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Berkeley, California

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NUCLEAR CROSS SECTIONS FOR 4.2-Bev NEGATIVE PIONS

Fredrick Wikner

(Thesis)

January 10, 1957

Printed for the U.S. Atomic Energy Commission

NUCLEAR CROSS SECTIONS FOR 4.2-Bev NEGATIVE PIONS

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NUCLEAR CROSS SECTIONS FOR 4.2-Bev NEGATIVE PIONS

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Berkeley, California

January 10, 1957

ABSTRACT

Simultaneous transmission measurements in good and poor geometry have been performed at the Berkeley Bevatron to measure the total and absorption cross sections of several nuclei. The negative pions were produced by bombarding a Be target with 6.2-Bev protons. The pions were analyzed by the Bevatron's magnetic field before leaving the quadrant vacuum tank thin window. The pions then passed through two strong-focusing quadrupoles and were further analyzed before entering the counter telescope. The pion beam incident on the attenuator was defined by two 1-inch-diameter counters 12 feet apart. Poor-and-good-geometry counters (5-inch diameter and 4-inch diameter respectively) were located 8 feet and 20 feet from the end of the defining telescope, resulting in a total telescope length of 32 feet.

The total cross sections σ_h and $\sigma_d - \sigma_h$ were measured by attenuation differences in good geometry of CH_2 - C and D_2O - H_2O , with the results:

$$\sigma_h = 28.7 \pm 2.6 \text{ mb,}$$

$$\sigma_d - \sigma_h = 23.0 \pm 2.6 \text{ mb.}$$

The cross sections of the following four elements were measured in good and poor geometry, and the following values of the total and absorption cross section are deduced (in units of millibarns):

	<u>Be</u>	<u>C</u>	<u>Al</u>	<u>Cu</u>
σ_{total}	302	386	763	1620
$\sigma_{\text{absorption}}$	177	219	407	725

The errors vary from 3% to 5% for these measurements.

An interpretation of these cross sections is given in terms of the optical model with two different density distributions, namely a uniform and a tapered density distribution. The absorption cross section data were fitted well with $R = (0.7 \pm 1.2 A^{1/3}) \times 10^{-13} \text{ cm}$ and $R_{\text{eq}} = 1.21 A^{1/3} \times 10^{-13} \text{ cm}$ respectively. The latter value obtained with tapered model density distribution is in quite good agreement with the electromagnetic determinations of the nuclear charge distribution.

I. INTRODUCTION

Information concerning the size and gross features of the interior of complex nuclei has been obtained in recent years when the results of high-energy scattering experiments have been analyzed in terms of the transparent* model of the nucleus.¹ (See Section VI for a brief discussion of the optical* or transparent* model used in this paper).

A considerable number of experiments using neutrons produced by the large cyclotrons at Berkeley, Harwell, Chicago, and elsewhere, with energies up to 400 Mev, have indicated nuclear transparency and a nuclear radius that depends on the atomic number A according to $R = r_0 A^{1/3} \times 10^{-13}$ cm. R is the nuclear radius, and r_0 , a constant, is called the radius parameter. These experiments have been summarized by Nedzel² and satisfactorily interpreted by the optical model.^{3*} Proton- and pion-scattering experiments are less numerous, but yield essentially the same values for the radius parameter r_0 .⁴

These values of the radius parameter r_0 are considerably larger than those obtained recently from electromagnetic measurements of the nuclear charge distribution,⁵ and have resulted in numerous proposals that attempt to understand this difference.⁶ Additional information for discussion has now come from the experiments performed in the Bev region following the completion of the cosmotron. The results of scattering 1.4-Bev neutrons⁷ and 860-Mev protons⁴ from nuclei give values of the nuclear radius parameter r_0 that are significantly smaller than obtained by the nucleons and pions in the 300-Mev region, and begin to approach the values obtained from the electromagnetic measurements.⁵

* Although there are many different kinds of optical model, the term "optical" or "transparent" model of a nucleus as used in this paper refers to that model developed by Fernbach, Serber, and Taylor.¹

Thus we felt it worth while to measure the pion-nucleus cross section at some high energy to see if negative pions would describe nuclei with radii similar to or different from the values found by the very recent high-energy nucleon experiments. As in the Cosmotron experiments, absorption cross sections were obtained directly, making the optical-model analysis quite simple since only a portion of the model was necessary for the calculations. In this analysis it is necessary to know the value of the pion-nucleon cross section at the energy used in the experiment. Therefore, in addition to the nuclear cross sections the negative pion-proton and pion-deuterium total cross sections were measured. (These latter measurements, being of fundamental importance in themselves, are part of a program to measure the negative pion-nucleon cross sections as a function of energy from 2.0 Bev to the maximum pion energy available with the Bevatron. As yet only very preliminary results are available except at the energy used in this experiment.) With the absorption cross sections, and with an assumption for the total pion-nucleon cross section in a nucleus, one can determine the nuclear radius R and the mass absorption coefficient K for nuclear matter. If, in addition, the total cross sections can be determined, one can obtain a value for the mean potential experienced by a negative pion in a nucleus.

We had planned to measure the absorption and total cross sections for Be, C, Al, and Cu, and the absorption cross sections only for Sn and Pb. We were unable to complete work on Sn and Pb because of difficulties in machine performance and subsequent limitations in running time. However, Ise, Lagarrigue, and Pyle have measured the absorption cross sections for Al and Pb at 5.0 Bev/c and at 3.2 Bev, using a multiple expansion cloud chamber.⁸ Assuming that the energy variation of these cross sections is negligible in this energy region, we have averaged their two Pb values for use in our analysis. The agreement between the two methods of measuring absorption cross sections is quite good, as is seen by a comparison of the values obtained for Al. We found that σ_a for Al is 407 ± 10 mb at 4.2 Bev, whereas Ise et al. obtained 400 ± 40 mb at 5.0 Bev/c and 430 ± 40 mb at 3.2 Bev.

II. EXPERIMENTAL METHOD AND EQUIPMENT

A. Experimental Method

To determine the total and absorption cross sections, transmission measurements were performed in good and in poor geometry.

In transmission experiments the cross section one measures is determined by what the detection system after the absorber does not "see." Let us explain this more fully. A beam of particles is defined in some manner--in our work by a counter telescope--and is allowed to strike an absorber. Any one beam particle while it is in the absorber may do one of the following: it may pass through unaffected, it may Coulomb-scatter, it may be elastically scattered by a nucleus in the absorber, or it may undergo a nuclear absorption with subsequent production of one or more secondary particles. (This last event occurs, of course, only in those experiments in which the beam particles have sufficient energy for particle production.)

Let us now consider the angular deflection experienced by each class of scattered particles, and also the variation in radius of the transmission detector as it records one or all types of particles scattered in the absorber. This detector, in our experiment, was a circular counter of radius R , centered on the incident-beam line some distance behind the absorber. (The actual radius of the counter and its location behind the absorber are considered quantitatively below.) Let θ , the angle of deflection of the scattered particles, be measured from the center of the absorber and from the line defined by the incident beam. Let us begin with the radius of the transmission counter large enough so all types of particles scattered in the absorber are recorded by the transmission detector. For large values of θ , the particles present in an annulus near the outer edge of the counter have resulted from particle production in the absorber, since this type of event involves large momentum transfers. This condition remains as θ is decreased until one reaches a value of θ corresponding to the first minimum for diffraction scattering for the element in the absorber. Therefore the outer ring of the counter, from the radius value R_m (corresponding to the first minimum for diffraction scattering) to the outer radius of the

counter, records only particles arising from particle production in the absorber. Now let us remove this ring and consider the central portion of the detector and the type of particles it records as R is decreased towards zero. At the moment it records predominantly pass-through particles, * and Coulomb and elastically scattered particles, although a small number of particles having their origin in inelastic events are present and are recorded. As R (or θ) is decreased, the number of elastically scattered particles detected also decreases, because the solid angle subtended by the transmission counter is decreasing until one finally arrives at a value for R (or θ) where only pass-through particles and particles that have Coulomb-scattered are present and therefore detected in large numbers.

To summarize, then, one hopes to have three separate angular regions, the first of which includes angles from $\theta = 0$ to $\theta = \theta_c$ and contains predominantly pass-through and Coulomb-scattered particles. (θ_c is the angle at which most of the Coulomb-scattered particles are included.) The second region includes angles from $\theta = 0$ to $\theta = \theta_d$, namely that of region one plus most of the nuclear diffraction scattering. (θ_d indicates the angle at which all diffraction scattering is included, that is, the angle at which the first minimum in the diffraction pattern occurs.) The third region would then include all events discussed above.

With this general qualitative picture in mind let us consider the measurement of the total nuclear cross section. This cross section σ_t , is composed of two parts, the absorption cross section σ_a , and the elastic or diffraction cross section σ_d . In order to measure the cross section σ_t ($= \sigma_a + \sigma_d$), in a transmission type experiment, a detector of some finite diameter must be located behind the absorber at a position such that it detects a minimum of those beam particles which have suffered a nuclear collision and it detects at the same time a maximum

* By "pass-through particle" we mean a particle that suffered no collision is passing through the absorber. This type of particle of course interacts with the atomic electrons in the absorber, suffering the usual $\frac{dE}{dx}$ loss in energy.

of the Coulomb-scattered particles. The requirement of minimizing the loss of beam particles due to multiple Coulomb scattering is obtained by using absorbers of limited thickness and by adequate corrections to the data. (The use of thin absorbers maximizes the probability of only one elastic scatter in the absorber. It also requires a high flux of particles for adequate measurements, since the rear counter records only small difference between the transmission with the absorber in the beam and the transmission with the absorber out.) Obviously it is impossible to achieve a situation in which no charged secondary* produced in the absorber will escape detection by the transmission detector, but that experimental arrangement which optimizes the conditions that allow the detector to "see" essentially pass-through and Coulomb-scattered particles only is called a condition of good geometry and therefore gives information concerning the total nuclear cross section σ_t .

The absorption cross section, σ_a , is obtained by locating the transmission counter with respect to the absorber so that it will detect all beam particles that pass through, which Coulomb-scatter and which elastically scatter in the absorber. This is the second region, as discussed above, where the detector subtends the angles from $\theta = 0$ to $\theta = \theta_d$. This experimental arrangement defines the condition of poor geometry and thus gives information concerning the absorption cross section. Actually, the condition described above is one of good geometry for the measurement of σ_a ; however, tradition has it that σ_t is measured in what is called "good" geometry and σ_a in what is called "poor" geometry.

Now it is obvious that there are not three separate and distinct angular regions occupied by particles resulting from one particular

* "Secondary" in this case means particles produced in an absorption-type collision and particles elastically scattered by nuclei. "Secondary" in absorption cross-section measurements refers to particles arising from inelastic events. The use and meaning of the term "secondary" depend, then, on the type of cross section under consideration.

type of event in the absorber. The regions overlap, and one then has the problem of correcting for the secondaries recorded by the transmission detector.

In this experiment we have attempted to make this correction experimentally. For the moment let us consider the measurement of the absorption cross section, σ_a for any nucleus. The principal problem in the determination of σ_a is the elimination from detection by the transmission counter of charged secondaries, produced in an inelastic collision in the absorber. At lower energies than used in this particular experiment this was accomplished by placing absorbers before the transmission detector to remove inelastically scattered particles.⁹ However, at the high energy used in this experiment, 4.2 Bev, the secondaries produced cannot be absorbed out easily because most of them have quite high energies. Therefore the following method was used to determine this effect.

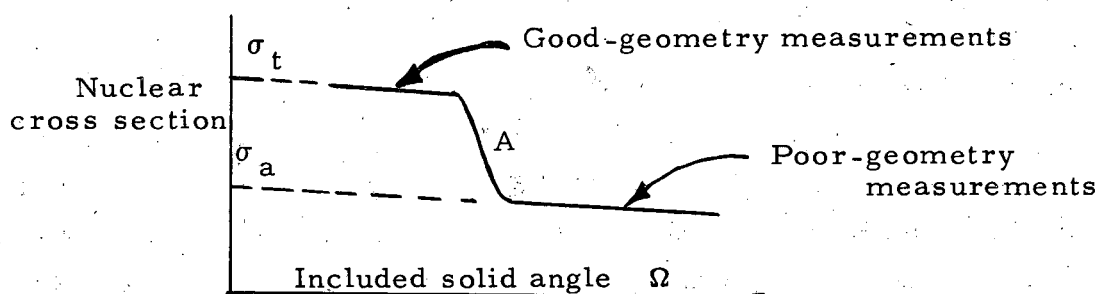
The beam particles were defined by a counter telescope, and some distance from the end of this telescope the poor-geometry transmission counter was located. The absorber was placed at various positions between the end of the defining telescope and the face of the transmission detector. Each absorber position defines a half-angle θ subtended by the transmission counter, and thus an apparent cross section

$$\sigma(\theta) = \frac{1}{N_x} \ln r(\theta), \quad (1)$$

where N_x is the number of atoms per cm^2 in the absorber and $1/(r(\theta)) = I(\theta)/I_0$ is the observed transmission within the included angle θ . This is the cross section for all interactions removing beam particles from the solid angle Ω defined by θ , minus the cross section for the production of secondaries entering the transmission counter. The effect of these particles is to cause the apparent cross sections $\sigma(\theta)$, measured at various subtended solid angles Ω , to decrease with increasing Ω even beyond those angles for which

elastic processes should have an effect. Now it is generally believed that the effect of these secondaries can be eliminated by extrapolating the values of the cross section measured at various solid angles to zero solid angle. It is assumed that the angular distribution of the secondaries is essentially uniform within the small angular region of concern, and thus that the cross section varies linearly with subtended solid angle. Although the discussion above has been concerned with the problem of charged secondaries in the determination of the absorption cross section σ_a , the same process can be and was used in the determination of the total cross section σ_t .

This method of measurement is more easily understood by reference to the figure below.



This figure is schematic and is meant only to illustrate the technique used in this experiment. The presence of the secondaries is shown by the negative slope of the lines in both geometries. The rapid rise in the curve at A reflects the loss from detection of the diffraction scattering by the transmission counter. This loss occurs as the solid angle subtended by this counter at the absorber is decreased, in the angular region where this type of scattering is important. It should be emphasized that extrapolation of measurements taken in poor geometry, as defined above, to zero included solid angle does not yield the total nuclear cross section, but rather the total absorption cross section.

