Title
The equivariant Euler characteristic of moduli spaces of curves

Author
Gorsky, Eugene

Publication Date
2014

DOI
10.1016/j.aim.2013.10.003

Peer reviewed

THE EQUIVARIANT EULER CHARACTERISTIC OF MODULI SPACES OF CURVES.

EUGENE GORSKY

ABSTRACT. We derive a formula for the S_n–equivariant Euler characteristic of the moduli space $\mathcal{M}_{g,n}$ of genus g curves with n marked points.

1. INTRODUCTION

Consider the moduli space $\mathcal{M}_{g,n}$ of algebraic curves of genus g with n marked points. The symmetric group S_n acts naturally on this space. Let V_λ denote the irreducible representation of S_n corresponding to a Young diagram λ, then one can decompose the cohomology of $\mathcal{M}_{g,n}$ into isotypic components:

$$\mathrm{H}^i(\mathcal{M}_{g,n}) = \bigoplus_{\lambda} a_{i,\lambda} V_\lambda.$$

The S_n-equivariant Euler characteristic of $\mathcal{M}_{g,n}$ is defined by the formula

$$\chi^{S_n}(\mathcal{M}_{g,n}) = \sum_{i,\lambda} (-1)^i a_{i,\lambda} s_\lambda,$$

where s_λ denotes the Schur polynomial labeled by the diagram λ. We calculate these equivariant Euler characteristics for all $g \geq 2$ and n.

Theorem 1.1. *The generating function for the S_n-equivariant Euler characteristics of $\mathcal{M}_{g,n}$ has the form*

$$\sum_{n=0}^{\infty} t^n \chi^{S_n}(\mathcal{M}_{g,n}) = \sum_{\underline{k}} c_{k_1,\ldots,k_r} \prod_{j=1}^{r} (1 + p_j t^j)^{k_j},$$

where p_j are power sums and the coefficients $c_{k_1,\ldots,k_r}$ are defined by the equation (6).

Consider the moduli space $\mathcal{M}_g(k_1 \ldots, k_r)$ of pairs (C, τ) where C is a genus g curve and τ is an automorphism of C such that for all i the Euler characteristic of the set of points in C having the orbit of length i under the action of τ equals ik_i. The coefficient $c_{k_1,\ldots,k_r}$ can be also defined as the orbifold Euler characteristic of $\mathcal{M}_g(k_1 \ldots, k_r)$.

This moduli space can be defined for any tuple of integers $(k_1, \ldots, k_r)$ of arbitrary size r, but we prove that (for a fixed genus g) it is non-empty only for a finite number of such tuples. In particular, r cannot exceed $4g + 2$.

Corollary 1.2. The generating function $\sum_{n=0}^{\infty} t^n \chi^{S_n}(\mathcal{M}_{g,n})$ is a rational function in t. Furthermore, for any n,

$$\chi^{S_n}(\mathcal{M}_{g,n}) \in \mathbb{Z}[p_1, \ldots, p_{4g+2}].$$

Theorem 1.1 can be compared with the computations of [4], [5], [8] and [10] in genus 2 and with the computations of [1], [2], [9], [17] and [18] in genus 3. A similar generating function for the moduli spaces of hyperelliptic curves was previously obtained in [11]. The non-equivariant Euler characteristics of moduli spaces of curves were computed by Bini and Harer in [3].

The paper is organized as follows. In Section 2 we consider a complex quasi-projective variety X with an action of a finite group G. Theorem 2.5 provides a formula for the S_n-equivariant

Euler characteristic of quotients $F(X,n)/G$, where $F(X,n)$ is a configuration space of n labeled distinct points on X. This theorem was previously proved in [10] using the results of Getzler [6, 7] concerning Adams operations over the equivariant motivic rings (see also [12]). The alternative proof presented here uses only the basic properties of Euler characteristic and seems to be more geometric. It also makes the proof of the main result self-contained.

