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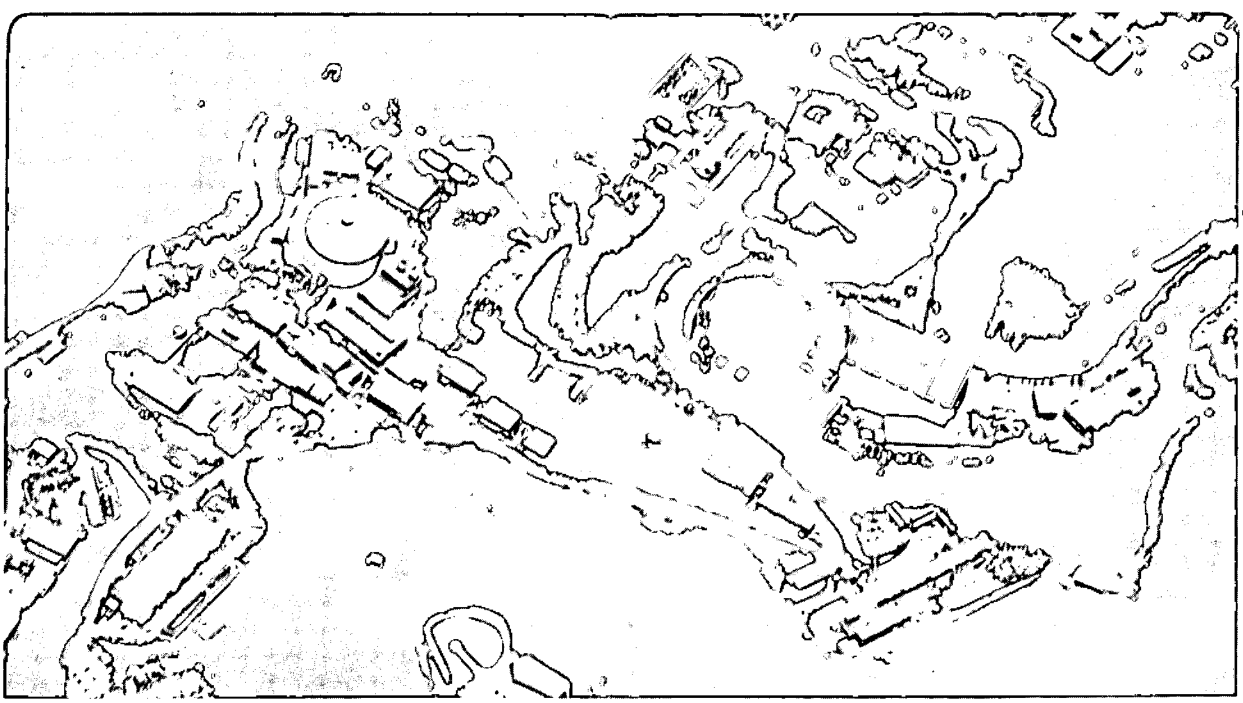

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Background Level and Counter Efficiencies Required for a Loophole-Free EPR-Experiment*

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ABSTRACT

An analysis is made of the background level and counter efficiencies actually necessary to perform a loophole-free EPR-experiment. Both requirements are correlated. Photon counters do not absolutely have to have more than 82.8% efficiency, if the signal-over-noise ratio is very high.

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Contents

1 Introduction.

In this paper, limits are set for the amount of background that can be tolerated in a loophole-free EPR-experiment, [1] [2] [3], as a function of η, the efficiency of the counters used. The experiment is assumed to be performed on entangled states of two photons and involve polarization measurements on them. It is possible to make a loophole-free experiment with $\eta < 82.8\%$, but it requires the background level to be very low.

The initial state is assumed to be prepared as a superposition of states of two photons, a and b, with correlated planes of polarization. One state is defined as $| \leftrightarrow\updownarrow >$, i.e. photon a polarized horizontally and photon b vertically; and another state as $| \updownarrow \leftrightarrow>$, i.e. a polarized vertically and b horizontally. For both photons, the polarization measurements are made with Nicol prisms set in such a way that the ordinary trajectory applies to a photon polarized in the horizontal plane and the extraordinary to a photon polarized vertically. In front of either Nicol, devices are disposed that rotate the plane of polarization of the photons. The angle by which the plane of polarization of a is rotated will be called α and, for b, β.