The experimental arrangement is shown schematically in Fig. 1. Figures 2 to 4 are included to show the equipment employed in this experiment. The general layout and appearance may be considered typical of many counting experiments performed at the Bevatron.

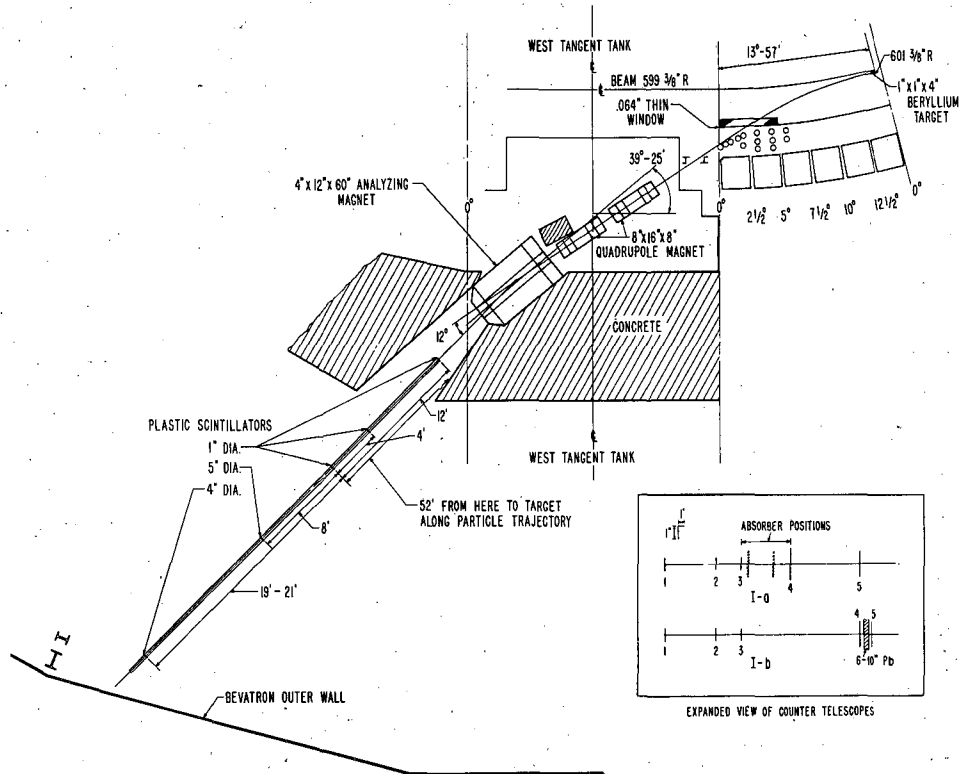


Fig. 1. Experimental arrangement.

Fig. 2. Bevatron west tangent tank and platform

West tangent tank

Southwest quadrant begins here

.064-in. thin window located back in
the quadrant structure

Remnant of the K-lifetime
experiment

Iron slabs located on side
of analyzing magnet to insure
a snug fit in the concrete shielding

4.3-Bev/c negative pions, produced by
protons striking a Be target
passed between two notched
I-beams of the support structure
of the southwest quadrant, and then

entered the two 4-in. -diameter
strong-focusing quadrupole
systems;

then passed through a 5-foot
analyzing magnet buried in the
concrete shielding surrounding
the Bevatron

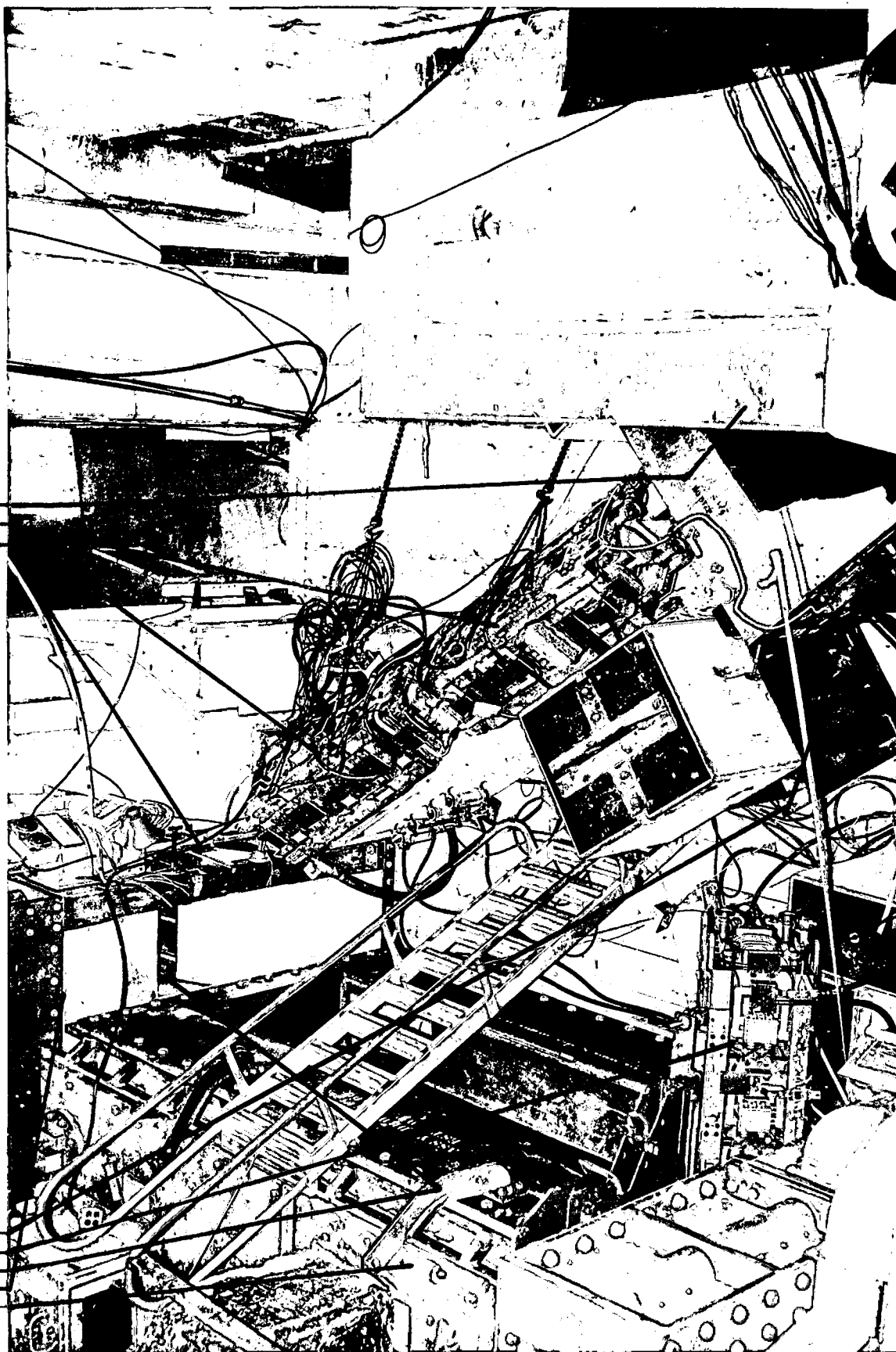


Fig. 3. Bevatron west tangent tank experimental area
early run experimental arrangement

Defining counters No. 2 and No. 3 - rear counters No. 4 and No. 5 located,
respectively 19 and 21 ft from counter No. 3

Electronics used in the experiment

This 5-foot magnet was located in a neutral beam for another
experiment and is identical to the one used in the negative-pion
experiment

Copper absorber

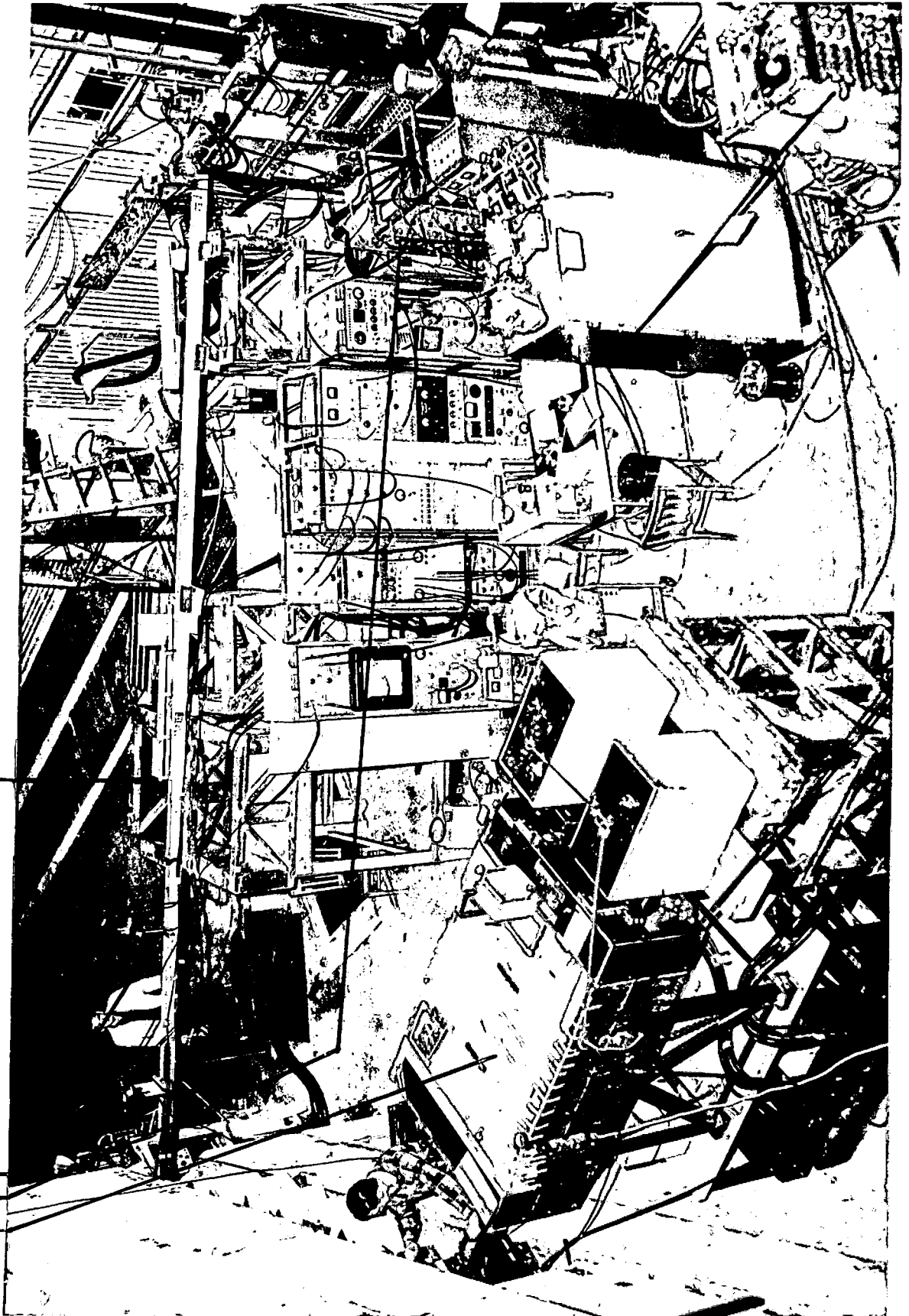
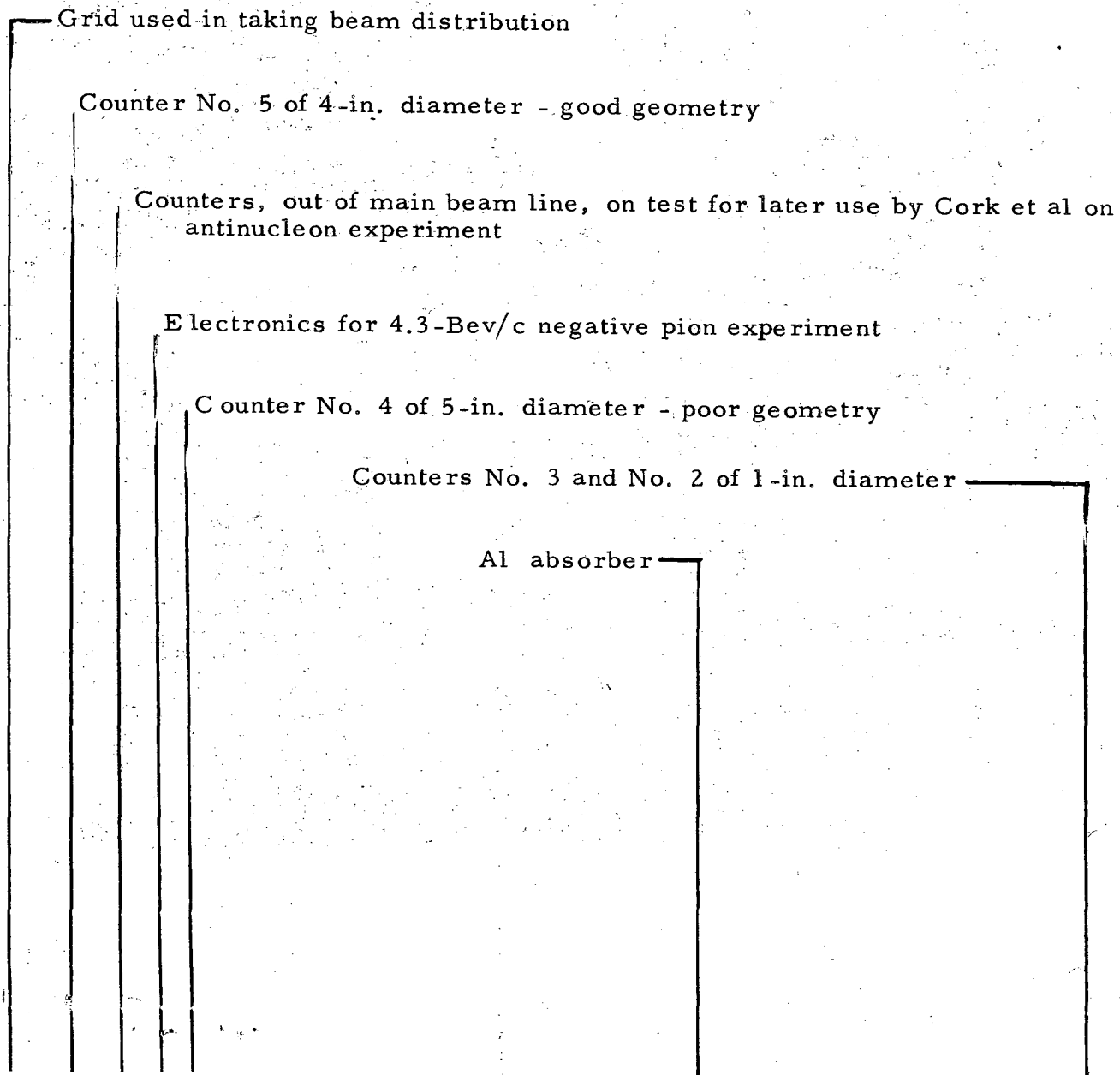
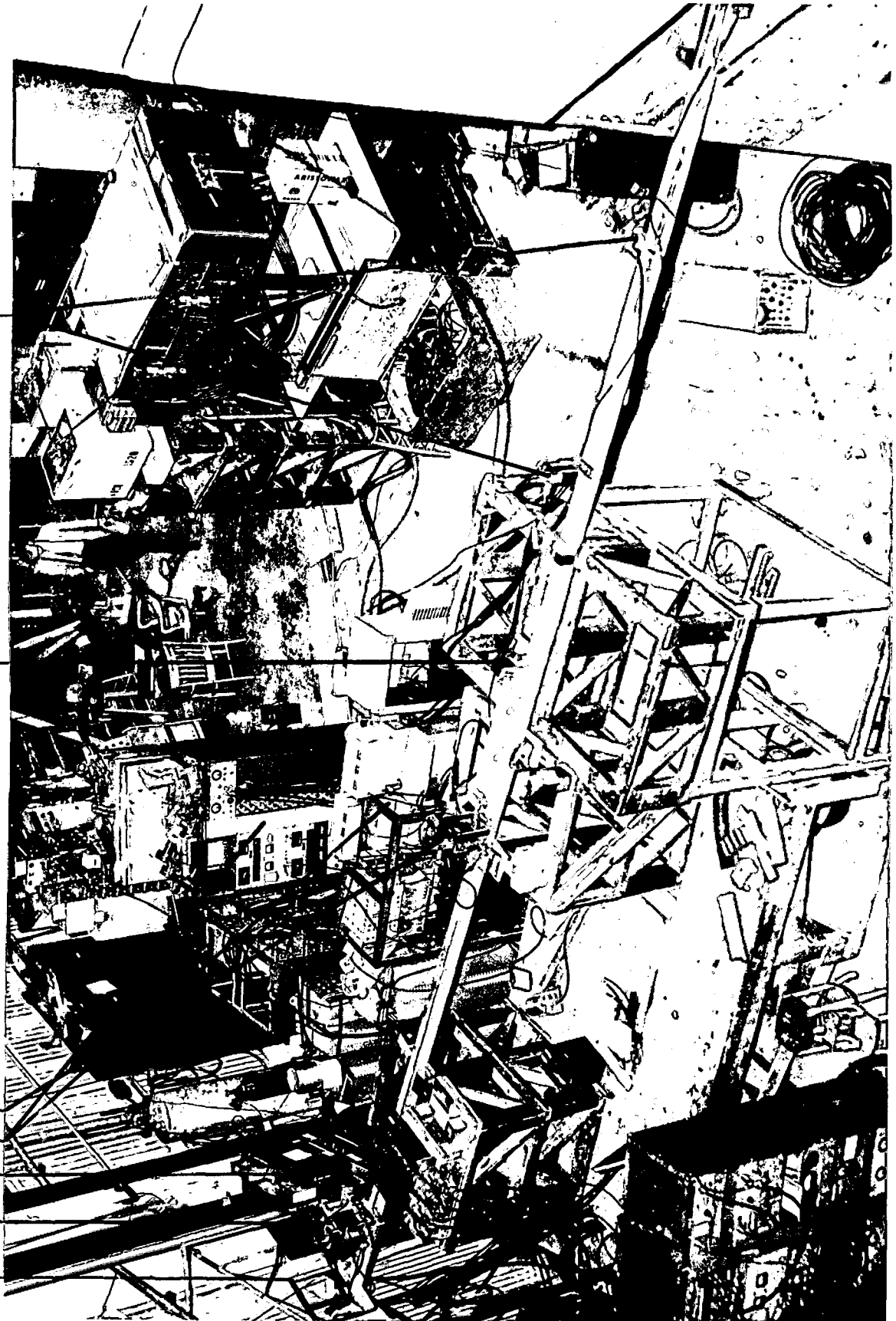


