The Discrete, the Continuous, and the Approximate Number System

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Abstract

This paper explores the value of skepticism towards the Approximate Number System (ANS). I sketch some of the main arguments levied against ANS-based interpretations of numerical cognition data and argue that there are empirical and conceptual reasons to reject wholesale replacement of the ANS with an Analog Magnitude System (AMS). To simplify the discussion, I focus for the most part on a recent critical review representative of this new wave of revisionist skepticism (Leibovich, T., Katzin, N., Harel, M., & Henik, A., 2017). I start with a brief review of some of the reasons offered to deny that experiments studying our numerical abilities reveal the presence of a system dedicated to representing quantities of discrete objects, before turning briefly to empirical responses to these worries. I then offer a few reflections on why even if the empirical rebuttal were to fail, there are conceptual reasons to doubt that we are only equipped with an AMS. While some of these reasons involve methodological implications of AMS-based theories, other conceptual reasons to doubt AMS skepticism revolve around how ANS-skepticism seems to go against the history of the relation between the continuous and the discrete, and how one cannot be derived from the other. I then end with a potential reply to my worries involving an appeal to the Object-File System (OFS) as a source of discrete content in our numerical abilities and find it wanting.

Keywords: Analog Magnitudes; Approximate Number System; Numerical Cognition; Object-Files; Discrete; Continuous; Representational content

Introduction

It's a great time to be interested in numerical cognition. In the past few years, research involving adults, infants, and nonhuman animals has accumulated mountains of data supporting the existence of (at least) two separate cognitive systems that appear to serve as building blocks for our formal arithmetical abilities.[1] Even if there are many outstanding issues concerning such systems – e.g. how many there are and which system plays what part in the development of mathematically-viable numerical content, among others – the received view in the study of numerical cognition relies on both the Object-File System (OFS) and the Approximate Number System (ANS) to explain where our ability to think about numbers comes from.

And yet, despite major progress being made in the study of numerical cognition, a voice of dissent has been growing. The problem has to do with the complications involved in studying abstract objects like numbers experimentally: given that numbers are generally considered to be as abstract and amodal as it gets, it is not always easy to isolate numerosity[2] as the only potential cause of behavior in experimental settings, and it is usually not difficult to find a non-numerical explanation of behavior in many experiments, prompting some to be skeptical of any interpretation of the data that does not rely on non-numerical cues. Such methodological skepticism has been around for a while (e.g. Clearfield & Mix 1999; Feigenson, L., Carey, S., & Spelke, E. 2002; Simon 1997), as illustrated by the (justified) skeptical response to Karen Wynn's (1992) celebrated violation-of-expectancy study and its alleged ability to establish that subjects were responding to numerical information.

However, while the methodological headaches that go hand in hand with numerical cognition studies are indeed considerable, the consensus has generally been that researchers manage to find ways of controlling for non-numerical confounds in their experiments, and that results obtained by those who fail to do so will eventually fail to be replicated. So while there has been some skepticism about numerical interpretations of behavioral and neuroimaging data for a long time, it has not prevented ANS-based approaches to numerical cognition from becoming the most widespread interpretation of the data, as skeptics readily acknowledge (Gebuis, T., Cohen Kadosh, R. & Gevers, W. 2016; Leibovich, T., Katzin, N., Harel, M., & Henik, A. 2017).

Despite the widespread acceptance of the ANS as a legitimate explanandum of numerical behavior, a handful of authors have recently taken this skepticism to a higher level, questioning to which degree it is possible to create *any* experimental conditions that can only be interpreted as evidence of the presence of innate cognitive systems that

[1] See Cohen Kadosh & Dowker 2015.

[2] While use of this term has come under fire (e.g. Clarke & Beck 2021), I will use it here to describe the number of *perceived* objects in a collection, in accordance with what seems to be standard usage.

