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Author
Gaillard, Mary K.

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Mary K. Gaillard

Physics Division

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ONE-LOOP REGULARIZATION OF SUPERGRAVITY II: THE DILATON AND THE SUPERFIELD FORMULATION*†

Mary K. Gaillard

Department of Physics, University of California, and
Theoretical Physics Group, Lawrence Berkeley Laboratory,
Berkeley, California 94720

Abstract

The on-shell regularization of the one-loop divergences of supergravity theories is generalized to include a dilaton of the type occurring in effective field theories derived from superstring theory, and the superfield structure of the one-loop corrections is given. Field theory anomalies and quantum contributions to soft supersymmetry breaking are discussed. The latter are sensitive to the precise choice of couplings that generate Pauli-Villars masses, which in turn reflect the details of the underlying theory above the scale of the effective cut-off. With a view to the implementation of the Green-Schwarz and other mechanisms for canceling field theory anomalies under a $U(1)$ gauge transformation and under the T-duality group of modular transformations, we show that the Kähler potential renormalization for the untwisted sector of orbifold compactification can be made invariant under these groups.

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†This paper is dedicated to the memory of Kamran Saririan.

1 Introduction

It has been shown [1]–[3], that Pauli-Villars (PV) regularization of one-loop ultraviolet divergences is possible for an $N = 1$ supergravity theory if Yang-Mills fields have canonical kinetic energy. In this paper those results are generalized to include their couplings to a dilaton. In Section 2 we summarize earlier results, and display the logarithmically divergent one-loop corrections in the form of superfield operators, which permits the extension of those results to fermionic terms [4, 5] in the one-loop corrected effective Lagrangian. This formulation will also be convenient for the subsequent analysis. In Section 3 the dilaton is incorporated in the Pauli-Villars regularization of anomaly-free supergravity described in Ref. [3], hereafter referred to as I. The application of PV regularization to determine soft supersymmetry breaking terms is also discussed in this section. It is shown that the contributions to A-terms are highly sensitive to the details of the regularization. In Section 4 we regulate effective theories of orbifold compactification with twisted sector fields set to zero in the background. We show that this regularization can be done in such a way that the renormalization of the Kähler potential is invariant under modular (T-duality) transformations; we have in mind the construction of an effective one-loop Lagrangian that is perturbatively modular invariant. In Section 5 the discussion of regularization and anomalies is extended to theories with an anomalous $U(1)$ gauge symmetry. The results are summarized in Section 6, where we discuss issues still to be addressed in order to achieve full anomaly cancellation. Many calculational details are relegated to the appendixes.

2 Preliminaries

In this paper we consider supergravity theories defined by the standard Lagrangian [8, 9] with N chiral multiplets $Z^i = \Phi^1, ...\Phi^{N-1}, S$, where S is a gauge singlet, and N_G gauge supermultiplets. The Kähler potential K, superpotential W and gauge kinetic function f are given by

$$\begin{aligned} K(Z,\bar{Z}) &= -\ln(S+\bar{S}) + G(\Phi,\bar{\Phi}) = k + G, \quad W(Z) = W(\Phi), \\ f_{ab}(Z) &= \delta_{ab}S = \delta_{ab}(x+iy), \end{aligned} \tag{2.1}$$

which are the classical functions found in string compactifications with affine level one.[1] In this section we briefly recall the results of [1, 3], and cast them in a superfield form that will allow us to short-cut some of the subsequent calculations.

2.1 One-loop logarithmic divergences in supergravity

The ultra-violet divergent part of the one-loop corrected supergravity Lagrangian for bosons was calculated in [10]-[12]. The result for the logarithmically divergent contribution is

$$\begin{aligned}
\mathcal{L}_{eff} &= \mathcal{L}\left(g_R, K_R\right) + \sqrt{g}\frac{\ln\Lambda^2}{32\pi^2}L \\
L &= \tilde{L}_0 + L_0' + \tilde{L}_1 + L_2 + L_3 + NL_\chi + N_G(\tilde{L}_g + L_g'), \\
\tilde{L}_0 &= L_0 + 41L_{GB}, \quad \tilde{L}_\chi = L_\chi + L_{GB}, \quad \tilde{L}_g = L_g - 3L_{GB}, \\
K_R &= K + \frac{\ln\Lambda^2}{32\pi^2}\left[e^{-K}A_{ij}\bar{A}^{ij} - 2\hat{V} + (N_G - 10)M^2 - 4\mathcal{K}^a_a - 16\mathcal{D}\right], \\
\mathcal{K}^a_b &= \frac{1}{x}(T^a z)^i (T_b \bar{z})^{\bar{m}} K_{i\bar{m}}, \quad A = e^K W = \bar{A}^\dagger, \quad A_{ij} = D_i D_j A.
\end{aligned} \tag{2.2}$$

where $\mathcal{L}(g, K)$ is the standard Lagrangian [8, 9] for $N = 1$ supergravity coupled to matter with space-time metric $g_{\mu\nu}$, Kähler potential K and superpotential W. $V = \hat{V} + \mathcal{D}$ is the classical scalar potential with $\hat{V} = e^{-K}A_i\bar{A}^i - 3M^2$, $A_i = D_i A$, $\mathcal{D} = (2x)^{-1}\mathcal{D}^a\mathcal{D}_a$, $\mathcal{D}_a = K_i(T_a z)^i$, $M^2 = e^{-K}A\bar{A}$ is the field-dependent squared gravitino mass, and D_i is the scalar field reparameterization covariant derivative. Scalar indices are lowered and raised with the Kähler metric $K_{i\bar{m}}$ and its inverse $K^{i\bar{m}}$.

The operators L_A in (2.2) are given in component form[2] in Eqs. (2.25–27) of I,

$$L_{GB} = \frac{1}{48}\left(r^{\mu\nu\rho\sigma}r_{\mu\nu\rho\sigma} - 4r^{\mu\nu}r_{\mu\nu} + r^2\right), \tag{2.3}$$

is the Gauss-Bonnet term which is a total derivative, and was not included explicitly in I. The operators L_A' are additional contributions that arise in the presence of a dilaton coupling

[1] The results can be generalized to the case $f_{ab} = \delta_{ab}k_a f$, k_a = constant, by making the substitutions $F^a_{\mu\nu} \to k_a^{\frac{1}{2}}F^a_{\mu\nu}$, $A^a_\mu \to k_a^{\frac{1}{2}}A^a_\mu$, $T^a \to k_a^{-\frac{1}{2}}T^a$.

[2] See Appendix D of I and Appendix E below for corrections to [10, 12]. There is an extraneous factor of x in the second line of (2.26) in I.

to the Yang-Mills terms. Their component field expressions read:

$$\begin{aligned}
L_0' &= 92\mathcal{D}M^2 - 2x^2\mathcal{W}_{ab}\overline{\mathcal{W}}^{ab} - 4x^2\mathcal{W}\overline{\mathcal{W}} \\
&\quad -\frac{\partial_\rho s\partial^\nu \bar{s}}{x}F^{+a}_{\mu\nu}F^{\mu\rho}_{-a} + 10\frac{\partial_\mu s\partial^\mu \bar{s}}{x^2}\mathcal{D} + 4i\frac{\partial_\mu s\partial_\nu \bar{s}}{x^2}\mathcal{D}^a F^{\mu\nu}_a \\
&\quad -\frac{6}{x}\left\{\left[i\partial_\nu s F^{\nu\mu}_{-a} + \frac{\partial^\mu s}{x}\mathcal{D}_a\right]\mathcal{D}_\mu \bar{z}^{\bar{m}} K_{i\bar{m}}(T^a z)^i + \text{h.c.}\right\} \\
&\quad +xF^{-a}_{\rho\mu}F^{\rho\nu}_{+a}\mathcal{D}_\nu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} K_{i\bar{m}} + 2i\mathcal{D}_\mu z^i \mathcal{D}_\nu \bar{z}^{\bar{m}} K_{i\bar{m}}\mathcal{D}^a F^{\mu\nu}_a \\
&\quad +4\mathcal{D}\hat{V} + 2\mathcal{D}K_{i\bar{m}}\mathcal{D}_\rho z^i \mathcal{D}^\rho \bar{z}^{\bar{m}}, \qquad (2.4) \\
L_g' &= -x\left(\mathcal{W}+\overline{\mathcal{W}}\right)\left(M^2+\hat{V}\right) - \frac{2}{3}M^2\left(\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} K_{i\bar{m}} + 4\hat{V} - 2\mathcal{D}\right) \\
&\quad -7M^4 + \frac{\partial_\mu s\partial^\mu s\partial_\nu \bar{s}\partial^\nu \bar{s}}{16x^4} - \frac{\partial_\mu s\partial_\nu \bar{s}}{2x^2}K_{i\bar{m}}\left(\mathcal{D}^\mu z^i \mathcal{D}^\nu \bar{z}^{\bar{m}} + \mathcal{D}^\mu \bar{z}^{\bar{m}} \mathcal{D}^\nu z^i\right) \\
&\quad +x^2\mathcal{W}\overline{\mathcal{W}} + \left[F^{+a}_{\rho\mu}F^{\rho\nu}_{-a} + \frac{2}{3}g^\nu_\mu\left(2K_{i\bar{m}}\mathcal{D}_\rho z^i \mathcal{D}^\rho \bar{z}^{\bar{m}} - \hat{V} - \mathcal{D}\right)\right]\frac{\partial_\nu s\partial^\mu \bar{s}}{4x} \\
&\quad +\frac{e^{-K}}{2x}\left(\partial_\mu \bar{s}\mathcal{D}^\mu z^i A_i \bar{A} + \text{h.c.}\right), \qquad (2.5)
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{W}_{ab} &= \frac{1}{4}\left(F_a\cdot F_b - i\tilde{F}_a\cdot F_b\right) - \frac{1}{2x}\mathcal{D}_a\mathcal{D}_b = -\frac{1}{2}\,\mathcal{D}^\alpha\mathcal{D}_\alpha W^\beta W_\beta\Big|, \\
F^{\mp}_{a\nu\mu} &= F_{a\nu\mu} \mp i\tilde{F}_{a\nu\mu}, \quad x = \text{Re}s, \quad \mathcal{W} = \mathcal{W}^a_a, \qquad (2.6)
\end{aligned}$$

with $F^a_{\mu\nu}$ the Yang-Mills field strength. As in I we have dropped total derivatives (except for the Gauss-Bonnet term) and other terms that do not contribute to the S-matrix, by virtue of the classical equations of motion of the physical fields.

It will be convenient here to display these operators in superfield form. θ-integration of the superfield operators gives expressions that include the various auxiliary fields. Replacing these by the solutions of their classical equations of motion gives the component expressions, up to terms that do not contribute to the S-matrix. We will display here the component expressions only for those operators that are not included in I. The component expressions for operators constructed from tensor-valued functions $T(Z,\bar{Z})$ are given in Appendix A.

In the Kähler $U(1)$ superspace formulation of supergravity, a general "F-term" Lagrangian takes the form [9]

$$L_A = L(\Phi_A) = \frac{1}{2}\int d^4\theta\frac{E}{R}\Phi_A + \text{h.c.}, \qquad (2.7)$$

where Φ is a chiral superfield of Kähler $U(1)$ weight $w(\Phi) = 2$. Here we construct these fields as bilinears in chiral superfields of weight 1, namely the Yang-Mills field strength superfield W_α^a, the curvature superfield $W_{\alpha\beta\gamma}$ (the lowest components of the totally symmetrized spinorial derivatives $\mathcal{D}_{\{\gamma}W_{\alpha\beta\gamma\}}\big|$ are elements of the Riemann tensor), and the superfields

$$T_\alpha = -\frac{1}{8}\left(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R\right)\hat{T}_\alpha, \quad \hat{T}_\alpha = T_i\mathcal{D}_\alpha Z^i, \tag{2.8}$$

where $T_i(Z, \bar{Z})$ is any (tensor-valued) zero-weight function of the chiral and anti-chiral superfields. In particular, the chiral superfield

$$K_\alpha = X_\alpha = -\frac{1}{8}\left(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R\right)\mathcal{D}_\alpha K, \tag{2.9}$$

was introduced in [9]; the lowest component of its spinorial derivative $-\frac{1}{2}\mathcal{D}^\alpha X_\alpha|$ is the kinetic term for matter fields in the classical Lagrangian. Then defining

$$\Phi_W = \frac{1}{6}W^{\alpha\beta\gamma}W_{\alpha\beta\gamma}, \quad \Phi_{YM}^a = \frac{1}{4}W_a^\alpha W_\alpha^a, \quad \Phi_\alpha = -\frac{1}{2}X^\beta X_\beta, \tag{2.10}$$

we may write (see Appendix A), up to total derivatives and field redefinitions,

$$\begin{aligned}
\tilde{L}_0 &= 41\tilde{L}_\chi + 6\left(L_\chi - C_a L_{YM}^a + \hat{L}_0\right) - \frac{20}{3}L_\alpha, \\
\tilde{L}_\chi &= L_W + \frac{1}{2}L_\chi + \frac{1}{9}L_\alpha, \quad \tilde{L}_G = -3\tilde{L}_\chi + 6L_\chi - \frac{1}{3}L_\alpha,
\end{aligned} \tag{2.11}$$

where C_a is the quadratic Casimir in the adjoint representation of the gauge subgroup $\mathcal{G}_a$: $\mathrm{Tr}(T_aT_b)_{\mathrm{adj}} = \delta_{ab}C_a$ with T_a a generator of $\mathcal{G}_a$ and T_b any generator. L_α is given in component form in (2.40) of I. The operators L_χ and

$$\begin{aligned}
\hat{L}_0 &= \left(\hat{V} + 2M^2\right)K_{i\bar{m}}\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^i + M^2\left(2\hat{V} + 3M^2 + 2\mathcal{D}\right) \\
&\quad + \mathcal{D}_\mu z^j\mathcal{D}^\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}}\mathcal{D}^\nu\bar{z}^{\bar{n}}K_{i\bar{n}}K_{j\bar{m}}
\end{aligned} \tag{2.12}$$

are "D-terms" of the form

$$L_A = L(\phi_A) = \int d^4\theta E\phi_A = -\frac{1}{16}\int d^4\theta\frac{E}{R}\left(\bar{\mathcal{D}}^2 - 8R\right)\phi_A + \text{h.c.}, \quad w(\phi_A) = 0. \tag{2.13}$$

To include these we define the zero-weight real superfields

$$
\begin{aligned}
T^{\alpha\dot\beta}_{\alpha\dot\beta} &= \frac{1}{16}\mathcal{D}^\alpha Z^i \mathcal{D}_\alpha Z^j \mathcal{D}_{\dot\beta}\bar{Z}^{\bar m}\mathcal{D}^{\dot\beta}\bar{Z}^{\bar n} T_{ij\bar m\bar n},\\
\phi_{WT} &= \frac{x}{2}W^\alpha_a \mathcal{D}_\alpha Z^i W^a_{\dot\beta}\mathcal{D}^{\dot\beta}\bar{Z}^{\bar m}T_{i\bar m}, \quad T^\alpha_\alpha = \frac{1}{2}\mathcal{D}^\alpha Z^i \mathcal{D}_\alpha Z^j T_{ij} + \text{h.c.},\\
\phi_{W^a_b} &= \frac{x^2}{4}W^\alpha_a W^b_\alpha W^a_{\dot\beta} W^{\dot\beta}_b, \quad \phi_W = \frac{x^2}{4}W^\alpha_a W^a_\alpha W^b_{\dot\beta} W^{\dot\beta}_b. \qquad (2.14)
\end{aligned}
$$

With these definitions we have

$$
\phi_\chi = \frac{1}{3}\hat\phi_0 - \frac{1}{6}\phi_{WK} + \frac{1}{3}\phi_{W^a_b}, \quad \hat\phi_0 = K^{\alpha\dot\beta}K_{\alpha\dot\beta} - e^K|W(Z)|^2. \qquad (2.15)
$$

The last term in $\hat\phi_0$ is equivalent to a renormalization of the Kähler potential; up to a field-dependent Weyl scaling and higher order terms in the loop expansion parameter, the shift in $\mathcal{L}/\sqrt{g}$ due to a shift $F(Z,\bar Z)$ in the Kähler potential is given by

$$
\begin{aligned}
\frac{1}{\sqrt{g}}\Delta_F\mathcal{L} &= \Delta_F L = -F\hat V + \left(e^{-K}\bar A^i A^{\bar m} + \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar z^{\bar m}\right)\partial_i\partial_{\bar m}F\\
&\quad - \left\{\partial_i F\left[e^{-K}\bar A^i A + \frac{1}{2x}\mathcal{D}_a(T^a z)^i\right] + \text{h.c.}\right\} = \frac{1}{\sqrt{g}}\int d^4\theta E F. \qquad (2.16)
\end{aligned}
$$

As shown in Appendix A, L_χ can be obtained as a linear combination of L_α and an operator generated by a metric field redefinition that eliminates terms quadratic in the space-time scalar curvature and the Ricci tensor. That is, it is equivalent to a linear combination of L_α and a D-term (2.13) constructed from the superfields that determine the elements of the super-Riemann and torsion tensors [9]: $\phi_a = R\bar R,\ G_a G^a, \ldots$. In addition we have the F-terms L_1, L_2 with

$$
\begin{aligned}
\Phi_1 &= 2C^M_a \Phi^a_{YM} - \frac{1}{2}\Gamma^{i\alpha}_j\left[\Gamma^j_{i\alpha} + 2(T_a)^j_i W^a_\alpha\right],\\
\Phi_2 &= \frac{1}{3}X^\alpha\left[\Gamma_\alpha + 2(T_a)^i_i W^a_\alpha\right], \quad \Gamma_\alpha = \Gamma^i_{i\alpha}, \qquad (2.17)
\end{aligned}
$$

where Z^i is a matter chiral superfield ($w(Z) = 0$), Γ^i_{jk} is an element of the affine connection associated with the Kähler metric, and C^M_a is the matter quadratic Casimir for the gauge subgroup $\mathcal{G}_a$: $(T_a T_b)^i_i = \delta_{ab}C^M_a$. These contributions to (2.2) are canceled by identical contributions from negative signature PV chiral superfields Z^I with the same gauge charges and Kähler metric as the matter fields.

