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Author
Crawford, Frank S.

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Frank S. Crawford

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Terrestial Timekeeping and General Relativity -- A Discovery?

Frank S. Crawford

Lawrence Berkeley Laboratory and Physics Department

University of California, Berkeley, California 94720

W.H. Cannon and O.G. Jensen (C & J) have written a paper with the above title but without the question mark.[1] Their theory is contained in their Eq. (31) which relates the proper time interval for a clock fixed to the earth to the time interval of a clock fixed in an inertial frame given approximately by the solar system with the earth removed. Their formula is

$$d\tau_o = [1 + \frac{\Phi}{c^2} + \frac{U}{c^2} - \frac{1}{2}\frac{r^2\Omega^2}{c^2}]\, d\tau \qquad (1)$$

where Φ is the Newtonian gravitational potential (due to mass sources), U is the centrifugal potential (due to acceleration), and $-\frac{1}{2} r^2 \Omega^2/c^2 = -\frac{1}{2}\beta^2$ is the correction due to special relativity. According to standard general relativity theory the clock correction formula would be identical to Eq. (1) but with the term U/c^2 removed. C & J believe that acceleration necessarily affects clock rates, because of the equivalence principle, and that therefore they should include the term U/c^2. They also claim experimental support for that hypothesis in their comparison of atomic clock rates for clocks at different elevations and latitudes around the world.

First I will explain why Eq. (1) is wrong from the standpoint of standard relativity theory. (Here they may agree with me). Then I will consider atomic clock experiments where the effect is 10^{13} times greater than those which they consider, and which conclusively rule out their theory.

As C & J point out, the centrifugal potential term U/c^2 and the special relativity term $-\frac{1}{2}r^2\ \Omega^2/c^2$ are numerically equal. That is no accident. They are equal because they are the very same correction. This correction can be called a special relativity correction, from the point of view of the solar system inertial frame; or it can be called a centrifugal potential contribution to the gravitational red shift demanded by the equivalence principle, from the point of view of a frame attached to the rotating earth. The correction should be put in once, not twice. At least that is so in standard relativity theory.

To see why the centrifugal potential term and the special relativity term are equal, and to prepare for comparison with experiments involving a different kind of atomic clock (mu mesons) we consider a beam of muons circling in a storage ring at velocity $v = \omega R$, where R is the radius of the ring and ω is the angular frequency. Provided that accelerations have no effect on clock rates the observed decay lifetime in the laboratory, τ, is related to the proper lifetime, τ_0, in the muon rest frame, by the special relativistic correction factor

$$\tau/\tau_0 = \gamma = (1 - \beta^2)^{-1/2} = 1 + \frac{1}{2}\beta^2 + \ldots \qquad (2)$$

(The same factor of course relates the laboratory and rest frame lifetimes for a linear beam of muons.) Now consider the circling muons from a frame that rotates with the mesons. A clock at the center of this frame reads time τ during a certain number of decays. A clock at radius R reads time interval τ_0. The mesons are at rest in this frame, at distance R from the center. The general relativistic correction factor relating τ and τ_0 is given by the gravitational red shift

$$\tau/\tau_0 = \Gamma = (1 + 2\Delta\phi/c^2)^{-1/2} = 1 - \Delta\phi/c^2 + \ldots \qquad (3)$$

where ϕ is the sum of the Newtonian potential Φ and the centrifugal potential U. In the present example the Newtonian potential is the same at center and periphery, and the entire effect is due to centrifugal potential. The difference in centrifugal potential is obtained by integrating centrifugal force per unit mass times distance, from $r = 0$ to $r = R$. The result is

$$\Delta\phi = \Delta U = -\frac{1}{2}\,\omega^2 R^2. \tag{4}$$

But of course $\omega R = v$, and we see that Γ and γ are exactly equal. They had to be equal because we were merely calculating the same ratio, τ/τ_0, using two difference reference frames.[2] It is clear that in standard relativity theory we should not give both a special relativity correction and a centrifugal potential gravitational red shift correction. In the lab frame there is no centrifugal force and there is a special relativistic correction. In the rotating frame there is a gravitational red shift, but the muons are at rest and there is no additional correction due to special relativity.