In J. Culbertson, A. Perfors, H. Rabagliati & V. Ramenzoni (Eds.), *Proceedings of the 44th Annual Conference of the Cognitive Science Society*.

track numerosities. For example, in a statement that sums up the concerns flagged by this recent wave of ANS skepticism, Leibovich and colleagues have argued that "the natural correlation between numerosities and continuous magnitudes makes it nearly impossible to study non-symbolic numerosity processing in isolation from continuous magnitudes, and therefore, the results of behavioral and imaging studies with infants, adults, and animals can be explained, at least in part, by relying on continuous magnitudes. (Leibovich et al. 2017, 1)

Much of this controversy concerns whether the data are best explained by appealing to a system specifically tuned to detecting quantities of discrete items (the ANS), or whether it is more prudent to appeal to a general sense of magnitude that is capable of responding to numerosity variations due to the fact that number co-varies with other magnitudes, in which case the system would be a more general analog magnitude system (the AMS). Considering that data supporting the existence of the ANS and the OFS in infants and animals are based on using non-symbolic stimuli, this ANS skepticism threatens to undermine an enormous body of evidence concerning the ontogenetic and phylogenetic origins of our ability to think about numbers.

In this paper I sketch some of the main arguments levied against ANS-based theories and argue that, on top of the empirical dispute, there are conceptual reasons to reject wholesale replacement of the ANS with an AMS. To simplify the discussion, I focus for the most part on a recent critical review representative of this new wave of revisionist skepticism (Leibovich et al. 2017)[3]. I start the paper with a brief recap of some of the reasons offered to deny that experiments studying our numerical abilities reveal the presence of a system dedicated to representing quantities of discrete objects, before turning to empirical responses to these worries. I follow with a few reflections on why even if the empirical rebuttal were to fail, there are conceptual reasons to doubt that we are only equipped with an AMS, given that the practice of arithmetic, unlike that of geometry, involves manipulation of discrete objects. This involves some discussion of the historical conception of the relation between the continuous and the discrete and how one cannot be derived from the other. I then end with a potential reply to my worries involving an appeal to the object-file system (OFS) as a source of discrete content in our numerical abilities and explain why I think it fails.

Doubting Numerosity

Nonplussed by the colossal progress made in the field of numerical cognition under an ANS-based framework, ANS skeptics like Leibovich et al. (2017) advocate overthrowing the dominant ANS-based interpretation of numerical cognition data and replacing it by one based on an AMS, which only processes continuous magnitudes like luminosity, average size, and duration, among others. Typically, an ANS-skeptical argument goes like this: since it is impossible to control for non-numerical cues in explanations of behavior in non-symbolic numerical tasks, and since number necessarily co-varies with one of these non-numerical cues, the evidence for the ANS is dubious, and should be abandoned in favor of a more general AMS whose domain is not specific to numerosity. To support this claim, they take apart the methods used to control for non-numerical magnitudes in non-symbolic numerical tasks, and show that none of them can eliminate non-numerical interpretations of the behavior.

The first method used to control for non-numerical magnitudes is to manipulate a single continuous magnitude (e.g. by keeping it constant) while varying numerosity throughout the experiment. This way, since only numerosity varies, behavioral change should be due to numerosity alone, rather than to the magnitude that was kept constant. The problem with this approach is that there is no way to manipulate one continuous magnitude without affecting others. For example, in experiments that change numerosities while controlling for reaction to total surface area of the display by keeping it constant, numerosity variation necessarily incurs average size variation.

A second method is to vary many continuous magnitudes throughout the experiment, though for each trial only one magnitude is manipulated. The same problem applies here too, since for each trial, participants could be responding to different non-numerical cues, so that their performance can be explained by a variety of non-numerical strategies throughout the experiment.

A third method is to create congruency conditions between numerical and non-numerical magnitudes, in a Stroop-like paradigm adapted to numerosity. The idea here is to see if manipulating numerosity and an associated continuous magnitude have cumulative effects on performance. If there are no behavioral differences between congruent and incongruent trials, then the manipulated magnitudes do not interact with each other. For example, when asking participants to compare numerosity or area of dot arrays, congruent displays would be those where both size of dots and their numerosity are larger in one display than another. In a study using such congruency manipulations (Nys & Content 2012), congruency effects were more marked for a size comparison task than a numerosity comparison task, which was interpreted by the authors as indicating that numerosity affected size comparison more than vice versa, and thus that numerosity is more salient in such tasks. The problem here, according to the skeptics, is that such results are difficult to replicate, and similar methods often support contradictory conclusions. In this case, a number of studies found, on the contrary, that numerosity was less salient than size (Leibovich et al. 2017, 5), which skeptics take as a sign of the task-dependence of many results of numerical cognition studies, thus indicating their unreliability.