The terms proportional to $L_{0,\chi,g}$ are partially canceled by the introduction of PV chiral superfields ϕ^C with Kähler metric

$$K_{C\bar{C}} = e^{\alpha_C K}, \quad \Gamma^C_{Di} = \alpha_C \delta^C_D K_i, \quad \Gamma^C_{D\alpha} = \alpha_C \delta^C_D X_\alpha, \tag{2.18}$$

some of which carry gauge charge. Assuming $\sum_C (T_a)^C_C = 0$, the ϕ^C-loop gives a contribution:

$$(L_1 + L_2)_{\phi^C} = \eta^C \left[2C_a L^a_{YM} + \left(\alpha^C - \frac{2}{3}\right) \alpha^C L_\alpha \right], \tag{2.19}$$

where $\eta^C = \pm 1$ denotes the signature of the PV field Φ^C. The operator L_3 depends both on elements $R_{i\bar{m}j\bar{n}}$ of the Kähler Riemann tensor and on covariant scalar derivatives of $A = e^K W$; it is the bosonic part of a D-term[3] (2.13):

$$\phi_3 = \frac{1}{2} R^{\alpha k}{}_{\alpha}{}^{l} R^{\dot{\beta}}{}_{k \dot{\beta} l} + \left(R^{\alpha k}{}_{\alpha}{}^{l} e^{-K/2} A_{kl} + \text{h.c.} \right). \tag{2.20}$$

Cancellation of this term and of the logarithmic divergence in the renormalization of the Kähler potential in (2.2) require PV chiral superfields Z^I with nonvanishing K_{IJ}, and with superpotential couplings to the light chiral multiplets. The part of K_R that depends on the gauge couplings of the light fields is canceled by superpotential couplings of the PV fields Φ^a to the Z^i and to PV chiral fields Y_I that transform according to the gauge group representation that is conjugate to the light matter representation. These couplings are given explicitly in Section 3, slightly modified with respect to those adopted in I, as required by the presence of the dilaton. The superfield form of the operator L'_0 is

$$\begin{aligned} L'_0 &= L(\phi'_0) + L(\Phi'_0), \quad \phi'_0 = \phi_{WK} - 4\phi_{Wk} - 2\phi_{W^a_b} - 4\phi_W, \\ \Phi'_0 &= 12 W^\alpha_a T^a_\alpha, \quad T^a_\alpha = -\frac{1}{8}\left(\bar{\mathcal{D}}^2 - 8\mathcal{R}\right)\left(x^{-1}\mathcal{D}^a \hat{f}_a\right), \quad f_i = \frac{\partial f}{\partial Z^i}. \end{aligned} \tag{2.21}$$

This term and the remaining contributions to $L_{0,\chi,g}$ are canceled by the introduction of massive Abelian gauge fields, some of which couple to the light Yang-Mills fields through a nontrivial gauge kinetic function, as described in I. The superfield structure of L'_g is less transparent. It is equivalent up to terms that vanish on shell to linear combinations of the

[3] Note that $T_{kl} = e^{-K/2} A_{kl}(Z, \bar{Z})$ is a superfield of weight $w(T_{kl}) = 2$; its spinorial derivatives satisfy $\mathcal{D}^{\dot{\beta}} T_{kl} = e^{K/2} \mathcal{D}^{\dot{\beta}} \bar{Z}^{\bar{m}} D_{\bar{m}} \left(e^{-K} A_{kl}\right)$, $\mathcal{D}_\alpha T_{kl} = e^{-K/2} \mathcal{D}_\alpha Z^i A_{kli}$. For general dilaton couplings, L_3 contains the additional term $\frac{1}{2} f^i e^{-K} \bar{A}^j R_i{}^k{}_j{}^l A_{kl} \overline{W}$ which vanishes in the model considered here since $A_{ss} = 0$.

the generic operators introduced above and D-terms that involve supergravity superfields: $\phi = G_{\alpha\dot\beta}K_{s\bar s}\mathcal{D}^\alpha S\mathcal{D}^{\dot\beta}\bar S,\ldots$. As shown in Appendix C, this term must be exactly canceled by PV Abelian gauge multiplets that couple to the dilaton.

2.2 PV regularization with a dilaton

The ultraviolet divergent one-loop corrections to supergravity were calculated [10]–[12] in the presence of a nontrivial gauge kinetic function of the form:

$$f_{ab}(Z) = \delta_{ab}f(Z)k_a, \quad f(z) = x + iy \neq \text{constant}. \tag{2.22}$$

In [1] it was shown that the dilaton-induced quadratically divergent contribution, given by [T_α is defined as in (2.8)]

$$\mathrm{STr}H \ni -\frac{2N_G f_i\bar f_{\bar m}}{(f+\bar f)^2}\left(\bar A^i A^{\bar m} + \mathcal{D}_\mu z^i\mathcal{D}^\mu\bar z^{\bar m}\right) = -N_G\,\mathcal{D}^\alpha T_\alpha|, \quad T_i = D_i\ln(f+\bar f), \tag{2.23}$$

can be regulated by the introduction of N_G additional Pauli-Villars chiral multiplets π^α with

$$K(\pi,\bar\pi) = \sum_\alpha (f+\bar f)|\pi^\alpha|^2, \quad W(\pi) = \sum_\alpha \mu^\pi_\alpha(\pi^\alpha)^2, \quad \eta^\pi_\alpha = +1. \tag{2.24}$$

The expression for the logarthmically divergent loop corrections [12] with an arbitrary holomorphic function $f(Z)$ is very complicated. Here we consider the much simpler case of the string dilaton, with the dilaton couplings defined by (2.1). For this model (2.23) takes the form

$$\mathrm{STr}H \ni -2N_G\left(M^2 + \frac{\partial_\mu s\partial^\mu\bar s}{4x^2}\right) = N_G\,\mathcal{D}^\alpha k_\alpha|, \tag{2.25}$$

and the gravitino mass is equal to the gaugino mass:

$$M^2_\lambda = M^2_\psi = M^2 = e^{-K}A_s\bar A^s. \tag{2.26}$$

In addition we have

$$f + \bar f = e^{-K(S,\bar S)} = e^{-k}, \tag{2.27}$$

so instead of introducing the additional PV fields in (2.24), we need only modify the Kähler potential for the gauge fields ϕ^C used in [1, 3] to regulate gravity loops:

$$K(\phi^C,\bar\phi^C) = \sum_C e^{\alpha_C K + \beta_C k}|\phi^C|^2, \tag{2.28}$$

where the case of canonical gauge kinetic energy, $f(Z)=1$, is recovered for $\beta_C=0$.

A term proportional to (2.25) is also generated if Abelian gauge PV superfields couple to the dilaton. We find that it is this latter mechanism that must be used in order to cancel the dilaton-dependent logarithmic divergences that arise from gauge loops. We will also need to introduce chiral PV multiplets with a Kähler potential of the form (2.28), with the constraints (see Appendix C)

$$\sum_C \eta^C \beta_C = \sum_C \eta^C \beta_C \alpha_C = 0. \tag{2.29}$$

3 Anomaly-free supergravity

Here we assume that there are no gauge or mixed gauge-gravitational anomalies: $\mathrm{Tr}T^a = \mathrm{Tr}(\{T_a, T_b\}T_c) = 0$, where T_a is a generator of the gauge group. This section closely follows I, and the reader is referred to that paper for the contributions that are unchanged when the dilaton is included.

We introduce Pauli-Villars chiral supermultiplets $Z^I_\alpha = \tilde{Z}^I_\alpha, \hat{Z}_\alpha$, that transform under the gauge group like Z^i_α, and $Y^\alpha_I = \tilde{Y}^\alpha_I, \hat{Y}^\alpha_I$, that transform according to the conjugate representation, as well as gauge singlets Y^0, Z^0, and chiral multiplets $\Phi^a_\alpha = \varphi^a_\alpha, \tilde{\varphi}^a_\alpha, \hat{\varphi}^a_\alpha$, that transform according to the adjoint representation of the gauge group. Additional charged fields X^A_β and U^β_A transform according to the representation R^a_A and its conjugate, respectively, under the gauge group factor $\mathcal{G}_a$, and V^A_β transforms according to a (pseudo)real representation that is traceless and anomaly-free. Their gauge couplings satisfy

$$\sum_{\beta,A} \eta^A_\beta C^a_A = \sum_i C^a_i \equiv C^a_M, \tag{3.1}$$

where

$$\mathrm{Tr}_R\left(T^a T^b\right) = \delta_{ab} C^a_R, \tag{3.2}$$

which may imply a constraint on the matter representations of the gauge group in the light spectrum, as discussed in I. In addition, we introduce gauge singlets φ^γ, as well as $U(1)$ gauge supermultiplets $W_\gamma = W^0_\gamma, W^s_\gamma$, with signatures $\eta^0_\gamma, \eta^s_\gamma$, respectively, that form massive vector supermultiplets with chiral multiplets $Z^{0,s}_\gamma = e^{\theta^{0,s}_\gamma}$ of the same signature and $U(1)_\beta$ charge $q_\gamma \delta_{\gamma\beta}$.

For the Pauli-Villars fields we take, for illustrative purposes, the Kähler potential

$$
\begin{aligned}
K_{PV} &= \sum_{\gamma}\left[e^{\alpha_{\gamma}^{\phi}K+\beta_{\gamma}^{\phi}k}\phi^{\gamma}\bar{\phi}_{\gamma}+\frac{1}{2}\nu_{\gamma}(\theta_{\gamma}+\bar{\theta}_{\gamma})^{2}+e^{K/2}\sum_{A}\left(|X_{\gamma}^{A}|^{2}+|U_{A}^{\gamma}|^{2}+|V_{\gamma}^{A}|^{2}\right)\right] \\
&\quad+\sum_{\alpha,a}\left(e^{G}\varphi_{\alpha}^{a}\bar{\varphi}_{a}^{\alpha}+e^{k}\hat{\varphi}_{\alpha}^{a}\bar{\hat{\varphi}}_{a}^{\alpha}+\tilde{\varphi}_{\alpha}^{a}\bar{\tilde{\varphi}}_{a}^{\alpha}\right)+\sum_{\alpha}\left(K_{\alpha}^{Z}+K_{\alpha}^{Y}\right), \\
K_{\alpha}^{Z} &= \sum_{I,J=i,j}\left[K_{i\bar{j}}Z_{\alpha}^{I}\bar{Z}_{\alpha}^{\bar{J}}+\frac{b^{Z}}{2}\left(K_{IJ}Z_{\alpha}^{I}Z_{\alpha}^{J}+\text{h.c.}\right)\right]+|Z_{\alpha}^{0}|^{2}, \\
K_{\alpha}^{Y} &= \sum_{I,J=i,j}K_{Y}^{I\bar{J}}Y_{I}^{\alpha}\bar{Y}_{\bar{J}}^{\alpha}-a_{\alpha}^{Y}\sum_{I=i}\left(Y_{I}^{\alpha}\bar{Y}_{\alpha}^{0}\kappa_{Y}^{i}+\text{h.c.}\right)+|Y_{0}^{\alpha}|^{2}\left[1+\left(a_{\alpha}^{Y}\right)^{2}\kappa_{Y}^{i}\kappa_{i}^{Y}\right], \\
K_{\widehat{Y}}^{I\bar{J}} &= K^{i\bar{j}}, \quad K_{\widetilde{Y}}^{I\bar{J}}=e^{\alpha_{I}K+\beta_{I}k}\delta^{i\bar{j}}, \quad \alpha_{I\neq S}=\frac{1}{2}, \quad \beta_{S}=-2, \quad \alpha_{S}=\beta_{I\neq S}=0, \\
K_{IJ} &= \partial_{i}\partial_{j}K-K_{i}K_{j}-\frac{1}{2x}\left(f_{i}K_{j}+f_{j}K_{i}\right)-\frac{1}{2x^{2}}f_{i}f_{j}, \quad b^{\widetilde{Z}}=1, \quad b^{\widehat{Z}}=0, \\
\kappa_{i}^{\widetilde{Y}} &= -\frac{1}{2x}f_{i}, \quad \kappa_{i}^{\widehat{Y}}=K_{i}+\frac{1}{2x}f_{i}, \quad \kappa_{Y}^{i}=K^{i\bar{m}}\kappa_{\bar{m}}^{Y}, \quad a_{\alpha}^{\widetilde{Y}}=1, \quad a_{\alpha}^{\widehat{Y}}=a_{\alpha},
\end{aligned}
\tag{3.3}
$$

and $K^{i\bar{j}}$ is the inverse metric. We take the superpotential

$$
\begin{aligned}
W_{PV} &= W_{1}+W_{2}, \\
W_{1} &= \sum_{\alpha,\beta}\left[\sum_{I}\mu_{\alpha\beta}^{Z}Z_{\alpha}^{I}Y_{I}^{\beta}+\mu_{\alpha\beta}^{0}Z_{\alpha}^{0}Y_{0}^{\beta}+\sum_{a}\mu_{\alpha\beta}^{\Phi}\Phi_{\alpha}^{a}\Phi_{\beta}^{a}\right] \\
&\quad+\frac{1}{2}\sum_{\gamma}\mu_{\gamma}^{\phi}(\phi^{\gamma})^{2}+\sum_{A\gamma}\left(\mu_{\gamma}^{X}U_{A}^{\gamma}X_{\gamma}^{A}+\frac{1}{2}\mu_{\gamma}^{V}(V_{A}^{\gamma})^{2}\right) \\
W_{2} &= \sum_{\alpha}\left[a_{\alpha}W_{i}\widehat{Z}_{\alpha}^{I}\widehat{Y}_{0}^{\alpha}+W\widehat{Z}_{\alpha}^{I}\widehat{Y}_{I}^{\alpha}+2g_{\alpha}\varphi_{\alpha+1}^{a}\widehat{Y}_{I}^{\alpha}(T_{a}Z)^{i}\right] \\
&\quad+\sum_{\alpha}\left[\frac{1}{2}\widetilde{Z}_{\alpha}^{I}\widetilde{Z}_{\alpha}^{J}W_{ij}+c_{\alpha}\widetilde{Z}_{\alpha}^{S}\widetilde{Y}_{S}^{\alpha}W\right],
\end{aligned}
\tag{3.4}
$$

where the index a refers to the light gauge degrees of freedom. Finally, we take for the gauge kinetic functions:

$$
\begin{aligned}
f^{ab} &= \delta^{ab}\left(S+\sum_{\alpha}h_{\alpha}f_{i}\widetilde{Z}_{\alpha}^{I}\widetilde{Y}_{0}^{\alpha}\right), \quad f_{s}^{a\gamma}=0, \\
f_{\gamma\beta}^{0} &= \delta_{\gamma\beta}, \quad f_{\gamma\beta}^{s}=\delta_{\gamma\beta}S, \quad f_{0}^{a\gamma}=\sum_{\beta}e^{\gamma\beta}\hat{\varphi}_{\beta}^{a},.
\end{aligned}
\tag{3.5}
$$

The matrices $\mu_{\alpha\beta}, d_{\alpha\beta}, e_{\alpha\beta}$, are nonvanishing only when they couple fields of the same signature. The parameters μ, ν, play the role of effective cut-offs. The parameters a, b, c, d, e, h, are of order unity, and are chosen to satisfy[4]:

$$\begin{aligned}
a &= \sum_\alpha \eta^{\widehat{Y}}_\alpha a^2_\alpha = -2, \quad a' = \sum_\alpha \eta^{\widehat{Y}}_\alpha a^4_\alpha = +2, \\
c &= \sum_\alpha \eta^{\widetilde{Z}}_\alpha c^2_\alpha = 5, \quad g = \sum_\alpha \eta^{\widehat{Y}}_\alpha g^2_\alpha a^2_\alpha = -1, \quad \sum_\alpha \eta^{\widehat{Y}}_\alpha g^2_\alpha = 1, \\
e &= \frac{1}{2}\sum_{\alpha,\beta} \eta^{\hat{\varphi}}_\alpha e^2_{\alpha\beta} = -4 = 3e', \quad e' = \frac{1}{4}\sum_{\alpha\beta\gamma\delta} \eta^{\hat{\varphi}}_\gamma e^\beta_\alpha e^\gamma_\beta e^\delta_\gamma e^\alpha_\delta, \\
h &= \sum_\alpha \eta^{\widetilde{Z}}_\alpha h^2_\alpha = 2, \quad w = \sum_\alpha \eta^{\widetilde{Z}}_\alpha h_\alpha c_\alpha = 1.
\end{aligned} \tag{3.6}$$

The signatures of the chiral PV multiplets satisfy

$$\begin{aligned}
\sum_\alpha \eta^\varphi_\alpha &= \sum_\alpha \eta^{\hat{\varphi}}_\alpha = \sum_\alpha \eta^{\tilde{\varphi}}_\alpha = 1, \quad \eta^\varphi_{1+\alpha} = \eta^{\widehat{Z}}_\alpha, \quad \eta^\varphi_1 = +1, \quad \eta^U_\alpha = \eta^X_\alpha, \\
\sum_\alpha \eta^{\widetilde{Z}}_\alpha &= -1, \quad \sum_\alpha \eta^{\widehat{Z}}_\alpha = 0, \quad \eta^{\widetilde{Z}}_\alpha = \eta^{\widetilde{Y}}_\alpha, \quad \eta^{\widehat{Z}}_\alpha = \eta^{\widehat{Y}}_\alpha, \\
\sum_\gamma \eta^0_\gamma &= -12, \quad \sum_\gamma \eta^s_\gamma = -N_G, \quad \sum_\gamma \eta^\theta_\gamma = -12 - N_G = N'_G,
\end{aligned} \tag{3.7}$$

and, from the results of I, we require for the exponents in (3.3)

$$\alpha = \sum_C \eta_C \alpha_C = -10, \quad \alpha' = \sum_C \eta_C \alpha^2_C = -4, \tag{3.8}$$

where in (3.8) and throughout this section ϕ^C is any chiral PV field except $Z, \widehat{Y}$, and $\alpha^{\widetilde{Y}s}_\alpha = 0$, $\beta^{\widetilde{Y}s}_\alpha = -2$. The Kähler potential for φ^a_α assures the Kähler anomaly matching condition for the term quadratic in the Yang-Mills field strength, as discussed in I and in Section 4 below, as well as the correct form of the gauge-dependent contribution to the renormalization of the Kähler potential.

[4]The contribution to K', Eq. (3.15) below, from the last term in (3.4) differs from that of I, where in (2.5) we set $c = -2 - N'_G = 10 + N_G$, by the term $-N_G M^2$ needed to cancel the $N_G M^2$ term in (2.2).

3.1 Quadratic divergences

For the class of supergravity theories considered here, the on shell[5] quadratically divergent contribution is proportional to

$$
\begin{aligned}
\mathrm{STr}H &= \frac{1}{2}(3+N_G-N)\,\mathcal{D}^\alpha X_\alpha| + \left(\hat{V}+M^2\right)(7+3N_G-N) \\
&\quad + N_G\,\mathcal{D}^\alpha k_\alpha| + \mathcal{D}^\alpha\Gamma_\alpha|, \qquad (3.9)
\end{aligned}
$$

where $X_\alpha = K_\alpha$, *etc.* are the chiral superfields defined in (2.8). The contribution of the Pauli-Villars fields to STrH is

$$
\begin{aligned}
\mathrm{STr}H^{PV} &= \left(3\sum_\gamma \eta^\theta_\gamma - \sum_P \eta_P\right)\left(\hat{V}+M^2\right) - \frac{1}{2}\left(\sum_P \eta_P - \sum_\gamma \eta^\theta_\gamma\right)\mathcal{D}^\alpha X_\alpha| \\
&\quad + \sum_\gamma \eta^s_\gamma\,\mathcal{D}^\alpha k_\alpha| + \sum_P \eta_P\,\mathcal{D}^\alpha \Gamma^P_{P\alpha}\Big|, \qquad (3.10)
\end{aligned}
$$

where P refers to all heavy chiral multiplets: $\phi^P = Z^I, Y_I, \phi^C$. From (2.28) we have

$$
\Gamma^I_{I\alpha} = \Gamma_\alpha, \quad \Gamma^C_{D\alpha} = (\alpha_C X_\alpha + \beta_C k_\alpha)\,\delta^C_D, \qquad (3.11)
$$

and we obtain for the contribution from heavy PV modes:

$$
\begin{aligned}
\mathrm{STr}H_{PV} &= -\frac{1}{2}(N' - N'_G - 2\alpha)\,\mathcal{D}^\alpha X_\alpha| + \left(\hat{V}+M^2\right)(3N'_G - N') - \mathcal{D}^\alpha\Gamma_\alpha| \\
&\quad + (\beta + f)\,\mathcal{D}^\alpha k_\alpha|, \\
\beta &= \sum_C \eta_C\beta_C, \quad N' = \sum_P \eta_P, \quad N'_G = \sum_\gamma \eta^\theta_\gamma, \quad f = \sum_\gamma \eta^s_\gamma. \qquad (3.12)
\end{aligned}
$$

Using (3.8), the absence of quadratic divergences requires

$$
\begin{aligned}
N' &= 3\alpha + 1 - N = -29 - N, \quad \beta + f = -N_G, \\
N'_G &= \alpha - 2 - N_G = -12 - N_G. \qquad (3.13)
\end{aligned}
$$

As explained in [1, 3] the $O(\mu^2)$ contribution to $S_0 + S_1 = \int d^4x\,(\mathcal{L}_0 + \mathcal{L}_1)$ takes the form of a correction to the Kähler potential, once additional finiteness contraints on the PV masses have been imposed. Throughout this section we set (see Appendix C)

$$
\beta = 0, \quad f = -N_G, \quad \beta' = \sum_C \eta_C\beta_C^2 = 2, \quad \sum_C \eta_C\alpha_C\beta_C = 0. \qquad (3.14)
$$

[5]Specifically, a contribution proportional to $r - \mathcal{D}^\alpha X_\alpha| - 6(\hat{V}+M^2)$, where r is the space-time curvature, can be removed to one-loop order by a scalar field dependent Weyl transfromation.