Fig. 4. Bevatron west tangent tank - experimental area
data-taking experimental arrangement





The particles used in this experiment were produced by allowing the internal proton beam of the Bevatron to strike a 1-in. -square 4-in. -long beryllium target located approximately 14° back inside the south-west quadrant of the Bevatron. The negative pions produced in the forward direction were analyzed by the Bevatron's magnetic field; the pions of 4.3 Bev/c passed through the thin window in the vacuum-tank wall, and, avoiding the supporting structure of the Bevatron, entered the auxiliary experimental magnetic fields (Figs. 1 and 2). The pions were focused by two sets of strong-focusing quadrupoles and magnetically analyzed again before impinging on the counter telescope, which began within the concrete shielding and extended to the outer wall of the Bevatron building.

The instantaneous counting rate was greatly reduced by allowing the beam to spill on the Be target over 130 milliseconds.* We achieved this spread by turning off the rf acceleration voltage slowly near the end of the acceleration cycle, aided, when necessary, by the simultaneous injection of noise into the rf system, properly timed, to achieve a uniform beam spill. We were then able to use the maximum beam intensity of the Bevatron's circulating proton beam, on occasions 2 to 3×10^{10} protons per pulse.** With the arrangement shown in Fig. 1, a triples counting rate of 150 to 200 pions per 10^{10} protons resulted.

A telescope of three 1-in. -diameter plastic scintillation counters, in triple coincidence, defined the beam of 4.2-Bev negative pions. The absorber was placed behind Counter 3 and the transmitted beam was measured by two counters separately in quadruple coincidence with the defining telescope. One of these counters (No. 4, of 5-in. -diameter) was located 8 ft. from Counter 3 and the other (No. 5, of 4-in. -diameter) was located 19 ft. from Counter 3. The location and size of Counter 4 were chosen so that the nuclear diffraction scattering of the elements under consideration in this experiment could be easily contained within the range of half angles θ available to an absorber placed at

* This time dispersion led to an energy spread in the proton beam of 5.7 to 6.2 Bev, though it had no effect on the energy of the pion beam.

** March - April - 1956. During Jan. 1957 proton beam intensities of 10^{10} particles/pulse were obtained quite frequently.

positions between Counters 3 and 4. That this is indeed the case can be seen in Table IA below, where the values of the first diffraction minimum are given for Be, C, Al, and Cu. The values were computed from the relation $\theta_m = 0.61 \frac{\lambda}{R}$. θ_m is the value of θ at the first diffraction minimum; λ is the wave length of the incident negative pion, and for this experiment has the value 2.883×10^{-14} cm; R is the radius of the nucleus. R was assumed to have the following $A^{1/3}$ dependence, $R = 1.3 \times A^{1/3} \times 10^{-13}$ cm. The table also gives the half-angle θ subtended by Counter 4 for absorbers located at 0, 1, 2, 3, 4, and 5 feet, respectively from Counter 3. From this table it is evident that the geometry selected is adequate for the determination of the absorption cross sections in the manner described above. (The method used for correcting for residual diffraction scattering can be found in Sec. VI-B.)

Table I

A			B		
	Values of θ_m (radians)	$A^{1/3}$	Absorber position (distance from Counter 3) (feet)	Values of the half-angle (in radians) subtended by	
				Counter 4	Counter 5
Be	0.0651	2.08	0	0.0261	0.00877
C	0.0591	2.29	1	0.0298	0.00926
			2	0.0347	0.00980
Al	0.0451	3.00	3	0.0415	0.0104
Cu	0.0339	3.99	4	0.0521	0.0111
			5	0.0694	0.0119

The location and size of Counter 5, were chosen so that the conditions of good geometry could be realized for both the pion-nucleon total cross sections and the pion-nucleus total cross sections with the various absorbers located at positions identical with those used in the absorption-cross-section measurements. The half-angles subtended by Counter 5 for these positions are also given in Table IB. The lengths of absorbers used in this experiment (see Table III) were such that the multiple Coulomb scattering correction was never greater than 10%.

(See Sec. V. for a discussion of this correction.) Here also it is evident that the geometry selected is adequate for the determination of the total nucleon and total nuclear cross sections as discussed above.

With the absorber located at various positions between Counters 3 and 4 simultaneous attenuation measurements could be made pertaining to the absorption and total cross sections for the elements considered. By moving the absorber, rather than the rear counters (numbers 4 and 5), maximum use was made of the total beam time, since it was not necessary to change any delay lengths for the rear counters as is required in those experiments in which the absorber is fixed and the transmission counter position varied to change Ω .

Therefore with the transmission measurements, coupled with suitable corrections and extrapolations, one can obtain σ_t from the poor-geometry measurements and σ_t from those taken in good geometry. The main problem in the determination of σ_a , as mentioned before, is the elimination of the recording of charged secondaries in the rear counter (number 4).

A more serious problem is encountered in the determination of σ_t , because of the unknown interference between Rutherford scattering and the diffraction scattering. At these high energies, 4.2 Bev, this interference makes the determination of the total cross section exclusive of Coulomb scattering difficult except for elements of low Z.

B. Equipment

All counters used in this experiment were made by using a plastic scintillator consisting of a solid solution of terphenyl and tetraphenyl-butadiene frequency shifter in a polystyrene medium. The three defining counters were 1-in. -diameter disks, 0.5 in. thick, viewed on edge by one RCA 1P21 photomultiplier. The two rear counters were 1 in. thick; No. 4 had a 5-in. -diameter crystal and No. 5 a 4-in. -diameter crystal; each scintillator was viewed by 4 RCA 1P21 photomultipliers symmetrically placed. Their outputs were passively added at the counter before amplification.

The signals from all counters were amplified by two Hewlett-Packard 460-A distributed amplifiers, or one 460-A and one 460-B, depending on the sign of the output used from the counter. The unclipped amplifier pulses then entered the various coincidence circuits that were used to form triple coincidences for the defining telescope and quadruple coincidences for the poor- and good-geometry arrangements. In addition, pulses from Counters 1 and 3 were used again to form a coincidence, which served as a check on the other electronics. Accidentals in the defining telescope were measured simultaneously by another circuit during each run. An electronic block diagram is shown in Fig. 5.

The coincidence circuits were of a type designed by Garwin,¹⁰ but modified in various different ways by Wiegand,¹¹ Madey,¹² and Evans.¹³ The resolving time (the half width at half maximum) of counters plus coincidence circuits was measured and found to be approximately 10^{-8} sec. This resolution proved quite adequate for the running conditions encountered. The negative output of the Wiegand and Evans circuits was amplified by a 460-B, the resultant pulse then being sufficient to drive a Hewlett-Packard 10-megacycle scaler (which was modified to allow gating). The final scaling was done by a UCRL-1024 scaler. (The output of the Madey circuit was handled as described in Ref. 12.)

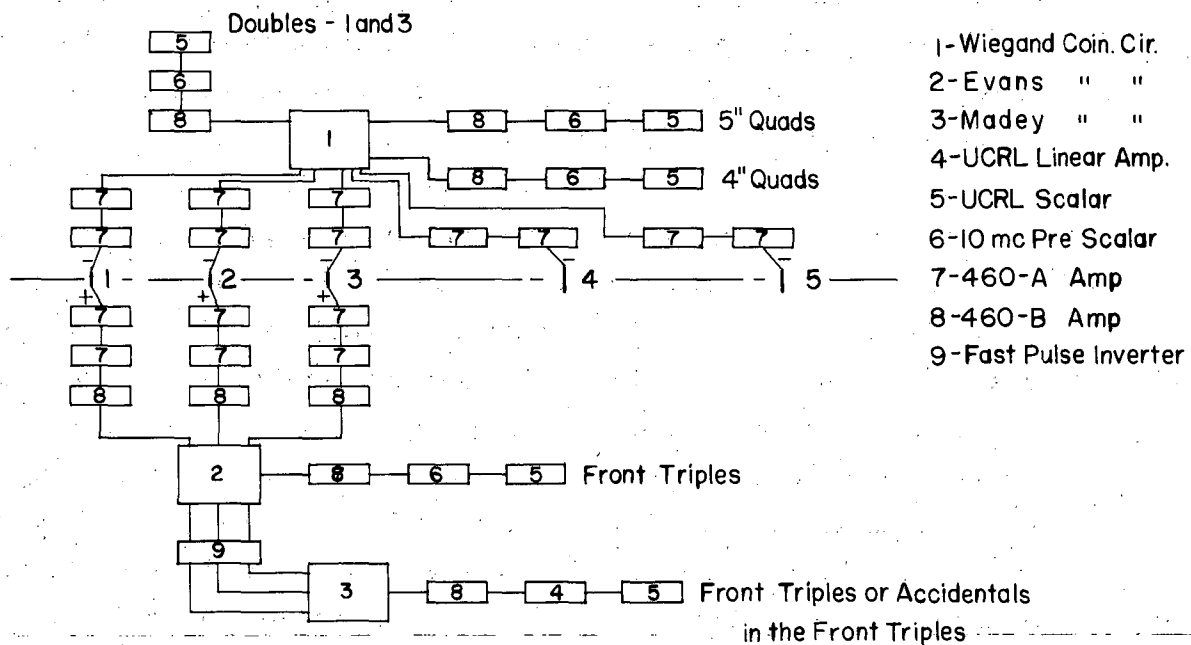


Fig. 5. Block diagram of electronics

An auxiliary monitor¹⁴ using a signal from the east induction electrode was employed as another rough check on the general operation of the electronic system.

The strong-focusing quadrupoles had an effective aperture slightly less than 4 in. in diameter and could produce gradients near 8000 gauss/in. The analyzing magnet had a 4-by-12-in. gap with an effective gap length of 60 in. A field of 20,000 gauss can be obtained with this magnet, and one could therefore deflect 4.3-Bev/c particles through $\sim 12^\circ$ quite easily, using less than the maximum field strength available.