In short, the skeptics claim is that it is 'virtually impossible' to control for non-numerical explanations of

[3] While the details of the positive proposal offered by this review differ from some of the other voices associated with the skeptical approach, the criticism of mainstream numerical cognition methods offered here is shared by virtually all of these revisionist accounts.

behavior, since there is always a continuous magnitude that co-varies with numerosity in non-symbolic numerical tasks.

Empirical Responses to ANS Skepticism

While the methodological humility advocated by many of these ANS skeptics is certainly warranted, especially considering the unfortunately liberal use of numerical terminology that permeates the study of numerical cognition in infants and animals, the drawbacks of such hardcore skepticism start to manifest themselves when it is confronted with stronger evidence for numerosity-based behavior. For example, a particularly strong line of evidence for the existence of the ANS is the cross-modal matching studies involving infants (e.g. Izard, V., Sann, C., Spelke, E. S. & Steri, A. 2009; Jordan & Brannon 2006; Starkey, P., Spelke, E. S., & Gelman, R. 1990) and animals (Meck & Church 1983; Jordan, K,. Maclean, E., & Brannon, E. 2008). In such cases, it is difficult to explain behavior without appealing to an abstract, amodal representation of discrete quantity, since any modality-specific confounds like average object size or total surface area could not transfer across modalities, suggesting that amodal numerosity representations produced by an ANS underlies behavior in these tasks. This would seem to nullify the ANS skeptic's weapon of choice, (intra-)perceptual confounds.

According to Leibovich and colleagues, "Such evidence, however, should be taken with a grain of salt" (Leibovich et al. 2017, 5), since these findings have been very difficult to replicate, with only 2 of 6 studies managing to find evidence of cross-modal matching in infants. The problem here is supposed to be that since the findings are hard to replicate, they should be dismissed. And yet, while the small number of studies that managed to find evidence of cross-modal matching does highlight the difficulty of testing for the presence of numerical representations in infants, simply negating the original finding because it has not been easy to replicate appears unjustified. The same authors similarly question the validity of these studies by appealing to perceptual limitations in infants. For example, they claim that in many cases (e.g. Izard et al. 2009), the fact that infants have poorly developed visual acuity means that "they are unlikely to be able to see objects that are placed relatively close to one another as being separate from one another, and they lack the ability to separate between object and background or between one object and another" (Leibovich et al. 2017, 6). If this is true, infants in cross-modal matching tasks could be reacting to MORE/LESS cross-modal matches: hearing more syllables, the infant expects to see more dots, and thus stares longer at the matching stimuli.

However, according to Hyde & Mou, "The claim that infants cannot perceptually individuate objects until 5 months is simply false" (Hyde & Mou 2017, 26). On the contrary, data suggest that while infant vision is poorly developed, it does not prevent them from individuating objects: "studies investigating newborns' visual perception have demonstrated that they are able to represent individual objects, at the same age as in the numerosity study" (de Hevia, M.D., Castaldi, E., Streri, A., Eger, E., & Izard, V. 2017, 21).

Conceptual and Methodological Issues with ANS Skepticism

Such debates surrounding the value and best interpretation of data are likely to continue for a long time. Apart from these empirically-oriented worries, however, there are a few methodological questions and conceptual problems facing such extreme skepticism with respect to what is now an incredibly diversified and well-documented body of research backing up the existence of the ANS. Given space constraints, I only sketch the methodological questions and offer a short discussion of the implications of ANS skepticism for our ability to explain the relation between the discrete and the continuous.

First, given that the ANS skeptic's bread and butter is to appeal to the fact that number co-varies with continuous magnitudes to explain behavior, the same method might as well be used to deny the existence of any of these other magnitudes as well: why would numerosity be the odd man out, instead of, say, convex hull or average size? Second, while it may appear plausible to deny that participants react to numerosity in many studies, one could be forgiven for being skeptical about the motley crew of magnitudes that has been recruited to replace it: how plausible is it that for the same numerical task (e.g. comparing dot arrays with respect to their numerosity), a variety of continuous magnitudes are recruited, depending on which one co-varies with number, instead of a single system like the ANS? Similarly, given that which member of this motley crew of magnitude representations is recruited depends on the potential confounds with numerosity, wouldn't there be a system required to determine which magnitude should be recruited in each case to give the appearance of numerosity-based responses, and if so, wouldn't this system basically be doing what an ANS is supposed to do?

