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Author
Muller, Rolf H.

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Rolf H. Muller and Michael L. Sand

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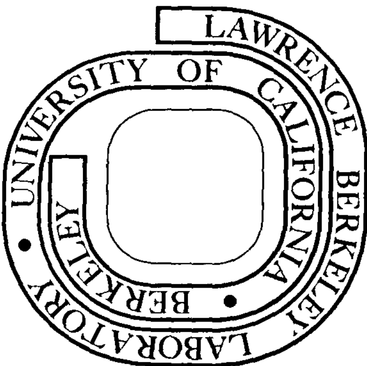

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Optimum Angle of Incidence for Monochromatic Interference in Transparent Films on Absorbing Substrates

Rolf H. Muller and Michael L. Sand*

Materials and Molecular Research Division, Lawrence Berkeley Laboratory
and Department of Chemical Engineering, University of California,
Berkeley, California 94720

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Abstract

Angles of incidence for s and p – polarized light have been computed and confirmed experimentally for which monochromatic interference in transparent thin films on absorbing substrates results in optimum interference fringe contrast (visibility = 1). By use of these angles of incidence and polarized light, film thickness determinations which are not possible at normal incidence or with unpolarized light, can be carried out with thin-film interference.

*Present address, Dept. of Chemical Engineering, University of Delaware, Newark, DE 19711

I. Introduction

The use of thin film interference is a convenient technique for the determination of the distribution of film thickness over extended surface areas with a minimum of disturbance. For the practical utility of the technique, the interference phenomenon has to result in high fringe contrast. While this requirement is usually met by interference in unsupported transparent films (such as soap films) under reflection near normal incidence, interference in films on highly reflecting substrates, such as metals, is often not observable in the same way. The qualitative reason for the deterioration of the interference phenomenon lies in the unequal amplitudes of the light waves reflected from the two film surfaces which result in only partial extinction under conditions of destructive interference. In an earlier, approximate analysis[1], the use of specific angles of incidence had been proposed where reflection from the two interfaces is equal.

This communication quantitatively defines conditions of thin-film interference which result in the best attainable fringe contrast. The film is assumed to be transparent, in contact with a gaseous incident medium and supported by a metallic substrate. The analysis shows that in order to establish well-defined phase relations, pure s or p-polarized light must be used. For given optical properties of film and substrate materials, an optimum angle of incidence is predicted for each state of polarization. An extension of the present discussion to white-light interference, in which the colorimetric purity of interference colors is optimized, is in preparation.

II. Thin Film Interference

Reflection from a film-covered surface is determined by the complex (amplitude) reflection coefficients of the two film surfaces,

$$\hat{r}_1 = r_1 e^{i\delta_1} \tag{1}$$

$$\hat{r}_2 = r_2 e^{i\delta_2} , \tag{2}$$

and the phase delay caused by propagation through the film,

$$\delta = \frac{4\pi}{\lambda} n_f d \cos \phi' , \tag{3}$$

which can be formulated as a function of λ, the vacuum wavelength of the light, n_f the refractive index of the film, d the film thickness and ϕ' the angle of refraction in the film (see also Fig. 1).

The reflectance of the film-substrate combination can be expressed as[2]

$$R = \frac{r_1^2 + r_2^2 + 2r_1 r_2 \cos(\delta_1 - \delta_2 + \delta)}{1 + r_1^2 r_2^2 + 2r_1 r_2 \cos(\delta_1 - \delta_2 + \delta)} . \tag{4}$$

Minima and maxima in reflectance, due to interference, occur for

$$\delta_1 - \delta_2 - \delta = m\pi \tag{5}$$

with

$$m = \pm 1 , \pm 3 , \pm 5 \text{ etc.} \quad \text{minima} \tag{6}$$

$$m = 0 , \pm 2 , \pm 4 \text{ etc.} \quad \text{maxima} . \tag{7}$$

It is characteristic of monochromatic thin-film interference (Eq. 5) that the effect of phase change due to reflection $(\delta_1 - \delta_2)$ can be interchanged with that due to optical path (thickness) in the film (δ). Also, under normal coherence conditions, the interference phenomenon, as a function of film thickness, is exactly repetitive past $\delta = 2\pi$. These facts are illustrated in Fig. 2.

The magnitudes of interference minima and maxima are obtained by differentiation of Eq. 4,

$$R_{min} = \left(\frac{r_1 - r_2}{1 - r_1 r_2}\right)^2 . \tag{8}$$

$$R_{max} = \left(\frac{r_1 + r_2}{1 + r_1 r_2}\right)^2 . \tag{9}$$

The reflection coefficients (Eqs. 1 and 2) depend on the refractive indices of the media, the angle of incidence and the polarization of the light. The use of pure s or p polarization greatly simplifies the consideration of phase and amplitude changes in reflection. The reflection coefficient for the gas-film interface is given by the Fresnel equations for s and p polarization,

$$r_{1s} = \left(\frac{n_o \cos\phi - n_1 \cos\phi'}{n_o \cos\phi + n_1 \cos\phi'}\right) \tag{10}$$

$$r_{1p} = \left(\frac{n_1 \cos\phi - n_o \cos\phi'}{n_1 \cos\phi + n_o \cos\phi'}\right). \tag{11}$$

The angle of refraction ϕ' is obtained from the angle of incidence ϕ and the refractive indices n_o and n_f of incident and film medium by use of Snell's law,

$$\phi' = \sin^{-1}\left(\frac{n_o}{n_f}\sin\phi\right) . \tag{12}$$

The phase change for reflection at the gas-film interface is constant for s-polarization. For the conventions and definitions used here[3] it is given by Eq. 13.

$$\delta_{1s} = \pi . \tag{13}$$

For p-polarization, it assumes different values below and above Brewster's angle,

$$\delta_{1p} = 2\pi \quad \text{for} \quad \phi < \phi_p \tag{14}$$

$$\delta_{1p} = \pi \quad \text{for} \quad \phi > \phi_p . \tag{15}$$

Reflection coefficients for the film-substrate interface can be described by expressions introduced by Koenig[4] and adapted for the present conventions and definitions[5]. They are

$$r_{2s} = \left(\frac{A^2 + B^2 - 2A\cos\phi' + \cos^2\phi'}{A^2 + B^2 + 2A\cos\phi' + \cos^2\phi'}\right)^{1/2} \tag{16}$$

$$r_{2p} = r_{2s}\left(\frac{A^2 + B^2 - 2A\sin\phi'\tan\phi' + \sin^2\phi'\tan^2\phi'}{A^2 + B^2 + 2A\sin\phi'\tan\phi' + \sin^2\phi'\tan^2\phi'}\right)^{1/2} , \tag{17}$$

where

$$A = \left[\frac{1}{2n_f^2}\left[\left((n^2-k^2-n_f^2\ \sin^2\ \phi')^2+4n^2k^2\right)^{1/2} + (n^2-k^2-n_f^2\ \sin^2\ \phi')\right]\right]^{1/2} \quad (18)$$

$$B = \left[\frac{1}{2n_f^2}\left[\left((n^2-k^2-n_f^2\ \sin^2\ \phi')^2+4n^2k^2\right)^{1/2} - (n^2-k^2-n_f^2\ \sin^2\ \phi')\right]\right]^{1/2} . \quad (19)$$

The phase changes for refection at the film-substrate interface are

$$\delta_{2s} = \tan^{-1}\left(\frac{-2B\ \cos\ \phi'}{A^2 + B^2 - \cos^2\ \phi'}\right) \quad 0 \leq \delta_s \leq \pi \quad (20)$$

$$\delta_{2p} = \tan^{-1}\left[\frac{2B\ \cos\ \phi'(A^2 + B^2 - \sin^2\ \phi')}{A^2 + B^2 - \frac{1}{n_f^4}(n^2 + k^2)^2\ \cos^2\ \phi'}\right] \quad (21)$$

III. Fringe Contrast

The observability of thin film interference can be expressed by the Michelson fringe visibility,

$$V = \frac{R_{max} - R_{min}}{R_{max} + R_{min}} . \quad (22)$$

Specific examples of how the fringe visibility and its component parts, R_{max} and R_{min} depend on angle of incidence are shown in Figures 3 and 4. The fringe visibility reaches its maximum value $V = 1$ where the minimum reflectance R_{min} is zero. For p-polarized light (Fig. 4) the fringe visibility is zero at Brewster's angle ϕ_p because the reflection coefficient for reflection from the gas-film interface is zero.

