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Limited Modified Gravity

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We systematically assess several limiting cases of modified gravity, where particular theoretical or observational conditions hold. This framework includes the well known scalar-tensor gravity and No Slip Gravity and No Run Gravity, and we extend it to three new limits: Only Run, Only Light, and Only Growth Gravities. These limits give simplifications that allow deeper understanding of modified gravity, including demonstration that gravitational effects on light and matter can have opposite signs in their deviation from general relativity. We also show observational predictions for the different cosmic structure growth rates $f\sigma_8$ and the ratio of gravitational wave standard siren luminosity to photon standard candle luminosity distance relations, defining a new statistic D_G that emphasizes their complementarity and ability to distinguish models.

I. INTRODUCTION

Modification of general relativity is a viable explanation for current cosmic acceleration and has several further predictable consequences beyond the expansion history, such as the change in large scale structure growth relative to the expansion history, change in light deflection behavior, gravitational slip (distinction between time-time and space-space metric gravitational potentials), and altered propagation of gravitational waves relative to light (even for the same speed of propagation).

This offers a rich array of signatures that current and upcoming observations can test (see recent reviews such as [1–4]). However, unlike the expansion history, which for a broad variety of models can be described by a few parameters (e.g. the matter density, dark energy equation of state today w_0, and dark energy time variation w_a have been shown to reconstruct the expansion history to 0.1% accuracy [5]), modified gravity theories need not just a few parameters but four functions of time in general, in addition to the expansion history [6–10].

Still, if our first goal is to detect any modifications of gravity, one can parametrize the effects on the observables. For example, binning the modified Poisson equation gravitational strength $G_{\rm matter}$ in redshift delivers subpercent accuracy for growth of structure with just 2–3 parameters [11, 12]. Comparison of observational data in expansion vs in growth offers another useful avenue for alerts, e.g. [13, 14].

A middle path is also desirable, where one has some closer connection to theory, and a way of seeing covariances between the several observable effects mentioned in the opening paragraph. One would like to narrow down the multiple functional freedom but still have a viable theory that can predict the multiple effects simultaneously. We call this limited modified gravity.

Section II introduces limited modified gravity in two forms, from the theory side and the phenomenology side, giving a systematic summary of the limiting cases. In Sec. III we investigate the main effects of the three new cases on gravitational strengths, sound speed stability,

etc. Section IV then propagates this to observables, showing how these models can be tested, and introducing the D_G statistic, revealing signatures beyond general relativity. We conclude in Sec. V.

II. THE SYSTEM OF LIMITED MODIFIED GRAVITY

Many individual theories of modified gravity exist, but under certain physical assumptions the most general scalar-tensor theory involving no more than second derivatives is the Horndeski class, e.g. [6, 8, 15, 16]. This will contain four functions of the scalar field ϕ and its kinetic term X, $G_i(\phi, X)$ for $i = 2$–5. No general principle for how to specify the functional forms is known. Within linear theory one can equivalently phrase the gravitational action in effective field theory terms, which can in turn be treated by property functions, which are functions of time, e.g. [9, 10]. No general principle for how to specify those functional forms is known.

The situation improves somewhat if one applies the constraint that gravitational waves must propagate at the speed of light, as strongly suggested by observations [17–19]. This (in the simplest interpretation) removes G_5 and makes $G_4(\phi)$, independent of X. On the property function side, it removes α_T. It is still difficult to see how to choose, or parametrize, the Horndeski functions, but there are some interesting perspectives on the property function side. Some modified gravity theories have specific relations between the property functions $\alpha_i(t)$, for example $f(R)$ gravity has $\alpha_B(t) = -\alpha_M(t)$.

For the theory avenue, therefore, we work with the property functions. Since the kineticity α_K generally has negligible observational impact [9, 20], we mostly ignore it (setting it to 10^{-4}, see [20]), leaving two functions: the Planck mass running α_M and the braiding α_B. We explore limited modified gravity through the limits where one or the other of these functions is zero (when both are zero then general relativity holds), or one is determined by the other. That is, we focus on how one function

determines the observational signatures.

For the phenomenology avenue, we work with the modified Poisson equations that relate the metric gravitational potentials to the observable large scale structure spatial density perturbations:

$$\nabla^2 \psi = 4\pi G_N \delta\rho \times G_{\text{matter}} \tag{1}$$

$$\nabla^2 (\psi + \phi) = 8\pi G_N \delta\rho \times G_{\text{light}} . \tag{2}$$

The first equation governs the growth of structure, with a gravitational strength G_{matter} relative to Newton's constant G_N, and the second governs the deflection of light, with a gravitational strength G_{light}. Both G_{matter} and G_{light} are functions of time. We explore limited modified gravity here through the limits where one or the other of these functions is unity (when both are unity then general relativity holds), or they are equal to each other.