In Section 3 we apply this theorem to the universal family over $\mathcal{M}_g$, the moduli space of genus g curves. This allows us to prove in Theorem 3.3 that the coefficient $c_{k_1,\ldots,k_r}$ is equal to the orbifold Euler characteristic of $\mathcal{M}_g(k_1\ldots,k_r)$. These Euler characteristics are then computed in Theorem 3.8 using the results of Harer and Zagier.

ACKNOWLEDGEMENTS

The author is grateful to J. Bergström, S. Gusein-Zade, M. Kazaryan and S. Lando for useful discussions. This work was partially supported by the grants RFBR-007-00593, RFBR-08-01-00110-a, NSh-709.2008.1 and the Möbius Contest fellowship for young scientists.

2. EQUIVARIANT EULER CHARACTERISTICS

Let X be a complex quasi-projective variety with an action of a finite group G. Let us denote by $F(X,n)$ the configuration space of ordered n-tuples of distinct points on X. For each n, the action of the group G on X can be naturally extended to the action of G on $F(X,n)$, commuting with the natural action of S_n.

In the computations below we will use the additivity and multiplicativity of the Euler characteristic, as well as the Fubini formula for the integration with respect to the Euler characteristic ([15, 19], see also [16]).

Lemma 2.1. *The following equation holds:* $\sum_{n=0}^{\infty}\frac{t^n}{n!}\chi(F(X,n))=(1+t)^{\chi(X)}$.

Proof. The map $\pi_n : F(X,n)\to F(X,n-1)$, which forgets the last point in the n-tuple, has fibers isomorphic to X without $n-1$ points. Therefore $\chi(F(X,n))=(\chi(X)-n+1)\cdot\chi(F(X,n-1))$, and $\chi(F(X,n))=\chi(X)\cdot(\chi(X)-1)\cdot\ldots\cdot(\chi(X)-n+1)$. □

Let p_k denote the kth power sum and let V_λ denote the irreducible representation of S_n labelled by the Young diagram λ. We define the S_n-equivariant Euler characteristic of $F(X,n)/G$ by the equation

$$\chi^{S_n}(F(X,n)/G)=\sum_{i,\lambda}(-1)^i a_{i,\lambda}s_\lambda,$$

where $\mathrm{H}^i(F(X,n)/G)=\bigoplus_\lambda a_{i,\lambda}V_\lambda$ and s_λ is the Schur polynomial.

Lemma 2.2. *The following equation holds:*

$$\chi^{S_n}(F(X,n)/G)=\frac{1}{n!}\sum_{\sigma\in S_n}p_1^{k_1(\sigma)}\cdot\ldots\cdot p_n^{k_n(\sigma)}\cdot\chi\left([F(X,n)/G]^\sigma\right),$$

where $k_i(\sigma)$ is the number of cycles of length i in a permutation σ.

Proof. It is well known that for every i

$$\sum_\lambda a_{i,\lambda}s_\lambda=\frac{1}{n!}\sum_{\sigma\in S_n}p_1^{k_1(\sigma)}\cdot\ldots\cdot p_n^{k_n(\sigma)}\cdot\mathrm{Tr}(\sigma)|_{\mathrm{H}^i(F(X,n)/G)},$$

hence

$$\chi^{S_n}(F(X,n)/G)=\frac{1}{n!}\sum_i(-1)^i\sum_{\sigma\in S_n}p_1^{k_1(\sigma)}\cdot\ldots\cdot p_n^{k_n(\sigma)}\cdot\mathrm{Tr}(\sigma)|_{\mathrm{H}^i(F(X,n)/G)}$$

Now the statement follows from the Lefschetz fixed point theorem. □

Lemma 2.3. *Let $\sigma \in S_n$. Then*

$$\chi\left([F(X,n)/G]^{\sigma}\right)=\frac{1}{|G|}\sum_{g\in G}\chi\left(F(X,n)^{g^{-1}\sigma}\right).$$