Fringe visibility at normal incidence ($\phi = 0$) for different substrate optical constants is shown in Fig. 5 for a low refractive index film. Except with substrates of low light absorption, fringe visibility at normal incidence is low. Visibility is substantially increased for a film of high refractive index (Fig. 6).

IV. Optimum Angle of Incidence

The angle of incidence at which the visibility of thin film interference fringes is one is defined here as the optimum angle of incidence ϕ_m . At this angle, according to Eqs. 8 and 22

$$R_{min} = \left(\frac{r_1 - r_2}{1 - r_1 r_2}\right)^2 = 0 . \qquad (23)$$

It follows from Eq. 23 that

$$\text{for} \quad \phi = \phi_m \quad r_1 = r_2 . \qquad (24)$$

The remarkably simple condition for the optimum angle of incidence for monochromatic thin-film interference, expressed by Eq. 24, is explored in Figs. 7-10. They illustrate, for a high and a low refractive index film, how the optimum angle of incidence depends on the complex refractive index $(n - ik)$ of the substrate and on the state of polarization (s or p) . The incident medium is assumed to be air ($n = 1$) . For the sake of clarity, overlapping curves associated with combinations of low values of n and k are not fully shown; also, not all of the substrate refractive indices used are physically realizable.

For p-polarized light (Figs. 8 and 10), an optimum angle of incidence always exists because the reflection coefficient for the air-film interface varies between 0 and 1 above Brewster's angle ϕ_p . A second optimum angle may exist if $r_{1p} > r_{2p}$ at normal incidence ($\phi = 0$) .

For s-polarized light (Figs. 7 and 9) there is at best one optimum angle of incidence. If none exists, the highest fringe visibility is found at normal incidence.

V. Experimental Test

Fringe visibilities have been derived from irradiance measurements in collimated light reflected from a vapor deposited tapered cryolyte film on a silicon substrate. The results, shown in Figs. 11 and 12 for s and p polarization confirm the theoretical analysis, considering some uncertainty in the refractive index of the film material. At high fringe visibilities, the measurements are affected by the finite width of the light probe.

Visual examination of similar films on Al substrates, where no interference is visible near normal incidence, has shown well-defined interference fringes at the predicted optimum angles of incidence.

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Figure Captions

Fig. 1. Interference in transparent film on absorbing substrate. Definition of terms: Refractive indices of incident medium, n_o , film, n_f , and substrate $\hat{n}$; film thickness, d ; angles of incidence ϕ and refraction ϕ' ; reflection coefficients for top and bottom film interfaces, $\hat{r}_1$ and $\hat{r}_2$.

Fig. 2. Reflection of monochromatic light (wavelength 570 nm) from a film-covered surface. Locus of interference minima (dotted lines) and maxima (dashed lines) as a function of optical path length $(2n_f d \cos \phi')$ in the film and phase change $(\delta_1 - \delta_2)$ due to reflection. Reflectance (solid curves) shown for 0° and 180° phase change and different amplitude reflection coefficients $(r \equiv r_1 = r_2)$.

Fig. 3. Variation of monochromatic interference fringe visibility V , reflectance at interference maxima R_{max} and minima R_{min} with angle of incidence. Dielectric film of low refractive index $(n_f = 1.35)$ on a highly-reflecting substrate $(\hat{n} = 2.07 - 4.4\, i)$. s-polarized light, optimum angle $\phi_m = 86.2°$.

Fig. 4. Variation of monochromatic interference fringe visibility V , reflectance at interference maxima R_{max} and minima R_{min} with angle of incidence. Dielectric film of low refractive index $(n_f = 1.35)$ on a highly-reflecting substrate $(\hat{n} = 2.07 - 4.4\, i)$. p-polarized light, optimum angle 85.4°.

Fig. 5. Michelson fringe visibility at normal incidence for interference in a dielectric film of low refractive index (n_f = 1.35) on absorbing substrates of complex refractive index $\hat{n} = n - ik$. Incident medium air.

Fig. 6. Michelson fringe visibility at normal incidence for interference in a dielectric film of high refractive index (n_f = 2.0) on absorbing substrates of complex refractive index $\hat{n} = n - ik$. Incident medium air.

Fig. 7. Optimum angle of incidence for monochromatic thin-film interference. Dielectric film of low refractive index (n_f = 1.35) on absorbing substrates of complex refractive index $\hat{n} = n - ik$, s-polarized light.

Fig. 8. Optimum angle of incidence for monochromatic thin-film interference. Dielectric film of low refractive index (n_f = 1.35) on absorbing substrates of complex refractive index $\hat{n} = n - ik$, p-polarized light, Brewster's angle $\phi_p = 53.5°$.

Fig. 9. Optimum angle of incidence for monochromatic thin-film interference. Dielectric film of high refractive index (n_f = 2.0) on absorbing substrates of complex refractive index $\hat{n} = n - ik$, s-polarized light.

Fig. 10. Optimum angle of incidence for monochromatic thin-film interference. Dielectric film of high refractive index (n_f = 2.0) on absorbing substrates of complex refractive index $\dot{n} = n - ik$, p-polarized light, Brewster's angle ϕ_p = 63.5° .

Fig. 11. Interference in a tapered cryolite film (n_f = 1.34) on a silicon substrate ($\hat{n}$ = 4.14 - 0.03 i) . Photometrically determined (circles) and computed (curve) fringe visibility as a function of angle of incidence. λ = 546 nm, s-polarization.

Fig. 12. Interference in a tapered cryolite film (n_f = 1.34) on a silicon substrate ($\hat{n}$ = 4.14 - 0.03 i) . Photometrically determined (circles) and computed (curve) fringe visibility as a function of angle of incidence. λ = 546 nm, p-polarization.