Other quantities that come from the two main functions are the gravitational slip,

$$\bar{\eta} = \frac{G_{\text{matter}}}{G_{\text{light}}} = \frac{2\psi}{\psi + \phi} , \tag{3}$$

measuring the offset between the metric potentials or gravitational strengths, and the sound speed of scalar fluctuations, c_s, important for testing stability of the theory, with $c_s^2 \geq 0$ required.

The relations between the phenomenological approach and the theory quantities are (with $\alpha_T = 0$):

$$G_{\text{matter}} = \frac{m_p^2}{M_\star^2} \frac{(2 + 2\alpha_M)(\alpha_B + 2\alpha_M) + 2\alpha_B'}{(2 - \alpha_B)(\alpha_B + 2\alpha_M) + 2\alpha_B'} \tag{4}$$

$$G_{\text{light}} = \frac{m_p^2}{M_\star^2} \frac{(2 + \alpha_M)(\alpha_B + 2\alpha_M) + 2\alpha_B'}{(2 - \alpha_B)(\alpha_B + 2\alpha_M) + 2\alpha_B'} \tag{5}$$

$$\bar{\eta} = \frac{(2 + 2\alpha_M)(\alpha_B + 2\alpha_M) + 2\alpha_B'}{(2 + \alpha_M)(\alpha_B + 2\alpha_M) + 2\alpha_B'} \tag{6}$$

$$c_s^2 = \frac{1}{\alpha_K + 3\alpha_B^2/2} \left[\left(1 - \frac{\alpha_B}{2}\right)(\alpha_B + 2\alpha_M) \right. \tag{7}$$

$$\left. + \frac{(H\alpha_B)'}{H} + \frac{\rho_m}{H^2}\left(1 - \frac{m_p^2}{M_\star^2}\right) + \frac{\rho_{\text{de}}(1 + w)}{H^2} \right] ,$$

where m_p is the usual (constant) Planck mass, $M_\star(t)$ is the running Planck mass, prime denotes $d/d\ln a$ with a the scale factor, H is the Hubble parameter, ρ_m is the matter density, ρ_{de} the effective dark energy density, and w the dark energy equation of state.

We can now identify several special limiting cases that will hopefully offer some interesting insights into how each limit affects observations, and provide benchmark theories that are tractable to test.

From the theory side, these are when $\alpha_M = 0$ and when $\alpha_B = 0$. The first of these is known as No Run Gravity, since the Planck mass is constant, and was investigated in [21]. The second has not been specifically studied, and is explored here as "Only Run Gravity". There are also well known theories relating nonzero α_M and α_B, such as various scalar-tensor theories like $f(R)$ gravity,

Brans-Dicke, or chameleon theories where $\alpha_B = -\alpha_M$, and No Slip Gravity where $\alpha_B = -2\alpha_M$ [20, 22]. One can write more generally $\alpha_B = R\alpha_M$ where R is constant [23], but this does not seem to have clear physical significance unlike the previous two cases. No Slip Gravity, as the name suggests, has the special property that the gravitational slip $\bar{\eta} = 1$, as in general relativity, despite modifying other parts of gravitation. The $\alpha_B = -\alpha_M$ scalar-tensor cases reduce the effect on light deflection to arise only from the Planck mass change. Thus, three of the four limited modified gravity classes of this sort are known, and we will explore the fourth.

From the phenomenology side, we can either set $G_{\text{light}} = m_p^2/M_\star^2$, i.e. the only effect on light is from the Planck mass, or do the same for $G_{\text{matter}} = m_p^2/M_\star^2$. The former leads to either the above scalar-tensor theories or to No Slip Gravity. The latter leads to No Slip Gravity. If we set $G_{\text{matter}} = G_{\text{light}}$ then the only solutions are either general relativity (where they are both unity) or No Slip Gravity (where they are both $m_p^2/M_\star^2$). These properties demonstrate that No Slip Gravity is a particularly physically meaningful limit (even apart from having no slip). If we force G_{matter} or G_{light} to be completely unaffected, i.e. to be unity, then we have two new theories, which we call Only Light Gravity and Only Growth Gravity, since only one of G_{light} or G_{matter} is modified. Of course if both are unity then we have general relativity. Imposing such conditions on G_{matter} or G_{light} defines a relation between α_M and α_B as we discuss below.

Table I summarizes the limited modified gravity classes, and gives the expressions for G_{matter}, G_{light}, $\bar{\eta}$, α_M, and α_B in each one.

III. GRAVITATIONAL FUNCTIONS

We focus in the rest of the article on the three new theories, and here relate the key quantities in each.