3.2 Logarithmic divergences

The Pauli-Villars contribution to (2.2) is, after an appropriate space-time metric redefinition,

$$\begin{aligned} \mathcal{L}_{PV} &= \sqrt{g}\frac{\ln\Lambda^2}{32\pi^2}\left[N'_G L_g - N_G L'_g + N' L_\chi + \sum_P \eta_P\left(L_1^P + L_2^P\right) + L_3^Z + L_{\mathcal{W}} + eL_e\right] \\ &\quad + \Delta_{K'}\mathcal{L}, \qquad K' = \frac{\ln\Lambda^2}{32\pi^2}e^{-K}\sum_{P,Q}\eta_P A_{PQ}\bar{A}^{PQ}. \end{aligned} \tag{3.15}$$

Using (3.6)–(3.8), (3.14) and (2.26), the PV contributions found in I are modified to read[6]

$$\begin{aligned} K' &= \frac{\ln\Lambda^2}{32\pi^2}\left[-e^{-K}A_{ij}\bar{A}^{ij} + 2(a+1)\hat{V} + (2c+4-2a+N'_G)M^2 + 4\mathcal{K}_a^a + 8g\mathcal{D}\right] \\ &= -\frac{\ln\Lambda^2}{32\pi^2}\left[e^{-K}A_{ij}\bar{A}^{ij} + 2\hat{V} + (N_G-6)M^2 - 4\mathcal{K}_a^a - 8\mathcal{D}\right], \end{aligned} \tag{3.16}$$

$$\begin{aligned} L_{\mathcal{W}} &= 6e'L(\phi_{\mathcal{W}_b^a}) + 2hL(\phi_{\mathcal{W}}) - 2wL_{\mathcal{W}} = 2eL(\phi_{\mathcal{W}_b^a}) + 4L(\phi_{\mathcal{W}}) - 2L'_{\mathcal{W}}, \\ L_e &= L(\Phi_e) + L(\phi_e), \quad \Phi_e = -\frac{1}{6}\Phi'_0, \\ \phi_e &= \phi_{WK} - \phi_{Wk} - 4\mathcal{D} - 4\phi_{\mathcal{W}_b^a}, \end{aligned} \tag{3.17}$$

$$\begin{aligned} \sum_P \eta_P L_2^P &= -L_2 - \frac{2}{3}\alpha L_\alpha, \quad \sum_P \eta^P L_1^P = -L_1 + 6L_{YM}^a + \alpha' L_\alpha + \beta' L_\beta + L_1^Y, \\ L_1^Y + L_3^Z &= \left(L_1^Y + L_3^Z\right)_I - \frac{1}{3}L(\Phi'_0) - 2L_\beta + 2L'_{\mathcal{W}} - 8\Delta_{M^2}L \\ &= -L_3 + 4\Delta_{\hat{V}}L + 4\Delta_{M^2}L + 8\Delta_{\mathcal{D}}L - \frac{1}{3}L(\Phi'_0) - 2L_\beta + 2L'_{\mathcal{W}} - 8\Delta_{M^2}L, \\ L_\beta &= L(\Phi_\beta), \quad \Phi_\beta = -\frac{1}{2}k^\alpha k_\alpha, \quad L'_{\mathcal{W}} = x\left(\mathcal{W} + \overline{\mathcal{W}}\right)\left(\hat{V} + M^2\right), \end{aligned} \tag{3.18}$$

where the subscript I refers to the result of I. The contributions L_α and L_β follow immediately from Eqs. (3.11) and (2.17). L_β and $L(\Phi'_0)$ are given explicitly in Appendix B, Eq. (B.26).

[6]See Appendix B of I and Appendix B below.

In I the logarithmic divergences were found to cancel[7] with $e = -3$. Hence we write

$$\begin{aligned} eL_e + L_{\mathcal{W}}(e) &= -3\,(L_e)_I + (L_{\mathcal{W}})_I + L'_e - 2L'_{\mathcal{W}} - 4(3+e)\Delta_{\mathcal{D}}L, \quad L'_e = L(\phi'_e) + L(\Phi'_e), \\ \Phi'_e &= -\frac{e}{6}\Phi'_0, \quad \phi'_e = 4\phi_{\mathcal{W}} + (3+e)\left(\phi_{WK} - 2\phi_{\mathcal{W}^a_b}\right) - \phi_{Wk}. \end{aligned} \tag{3.19}$$

The renormalization of the Kähler potential is now finite if $e = -4$. Complete cancellation of the ultra-violet divergences then requires, once the conditions (3.8), (3.14) are imposed,

$$L'_0 + L'_e - \frac{1}{3}L(\Phi'_0) = 0, \tag{3.20}$$

which is achieved for $e = -4$.

3.3 Soft supersymmetry breaking terms

Pauli-Villars regularization can be used to calculate one-loop contributions to soft supersymmetry breaking. The calculation of gaugino masses has been given in [11] for string-derived supergravity with the dilaton in a linear supermultiplet, and including a Green-Schwarz (GS) term. These include the "anomaly mediated" contribution [12, 13] as well as additional model-dependent contributions. A general analysis of soft supersymmetry breaking terms in this class of models will be given elsewhere [14]. As an example, we calculate here the one-loop induced A-term for supergravity theories with matter in chiral supermultiplets. To obtain this contribution we take constant background fields, and the effective one-loop potential is given simply by

$$\begin{aligned} \mathcal{L} &= \frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\mathrm{STr}\ln\eta\left(p^2 - m^2 - H\right) = -\frac{1}{32\pi^2}\mathrm{STr}\eta\Big[\left(hm^2 + \frac{1}{2}g^2\right)\ln(m^2) \\ &\quad + \frac{1}{2}h^2\ln\left(\frac{m^2}{\mu^2}\right) + \frac{3hg^2}{6m^2} - \frac{g^4}{24m^4} + O(\frac{1}{m^2})\Big], \end{aligned} \tag{3.21}$$

[7]These operators are the bosonic parts of D-terms of the form (2.13) with:

$$(\phi_{\mathcal{W}})_I = -6\phi_{\mathcal{W}^a_b}, \quad (\phi_e)_I + 4\mathcal{D} = \phi_{WK} - 4\phi_{\mathcal{W}^a_b} = 2\hat{\phi}_0 - 6\phi_\chi - 2\phi_{\mathcal{W}^a_b},$$

from which it follows immediately that the conditions (2.20) and (2.46) of I give $L + L_{PV} = 0$. The term $-3C^a\delta_{ab}\left(\mathcal{W}^{ab} + \mathrm{H.c.}\right) + 4\Delta_{\mathcal{D}}L$ is missing from the right hand side of the third of Eqs. (2.43) of I, and $8\mathcal{D}$ should be replaced by $(8-4e)\mathcal{D}$ in the second of those equations.

where μ is the normalization scale, and $h+g$ is the effective field-dependent squared mass with the PV mass term removed:

$$H_{PV} = H + m^2, \quad H = h + g, \quad h \sim m^0, \quad g \sim m^1. \tag{3.22}$$

We dropped a term $-\frac{1}{6}r\mathrm{STr}H$ in the integrand; in the constant background field approximation $r \to V$ after a Weyl transformation. Assuming $< V >= 0$, terms proportional to V can at most contribute small corrections to soft terms already present at tree level. The second equality in Eq. (3.21) is schematic: $[H, m^2] \neq 0$ in general.

Soft terms are generated by the PV fields $\Phi^A = \widetilde{Z}^I, \widehat{Y}_I, \varphi^a$ that govern the wave function renormalization through the dimension three operators in W_2, Eq. (3.4). We denote by $\Phi^\alpha = \widetilde{Y}^I, \widehat{Z}^I, \varphi'^a$, respectively, the fields to which to which they couple in W_1:

$$W_1(\Phi^A, \Phi^\alpha) = \sum_{A=\alpha} \mu_A \Phi^A \Phi^\alpha. \tag{3.23}$$

Setting

$$K_{A\bar{A}} = h_A(z), \quad K_{\alpha\bar{\alpha}} = h_\alpha(z), \tag{3.24}$$

we have

$$m_A^2 = m_\alpha^2 = f_A \mu_A^2, \quad f_A = e^K g_A^{-1} g_\alpha^{-1}. \tag{3.25}$$

The first two terms in Eq. (3.21) are the shift in the potential due to the shift δK in the Kähler potential. The first term, proportional to m^2, corresponds [1] to $\delta K = \sum_P c_P m_P^2$, $c_P =$ constant. They contribute A-terms and scalar masses proportional to those already contained in the tree potential, with coefficients suppressed by the factor $1/32\pi^2$ ($m^2 \sim 1$ in reduced Planck units), and we neglect them.

From the general matrix elements evaluated in Appendix C of [10], assuming D-terms vanish, dropping derivatives, space-time curvature and gauge fields, we have

$$\begin{aligned}
(H^\chi)^A_B &= \; = h^A_B = e^{-K} A_{AB} \bar{A}^{AB}, \quad (H^\chi)^\alpha_\beta = 0, \\
(H^\chi)^\alpha_D &= K^{\alpha\bar{\beta}} \mu_B K^{\bar{B}C} A_{CD} = g^\alpha_D = e^{-K} f_A \mu_A A_{AD}, \\
(H^\chi)^A_\beta &= \bar{A}^{AB} \mu_B = g^A_\beta, \quad (H^\phi)^P_Q = (H^\chi)^P_Q + \delta^P_Q \left(\hat{V} + M^2\right),
\end{aligned} \tag{3.26}$$

for fermions χ and scalars ϕ, respectively. For the reasons given above we can neglect the $\hat{V}$ term, and terms containing only powers of H^χ cancel in the supertrace. The M^2 term does not contribute in leading order to the A-terms, so they get contributions only from the scalar trace terms that have factors of H^{ϕ}_{PQ}:

$$
\begin{aligned}
H^{\phi}_{AB} &= h_{AB} = e^{-K}\left(\bar{A}^i D_i A_{AB} - A_{AB}\bar{A}\right), \quad h_{\alpha\beta} = 0, \\
H^{\phi}_{A\beta} &= g_{A\beta} = e^{-K}\bar{A}^i D_i\left(e^K W_{A\beta}\right) - \bar{A}W_{A\beta} = -\delta_{A\beta}\mu_A\left(\bar{A} - \bar{A}^i\partial_i f_A\right). \qquad (3.27)
\end{aligned}
$$

Taking into account the fact that $[H, m^2] \neq 0$ in the integral Eq. (3.21), we have on the right hand side:

$$
\begin{aligned}
\mathrm{Tr} h^2 \ln(m^2/\mu^2) &\to 2\sum_{AB}\eta_A\left(h^A_B h^B_A + h_{AB}h^{AB}\right)\left[q(m_A^2, m_B^2) - \ln\mu^2\right], \\
q(m_A^2, m_B^2) &= \frac{m_A^2\ln(m_A^2/\mu^2) - m_B^2\ln(m_B^2/\mu^2)}{m_A^2 - m_B^2} - 1, \qquad (3.28)
\end{aligned}
$$

and

$$
\begin{aligned}
\mathrm{Tr}\frac{3hg^2}{6m^2} &\to \sum_{AB}\eta_A h^A_{\bar{B}}\left[g^{\bar{B}}_{\beta}g^{\beta}_{A}\frac{\partial}{\partial m_B^2}q(m_A^2, m_B^2) + g^{\bar{B}}_{\bar{\alpha}}g^{\bar{\alpha}}_{A}\frac{\partial}{\partial m_A^2}q(m_A^2, m_B^2)\right] + \mathrm{h.c.} \\
&= -e^{-3K/2}\sum_{AB}\eta_A\Big[\left(m_{\tilde{G}} + \bar{F}^{\bar{m}}\partial_{\bar{m}}\ln f_B\right)m_B^2\frac{\partial}{\partial m_B^2}q(m_A^2, m_B^2) \\
&\quad + \left(m_{\tilde{G}} + \bar{F}^{\bar{m}}\partial_{\bar{m}}\ln f_A\right)m_A^2\frac{\partial}{\partial m_A^2}q(m_A^2, m_B^2)\Big]\eta_A\bar{A}^i(D_i A_{AB})\bar{A}^{AB} \\
&\quad + \mathrm{h.c.} + \cdots, \qquad (3.29)
\end{aligned}
$$

where $\bar{F}^{\bar{m}} = -e^{-K/2}A^{\bar{m}}$ is the auxiliary field of the superfield $\bar{Z}^{\bar{m}}$. In (3.29) we have explicitly retained only contributions to A-terms (and "B-terms"). Scalar masses get contributions from additional terms in (3.29) as well as from $\mathrm{Tr}g^4/24m^2$ in (3.21). The one-loop corrected scalar kinetic term is

$$
\begin{aligned}
\mathcal{L}_{KE} &= \mathcal{D}_\mu\bar{z}^i\mathcal{D}^\mu\bar{z}^{\bar{m}}\left(K_{i\bar{m}} + \delta K_{i\bar{m}}\right) = \mathcal{D}_\mu\bar{z}^i\mathcal{D}^\mu\bar{z}^{\bar{m}}(Z^{\frac{1}{2}})^j_i K_{j\bar{n}}(Z^{\frac{1}{2}})^{\bar{n}}_{\bar{m}}, \\
(Z^{\frac{1}{2}})^j_i &= \delta^i_j + \frac{1}{2}K^{j\bar{n}}\delta K_{i\bar{n}}, \\
\delta K &= -\frac{1}{32\pi^2}e^{-K}\sum_{AB}\eta_A\bar{A}^{AB}A_{AB}\left[q(m_A^2, m_B^2) - \ln\mu^2\right], \qquad (3.30)
\end{aligned}
$$

where z_R is the renormalized field, and the matrix-valued anomalous dimension is

$$
\begin{aligned}
\gamma^j_i &= K^{j\bar{n}}D_{\bar{n}}D_i\frac{\partial}{\partial\mu^2}\delta K = \frac{1}{32\pi^2}D^j D_i(e^{-K}\sum_{AB}\eta_A\bar{A}^{AB}A_{AB}) = e^{-K}\sum_{AB}\eta_A\bar{A}^{jAB}A_{iAB} + \cdots, \\
&= \frac{1}{32\pi^2}\left[e^K W_{ikl}\overline{W}^{jkl} - 4g^2(T^a\phi)^i K_{j\bar{m}}(T_a\bar{\phi})^{\bar{m}}\right] + \ldots. \qquad (3.31)
\end{aligned}
$$

where the ellipses represent higher order terms. To evaluate the A-terms we expand Eqs. (3.28) and (3.29) in terms of the light gauge-charged fields ϕ^i. For example we have

$$\begin{aligned}
\mathrm{STr}h^2 &= 2\left(h_{AB}h^{AB} + h^A_B h^B_A\right) \\
&= -2m_{\tilde{G}}e^{-3K/2}\bar{A}^i(D_iA_{AC})\bar{A}^{AC} + \text{h.c.} + \cdots \\
&= -2m_{\tilde{G}}e^{-K/2}\bar{A}^i\phi^{\bar{m}}K_{j\bar{m}}\gamma^j_i + \text{h.c.} + \cdots .
\end{aligned} \tag{3.32}$$

If at tree level we have

$$K_{i\bar{m}} = h_i(z)\delta_{im} + O(|\phi|^2), \quad A_i = e^K\left(c_{ijk}\phi^j\phi^k + \mu_{ij}\phi^j\right) + O|\phi|^3, \tag{3.33}$$

using

$$\left(m^2_B\frac{\partial}{\partial m^2_B} + m^2_A\frac{\partial}{\partial m^2_A}\right)q(m^2_A, m^2_B) = 1, \tag{3.34}$$

we get a one-loop contribution to the A-term

$$\begin{aligned}
\mathcal{L}^1_A &= \frac{1}{32\pi^2}e^{-K/2}\phi^{\bar{m}}\bar{A}^iK_{j\bar{m}}\gamma^j_i\left[m_{\tilde{G}}\left(1+\ln(m^2_{ij}/\mu^2)\right) + \bar{F}^{\bar{p}}\partial_{\bar{p}}\ln m^2_{ij}\right] + \text{h.c.} \\
&= \sum_{ijk}e^{-K/2}h_j^{-1}\phi^i_R\phi^k_R\left[\sum_l\phi^l_Rc_{jkl}(h_i/h_kh_l)^{\frac{1}{2}} + m_{jk}(h_i/h_k)^{\frac{1}{2}}\right]\gamma^i_j \times \\
&\qquad \left[m_{\tilde{G}}\left(1+\ln(m^2_{ij}/\mu^2)\right) + F^{\bar{m}}\partial_{\bar{m}}\ln f_{ij}\right] + \text{h.c.} + \cdots , \\
\ln m^2_{ij} &= \frac{1}{32\pi^2}\sum_{AB}q(m^2_A, m^2_B)D^jD_i(e^{-K}\bar{A}^{AB}A_{AB})/\gamma^j_i, \\
\partial_{\bar{m}}\ln f_{ij} &= \frac{1}{32\pi^2}\sum_{AB}\left[\partial_{\bar{m}}q(m^2_A, m^2_B)\right]D^jD_i(e^{-K}\bar{A}^{AB}A_{AB})/\gamma^j_i.
\end{aligned} \tag{3.35}$$

Note that the term linear in ϕ in A_i can arise from a quadratic term in the superpotential or in the Kähler potential; the relation between the corresponding supermultiplet mass and the one-loop induced "B-term" is the same in both cases. If $m^2_{ij} = \mu^2$ and one assumes canonical kinetic energy for both the light fields and the PV fields, $\mathcal{L}_A$ reduces to the "anomaly mediated" term found in [7]. The contributions that depend explicitly on the PV masses are contained in the component field expression of the superfield operator (2.16) that determines the renormalization of the Kähler potential. The term proportional to $\ln(m^2_{ij}/\mu^2)$ is not negligible if the scale of supersymmetry breaking is significantly below the Planck scale. A further model dependence is in the $\partial_{\bar{m}}lnf_{ij}$ terms.

In contrast to the case of gaugino masses studied in [13], the one-loop corrections to the soft terms in the scalar potential are sensitive to the details of the Pauli-Villars regularization. In the gaugino mass case, the PV squared mass matrix commutes with relevant (gauge superfield dependent) matrix elements. The regulator masses appear only through the $\ln m^2$ term, averaged over all charged PV fields, and only the field dependent part $f_P(z)$ of $m_P^2 = f_P(z)\mu_P^2$ contributes to gaugino masses. The field dependence (i.e., the dependence on fields that do not vanish in the vacuum, such as the dilaton and moduli) on this "average" $\ln m^2$ is completely fixed in terms of the field-dependence of the light field Kähler metrics. Both the requirements of finiteness discussed in Section 3 above and the supersymmetry of the Kähler anomaly [16] uniquely determine the field dependence of $\ln m^2$ once the tree-level theory (including possible couplings of charged matter to a GS term) is specified. However, only a subset of charged PV fields contribute to the renormalization of the Kähler potential. While the Kähler metrics of the fields Φ^A that appear in W_2 is determined by the finiteness requirement, the metrics of the fields Φ^α to which they couple in W_1 is arbitrary. Since the associated Kähler anomaly is a D-term, it is supersymmetric by itself and there is no constraint analogous to the conformal/chiral anomaly matching in the case of gauge field renormalization with an F-term anomaly. As a consequence the "non-universal" terms appearing in $\mathcal{L}^1_{soft}$ cannot be determined precisely in the absence of a detailed theory of Planck scale physics. In the following sections we give examples in which the PV masses that contribute to $\mathcal{L}^1_{soft}$ are field independent.