Demonstrations exist showing that the maximum violation by predictions of quantum mechanics of an inequality of the type Bell-CHSH for a two-particle system is $2(\sqrt(2) - 1)$, [4] [5] [6]. That limit is reached in particular if the initial state is described by a state vector

$$\psi = \frac{1}{\sqrt{2}} \left(| \leftrightarrow\updownarrow > \; + | \updownarrow \leftrightarrow> \right) \; ; \tag{1}$$

if the two values of α used in the relevant experimental setups are

$$\alpha_1 \quad = \quad 78.75^\circ \; , \tag{2}$$

$$\alpha_2 = -56.25^\circ ; \tag{3}$$

and, if the two values of β are

$$\beta_1 = 11.25^\circ , \tag{4}$$

$$\beta_2 = -33.75^\circ . \tag{5}$$

These demonstrations make use of an operator called "Bell operator", $\mathcal{B}$, which is related to the expectation value $\mathcal{J}_\mathcal{B}$ of a Bell inequality and any initial state ψ by the relation

$$\mathcal{J}_\mathcal{B} = \psi^\dagger \mathcal{B}\, \psi . \tag{6}$$

In these demonstrations, the inequality $\mathcal{J}_\mathcal{B}$ and the operator $\mathcal{B}$ are written for the case of a 100% efficiency. It is possible to modify the inequality to take into account the case of a less than 100% efficiency. If this is done but the initial state ψ and the values of α_1, α_2, β_1, and β_2, are kept at the values of Eqs. (1), (2), (3), (4), and (5), i.e., if the optimization of $\mathcal{J}_\mathcal{B}$ is made <u>before</u> setting η to a value less than 100%, it can be shown, [7], that Bell's inequality in these particular conditions requires an efficiency of the counters

$$\eta > 2 \left(\sqrt{2} - 1\right) \approx 82.8\% . \tag{7}$$

However, if $\mathcal{J}_\mathcal{B}$ is optimized changing ψ, α_1, α_2, β_1 and β_2 <u>after</u> introducing the correction for $\eta < 100\%$, a lower requirement for the efficiency may be expected. This is the subject of this paper. The Bell operator $\mathcal{B}$ is first modified to take into account values of η less than 100%. Then all parameters ψ, α_1, α_2, β_1, and β_2 are changed to optimize $\mathcal{J}_\mathcal{B}$.

2 Bell Inequalities for $\eta < 100\%$.

Bell inequalities concern expectation values of quantities that can be measured in four different experimental setups, defined by specific values α_1, α_2, β_1, and β_2 of α and β. The setups will be referred to by the symbols (α_1, β_1), (α_1, β_2), (α_2, β_1), and (α_2, β_2), where the first index designates the value of α and the second index the value of β.

In any setup, the *fate* of the photon a and the *fate* of photon b is referred to by an index

$(o) \equiv$ photon detected in the ordinary beam, or

$(e) \equiv$ photon detected in the extraordinary beam, or

$(u) \equiv$ photon undetected.

There are nine types of events defined as (o,o), (o,u), (o,e), (u,o), (u,u), (u,e), (e,o), (e,u), and (e,e), where the first index designates the fate of photon a and the second the fate of photon b. Table 1 shows a display of boxes corresponding to the nine types of event in each setup. The value of α and the fate of photon a designate a row. The value of β and the fate of photon b designate a column. Any event obtained in one of the setups corresponds to one box in Table 1.

For a given theory, we consider all the possible sequences of N events that can occur in each setup. N is arbitrarily large. As in [8] and [9], a theory is defined as being "local" if it predicts that, among these possible sequences of events, one can find four sequences (one for each setup) where every event satisfies the following conditions :

α \\ β		β_1			β_2		
		o	u	e	o	u	e
α_1	o	•				×	×
	u						
	e						
α_2	o				×		
	u	×					
	e	×					

Table 1: *Possible results expected in the four setups.*

(i) the fate of photon a is independent of the value of β, i.e., is the same in event numbered k of the sequence corresponding to setup (α_1, β_1) as in event k for (α_1, β_2); also same fate for a in (α_2, β_1) and (α_2, β_2); this is true for all k's for these carefully selected sequences;

(ii) the fate of photon b is independent of the value of α, i.e., is the same in event k of sequences (α_1, β_1) and (α_2, β_1); also same fate for b in sequences (α_1, β_2) and (α_2, β_2);

(iii) all four sequences that one has been able to find with conditions (i) and (ii) are among those for which all averages and correlations differ from their expectation values given by the theory by less than, let us say, 10 standard deviations.