A. Only Run Gravity

When $\alpha_B = 0$ then the scalar sector is not braided with the tensor sector and the effective dark energy does not cluster on subhorizon scales (see [9] for details), however there is still modification to matter perturbation growth. The main quantities are determined by α_M (and the expansion history), with

$$G_{\text{matter}} = \frac{m_p^2}{M_\star^2}(1 + \alpha_M) \tag{8}$$

$$G_{\text{light}} = \frac{m_p^2}{M_\star^2}\left(1 + \frac{\alpha_M}{2}\right) \tag{9}$$

$$\bar{\eta} = \frac{1 + \alpha_M}{1 + \alpha_M/2} . \tag{10}$$

Note that $\alpha_B = 0$ implies in the Horndeski approach that $G_{4\phi} = XG_{3X}$. This can be compared to No Slip Gravity

Model	G_{matter}	G_{light}	$\bar{\eta}$	α_M	α_B	Reference
Scalar-tensor	G_{matter}	$\frac{m_p^2}{M^2}$	$\bar{\eta}$	α_M	$-\alpha_M$	e.g. [24–27]
No Slip	$\frac{m_p^2}{M_\star^2}$	$\frac{m_p^2}{M_\star^2}$	1	α_M	$-2\alpha_M$	[22]
No Run	G_{matter}	G_{matter}	1	0	α_B	[21]
Only Run	$\frac{m_p^2}{M_\star^2}(1+\alpha_M)$	$\frac{m_p^2}{M_\star^2}\left(1+\frac{\alpha_M}{2}\right)$	$\frac{1+\alpha_M}{1+\alpha_M/2}$	α_M	0	new
Only Light	1	G_{light}	$1/G_{\text{light}}$	α_M	dif.eq.(α_M)	new
Only Growth	G_{matter}	1	G_{matter}	α_M	dif.eq.(α_M)	new

TABLE I. Several limited modified gravity models are of special interest, having specific characteristics in either theoretical or observational functions. The lower three are new and analyzed in this article. The notation "dif.eq.(α_M)" denotes that α_B is determined from α_M by a differential equation.

which has $G_{4\phi} = -XG_{3X}$.

In the early universe we wish all quantities to restore to general relativity, to preserve primordial nucleosynthesis and the cosmic microwave background (CMB) results, so we want $\alpha_M(a \ll 1) \to 0$. At late times, if we seek a de Sitter state then all time variations must stop, so $M_\star^2$ freezes and again $\alpha_M(a \gg 1) \to 0$, since $\alpha_M \equiv d\ln M_\star^2/d\ln a$. This tells us that at early times $G_{\text{matter}} \to G_{\text{light}} \to 1$, $\bar{\eta} \to 1$ and at late times $G_{\text{matter}} \to G_{\text{light}} \to m_p^2/M_\star^2$, $\bar{\eta} \to 1$. Moreover, $\bar{\eta}$ will have its maximum deviation from unity when α_M does. Thus a deviation of α_M (and hence growth and other effects) from general relativity in the observable epoch will also give slip in that epoch.

One can also see from Eq. (7) that the sound speed squared will be proportional to α_M at early times, requiring $\alpha_M(a \ll 1) \geq 0$ for stability. The same holds in the approach to a de Sitter state. Thus the simplest model will be of a hill form, as in [22]. We adopt that here:

$$\alpha_M = \frac{4c_M\,(a/a_t)^\tau}{[(a/a_t)^\tau + 1]^2}\ , \tag{11}$$

which also gives an analytic expression

$$\frac{M_\star^2}{m_p^2} = e^{(2c_M/\tau)(1+\tanh[(\tau/2)\ln(a/a_t)])}\ . \tag{12}$$

The Planck mass squared therefore goes from 1 in the past to $e^{4c_M/\tau}$ in the future. Note the maximum of α_M is c_M, occurring at $a = a_t$, and τ is a measure of the transition width.

Unlike in No Slip Gravity and No Run Gravity, stability at early times only depends on the sign of c_M, requiring it to be positive, and not on the value of τ. We also assume $\alpha_K > 0$. What is especially interesting is the influence of τ on G_{matter} and G_{light}. At early times $\alpha_M \to 4c_M(a/a_t)^\tau$ and

$$G_{\text{matter}} \to 1 + (\tau-1)\alpha_M \tag{13}$$
$$G_{\text{light}} \to 1 + (\tau/2-1)\alpha_M\ . \tag{14}$$

This implies that for $1 < \tau < 2$, the gravitational strength deviations from general relativity $G_{\text{matter}} - 1$

and $G_{\text{light}} - 1$ will have opposite signs, one being weaker and one being stronger than Einstein gravity. This gives a direct proof that such a condition is possible, as argued by [28] regarding the conjecture requiring same signs [29].