On top of these methodological questions, a conceptual worry relates to whether or not it is possible to claim that our ability to think about numbers is rooted only in systems that produce content concerning continuous magnitudes. The worry is that there seems to be a fundamental incommensurability between continuous and discrete content, as can be seen in both the history of mathematics and in developmental psychology. For example, in a number of infant studies used to probe our numerical abilities, infants are shown two stimuli and then allowed to reach for the one they want[4]. In such tasks, infants are reaching using their hands to grab an individual object, so it is difficult to understand how representations for only continuous magnitudes can underlie their behavior, given that hands do not grab convex hulls nor average luminosities, nor any other continuous magnitudes. In such manual search paradigms, hands are used to reach for *things* (as opposed to *stuff*). This

[4] Carey 2009 contains numerous examples of this method.

means infant behavior here would be expected to be based on representation of a discrete magnitude, and the only discrete magnitude that can account for such quantity-based grabbing behavior is numerosity.

This observation reflects one of the fundamental aspects of the practice of arithmetic: it deals with discrete entities – namely, natural numbers. On the other hand, ANS skeptics, by denying that we have a cognitive system dedicated to tracking variations of quantities of discrete objects in our environment, end up denying that there is any discrete content in the systems from which our numerical abilities are built. After all, the AMS tracks only continuous magnitudes. This means that ANS-skeptics are dedicated to the claim that we derive discrete content of the sort used in arithmetic from representations of continuous magnitudes. In other words, they are committed to the claim that it is possible to derive discrete content from continuous content.

This goes against the well-known dichotomy between the discrete and the continuous in the practice of mathematics. Indeed, the relation between the continuous and the discrete has fueled debate throughout the history of mathematics, from Zeno's paradoxes to Aristotle's framing both as distinct species of quantities in book VI of the *Categories* to Bergson's reaction to infinitesimals to the intuitionist backlash against attempts to reduce the continuum to infinite sets of real numbers (e.g. Brouwer 1907). A theme that runs through the historical discussion of the relation between the discrete and the continuous is that there is a fundamental distinction between these and that it is impossible to reduce one to the other[5]. While there is reason to doubt that this is a formal truth of mathematics, given the overall success of analysis, it does seem to describe the experience of doing arithmetic accurately, especially compared to, say geometry, where we manipulate lines and figures that are continuous in nature.

Also, perhaps more importantly here, this distinction seems to describe our experience generally: we do not interact with discrete objects in the same way that we do with substances. This is true from a very young age, as illustrated by violation-of-expectancy studies showing that young infants react differently to sand than they do to objects (Huntley-Fenner, Carey, & Solimando, 2002). If these reflections are true, then beyond any empirically-oriented disputes, there is a conceptual problem with eliminating the ANS from our interpretation of numerical cognition data: it suggests that we can derive formal discrete numerical content using representations of continuous magnitudes as our starting point, which runs counter to both our experience of the world and to how the relation between the discrete and the continuous has been characterized throughout history.

Rebuttal and Reply: Object-Files

The ANS-skeptic may object here that I am making a strawman of their position by claiming that denying the existence of the ANS is tantamount to claiming that discrete numerical content develops only from representations of continuous magnitudes: after all, one could easily argue that we get the discrete content from the Object-File System (OFS). There is value to this objection: on (a simplified version of) the orthodox view, the ANS essentially supplies the quantitative — sometimes called 'semantic' (e.g. Dehaene 1997/2011) background — to the development of numerical content, while the OFS plays a major role in individuating quantities of discrete objects as such, with number words only being mapped to the ANS much later in the learning process (e.g. Carey 2009: 307-234). If this is true, the reply goes, then the ANS skeptic can claim the discrete content from which we derive numerical representations doesn't reduce to the content produced by representations of analog magnitudes, it comes from the OFS, and there is no need to go from denying the existence of an ANS to claiming that we can get discrete content from representations of continuous magnitudes.