These conditions are fulfilled by a deterministic *local* hidden-variable theory, i.e., one where the fate of photon a does not depend on β and the fate of b does not depend on α. For such a theory, these four sequences could be just four of the most common sequences of events generated by the same values of the hidden variables in the different setups. Conditions (i), (ii), and (iii) are also fulfilled by probabilistic *local* theories, which assign probabilities to various outcomes in each of the four setups and assume no "influence" of the angle β on what happens to a and no "influence" of α on b. With such theories, one can generate sequences of events as mentioned above by Monte Carlo, using an algorithm that decides the fate of a without using the value of β and, for b, without using the value of α. If the same random numbers are used for the four different setups, the sequences of events will necessarily have properties (i) and (ii), and the vast majority of them will have property (iii).

Let us follow an argument first used by Ref. [10]. The four events numbered k in the four sequences correspond to four boxes in Table 1, one box in the set of nine corresponding to each setup. Because of condition (i), the two events corresponding to setups (α_1, β_1) and (α_1, β_2) lay on the same row. Samething for (α_2, β_1) and (α_2, β_2). Because of (ii), events for (α_1, β_1) and (α_2, β_1) lay on the same column, as well as events for (α_1, β_2) and (α_2, β_2). The four boxes corresponding to the four events numbered k lay at the corners of a rectangle on Table 1, each corner in the set of nine boxes reserved for the proper setup. It follows that, everytime an event labeled k in setup (α_1, β_1) falls into the (o, o)-box (the one marked with a $\bullet$), in a local theory, it corresponds to the upper-left corner of a rectangle in Table 1. Then at least one of the other corners is in one of the boxes marked with an $\times$ on that Table, i.e.: either event numbered k in sequence (α_1, β_2) lands in one of the boxes (o, e) or (o, u) in setup (α_1, β_2), and/or event k in sequence (α_2, β_1) is in box (e, o) or (u, o), or event k in sequence (α_2, β_2)

lays in box (o, o). It follows that the number $n_{oo}(\alpha_1, \beta_1)$ of events of type (o, o) in setup (α_1, β_1), i.e., in the box marked by a $\bullet$, cannot be larger than the sum of the number $n_{oe}(\alpha_1, \beta_2)$, $n_{ou}(\alpha_1, \beta_2)$, $n_{eo}(\alpha_2, \beta_1)$, $n_{uo}(\alpha_2, \beta_1)$, and $n_{oo}(\alpha_2, \beta_2)$ of events of types (o, e) and (o, u) in setup (α_1, β_2), of types (e, o) and (u, o) in setup (α_2, β_1), and type (o, o) in setup (α_2, β_2), i.e., the boxes marked with an $\times$. Thus,

$$\begin{aligned} n_{oe}(\alpha_1, \beta_2) &+ n_{ou}(\alpha_1, \beta_2) + n_{eo}(\alpha_2, \beta_1) + n_{uo}(\alpha_2, \beta_1) \\ &+ n_{oo}(\alpha_2, \beta_2) - n_{oo}(\alpha_1, \beta_1) \quad \geq 0 \; . \end{aligned} \tag{8}$$

This inequality applies to the number of events of various types in the four sequences. It was arrived at using conditions (i) and (ii) above applied to the four carefully selected sequences. For condition (iii) to be true no matter how large the number of events N is, Ineq. (8) has also to apply to the expectation values of these numbers. It is the form of Bell inequality that will be used hereafter.

A similar inequality to Ineq. (8) can be derived where all (o)'s are changed into (e)'s and vice versa. In principle, one could average this new inequality with Ineq. (8) to improve statistics. (By doing so, one arrives directly at an inequality almost identical to the Bell-CHSH inequality.) However, optimizing the averaged inequality leads to the conditions of Eqs. (1), (2), (3), (4), and (5) regardless of the efficiency η. The minimum efficiency required is then 82.8%. The optimizing procedure of Sect. 3 actually makes an improvement on only one of the inequality of the average, at the expense of the other inequality.

3 Predictions of Quantum Mechanics.

The goal of a loophole-free experiment is to find results in a case where the predictions of quantum theory contradict the prediction of all *local* theories, i.e., where the predictions of quantum mechanics violate a form of Bell inequality. If the predictions of quantum theory are upheld, the existence of non-local effects in nature will thus be proven.