Figure 1 exhibits the evolution of the key quantities G_{matter}, G_{light}, $\bar{\eta}$, and c_s^2. Indeed we see that $G_{\text{matter}} - 1$ and $G_{\text{light}} - 1$ can have opposite signs – gravity for growth is strengthened while gravity for light deflection is weakened – at early times. Gravitational slip is present and $c_s^2 > 0$ shows the theory is stable. Extending the numerical evolution further to the future than shown verifies that all quantities freeze to constant values (with $\bar{\eta} \to 1$).

Figure 2 explores the opposite sign conditions on $G_{\text{matter}} - 1$ and $G_{\text{light}} - 1$ in further detail, verifying the analytic derivation that this occurs at early times when $1 < \tau < 2$. Note however that the opposite signs can occur more generally at later times, including during the key observational window $a \approx 0.25$–0.6.

B. Only Growth Gravity

When we limit the modification of gravity to the growth sector, leaving light deflection unchanged from general relativity, $G_{\text{light}} = 1$, this imposes a condition relating α_B to α_M through a differential equation,

$$\alpha_B' = (\alpha_B + 2\alpha_M)\left[-1 + \frac{\alpha_B + \mu\alpha_M}{2(1-\mu)}\right]\ , \tag{15}$$

where $\mu = m_p^2/M_\star^2$.

We can plug this back into Eq. (4) to obtain

$$G_{\text{matter}} = \frac{\alpha_B + \alpha_M(2-\mu)}{\alpha_B + \alpha_M}\ . \tag{16}$$

Note however that one must solve the differential equation to obtain $\alpha_B(\alpha_M)$. The early universe limit is $G_{\text{matter}} \to 1$ so $\mu \to 1$, $\alpha_M \to 0$, $\alpha_B \to 0$. The de Sitter limit is $G_{\text{matter}} \to 1$ with $\alpha_B \to 2(1 - m_p^2/M_{\star,\text{dS}}^2)$ since $\alpha_B' \to 0$. The stability condition $c_s^2 \geq 0$ at early times requires $\alpha_M > 0$. The differential equation is straightforward to solve and we present the numerical results below, also checking stability for all times. The results are insensitive to initial conditions, as long as $\alpha_{B,i} < -\alpha_{M,i}$
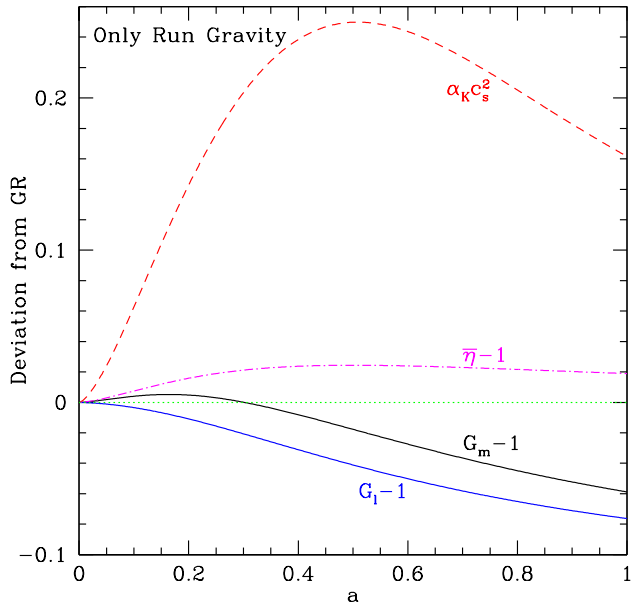
FIG. 1. The deviations from general relativity (GR) for the gravitational coupling strength for matter $G_{matter} - 1$, for light $G_{light} - 1$, the gravitational slip $\bar{\eta} - 1$, and the sound speed squared c_s^2 for Only Run Gravity are plotted vs scale factor. Here we take the hill form Eq. (11) for α_M with $c_M = 0.05$, $a_t = 0.5$, $\tau = 1.5$.
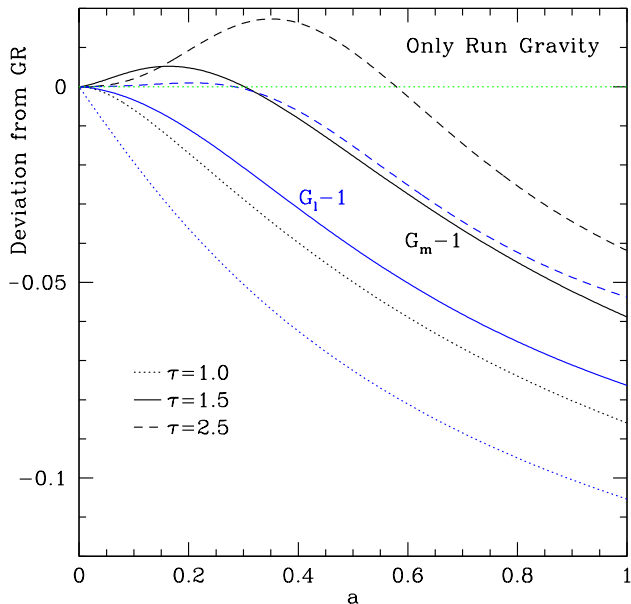

