. 4)

### 4.3. ERICSON-MYHRER DEEPLY-BOUND PION ORBITING

One rather striking aspect of the low-energy pion bump is that it tends to peak at a $p_\perp$ value of 0.4 to 0.5 $m_\pi c$. The reciprocal of this gives an uncertainty relation size of 2.5 to 2.0 pion Compton wave lengths, or 3.5 to 2.8 fm. Now this is very close to the nuclear size of the neon or argon projectiles or fireclouds they would produce. The relation seems too remarkable to me to dismiss as mere coincidence. A fancier way of saying this is that the pion bump profile observed in Fig. 7 and in prior studies is a snapshot of a pion wave function in momentum space. If we perform a 3-dimensional Fourier transform, we get an estimate of the wave function in configuration space. With present data it is not justified to push this analysis too far. Qualitatively, the results are the same as the simple uncertainty relation argument above. If the data are really like those of Fig. 7, with the bump displaced from zero, then the Fourier-transformed wave function, using harmonic oscillator forms to fit, will peak at the uncertainty relation value and have a low value at the origin. Other data suggest the bump may be centered at the origin, and in such case the Fourier transform of a Gaussian in momentum space is a Gaussian in configuration space, with the uncertainty relation connecting the two widths. Further work at much higher statistics is called for; especially valuable would be more finely tagged data instead of impact-parameter-averaged inclusive data.

The low-energy pion bump, or rather this interpretation of it, may have been anticipated in the 1979 theoretical paper of Zimanyi, Fai, and Jakobsson.[16] They were working with the equations of thermodynamic equilibrium in hot nuclear matter among the constituent nucleons, pions, etc. Since pions obey Bose-Einstein statistics, they obtained pion spectra dividing into two components, a "zero-energy" component and a finite-energy component, approaching the Boltzmann-like exponential behavior asymptotically. Kitazoe and Sano[17] had noted this behavior also, but Zimanyi *et al.*[16] went a step further to suggest that the "zero-energy" pions would not be literally zero, as in infinite nuclear matter, but would have an energy distribution of width given by the uncertainty principle and the nuclear size. They stressed that these pions represented a boson-condensation of real pions, not to be confused with the virtual pions of the much-sought "pion condensation." Fig. 8 from their paper shows low energy bumps as square peaks of 5 MeV width, corresponding by uncertainty principle to the size of the U + U system.

Now if the observed pion bumps do bear a relation to these theoretical predictions, it does not necessarily imply thermochemical equilibrium nor boson condensation. It does raise two deep questions: (1) what is the origin of the confining potential for the intranuclear pionic orbits and (2) why is nuclear absorption and scattering not so strong as to make orbiting meaningless? The latter question is reminiscent of the question that probably delayed the serious consideration of nuclear shell models for more than a decade after Elsasser's original proposals.[18]

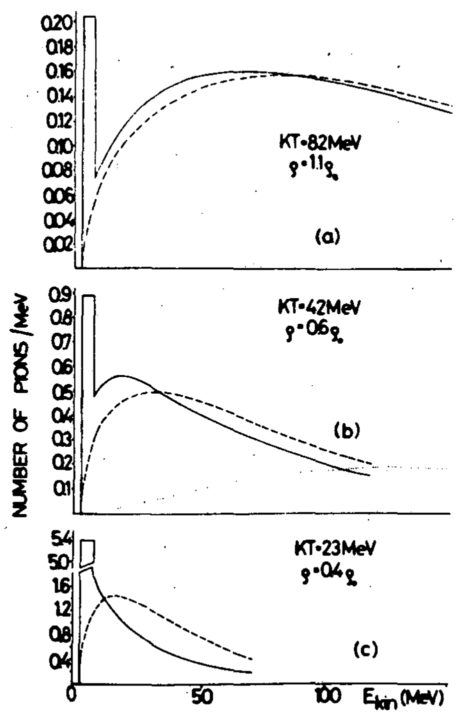


Fig. 8 Theoretical pion energy spectra (c.m.) from 1.4 A GeV (lab) uranium on uranium at three different stages of expansion, with the densities and temperatures noted. The (c) part is stated as probably unrealistic in that chemical and thermal equilibrium among pions and nucleons would probably be lost before the expansion reaches those low densities and temperatures. The solid curve is the Bose distribution, while the dashed curve is the Boltzmann distribution. The faint dotted line of the (b) part represents the contribution of Δ decay after break-up. (From ref. 16)