To compute the predictions of quantum mechanics, let us use a representation where the helicities of the two photons are diagonal operators. Given an initial state ψ, a value for η and a set of angles α_1, α_2, β_1, and β_2, predictions for the number of events involved in Ineq. (8) can be computed. For N pairs of photon emitted in the superposition state and assuming an ideal case where there is no background, these predictions are

$$
\begin{aligned}
n_{oo}^{ideal}(\alpha_1,\beta_1) &= N\frac{\eta^2}{4}\psi^\dagger\left(I+\sigma(\alpha_1)\right)\left(I+\tau(\beta_1)\right)\psi\ , && (9)\\
n_{oe}^{ideal}(\alpha_1,\beta_2) &= N\frac{\eta^2}{4}\psi^\dagger\left(I+\sigma(\alpha_1)\right)\left(I-\tau(\beta_2)\right)\psi\ , && (10)\\
n_{ou}^{ideal}(\alpha_1,\beta_2) &= N\frac{\eta(1-\eta)}{2}\psi^\dagger\left(I+\sigma(\alpha_1)\right)\psi\ , && (11)\\
n_{eo}^{ideal}(\alpha_2,\beta_1) &= N\frac{\eta^2}{4}\psi^\dagger\left(I-\sigma(\alpha_2)\right)\left(I+\tau(\beta_1)\right)\psi\ , && (12)\\
n_{uo}^{ideal}(\alpha_2,\beta_1) &= N\frac{\eta(1-\eta)}{2}\psi^\dagger\left(I+\tau(\beta_1)\right)\psi\ , && (13)\\
n_{oo}^{ideal}(\alpha_2,\beta_2) &= N\frac{\eta^2}{4}\psi^\dagger\left(I+\sigma(\alpha_2)\right)\left(I+\tau(\beta_2)\right)\psi\ ; && (14)
\end{aligned}
$$

where, in our helicity representation, the phases are chosen in such a way that the elements of $\sigma(\alpha_1)$ and of $\tau(\beta_1)$ are real :

$$\sigma(\alpha) = \begin{vmatrix} 0 & e^{-2i(\alpha-\alpha_1)} & 0 & 0 \\ e^{2i(\alpha-\alpha_1)} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-2i(\alpha-\alpha_1)} \\ 0 & 0 & e^{2i(\alpha-\alpha_1)} & 0 \end{vmatrix} \tag{15}$$

$$\tau(\beta) = \begin{vmatrix} 0 & 0 & e^{-2i(\beta-\beta_1)} & 0 \\ 0 & 0 & 0 & e^{-2i(\beta-\beta_1)} \\ e^{2i(\beta-\beta_1)} & 0 & 0 & 0 \\ 0 & e^{2i(\beta-\beta_1)} & 0 & 0 \end{vmatrix} \tag{16}$$

$$\begin{aligned} \mathcal{J}_{\mathcal{B}}{}^{ideal} = n_{oe}^{ideal}(\alpha_1,\beta_2) &+ n_{ou}^{ideal}(\alpha_1,\beta_2) + n_{eo}^{ideal}(\alpha_2,\beta_1) + n_{uo}^{ideal}(\alpha_2,\beta_1) \\ &+ n_{oo}^{ideal}(\alpha_2,\beta_2) - n_{oo}^{ideal}(\alpha_1,\beta_1) \ . \end{aligned} \tag{17}$$

The above computation assumes that only the N photons in the entangled state contribute to the counting rates and that the polarization analyzers are perfect. A correction has to be made to formula (17) to take into account deviations from that ideal case. The sample of events of type (o,e), (o,u), (e,o), and (u,o) actually counted in the experiment and introduced in Ineq. (8) will include not only the $n_{oe}^{ideal}(\alpha_1,\beta_2)$, $n_{ou}^{ideal}(\alpha_1,\beta_2)$, $n_{eo}^{ideal}(\alpha_2,\beta_1)$, and $n_{uo}^{ideal}(\alpha_2,\beta_1)$ events of Eqs. (10), (11), (12), and (13), but other events with a less sharp dependence on α and β. It is a *background.* We will take that background into account by an α- and β independent term, $N\zeta$, to be added to the $n_{oe}^{ideal}(\alpha_1,\beta_2) + n_{ou}^{ideal}(\alpha_1,\beta_2)$ events of Eqs. (10)and (11), and to the $n_{eo}^{ideal}(\alpha_2,\beta_1)+n_{uo}^{ideal}(\alpha_2,\beta_1)$ events of Eqs. (12) and (13). In principle there is also a background in samples of type (o, o) events in setup (α_1, β_1) and (α_2, β_2). However, since we assume no dependence of the background on α and β, the effect of the type (o, o) background cancels in Ineq. (8). After correction for background, we write