Proof. For a point $\mathbf{y}\in F(X,n)$ whose projection on $F(X,n)/G$ is σ-invariant there exists an element $g\in G$ such that $\sigma\mathbf{y}=g\mathbf{y}$. Consider the set of pairs

$$S=\{(g,\mathbf{y})|g\in G,\mathbf{y}\in F(X,n)|\sigma\mathbf{y}=g\mathbf{y}\}$$

and its two-step projection $S\to F(X,n)\to F(X,n)/G$. The fiber of the first projection over a point $\mathbf{y}$ is isomorphic to G-stabiliser of $\mathbf{y}$ or empty, the fiber of the second projection containing $\mathbf{y}$ is exactly the orbit of $\mathbf{y}$. Therefore the cardinality of every fiber of the composition is equal to $|G|$. □

Definition 2.4. For any $g\in G$ we denote by $X_k(g)$ the subset of X consisting of points with g-orbits of length k. For example, $X_1(g)$ is a set of g-fixed points. Let $\widetilde{X}_k(g)=X_k(g)/(g)$, where (g) is a cyclic subgroup in G generated by g.

The following theorem was deduced in [10] from the results of Getzler [6, 7], here we would like to present a more geometric and straightforward proof.

Theorem 2.5. *The generating function for the S_n-equivariant Euler characteristics of the quotients $F(X,n)/G$ is given by the following equation:*

$$\sum_{n=0}^{\infty}t^n\chi^{S_n}(F(X,n)/G)=\frac{1}{|G|}\sum_{g\in G}\prod_{k=1}^{\infty}(1+p_kt^k)^{\frac{\chi(X_k(g))}{k}}. \tag{1}$$

Proof. Since all points in $X_k(g)$ have g-orbit of length k, we have $\chi(\widetilde{X}_k(g))=\chi(X_k(g))/k$. From Lemma 2.1 one gets:

$$(1+p_jt^j)^{\chi\left(\widetilde{X}_j(g)\right)}=\sum_{k_j=0}^{\infty}\frac{p_j^{k_j}t^{jk_j}}{(k_j)!}\chi\left(F\left(\widetilde{X}_j(g),k_j\right)\right),$$

Therefore the coefficient at t^n in the right hand side of (1) equals to:

$$\frac{1}{|G|}\sum_{g\in G}\sum_{\sum jk_j=n}\prod_j\frac{p_j^{k_j}}{k_j!}\chi\left(F\left(\widetilde{X}_j(g),k_j\right)\right).$$

On the other hand, by Lemma 2.2 and Lemma 2.3, the left hand side of (1) can be rewritten as following:

$$\frac{1}{|G|}\sum_{g\in G}\frac{1}{n!}\sum_{\sigma\in S_n}p_1^{k_1(\sigma)}\cdot\ldots\cdot p_n^{k_n(\sigma)}\cdot\chi([F(X,n)]^{g^{-1}\sigma}).$$

If for a tuple $\mathbf{y}\in F(X,n)$ we have $\sigma(\mathbf{y})=g(\mathbf{y})$, the action of (g) at this tuple has $k_j(\sigma)$ cycles of length j. Every cycle of length j corresponds to a point in $\widetilde{X}_j(g)$, hence for every g we can define a map

$$\alpha_g:\sqcup_{\sigma\in S_n}[F(X,n)]^{g^{-1}\sigma}\to\prod_j F(\widetilde{X}_j(g),k_j)/S_{k_j}.$$

Given a g-invariant n-tuple of distinct points in X, there are $n!$ ways to label them and make an ordered tuple $\mathbf{y}$. Every such ordering defines a unique permutation σ such that $\sigma(\mathbf{y})=g(\mathbf{y})$, therefore all fibers of α_g have cardinality $n!$ and

$$\frac{1}{n!}\sum_{\sigma\in S_n}\chi([F(X,n)]^{g^{-1}\sigma})=\prod_j\chi\left(F(\widetilde{X}_j(g),k_j)/S_{k_j}\right)=\prod_j\frac{\chi\left(F(\widetilde{X}_j(g),k_j)\right)}{k_j!}.$$