III. AUXILIARY CALCULATIONS AND MEASUREMENTS

In order to locate the experimental equipment, described above, theoretical negative-pion orbits of the desired momentum were drawn by use of quarter-scale drawings of the Bevatron's magnetic field and a mechanical analyzer (affectionately known as "The Bug").¹⁵ The magnetic field measurements were taken by the UCRL Magnetic Measurements Group. This series of measurements gave values of the field at various positions as a function of time during the acceleration cycle, and those points of interest to us were reduced from the original data for use in this experiment.¹⁶ Negative pions produced near 0° from the 14° Be target, of 4.3 Bev/c, came out of the machine at a convenient place and were selected for use in this experiment. With this momentum the currents in the strong-focusing magnets were calculated by use of an analog computer, and the current in the analyzing magnet was selected using known data for the magnet to deflect negative pions $\sim 12^\circ$. No experimental time was taken to tune the strong-focusing magnets, and the final current setting in the analyzing magnet was chosen in accordance with previous calculations after some brief experimental investigations to show that the beam was as anticipated in the experimental area. Estimates were made of the errors involved in the machines and methods used in the calculations, plus the momentum spread accepted by the telescope, and the momentum in this experiment was taken to be 4.3 ± 0.2 Bev/c.

The beam of particles used was predominantly negative pions; contamination was due mainly to muons and a negligible number of electrons. The muon contamination was found by calculation to be 4%. The calculation was quite simple and was done considering the mean distance against decay for the pions, the solid angle subtended by the counting system along the path of the pion, and the limitations imposed by the magnetic field on the total decay volume available to the resulting μ meson compatible with subsequent detection. Similar considerations yield an electron contamination resulting from π^0 decay less than 1%. In a short preliminary run in which the π^- -proton cross section was

under consideration, using a counting arrangement similar to that in Fig. 1-B, the cross-sectional value measured with 6 in. and 10 in. of Pb between the two rear counters was found to be the same as that determined with no Pb present. The error in these measurements was 3 to 5%.

For the purpose of computing the multiple-scattering correction, it is necessary to know the distribution of the beam over the 1-in. diameter of the defining telescope. This was done by counting with a small counter (3/4-in. diameter), at different positions on a grid system at the end of the Al rail, in quadruple coincidence with Counters 1, 2, and 3. No appreciable divergence of the beam was found. The measurement in the horizontal plane is shown in Fig. 6.

In transmission-type experiments it is essential that the large-diameter rear counter be of high efficiency and uniform in response over its surface. If it is not, all transmissions measured will be too small and the resulting cross sections too large. Counters 4 and 5, when viewed with four photomultipliers symmetrically placed, yielded an efficiency and uniformity of response of essentially 100%. This was determined by exploring the face of each counter with a 1-in. -diameter high-energy cosmic-ray telescope. The statistical accuracy of the measurements was 2.5% to 4%.

Accidental coincidences were measured by delaying in time any one counter by inserting 2 to 5×10^{-8} sec of cable into the counter in question. With the beam spilled over 130 milliseconds, and with the counting rates described, it was not surprising to find the accidental rate usually negligible. If we noted a rise to something like 1% it could usually be traced to a fluctuation in the machine operation, which allowed bunching and large irregularities in the uniformity of the beam spill. As the experiment progressed we learned how to regulate the behavior of the final stages of the acceleration so that essentially no accidentals were present.

This method of measuring accidental rates yields partial rates, so that one is left with the problem of how to combine them. In addition, the rate is overestimated, since a large fraction of the

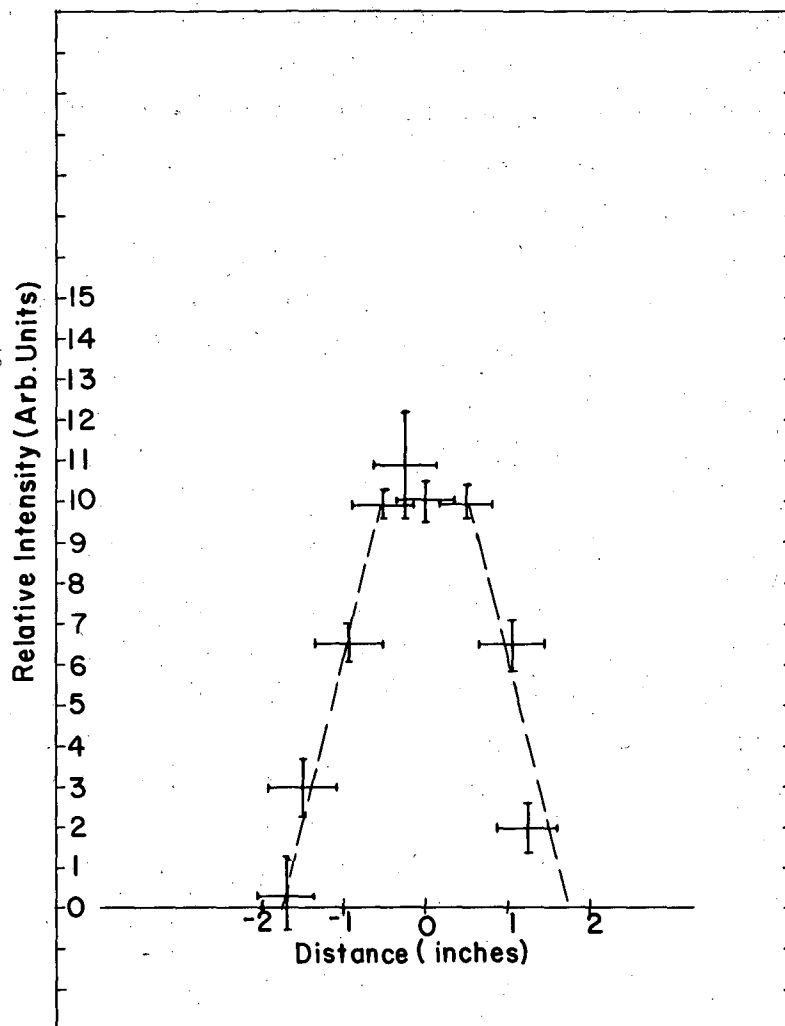


Fig. 6. Distribution of the beam defined by the first three counters as seen by the probe counter at the end of the counter telescope (see Fig. 4.) The size of the probe counter is indicated by the horizontal limits. (Measurement made in horizontal plane.)

counting rate is due to true beam particles. Thus a displacement or delay of one counter can still allow a beam particle to give rise to an accidental count that under normal circumstances would be a true coincidence. Since the rate as measured was small, this overestimation is not a serious problem and was ignored.

With the analyzing magnet turned off no coincidences were observed. No coincidences were observed when, with the magnet on, the target failed to operate.

The counting system was plateaued before the run by the use of high-energy cosmic rays. Occasion was taken during the experiment to check these high-voltage plateaus, and exact agreement was found with the cosmic-ray results.

IV. EXPERIMENTAL PROCEDURE

The strong-focusing quadrupoles, analyzing magnet, and counter telescope were located in the median plane of the Bevatron along the line calculated from the magnetic field plots. The photomultiplier high voltages were set at the values obtained from the cosmic-ray measurements; each counter within the counter telescope had an appropriate amount of cable between itself and the coincidence circuit consistent with its location in the flight time of the pions. During the experiment the electronics was frequently pulsed through by using a millimicrosecond pulser to check the triggering level of the counting system; appropriate adjustments were made when necessary. The efficiency of the counting system is the ratio of the number of particles detected by the defining telescope (i. e., front triples, Fig. 5) to the number of particles detected by the total telescope (i. e., front triples plus either transmission counter Quads in Fig. 5 in the absence of an absorber. This efficiency was 100% for the poor-geometry arrangement and 98% to 100% for the good-geometry arrangement, throughout the run. Scattering by the defining counters and by the air probably accounts for the 2% variation, since the counter itself was found to be 100% efficient. Accidental rates and magnet current settings were frequently checked during the experiment.

The absorber position was varied from a location immediately following Counter 3 to the face of Counter 4. Air runs--i. e., absorber out--were usually taken at the beginning, middle, and end of a series of measurements for each element. In the computation of the transmissions for each element all air runs were averaged for those cases in which the electronics had no known drifts or difficulties. On those few occasions when the electronics was not operating as well as we liked, but still in acceptable manner, an air run was taken immediately before or after an absorber run (or both), and the transmission was computed from these data. Some experimental points were repeated on separate days during the experiment and were found to be the same within statistical error. The hydrogen and deuterium cross sections were obtained with two different lengths and types of absorber, and in a few cases the nuclear cross sections were measured with two different absorber thicknesses.

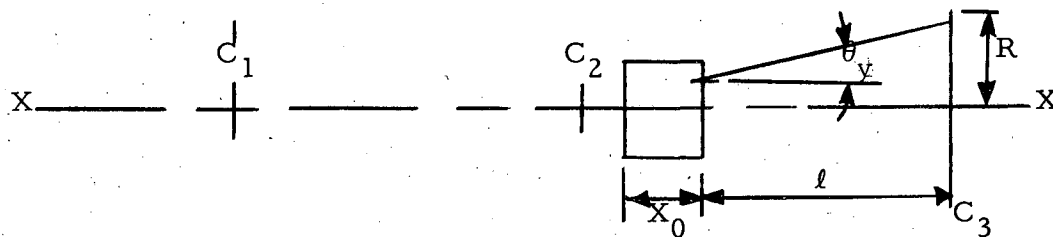
V. GENERAL CORRECTION TO THE DATA

In this section those errors which might be considered of a general nature are discussed, whereas errors particular to one cross section measurement are discussed in the section on Experimental Results (Sec. VI).

The precision of the measurement of negative-pion total nuclear cross sections depends, principally, on the treatment of the corrections necessitated by Coulomb scattering in the absorber and by interference effects between Coulomb and diffraction scatterings. (The effect of the Coulomb interaction on the poor-geometry measurements is discussed in Sec. VI-B).

The method used to calculate the fraction of the incident particles that miss the transmission counter behind the absorber, due to single, plural, and multiple Coulomb scattering in the absorber is due to a paper by R. M. Sternheimer.¹⁷

He considers a general experimental arrangement having an arbitrary number of counters defining the beam incident on the absorber of length X_0 , in radiation lengths, and a transmission counter of radius R located a distance l from the absorber. Circular symmetry about $X-X$ is assumed. The only relevant properties of the incident beam are the radius of the circle that would be covered by the beam at C_3 in the absence of scattering and the radial distribution of the particles inside the circle. When this distribution is assumed to be uniform the results of Sternheimer's paper can be used directly. This was done in this experiment--i.e. a uniform distribution was assumed--for these corrections.



If the beam had been extremely narrow, the particles that have experienced multiple Coulomb collisions when they emerge from the absorber would have a Gaussian distribution in the deflected angle, and this Gaussian can easily be integrated over the solid angle subtended by the transmission counter. For a beam of finite extent, however,

particles off the axis of the beam will have Gaussians centered about different points in the cross section of the beam, and this then necessitates an additional integration over the beam distribution. The results of Sternheimer's calculations are in the form of curves of F as a function two parameters r_0 and ρ_0 ($1 - F$ is the fraction of the beam lost due to multiple Coulomb scattering; that is, the observed transmission is $F \cdot I/I_0$ instead of I/I_0 , the ratio desired). Here r_0 is a function of the rms scattering angle and the geometrical arrangement (not a radius, but rather a quantity characterizing the geometry and amount of multiple Coulomb scattering in the absorber), and ρ_0 is the radius of a uniform circular beam in units of the radius of the transmission counter. For each experimental point, we computed r_0 using radiation lengths given by Rossi,¹⁸ and then $F(\rho_0)$ was found for each ρ_0 from Sternheimer's curves.

The results of Sternheimer's paper also enable one to conclude that losses due to single and plural Coulomb scattering are very small and negligible.

Accidental coincidence corrections are hard to make, since the accidental rate is difficult to measure and fluctuates with beam bunching and beam intensity. However as the measurements indicated a small accidental rate, only a few values of $\sigma(\theta)$ out of all the measurements were corrected. This correction amounted to a simple subtraction with a 10% increase in the statistical error.

The possibility of double diffraction scattering was minimized by using absorber thickness of $1/4$ to $1/2$ a mean free path. (The thickness of the Cu absorber was chosen considerably smaller than $1/4$ of a mean free path in an effort to keep the multiple Coulomb scattering corrections in good geometry less than 10%). The mean free path λ referred to is the mean free path for total nuclear events. That the probability for double diffraction scattering is small can be seen from the following argument.

The probability for an elastic collision is

$$P_1 = (x/\lambda_d) e^{-x/\lambda_t},$$

where x is the absorber thickness in mean free paths, λ_d is the

mean free path for diffraction scattering, and λ_t is the mean free path for total nuclear collisions. The probability of two diffraction scatters is

$$P_2 = \frac{1}{2!} \left(\frac{x}{\lambda_d} \right)^2 e^{-x/\lambda_t}$$

and the ratio of P_1 to P_2 is $2 \lambda_d/x$. Now consider a total cross section σ_t for some nucleus in which $\sigma_t/2 = \sigma_a = \sigma_d$. For this case we have $\lambda_d = \lambda_a = \frac{\lambda_t}{2}$. Therefore if we are using an absorber with $x = \frac{\lambda_t}{4}$ or $\frac{\lambda_t}{2}$ then $P_1/P_2 = 16$ and 8 respectively. However, in no case was the absorber thickness $x = \frac{\lambda_t}{2}$, as can be seen in Table II. Here are recorded the values of λ_t and λ_d as calculated by using the absorber lengths and cross-section values given in Table IV. It is quite evident that the probability for double scattering is less than 10% of that for a single scattering. Double scattering has its effect on the angular distribution of elastic scattering, and here it merely smears out and widens the diffraction pattern slightly, since the doubly scattered particles have a distribution which is essentially the convolution of the diffraction pattern by itself. The effect on $\sigma(\theta)$ is small and was therefore neglected.

Table II

Mean free paths for C, Al, and Cu computed from data of Table IV. The final two columns give the lengths of the absorber used in the experiment in units of the total and diffraction mean free paths.