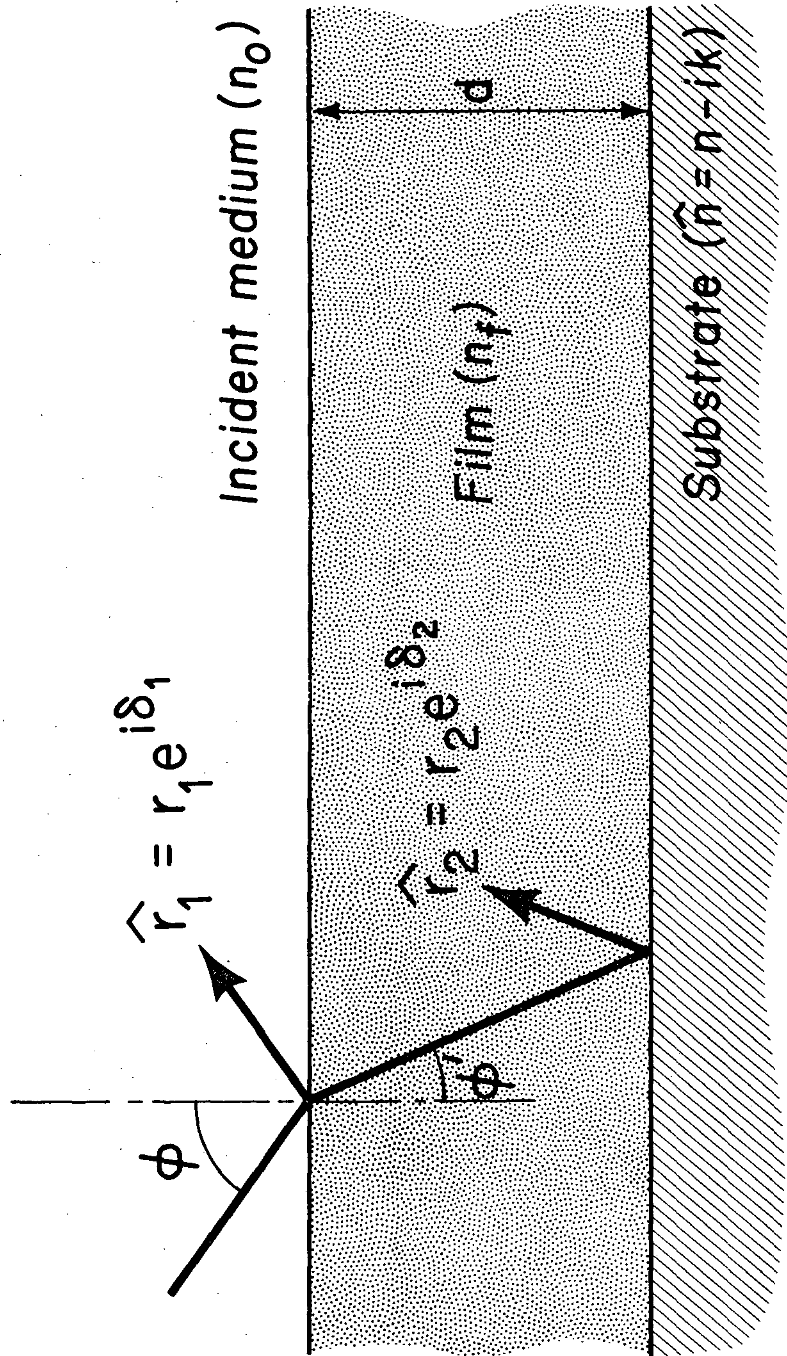
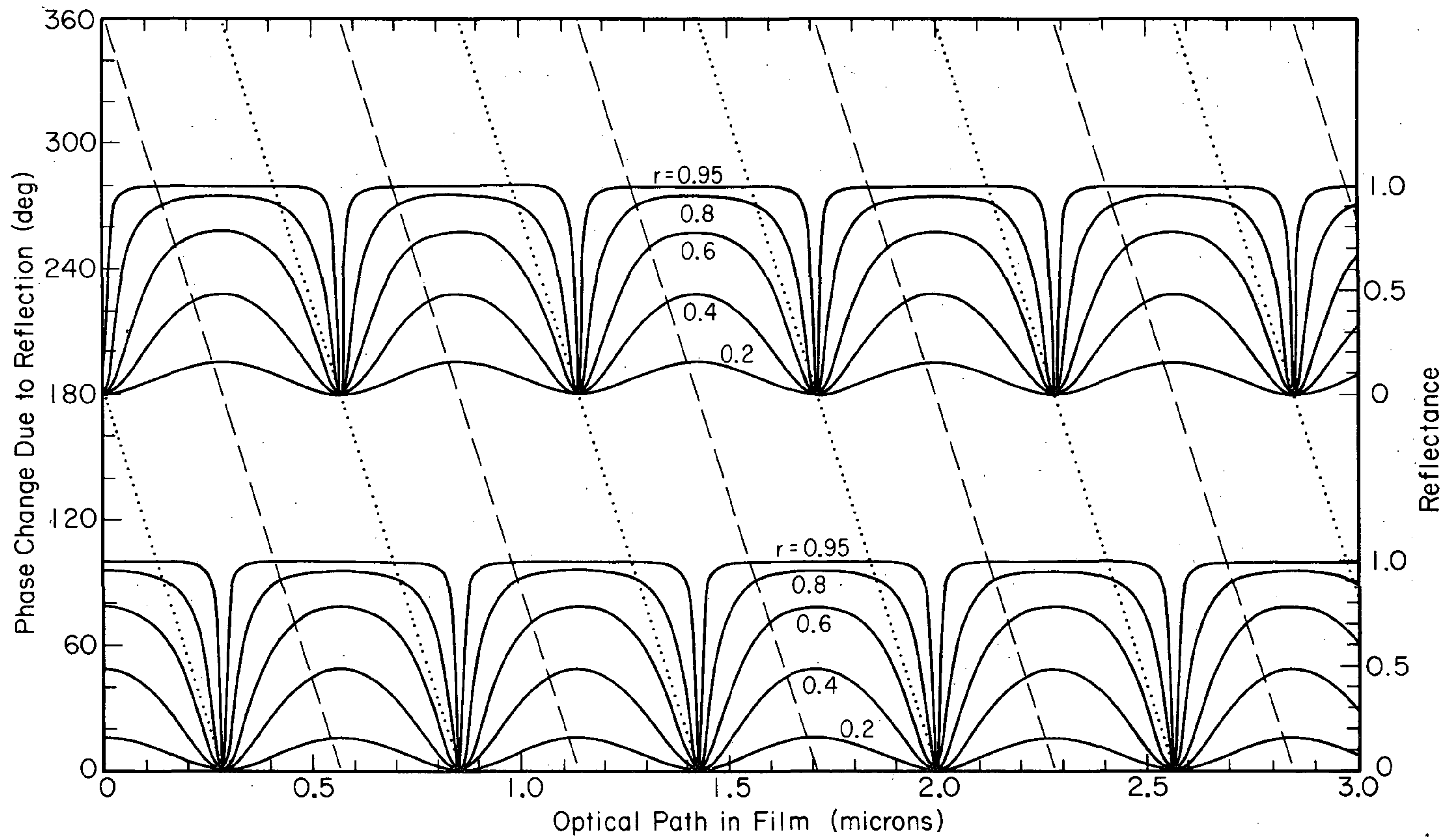


FIGURE 1.