There are at least two problems with this rebuttal. The first is a simple one: evidence suggests that the OFS does not have any quantitative content: "In this alternative representational system, number is only implicitly encoded; there are no symbols for number at all, not even analog magnitude ones." (Carey 2009, 138). There is no reason to think the OFS individuates content based on quantitative information — especially not if we adopt the strict approach to interpreting data suggested by ANS skeptics. More parsimonious, non-quantificational explanations can account for all the data giving the appearance of numerical abilities in the subitizing range (i.e. between 1 and 4 objects) by appealing to operations on objects files (Kahneman, D., Treisman, A. & Gibbs, B. J. 1992). On these non-numerical explanations, the OFS can be described as supporting one-to-one correspondence between individuated object files or groups of these, which can lead to representations with the content SAME or DIFFERENT. The underlying reason for this difference or sameness, numerical inequality, does not need to be explicitly represented here. If this is true, then we cannot appeal to the OFS for the origins of the concept of DISCRETE QUANTITY, since it doesn't even produce QUANTITY.

Second, the OFS is an *individuation* system: it parses our environment into discrete entities and labels them as objects, so that we can then track some of their individual properties (Kahneman et al. 1992). While it may allow for aggregate computations, there is no reason to think that it does this beyond the subitizing range, so whatever discrete content it can generate, it does not seem to apply to the majority of data

[5] For example, in describing the fundamental intuition on which intuitionist mathematics are built, Brouwer writes that "the continuous and the discrete appear as inseparable complements, each with equal rights and equally clear, it is impossible to avoid one as a primitive entity and construct it from the other, posited as the independent primitive." (Brouwer, 1907, 8)

collected that is typically associated with the ANS, which goes well beyond the subitizing range.

To illustrate how these issues prevent us from relying on the OFS to explain where we get the discrete aspect of our numerical representations, imagine you are faced with a scene where there are around seven lights on your left side, around twelve on your right side, and there is a fire burning in front of you. Faced with such a scene, we would expect a person to be able to compare *how much* overall luminosity is present of the left side vs the right side vs the fire in front of them: this is what something like an AMS would produce. However, on top of this, we would also expect a person to be able to compare each scene in terms of *how many* light sources they have (say, seven-ish vs one vs 12-ish). However, under the AMS-enthusiast's rebuttal, it is unclear how we can do this. We know we can compare overall luminosity with the AMS. We know we can individuate light sources as objects via the parallel individuation of the OFS. But where does our ability to evaluate the *number* of *discrete light sources* in each scene *as distinct from their aggregate luminosity*? The OFS does not seem to produce content capable of supporting this thought.

If we only have an AMS and the OFS playing a part in the development of our numerical abilities, then, can we say that a person would have the ability to compare scenes in terms of the number of (discrete) light sources? This is now far from obvious: the AMS is responding to luminosity here, which should allow an individual to say which scene produces the most light (a continuous magnitude). But how then can we explain a person's ability to compare the scenes in terms of the number of discrete light sources? The OFS does not help here, since even if we accept that it allows us to individuate collections of discrete objects, it does not allow us to do this with respect to quantity. And under the AMS-only framework, we do not have an abstract notion of quantity, we only have magnitude-specific quantities, like quantity of light, or average size.

This absence of an ability to get a rough idea of the number of individual light sources seems to run counter to how we experience such scenes, in that we would expect our experience to include our ability to both compare overall luminosity and the number of light sources. Similarly, for studies that expose animals to recordings containing many discrete sound samples of predators to gage their responses (e.g. Benson-Amram, S., Heinen, V.K., Dryer, S. L., & Holekamp, K.E. 2011), the ANS-skeptic could not accept an interpretation of the data where an animal is receiving the auditory stimulus as giving it information on the quantity of predators available. Rather, it would be limited to the overall 'predatoriness' of the scene. Again, this seems to run counter to how we would expect an animal to experience the world.

Here, the ANS-skeptic could respond that I am confusing numbers as conceptual entities with numbers as perceptual entities. The response would claim that how we conceive of numbers (as separate objects) does not need to reflect the (continuous) content from which we build our thoughts about numbers. That is: how we think about numbers doesn't need to mirror how we perceive them, and we could just explain the discrete character of numbers via some computations made on the output of the AMS[6]. After all, while we can easily think of the difference between 31 and 32, we cannot perceive the numerical difference between a stimulus composed of 31 blue dots and one composed of 32 blue dots, so there is clearly a difference between the output of the systems used to build numerical content and properties of numbers.