	Density (g/cm ³)	Length used in experiment (g/cm ²)	λ_t (g/cm ²)	λ_d (g/cm ²)	x_t (mean free paths)	x_d (mean free paths)
C	1.60	19.54	52.8	120	0.37	0.163
Al	2.70	26.26	58.9	126	0.456	0.209
Cu	8.95	19.5	90.5	146.5	0.215	0.134

The beam has a nonnegligible diameter, in the sense that it is not extremely narrow. Because of this the observed angular distribution is a superposition of diffraction patterns centered on different points in the beam cross section. The effect is to increase $\sigma(\theta)$ for intermediate angles and to leave $\sigma(\theta)$ unchanged in the limits of both good and poor geometry. Sternheimer has shown that the computation of this correction is similar to that of finding the multiple scattering correction, and that one can use the same curves for F if the diffraction pattern is replaced by a Gaussian

$$1/4 K^2 R^2 \exp \left[-1/4 (KR\theta)^2 \right] \approx \left[J_1(KR\theta)/\theta \right]^2$$

which closely approximates the optical-model pattern up to the first minimum.¹⁷ Here R is the nuclear radius, K the propagation constant, and J the first-order Bessel function. The error $\Delta\sigma$ in $\sigma(\theta)$ is given roughly by

$$\Delta\sigma = -\sigma_d \left[F(r_0, \theta) - F(r_0, \rho_0) \right],$$

where σ_d is the total diffraction cross section, and r_0 is given by $2/KR\theta$. As this correction is important only at points intermediate between the good- and poor-geometry conditions, and, since we believed that the geometries achieved in this experiment were quite good, no correction was made to those few values of $\sigma(\theta)$ taken at points in transition from the conditions of good or poor geometry to intermediate geometry.

The presence of Counter 4 had a negligible effect on the transmitted beam, and no corrections to the total cross sections were necessary because of its position between the absorber and the good-geometry counter.

All values of $\sigma(\theta)$ taken in this experiment were corrected for the calculated 4% contamination of μ mesons present in the negative-pion beam incident on the absorber.

VI. EXPERIMENTAL RESULTS

A. The Nucleon Total Cross Sections

In the determination of the pion-proton, pion-"neutron" total cross sections we were unable to measure $\sigma(\theta)$ as a function of the solid angle subtended by the transmission counter, because of insufficient running time. In order to correct for those charged secondaries which passed through the rear counter (No. 5), we used data obtained with a high-pressure hydrogen diffusion cloud chamber concerning the angular distributions of elastically scattered and charged secondary particles. This total correction is quite small, as was expected with the geometry used in this experiment.*

By measuring the difference in attenuation between polyethylene, $(\text{CH}_2)_n$, and carbon, and between $(\text{CD}_2)_n$ and $(\text{CH}_2)_n$, or D_2O and H_2O , one can obtain the pion-proton and pion-"neutron" cross sections, respectively. However, it has been shown both by experiment¹⁹ and by theory²⁰ that one does not really measure the "free-neutron" cross section by a deuterium-hydrogen difference. The nucleons in deuterium can shield each other a portion of the time, and this "shadowing" effect reduces the deuterium cross section so that it is not equal to the sum of the two elementary cross sections. Thus, the value of the π^- -neutron cross section obtained in this way is less than the π^+ -proton cross section; these two values are necessarily the same if the principle of charge symmetry is to hold. The correction $\delta\sigma = \sigma_d - \sigma_n - \sigma_p$ has been computed by Glauber²⁰ as 3.3 to 5.3 mb, and measured by Cool, Piccioni, and Clark¹⁹ as 6 ± 2 mb. This is an average value obtained

* An examination of all the cloud chamber inelastic events, obtained with 5.0-Bev/c pions, failed to yield one prong with the requirements of angle and direction such that it would have been detected by a transmission counter in the geometrical arrangement used in this experiment. The numbers of events were 60 two-prong, 38 four-prong, and 3 six-prong, making a total of 101 events containing 240 particles.²²

from measurements with pions between 0.79 and 1.5 Bev. The variation in the computed value of the $\delta\sigma$ arises from the following uncertainties. (a) The deuteron wave function is not known very accurately at small distances, and the use of different wave functions produces the variation in $\delta\sigma$. (b) The shielding calculation was developed from a general form of interaction, but in order to evaluate the effect it was necessary to assume a black-sphere model for the particles. It is by no means clear that such a description is adequate at these high energies; however, it appears that the calculation provides a reasonable explanation for the observed affect. (Cool et al. feel that their results are in substantial agreement with the calculations.¹⁹).

The results of the nucleon measurements are given in Table III, where the essential parameters and the absorber thicknesses are given. The "corrected" pion-neutron value is obtained by adding 5 mb to the value obtained in the deuterium-hydrogen subtraction. For comparison, the value of the pion-proton cross section as determined with a 35-atmosphere hydrogen diffusion cloud chamber and 5.0-Bev/c negative pions is included.²¹ This value has been corrected for muon contamination but, as yet, no correction (if any) has been made for a low scanning efficiency for the detection of events having no charged secondaries. This might amount to 2 or 3 mb at the very most,²² and would leave unexplained the remaining small difference in the values of the cross sections as determined by the two method of measurement.* (See Section III on rear-counter uniformity and efficiency.) Table III also includes the average value of the cross sections where the error includes an estimate, 0.5 mb, to include any systematic errors present in the experiment.

* A more detailed explanation of the errors involved in diffusion cloud chamber measurements can be obtained from George Maenchen UCRL Berkeley, California.

Table III

Total nucleon cross sections for 4.2-Bev negative pions

Four-inch-square blocks

$$\left. \begin{array}{l} \text{Ch}_2, 45.53 \text{ g/cm}^2 \\ \text{C}, 39.03 \text{ g/cm}^2 \end{array} \right\}$$

$$\sigma_h = (29.8 \pm 2.1) \text{ mb}$$

Half-angle subtended by the transmission counter, $\theta = 0.5^\circ$

Two-inch square blocks

$$\left. \begin{array}{l} \text{CD}_2, 58.00 \text{ g/cm}^2 \\ \text{CH}_2, 50.70 \text{ g/cm}^2 \\ \text{C}, 43.30 \text{ g/cm}^2 \end{array} \right\}$$

$$\sigma_d = (50.8 \pm 2.8) \text{ mb}$$

$$\sigma_h = (27.6 \pm 2.7) \text{ mb}$$

$$\sigma_d - h = (23.3 \pm 2.5) \text{ mb}$$

$$\theta_{1/2} = 0.5^\circ$$

D₂O - H₂O Subtraction

$$\left. \begin{array}{l} \text{D}_2\text{O}, 67.0 \text{ g/cm}^2 \\ \text{H}_2\text{O}, 60.9 \text{ g/cm}^2 \end{array} \right\}$$

$$\sigma_d - h = (22.7 \pm 2.5) \text{ mb}$$

$$\theta_{1/2} = 0.5^\circ$$

Average of Results (Error includes an estimate of any systematic errors.)

All values in millibarns.)

$$\sigma_d = 50.8 \pm 2.8$$

Diffusion Cloud Chamber
Result at 5.0 Bev/c

$$\sigma_h = 28.7 \pm 2.6$$

$$20.7 \pm 2$$

$$\sigma_d - h = 23.0 \pm 2.6$$

σ_{un} (obtained by adding 5 mb to d-h value)

$$= 28.0 \pm 2.6 \text{ mb}$$

B. Absorption Cross Sections for Nuclei

Absorption cross sections were determined in this experiment for Be, C, Al, and Cu. The most serious problem encountered in the measurement of σ_a , as we have mentioned before, is the elimination of charged secondaries from detection by the transmission counter. We recall that the effect of these secondaries is to increase the apparent transmission and thus decrease the apparent cross section as the angle θ subtended by the rear counter is increased. The method employed to correct for this effect, as described above, was to plot the values of $\sigma(\theta)$ obtained at the various values of Ω against Ω , in the region of Ω where diffraction scattering is included, and to then extrapolate the resulting straight line to zero solid angle. (See Sec. II-A for a more complete discussion.) Figure 7 gives two examples of this type of extrapolation. The points plotted were corrected for the beam contamination by μ mesons, and, when necessary, for any residual diffraction scattering present. (This latter correction is described briefly below).

It is apparent from the discussion in Sec. II-A that there are absorber positions in the poor-geometry range where the solid angle Ω included by the detector is not sufficient to contain all the diffraction scattering. This elastic scattering, which is not detected by the transmission counter, is referred to as residual diffraction scattering and was corrected for in this experiment by using the optical-model expression for $d\sigma_{el}/d\Omega$, which is²³

$$\frac{d\sigma(\theta)}{d\Omega} \text{ }_{el} = \frac{\sigma_d}{\pi} \left[\frac{J_1(K_0 R \sin \theta)}{\sin \theta} \right]^2,$$

where K_0 is the wave number of the incident pion; R is the nuclear radius and was determined from a preliminary analysis of the data.

The result for Be was determined from a transmission measurement at one value of Ω only. It was assumed that if values of the transmission had been obtained at other values of Ω , the resulting line drawn

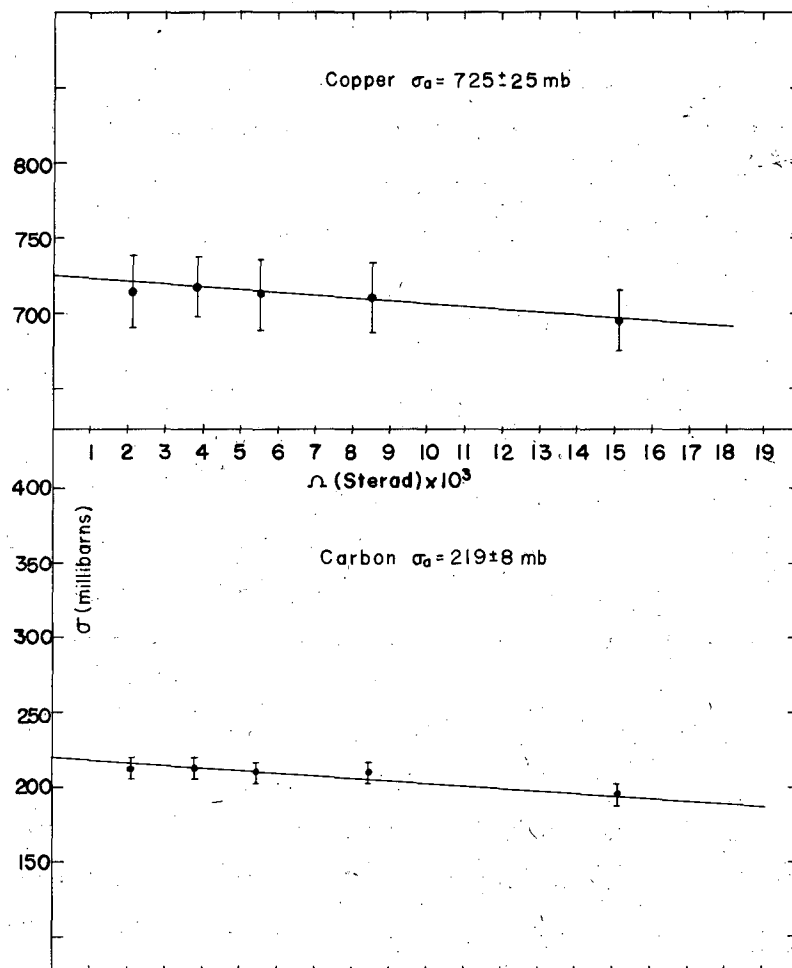


Fig. 7. Poor-geometry cross sections for Cu and C, corrected for μ -meson contamination and residual diffraction scattering, plotted against the solid angle subtended by the transmission counter.

through the points of a plot of $\sigma(\theta)$ vs Ω would have had a slope very much the same as that obtained for C. The value of the Be cross section was then obtained by drawing a line with the C slope through the median value of Be, using the resulting intercept for the value of σ_a for Be. The error quoted is twice the statistical error of the single measurement.

The Coulomb field of the nucleus can slightly affect the measurement of the absorption cross section because the negative pion trajectory will be bent toward the nuclei in the absorber. Thus the desired cross section, σ'_a , is related to the measured cross section by a factor α :

$$\alpha \sigma'_a = \sigma_a \text{ (measured),}$$

where

$$\alpha = 1 + 2e V_c / p \beta c, \quad (3)$$

V_c is the Coulomb potential at the nuclear surface, and p and βc are the momentum and velocity respectively of the incident pion. This is only a 2% correction for the heaviest element planned for use in this experiment (Pb), and was neglected for the cross sections actually measured in this experiment.

The values of the absorption cross sections obtained in the above manner are given in Table IV. The error given includes the error in the intercept given by the least-squares method only. No attempt was made to estimate the errors involved in the neglect of the finite size of the beam, of the Coulomb factor α , and of the essential point of the validity of the extrapolation method employed.

Table IV

Total and absorption cross sections for 4.2-Bev π^- mesons scattered from various nuclei. The prime indicates the values obtained by Ise, Lagarrigue, and Pyle,⁸ using a multiplate expansion cloud chamber.

Element	$A^{1/3}$	g/cm^2	$\sigma_a(\text{mb})$	$\sigma_t(\text{mb})$	σ_t/σ_a
Pb'	5.92		1710' $\pm$ 130 (5.0 Bev/c) 1850' $\pm$ 130 (3.2 Bev.)		---- (----)
Cu	3.99	19.50	725 $\pm$ 25	1620 $\pm$ 90	2.24 $\pm$ 0.15
Al	3.00	26.26	407 $\pm$ 10	763 $\pm$ 40	1.875 $\pm$ 0.107
C	2.29	19.54	219 $\pm$ 8	386 $\pm$ 20	1.76 $\pm$ 0.11
Be	2.08	31.50	177 $\pm$ 9	302 $\pm$ 15	1.71 $\pm$ 0.11

C. Total Cross Sections for Nuclei

The total cross sections obtained in this experiment are given in Table IV. The values given there were obtained with an extrapolation method as described in the section on the experimental method. Using this extrapolation method results in overestimating the value of the total cross section. This can be seen by integrating Eq. (2) over solid angle. One will note that the resulting function is approximately linear in θ in the region $0.7 \theta_m$ to $0.1 \theta_m$, where θ_m is the angle of the first diffraction minimum. Since the function is rounded off near 0° , a linear extrapolation tends to overestimate the total cross section σ_t . (Of course, one did not have to use a linear extrapolation; however, it was used here for convenience).

The datum points used in making the extrapolations were corrected for multiple Coulomb losses, when necessary, and for the beam contamination by μ mesons. No correction was made for the interference effect between Rutherford and elastic nuclear scattering, since its magnitude and sign are unknown. The data used for the extrapolations and the extrapolations themselves were not included in this report because they are essentially the same as those included in the previous

section on the absorption cross sections; the only differences are in the magnitude of the abscissa and ordinate.