At this juncture it behooves us to take a new hard look at the curious deeply-bound pionic states theoretically studied by Ericsson and Myhrer some years ago.[19] The deeply-bound states come about as follows: their tractable pion optical potential (an expansion from a non-local potential) has static terms that are repulsive, isospin dependent, with a part dependent linearly on the local nuclear density and a part quadratically dependent on the density. The optical potential also has attractive Kisslinger potential terms that have a $k^2$ dependence just like the kinetic energy term in the Schrödinger equation. The potential has small imaginary components associated with inelastic scattering (linearly dependent on density) and large components associated with true absorption (quadratically dependent.) For $\pi^-$ and neutron-rich species (or for $\pi^+$ and proton-rich species) of ordinary nuclei the attractive Kisslinger term can exceed the kinetic energy term in the nuclear interior. That is, the effective mass is negative there and it passes through a singularity in the surface region, finally decreasing to its asymptotic value of 140 MeV at large distance. The behavior of the wave function near the singularity, where the curvature becomes infinite, poses something of a problem. Mandelzweig, Gal, and Friedman[20] restudied the problem, dropping the repulsive static potential, but looking in great deal at the role of the singularity. They show that the singularity serves as a kind of boundary dividing solutions into two classes, interior and exterior solutions, though the actual wave functions of the two classes are not exclusively confined to those regions. They give useful expressions for WKB solutions in terms of the Bohr-Sommerfeld phase integral; whereas for an ordinary sloping potential wall the WKB solution in the classically allowed region has a phase representing penetration of $\pi/4$ into the wall, the mass singularity has the opposite effect of adding $\pi/4$ to the phase integral.

Evidently the surface defined by the effective mass singularity would serve as a confining potential for intranuclear pion orbits. It is a touchy matter whether the singularity is attained in most nuclei. However, with only modest increases in the nuclear density, such as would be exceeded in high-energy heavy ion collisions, the effective mass is driven negative for all reasonable values of the optical potential parameters. The remaining big question is whether true nuclear absorption is too strong for such states to be observed. The widths derived in the cited studies are probably not too large for such states to be the possible explanation of the observed low-energy pion bumps as orbiting within the firecloud. Whether or not the boson tendency for multiple occupation of the same state plays a role is a more open question.

To reexamine these Ericson-Myhrer inner solutions we choose the most up-to-date pion optical potentials, those of Carr, McManus, and Stricker-Bauer (hereafter their published potential set F will be referred to here as the CMS potential.)[21]

To get some orientation on pion orbiting conditions I have developed a computer code to solve for eigenvalues of pions in a nuclear optical potential. The methods of Ericson and Myhrer[19] have been employed except that we obtain eigenvalues first in the real part of the potential, using the WKB approximation and applying Bohr-Sommerfeld quantization conditions, with modifications of ref. 20 where there is a singularity in the effective mass. The widths of the states are calculated perturbatively from the imaginary parts of the potential; the states of greatest interest to us will have rather small widths and thus be adequately estimated by the perturbative treatment of the imaginary phase. I have taken potential-parameter starting values from the recent work of Carr, McManus, and Stricker-Bauer,[21] though they are not very different from the pionic atom values used by Ericson and Myhrer[19] except in the isospin dependence of the static repulsive potential. The optical potential in the notation of ref. 21 is as follows:

$$2\omega U = -4\pi \left[ b + B - \nabla \cdot \frac{L}{1 + 4\frac{\pi}{3}\lambda L} \nabla \right] \qquad (4)$$