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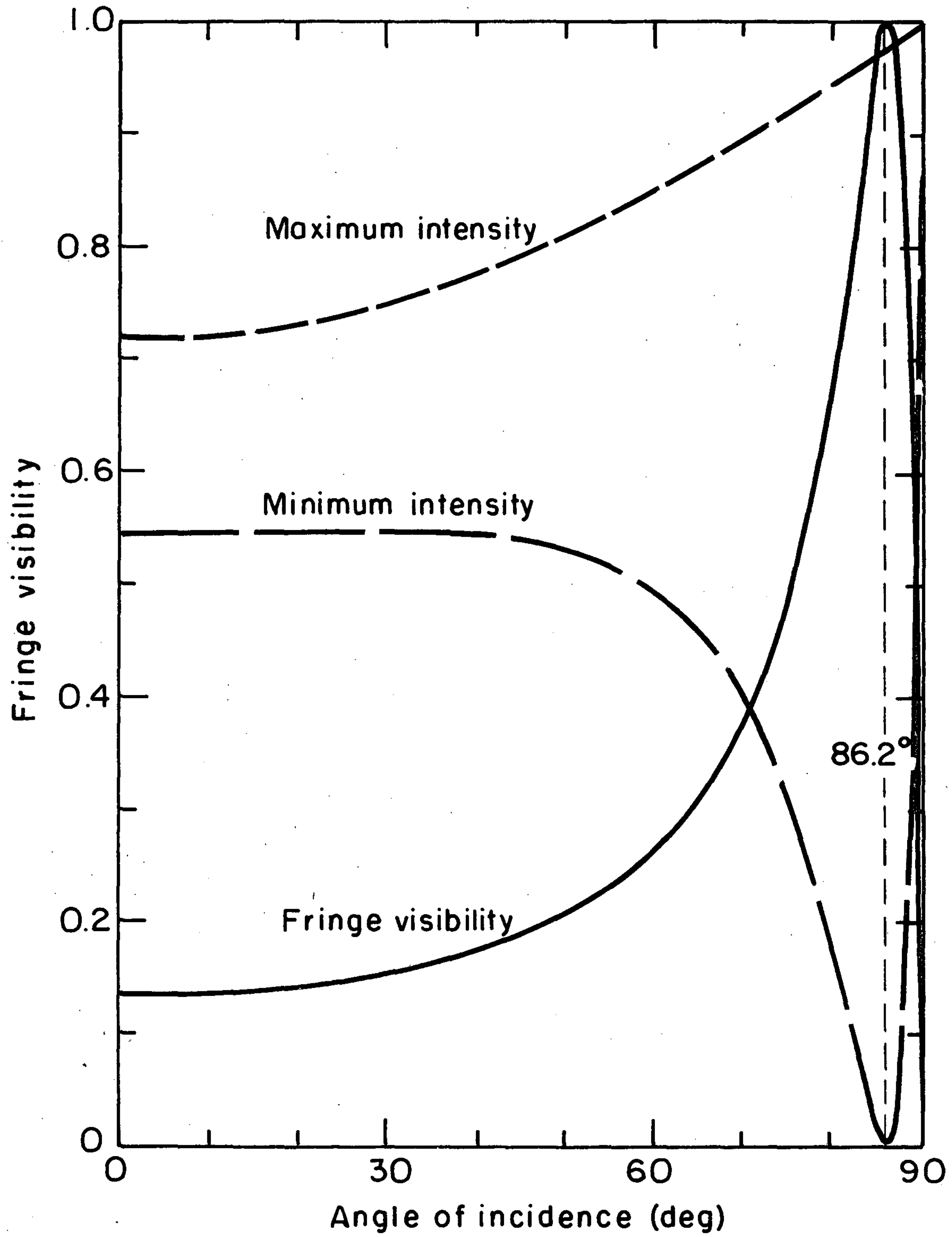
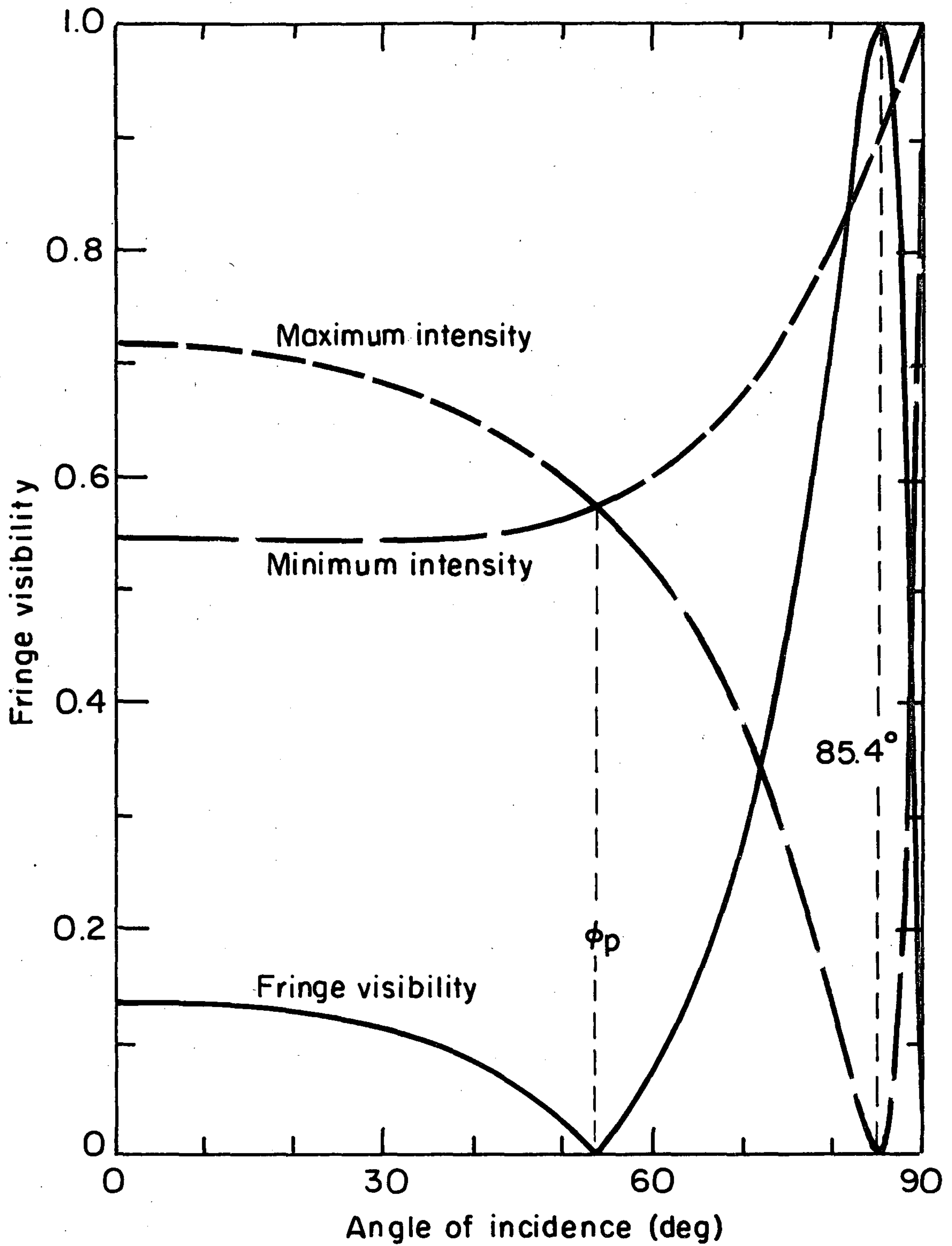


FIGURE 3.



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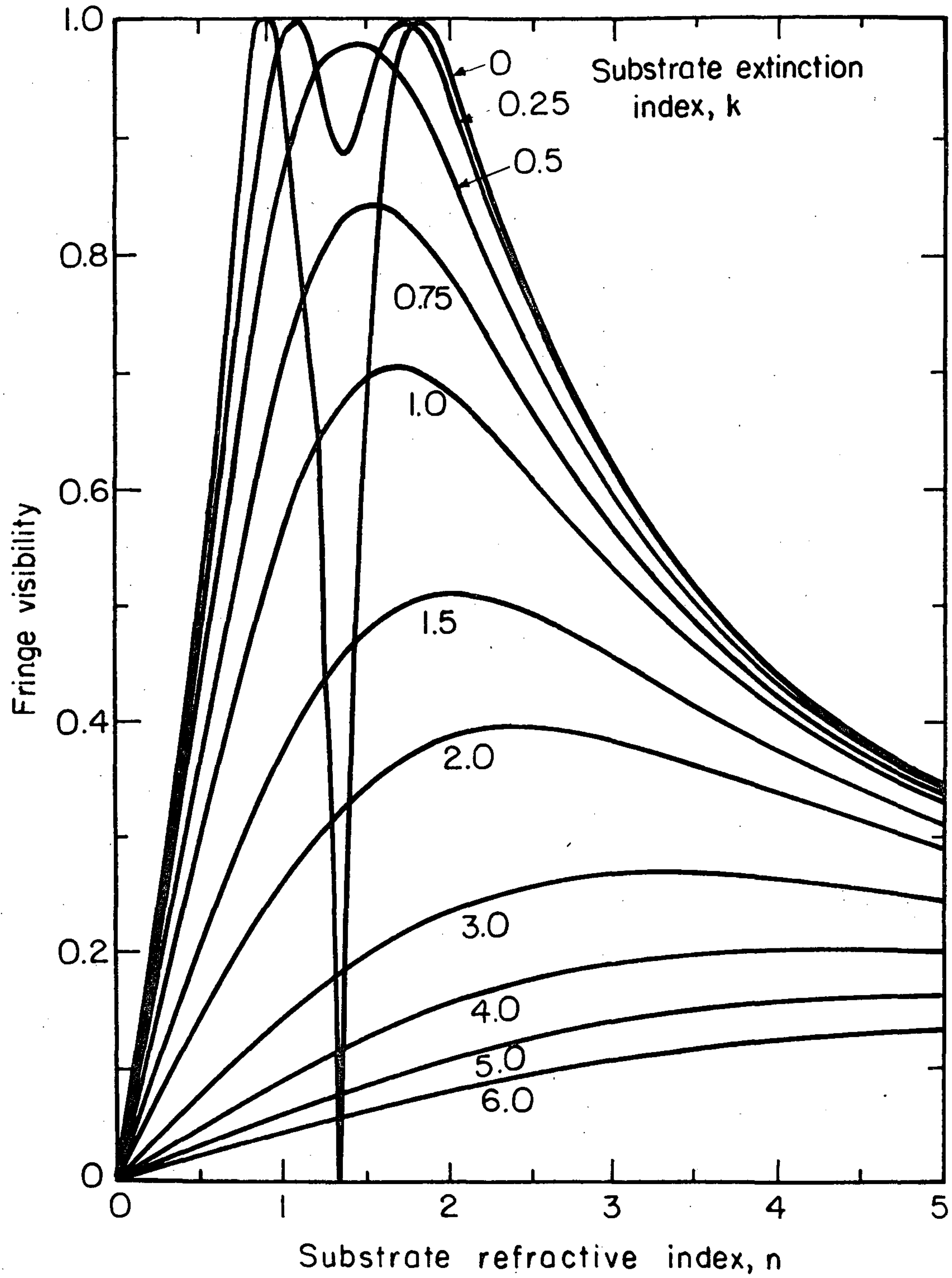