□

3. Moduli spaces of curves

Let us apply Theorem 2.5 to the study of moduli spaces of curves. Let $\mathcal{M}_g$ denote the moduli space of genus g algebraic curves and let $\mathcal{M}_{g,n}$ denote the moduli space of genus g algebraic curves with n parked points (we will always assume $g \geq 2$). Let $\mathcal{M}_g(k_1, \ldots, k_r)$ be the moduli space of pairs (C, τ) where C is a genus g curve and τ is an automorphism of C such that $\chi(C_i(\tau)) = ik_i$ for all i. Since $g \geq 2$, every automorphism of C has finite order, hence one can choose r such that $k_r \neq 0$ and $k_i = 0$ for $i > r$.

There is a natural forgetful map $\pi_{g,\underline{k}} : \mathcal{M}_g(k_1, \ldots, k_r) \to \mathcal{M}_g$ sending (C, τ) to C. For a curve C we define $\mathrm{Aut}_{\underline{k}}(C) = \pi_{g,\underline{k}}^{-1}(C) \subset \mathrm{Aut}(C)$.

Proposition 3.1. Suppose that $\mathcal{M}_g(k_1, \ldots, k_r)$ is not empty. Then $k_r < 0, k_i = 0$ for $i \nmid r$ and $k_i \geq 0$ for $i \mid r, i < r$. Moreover, we have the following bounds on r and k_i:

$$r \leq 4g + 2,\ |k_r| \leq 2g,\ \sum_{i=1}^{r-1} k_i \leq 2g + 2.$$

Proof. Let τ be an automorphism of a genus g curve C such that $\chi(C_i(\tau)) = ik_i$ for all i. Note that $C_i(\tau)$ are finite sets for $i < r$ and

$$\chi(C) = 2 - 2g = \sum_{i=1}^{r-1} ik_i - r|k_r| \tag{2}$$

The quotient $C_1 = C/\tau$ is a smooth curve of some genus h, and the Riemann-Hurwitz formula yields its Euler characteristic:

$$\chi(C_1) = 2 - 2h = \sum_{i=1}^{r-1} k_i - |k_r|. \tag{3}$$

The projection of C to C_1 is a ramified covering of order r with $s = \sum_{i=1}^{r-1} k_j$ ramification points. The automorphism τ has order r, so $i|r$, if $k_i \neq 0$. By a theorem of Wiman ([20], see also [14]), the maximal order for an automorphism of a genus g curve equals $4g + 2$, hence $r \leq 4g + 2$.

Since proper divisors of r cannot exceed $r/2$, equation (3) implies:

$$\sum_{i=1}^{r-1} ik_i \leq \frac{r}{2} \sum_{i=1}^{r-1} k_i = \frac{r}{2}(2 - 2h + |k_r|),$$

hence by (2):

$$2g - 2 = r|k_r| - \sum_{i=1}^{r-1} ik_i \geq \frac{r}{2}(2h + |k_r| - 2). \tag{4}$$

Therefore $|k_r| - 2 \leq 2g - 2$ and $|k_r| \leq 2g$. Finally, $\sum_{i=1}^{r-1} k_i = |k_r| + 2 - 2h \leq 2g + 2$. □

Remark 3.2. The bounds on r and on k_i are sharp. Indeed, consider a hyperelliptic curve P covering $\mathbb{CP}^1$ with ramifications at the vertices of a regular $(2g+1)$-gon and at its center. The covering can be chosen such that the automorphism of P induced by the rotation of this polygon acts nontrivially in the fibers and hence has order $r = 2(2g + 1) = 4g + 2$.

On the other hand, consider a hyperelliptic curve C with involution τ. We have

$$\chi(C_1(\tau)) = 2g + 2, \chi(C_2(\tau)) = 2 - 2g - (2g + 2) = -4g,$$

hence a pair (C, τ) belongs to the moduli space $\mathcal{M}_g(2g + 2, -2g)$.