4 String-derived supergravity and T-duality

Effective field theories from superstring compactifications are perturbatively invariant [17] under an $SL(2,Z)$ group (T-duality) of transformations on the chiral superfields $Z \to Z'(Z)$, which is a subgroup of a continuous $SL(2,R)$ group, itself a symmetry of the classical Lagrangian. Here we will refer to both groups as modular transformations. They effect a Kähler transformation:

$$\begin{aligned} K(Z,\bar{Z}) &\to K(Z',\bar{Z}') = K'(Z,\bar{Z}) = K(Z,\bar{Z}) + F(Z) + \bar{F}(\bar{Z}), \\ W(Z) &\to W(Z') = W'(Z) = e^{-F(Z)}W(Z), \end{aligned} \tag{4.1}$$

and therefore leave the classical Lagrangian invariant. Because (4.1) includes phase transformations on chiral fermions, the symmetry is anomalous at the quantum level. Ungauged nonlinear σ-models were considered in I, where it was shown that, while the PV Kähler potential can be chosen to be invariant under (4.1), regularization of the theory with invariant PV masses requires constraints on the light spectrum. Moreover, for gauged σ-models, invariant regularization does not appear to be possible for any choice of spectrum. Indeed, in supergravity theories obtained from orbifold compactifications of string theory, the (weighted average) masses of gauge nonsinglet PV chiral multiplets are fixed [16] by matching field theory and string theory loop corrections to the moduli-Yang-Mills couplings, and cannot all be invariant under T-duality transformations.

Specifically, we consider a class of orbifold compactifications with, in addition to the dilaton, the chiral superfields $Z^p = T^i, \Phi^p$, where T^i, $i = 1,2,3$, are the untwisted moduli, and the Kähler potential

$$\begin{aligned} G &= \sum_i g^i + e^{g^p}|\Phi^p|^2 + O\left(|\Phi^p|^4\right), \\ g^p &= \sum_i q_i^p g^i, \quad g^i = -\ln\left(T^i + \bar{T}^{\bar{i}}\right). \end{aligned} \tag{4.2}$$

The modular transformation

$$\begin{aligned} T^i &\to T'^i = \frac{aT^i - ib}{icT^i + d}, \quad S \to S' = S, \quad ad - bc = 1, \\ \Phi^p &\to \Phi'^p = e^{-q_p^i F^i}\Phi^p, \quad F^i = \ln\left(icT^i + d\right), \end{aligned} \tag{4.3}$$

where q_p^i are the modular weights of Φ^p, effects the Kähler transformation (4.1) with

$$F(Z) = \sum_i F^i(T^i). \tag{4.4}$$

Setting to zero the gauge-charged background fields, the one-loop corrected Lagrangian contains the term[8]:

$$\begin{aligned} \mathcal{L}_1 &\ni \frac{1}{64\pi^2}\sum_a F_a^{\mu\nu}F_{\mu\nu}^a \sum_\alpha \eta_\alpha \mathrm{Tr}\left(C_a^\phi \ln M^2\right)_\alpha, \\ (M^2)_Q^P &= e^K K^{P\bar{M}}\mu_{\bar{M}\bar{N}}K^{\bar{N}R}\mu_{RQ}, \quad (C_a^\phi)_Q^P = \delta_Q^P C_a^P, \end{aligned} \tag{4.5}$$

[8]The sign of this term in (3.3)–(3.7) of I is incorrect

and $C_a^P = (\mathrm{Tr}T_a^2)_P$ is the eigenvalue of the quadratic Casimir operator on ϕ^P. Since the parameters μ_{PQ} of the superpotential (3.4) and – for vanishing gauge-charged background fields – the elements $K_{P\bar{M}}$ of the metric connect only fields ϕ^P with the same values of C_a^P, we have

$$\sum_\alpha \eta_\alpha \mathrm{Tr}\left(C_a^\phi \ln M^2\right)_\alpha = \sum_P \eta^P C_a^P \mathrm{Tr}\ln M_P^2 = \sum_P \eta^P C_a^P \ln \mathrm{Det} M_P^2. \tag{4.6}$$

With the choice of Kähler potential (3.3) we have, for $P, M \neq T^I, S$:

$$\begin{aligned}
K_Z^{P\bar{M}} &= \delta^{PM} e^{q_P^i g^i}, \quad K_{\widehat{Y}}^{P\bar{M}} = \delta^{PM} e^{-q_P^i g^i}, \quad K_{X,U,V,\widetilde{Y}}^{P\bar{M}} = e^{K/2}\delta^{PM}, \\
M_{X,U,V}^2 &= \mu_{X,U,V}^2, \quad M_{\widehat{Z},\widehat{Y}}^2 = e^K \mu_{\widehat{Z},\widehat{Y}}^2, \quad \mathrm{Det} M_{\Phi^a}^2 = e^K \mathrm{Det}\mu_{\Phi^a}^2, \\
M_{\widetilde{Z},\widetilde{Y}}^2 &= e^{\frac{1}{2}(K - 2\sum_i q^i g^i)} \mu_{\widetilde{Z},\widetilde{Y}}^2, \quad q^i = \mathrm{diag}(q_{p_1}^i, \cdots, q_{P_{N-4}}^i).
\end{aligned} \tag{4.7}$$

Then using the constraints (3.7) we obtain

$$\mathcal{L}_1 \ni \frac{1}{64\pi^2} \sum_a F_a^{\mu\nu} F_{\mu\nu}^a \left[\sum_P \eta^P C_a^P \ln \mathrm{Det}\mu_P^2 - \sum_p C_p^a \left(K - 2\sum_i q_p^i g^i \right) + C^a K \right]. \tag{4.8}$$

As is well known [18]–[20], [16], invariance under (4.3) is restored by the GS mechanism; the Kähler potential of the dilaton[9] and its modular transformation property are modified to read

$$k = -\ln\left(S + \bar{S} + \frac{C_{E_8}}{8\pi^2} G \right), \quad S' = S - \frac{C_{E_8}}{8\pi^2} F, \tag{4.9}$$

so that the variation of $\mathcal{L}_1$, and of model-dependent threshold corrections, are canceled by a variation in the tree-level coupling of the dilaton to the Yang-Mills fields. The contribution in (4.8) satisfies the string matching condition [16] when the Green-Schwarz term and the string-loop threshold corrections are included. Threshold corrections [18, 21] can be included as moduli-dependent terms in the PV superpotential $W_1 : \mu_P = \mu_P(T^i)$.

In order to achieve full perturbative modular invariance, we must investigate more completely the anomaly structure of the one-loop corrected effective theory, including gauge nonsinglet background fields. Supersymmetry relates conformal anomalies, associated with logarithmic divergences, to chiral anomalies that arise from linearly divergent integrals in

[9]The Kähler potential k no longer satisfies $\rho_{ij} = a_i = 0$, in the notation of [12], resulting in additional contributions to the loop corrections. However the modification of k is of one-loop order, and hence the corresponding one-loop corrections are of two-loop order, which we do not consider here.

quantum corrections to the low energy effective theory. When the theory is regulated in such a way that all integrals are finite, there are strictly speaking no anomalies, but a corresponding noninvariance of the quantum corrected theory results from the noninvariance of the regulator masses. For example, only light quark loops contribute to the chiral anomaly that permits neutral pion decay; the anomaly from heavy quark loops is exactly canceled by the explicit chiral symmetry breaking due to the quark mass term. The contribution of a PV quark with negative signature has the opposite sign; it's anomaly cancels the light quark anomaly and one is left with the explicit breaking term that exactly reproduces the light quark anomaly.

Provided we can define modular transformations on the PV fields such that K_{PV} is invariant and W_2 is covariant ($W \to e^{-F}W$), the noninvariance of the regulated one-loop Lagrangian will arise solely from the noncovariance of W_1 which governs the PV mass-matrix M_{PV}. The Kähler potential for the θ_γ in (3.3) is modular invariant provided the chiral superfields $\theta'_\gamma = \theta_\gamma$ under (4.1). In addition, if we take for the Φ^a mass term in (3.4)

$$W_1(\Phi^a) = \sum_{\alpha,a}\left[\mu^\varphi_\alpha \varphi^a_\alpha \hat{\varphi}^a_\alpha + \frac{1}{2}\mu^{\tilde{\varphi}}_\alpha \tilde{\varphi}^a_\alpha \tilde{\varphi}^a_\alpha\right], \tag{4.10}$$

the superpotential for chiral fields $\hat{\varphi}^a$ with dilaton-like couplings is modular covariant. Then the one loop action can be written as

$$\mathcal{L}_1 = \mathcal{L}_{inv} + \mathcal{L}_\chi, \quad \mathcal{L}_\chi = \frac{i}{2}\mathrm{STr}\ln\left[D^2 + H(M_{PV})\right]_\chi + T_-(M_{PV}), \tag{4.11}$$

where $\mathcal{L}_{inv}$ is modular invariant and $\mathcal{L}_\chi$ contains only chiral supermultiplet loop contributions. As a result the masses and covariant derivatives appearing in the noninvariant contribution contain no Dirac matrices except in the spin connection, and their contributions are straightforward to evaluate.

As shown in I, under a transformation on the PV fields that leaves the tree Lagrangian and the PV Kähler potential invariant, with W_2 covariant:

$$\begin{aligned}\Phi' &= g\Phi, \quad M'_{PV}(\Phi) = M_{PV}(\Phi') \\ \mathcal{L}' &= \mathcal{L}_{inv} + \mathcal{L}_\chi(\widetilde{M}_{PV}), \quad \widetilde{M}_{PV} = g^{-1}M'_{PV}g,\end{aligned} \tag{4.12}$$

because all the operators in the determinants except M_{PV} are covariant. Therefore the anomalous shift in the Lagrangian is given simply by

$$\Delta\mathcal{L}_1 = \mathcal{L}_\chi(\widetilde{M}_{PV}) - \mathcal{L}_\chi(M_{PV}). \tag{4.13}$$

As discussed in I, the quadratically divergent terms may be made invariant by constraints on the PV mass parameters. In this paper we consider only anomalies arising from logarithmic divergences and the associated chiral anomalies. As a first step toward the construction of a modular invariant one-loop effective Lagrangian, we give examples below of regularization prescriptions with modular covariant PV couplings except in the PV mass terms. In addition we choose the mass terms such that the renormalization of the Kähler potential is modular invariant.

4.1 No-scale supergravity

First we consider a toy "superstring-inspired" model [22] with a single modulus T; the Kähler potential and superpotential given by

$$K = k + G, \quad G = -3\ln\left(T + \bar{T} - \sum_{p=1}^{N-2}|\Phi^p|^2\right), \quad W = d_{pqr}\Phi^p\Phi^q\Phi^r. \tag{4.14}$$

The modular transformations are defined by

$$\begin{aligned} T &\to T' = \frac{aT - ib}{icT + d}, \quad S \to S' = S, \quad ad - bc = 1, \\ \Phi^p &\to \Phi'^p = e^{-F/3}\Phi^p, \quad F = 3\ln(icT + d), \end{aligned} \tag{4.15}$$

To construct a modular invariant PV Kähler potential and a modular covariant superpotential W_2, we note that if the PV Kähler potential and superpotential of (3.3) and (3.4) are modified by the additional terms

$$\begin{aligned} K_{PV} &= K_{PV}^{(3.3)} + \sum_\alpha\left[\rho_\alpha\sum_{I=i}K_i Z_\alpha^I Z_\alpha^0 + \frac{1}{2}\rho'_\alpha\left(Z_\alpha^0\right)^2 + \text{h.c.}\right] \\ W_2 &= W_2^{(3.4)} + \sum_\alpha\left[\rho_\alpha\sum_{I=i}W_i Z_\alpha^I Z_\alpha^0 - \frac{1}{2}\rho'_\alpha\left(Z_\alpha^0\right)^2 W\right]. \end{aligned} \tag{4.16}$$

the one-loop corrections are unchanged:

$$A_{I0}^{\widetilde{Z}} = R_{\bar{n}I0\bar{m}}^{\widetilde{Z}} = A_{00}^{\widetilde{Z}} = R_{\bar{n}00\bar{m}}^{\widetilde{Z}} = 0. \tag{4.17}$$

For the Kähler potential (4.14) we have, for $Z^i = T, \Phi^q$

$$K_i = G_i, \quad \partial_i\partial_j G = \frac{1}{3}G_iG_j, \quad K_{IJ} = -\frac{2}{3}G_iG_j, \tag{4.18}$$

and, under (4.15),

$$\begin{aligned} K'_i &= \frac{\partial K(Z')}{\partial Z'^i} = N_i^j\left(K_j + F_j\right), \quad K'_{i\bar{m}} = N_i^k N_{\bar{m}}^{\bar{n}} K_{k\bar{n}}, \\ K'_{IJ} &= N_i^n N_j^m \left[K_{NM} - \frac{2}{3}\left(F_n K_m + F_m K_n + F_m F_n\right)\right]. \end{aligned} \tag{4.19}$$

In addition,

$$\begin{aligned} W'_i &= e^{-F} N_i^j \left(W_j - F_j W\right), \\ W'_{ij} &= \frac{\partial^2 W(Z')}{\partial Z'^i \partial Z'^j} = N_i^k \partial_k \left[N_j^m e^{-F}\left(W_m - F_m W\right)\right] \\ &= e^{-F} N_i^k N_j^m [W_{km} - F_k W_m - F_m W_k - \left(F_{km} - F_k F_m\right) W \\ &\quad -N_n^l \left(W_l - F_l\right) \partial_k M_m^n \end{aligned} \tag{4.20}$$

Writing the transformation (4.15) in the form

$$\begin{aligned} Z &= \begin{pmatrix} \Phi^p \\ T \end{pmatrix} \to Z'(Z), \quad M_j^i = \frac{\partial Z'^i}{\partial Z^j}, \quad N_i^j = \frac{\partial Z'^j}{\partial Z^i}, \\ M &= \begin{pmatrix} e^{-F/3}\delta_q^p & -\frac{1}{3}F_t e^{-F/3}\Phi^q \\ 0 & e^{-2F/3} \end{pmatrix}, \quad N = \begin{pmatrix} e^{F/3}\delta_q^p & \frac{1}{3}F_t e^{2F/3}\Phi^q \\ 0 & e^{2F/3} \end{pmatrix}, \end{aligned} \tag{4.21}$$

and using

$$W_p \Phi^p = 3W, \quad F_{ij} = -\frac{1}{3}F_iF_j, \tag{4.22}$$

we obtain

$$W'_{ij} = N_i^n N_j^m \left[W_{mn} - \frac{2}{3}\left(F_n W_m + F_m W_n - F_m F_n W\right)\right]. \tag{4.23}$$

If we also modify the Kähler metric for $\widetilde{Z}$ to read

$$K^{\widetilde{Z}}_{I\bar{J}} = K_{i\bar{j}} + a^2 G_i G_{\bar{j}}, \quad K^{\widetilde{Z}}_{I\bar{0}} = aG_i, \quad K^{\widetilde{Z}}_{0\bar{J}} = aG_{\bar{j}}, \tag{4.24}$$

the metric for $\widetilde{Z}$ is just the inverse of that for $\widehat{Y}$ (see Appendix A of I), *i.e.* its inverse is given by

$$K^{I\bar{J}}_{\widetilde{Z}} = K^{i\bar{j}}, \quad K^{0\bar{J}}_{\widetilde{Z}} = -aG^{\bar{j}}, \quad K^{0\bar{0}}_{\widetilde{Z}} = 1 + a^2 G^i G_i. \tag{4.25}$$

Because of (4.17), there is no additional contribution to L_3^Z or K', but there is now a contribution $L_1^{\widetilde{Z}}$ similar to $L_1^{\widehat{Y}}$. We can incorporate this contribution if we change the values of $a_{\widetilde{Y}}$ and $a'_{\widetilde{Y}}$ and the parameters in $W_2(\widehat{Y})$. Moreover, nothing is changed if we substitute $W_2(\widehat{Y},\widehat{Z}) \to W_2(\widehat{Y},\widetilde{Z})$. Finally, because of the property (4.18), the derivatives of the Kähler metric $G_{i\bar{m}}$ satisfy:

$$\begin{aligned} \Gamma_j^{i\alpha}\Gamma_{i\alpha}^j &= \frac{1}{9}\left[(N+1)\,G^\alpha G_\alpha + G_j^{i\alpha}G_{i\alpha}^j\right], \quad \Gamma_{i\alpha}^i = \frac{N}{3}G_\alpha, \\ G_\alpha &= G_{i\alpha}^i, \quad G_{i\alpha}^j = -\frac{1}{8}\left(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R\right)\left(G_i\mathcal{D}_\alpha Z^j\right), \\ \Gamma_{i\alpha}^j(T^a)_j^i &= \frac{1}{3}G_\alpha^a, \quad G_\alpha^a = -\frac{1}{8}\left(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R\right)\left[G_i\mathcal{D}_\alpha(T^aZ)^i\right], \end{aligned} \tag{4.26}$$

while the derivatives of the metrics for ϕ^C with $\beta_C = -1$, and for $\widehat{Y}_{P\neq S}$, satisfy [see (2.41) of I]

$$\begin{aligned} &\Gamma_D^{C\alpha}\Gamma_{C\alpha}^D = \alpha_C^2 G^\alpha G_\alpha, \quad \Gamma_{D\alpha}^C = \alpha_C G_\alpha \delta_D^C, \\ &\Gamma_{P\alpha}^Q (T^a)_Q^P = \Gamma_{i\alpha}^j (T^a)_j^i + a^2 G_\alpha^a, \quad \Gamma_{P\alpha}^P = -\Gamma_{i\alpha}^i, \\ &(\Gamma)_Q^{P\alpha}(\Gamma)_{P\alpha}^Q = \Gamma_j^{i\alpha}\Gamma_{i\alpha}^j + \left(\frac{2}{3}a^2 + a^4\right)\left(G^\alpha G_\alpha + G_j^{i\alpha}G_{i\alpha}^j\right) - 2\left(\frac{1}{3}a^2 + a^4\right)G_{i\alpha}Z_\alpha^i, \\ &G_{i\alpha} = -\frac{1}{8}\left(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R\right)G_i\mathcal{D}_\alpha G, \quad Z_\alpha^i = -\frac{1}{8}\left(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R\right)\mathcal{D}_\alpha Z^i. \end{aligned} \tag{4.27}$$

Therefore, since in addition $k_{s\bar{s}} = e^{2k}$, the contribution of fields with metric $K_{i\bar{m}}$ can be canceled by an appropriate combination of $\widehat{Y}, \widetilde{Z}, \phi$, provided some ϕ are gauge-charged.