FIG. 2. The gravitational strengths G_{matter} and G_{light} for Only Run Gravity can exhibit opposite deviations from general relativity (opposite signs in $G_{matter} - 1$ and $G_{light} - 1$) for some of their evolution. We show cases for three different values of τ, with the behavior following the analytic predictions of Eqs. (13)–(14).
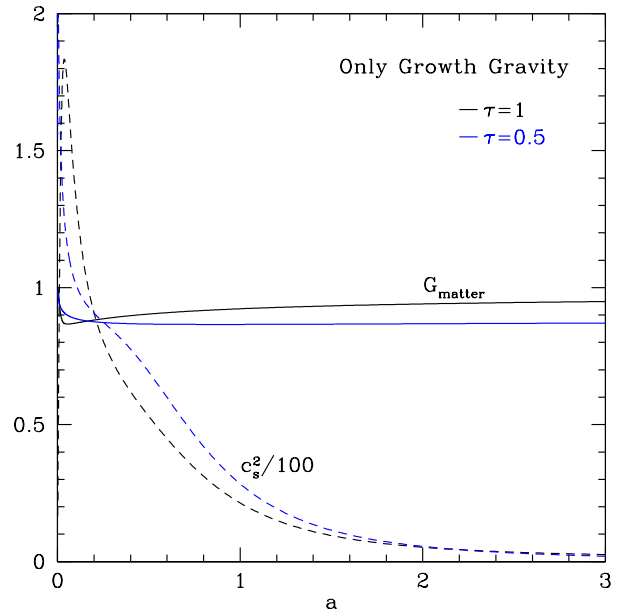

FIG. 3. The gravitational coupling strength for matter G_{matter} in Only Growth Gravity can be weaker than in general relativity. The evolution $G_{matter}(a)$ is shown with the solid curves, and the sound speed squared $c_s^2(a)$ by the dashed curves. For each, two values of τ are exhibited for α_M in the hill form, with $c_M = 0.03$, $a_t = 0.5$.

(otherwise G_{matter} will diverge when $\alpha_B + \alpha_M$ crosses zero).

Figure 3 shows G_{matter} and c_s^2 for the hill form of α_M with $c_M = 0.03$, $a_t = 0.5$, and two different values of τ. (Recall that for this model $\bar{\eta} = G_{matter}$.) Note that the modified Poisson equation for growth shows weaker gravity than general relativity (while the Poisson equation for light deflection is the same as general relativity by construction). The minimum strength $G_{matter,min} \approx 1 - 4c_M$ and then it slowly approaches $G_{matter} \to 1$ in the de Sitter limit (verified by extending the integration to $a \gg 1$); also $c_s^2 \to 0$ in that limit. Only Growth Gravity, like No Slip Gravity but unlike many scalar-tensor theories, suppresses growth – this can ease some tensions in $f\sigma_8$ measurements from redshift space distortions with respect to ΛCDM cosmology, and possibly also σ_8 tensions from weak lensing – making it a theory worth further study.

C. Only Light Gravity

The third new theory is limiting the modification of gravity to the light deflection sector, leaving growth unchanged from general relativity, $G_{matter} = 1$. This again gives a relation for α_B in terms of α_M, in the form of

$$\alpha'_B = (\alpha_B + 2\alpha_M)\left[-1 + \frac{\alpha_B + 2\mu\alpha_M}{2(1-\mu)}\right] . \quad (17)$$

We can plug this back into Eq. (5) to obtain

$$G_{\text{light}} = \frac{\alpha_B + \alpha_M(1 + \mu)}{\alpha_B + 2\alpha_M} \ . \qquad (18)$$

Again note that one must solve the differential equation to obtain $\alpha_B(\alpha_M)$. The early universe limit is $G_{\text{light}} \to 1$ so $\mu \to 1$, $\alpha_M \to 0$, $\alpha_B \to 0$. The de Sitter limit is $G_{\text{light}} \to 1$ with $\alpha_B \to 2(1 - m_p^2/M_{\star,\text{dS}}^2)$, as in the Only Growth Gravity case, and again the differential equation is straightforward to solve.

Only Light Gravity is more difficult, however, in that the denominator of G_{light} involves $\alpha_B + 2\alpha_M$ and this is exactly the prefactor in the α_B' equation. This means that if at some point in the evolution of α_B it reaches or crosses $-2\alpha_M$, as the dynamical equation motivates, then the gravitational strength diverges. We have not been able to find cases yet where this does not occur (e.g. trying the hill form for α_M, or power law times Gaussians), though we also have not found a proof there is no nondivergent solution.