The error given for the total cross sections in Table IV includes the error in the intercept given by the least-squares method. No estimate was made of any error which might be introduced owing to the extrapolation method employed for the determination of the values for the total cross section for elements considered in this experiment, or for the effects discussed above.

VII. OPTICAL-MODEL ANALYSIS OF THE NUCLEAR CROSS SECTIONS

A. General Discussion of the Optical Model

The availability of transmission measurements has led to the development and use of a quite abstract and general nuclear model which describes gross features of the nucleus, e.g., its radius and opacity, without concern for the details of the nuclear collisions. The nucleus is thought of as a grey refracting sphere having a static potential without internal structure. Its only refinement is that the potential may have both a real and an imaginary part, to allow for absorption as well as for elastic scattering. Thus we can generally compute the amplitude of the scattered wave and the damping due to absorption neglecting all interface reflections and considering only volume effects.

We have been able to measure the total and absorption cross sections of Be, C, Al, and Cu. In terms of the optical model, σ_a is a function of the nuclear radius R and the mass absorption coefficient K , while σ_t and the shape of the diffraction curve are functions of R, K , and k_1 , the increment in the wave number of the incident pion wave received upon entering the nuclear potential well. In addition, the two cross sections and the shape of the diffraction pattern depend on the assumed nuclear density distribution and on the wave number k of the incident mesons.

It is quite clear that not all parameters can be deduced from the two measured cross sections. Now, our main interest is in the determination of a value of the nuclear radius parameter r_0 in the expression $R = r_0 \cdot A^{1/3}$. This can be done quite easily by using the absorption cross sections determined in this experiment, and, of course, the Pb value determined by Ise et al.⁸ in conjunction with that subportion of the optical-model formalism which describes this type of cross section. In this formalism the absorption cross section is simply related to the elementary pion-nucleon cross section $\bar{\sigma}$ in a nucleus and the average density of nucleons, $\rho(r)$, at the radius r . Using this method, the only unknown parameter we must choose is the average pion-nucleon cross section in a nucleus, $\bar{\sigma}$. At the high energy used in this experiment

we believe that the value of $\bar{\sigma}$ will be close in value to the free pion-nucleon total cross section. (The total pion-nucleon cross section, or some fraction of it (see below) is used for $\bar{\sigma}$, since both elastic and inelastic collisions of a pion with a single nucleon in a nucleus looks to the whole nucleus like an inelastic event). If one could consider the nucleons in a nucleus as being entirely independent, $\bar{\sigma}$ would be equal to the "free" pion-nucleon total cross section. However, Serber has pointed out that the effects of nuclear binding inhibit collisions involving small momentum transfers, with the result that we have $\bar{\sigma} < \sigma_{\text{free}}$.²⁵ For 4.2-Bev pions the inelastic collisions involve very high momentum transfers, and also comprise 75% of the total cross section. Therefore only elastic collisions near 0° , i.e. those involving small momentum transfers, are inhibited. This, then, is the only source by which $\bar{\sigma}$ might be made less than σ_{free} . No quantitative calculations have been done for $\bar{\sigma}$ in this energy region, therefore the dependence of the analysis on $\bar{\sigma}$, and the resulting variation in the radius R , are considered below.

That the absorption cross section can be related to $\bar{\sigma}$ and $\rho(r)$ is dependent on the satisfaction of the following conditions: (a) the nucleons in the nucleus must scatter or absorb the pion wave independently of one another; (b) the mass number of the nucleus must not be too small. Let us briefly consider these restrictions.

For the independence approximation to hold it is necessary that both the pion reduced wave length, λ , and the range of the pion-nucleon interaction be smaller than the average internucleon spacing. At 4.3 Bev/c $\lambda = 0.046 \times 10^{-13}$ cm, and since the average nucleon-nucleon separation is 1.8×10^{-13} cm, the smallness condition on λ is well satisfied. The internucleon spacing is dependent on the density distribution one assumes for the analysis. If we consider a tapered model (this term will be defined more explicitly below) for $\rho(r)$, one finds that in the central region of this distribution the range of nuclear forces is about equal to the internucleon spacing. However, this average internucleon distance increases near the edge of the nucleus and is the important region for the calculation. A uniform-density model yields results that can be interpreted as implying an internucleon spacing

larger than the interaction distance. Although the assumption that the range of the pion-nucleon interaction is smaller than the internucleon spacing is not quantitatively supported, we will assume it to be valid in this discussion, for both the tapered and uniform densities employed, and thus assume that any nonadditive effects due to the failure of the independence condition will be small.* This has been the general assumption in past calculations--namely that the nucleons in the nucleus scatter or absorb the pion wave independently of one another--since one is not quite sure how to retain the simple calculational procedure of the optical model used in this discussion if this were not the case!

The requirement that the mass number be large is related to the statistical nature of the exponential law of attenuation (extinction).²⁶ This law is usually derived from a classical gas-kinetic argument or from the theory of multiple scattering of sound waves. In both these cases the scatters are assumed to be large in number and to occupy a large volume. Therefore, an optical-model analysis should draw its conclusions from the results obtained with nuclei of large A . It will be noted below--and also in other optical-model analyses of nuclear cross sections--that, in most cases, nuclei of small A give results which support the conclusions obtained with large- A nuclei. This is perhaps because in an experiment an absorber is bombarded with many particles, with the result that the desired statistical effect is obtained. However, results are still drawn predominantly from the values obtained with nuclei of large A .

With this general discussion of the optical model in mind we can now obtain an expression for the absorption cross section σ_a from the

* High-energy nucleons, when scattered from nuclei, exhibit the general result that they interact with single nucleons in the bombarded nucleus with no interference between elastic and absorption events. A specific search for this interference term in the scattering of nucleons from deuterium has shown that for this element no such interference term exists.

following simple semiclassical considerations. The pion wave is attenuated exponentially²⁶ as it passes through the nuclear matter with a mass absorption coefficient $K(r) = \rho(r) \bar{\sigma}$. The size and shape of the nucleus are included in the density distribution $\rho(r)$. The total attenuation is obtained from an integral over s , the coordinate along the pion trajectory (see Fig. 8). The cross section is then the probability of interaction integrated over the impact parameter b . It is evident from Fig. 8 that $r^2 = s^2 + b^2$. Using this relation one can write the expression for σ_a as follows:

$$\sigma_a = 2\pi \int_0^\infty \left\{ 1 - \exp \left[- \int_{-\infty}^\infty K \left((s^2 + b^2)^{1/2} \right) ds \right] \right\} b db. \quad (4)$$

the coordinates s, b , and r are defined in Fig. 8.

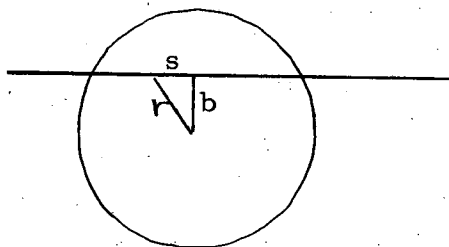


Fig. 8. The path of a pion through the nucleus is designated by the coordinate s ; b is the impact parameter; r is a radius vector.

The one problem remaining before any calculations can be done is the selection of the nucleon density distribution $\rho(r)$. This experiment did not yield any information on the form of the density distribution; in fact any reasonable form is compatible with the data. Since the Stanford electron-scattering experiments⁵ do yield the shape for the charge distribution in medium and heavy nuclei, we will assume a shape for the nucleon distribution similar to those suggested by the electron experiments except for a scale factor to be determined from our data. Also, it is of interest to include the results obtained by using a uniform-density distribution so that easy comparison can be made with the many early nucleon-scattering experiments, which were analyzed with the optical model having a uniform $\rho(r)$.

The information concerning $\rho(r)$ that we summarize here was obtained by the Stanford group, using in their analysis what is called the smoothed, uniform Fermi distribution* for the charge density of a nucleus. This form of the charge density is given by

$$\rho(r) = \rho_1 / (\exp [r - c / Z_1] + 1)$$

and describes all nuclei by a charge density which is uniform in the central region and which drops off smoothly at the edge. In this expression for $\rho(r)$, c is the radius parameter, which is proportional to $A^{1/3}$, while $Z_1 = t/4.40$, where t is the thickness of the surface of the charge distribution. The thickness t , or the fall-off distance--i.e., the distance in which the charge density falls from 90% to 10% of its central value--is essentially a constant for all nuclei and has the value

$$t = \Delta r_{90-10} = 2.4 \times 10^{-13} \text{ cm.}$$

For orientation, the results of the Stanford analysis of nuclei in terms of the Fermi smoothed, uniform charge distribution*--the $\rho(r)$ defined above--are tabulated below. All lengths are given in Fermi units, i.e., 10^{-13} cm. The accuracy of the surface thickness parameter t is $\pm 10\%$, and that of the radial parameter c is $\pm 2\%$. R is the value of the uniform charge distribution having the same rms radius as the Fermi distribution. (This quantity is defined in Eq. (7) below).

c = radius parameter = $kA^{1/3}$. (Measured from $r = 0$ to the midpoint of t .)

$t = \Delta r_{90-10}$ = fall-off distance or thickness parameter.

* The specification of the type of charge distribution employed is important for reference to any paper concerning these measurements, since various different distributions give equally good fits to the data but different values for the parameters c and t .

Table V

Nucleus	c	t	R	$\left(\frac{c}{A^{1/3}}\right)=r_1$	$\left(\frac{R}{A^{1/3}}\right)=r_0$
$^{40}_{20}\text{Ca}$	3.64	2.5	4.54	1.06	1.32
$^{51}_{23}\text{V}$	3.98	2.2	4.63	1.07	1.25
$^{59}_{27}\text{Co}$	4.09	2.5	4.94	1.05	1.27
$^{115}_{49}\text{In}$	5.24	2.3	5.80	1.08	1.19
$^{122}_{51}\text{Sb}$	5.32	2.5	5.97	1.07	1.20
$^{197}_{79}\text{Au}$	6.38	2.32	6.87	1.096	1.180
$^{204}_{83}\text{Bi}$	6.47	2.7	7.13	1.09	1.20

From such data as given in the above table, the Stanford group also concludes, from a comparison of the results obtained for r_0 for nuclei of low A with those of high A , that the findings are not compatible with an assumed constancy of r_0 for all A . Rather, the trend appears to be that heavy nuclei are described with an r_0 that is smaller than the r_0 characterizing nuclei of small A . However, they caution that further data on other nuclei are required before this conclusion may be accepted for all nuclei. It is indeed possible, in view of shell structure, that local variations of r_0 may show exceptions to the rule.

The Stanford group also feels that for the lighter nuclei--i.e., where the flat central regions shrinks to nothing, and where a gaussian form for the charge distribution gives a better fit to the data--the significance of t can be expected to decline, for (as might be expected) in elements lighter than carbon, the whole nucleus is not even as large as the skin, t ($= 2.4$ fermis).

B. Tapered-Density Model

The tapered-density model described here is due to R. W. Williams.²⁷ The specific form for the nuclear density distribution was chosen in accordance with the high energy scattering results, but with one important modification. He assumed that any nucleus could be described by a radius R which is proportional to $A^{1/3}$ and in addition by a Δr_{90-10} which is also proportional to $A^{1/3}$, not a constant as the Stanford experiments have shown. However, if in the analysis one considers the results obtained from the heavy-element data only, the model by Williams gives values for Δr_{90-10} in the same range of values as found in the Stanford experiments. (See Fig. 9.) Even with this weakness (or fault, if you like), when the whole range of A is viewed the model is a better approximation to reality than a uniform density distribution. This particular model was used because of the essential "ease" with which one can calculate R by use of the $\rho(r)$ defined in Eq. (5).

The form chosen has a constant central region attached to a smoothly dropping polynomial:

$$\begin{aligned} \rho(r) &= \rho_0, & \text{for } r \leq Q; \\ \rho(r) &= \rho_0 \left(\frac{2r^3}{Q^3} - \frac{9r^2}{Q^2} + 12 \frac{r}{Q} - 4 \right), & \text{for } Q \leq r \leq 2Q; \\ \rho(r) &= 0 & \text{for } r \geq 2Q. \end{aligned} \quad (5)$$

This form is illustrated in Fig. 9 for the elements Al, Cu, and Pb on the basis of experimentally determined quantities discussed below. The use of a polynomial proved easier to handle than a function involving exponentials, i. e., a Fermi-Type density function mentioned above. The density function has the form

$$\rho(r) = \rho_0 f(r/Q) \text{ or } \rho_0 f(x),$$

where $x = r/Q$ and where $f(x)$ is unity for $0 \leq x \leq 1$, and zero for $x > 2$. The radius parameter Q for any nucleus is assumed to have the

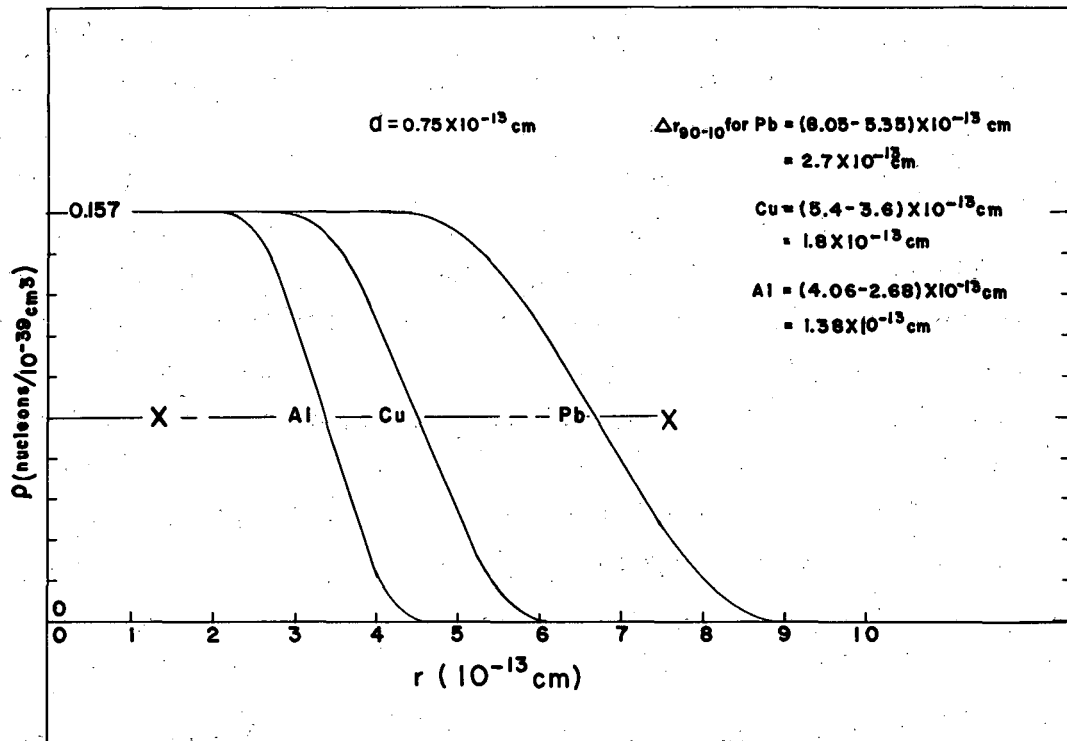


Fig. 9. Tapered-density distributions for Al, Cu, and Pb, plotted as a function of r .

form $Q = aA^{1/3}$, where a is a constant to be determined from the data. The central density ρ_0 -- i.e. the number of nucleons occupying the volume in the central, uniform region -- can be described as $\rho_0 = \frac{1}{a^3 v}$, where v is the volume-normalization constant characteristic of the particular $f(x)$ under consideration. In particular, we have

$$v \equiv \int_0^\infty f(x) 4\pi x^2 dx = \frac{24\pi}{5}.$$

If we consider a uniform density distribution, then we have

$$f(x) = 1 \quad \text{and} \quad v = 4/3 \cdot \pi$$

so that $\rho_0 = A/(4/3)\pi R^3$. For a specific $f(x)$, Eq. (4) reduces to the form

$$\sigma_a = \pi Q^2 O \left(\frac{\ell_0}{Q} \right), \quad (6)$$

where the opacity O is a function of the mean free path at the center of the nucleus, $\ell_0 \equiv K_0^{-1}$, divided by Q .