As a consequence of the above, the ultraviolet divergences are still canceled if we modify (3.3), (3.4), (3.6) and (3.7) to read [note that $k^ik_i = 1$, and $G^iG_i = 3$ is invariant under the modular transformations (4.15)]

$$\begin{aligned} K_{PV} &= \sum_\gamma \left[e^{\alpha_\gamma^\phi K + \beta_\gamma^\phi k}\phi^\gamma\bar\phi_\gamma + \frac{1}{2}\nu_\gamma(\theta_\gamma + \bar\theta_\gamma)^2\right] + e^{K/2}\sum_A\left(|X_\gamma^A|^2 + |U_A^\gamma|^2 + |V_\gamma^A|^2\right) \\ &+ \sum_\alpha\left[\sum_a\left(e^G\varphi_\alpha^a\bar\varphi_a^\alpha + e^k\hat\varphi_\alpha^a\bar{\hat\varphi}_a^\alpha + \tilde\varphi_\alpha^a\bar{\tilde\varphi}_a^\alpha\right) + e^K\sum_{r=1}^3 e^{\beta_\alpha^r k}|\phi_\alpha^r|^2\right] \\ &+ \sum_\alpha\left[e^{-2k}|\phi_S^\alpha|^2 + 2|\phi_0^\alpha|^2 - e^{-k}\left(\bar\phi_S^\alpha\phi_0^\alpha + \text{h.c.}\right) + e^{2k}|\phi_\alpha^S|^2\right] \\ &+ \sum_\alpha\left[\sum_{I\neq S}\left(e^{G/3}|\phi_\alpha^I|^2 + e^{K/2}|\phi_I^\alpha|^2\right) + e^{G/3}|\phi_\alpha^0|^2 + K_\alpha^Z + K_\alpha^Y\right] \end{aligned}$$

$$
\begin{aligned}
K^Z_\alpha &= e^{\alpha^Z G}\Bigg\{ \sum_{I,J=i,j}\left[Z^I_\alpha \bar{Z}^{\bar{J}}_\alpha\left(G_{i\bar{j}}+a^2_\alpha G_i G_{\bar{j}}\right)-\frac{b}{3}\left(G_iG_jZ^I_\alpha Z^J_\alpha+\text{h.c.}\right)\right]+|Z^0_\alpha|^2 \\
&\quad -\left[\sum_{I=i} G_i Z^I_\alpha\left(\frac{2b}{3a_\alpha}Z^0_\alpha - a_\alpha \bar{Z}^0_\alpha\right)+\frac{b}{3a^2_\alpha}\left(Z^0_\alpha\right)^2+\text{h.c.}\right]\Bigg\}, \\
K^Y_\alpha &= e^{\alpha^Y G}\left[\sum_{I,J=i,j} G^{i\bar{j}} Y^\alpha_I \bar{Y}^\alpha_{\bar{J}} - a_\alpha \sum_{I=i}\left(Y^\alpha_I \bar{Y}^0_\alpha G^i+\text{h.c.}\right)+|Y^\alpha_0|^2\left(1+3a^2_\alpha\right)\right], \\
\tilde{b} &= 1, \quad \hat{b}=0, \quad \alpha^{\widetilde{Z}}=\alpha^{\widehat{Y}}=0, \quad \alpha^{\widehat{Z}}=\alpha^{\widetilde{Y}}=1, \qquad (4.28) \\
W_1 &= \sum_\alpha\left[\sum_{P\neq S}\left(\tilde{\mu}_\alpha \widetilde{Z}^P_\alpha \widetilde{Y}^\alpha_P+\hat{\mu}_\alpha \widehat{Z}^P_\alpha \widehat{Y}^\alpha_P\right)+\sum_{I\neq S}\mu^\phi_\alpha\left(\phi^I_\alpha\phi^\alpha_I+\frac{1}{2}(\phi^0_\alpha)^2\right)\right] \\
&\quad +\sum_\alpha\left[\sum_a\left(\mu^\varphi_\alpha \varphi^a_\alpha \hat{\varphi}^a_\alpha+\frac{1}{2}\mu^{\tilde{\varphi}}_\alpha \tilde{\varphi}^a_\alpha \tilde{\varphi}^a_\alpha\right)+\sum_r \mu^S_\alpha \phi^r_\alpha \hat{\phi}^\alpha_r\right] \\
&\quad +\sum_{A\gamma}\left(\mu^X_\gamma U^\gamma_A X^A_\gamma+\frac{1}{2}\mu^V_\gamma (V^\gamma_A)^2\right)+\frac{1}{2}\sum_{C,D}\mu^\phi_{CD}\phi^C\phi^D, \qquad (4.29) \\
W_2 &= \sum_\alpha\left\{\frac{1}{2}W_{ij}\widetilde{Z}^I_\alpha \widetilde{Z}^J_\alpha+\frac{1}{3a^2_\alpha}\left(\widetilde{Z}^0_\alpha\right)^2 W-\frac{2}{3a_\alpha}W_i \widetilde{Z}^I_\alpha \widetilde{Z}^0_\alpha+2\sum_a \varphi^a_\alpha \widehat{Y}^\alpha_I (T_a Z)^i\right\} \\
&\quad +\sqrt{2}\sum_{\alpha>1}\widetilde{Z}^I_\alpha\left(\widehat{Y}^\alpha_I W+\hat{a}_\alpha W_i \widehat{Y}^\alpha_0\right)+\sum_\alpha c_\alpha \phi^S_\alpha \phi^\alpha_S W, \qquad (4.30) \\
f^{ab} &= \delta^{ab}\left(s+\sum_\alpha h_\alpha \phi^S_\alpha \phi^\alpha_0\right), \quad f^{a\alpha}_s = 0, \\
f^0_{\alpha\beta} &= \delta_{\alpha\beta}, \quad f^s_{\alpha\beta}=\delta_{\alpha\beta}S, \quad f^{a\alpha}_0=\sum_\beta e^{\alpha\beta}\hat{\varphi}^a_\beta, \qquad (4.31) \\
\tilde{a} &= -\frac{1}{6}, \quad \tilde{a}'=\frac{1}{18}, \quad \hat{a}=-\hat{a}'=-1, \quad \hat{a}_1=0, \\
h &= 2, \quad e=-4=3e', \quad c=5, \quad w=1, \qquad (4.32) \\
\sum_\alpha \eta^\varphi_\alpha &= \sum_\alpha \eta^{\hat{\varphi}}_\alpha=\sum_\alpha \eta^{\tilde{\varphi}}_\alpha=\sum_\alpha \eta^{\widehat{Z}}_\alpha=-\sum_\alpha \eta^{\widetilde{Z}}_\alpha=-\sum_\alpha \eta^r_\alpha=+1, \\
\eta^\varphi_\alpha &= \eta^{\widehat{Z}}_\alpha=\eta^{\widehat{Y}}_\alpha, \quad \eta^{\hat{\varphi}}_\alpha=\eta^\varphi_\alpha, \quad \eta^r_\alpha=\eta^{\phi^{I,0}}_\alpha=\eta^{\phi_{I,0}}_\alpha, \quad \eta^U_\alpha=\eta^X_\alpha, \\
\eta^{\widetilde{Y}}_\alpha &= \eta^{\widetilde{Z}}_\alpha, \quad \eta^{\widetilde{Z}}_{\alpha+1}=\eta^{\widehat{Y}}_{\alpha+1}, \quad \eta^{\widetilde{Z}}_1=-\eta^{\widehat{Y}}_1=-1, \\
\sum_\gamma \eta^0_\gamma &= -12, \quad \sum_\gamma \eta^s_\gamma=-N_G, \quad \sum_\gamma \eta^\theta_\gamma=-12-N_G=N'_G, \qquad (4.33)
\end{aligned}
$$

where ϕ^I, ϕ_I transform like Z^I, Y_I, respectively, under the gauge group, $\hat{\phi}^r = \phi^S, \phi_S, \phi_0$, and in W_1 the sum over ϕ^C includes $\phi_{P=I,0}$ but not $\phi^{P=I,0}$, and a, a' are defined as in (3.6).

The metric derivatives for $\widetilde{Z}$ are the same as for $\widehat{Y}$ in (4.27) except that $(\Gamma^Q_{P\alpha})_Z = -(\Gamma^P_{Q\alpha})_Y$, and the derivatives for $X' = \widetilde{Y}, \widehat{Z}$ are related to those for $X = \widetilde{Z}, \widehat{Y}$ by

$$
\begin{aligned}
&(\Gamma_{X'})^P_{P\alpha} = -(\Gamma_X)^P_{P\alpha} + NG_\alpha, \quad (\Gamma_{X'})^Q_{P\alpha}(T_a)^P_Q = (\Gamma_X)^Q_{P\alpha}(T_a)^P_Q \\
&(\Gamma_{X'})^{P\alpha}_Q(\Gamma_{X'})^Q_{P\alpha} = (\Gamma_X)^{P\alpha}_Q(\Gamma_X)^Q_{P\alpha} + N\left(1 \pm \frac{2}{3}\right)G^\alpha G_\alpha.
\end{aligned} \tag{4.34}
$$

Since $\widehat{Z}$ and $\widetilde{Y}$ have opposite signature, the additional surviving contributions are equivalent to that of a set ϕ^C with $\alpha = \beta = 0$, $\alpha' = \beta' = -\sigma = -4N/3$. To cancel this contribution we must modify (3.8) and (3.14) to read

$$
f = -N_G, \quad 4 + \alpha' = \beta' = -\sigma = +4N/3, \tag{4.35}
$$

where in the sums defining these quantities

$$
\phi^C = \phi_\gamma, X^A_\gamma, U^\gamma_A, V^A_\gamma, \phi^\gamma_P, \Phi^a_\gamma. \tag{4.36}
$$

In other words [see (C.2) of Appendix C],

$$
\begin{aligned}
\sum_I L_2^{\phi^I} &= -L_2, \quad L_2^Y + L_2^Z = 0, \\
\sum_I L_1^{\phi^I} + L_1^Y + L_1^Z &= -L_1 - \frac{4N}{3}\left(L_\alpha + L_\beta + L'_\beta\right).
\end{aligned} \tag{4.37}
$$

Since $(T_a Z)^i F_i = 0$, K_{PV} is invariant and W_2 is covariant ($W_2 \to e^{-F} W_2$) under (4.15) provided the PV chiral multiplets transform as

$$
\begin{aligned}
\phi'^C &= e^{-\alpha_C F}\phi^C, \quad Y'^\alpha_I = e^{-\alpha^Y F} N^j_i \left(Y^\alpha_J + a_\alpha F_j Y_0\right), \quad Y'_0 = e^{-\alpha^Y F} Y_0 \\
Z'^J_\alpha &= e^{-\alpha^Z F} M^j_i Z^I_\alpha, \quad Z'^0_\alpha = e^{-\alpha^Z F}\left(Z^0 - a_\alpha Z^I_\alpha F_i\right),
\end{aligned} \tag{4.38}
$$

with all other PV superfields invariant. Note that we have chosen W_1 such that all masses are covariant for fields that appear in the gauge kinetic functions f^{AB}. Provided each ϕ^C appears in only one term in W_1 [*i.e.* $\mu_{CC'}\phi^C\phi^{C'}$ or $\frac{1}{2}\mu_C(\phi^C)^2$], the squared-mass matrix defined in (4.5) is block diagonal Thus, for example, if we include a modular covariant T-dependence in the mass terms for some $\phi^P \neq Z, Y$, we have

$$
\begin{aligned}
(M^2_{\widehat{Z}})^P_Q &= (M^2_{\widetilde{Y}})^Q_P = \tilde{\mu}^2\delta^P_Q, \quad (M^2_{\widetilde{Z}})^P_Q = (M^2_{\widehat{Y}})^Q_P = \hat{\mu}^2\delta^P_Q, \\
M^2_{\phi^P} &= M^2_{\phi^{P'}} = \tilde{\mu}^2_{PP'}|\eta(it)|^{4b_P} e^{K(1-\alpha_P-\alpha_{P'})-k(\beta_P+\beta_{P'})},
\end{aligned} \tag{4.39}
$$

where $\eta(it)$ is the Dedekind function:

$$\eta(iT'^i) = e^{\frac{1}{2}F^i}\eta(iT^i),$$

and

$$\widetilde{M}^2_{\phi^P} = M'^2_{\phi^P} = e^{(F+\bar{F})(1-\alpha_P-\alpha_{P'}+b_P)}M^2_{\phi^P}, \quad \phi^P \neq Z, Y, \tag{4.40}$$

with $\widetilde{M}^2 = M^2$ otherwise. The $\eta(it)$ factor can be interpreted as a parameterization of string loop threshold corrections, as mentioned in Section 4.1. We now turn to a more realistic model from string theory.

4.2 The untwisted sector of orbifold compactifications

Consider next the classical Lagrangian for the untwisted sector of orbifold compactifications with three untwisted moduli. It is defined by the Kähler potential and the superpotential[10]

$$\begin{aligned} K &= k + G, \quad G = G^u, \quad W = d_{abc}\Phi^{a1}\Phi^{b2}\Phi^{c3}, \\ G^u &= \sum_{i=1}^{3} G^{(i)}, \quad G^{(i)} = -\ln\left(T^i + \bar{T}^{\bar{\imath}} - \sum_{a=1}^{n}|\Phi^{ai}|^2\right), \end{aligned} \tag{4.41}$$

Setting $Z^p = \{T^i, \Phi^{(ai)}\}$, we now have the properties

$$\begin{aligned} \partial_p\partial_q G &= \delta_{ij}G^{(i)}_p G^{(i)}_q, \quad K_{pq} = -\sum_{i\neq j} G_p G_q, \\ \sum_p W_p G^p &= 0, \quad \sum_a W_{(ai)}\Phi^{(ai)} = W. \end{aligned} \tag{4.42}$$

The Lagrangian is invariant under modular transformations:

$$\begin{aligned} G &\to G' = F + \bar{F}, \quad F = \sum_i F^i, \quad F^i = \ln(icT^i + d), \\ Z &= \begin{pmatrix} \Phi^p \\ T \end{pmatrix} \to Z'(Z), \quad M^p_q = \frac{\partial Z'^p}{\partial Z^q}, \quad N^q_p = \frac{\partial Z'^q}{\partial Z^p}, \\ M &= \delta^i_j \begin{pmatrix} e^{-F^i}\delta^a_b & -F_i e^{-F^i}\Phi^{(ai)} \\ 0 & e^{-2F^i} \end{pmatrix}, \quad N = \delta^i_j \begin{pmatrix} e^{F^i}\delta^a_b & F_i e^{2F^i}\Phi^{(ai)} \\ 0 & e^{2F^i} \end{pmatrix}, \\ F_i &\equiv F_{t^i} = F^i_{t^i}, \quad F_{ij} = -\delta_{ij}F_i^2. \end{aligned} \tag{4.43}$$

[10]It is straightforward, but slightly more cumbersome, to generalize the results to the case of a Kähler potential as in (4.41) with $n \to n_i$, $n_i \neq n_j$.

Properties analogous to (4.19), (4.20), (4.26) and (4.27) are given in Appendix D. In analogy with the discussion of the preceding section, we introduce PV superfields $Z^{(0I)}, Y_{(0I)}$, and modify $K^{Y,Z}$ and $W_2(Z)$ in (4.28) and (4.30) to read [here we suppress the index α, and now $G^p_{(i)}G^{(i)}_p = 1$ for fixed i is invariant under (4.43)]

$$\begin{aligned}
K^Z &= e^{\alpha^Z G}\sum_{I=1}^{3} K^Z_I - \frac{b_Z}{2}\sum_{I\neq J}\left(G^I_Z G^J_Z + \text{h.c.}\right), \quad K^Y = e^{\alpha^Y G}\sum_{I=1}^{3} K^Y_I, \\
K^Z_I &= \sum_{P\bar{M}} Z^P \bar{Z}^{\bar{M}} G^{(i)}_{p\bar{m}} + |aG^I_Z|^2, \quad G^I_Z = \sum_P Z^P G^{(i)}_p + a^{-1}Z^{(0I)}, \\
K^Y_I &= \left[\sum_{P\bar{M}} Y_P \bar{Y}_{\bar{M}} G^{p\bar{m}}_{(i)} - a\sum_P\left(Y_P \bar{Y}_{(\bar{0}I)} G^p_{(i)} + \text{h.c.}\right) + Y_{(0I)}\bar{Y}_{(\bar{0}I)}\left(1+a^2\right)\right], \\
b_{\widetilde{Z}} &= -2\tilde{a} = 2\tilde{a}' = -\hat{a} = \hat{a}' = 1, \quad b_{\widehat{Z}} = 0, \qquad (4.44) \\
W_2(\widetilde{Z}) &= \frac{1}{2}W_{pq}\widetilde{Z}^P\widetilde{Z}^Q - a^{-1}\sum_{i\neq j}\widetilde{Z}^{0J}\left(W_{(ai)}\widetilde{Z}^{(AI)} - \frac{1}{2}a^{-1}W\widetilde{Z}^{(0I)}\right) \\
&\quad + \sqrt{2}\sum_{P,\alpha>1}\widetilde{Z}^P\left(aW_p\widehat{Y}_0 + \widehat{Y}_P W\right). \qquad (4.45)
\end{aligned}$$

In addition we replace the fields $\phi^{I,0}, \phi_I,\ I \neq S$ by $\phi^{(PI),(P0)}, \phi_{(0I)},\ P \neq S$, with Kähler potential

$$K^\phi = \sum_i\left[e^{G^{(i)}}\left(\sum_P|\phi^{(PI)}|^2 + |\phi^{(0I)}|^2\right) + e^{K/2}\sum_P|\phi_{(PI)}|^2\right]. \qquad (4.46)$$

The mass terms for theses fields are determined by W_1 in (4.29) with

$$Z^P, Y_P \to T^I, Z^{(AI)}, T_I, Y_{(AI)}, \quad Z^0, Y_0 \to Z^{(0I)}, Y_{(0I)},$$

$$\phi^{I,0}, \phi_I,\ I \neq S, \ \to \phi^{(PI),(0I)}, \phi_{(PI)},\ P \neq S,$$

and the sum over C in the definitions of α, α' now includes $\phi_{(PI)}$. The Kähler potential is invariant and W_2 is covariant under modular transformations provided

$$\begin{aligned}
\phi'^C &= e^{-\alpha_C F}\phi^C, \quad Y'_{P=T_I,(AI)} = e^{-\alpha^Y F}N^q_p\left(Y_Q + aF^i_q Y_{(0I)}\right), \\
Z'^Q &= e^{-\alpha^Z F}M^q_p Z^P, \quad Z'^{(0I)} = e^{-\alpha^Z F}\left(Z^{(0I)} - aZ^P F^i_p\right), \\
Y'_{(0I)} &= e^{-\alpha^Y F}Y_{(0I)}, \quad \phi'^{(NI)} = e^{-F^i}\phi^{(NI)}, \quad N = P, 0. \qquad (4.47)
\end{aligned}$$

The renormalization of the Kähler potential (3.16) arises from $Z, Y, \varphi, \theta, \phi^S, \phi_S$, contributions and is modular invariant, since we have chosen the PV couplings such that their masses are covariant. Writing, for $\phi^P \neq Z, Y$,

$$\begin{aligned} K(\phi^P, \bar{\phi}^P) &= e^{G_P+\beta_P k}|\phi^P|^2, \quad G_P = \sum_i \alpha^i_P G^{(i)}, \\ W_1(\phi^P, \phi^{P'}) &= \mu_P \prod_i [\eta(it^i)]^{2b^i_P} \phi^P \phi^{P'}, \end{aligned} \tag{4.48}$$

we have

$$M^2_{\phi^P} = M^2_{\phi^{P'}} = \mu^2_P \prod_i |\eta(it^i)|^{4b^i_P} e^{K-G_P-G_{P'}-k(\beta_P+\beta_{P'})}, \tag{4.49}$$

and

$$\widetilde{M}^2_{\phi^P} = M'^2_{\phi^P} = e^{\sum_i (F^{(i)}+\bar{F}^{(i)})(1-\alpha^i_P-\alpha^i_{P'}+b^i_P)} M^2_{\phi^P}, \quad \phi^P \neq Z, Y. \tag{4.50}$$

4.3 Including the twisted sector

The Kähler potential for orbifolds is not known beyond leading (quadratic) order in the fields $Z^a \neq S, T$, except for the untwisted sector, whose Kähler potential (4.41) is determined by the metric on the compact space. As a consequence, we cannot determine the one-loop effective action for the twisted sector, but we can include twisted sector loop contributions to the untwisted sector action, provided the superpotential contains no terms quadratic in the twisted sector fields. The general modular invariant superpotential[11]

$$W = \sum_\alpha w_\alpha \prod_{j=1}^3 \eta^{-2}(T^j) \prod_a \left[Z^a \prod_{i=1}^3 \eta^{2q^a_i}(T^i) \right], \tag{4.51}$$

depends on the moduli through the Dedekind η-function, interpreted as arising from string world-sheet instanton effects. In the absence of these effects, which we neglect here, there is no superpotential for twisted sector fields. We will set background twisted sector fields to zero, and include only quantum corrections due to the (modular invariant) quadratic term in $Z^a \neq S, T^i, \Phi^{ia}$ in the superpotential:

$$\begin{aligned} K &= k + G^u + \sum_a e^{g^a}|Z^a|^2, \\ g^a &= -\sum_i q^i_a \ln(T^i + \bar{T}^{\bar{i}}) + f^a[|\Phi^{bi}|^2/(T^i + \bar{T}^{\bar{i}})] \end{aligned} \tag{4.52}$$

[11]There can be additional factors which are holomorphic, modular invariant functions of the moduli.