Theorem 3.3. *The following equation holds:*

$$\sum_{n=0}^{\infty} t^n \chi^{S_n}(\mathcal{M}_{g,n}) = \sum_{\underline{k}} \chi^{orb}(\mathcal{M}_g(k_1,\ldots,k_r)) \cdot \prod_{j=1}^{r}(1+p_j t^j)^{k_j}. \tag{5}$$

Proof. Consider the forgetful map $\pi_{g,n} : \mathcal{M}_{g,n} \to \mathcal{M}_g$. Its fiber over a point representing a curve C is isomorphic to $F(C,n)/\operatorname{Aut}(C)$, hence one can apply Theorem 2.5 to compute its equivariant Euler characteristic:

$$\sum_{n=0}^{\infty} t^n \chi^{S_n}(\pi_{g,n}^{-1}(C)) = \sum_{n=0}^{\infty} t^n \chi^{S_n}(F(C,n)/\operatorname{Aut}(C)) =$$

$$\frac{1}{|\operatorname{Aut}(C)|}\sum_{\tau\in\operatorname{Aut}(C)}\prod_i (1+p_i t^i)^{\frac{\chi(C_i(\tau))}{i}} = \sum_{\underline{k}} \frac{1}{|\operatorname{Aut}(C)|}\sum_{\tau\in\operatorname{Aut}_{\underline{k}}(C)}\prod_i (1+p_i t^i)^{k_i}.$$

Therefore:

$$\sum_{n=0}^{\infty} t^n \chi^{S_n}(\mathcal{M}_{g,n}) = \int_{\mathcal{M}_g}\sum_{n=0}^{\infty} t^n \chi^{S_n}(\pi_{g,n}^{-1}(C))d\chi =$$

$$\sum_{\underline{k}}\prod_i (1+p_i t^i)^{k_i}\int_{\mathcal{M}_g}\frac{|\operatorname{Aut}_{\underline{k}}(C)|}{|\operatorname{Aut}(C)|}d\chi.$$

On the other hand,

$$\chi^{orb}(\mathcal{M}_g(k_1,\ldots,k_r)) = \int_{\mathcal{M}_g}\frac{|\pi_{g,\underline{k}}^{-1}(C)|}{|\operatorname{Aut}(C)|}d\chi = \int_{\mathcal{M}_g}\frac{|\operatorname{Aut}_{\underline{k}}(C)|}{|\operatorname{Aut}(C)|}d\chi \quad \square$$

Using the Proposition 3.1, we conclude that the sum in the right hand side of (5) is finite.

Corollary 3.4. The generating function $\sum_{n=0}^{\infty} t^n \chi^{S_n}(\mathcal{M}_{g,n})$ is a rational function in t. Furthermore, for any n,

$$\chi^{S_n}(\mathcal{M}_{g,n}) \in \mathbb{Z}[p_1,\ldots,p_{4g+2}].$$

The orbifold Euler characteristic of $\mathcal{M}_g(k_1,\ldots,k_r)$ can be computed using the combinatorial results of Harer and Zagier [13]. We will denote the greatest common divisor of integers a and b by (a,b). Let $\varphi(n)$ and $\mu(n)$ denote the Euler function and the Möbius function respectively. Define

$$c(k,l,d) := \mu\left(\frac{d}{(d,l)}\right)\frac{\varphi(k/l)}{\varphi(d/(d,l))},$$

Definition 3.5. Let $\lambda = (\lambda_1,\ldots,\lambda_s)$ be a partition. We define a number

$$N(r;\lambda) = |\{(x_1,\ldots,x_s)\in(\mathbb{Z}/r\mathbb{Z})^s : x_1+\ldots+x_s \equiv 0 \pmod r, (x_i,k)=\lambda_i\}|.$$

Lemma 3.6. ([13]) *The following equation holds:*

$$N(r;\lambda) = \frac{1}{r}\sum_{d|r}\varphi(d)\prod_{i=1}^{s}c(k,\lambda_i,d).$$

Theorem 3.7. ([13]) *The orbifold Euler characteristic of the moduli space $\mathcal{M}_{h,s}$ of genus h curves with s marked points is given by the formula:*

$$\chi^{orb}(\mathcal{M}_{h,s}) = (-1)^s\frac{(2h-1)B_{2h}}{(2h)!}(2h+s-3)!$$

where B_k denote Bernoulli numbers.