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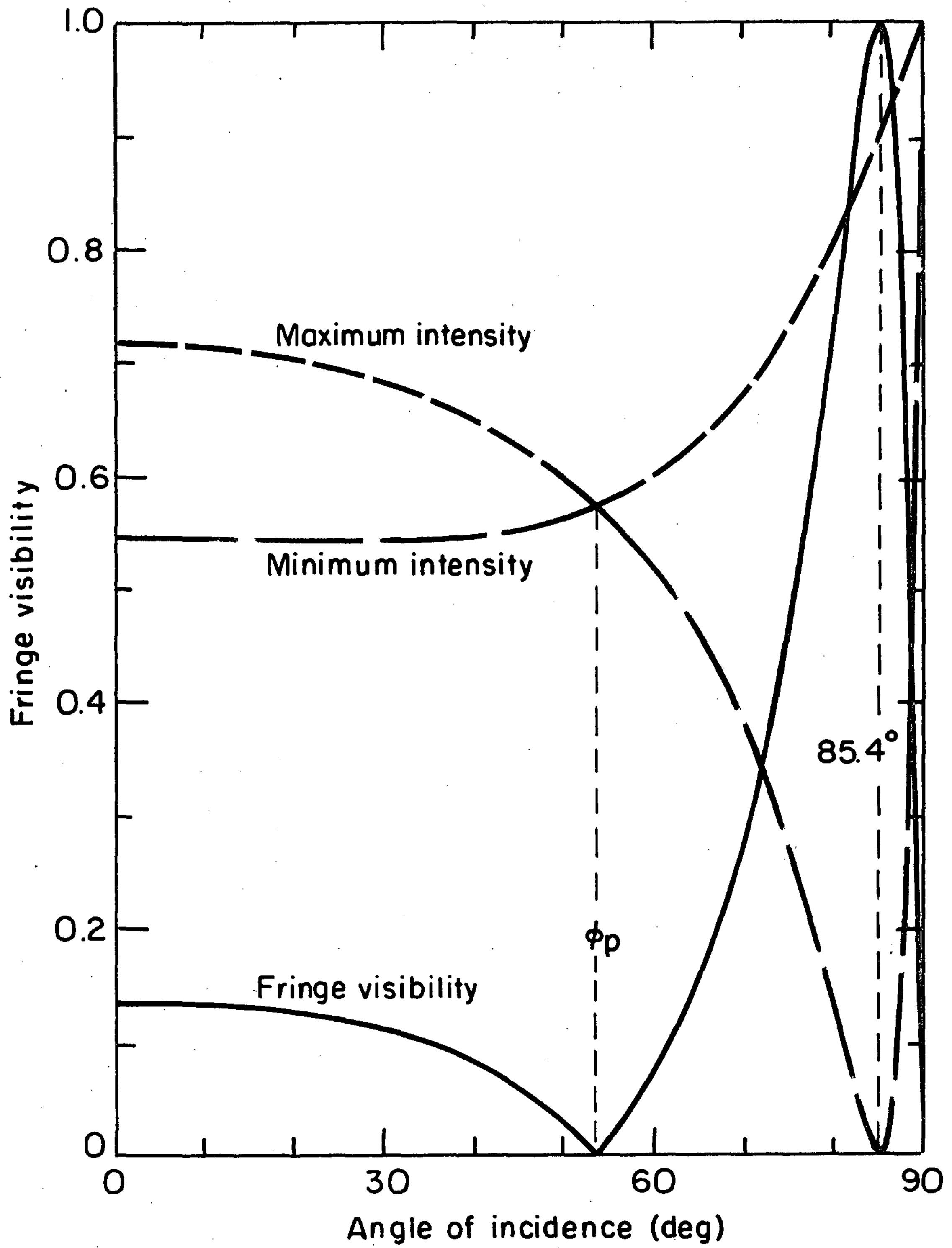
FIGURE 5.



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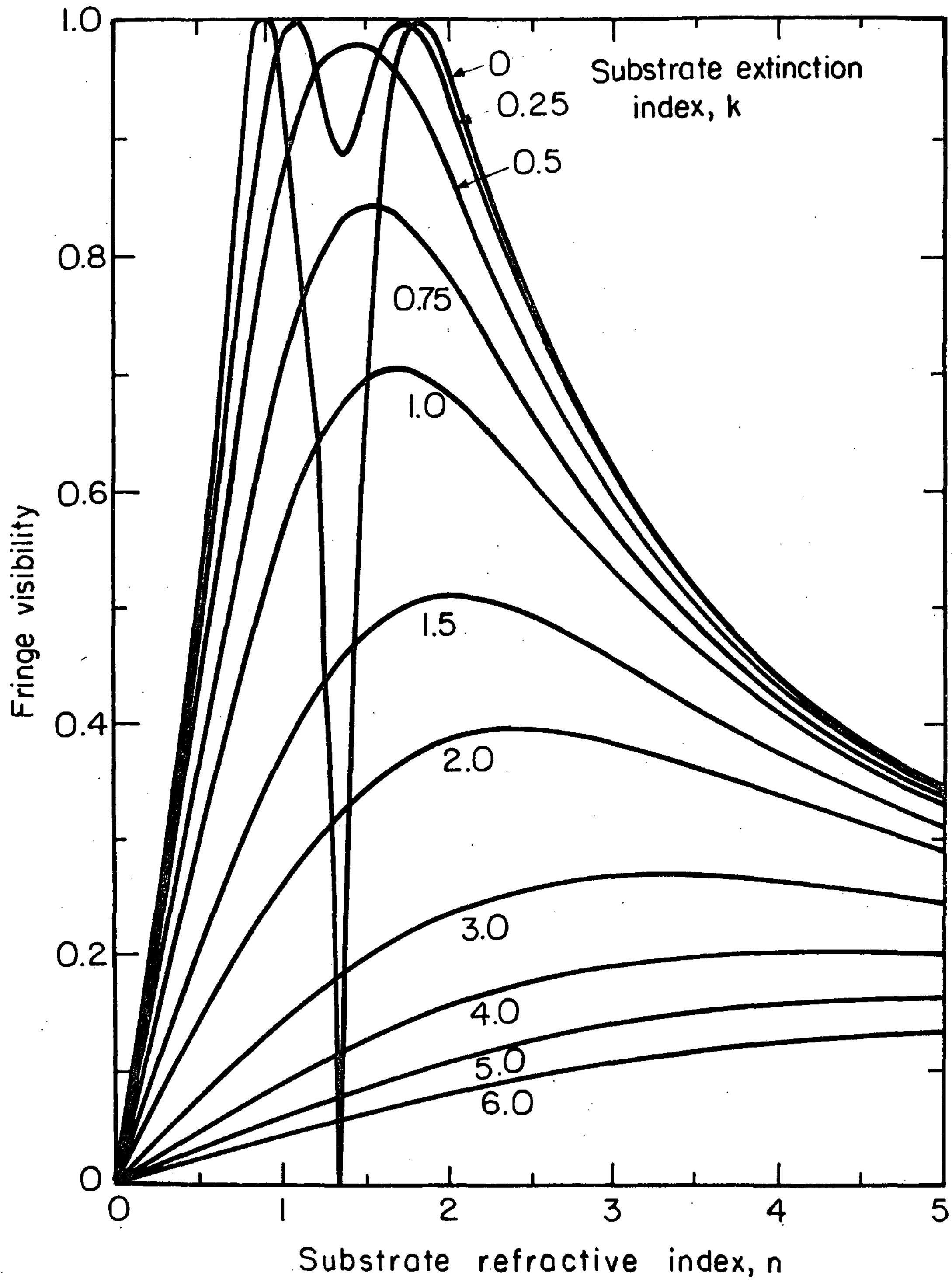