plus the Coulomb term. Here

$$b = \bar{b}_0\rho - \epsilon_\pi b_1\delta\rho \quad (5a)$$

$$L = c + C \quad (5b)$$

$$c = c_0\rho - \epsilon_\pi c_1\delta\rho \quad (5c)$$

$$B = B_0\rho^2 - \epsilon_\pi B_1\rho\delta\rho \quad (5d)$$

and

$$C = C_0\rho^2 - \epsilon_\pi C_1\rho\delta\rho \quad (5e)$$

where $\rho$ is the nucleon density and $\delta\rho$ is the neutron density minus the proton density. Capitalized parameters B and C are coefficients of terms arising from true pion absorption and are dependent on the square of the nuclear density. The lower case parameters b and c denote terms arising from single nucleon scattering and are dependent linearly on the density. Isoscalar and isovector terms are distinguished by the subscripts zero and one, respectively. The Lorenz-Lorentz-Ericson-Ericson (LLEE) parameter for polarization of the medium is denoted by lambda. In writing the above equations I have suppressed the radial argument r, which affects all the densities and density dependent variables. Furthermore, I have set the kinematic factors of ref. 21 to unity for simplicity, since they differ from unity only by terms of the order of the pion-nucleon mass ratio. The quantity $\omega$ is the relativistic total energy of the pion at infinity. The quantity $\epsilon_\pi$ is $+1$ for $\pi^+$ and $-1$ for $\pi^-$. First, with the nuclear potential turned off I have verified that the code gives about the right Bohr value for the pionic binding energy in lowest s- and p-states. Then the first calculations with the double density of a fireball made clear that the central part of the nucleus will have a negative effective pion mass for both $\pi^+$ and $\pi^-$ under a broad range of neutron-to-proton ratios. The mass-singularity near the surface thus provides always a confining boundary for inner solutions. In general they are deeply bound states, but they show large widths. The calculation of widths or lifetimes is quite uncertain, since the imaginary potential will depend on the depth of binding. Furthermore, the imaginary terms in the velocity-dependent part produce spurious source terms in classicially forbidden regions.

### 5. Speculations on pionic atoms and anomalons

Although I have looked at many double-density solutions in the CMS potential, I will conclude here by showing Ericson-Myhrer inner solutions at rather ordinary nuclear densities. (The expanding firecloud will in any event pass through the density region of ordinary nuclei.) These solutions were made with Prof. Wm McHarris to explore the idea that anomalons might be Ericson-Myhrer solutions of $\pi^-$ on neutron-rich projectile fragments.

Anomalons are heavy-ion fragments observed in nuclear emulsions to have abnormally short mean-free-paths for interactions in the first 2 or 3 cm of their range. These observations have been made first in cosmic rays and later in exposures at the Berkeley Bevalac and the Dubna synchrophasotron. There is not time here to list extensive references. I note here only the article of Friedlander *et al.* and several adjoining articles in the Proceedings of the 5th High Energy Heavy Ion Study at Berkeley last year.[22]) McHarris and I have described our model[23]) of anomalons as pineuts[24]) orbiting nuclear fragments, but our quantitative estimates are made by calculating Ericson-Myhrer solutions in their test nucleus $^{34}Na$. We, however, use the newer CMS potential and an enhanced effective neutron density brought about locally by the presence of the $\pi^-$. It is this local neutron-density build-up that insures attaining a negative effective mass in the nuclear interior, and this effective-density feature is the reason for using the language "orbiting pineuts" to describe our picture of anomalons.

Fig. 9 shows the potentials and eigenvalues for the nodeless $\pi^-$ solutions for angular momenta 0-4. Also shown is the effective mass behavior. Note that the centrifugal potential of the radial wave equation is negative inside the singularity, and the kinetic energy is also negative.

For the anomalon explanation there is the serious constraint of a long lifetime, i.e., $>10^{-10}$sec. We therefore look to the deepest bound solutions, where all or most channels are closed for the true pion absorption process that involves a $\pi^-$ converting a proton to a neutron in the close proximity of another nucleon, with the pair of nucleons carrying off the available energy of 140 MeV rest mass energy less the pion binding energy. In the calculations illustrated in Fig. 9 the imaginary parts of the optical potential were therefore set to zero. To furnish an explanation for the mid-rapidity bumps there is no such tight constraint on lifetimes, but we have not paid much attention to the widths given by the CMS potential imaginary parts, since the true absorption will surely be different under firecloud conditions than for normal cold nuclei.