IV. OBSERVATIONAL FUNCTIONS

These modified gravity theories are highly predictive (in the linear regime at least). With the expressions for G_{matter}, G_{light}, and $M_\star^2$ one can calculate observables in growth and light propagation. Furthermore, [22] identified a clear link between predictions for cosmic growth and for gravitational wave propagation. Basically, deviations in cosmic growth predict deviations in gravitational waves and vice versa.

This allows an important test for modified gravity – if a signature is seen in growth of large scale structure, it could be seen as well in the luminosity distances of gravitational wave standard sirens vs standard candles. Such a crosscheck is a valuable systematics test; while one might find other cosmological model parameters or astrophysical uncertainties that could change growth and, say, the CMB or lensing dynamics in a way that mimics modified gravity (e.g. neutrinos or selection effects), such common systematics are much less likely with a gravitational wave comparison.

Therefore in this section we not only look at the observational effects on large scale structure growth through the growth rate $f\sigma_8$, but also their connection to observational effects on gravitational wave propagation. Recall that luminosity distances for photon sources, such as supernovae, only depend on the background expansion, which we are holding fixed when we change gravity from general relativity. However gravitational wave propagation is sensitive to the Planck mass running [22, 30–37], and so

$$\frac{d_{L,\text{GW}}(a)}{d_{L,\gamma}(a)} = \left[\frac{M_\star^2(a=1)}{M_\star^2(a)}\right]^{1/2} \ . \qquad (19)$$

Figure 4 shows the prediction for both probes for No Slip Gravity. We see the characteristic suppression of
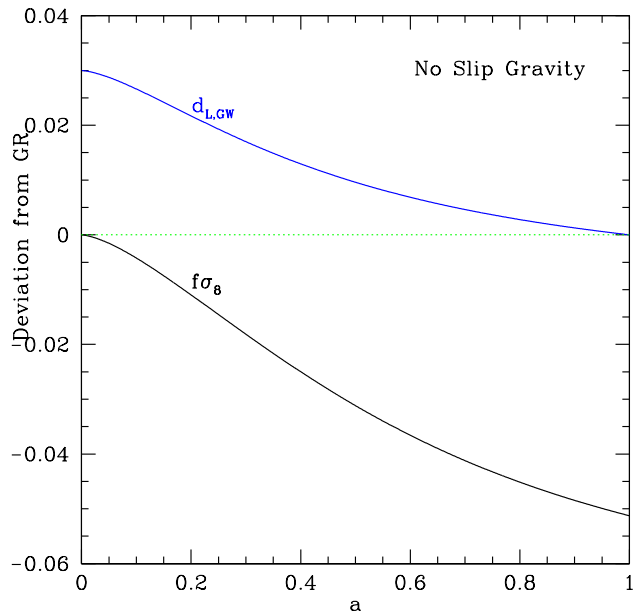

FIG. 4. Deviations from general relativity in the cosmic growth and gravitational wave distance predictions are connected, and serve as a valuable crosscheck. Here the relations are shown for $d_{L,\text{GW}}^{\text{MG}}/d_L^{\text{GR}} - 1$ and $f\sigma_8^{\text{MG}}/f\sigma_8^{\text{GR}} - 1$ for No Slip Gravity, with model parameters $c_M = 0.03$, $a_t = 0.5$, $\tau = 1.5$. Deviations will scale linearly with c_M.

growth, at the 3–5% level, relative to general relativity, over the currently measured range of redshifts using redshift space distortions as in Fig. 3 of [22]. But in addition we plot the deviation in luminosity distance to gravitational wave standard sirens relative to photon luminosity distances, e.g. from standardized candles such as Type Ia supernovae. At redshift $z = 1$ this model predicts a 1% deviation in d_L, concomitant with a 3% deviation in $f\sigma_8$. As measurements move to higher redshift, say $z = 2$, the deviations become 1.6% in d_L and 2% in $f\sigma_8$. The numbers given are for $c_M = 0.03$ and will scale linearly with c_M. The key point is that the gravity model predicts exactly how they should be related at all redshifts, allowing for leverage by combining several low signal to noise measurements.