For a linear beam of muons, C & J would predict, using Eq. (1) with $\Phi = 0$ (no Newtonian potential), $U = 0$ (no acceleration), and $\beta^2 = r^2\Omega^2/c^2$,

$$\tau/\tau_0 = \gamma = 1 + \frac{1}{2}\,\beta^2 + \ldots\quad . \tag{5}$$

For a circulating beam of muons at low velocities they would predict

$$\tau/\tau_0 = 1 - \Delta U/c^2 + \frac{1}{2}\,\beta^2 + \ldots \tag{6}$$

$$= 1 + \beta^2 + \ldots\quad .$$

C & J only give their theory for $\beta \ll 1$, i.e., for γ ($=\Gamma$) close to unity. What would they predict for high velocities and accelerations? The most plausible extrapolation of Eq. (6) is

$$\tau/\tau_0 = \gamma\,\Gamma \tag{7}$$

This reduces to Eq. (6) for low velocities; and to Eq. (2) for all velocities, when there is no gravitational or centrifugal potential ($\Gamma = 1$).

An alternative possible extrapolation of Eq. (6) is

$$\tau/\tau_0 = \gamma + \Gamma - 1, \qquad (8)$$

with the same limiting behavior. Note that Eq. (5) applied to a circulating muon beam gives the "Twin Paradox" prediction of standard relativity theory. We might call Eq. (7) or Eq. (8) the prediction of a "Super Twin Paradox", since the decay rate is predicted to be slowed by more than a factor of γ.

The formulas that Cannon and Jensen compare with experiment are essentially Eqs. (5) and (6), with the addition of the terms due to Newtonian potential differences. In their comparisons of atomic clock rates, $\beta^2 \simeq 10^{-12}$. Their experimental task is therefore difficult, and their results are inconclusive. We turn to easier experiments, where $\gamma - 1$ is larger by a factor of 10^{13}. For a linear beam of muons Eq. (5) was first used by Bruno Rossi[3] to estimate the rest-frame lifetime τ_0 of hard cosmic ray muons of momentum ≈ 1 BeV/c, and hence $\gamma \approx 10$. He found $\tau_0 = 3 \times 10^{-6}$ sec. Later experiments with muons at rest give $\tau_0 = 2.20 \times 10^{-6}$ sec. Thus Rossi got the right result within experimental errors using Eq. (5). Later experiments with linear beams of accelerator produced mesons have verified Eq. (5) with much higher precision. On this there is no disagreement.

For circulating muons we turn to the "g-2 experiments" performed using the C.E.R.N. muon storage ring.[4] The muons have momentum 1.27 GeV/c and hence $\gamma = \Gamma = 12.1$. The mean life predicted by Eq. (5), which assumes accelerations with respect to an inertial frame do not affect clock rates, is $\tau = \gamma\, \tau_0 = 26.6 \times 10^{-6}$ sec. The experimental decay rate agrees with this prediction to within 1%. The mean life predicted by Eq. (7) is $\tau = 322 \times 10^{-6}$ sec; that predicted by Eq. (8) is 51×10^{-6} sec. Either of these plausible extrapolations of Eq. (6) to high velocities and accelerations is in complete disagreement with experiment. Accelerations have no detectable effect on the rates of muon clocks and the hypothesis of Cannon and Jensen is untenable.

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References

1. W.H. Cannon and O.G. Jensen, Science 188, 317 (1975).

2. τ is the time interval read by a clock in a particular frame, the laboratory, during the time it takes a beam of 1000 mesons to decay to about 368 mesons. Any frame of reference can be used to calculate τ, and all calculations must give the same result. A similar statement holds for τ_0. Hence the ratio τ/τ_0 is independent of the frame used to perform the calculation.

3. B. Rossi, Rev. Mod. Phys. 11, 296 (1939).

4. J. Bailey, W. Bartl, G. Von Bochmann, R.C.A. Brown, F.J.M. Farley, M. Giesch, H. Jostlein, S. Van Der Meer, E. Picasso, and R.W. Williams, Nuovo Cimento 9A, 369 (1972).