$$\mathcal{J}_B = \mathcal{J}_B{}^{ideal} + N\zeta \ . \tag{18}$$

Eq. (8) stipulates that local theories predict

$$\mathcal{J}_B \geq 0 \ , \tag{19}$$

while quantum theory predicts

$$\mathcal{J}_B = \psi^\dagger \mathcal{B} \ \psi \ , \tag{20}$$

where

$$\mathcal{B} = N\frac{\eta}{2} \begin{vmatrix} 2-\eta+\xi & 1-\eta & 1-\eta & A^*B^*-\eta \\ 1-\eta & 2-\eta+\xi & AB^*-\eta & 1-\eta \\ 1-\eta & A^*B-\eta & 2-\eta+\xi & 1-\eta \\ AB-\eta & 1-\eta & 1-\eta & 2-\eta+\xi \end{vmatrix} \tag{21}$$

and

$$A = \frac{\eta}{2}\,(e^{2i(\alpha_2-\alpha_1)} - 1) \ , \tag{22}$$

$$B = \frac{\eta}{2}\,(e^{2i(\beta_2-\beta_1)} - 1) \ , \tag{23}$$

$$\xi = \frac{4\zeta}{\eta} \ . \tag{24}$$

To perform a loophole-free experiment, we need experimental conditions in which the prediction for $\mathcal{J}_B$ of Eq. (20) is negative. That is possible as long as the operator $\mathcal{B}$ has a negative eigenvalue. That is impossible if all eigenvalues are positive, even if one uses incoherent mixtures of pure states. The maximum amount of background that can be tolerated corresponds to that value of ζ that makes the last negative eigenvalue of $\mathcal{B}$ turn from negative to positive, i.e. when the determinant of $\mathcal{B}$ of Eq. (21) ceases to be negative to become zero.

A computer program was written to compute the determinant of $\mathcal{B}$ of Eq. (21), for any given value of the efficiency η. The program varied $\alpha_2-\alpha_1$, $\beta_2-\beta_1$, and ζ to find the maximum value of the background ζ that kept the determinant negative. For $\eta < \frac{2}{3}$, there is none. For $\eta > \frac{2}{3}$ there are negative values of the determinant for small values of ζ, increasing from 0 to $\frac{\sqrt{2}-1}{4}$ as η increases from $\frac{2}{3}$ to 1. The maximum value of ζ as a function of η is given in Table 2. It is plotted on Fig. 1, as well as the maximum affordable value of ζ if the conditions were not the optimum ones, but these of Eqs. (1), (2), (3), (4), and (5) instead.

The program also recorded the values of $\alpha_2-\alpha_1$ and $\beta_2-\beta_1$ and computed the relevant eigenvector ψ, i.e. the conditions that makes $\mathcal{J}_\mathcal{B}$ of Eq. (20) equal to zero for the maximum ζ. There were degeneracies in the solutions. The two angles $\alpha_2-\alpha_1$ and $\beta_2-\beta_1$ could always be taken to be the same, or the opposite of one another, as can be understood from an analytic study of Eqs. (6) and (21). The vector ψ turned out to be of the form

$$\psi = \left| \begin{array}{c} (1+r)e^{-i\omega} \\ -(1-r) \\ -(1-r) \\ (1+r)e^{i\omega} \end{array} \right| \tag{25}$$