Theorem 3.8. *The generating function for the S_n-equivariant Euler characteristics of $\mathcal{M}_{g,n}$ has the form*

$$\sum_{n=0}^{\infty} t^n \chi^{S_n}(\mathcal{M}_{g,n}) = \sum_{\underline{k}} c_{k_1,\ldots,k_r} \prod_{j=1}^{r} (1+p_j t^j)^{k_j},$$

where p_j are power sums and the coefficients $c_{k_1,\ldots,k_r}$ are defined by the equation:

$$c_{k_1,\ldots,k_r} = \chi^{orb}(\mathcal{M}_{h,s}) r^{2h} \prod_{p|\gamma} (1-p^{-2h}) \cdot \frac{N(r;\lambda)}{r\prod_{i=1}^{r-1} k_i!}. \tag{6}$$

Here $h = \frac{1}{2}(1-\sum_{j=1}^{r} k_j)$, $s = \sum_{j=1}^{r-1} k_j$, $\gamma = \mathrm{GCD}(i : k_i > 0)$, $\lambda = \left(1^{k_1} 2^{k_2} \ldots (r-1)^{k_{r-1}}\right)$

Proof. By Theorem 3.3 one has $c_{k_1,\ldots,k_r} = \chi^{orb}(\mathcal{M}_g(k_1,\ldots,k_r))$. Consider the moduli space $\mathcal{M}_g(k_1,\ldots,k_r)$ of pairs (C,τ). As in Proposition 3.1, to such a pair one can associate a genus h curve $C_1 = C/\tau$. The projection from C to C_1 is ramified in s points subdivided into groups of size $k_1,\ldots,k_{n-1}$. The orbifold Euler characteristic of the moduli space of genus h curves with such markings equals $\chi^{orb}(\mathcal{M}_{h,s})/\prod_{i=1}^{r-1} k_i!$.

The number of pairs (C,τ) associated to a curve C_1 with fixed marked points was computed in [13, pages 478–479] and equals

$$\frac{1}{r} r^{2h} \prod_{p|\gamma} (1-p^{-2h}) \cdot N(r;\lambda).$$

This completes the proof. □

The non-equivariant Euler characteristic of $\mathcal{M}_{g,n}$ has been computed in [3, Theorem 4.3]. It can be compared with Theorem 3.8 since

$$\chi(\mathcal{M}_{g,n}) = n! \cdot \chi^{S_n}(\mathcal{M}_{g,n})[p_1 = 1,\ p_k = 0 \text{ for } k > 1].$$

Example 3.9. The generating function for the S_n-equivariant Euler characteristics of the moduli spaces of genus 2 curves with marked points has a form [10]:

$$\sum_{n=0}^{\infty} t^n \chi^{S_n}(\mathcal{M}_{2,n}) = -\frac{1}{240}(1+p_1t)^{-2} - \frac{1}{240}(1+p_1t)^6(1+p_2t^2)^{-4} +$$
$$+\frac{2}{5}(1+p_1t)^3(1+p_5t^5)^{-1} + \frac{2}{5}(1+p_1t)(1+p_2t^2)(1+p_5t^5)(1+p_{10}t^{10})^{-1} +$$
$$+\frac{1}{6}(1+p_1t)^2(1+p_2t^2)(1+p_6t^6)^{-1} - \frac{1}{12}(1+p_1t)^4(1+p_3t^3)^{-2} -$$
$$-\frac{1}{12}(1+p_2t^2)^2(1+p_3t^3)^2(1+p_6t^6)^{-2} + \frac{1}{12}(1+p_1t)^2(1+p_2t^2)^{-2} +$$
$$+\frac{1}{4}(1+p_1t)^2(1+p_4t^4)(1+p_8t^8)^{-1} - \frac{1}{8}(1+p_1t)^2(1+p_2t^2)^2(1+p_4t^4)^{-2}.$$

These coefficients can be matched with the ones defined in Theorem 3.8.