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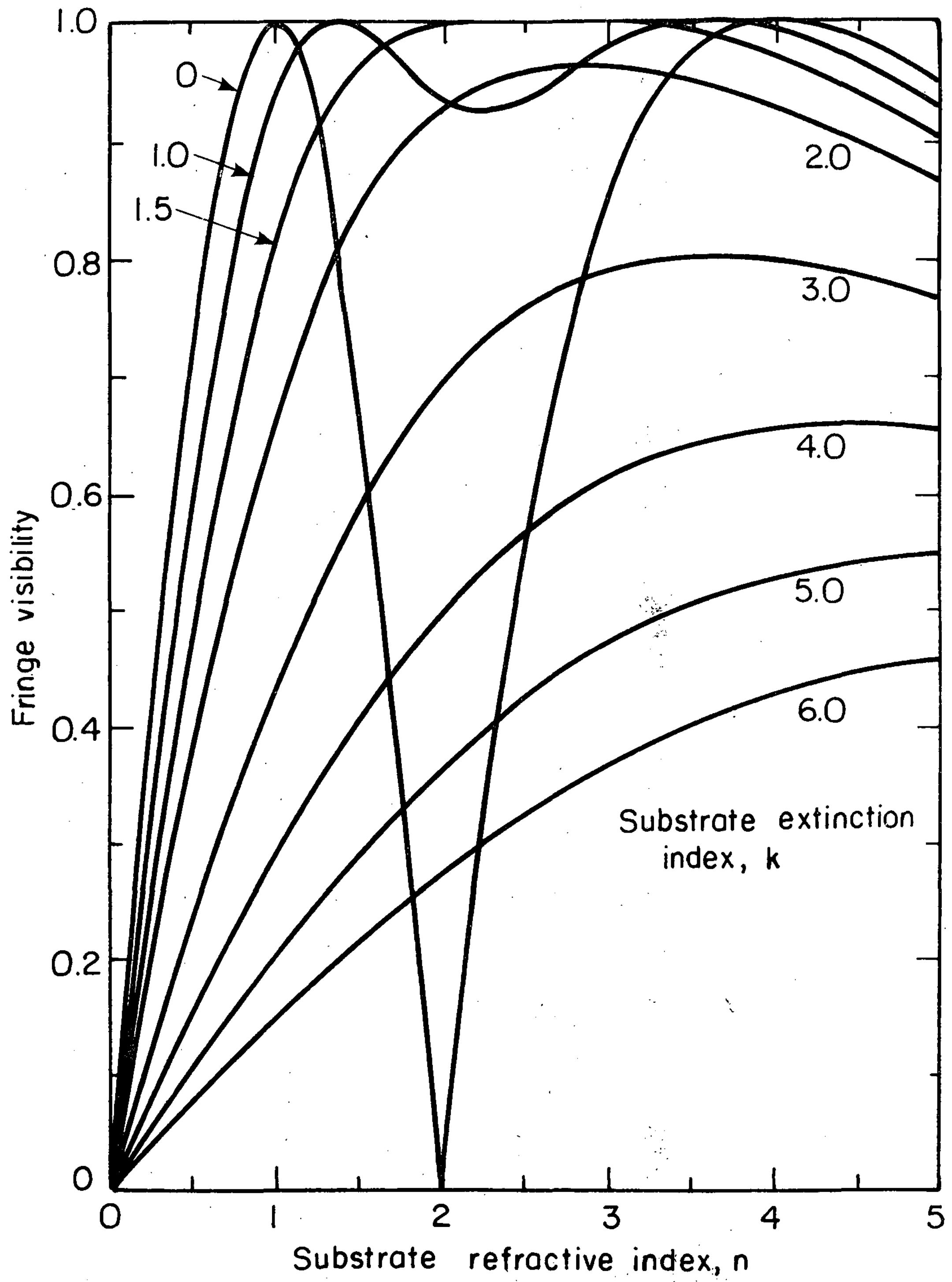

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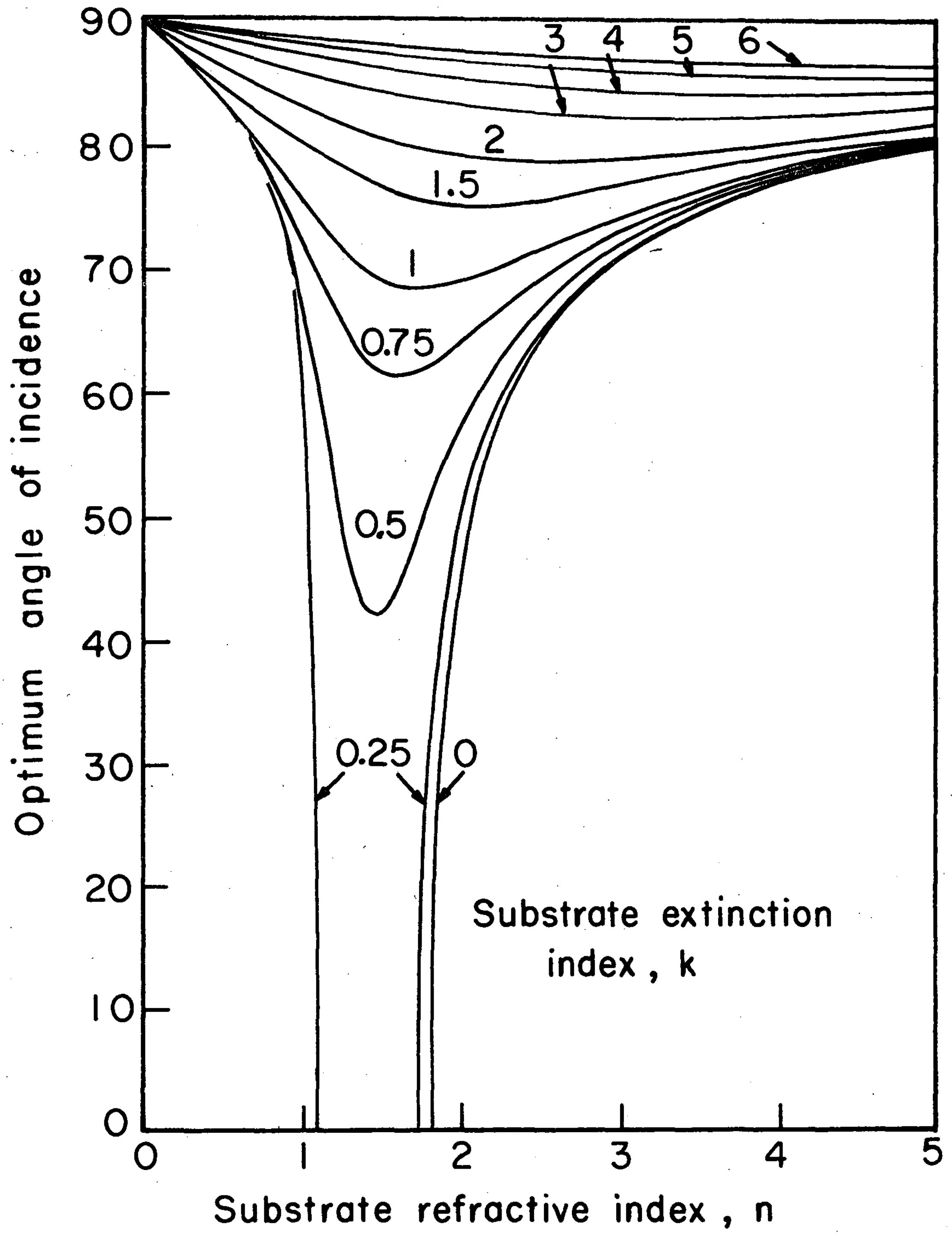

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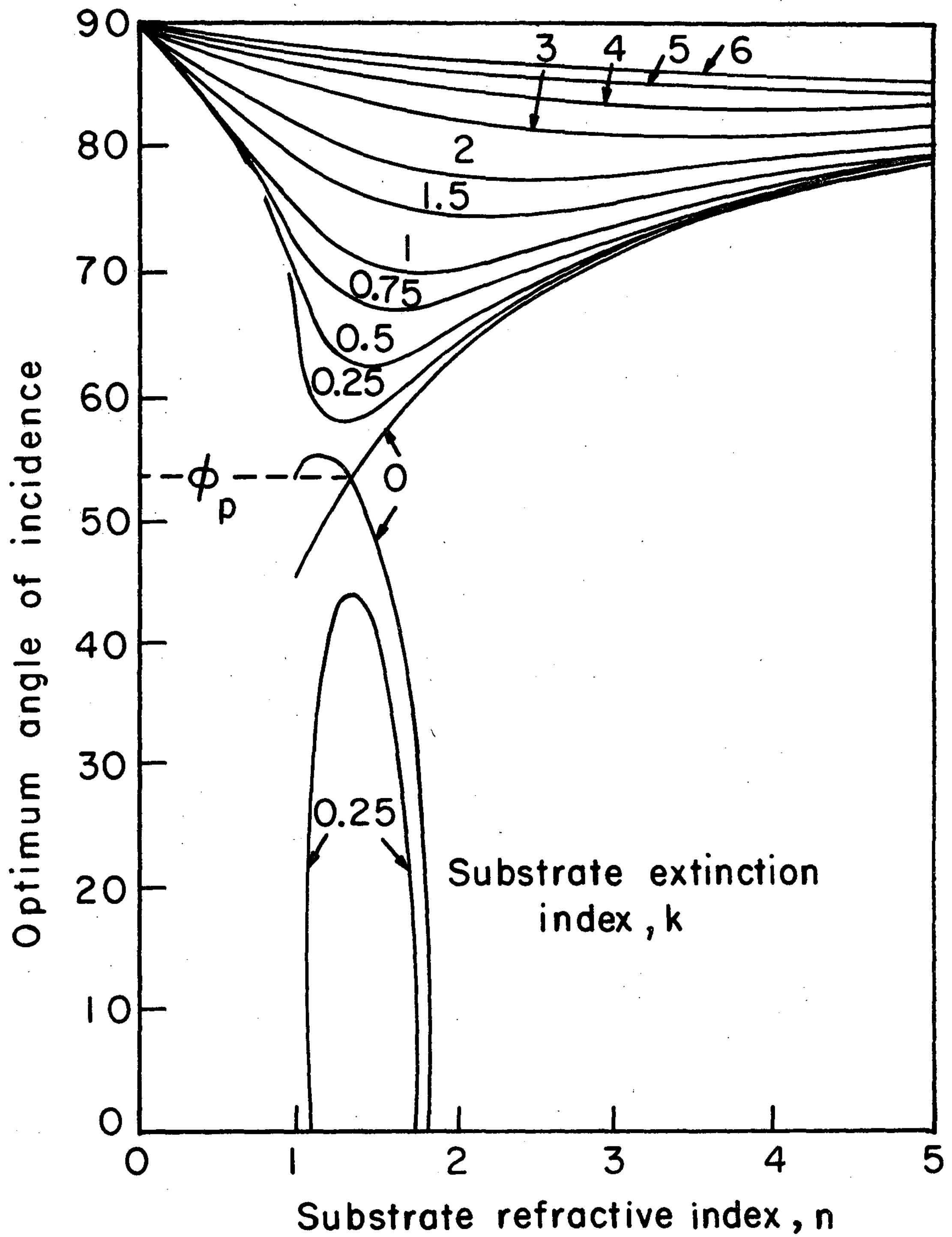