which can be reached in the two-photon experiment considered in this paper

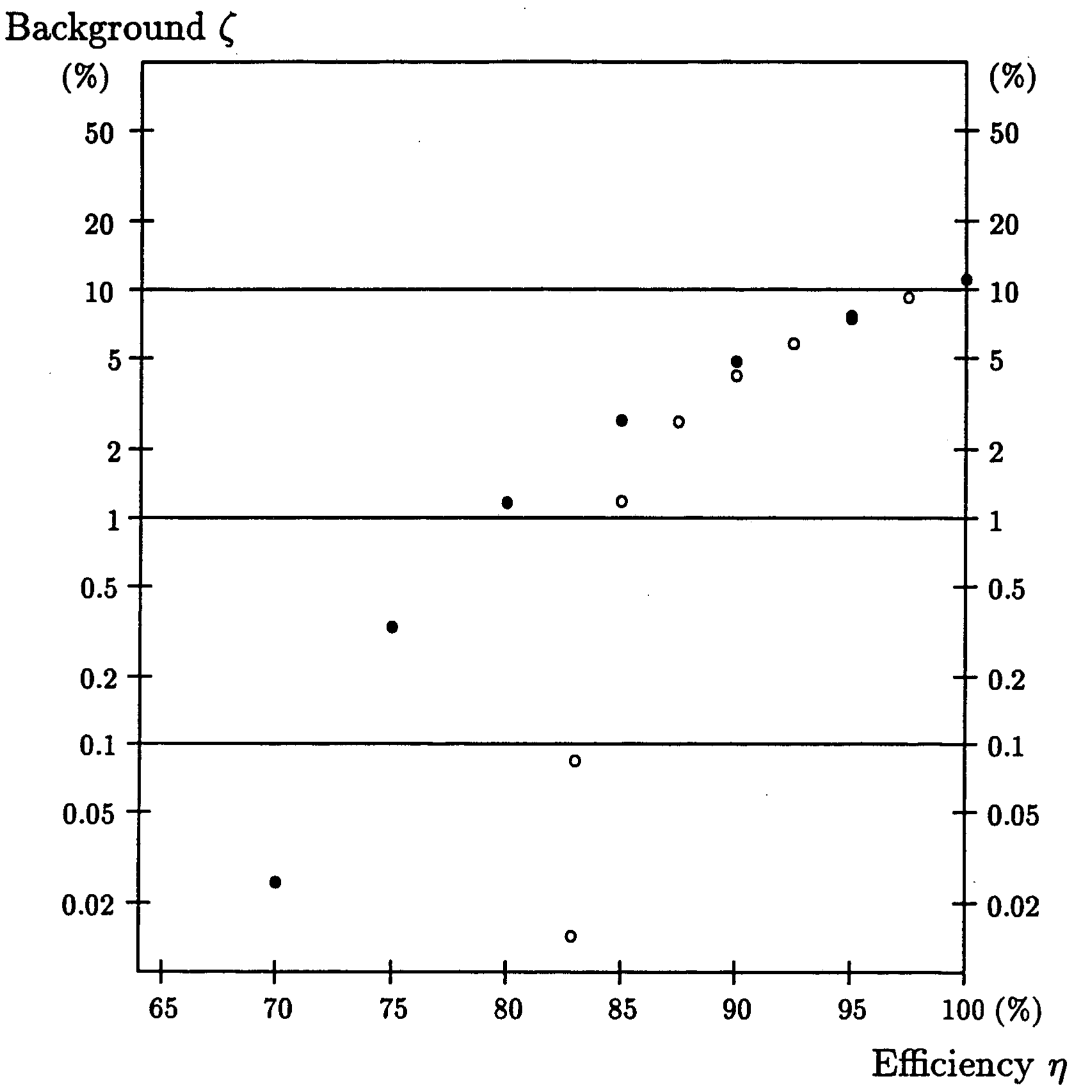

Figure 1: *Maximum affordable background versus efficiency; •: optimized conditions; ∘: conditions of Eqs. (1) to (5).*

$\eta(\%)$	$\zeta(\%)$	r	ω (°)	$\alpha_2 - \alpha_1$ (°)
66.7	0.00	0.001	0.0	2.2
70	0.02	0.136	3.4	21.4
75	0.31	0.311	9.7	32.0
80	1.10	0.465	14.9	37.9
85	2.48	0.608	18.6	41.5
90	4.50	0.741	20.9	43.6
95	7.12	0.871	22.1	44.7
100	10.36	1.000	22.5	45.0

Table 2: *Extreme Conditions for a Loophole-Free Experiment.*

by first superposing states $|\leftrightarrow\updownarrow>$ and $|\updownarrow\leftrightarrow>$ in unequal amounts,

$$\psi_0 = \frac{1}{4\sqrt{1+r^2}}\Big(|\leftrightarrow\updownarrow> \; + r|\updownarrow\leftrightarrow>\Big) \; ; \tag{26}$$

then rotating the planes of polarization of a and of b in setup (α_1, β_1) by the angles

$$\alpha_1 = 90° - \frac{\omega}{2}, \tag{27}$$

$$\beta_1 = \frac{\omega}{2} \tag{28}$$

respectively, with the values of r, ω and $\alpha_2 - \alpha_1$ $(\equiv \beta_1 - \beta_2)$ given in Table 2.

In conclusion, it is possible to perform a loophole-free experiment if the efficiency η of the photon counters is higher than 66.7% and the background is less than the value indicated on Fig. 1 for that value of η. For small background levels, it is possible to perform a loophole-free EPR-experiment with less than 82.8% counter efficiency.

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