References

[1] J. Bergström. Cohomologies of moduli spaces of curves of genus three via point counts. J. Reine Angew. Math. **622** (2008), 155–187.

[2] J. Bergström, O. Tommasi. The rational cohomology of $\overline{\mathcal{M}}_4$. Math. Ann. **338** (2007), no. 1, 207–239.

[3] G. Bini, J. Harer. Euler Characteristics of Moduli Spaces of Curves. J. Eur. Math. Soc. (JEMS) **13** (2011), no. 2, 487–512.

[4] G. Bini, G. Gaiffi, M. Polito. A formula for the Euler characteristic of $\overline{\mathcal{M}}_{2,n}$. Math. Z. **236** (2001) 491–523.

[5] C. Faber, G. van der Geer. Sur la cohomologie des systèmes locaux sur les espaces de modules des courbes de genre 2 et des surfaces abéliennes. I, II. C. R. Math. Acad. Sci. Paris **338** (2004), no. 5, 381–384; no. 6, 467–470.

[6] E. Getzler. Mixed Hodge structures of configuration spaces. arXiv:math.AG/9510018

[7] E. Getzler. Resolving mixed Hodge modules on configuration spaces. Duke Math. J. **96** (1999), no. 1, 175–203.

[8] E. Getzler. Euler characteristics of local systems on $\mathcal{M}_2$. Compositio Math. **132** (2002), 121–135.

[9] E. Getzler, E. Looijenga. The Hodge polynomial of $\overline{\mathcal{M}}_{3,1}$. arXiv:math.AG/9910174

[10] E. Gorsky. On the S_n - equivariant Euler characteristic of $\mathcal{M}_{2,n}$. arXiv:0707.2662.

[11] E. Gorsky. On the S_n-equivariant Euler characteristic of moduli spaces of hyperelliptic curves. Math. Res. Lett. **16** (2009), no. 4, 591–603.

[12] E. Gorsky. Adams operations and power structures. Mosc. Math. J. **9** (2009), no. 2, 305–323.

[13] J. Harer, D. Zagier. The Euler characteristic of the moduli space of curves. Invent. Math. **85** (1986), 457–485.

[14] W. J. Harvey. Cyclic groups of automorphisms of a compact Riemann surface. Quart. J. Math. Oxford Ser. (2) **17** (1966), 86–97.

[15] A. Khovanskii, A. Pukhlikov. Integral transforms based on Euler characteristic and their applications. Integral Transform. Spec. Funct. **1** (1993), no. 1, 19–26.

[16] R. MacPherson. Chern classes for singular algebraic varieties. Ann. of Math. (2) **100** (1974), 423–432.

[17] O. Tommasi. Rational cohomology of the moduli space of genus 4 curves. Compos. Math. **141** (2005), no. 2, 359–384.

[18] O. Tommasi. Rational cohomology of $\mathcal{M}_{3,2}$. Compos. Math. **143** (2007), no. 4, 986–1002.

[19] O. Viro. Some integral calculus based on Euler characteristic. Topology and geometry – Rohlin Seminar, 127–138, Lecture Notes in Math., **1346**, Springer, Berlin, 1988.

[20] A. Wiman. Ueber die hyperelliptischen Curven und diejenigen vom Geschlechte $p = 3$ welche eindeutigen Transformationen in sich zulassen. Bihang Till. Kongl. Svenska Veienskaps-Akademiens Hadlingar **21** (1895-6) 1–23.

MATHEMATICS DEPARTMENT, STONY BROOK UNIVERSITY, STONY BROOK NY, 11794-3651, USA
E-mail address: egorsky@math.sunysb.edu