FIGURE 8.

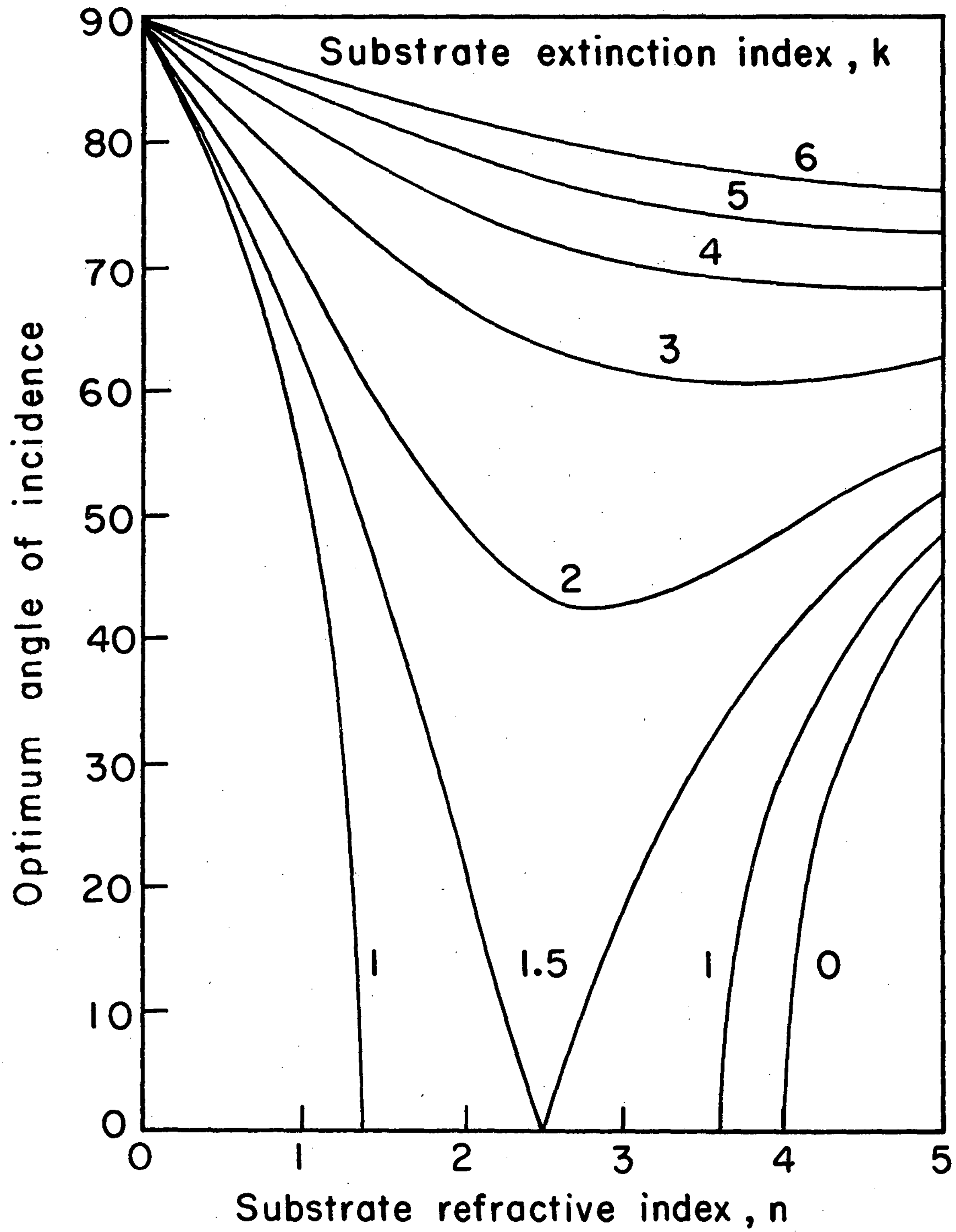

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FIGURE 9.

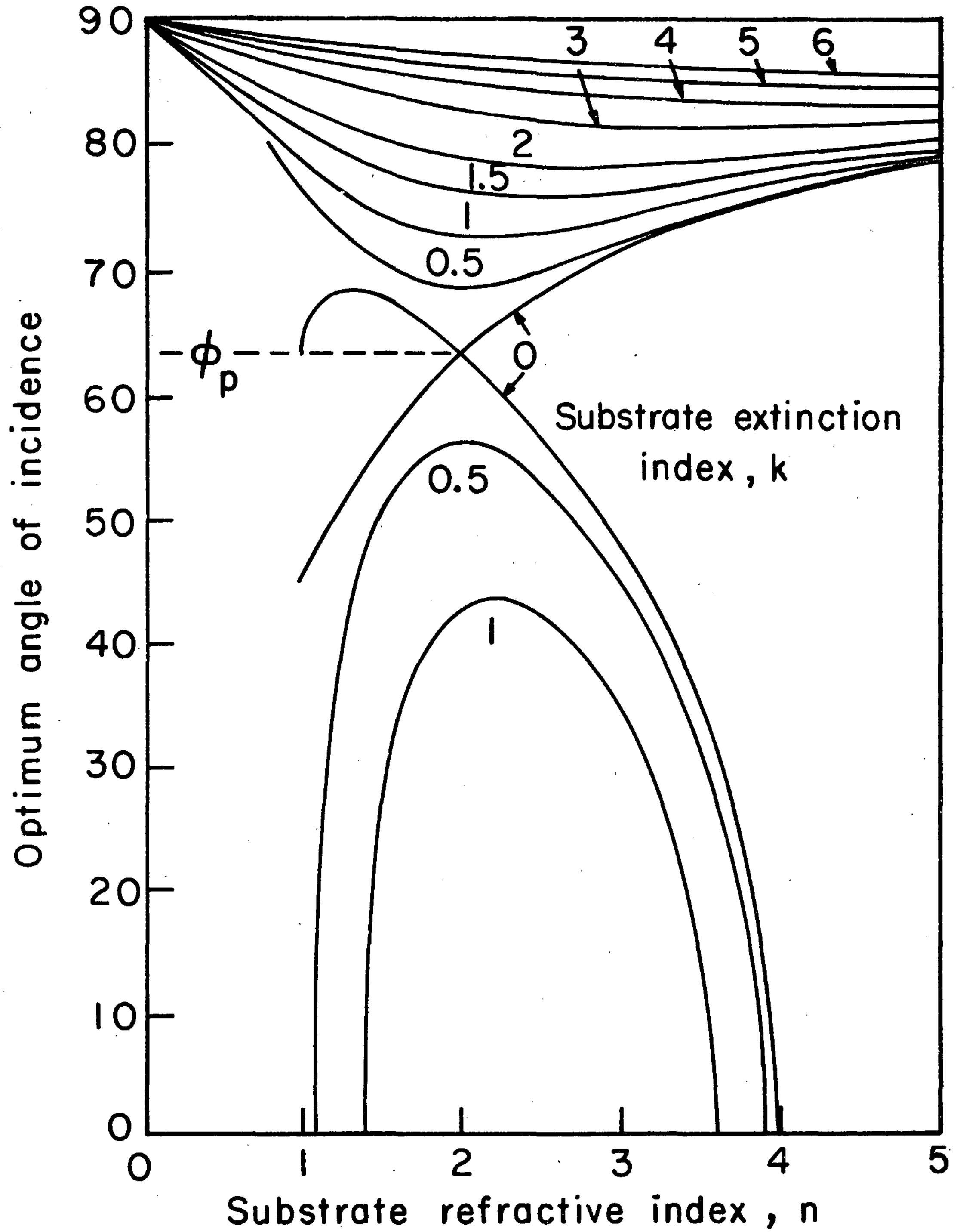

FIGURE 10.