Perhaps it may seem we are overworking the Ericson-Myhrer pionic states to invoke them in explanation for both the mid-rapidity pion bumps and the anomalons. That remains for future experiment and theory to explore.

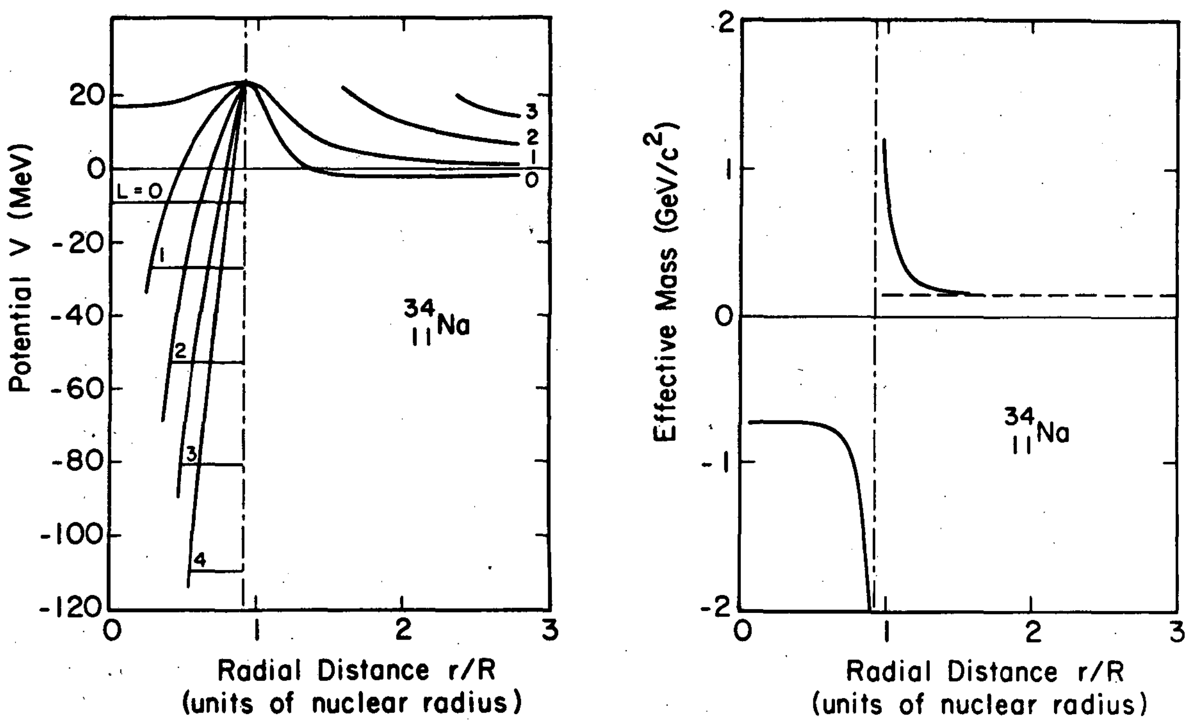


Fig. 9 The left side shows the radial potentials for $\pi^-$ in neutron-density-enhanced $^{34}Na$ for L values 0 through 4. The eigenvalues are indicated by horizontal bars at the appropriate energy running between the inner turning point and the mass singularity. The right side shows the effective mass dependence on distance.

We would note also that pionic states with two or more pions may gain extra stability from admixtures of $\Delta$ pairs, which are predicted to have very strong binding in certain spin-isospin states. The free pineut may have a better chance of being bound with two $\pi^-$ than with one. Other exotics, such as, the neutral species of two $\pi^-$ bound to a $^8$He core might be sought. Even if this anomalon explanation is wrong, it may stimulate the interesting, if difficult, investigation of external pionic atom states in which two $\pi^-$ are present. Only high energy heavy ion physics offers a realistic opportunity for such studies, which could shed light on the fundamental interaction between two $\Delta$ particles in the isospin -3 channel.

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