In Fig. 10 we see a plot of a , determined by numerical calculation of Eq. (4) using $\rho(r)$ in Eq. (5), as a function of $A^{1/3}$ for various values of $\bar{\sigma}$. The values of the nuclear radius parameter a are included for each element and from the numbers given the number 0.75×10^{-13} cm has been selected as representative for a when $\bar{\sigma} = (28 \pm 2)$ mb. Figure 11 shows a semi-log plot of the opacity O as a function of $(K_0 Q)^{-1}$ when $\bar{\sigma}$ has a value of 28 mb. From Fig. 9 the following information can be obtained (all units in fermis, 10^{-13} cm):

Nucleus	c	Δr_{90-10}	$c/A^{1/3} = r_1$
Al	3.38	1.38	1.126
Cu	4.50	1.8	1.127
Pb	6.66	2.7	1.125

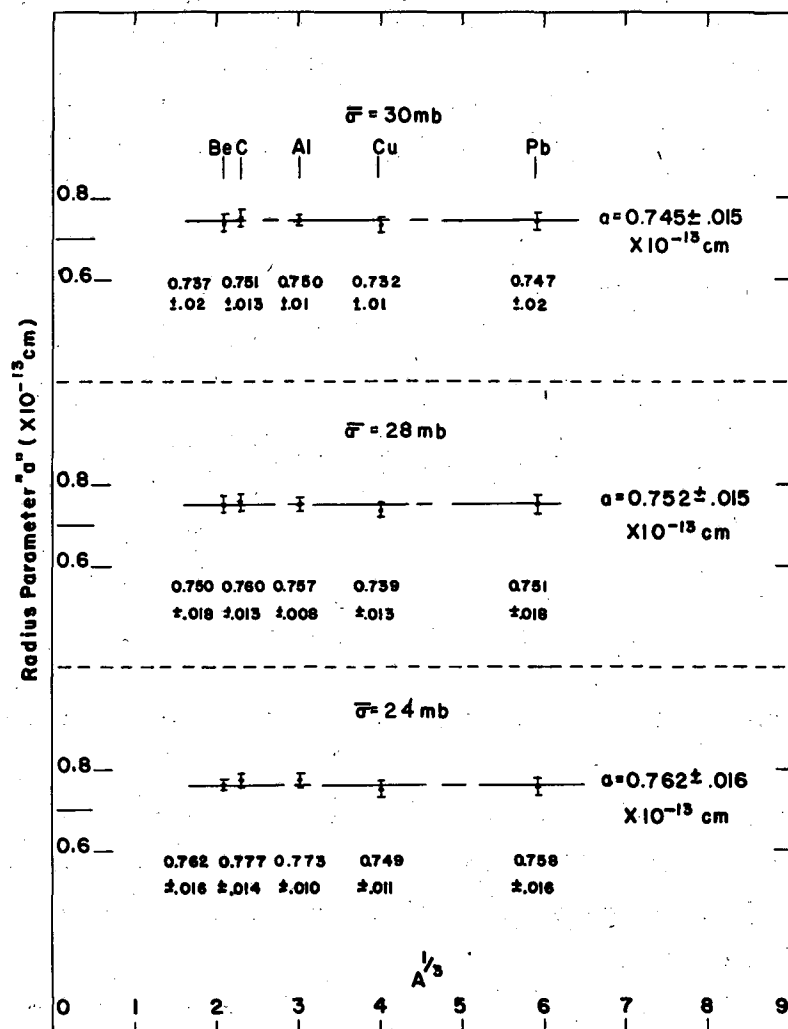


Fig. 10. Tapered-density model. Nuclear-radius parameter a as a function of $A^{1/3}$ for various values of $\bar{\sigma}$. Value of a given at right obtained without using the Be and C values.* The variation in the results with different choices of $\bar{\sigma}$ is small, as can be seen. * The Be and C points were not used since these nuclei have very low A . However including them does not change the value of a significantly.

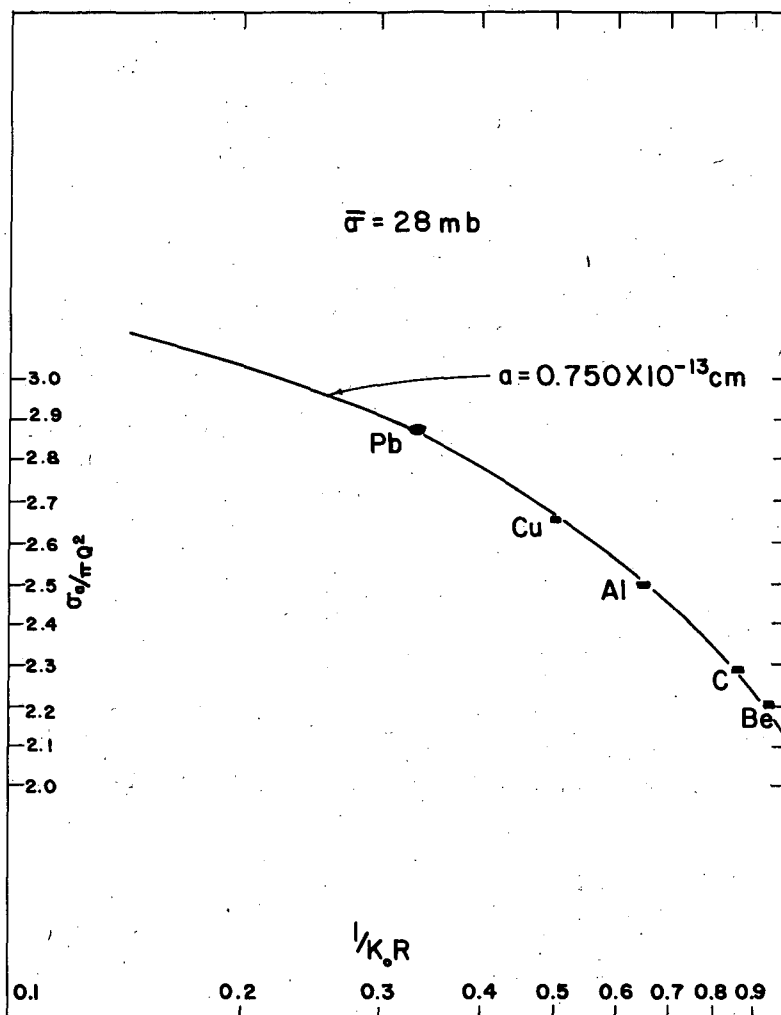


Fig. 11. Tapered density model; $\bar{\sigma} = 28 \text{ mb}$. Opacity $\frac{\sigma_a}{\pi Q^2} = \frac{\sigma_a}{\pi a^2 A^{2/3}}$ plotted as a function of $\frac{1}{K_0 R} = \frac{4.8 \pi a^2}{\bar{\sigma} A^{1/3}}$.

Here c = distance from $r = 0$ to midpoint of Δr_{90-10} , i.e. measured along X-X of Fig. 9. Then, following Ford and Hill,²⁸ we have expressed the nuclear radius determined from the tapered-density model in terms of the uniform density distribution, which yields the same $\langle r_1^2 \rangle_{av}$ value. Using

$$R = r_0 A^{1/3} = \left[\frac{5}{3} \langle r_1^2 \rangle_{av} \right]^{1/2}, \quad (7)$$

we find

R for Al = 3.613 fermis,	r_0 for Al = 1.205 fermis;
R for Cu = 4.81 fermis,	r_0 for Cu = 1.205 fermis;
R for Pb = 7.13 fermis,	r_0 for Pb = 1.205 fermis.

The root-mean-square value was computed by using Eq. (5) for $\rho(r)$ in the definition

$$\langle r^2 \rangle_{av} = \frac{\int_0^\infty \rho(r) 4\pi r^2 \cdot r^2 dr}{\int_0^\infty \rho(r) 4\pi r^2 dr};$$

from this it follows that $\langle r^2 \rangle_{av} = 1.55 Q^2$. Therefore from this analysis we can conclude that the relation for R is

$$R = r_0 A^{1/3} = (1.21 \pm .04) \cdot A^{1/3} \cdot 10^{-13} \text{ cm}. \quad (8)$$

The error includes the uncertainty in the determination of the pion-proton cross section, and therefore in $\bar{\sigma}$, and the error in the measurement of the absorption cross sections. This value of the nuclear radius is in quite good agreement with the electromagnetic experiments that determine the charge radius. (Table VI contains a summary of the Cosmotron and electromagnetic determinations of nuclear size.) As we mentioned above (Table V) the Stanford group feels there is some indication that not all nuclei--i.e., high- A and low- A nuclei--can be described by the same r_0 . Within the range of A under common consideration in these two experiments, the analysis of the measurements we have taken, using the tapered model defined above, does not yield a

Table VI

Partial summary of the radii determined for the distributions of nucleons and of charge in nuclei

Un - Radius determined with a uniform density distribution.

T - Radius determined with a tapered density distribution; the value given is the radius of the uniform density distribution, which yields the same $\langle r^2 \rangle_{av}$ value.

Nucleon Distribution

Energy and type of probe	Radius ($\times 10^{-13}$ cm)	Value of $\bar{\sigma}$ used in analysis (mb)
4.2-Bev negative pions	Un. $0.63 + 1.2 A^{1/3}$	$\bar{\sigma} = 30$
	Un. $0.8 \pm 1.18 A^{1/3}$	$\bar{\sigma} = 28$
	T $(1.21 \pm 0.4) A^{1/3}$	$\bar{\sigma} = 28 \pm 2$
1.4-Bev neutrons	Un. $1.28 A^{1/3}$	$\bar{\sigma} = 43$
	T $1.19 A^{1/3}$	$\bar{\sigma} = 43$
860-Mev neutrons	Un. $1.25 A^{1/3}$	$\bar{\sigma} = 45$

Charge Distribution*

μ -mesonic x-rays	T $1.17 A^{1/3}$
High-energy electron scattering	T $1.20 A^{1/3}$
Semiempirical mass formula	T $1.22 A^{1/3}$

* Here T refers to tapered density distributions different from that described in this paper. The particular type employed can be determined by reading those references which apply to the specific analysis involved. See References on pg. 66-67.

Table VII

The optical-model parameters R , the radius; O , the opacity; K , the mass absorption coefficient; and k_1 , the increment in the wave number, obtained by using a uniform density distribution for $\rho(r)$. For comparison the average value of the optical model parameters R , O , K and k_1 are as determined by other experiments are included.

4.2-Bev Negative Pions

$$\bar{\sigma} = 30\text{-mb}$$

Element	R	O	K	KR
Pb	$(7.86 \pm .14) \times 10^{-13} \text{ cm}$	0.92	$0.261 \times 10^{-13} \text{ cm}^{-1}$	2.05
Cu	$5.21 \pm .12$	0.85	0.255	1.33
Al	$4.19 \pm .1$	0.73	0.261	1.1
C	$3.39 \pm .15$	0.61	0.261	0.885
Be	$3.22 \pm .2$	0.55	0.261	0.84

Average Value

$$0.63 \pm 1.2 A^{1/3}$$

$$0.261$$

From Fig. 14.

$$k_1 = (1.67 \pm .33) \times 10^{+12} \text{ cm}^{-1}$$

$$V = 33 \pm 7 \text{ Mev}$$

	$R(10^{-13} \text{ cm})$	$K(10^{13} \text{ cm}^{-1})$	$k_1(10^{13} \text{ cm}^{-1})$	$V(\text{Mev})$
1.4-Bev neutrons ⁷	$1.28 A^{1/3}$	0.49	$1.5 \pm .05$	26 ± 9
860-Mev protons ⁴	$1.25 A^{1/3}$	0.56	nonvanishing	≈ 50
410-Mev neutrons ^{2*}	$1.23 A^{1/3}$	0.51		
140-Mev to 400 Mev ⁴	$1.42 A^{1/3}$			
Negative Pions				

* * Taylor³ in an analysis of all neutron data from 50 to 400 Mev finds that $R = (0.75 \pm 1.26 A^{1/3}) \times 10^{-13} \text{ cm}$, or that $r_0 = 1.45 A^{1/3}$ for $A \sim 60$ and $r_0 = 1.39 A^{1/3}$ for $A \sim 200$. Taylor includes the 410-Mev neutron data in his analysis.

similar conclusion. However, as we see below, a uniform density distribution, when employed for $\rho(r)$, does give results on the variation of r_0 with A similar to those from the electron-scattering experiments.