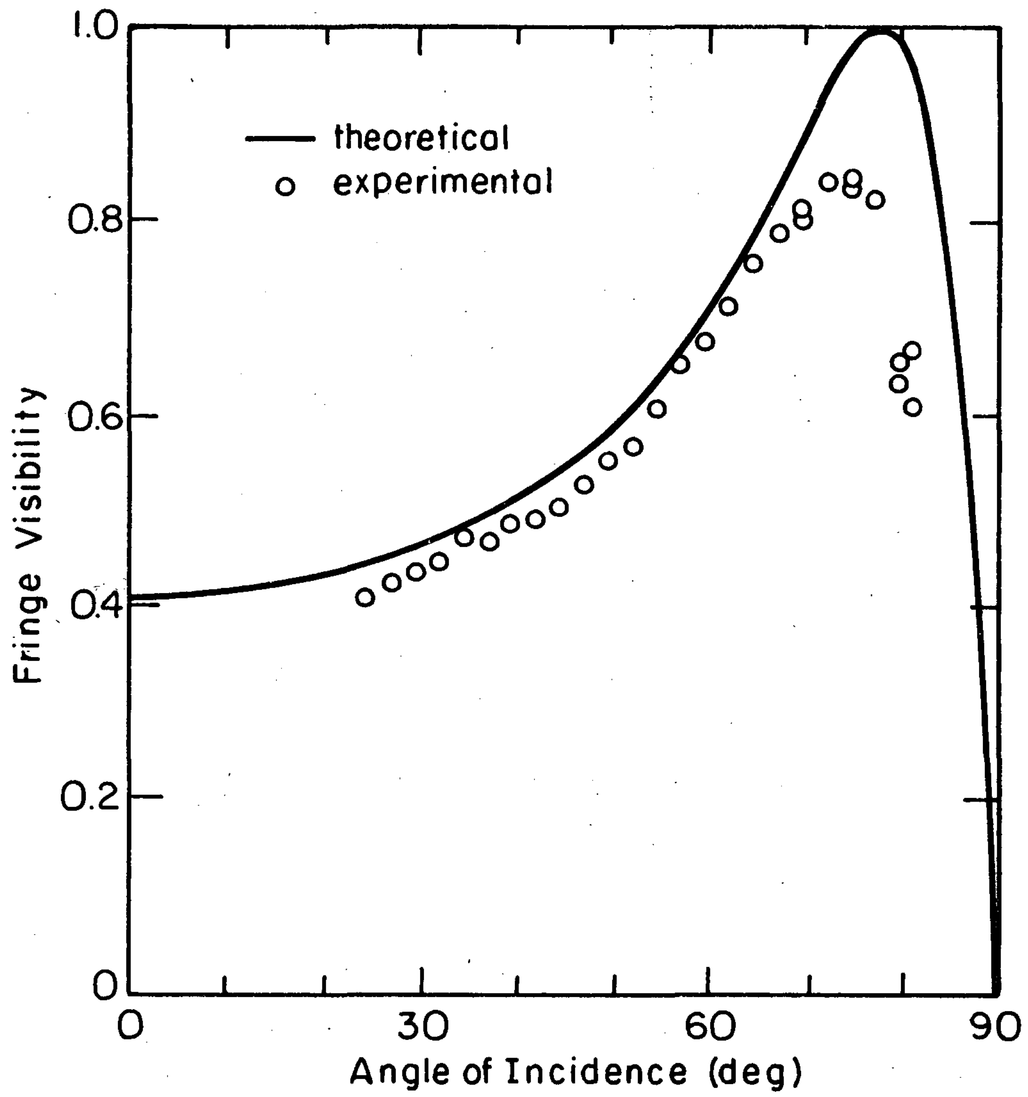

FIGURE 11.

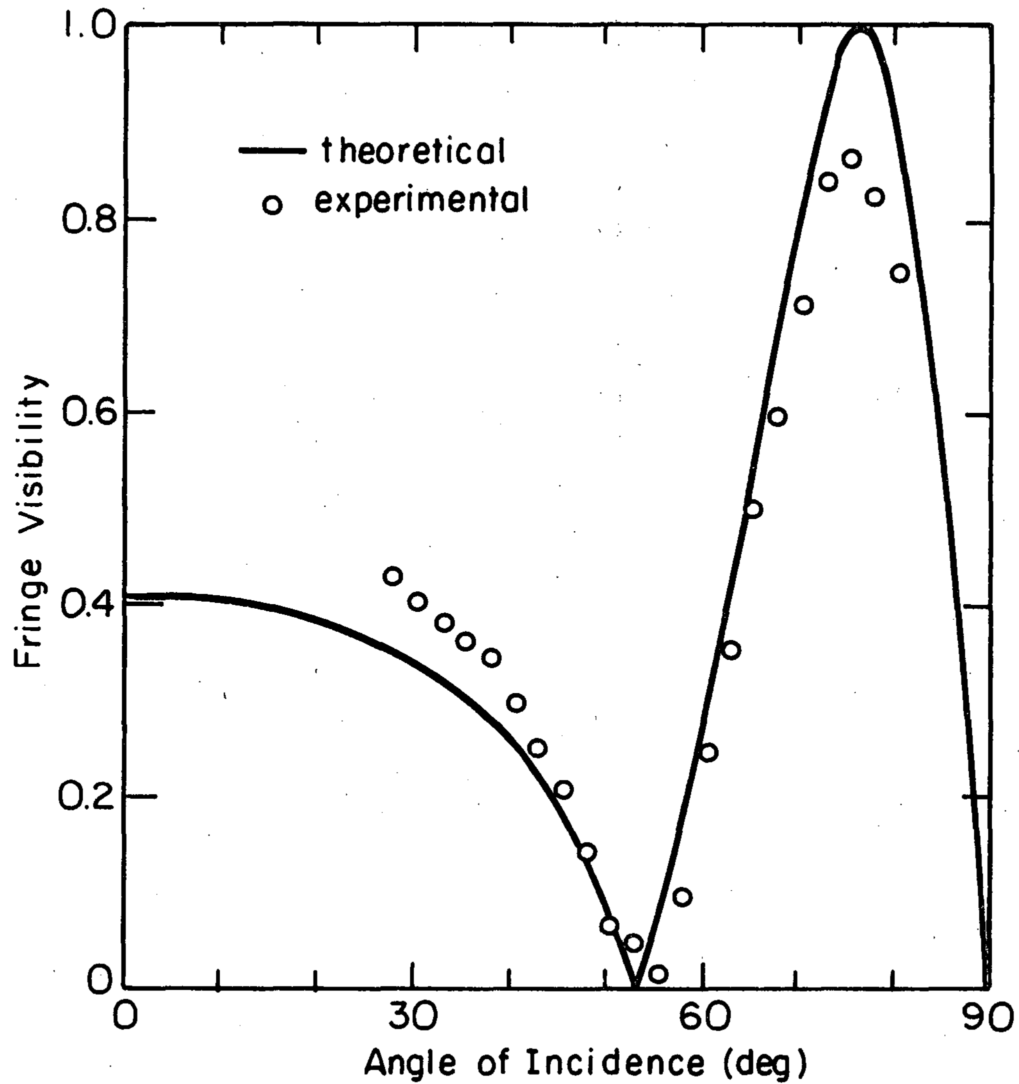

FIGURE 12.

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