Figure 5 shows the growth and gravitational wave quantities for Only Run Gravity. Here, the deviation of the growth from general relativity is partially canceled because the gravitational strength G_{matter} is enhanced at high redshift, but suppressed at low redshift, as seen in Fig. 2. This increases $f\sigma_8$ relative to general relativity for $a \lesssim 0.5$ but decreases it for $a \gtrsim 0.5$. That allows higher values of Planck mass running amplitude c_M to be viable for growth observations. However, the hiding of the deviation in growth due to the cancellation does not hold for the gravitational wave luminosity distance, which sees simply the enhancement of $M_\star^2$ relative to m_p^2. Thus the two observational probes work extremely well
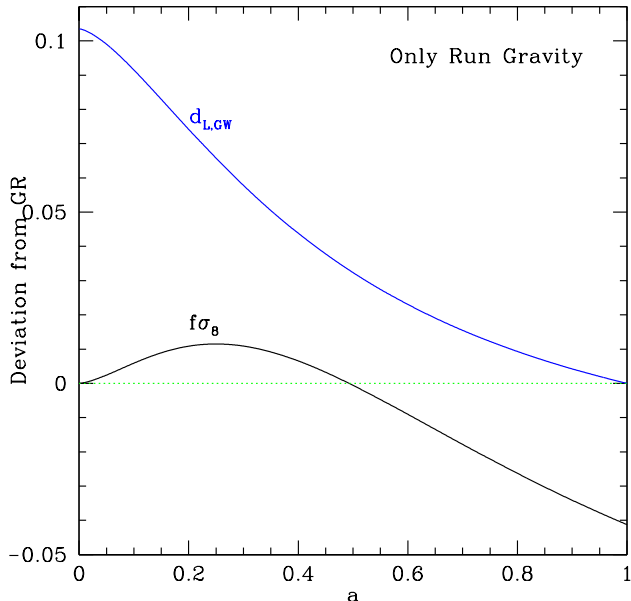
FIG. 5. As Fig. 4 but for Only Run Gravity, with model parameters $c_M = 0.1$, $a_t = 0.5$, $\tau = 1.5$. Relatively large values of c_M still give viable results for growth, allowing for strong effects on gravitational waves.
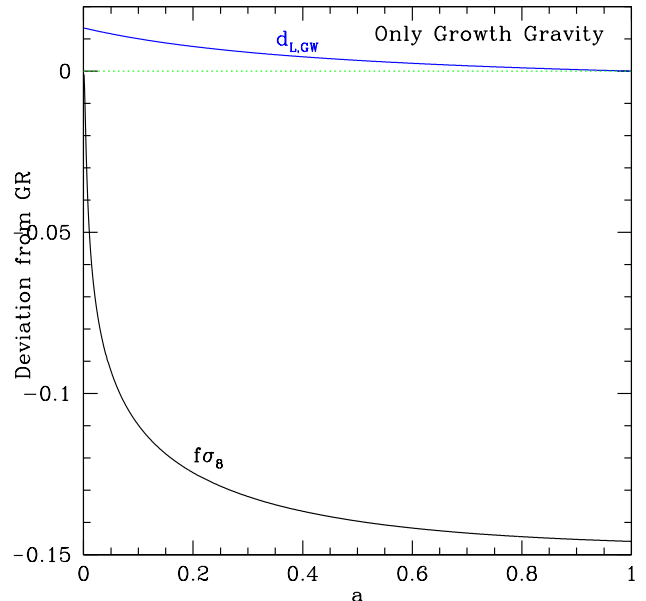

FIG. 6. As Fig. 4 but for Only Growth Gravity, with model parameters $c_M = 0.01$, $a_t = 0.5$, $\tau = 1$. Note that the early time, and sustained, weakening of G_{matter} as seen in Fig. 3 have a strong effect to suppress growth. This indicates that even small values of c_M can have an observable effect on growth, though then the effect on gravitational waves becomes negligible.

together.

Figure 6 shows the growth and gravitational wave quantities for Only Growth Gravity. This has a third, distinct behavior for the relation between growth and gravitational waves. Due to the rapid suppression of G_{matter} at early times, the growth gets off to a slow start, and the continued weakness of gravity does not allow it to recover, giving a strongly suppressed growth rate in the observational epoch. This requires a small value of c_M for viability, which substantially reduces the signature of deviation in gravitational waves. However this does mean that cosmic growth measurements can probe much smaller c_M values than the other models discussed.

Thus we have seen that cosmic growth rate measurements through redshift space distortions and gravitational wave luminosity distance measurements through standard sirens have great complementarity. The three models we discussed in this section have distinct signatures in each, with predictions for their respective redshift dependences. Measurements through both probes could not only test general relativity but distinguish between these classes of gravity models: No Slip Gravity gives discernible deviations in each, Only Run Gravity has a larger effect on gravitational waves, and Only Growth Gravity has a larger effect on the cosmic growth rate. (And of course Only Light Gravity has no effect on growth, only on gravitational waves, while No Run Gravity has no effect on gravitational waves, but enhances growth.)