If K depends on the moduli only through the compact radii, we have

$$g^a = G^a = \sum_i q_a^i G^i, \quad f_a = -\sum_i q_a^i \ln[1 - \sum_b |\Phi^{bi}|^2/(T^i + \bar{T}^{\bar{i}})]. \tag{4.53}$$

Under a modular transformation

$$Z'^a = e^{-F^a} Z^a, \quad F^a = \sum_i q_a^i F^i. \tag{4.54}$$

To regulate the twisted sector contribution, we introduce negative signature PV fields Φ^A, Φ_A that transform under the gauge group like Φ^a and its conjugate, respectively, with Kähler potential and superpotential

$$K_{PV}^T = \sum_A \left(e^{g^a}|\Phi^A|^2 + e^{K/2}|\Phi_A|^2\right), \quad W_1^T = \sum_A \prod_i |\eta(it^i)|^{2b_A^i} \mu_A \Phi^A \Phi_A. \tag{4.55}$$

Under (4.3) we have

$$\begin{aligned} \Phi^A &\to e^{-F^a}\Phi^A, \quad \Phi_A \to e^{-\frac{1}{2}F}\Phi_A, \\ (\widetilde{M}^A)^2 &= \widetilde{M}_A^2 = e^{\sum_i (F^{(i)} + \bar{F}^{(i)})(\frac{1}{2} - q_a^i + b_A^i)} M_A^2. \end{aligned} \tag{4.56}$$

Combining this with (4.49), (4.50), the one-loop Yang-Mills Lagrangian (4.8) takes the form[12]

$$\mathcal{L}_1 \ni \frac{1}{64\pi^2} \sum_a F_a^{\mu\nu} F_{\mu\nu}^a \left[\sum_P \eta^P C_a^P b_P^i \ln|\eta(it^i)|^4 - \sum_p C_p^a \left(K - 2\sum_i q_p^i g^i\right) + C^a K\right], \tag{4.57}$$

where the sum over P now included the twisted sector fields. The first term in (4.57) correctly reproduces the threshold effects (neglecting the universal, modular invariant term [21]) provided

$$b_a^i = \sum_P \eta^P C_a^P b_P^i = C_{E_8} + \sum_p C_p^a \left(1 - 2q_p^i\right) - C^a. \tag{4.58}$$

Then the variation in (4.57) is cancelled by the variation in the classical Yang-Mills Lagrangian due to the transformation property (4.9) of the dilaton.

[12]There are additional dilaton-dependent terms (formally of two-loop order) if the gauge charged fields couple to the GS term (refgsterm).

5 Anomalous $U(1)$

The modifications needed for regulating one-loop supergravity in the presence of an anomalous $U(1)$ gauge group $\mathcal{G}_X$ are described in detail in I. The light matter loops generate a quadratically divergent term proportional to $2x^{-1}D_X\mathrm{Tr}T_X$ and logarithmic divergences proportional to $\mathrm{Tr}T_X$ associated with the operators $\Phi_{1,2}$ in (2.17). To regulate these terms we must introduce PV chiral multiplets ϕ^P with superpotential terms that are not invariant under $U(1)_X$. As discussed in I, in order for the superpotential to remain holomorphic under a $U(1)_X$ gauge transformation, we require the transformation properties

$$\begin{aligned} \mathcal{A}^X_M &\to \mathcal{A}^X_M - g^{-1}D_M g, \quad V_X \to V'_X = V_X + \frac{1}{2}\left(\Lambda + \bar{\Lambda}\right), \\ Z^i &\to g^{q^i_X} Z^i, \quad Z^I \to g^{-q^i_X\Lambda} Z^I, \quad g = (g^\dagger)^{-1} = e^{\frac{1}{2}(\bar{\Lambda}-\Lambda)}. \end{aligned} \tag{5.1}$$

The chiral Yang-Mills superfield W^α is obtained as a component [9] of the two-form $\mathcal{F}_{MN}$, which is the super-curl of the Yang-Mills one-form potential $\mathcal{A}_M$, and is also the chiral projection of the commonly used Yang-Mills superfield potential V_X: $W_\alpha = -\frac{1}{4}\left(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R\right)\mathcal{D}^\alpha V_X$. While the light fields are defined to be covariantly chiral [9] under $U(1)_X$, the $U(1)_X$-charged PV fields are covariantly chiral only with respect to the nonanomalous gauge group; their invariant superpotential takes the form

$$K_{PV}(|\phi^P|^2) = e^{g^P(Z)+2q^P_X V_X}|\phi^P|^2. \tag{5.2}$$

5.1 General supergravity

If we assume that the $U(1)_X$ generator commutes with the Kähler metric in the general supergravity model of Section 2, we can simply assign zero $U(1)_X$, charge to $X_\gamma, U_\gamma, V_\gamma$, and to $\widehat{Y}^\alpha_I$ for a set of values $\alpha = \alpha_0$ with $\sum_\alpha \eta_{\alpha_0} = -1$. $U(1)_X$ gauge invariance of K_{PV} and W_2 as defined in Eq. (3.4) requires $a_{\alpha_0} = g_{\alpha_0}$. We must also remove $\widehat{Y}^{\alpha_0}_I, \widehat{Z}^I_{\alpha_0}$ as well as a pair with $\alpha \neq \alpha_0$ and net positive signature from the second term in W_2, Eq. (3.4). With this choice the linear divergences associated with the $U(1)_X$ anomaly are canceled. The chiral anomaly reappears due to the noninvariance of the mass terms coupling the $\widehat{Y}^{\alpha_0}$ to fields $\widehat{Z}^I_{\alpha_0}$ with the same $U(1)_X$ charge as Z^i, and forms a supersymmetric F-term with the chiral anomaly. Note that the renormalization of the Kähler potential is $U(1)_X$ invariant in this general case, since $\widehat{Y}_{\alpha_0}, \widehat{Z}^I_{\alpha_0}$ do not appear in W_2.

5.2 Orbifold compactification

In this case we cannot impose the condition that the Kähler metric commutes with the $U(1)_X$ generator, but with an appropriate choice of PV $U(1)_X$ charges and superpotential, the $U(1)_X$ generator does commute with the Kähler metric for PV fields with PV masses that are not $U(1)_X$ covariant. For the untwisted sector of the orbifold model of Sections 4.2-3, we have

$$\begin{aligned} \Gamma^{(pi)}_{(qi)\alpha}(T_X)^{(pi)}_{(qi)} &= G^{(i)}_\alpha(T_X)^{(pi)}_{(pi)} + \mathcal{G}^X_\alpha = \Gamma^{(PI)}_{(QI)\alpha}(T_X)^{(PI)}_{(QI)} + \mathcal{G}^X_\alpha , \\ \Gamma^b_{a\alpha}(T_X)^a_b &= g^a_\alpha(T_X)^a_a = \Gamma^B_{A\alpha}(T_X)^A_B. \end{aligned} \tag{5.3}$$

The contribution from $\mathcal{G}^X_\alpha$, which is defined in (4.26), is canceled as before provided $q_X^{Z(AI)} = -q_X^{Y(AI)} = q_X^{Z(ai)}$ and to cancel the new contributions, we assign $U(1)_X$ charge to $\phi^{(AI)}$: $q_X^{(AI)} = q_X^{(aI)}$ and to ϕ^C: $q_X = \sum_C \eta_C \alpha_C q^C_X = -2$, $\sum_C \eta_C \beta_C q^C_X = \sum_C \eta_C q^C_X = 0$, where q_X is chosen to cancel the contribution from the last term in (D.4) of Appendix 4. We also require $q^A_X = q^a_X$ for the PV regulator fields for the twisted sector. With these choices, the renormalization of the Kähler potential is $U(1)_X$ invariant.

6 Summary of results

We have shown that it is possible to regulate supergravity at one loop by introducing Pauli-Villars fields in chiral multiplets and Abelian gauge multiplets. For calculational simplicity, we restricted the dilaton couplings to those of the classical limit of supergravity derived from the heterotic string, but there is no impediment in principle to extending our results to the more general case. In the context of string theory, this generalization is required, for example, when nonperturbative string effects and/or GS terms are included in the effective "tree" Lagrangian. It would also be useful to know the full one-loop corrections in the linear multiplet formulation. However, certain one loop-effects such as the soft supersymmetry breaking terms and the anomalous contributions to the Yang-Mills kinetic term, depend only on gauged-charged matter and Yang-Mills loops. In this case, with the dilaton appearing only as a background field, it is fairly straightforward [13, 14] to include the above-mentioned terms, and to generalize the results to the linear multiplet formulation for the dilaton. The A-terms for general supergravity without a GS term were calculated in Section 3, and were found to be very sensitive to the details of the precise choice of the Pauli-Villars couplings, which in turn can be determined only with a detailed understanding of Planck-scale physics.

String-derived supergravity is anomalous at the quantum level under perturbatively exact symmetries such as T-duality and $U(1)_X$ of the underlying string theory. When appropriate Green-Schwarz terms are included, the effective field theory should be invariant, up to nonperturbative string effects, at the quantum level. One could, for example, make the regulated tree Lagrangian fully modular invariant by including appropriate factors of the modular covariant Dedekind function $\eta(iT)$ in the PV mass term W_1. These would be interpreted as threshold corrections from heavy string and Kaluza-Klein modes. However, string-loop calculations show that at least a part of the modular anomaly is canceled by a GS term; in particular, for orbifolds like Z_3 and Z_7 with no $N=2$ supersymmetric twisted sector, there are no (modular noninvariant) threshold corrections [24] to the gauge kinetic term: $b^i_a = 0$ in (4.58). Moreover, cancellation of the $U(1)_X$ anomaly other than by a GS mechanism seems problematic.

A part of the conformal anomaly can be directly inferred by replacing $\ln\Lambda^2$ in (2.2) by the real superfield $\ln M^2(Z^i,\bar{Z}^{\bar{m}})$, where the lowest component $M^2(z^i,\bar{z}^{\bar{m}}) = M^2(Z^i,\bar{Z}^{\bar{m}})|$ is the PV squared mass matrix. Under a transformation that leaves the regulated tree Lagrangian invariant except for the PV mass terms, the shift in (2.2) is determined by [see (4.12)] $M^2(Z^i,\bar{Z}^{\bar{m}}) \rightarrow \widetilde{M}^2(Z^i,\bar{Z}^{\bar{m}}) = e^{H(Z)+\bar{H}(\bar{Z})}M^2(Z^i,\bar{Z}^{\bar{m}})$, where $H(Z)$ is a holomorphic function of the chiral fields. The supersymmetric anomalies associated with the F-term operators given in Section 2 are also F-terms which contain the associated chiral anomalies; the general form of these operators is given in Appendix 2. It has been conjectured [19] that all of these anomalies might be canceled entirely or in part, depending on the string threshold corrections in specific models, by the GS term included in (4.9). This would require a tree-level coupling of the dilaton to the chiral superfields Φ_W, Φ_α in (2.10), for example, inducing additional operators (and potential anomalies) at the one-loop level. The D-term operators of Section 2 give rise to D-term anomalies, also displayed in Appendix A. In principle these could also be canceled by a tree-level coupling of the dilaton to real superfields such as those in (2.14) *via* a D-term of the form (2.13), again implying additional operators at one loop. One such D-term is the shift in the Kähler potential, (2.16). We have shown that the regularization of this term can be made free of modular and $U(1)_X$ anomalies for supergravity from orbifold compactification with background twisted sector fields set to zero. It is not clear that this can be achieved with twisted sector fields in the background.

The full set of anomalous operators contains additional terms that arise due to the fact that the PV masses are not constant; $D_\mu\widetilde{M} \neq 0$. Determining these requires keeping higher order terms in the derivative expansion (as in the calculation of soft terms in Section 3.3) and retaining total derivatives (like the Gauss-Bonnet term) in the coefficient of $\ln\Lambda^2$. In addition it is necessary to verify the cancellation of linear divergences – or equivalently[13] to show that (4.12) is satisfied by comparing that expression with with the anomaly calculated from $L(\Phi') - L(\Phi)$. These issues will be addressed elsewhere.

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Appendix

A. Component expressions of general superfields operators

After eliminating the auxiliary fields using their tree-level equations of motion [9]:

$$F^i = -e^{-K/2}\bar{A}^i, \quad 2R| = e^{-K/2}A, \quad -x\mathbf{D_a} = \mathcal{D}_a, \tag{A.1}$$

we obtain for the bosonic terms for the superfield operators introduced in Section 2.1:

$$\mathcal{D}_\beta T_\alpha| \ = \ \epsilon_{\beta\alpha}T_0 + (\sigma^{mn}\epsilon)_{\beta\alpha}T_{mn}, \quad T_0 = \frac{1}{2}\,\mathcal{D}^\alpha T_\alpha|\,, \quad T_{mn} = \epsilon^\mu_m\epsilon^\nu_n T_{\mu\nu},$$

[13]However this procedure applied to modular and $U(1)_X$ anomalies will not insure, for example, the correct dilaton dependence of the Kähler metric.

$$\begin{aligned} \mathcal{D}^\alpha T_\alpha| &= -2D_{\bar m}T_i\left(e^{-K}\bar A^i A^{\bar m} + \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar z^{\bar m}\right) + 2x^{-1}\mathcal{D}_a T_i (T^a z)^i, \\ T_{\mu\nu} &= \left[\left(\mathcal{D}_\mu z^i \mathcal{D}_\nu \bar z^{\bar m} - \mathcal{D}_\nu z^i \mathcal{D}_\mu \bar z^{\bar m}\right) D_{\bar m} - iF^a_{\mu\nu}(T_a z)^i\right] T_i, \end{aligned} \tag{A.2}$$

where m, n are tangent space Lorentz indices, and α, β are spinor indices in the two-component spinor notation[14] of [9]. For the bosonic parts of the F-terms of Section 2.1 we obtain:

$$\begin{aligned} L(W_a^\alpha T_\alpha^a) &= \left[\frac{\mathcal{D}_a}{x}\left(\mathcal{D}_\rho z^i \mathcal{D}^\rho \bar z^{\bar m} + e^{-K} A^i \bar A^{\bar m}\right) + i\mathcal{D}_\mu z^i \mathcal{D}_\nu \bar z^{\bar m} F^{\mu\nu}_{a-}\right] D_{\bar m} T_i^a \\ &\quad + \mathcal{W}_{ab}(T^b z)^i T_i^a + \text{h.c.}, \end{aligned} \tag{A.3}$$

$$\begin{aligned} & L(T^\alpha T'_\alpha) \equiv L^{ij} T_i T'_j = \left(\mathcal{W}^{ab} + \overline{\mathcal{W}}^{ab}\right)(T_a z)^i (T_b z)^j T_i T'_j \\ & + \left[\frac{\mathcal{D}_a}{x}\left(\mathcal{D}_\rho z^i \mathcal{D}^\rho \bar z^{\bar m} + e^{-K} A^i \bar A^{\bar m}\right) + i\mathcal{D}_\mu z^i \mathcal{D}_\nu \bar z^{\bar m} F^{\mu\nu}_a\right](T^a z)^j \left(T_j D_{\bar m} T'_i + T_j D_{\bar m} T'_i\right) \\ & - \left(\mathcal{D}_\mu \bar z^{\bar m} \mathcal{D}^\mu z^i + e^{-K} A^{\bar m} \bar A^i\right)\left(\mathcal{D}_\nu \bar z^{\bar n} \mathcal{D}^\nu z^j + e^{-K} A^{\bar n} \bar A^j\right) D_{\bar m} T_i D_{\bar n} T'_j \\ & - \mathcal{D}^\mu z^i \mathcal{D}^\nu \bar z^{\bar m}\left(\mathcal{D}_\mu z^j \mathcal{D}_\nu \bar z^{\bar n} - \mathcal{D}_\mu \bar z^{\bar n} \mathcal{D}_\nu z^j\right) D_{\bar m} T_i D_{\bar n} T'_j + \text{h.c.}. \end{aligned} \tag{A.4}$$

In section 4 we also introduced F-terms of the form

$$L(T, T')^\alpha_\alpha = L^{ij} T_j T'_i, \tag{A.5}$$

that is, they are they same as (A.4) except for the signs of two four-derivative terms. In addition we have, with $X_{\mu\nu} = K_{\mu\nu}$

$$\begin{aligned} L(6\Phi_W) &= \frac{1}{2}\int d^4\theta \frac{E}{R} W^{\alpha\beta\gamma} W_{\alpha\beta\gamma} + \text{h.c.} \\ &= \frac{1}{2}\mathcal{D}_\alpha W_{\beta\gamma\delta}\mathcal{D}^\alpha W^{\beta\gamma\delta} + \text{h.c.} + \text{fermions} \\ &= 6L_{GB} + \frac{1}{4}r_{\mu\nu}r^{\mu\nu} - \frac{1}{12}r^2 + \frac{1}{12}X_{\mu\nu}X^{\mu\nu} + \text{fermions}. \end{aligned} \tag{A.6}$$

Up to terms that vanish on shell due to the graviton tree-level equations of motion, we have the identity [see (2.23)–(2.25) of [10]]

$$\frac{1}{12}\left(3r_{\mu\nu}r^{\mu\nu} - r^2\right) = 3L_\chi - \frac{1}{3}L_\alpha - \frac{1}{12}X_{\mu\nu}X^{\mu\nu}, \tag{A.7}$$

[14]The component field expressions use the metric $g_{\mu\nu} = \text{diag}(+---)$, the opposite of the metric of [9].

and we obtain

$$\frac{1}{2}L_\chi + L_{GB} = \frac{1}{12}\int d^4\theta\frac{E}{R}\left(W^{\alpha\beta\gamma}W_{\alpha\beta\gamma} - \frac{1}{3}X_\alpha X^\alpha\right) + \text{h.c.}. \tag{A.8}$$

For the D-terms we obtain

$$\begin{aligned}
L(\phi_{WT}) &= \Big(xF^{-a}_{\rho\mu}F^{\rho\nu}_{+a}\mathcal{D}_\nu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}} + 4e^{-K}\mathcal{D}\bar{A}^iA^{\bar{m}} \\
&\quad +2i\mathcal{D}_\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}}\mathcal{D}^aF^{\mu\nu}_a + 2\mathcal{D}\mathcal{D}_\rho z^i\mathcal{D}^\rho\bar{z}^{\bar{m}} + \Big)T_{i\bar{m}}, &\text{(A.9)}\\
L(T^\alpha_\alpha) &= e^{-K}\left(\bar{A}\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^J + \frac{1}{x}\mathcal{D}_a(T^az)^i\bar{A}^j\right)t_{ij} \\
&\quad +e^{-K}\left(e^{-K}A\bar{A}^i\bar{A}^j - \frac{1}{2}\bar{f}^i\overline{W}\bar{A}^j\right)t_{ij} \\
&\quad -\left(\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^i + e^{-K}A^{\bar{m}}\bar{A}^i\right)D_{\bar{m}}(e^{-K}\bar{A}^jt_{ij}) \\
&\quad +e^{-K}\mathcal{D}_\mu\bar{z}^k\mathcal{D}^\mu z^i\bar{A}^j\left(D_kt_{ij} - D_jt_{ik}\right), \\
T_{ij} &= e^{-K/2}t_{ij}, \quad w(T_{ij}) = w(t_{ij}) - 2 = 2, &\text{(A.10)}\\
L(T^{\alpha\dot\beta}_{\alpha\dot\beta}) &= \Big(\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}_\nu z^i\mathcal{D}^\mu\bar{z}^{\bar{n}}\mathcal{D}^\nu z^j + e^{-2K}A^{\bar{m}}\bar{A}^iA^{\bar{n}}\bar{A}^j \\
&\quad +2e^{-K}A^{\bar{m}}\bar{A}^i\mathcal{D}_\nu\bar{z}^{\bar{n}}\mathcal{D}^\nu z^j\Big)T_{ij\bar{m}\bar{n}}, &\text{(A.11)}
\end{aligned}$$