C. Uniform-Density Model

This form of the density distribution has the virtue of simplicity, since now we have

$$\rho = \rho_0 = A / \frac{4}{3} \pi R^3. \quad (9)$$

Then we obtain

$$\sigma_a = \pi R^2 \left\{ 1 - \frac{1}{2k^2 R^2} (1 - (1 + 2kR)e^{-2kR}) \right\},$$

$$K = A\bar{\sigma} / \frac{4}{3} \pi R^3. \quad (10)$$

Figures 12 and 13 show the value of the parameter r_0 , determined with a uniform density distribution for two values of $\bar{\sigma}$, and these values can be compared with other determinations in Table IV. (The graphs and table are self-explanatory.)

If we plot the experimental points of σ_t/σ_a vs KR and the corresponding theoretical curves for various values of k_1/K , using the uniform-density model, we can obtain a rough value of the optical model parameter k_1 . This parameter is related to the average pion-nucleon forward scattering amplitude, as seen from

$$k_1 = \frac{2\pi\rho}{k} \text{Re } f(0),^{29} \quad (11)$$

where ρ is the nucleon density and k is the wave number of the incident pion. From Fig. 14 we see that $k_1 = 0.167 \times 10^{-13}$ cm, and thus that $\text{Re } f(0) = (4.18 \pm .84) \times 10^{-13}$ cm. This implies a minimum value for the forward elastic pion-nucleon cross section of 175^{+87}_{-63} mb/steradian. This high value implies that the elastic pion-nucleon scattering is strongly

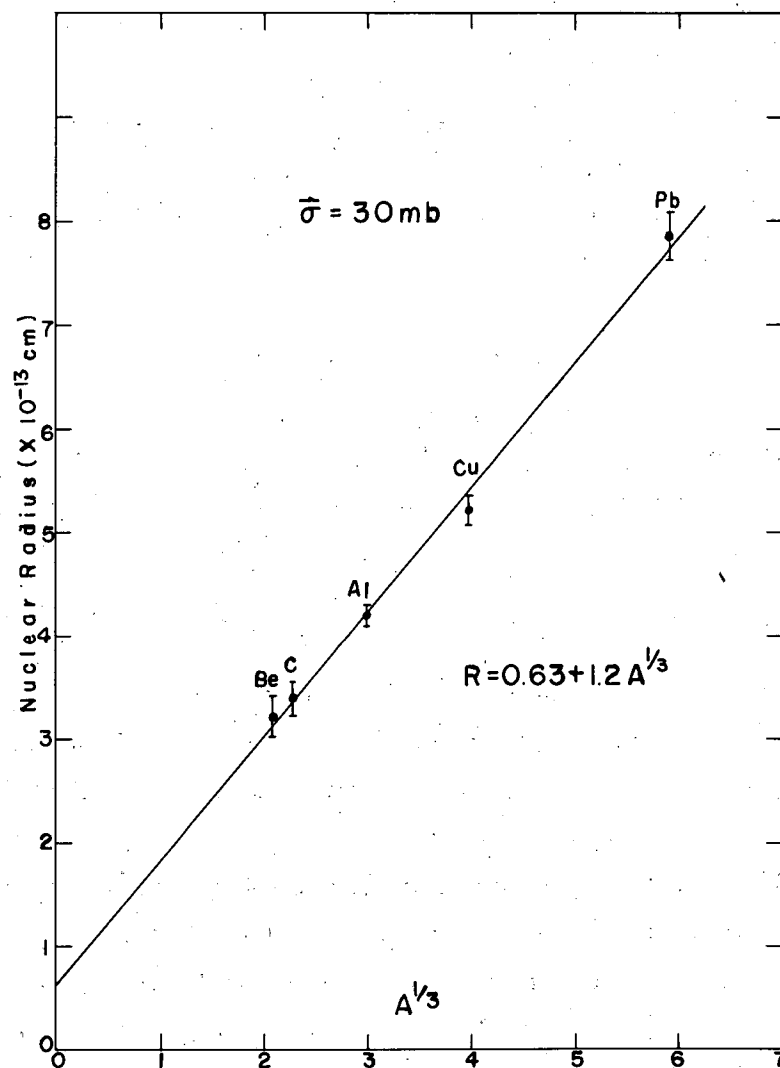


Fig. 12. Uniform-density model; $\bar{\sigma} = 30 \text{ mb}$.

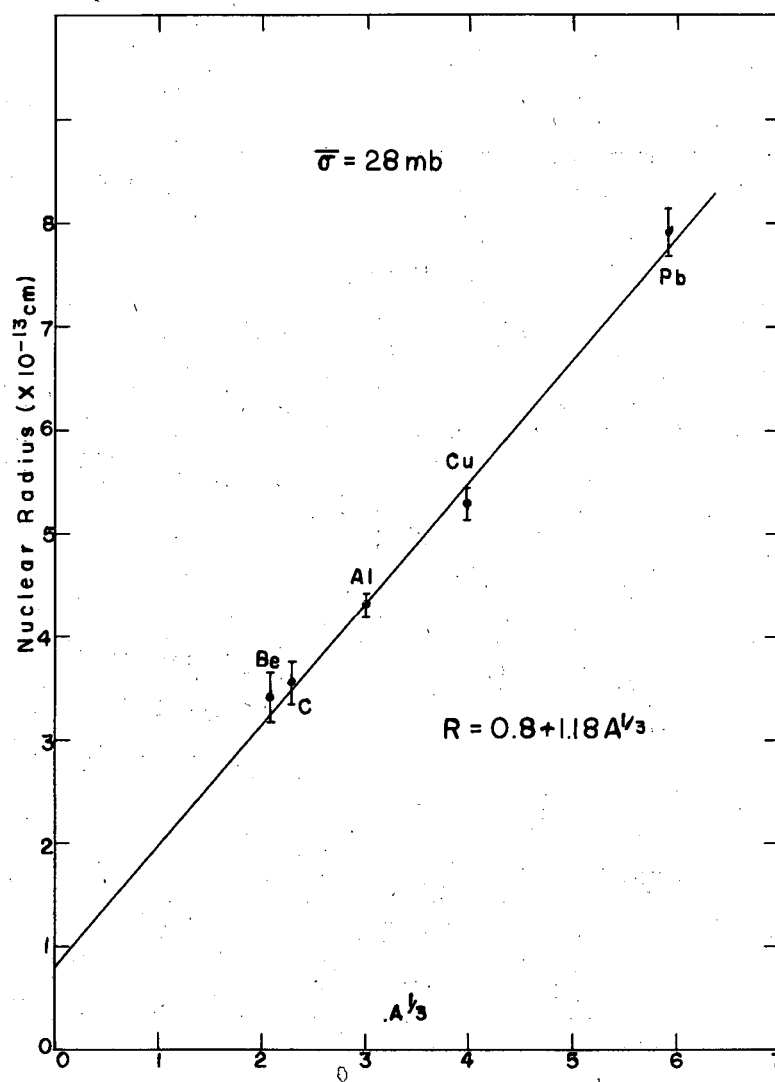


Fig. 13. Uniform-density model; $\bar{\sigma} = 28 \text{ mb}$.

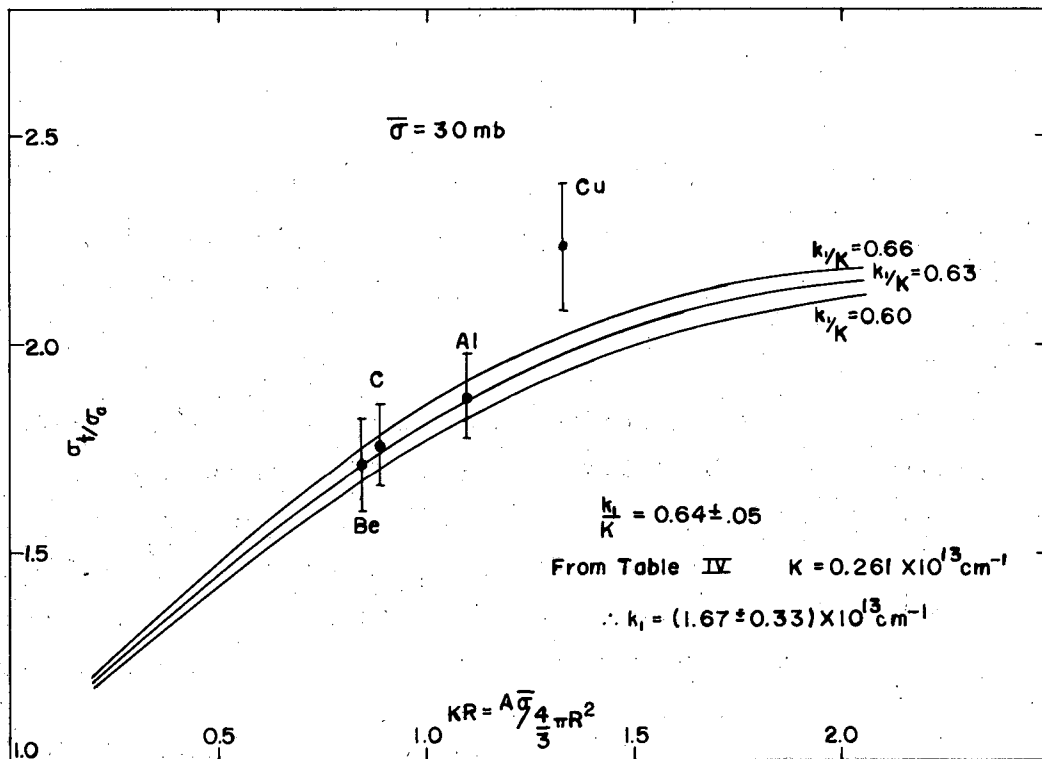


Fig. 14 $\frac{\sigma_t}{\sigma_a}$ versus KR for uniform-density model;
 $\bar{\sigma} = 30 \text{ mb.}$

peaked in the forward direction. As has been customary in the low-energy regions, we can relate the constant k_1 to the average nuclear potential, assuming that an average nuclear potential has meaning at these energies. Using

$$k_1 = \frac{\bar{V}}{\hbar\beta c}, \quad (12)$$

we find

$$\bar{V} = 33 \pm 7 \text{ Mev.}$$

Because a uniform density distribution in nuclei is not considered realistic, only a few comments will be made about the optical model parameters obtained with such a distribution with the data of this experiment.

The mass absorption coefficient K ($= 0.261 \times 10^{13} \text{ cm}^{-1}$) obtained in the above analysis is similar in value to those used by Taylor to fit the neutron data in the 300-Mev region. This similarity can be traced to the similarity in the value of $\bar{\sigma}$ used in both analysis; the value of K is completely determined by the assumed value of $\bar{\sigma}$ and the deduced value of the nuclear radius. The high value of K found in the Cosmotron nucleon experiments reflects the increased nucleon-nucleon cross section above 500 Mev.

The values of k_1 , $\frac{d\sigma_{el}}{d\Omega}$, and $\bar{V}$ determined by this experiment are essentially the same as those found by the nucleon probes at Brookhaven, as might be expected. (See Table VI).

The same is true for the values obtained for r_0 . (Tables VI and VII contain values of r_0 from a representative selection of other experiments.)

The dependence of the values determined for the radius R on $\bar{\sigma}$ can be seen from Figs. 7 and 8. A change in $\bar{\sigma}$ produces the greatest change in the radius values of the light elements. This results in quite a large change in the intercept c , rather than in r_0 , in the expression $R = c + r_0 A^{1/3}$.

One can therefore conclude that for the uniform density distribution the radii determined for the medium and heavy nuclei are not a sensitive function of $\bar{\sigma}$. (This, of course, was expected, since for these elements the opacity is high.) One hesitates to discuss the significance of the intercept c as determined in the above analysis since a 6.7% change in $\bar{\sigma}$ results in a 21% change in c . Some authors have felt that c is related to the range of nuclear forces,³⁰ and if this is true then we have determined this range to be $(0.63 \pm .2) \times 10^{-13}$ cm.

VIII DISCUSSION

Measurements of the noncoherent part of the total cross section-- what we have called the absorption cross section--of high-energy pions on nuclei has been shown to be an easily interpreted nuclear method for obtaining the spatial extension of nuclear matter. As we have seen above, it is straightforward and involves the use of few unknown parameters. The least-known quantity entering the determination of the nuclear size is the effective elementary cross section $\bar{\sigma}$, which might well be less than $\sigma_{\pi sp}$, the value we have used. For a tapered-density model the outer edge of the nucleus is always somewhat transparent, so that even the cross sections for heavy nuclei depend on $\bar{\sigma}$. Therefore it is essential to investigate the sensitivity of the radius parameter a , or r_0 to changes in the value of $\bar{\sigma}$. As can be seen from Fig. 10, these parameters are very insensitive to a change in $\bar{\sigma}$. Since it is unlikely that $\bar{\sigma}$ can be reduced by more than 25%, the value of r_0 given in Eq. (8) is probably not more than 3% too low on this account. We therefore conclude that radius parameter, r_0 , as determined by this experiment is in quite good agreement with the value determined by the electromagnetic measurements of nuclear charge. We therefore conclude that the nuclear radius parameter, r_0 , as determined by this experiment is in quite good agreement with the value of r_0 obtained by the electromagnetic experiments, which are concerned with the spatial extent of the nuclear charge distribution.

This conclusion is in contrast to the results obtained by the investigations by Taylor³ of all available neutron data from 50 to 400 Mev, in which he finds $r_0 = 1.37 \times 10^{-13}$ for Pb, and larger values for lighter nuclei, up to 1.5×10^{-13} for Al. Feshbach, Porter, and Weisskopf,³⁰ in the Mev range, find $r_0 = 1.45 \times 10^{-13}$ cm, and negative pions at these lower energies, ~200 Mev, describe nuclei with an r_0 of 1.42×10^{-13} cm.⁴ All these investigations describe nuclei with radii 15% to 25% larger than the very-high-energy pion, proton, neutron, and electromagnetic determinations of the spatial and charge extends in nuclei.

Although the above discrepancy is not completely understood, it has been proposed that the contrast between the low-energy and high-energy nucleon results suggests a real difference in the effective range of the nucleon-nucleon interaction in the two energy regions. The same might now be said for the range of the pion-nucleon interaction. However, a complete understanding of the results of these measurements is not yet available.

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