We demonstrate the clear leverage for distinguishing

the classes of gravity by defining a new statistic,

$$D_G(a) = \frac{d_{L,\text{GW}}^{\text{MG}}/d_L^{\text{GR}}}{f\sigma_8^{\text{MG}}/f\sigma_8^{\text{GR}}} \ . \qquad (20)$$

In general relativity this is simply a constant with value unity for all a. However each of the classes of modified gravity we discussed will not only show in the D_G statistic deviations from unity (testing general relativity), but have a distinct shape with redshift. While scaling c_M will change the amplitude, it will not mix the shapes. Figure 7 illustrates that indeed the different models are highly distinct in the D_G statistic.

V. CONCLUSIONS

We assessed in a systematic way limits of modified gravity in terms of property functions and observational functions, including introducing three new classes of modified gravity. Such limits are simpler than the full freedom of gravity theories but are more predictive, and display clear signatures that observations can use to test general relativity and distinguish between theory classes.

For the three new theories – Only Run Gravity, Only Growth Gravity, and Only Light Gravity – we compute the key functions of the gravitational strengths for cosmic growth and for light deflection, G_{matter} and G_{light}, and
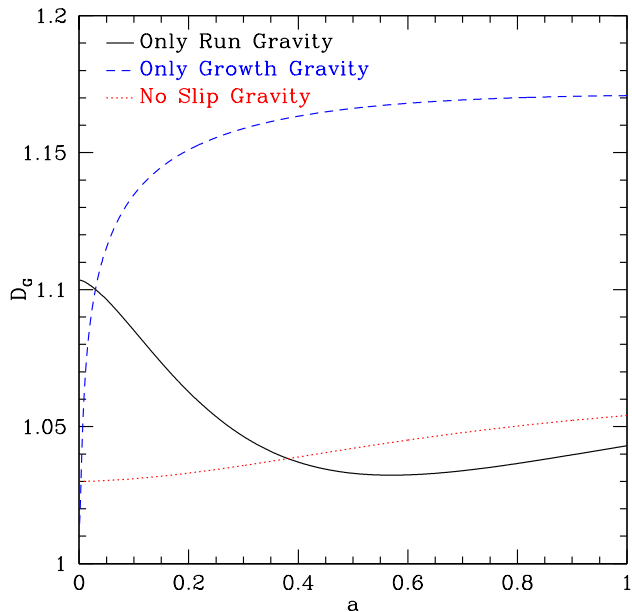
FIG. 7. The new D_G statistic, using the complementarity of the gravitational wave luminosity distance $d_{L,\mathrm{GW}}$ and the cosmic matter growth rate $f\sigma_8$, can clearly distinguish different classes of gravity. Each class has a distinct shape in its redshift dependence $D_G(a)$. General relativity has constant $D_G = 1$.

the gravitational slip $\bar{\eta}$ and scalar perturbation sound speed squared c_s^2. Interestingly, Only Run Gravity provides a definite demonstration that the deviations from general relativity $G_{\mathrm{matter}} - 1$ and $G_{\mathrm{light}} - 1$ for matter and light can have opposite signs, which has been a topic of conjecture. These theories can also provide suppressed matter growth, in contrast to many scalar-tensor theories and in some accord with observations.

In addition to solving for the evolution of these key functions, we also calculate two observational quantities. One is $f\sigma_8$, the cosmic growth rate for large scale structure perturbations, measurable through redshift space distortions in galaxy surveys such as DESI [38]. The other is the luminosity distance to gravitational wave standard siren events, $d_{L,\mathrm{GW}}$, which can differ from the photon luminosity distance to standard candles such as Type Ia supernovae, despite a gravitational wave propagation speed equal to the speed of light.

Conjoined analysis of the two observables, $f\sigma_8$ and $d_{L,\mathrm{GW}}$, as introduced by [22], is highly insightful. For one thing, they offer a critical crosscheck for systematic control. As well, there is a diversity of behaviors between the classes of gravity in the magnitude of deviations in one vs the other, and predictive power in the specific redshift dependence between the two. This enables even low signal to noise measurements at individual redshifts to combine to give significant evidence to test general relativity and distinguish classes of gravity. We defined a new statistic D_G to use for the conjoined analysis of the two probes, illustrating that it has distinct redshift dependence for different classes. Future measurements will demonstrate the strong complementarity of these probes. Other combinations of gravitational wave and large scale structure information are discussed in, e.g., [39, 40].

There is still much to understand about modified gravity, especially if one starts furthest from the observations with the $G_i(\phi, X)$ functions in the Horndeski lagrangian. The relation between these functions exhibited by, e.g., Only Run Gravity and No Slip Gravity may provide some direction to future investigations, but here we focused on quantities closer to the observations. The approach of Limited Modified Gravity gives a framework that is tractable, predictive, and yet with a range of important characteristics that can yield insights when confronted with forthcoming data.

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