In the fully regulated Lagrangian, $\ln\Lambda^2$ in (2.2) is replaced by the real superfield $\ln M^2(Z^i,\bar{Z}^{\bar{m}})$, where the lowest component $M^2(z^i,\bar{z}^{\bar{m}}) = M^2(Z^i,\bar{Z}^{\bar{m}})|$ is the PV squared mass matrix. Under a transformation that leaves the regulated tree Lagrangian invariant except for the PV mass terms: $M^2(Z^i,\bar{Z}^{\bar{m}}) \to e^{H(Z)+\bar{H}(\bar{Z})}M^2(Z^i,\bar{Z}^{\bar{m}})$, where $H(Z)$ is a holomorphic function of the chiral fields, the full anomaly associated with the one-loop generated F-term operators given in Section 2 can be expressed in term of supersymmetric field operators of the form

$$\begin{aligned}
L(T,T',H) &= \frac{1}{2}\int d^4\theta\frac{E}{R}T^\alpha T'_\alpha H(Z) + \text{h.c.} \\
&= \frac{1}{2}H(z)\mathcal{D}_\alpha T'_\beta\mathcal{D}^\alpha T^\beta + \text{h.c.} + \text{fermions} \\
&= -2\text{Re}HT^0T'_0 - \text{Re}HT'_{\mu\nu}T^{\mu\nu} - \text{Im}H\tilde{T}'_{\mu\nu}T^{\mu\nu} + \text{fermions} \\
&= -2\text{Re}HL(T,T') - \text{Im}H\tilde{T}'_{\mu\nu}T^{\mu\nu} + \text{fermions}, &\text{(A.12)}
\end{aligned}$$

where $L(T,T',1) = -\frac{1}{2}L(T^\alpha T'_\alpha)$ is defined by (A.4), and

$$\text{Re}H\left(\frac{1}{2}L_\chi + L_{GB}\right) + \text{Im}H\frac{r\tilde{r}}{48} = \frac{1}{12}\int d^4\theta\frac{E}{R}H(Z)\left(W^{\alpha\beta\gamma}W_{\alpha\beta\gamma} - \frac{1}{3}X_\alpha X^\alpha\right) + \text{h.c.}. \tag{A.13}$$

The chiral anomalies in the above expressions arise from the standard nonlocal operators generated by fermion loops. For the D-terms operators of Section 2, the corresponding anomalies are also D-terms:

$$L(\phi, H) = \int d^4\theta E\phi H + \text{h.c.}. \tag{A.14}$$

In addition there are contributions from terms involving derivatives of the Pauli-Villars masses that do not grow with the cut-off and were not included in Section 2.

B. Modifications of the Z^I, Y_I contributions

The fields $\widetilde{Z}^I_\alpha$ play the same role as Z^I_1 in I. However, if we were to use the Kähler potentials K^Z, K^Y adopted in I, we would have for the covariant derivatives of the gauge kinetic function $f(z) = s$:

$$f^{\widetilde{Z}}_{IJ} = D_I D_J f = -\Gamma^k_{IJ} f_k \neq 0, \tag{B.1}$$

which would generate unwanted contributions from $\widetilde{Z}^I_1$-loops. The effect of the f^i-dependent terms in $K^{\widetilde{Z}}$ is to eliminate these contributions; their presence in turn requires compensating modifications of $K^{\widetilde{Y}}$ and $K^{\widehat{Y}}$. In this appendix we calculate the modifications with respect to I of the Z^I, Y_I loop contributions.

Denoting by a tilde quantities derived from the Kähler potentials $K^{Z,Y}_\alpha$ in (3.3) with $f_i = 0$, that is

$$\tilde{K}^{Z,Y}_\alpha = K^{Z,Y}_\alpha\Big|_{f_i=0}, \tag{B.2}$$

we have

$$L^Z_3 + L^Y_1 = \tilde{L}^Z_3 + \tilde{L}^Y_1 + \Delta\left(L^Z_3 + L^Y_1\right), \tag{B.3}$$

In I we found

$$\begin{aligned}\left(L^Z_3 + L^Y_1\right)_I &= -L_3 - \frac{2}{\sqrt{g}} e^{-K}\left(A_i \bar{A} \mathcal{L}^i_I + \text{h.c.}\right) \\ &\quad - \frac{2}{x\sqrt{g}}\left[\mathcal{D}_a (T^a z)^i \mathcal{L}^I_i + i\mathcal{D}_\mu \bar{z}^{\bar{m}} (T_a z)^i K_{i\bar{m}} \mathcal{L}^{\mu a}_I + \text{h.c.}\right] \\ &\quad + 4\Delta_{\hat{V}} L_I + 12\Delta_{M^2} L_I + 8\Delta_{\mathcal{D}} L_I,\end{aligned} \tag{B.4}$$

where the subscript I denotes the Lagrangian with $f =$ constant. We have

$$\begin{aligned} A_i\mathcal{L}^i &= A_i\mathcal{L}^i_I + x\mathcal{W}A, \quad \Delta_{\hat{V}}L = \Delta_{\hat{V}}L_I, \quad \Delta_{M^2}L = \Delta_{M^2}L_I, \\ \frac{1}{x\sqrt{g}}\mathcal{L}^{\mu a} &= \frac{1}{x\sqrt{g}}\mathcal{L}^{\mu a}_I + \frac{\partial_\nu x}{x}F^{a\mu\nu} + \frac{\partial_\nu y}{x}\tilde{F}^{a\mu\nu}, \end{aligned} \tag{B.5}$$

and, from the results in (B.18) and (B.20) of [12],

$$\begin{aligned} &-\frac{2}{x\sqrt{g}}\left[\mathcal{D}_a(T^a z)^i\mathcal{L}_i + \text{h.c.}\right] + 8\Delta_D L = -\frac{2}{x\sqrt{g}}\left[\mathcal{D}_a(T^a z)^i\mathcal{L}^I_i + \text{h.c.}\right] \\ &\quad +8\Delta_D L_I - 2\frac{\partial_\mu x}{x}\mathcal{D}^a K_{j\bar{m}}\left[\mathcal{D}^\mu z^j(T_a\bar{z})^{\bar{m}} + (T_a z)^j\mathcal{D}^\mu\bar{z}^{\bar{m}}\right] + 32M^2\mathcal{D} \\ &\quad -4i\frac{\partial^\mu y}{x^2}\mathcal{D}^a\left[K_{i\bar{m}}(T_a z)^i\mathcal{D}_\mu\bar{z}^{\bar{m}} - \text{h.c.}\right] + \frac{4}{x^2}\mathcal{D}\left[\partial_\mu x\partial^\mu x + \partial_\mu y\partial^\mu y\right] \\ &\quad +\text{total derivative}. \end{aligned} \tag{B.6}$$

Combining these results, we obtain

$$\tilde{L}^Z_3 + \tilde{L}^Y_1 = \left(L^Z_3 + L^Y_1\right)_I + 2xM^2\left(\mathcal{W} + \overline{\mathcal{W}}\right) - \frac{1}{3}L(\Phi'_0), \tag{B.7}$$

where we used (C.76) of [12].

Writing

$$\begin{aligned} K &= k + G, \quad k = -\ln(s+\bar{s}), \quad k_i = -f_i/2x^2, \\ K_{IJ} &= \tilde{K}_{IJ} + \hat{K}_{IJ} \quad \tilde{K}_{IJ} = K_{ij} - K_iK_j, \\ \hat{K}_{IJ} &= -\frac{1}{2x}\left(f_iK_j + f_jK_i\right) - \frac{1}{2x^2}f_if_j, \end{aligned} \tag{B.8}$$

the effect of the f_i-dependent terms in $K^{\widetilde{Z}}$ is to eliminate the contributions to K_{IJ} with $IJ = LS, SL,\ L \neq S$ (note that $\hat{K}_{SS} = \tilde{K}_{SS} = K_{SS} = 0$). Since

$$\hat{K}_{IJ}\tilde{K}^{IJ} = -\hat{K}_{IJ}\hat{K}^{IJ}, \tag{B.9}$$

we simply need to subtract the terms quadratic in $\hat{K}_{IJ}$ in products of K_{IJ} and its derivatives. We have

$$\begin{aligned} \hat{K}_{IJ} &= K_jk_i + K_jk_i - 2k_ik_j, \\ \hat{\Gamma}^k_{IJ} &= \delta^k_ik_j + \delta^k_jk_i + k^k_iK_j + k^k_jK_i - 4k^k_ik_j, \end{aligned} \tag{B.10}$$

where $k^i_j = k^{i\bar{m}}k_{\bar{m}j}$ projects out s-components. Then using

$$\begin{aligned} k_{i\bar{m}} &= k_i k_{\bar{m}}, \quad k_{ij} = k_i k_j, \quad D_j k_i = -k_i k_j, \quad k^i A_i = A, \quad k^j_i A_j = k_i A, \\ A_s &= k_s A, \quad A_{sk} = k_s A_k - k_s k_k A, \quad k^i A_{ij} = A_j - k_j A, \end{aligned} \tag{B.11}$$

we obtain

$$\begin{aligned} \hat{A}_{IJ} &= \hat{K}_{IJ}A - \hat{\Gamma}^k_{IJ}A_k = -A_j k_i - A_i k_j + 2Ak_i k_j, \\ \hat{R}^k_{IJ\bar{m}} &= \partial_{\bar{m}}\hat{\Gamma}^k_{IJ} = \delta^k_i k_{j\bar{m}} + \delta^k_j k_{i\bar{m}} + k^k_i K_{j\bar{m}} + k^k_j K_{i\bar{m}} - 4k^k_i k_{j\bar{m}}, \end{aligned} \tag{B.12}$$

and [with $\hat{k}_\alpha$, *etc.*, defined as in (2.8), (2.14)]

$$\begin{aligned} & R^{\dot{\beta}}{}_{IJk\dot{\beta}}R^{\alpha IJ}{}_{\alpha} = \tilde{R}^{\dot{\beta}}{}_{IJk\dot{\beta}}\tilde{R}^{\alpha IJ}{}_{\alpha} + 8k^{\alpha\dot{\beta}}\left(k_{\alpha\dot{\beta}} - K_{\alpha\dot{\beta}}\right), \\ & \bar{A}_{IJ}R^{I\alpha J}{}_{\alpha} = \bar{\tilde{A}}_{IJ}\tilde{R}\bar{A}_{IJ}R^{I\alpha J}{}_{\alpha} + 4\bar{A}\hat{k}^\alpha\hat{k}_\alpha. \end{aligned} \tag{B.13}$$

Then from the expression for L_3 given by Eqs. (2.13), (2.20), (A.10) and (A.11) [or explicitly in (2.27) of I], we obtain, with $\eta^Z_1 = -1$,

$$\begin{aligned} \Delta L^Z_3 = & -\frac{\partial_\mu s\partial^\mu s\partial_\nu\bar{s}\partial^\nu\bar{s}}{2x^4} + \frac{2}{x^2}K_{i\bar{m}}\mathcal{D}_\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}}\partial^\mu s\partial^\nu\bar{s} - 8M^4 \\ & -12M^2\hat{V} - 2M^2\frac{\partial_\mu s\partial^\nu\bar{s}}{x^2} + 2\frac{e^{-K}}{x}\left(\mathcal{D}_\mu z^i\partial^\mu sA_i\bar{A} + \text{h.c.}\right) \\ & + \left(\frac{\partial^\mu s\partial_\mu s}{2x^2} + \text{h.c.}\right)\left(\hat{V} + M^2\right) - 4e^{-K}\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}A_i\bar{A}_{\bar{m}} \\ & +2e^{-K}\frac{\partial^\mu x}{x}\left[\mathcal{D}_\mu z^i\left(A_{ij}\bar{A}^j - 3A_i\bar{A}\right) + \text{h.c.}\right] - 4\hat{V}^2 \\ & +8M^2\mathcal{D} + 2e^{-K}\left(\bar{A}A_{ij}\mathcal{D}_\mu z^j\mathcal{D}^\mu z^i - A_{ij}\bar{A}^j\bar{A}^iA + \text{h.c.}\right). \end{aligned} \tag{B.14}$$

As noted in I, the derivatives of the metric defined by

$$K_{PQ}Y^P\bar{Y}^Q = \sum_{I,J=i,j} K^{i\bar{j}}Y_I\bar{Y}_{\bar{J}} - a\sum_{I=i}\left(Y_I\bar{Y}^0\kappa^i + \text{h.c.}\right) + |Y_0|^2\left(1 + a^2\kappa^i\kappa_i\right), \tag{B.15}$$

are most easily easily evaluated in terms of the derivatives of the inverse metric

$$K^{PQ}Y_P\bar{Y}_Q = \sum_{I,J=i,j}\left(K_{i\bar{j}} + a^2\kappa_i\kappa_{\bar{j}}\right)\bar{Y}^IY^{\bar{J}} + a\sum_{I=i}\left(\bar{Y}^IY^0\kappa_i + \text{h.c.}\right) + |Y_0|^2. \tag{B.16}$$

One finds for the elements of the affine connection

$$\begin{aligned}\Gamma^I_{0k} &= -aD_i\kappa_k + a^3K^{j\bar m}\kappa_j\kappa_i\partial_{\bar m}\kappa_k, \quad \Gamma^0_{Ik} = -aK^{i\bar m}\partial_{\bar m}\kappa_k,\\ \Gamma^I_{Jk} &= -\Gamma^j_{ik} - a^2K^{j\bar m}\kappa_i\partial_{\bar m}\kappa_k, \quad \Gamma^0_{0k} = a^2K^{i\bar m}\kappa_i\partial_{\bar m}\kappa_k.\end{aligned} \tag{B.17}$$

It follows immediately that $\Gamma^P_{P\alpha} = \tilde\Gamma^P_{P\alpha}$, so there are no changes to H^2_Y, L^Y_2. For $K^{\widehat Y}$ we have $\kappa_i = K_i - k_i$, $(T_a)^P_Q\Gamma^Q_{P\alpha} = (T_a)^P_Q\tilde\Gamma^Q_{P\alpha}$, and

$$\begin{aligned}D_I(T_ay)^J &= \tilde D_I(T_ay)^J - a^2k_j(T_az)^i, \quad D_I(T_ay)^0 = \tilde D_I(T_ay)^0,\\ D_0(T_ay)^J &= \tilde D_0(T_ay)^J - a^3k_j\mathcal{D}_a, \quad D_0(T_ay)^0 = \tilde D_0(T_ay)^0,\\ R^0_{0i\bar m} &= \tilde R^0_{0i\bar m} - a^2k_{i\bar m}, \quad R^0_{Ii\bar m} = \tilde R^0_{Ii\bar m} = 0,\\ R^J_{Ik\bar m} &= \tilde R^J_{Ik\bar m} + a^2\left(\delta^i_k k_{j\bar m} + k^i_k K_{j\bar m} - k^i_k k_{j\bar m}\right),\end{aligned} \tag{B.18}$$

with the result that for $P, Q = \widehat Y$,

$$\begin{aligned}D_P(T_ay)^Q D_Q(T_ay)^P &= \tilde D_P(T_ay)^Q\tilde D_Q(T_ay)^P,\\ D_P(T_ay)^Q R^P_{Qk\bar m} &= \tilde D_P(T_ay)^Q\tilde R^P_{Qk\bar m},\end{aligned} \tag{B.19}$$

and the modifications to $L^{\widehat Y}_1$ are determined by

$$\begin{aligned}R^P_{Qk\bar m}R^Q_{Pj\bar n} &= \tilde R^P_{Qk\bar m}\tilde R^Q_{Pj\bar n} + 2a^2R^{(k)}_{k\bar m j\bar n}\\ &\quad + a^4\left(2k_kk_{\bar m}k_jk_{\bar n} - k_{k\bar m}K_{j\bar n} - K_{k\bar m}k_{j\bar n} - k_{j\bar m}K_{k\bar n} - K_{j\bar m}k_{k\bar n}\right)\\ &= \tilde R^P_{Qk\bar m}\tilde R^Q_{Pj\bar n} + 2\left(a^4 - 2a^2\right)k_kk_{\bar m}k_jk_{\bar n}\\ &\quad - a^4\left(k_{k\bar m}K_{j\bar n} + K_{k\bar m}k_{j\bar n} + k_{j\bar m}K_{k\bar n} + K_{j\bar m}k_{k\bar n}\right),\end{aligned} \tag{B.20}$$

where $R^{(k)}_{k\bar m j\bar n}$ is the Riemann tensor derived from k.

The Kähler metric for $\widetilde Y_S, \widetilde Y_0$ has $K^{i\bar m} \to k^{i\bar m}$ and $\kappa_i = k_i$, and is the same as the Y-metric in I, with the Kähler potential $K(z,\bar z) \to k(s,\bar s)$ and $a = 1$. Since $(\Gamma^{\widetilde Y})^0_{S\alpha} = S_\alpha$, and $\mathcal{D}_\alpha S_\beta|$ has no bosonic terms we need only consider

$$(\Gamma^{\widetilde Y})^S_{S\alpha} = -\Gamma^s_{s\alpha} - a^2k_\alpha = -3k_\alpha, \quad (\Gamma^{\widetilde Y})^0_{0\alpha} = a^2k_\alpha = k_\alpha, \quad \Phi^{\widehat Y}_1 = 10\Phi_\beta. \tag{B.21}$$

Since $\sum_\alpha \eta^{\widetilde Y}_\alpha = -1$, (B.21) gives a total contribution equal to $-10L_\beta$ to L^P_1, (3.18), but a portion $-4L_\beta$ of this is included in $\beta' L_\beta$. Using (2.25) of I to evaluate the contribution from

(B.20), with $a^2 \to a = -2$, $a^4 \to a' = +2$, we obtain a net contribution:

$$
\begin{aligned}
\Delta L_1^Y &= 3\frac{\partial_\mu s \partial^\mu s \partial_\nu \bar{s} \partial^\nu \bar{s}}{4x^4} - \frac{2}{x^2} K_{i\bar{m}} \mathcal{D}_\mu z^i \mathcal{D}_\nu \bar{z}^{\bar{m}} \partial^\mu s \partial^\nu \bar{s} \\
&+ \left(3M^2 - \hat{V}\right) \frac{\partial_\mu s \partial^\mu \bar{s}}{x^2} + 2\frac{e^{-K}}{x} \left(\mathcal{D}_\mu z^i \partial^\mu \bar{s} A_i \bar{A} + \text{h.c.}\right) \\
&-4M^2 K_{i\bar{m}} \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} - 4M^2 \left(3M^2 + 2\hat{V}\right) - 6L_\beta, \qquad \text{(B.22)}
\end{aligned}
$$

and

$$
\begin{aligned}
\Delta\left(L_3^Z + L_1^Y\right) &= -4M^2 K_{i\bar{m}} \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} + \frac{\partial_\mu s \partial^\mu s \partial_\nu \bar{s} \partial^\nu \bar{s}}{4x^4} - 4\hat{V}^2 \\
&-20M^2\left(M^2 + \hat{V}\right) + \left(M^2 - \hat{V}\right) \frac{\partial_\mu s \partial^\nu \bar{s}}{x^2} \\
&+ \left(\frac{\partial^\mu s \partial_\mu s}{2x^2} + \text{h.c.}\right)\left(\hat{V} + M^2\right) - 4e^{-K} \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} A_i \bar{A}_{\bar{m}} \\
&+8M^2 \mathcal{D} + 2e^{-K}\left(\bar{A} A_{ij} \mathcal{D}_\mu z^j \mathcal{D}^\mu z^i - A_{ij} \bar{A}^j \bar{A}^i A + \text{h.c.}\right) \\
&+2e^{-K} \frac{\partial^\mu x}{x} \left[\mathcal{D}_\mu z^i \left(A_{ij} \bar{A}^j - A_i \bar{A}\right) + \text{h.c.}\right] - 6L_\beta. \qquad \text{(B.23)}
\end{aligned}
$$

Using the relations [see (B.18) of [12]]

$$
\begin{aligned}
\frac{2}{\sqrt{g}} \mathcal{L}_i \bar{A}^i A e^{-K} + \text{h.c.} &= 2e^{-2K}\left(\mathcal{D}_\mu z^i \mathcal{D}^\mu z^j A_{ij} \bar{A} - A_{ij} \bar{A}^i \bar{A}^j A + \text{h.c.}\right) \\
&+8M^2\left(\hat{V} + 3M^2 - \mathcal{D}\right) + 2xM^2\left(\mathcal{W} + \overline{\mathcal{W}}\right) \\
&+4e^{-K} \mathcal{D}^\mu z^i \mathcal{D}_\mu \bar{z}^{\bar{m}} (\bar{A}_i A_{\bar{m}} + K_{i\bar{m}} A \bar{A}), \\
2\partial_\mu\left[\left(\hat{V} + M^2\right) \frac{\partial^\mu x}{x}\right] &= 2\partial_\mu\left[e^{-K} \frac{\partial^\mu x}{x}\left(A_i \bar{A}^i - 2A\bar{A}\right)\right] \\
&= 2e^{-K} \frac{\partial^\mu x}{x}\left[\mathcal{D}_\mu z^i \left(A_{ij} \bar{A}^j - A_i \bar{A}\right) + \text{h.c.}\right] \\
&+2\left(\hat{V} + M^2\right)\left(\frac{\nabla^2 x}{x} - \frac{\partial^\mu x \partial_\mu x}{x^2}\right) \\
&= 2e^{-K} \frac{\partial^\mu x}{x}\left[\mathcal{D}_\mu z^i \left(A_{ij} \bar{A}^j - A_i \bar{A}\right) + \text{h.c.}\right] \\
&+ \left(\hat{V} + M^2\right)\left(\frac{\partial_\mu s \partial^\mu s}{2x^2} - \frac{1}{x\sqrt{g}} f_i \mathcal{L}^i + x\mathcal{W} + \text{h.c.}\right) \\
&+4\left(\hat{V}^2 + M^2 \hat{V}\right) - \left(\hat{V} + M^2\right) \frac{\partial_\mu s \partial^\mu \bar{s}}{x^2},
\end{aligned}
$$

$$\Delta L_{M^2} = 2e^{-K}\frac{\partial^\mu x}{x}\left(\mathcal{D}_\mu z^i A_i \bar{A} + \text{h.c.}\right)$$
$$-2M^2\mathcal{D} + \hat{V}^2 + 4M^2\hat{V} + 6M^4, \quad \text{(B.24)}$$

and dropping total derivatives we get

$$\Delta\left(L_3^Z + L_1^Y\right) = \frac{1}{\sqrt{g}}\left\{\mathcal{L}^i\left[\frac{f_i}{x}\left(\hat{V}+M^2\right) + 2e^{-K}A_i\bar{A}\right] + \text{h.c.}\right\}$$
$$-8\Delta L_{M^2} - 2L_\beta + 2x\hat{V}\left(\mathcal{W}+\overline{\mathcal{W}}\right). \quad \text{(B.25)}$$

Combining (B.25) with (B.7) gives the result in (3.18), with

$$-\frac{1}{3}L(\Phi_0') = -32M^2\mathcal{D} - \frac{4}{x^2}\mathcal{D}\partial_\mu s\partial^\mu\bar{s} - \frac{2i}{x^2}\partial^\mu s\partial^\nu\bar{s}\mathcal{D}_a F^a_{\mu\nu}$$
$$+2\left[\partial^\mu s\left(iF^{-a}_{\mu\nu} + g_{\mu\nu}\frac{1}{x}\mathcal{D}^a\right)K_{i\bar{m}}\mathcal{D}^\nu\bar{z}^{\bar{m}} + \text{h.c.}\right],$$
$$L_\beta = M^4 + M^2\frac{\partial_\mu s\partial_\nu\bar{s}}{2x^2} + \frac{\partial_\mu s\partial^\mu s\partial_\nu\bar{s}\partial^\nu\bar{s}}{16x^4}. \quad \text{(B.26)}$$

In addition we have, using $k^iW_i = 0$,

$$A^{\widetilde{Z}}_{IJ}\bar{A}^{IJ}_{\widetilde{Z}} = \tilde{A}^Z_{IJ}\bar{\tilde{A}}^{IJ}_Z - \bar{A}_{is}\bar{A}^{is} = A_{ij}\bar{A}^{ij} - 2\left(A_i\bar{A}^i - A\bar{A}\right)$$
$$= A_{ij}\bar{A}^{ij} - 2e^K\left(\hat{V}+2M^2\right),$$
$$A^{\widehat{Z}_\alpha,\widehat{Y}_\alpha}_{IJ} = A\delta^j_i, \quad \bar{A}^{IJ}_{\widehat{Z}_\alpha,\widehat{Y}_\alpha} = \delta^i_j\bar{A} + a^2_\alpha e^K\left(K_j - k_j\right)\left(\bar{A}^i - k^i\bar{A}\right),$$
$$\bar{A}^{I0}_{\widehat{Z}_\alpha,\widehat{Y}_\alpha} = a_\alpha\left(\bar{A}^i - k^i\bar{A}\right), \quad A^{\widehat{Z}_\alpha,\widehat{Y}_\alpha}_{I0} = a_\alpha e^K W_i,$$
$$A^{\widehat{Z}\widehat{Y}}_{PQ}\bar{A}^{PQ}_{\widehat{Z}\widehat{Y}} = 2\tilde{A}^{ZY}_{PQ}\bar{\tilde{A}}^{PQ}_{ZY} - 2a^2_\alpha A\bar{A}, \quad A^{\widetilde{Z}\widetilde{Y}}_{PQ}\bar{A}^{PQ}_{\widetilde{Z}\widetilde{Y}} = 2c^2_\alpha A\bar{A}, \quad \text{(B.27)}$$

giving the result in (3.16). Note that the overall normalization of $A^{\widehat{Z}\widehat{Y}}_{PQ}$ differs from that[15] used in I for A^{ZY}_{PQ}.

Finally, there is a contribution from the diagonal part $f\delta_{ab}$ of the gauge kinetic function f_{ab}:

$$f^{\widetilde{Z}\widetilde{Y}}_{I0} = h_\alpha f_i, \quad \bar{f}^{I0}_{\widetilde{Z}\widetilde{Y}} = h_\alpha\bar{f}^{\bar{i}}, \quad \bar{f}^{IJ}_{\widetilde{Z}\widetilde{Y}} = h_\alpha\bar{f}^i k_j,$$

[15]There are extraneous factors of e^K and W in the last term of the expression for $\bar{A}^{IJ}_{Z_\alpha,Y_\alpha}$ in I.

$$
\begin{aligned}
f_{PQ}^{\widetilde{Z}\widetilde{Y}} \bar{f}_{\widetilde{Z}\widetilde{Y}}^{PQ} &= 2h_\alpha^2 \bar{f}^i f_i = 8x^2 h_\alpha^2, \quad \bar{f}_{\widetilde{Z}\widetilde{Y}}^{PQ} A_{PQ}^{\widetilde{Z}\widetilde{Y}} = 2h_\alpha c_\alpha \bar{f}^s k_s A = -4xA, \\
A_{kIQ}^{\widetilde{Z}\widetilde{Y}} &= D_k A_{IQ}^{\widetilde{Z}\widetilde{Y}} = \partial_k A_{IQ}^{\widetilde{Z}\widetilde{Y}} - \Gamma_{ki}^l A_{LQ}^{\widetilde{Z}\widetilde{Y}} - \left(\Gamma^Y\right)_{kQ}^P A_{IP}^{\widetilde{Z}\widetilde{Y}} \\
A_{kI0}^{\widetilde{Z}\widetilde{Y}} &= -a\left(1+a^2\right) k_i k_k A = -2k_i k_k A, \\
A_{kIJ}^{\widetilde{Z}\widetilde{Y}} &= A_k + a^2 k_k = \left(A_k + k_k A\right), \\
\bar{f}_{\widetilde{Z}\widetilde{Y}}^{PQ} A_{kPQ}^{\widetilde{Z}\widetilde{Y}} &= -4x h_\alpha c_\alpha \left(A_k - k_k A\right).
\end{aligned}
\tag{B.28}
$$

Then the scalar mass-matrix element $H_{PQ}^{\widetilde{Z}\widetilde{Y}}$ takes the form [12]

$$
\begin{aligned}
H_{PQ}^{\widetilde{Z}\widetilde{Y}} &= e^{-K}\left(A_{PQk}\bar{A}^k - A_{PQ}\bar{A}\right) + \frac{1}{2} f_{PQ}\mathcal{W}, \\
H_{PQ}^{\widetilde{Z}\widetilde{Y}} H_{\widetilde{Z}\widetilde{Y}}^{PQ} &= -2x h_\alpha c_\alpha \left(\hat{V} + M^2\right)\left(\mathcal{W} + \overline{\mathcal{W}}\right) + 2x^2 h_\alpha^2 \mathcal{W}\overline{\mathcal{W}} \\
&\quad + \cdots,
\end{aligned}
\tag{B.29}
$$

where the dots represent contributions independent of $\mathcal{W}$ that have already been included. Together with the results given in Appendix B of I, we obtain the contribution (3.17).

C. Parameter constraints

Defining

$$
\beta = \sum_C \eta^C \beta_C, \quad \sigma = 2\sum_C \eta^C \beta_C \alpha_C,
\tag{C.1}
$$

if $\beta = -N_G - f$ and/or $\sigma \neq 0$ there are additional contributions to the logarithmic divergences:

$$
\begin{aligned}
\mathcal{L}_{PV} &\ni \sqrt{g}\frac{\ln \Lambda^2}{32\pi^2}\left[\left(\sigma - \frac{2}{3}\beta\right) L'_\beta - \beta L'_g\right], \\
L'_\beta &= M^2\left(\hat{V} + 3M^2\right) + M^2 K_{i\bar{m}} \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} + i\frac{\partial_\mu s \partial_\nu \bar{s}}{4x^2} F_a^{\mu\nu} \mathcal{D}^a \\
&\quad + 2\mathcal{D}M^2 + \left(K_{i\bar{m}} \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} + \hat{V} + 3M^2 + 2\mathcal{D}\right)\frac{\partial_\nu s \partial^\nu \bar{s}}{4x^2} \\
&\quad + \frac{\partial_\mu s \partial_\nu \bar{s}}{4x^2} K_{i\bar{m}}\left(\mathcal{D}^\mu z^i \mathcal{D}^\nu \bar{z}^{\bar{m}} - \mathcal{D}^\mu \bar{z}^{\bar{m}} \mathcal{D}^\nu z^i\right),
\end{aligned}
\tag{C.2}
$$

where L'_g is given in (2.5). The contribution from (C.2) contains for example the terms

$$
\left(\sigma - \frac{2}{3}\beta\right) L'_\beta - \beta L'_g \ni (\sigma - 2\beta) K_{i\bar{m}} \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} \frac{\partial_\nu s \partial^\nu \bar{s}}{4x^2}
$$

$$-\left(\sigma-\frac{8}{3}\beta\right)K_{i\bar{m}}\mathcal{D}^{\mu}\bar{z}^{\bar{m}}\mathcal{D}^{\nu}z^{i}\frac{\partial_{\mu}s\partial_{\nu}\bar{s}}{4x^2}, \tag{C.3}$$

that are not generated by any other contribution, requiring $\sigma=\beta=0$.

D. The untwisted sector in orbifold compactifications

Here we give explicitly the relations needed for modular covariant ($K'_{PV}=K_{PV}$, $W'_2=e^{-F}W_2$) regularization of the theory defined by (4.41). Under the modular transformation (4.43) we have

$$\begin{aligned}
K'_p &= N^q_p\left(K_q+F_q\right), \quad K'_{p\bar{m}}=N^q_pN^{\bar{n}}_{\bar{m}}K_{q\bar{n}},\\
K'_{PQ} &= N^n_pN^m_q\left[K_{NM}-\sum_{i\neq j}\left(F^i_nG^{(j)}_m+F^j_mG^{(i)}_n+F^i_mF^j_n\right)\right],\\
W'_{pq} &= e^{-F}N^n_pN^m_q\left[W_{mn}-\sum_{i\neq j}\left(F^j_nW_{m=ia}+F^i_mW_{n=(aj)}-F^i_mF^j_nW\right)\right]\\
W'_p &= e^{-F}N^q_p\left(W_q-F_qW\right).
\end{aligned}\tag{D.1}$$

The operators that determine scalar curvature dependent quadratic divergences and the logarithmically divergent contributions $L_{1,2}$ are:

$$\begin{aligned}
\Gamma^p_{p\alpha} &= \tilde{N}G_\alpha, \quad \Gamma^{p\alpha}_q\Gamma^q_{p\alpha}=\left(\tilde{N}+1\right)G^{\alpha}_{(i)}G^{(i)}_\alpha+\hat{G}^{p\alpha}_q\hat{G}^q_{p\alpha},\\
G^{(i)}_\alpha &= -\frac{1}{8}\left(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha}-8R\right)\mathcal{D}_\alpha G^{(i)}, \quad \hat{G}^p_{q\alpha}=-\frac{1}{8}\left(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha}-8R\right)\left(G^{(i)}_q\mathcal{D}_\alpha Z^p\right)\delta_{i_p,i_q},\\
\tilde{N} &= n+2, \quad \Gamma^q_{p\alpha}(T^a)^p_q=\sum_i T^a_iG^{(i)}_\alpha+G^a_\alpha, \quad T^a_i=\sum_b(T^a)^{ib}_{ib}.
\end{aligned}\tag{D.2}$$

The corresponding operators from $\tilde{Z}^P$, $P\neq S$, are

$$\begin{aligned}
&\left(\Gamma_{\tilde{Z}}\right)^{P\alpha}_Q\left(\Gamma_{\tilde{Z}}\right)^Q_{P\alpha}=\Gamma^{p\alpha}_q\Gamma^q_{p\alpha}+\left(2a^2+a^4\right)\left(G^{\alpha}_{(i)}G^{(i)}_\alpha+\hat{G}^{p\alpha}_q\hat{G}^q_{p\alpha}\right)-2\left(a^2+a^4\right)\hat{G}_{p\alpha}Z^p_\alpha,\\
&\hat{G}_{p\alpha}=-\frac{1}{8}\left(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha}-8R\right)G_p\mathcal{D}_\alpha G^{(i)}\delta_{i_p,i},\\
&\left(\Gamma_{\tilde{Z}}\right)^P_{P\alpha}=\Gamma^p_{p\alpha}, \quad \left(\Gamma_{\tilde{Z}}\right)^Q_{P\alpha}(T^a)^P_Q=\Gamma^j_{i\alpha}(T^a)^i_j+a^2G^a_\alpha,
\end{aligned}\tag{D.3}$$

and the metric derivatives for $\hat{Y}_P$, $P \neq S$, are related to these by $\left(\Gamma_{\hat{Y}}\right)^Q_{P\alpha} = -\left(\Gamma_{\tilde{Z}}\right)^P_{Q\alpha}$. The derivatives for $X' = \hat{Z}, \tilde{Y}$ are now related to those for $X = \tilde{Z}, \hat{Y}$ by

$$\begin{aligned}
&(\Gamma_{X'})^{P\alpha}_Q (\Gamma_{X'})^Q_{P\alpha} = (\Gamma_X)^{P\alpha}_Q (\Gamma_X)^Q_{P\alpha} + \tilde{N}(1 \pm 2) G^\alpha G_\alpha, \\
&(\Gamma_{X'})^P_{P\alpha} = -(\Gamma_X)^P_{P\alpha} + \tilde{N} G_\alpha, \\
&(\Gamma_{X'})^Q_{P\alpha} (T_a)^P_Q = (\Gamma_X)^Q_{P\alpha} (T_a)^P_Q \pm X_\alpha \mathrm{Tr} T_a.
\end{aligned} \tag{D.4}$$

The divergences from matter loops are canceled loops from Z^I, Y_I and $\phi^{(NI)}$, $N = 0, P = 0, T, A = 1, \cdots, n$:

$$\sum_I \Gamma^{(NI)\alpha}_{(MI)} \Gamma^{(MI)}_{(NI)\alpha} = \tilde{N} G^\alpha G_\alpha, \quad \Gamma^{(NI)}_{(NI)\alpha} = \tilde{N} G_\alpha, \tag{D.5}$$

with additional contributions that require a modification of the constraints on the parameters α', β', σ, as in Section 4.1, with $N \to 3\tilde{N}$ in (4.35). When an anomalous $U(1)$ is present we require that some Φ_C carry $U(1)$ charge so as to cancel the last term in (D.4), as described in Section 5.2.

E. Errata

Here we list additional corrections to [12] that involve dilaton couplings, and were not reported in I.

1. The second line of the RHS of the expression (C.48) for $\mathrm{Tr} Y^2$ should read

$$+\frac{x^4 \rho_i \rho^i}{8}\left[\left(F^a_{\mu\nu} F^{\mu\nu}_b\right)^2 + \left(F^a_{\mu\nu} \tilde{F}^{\mu\nu}_b\right)^2 - \left(F^a_{\mu\nu} F^{\mu\nu}_a\right)^2 - \left(F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a\right)^2\right].$$

2. A contribution is missing from T_3^{g+G} in (C.59), namely

$$T_3^{g\alpha} = \left[\hat{L}_{\mu\nu}, \bar{m}\right]^\alpha_a (M^{\mu\nu})^a_\alpha - \left[\hat{L}_{\mu\nu}, m\right]^\alpha_a \left(\bar{M}^{\mu\nu}\right)^a_\alpha + (a \leftrightarrow \alpha) = \frac{\partial_\mu x \partial_\nu y}{x^2} D^a F^{\mu\nu}_a.$$

3. There is a term missing from the expression (C.43), namely a contribtion

$$-3\frac{\partial_\mu x \partial_\nu y}{x^2} D^a F^a_{\mu\nu}$$

involving the graviton-gaugino connection in $2\left(\tilde{D}_\mu \bar{m}\right)^i_a \left(\tilde{D}_\mu m\right)^a_i$.

4. The sign of the first term on the RHS in the expression for $\tau_3^{\chi g}$ in (C.44) is incorrect.

5. A contribution to T^{g+G} is missing from (C.59), namely

$$T_3^{g\alpha} = \frac{\partial_\mu x \partial_\nu y}{x^2} \mathcal{D}^a F^a_{\mu\nu}.$$

6. As noted in I, there are errors in the coefficients of $M^2\mathcal{D}$ in the traces given in Appendix C. For the string dilaton case considered here the changes with respect to the canoncial gauge kinetic energy case considered in I are: -18 in $\frac{1}{2}\mathrm{STr}H^2_\chi$, Eq. (C.36); -14 in $\frac{1}{8}\mathbf{Tr}\,(H_1^{\chi g})^2$, Eq. (C.41); $+2$ in $-T_4^{\chi g}$, Eq. (C.44); $+58$ in $\frac{1}{2}\mathrm{STr}H^2_{\chi g}$, Eq. (C.47); $+52$ in $\frac{1}{2}\mathrm{STr}H^2_{\chi g}$, Eq. (C.62).

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ERNEST ORLANDO LAWRENCE BERKELEY NATIONAL LABORATORY
ONE CYCLOTRON ROAD | BERKELEY, CALIFORNIA 94720

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