This response seems problematic in at least two ways: first, there are no numbers in the world for us to perceive. Numbers are abstract entities, we cannot track or perceive them. Second, this response conflates two types of experience, one that focuses on continuous aspects of our environment, and one that focuses on discrete ones, as illustrated by the light source example. While it is undoubtedly true that we cannot perceive the numerical difference between a stimulus composed of 31 blue dots and one composed of 32 blue dots, we can certainly tell the difference between wondering which stimulus has more dots and which has more blue. It is this difference that I claim the ANS-skeptic is not able to account for, since the AMS does not produce any discrete content.

So even if there is an important difference between what we are responding to in the world and what we end up thinking, I argue that the difference cannot be such that it takes us from thinking about continuous magnitudes to thinking about discrete objects. Perhaps more importantly, as mentioned above, even if we allow there to be such a difference between what we are perceiving and what we end up thinking, we would still have to say that the discrete content is computed by another system operating on the continuous output of the AMS, in which case it sure sounds like we are saying we need the ANS after all.

Conclusion

The ANS-skeptic is right to be weary of over-interpreting data, as shown by mistakes made in the past. However, this methodological commitment to strict interpretation of data should not come at the cost of eliminating important aspects of how we experience the world. I argue that eliminating the ANS entirely is too costly in terms of its conceptual implications. When methodological skepticism leads to disregarding the fundamental incommensurability between the discrete and the continuous, as proponents of the AMS-only approach do, one could argue it has gone too far. While it is important not to underestimate the potential influence of continuous magnitudes in numerical cognition studies, it is equally important not to overstate it, lest we only see the continuous forest and forget about the discrete trees.

References

Benson-Amram, S., Heinen, V.K., Dryer, S. L., & Holekamp, K.E. (2011). Numerical assessment and individual call

[6] I am grateful to an anonymous referee for highlighting this point.

discrimination by wild spotted hyaenas, Crocuta crocuta. *Animal Behaviour*, 82, 743–752.
Brouwer, L.E.J. (1907). *On the foundations of mathematics*. In A. Heyting. (Ed.) (1975) *L.E.J Brouwer Collected Works Vol l,* North-Holland, Amsterdam
Carey, S. (2009). *The origin of concepts*. New York: Oxford University Press.
Clarke S, Beck J. (2021) The number sense represents (rational) numbers. *Behavioral and Brain Sciences* 44, e178: 1–62. doi:10.1017/S0140525X21000571
Clearfield, M. W., & Mix, K. S. (1999). Number versus contour length in infants' discrimination of small visual sets. *Psychological Science*, 10, 408–411.
Cohen Kadosh, R. & Dowker, A. (Eds.) (2015) *The Oxford Handbook of Numerical Cognition*. Oxford: Oxford University Press.
Dehaene, S. (1997/2011) The Number Sense: How the Mind Creates Mathematics. New York: Oxford University Press.
Feigenson, L., Carey, S., & Spelke, E. (2002). Infants' discrimination of number vs. continuous extent. *Cognitive Psychology,* 44, 33–66
Gebuis, T. & Reynvoet, B. (2012a). Continuous visual properties explain neural responses to nonsymbolic number. *Psychophysiology* 49(11):1481–91. doi: 10.1111/j.1469-8986.2012.01461.
Gebuis, T. & Reynvoet, B. (2012b). The interplay between nonsymbolic number and its continuous visual properties. *Journal of Experimental Psychology: General* 141(4):642–48. doi: 10.1037/a0026218.
Gebuis, T., Gevers, W. & Cohen Kadosh, R. (2014). Topographic representation of high-level cognition: Numerosity or sensory processing? *Trends in Cognitive Sciences* 18(1):1–3. doi: 10.1016/j.tics.2013.10.002.
Gebuis, T., Cohen Kadosh, R. & Gevers, W. (2016) Sensory-integration system rather than approximate number system underlies numerosity processing: A critical review. *Acta Psychologica* 171:17–35.
de Hevia, M.D., Castaldi, E., Streri, A., Eger, E., & Izard, V. (2017). Perceiving numerosity from birth. *Behavioral and Brain Sciences*, 40, E169. doi:10.1017/S0140525X16002090
Huntley-Fenner, G., Carey, S., & Solimando, A. (2002). Objects are individuals but stuff doesn't count: Perceived rigidity and cohesiveness influence infants' representations of small groups of discrete entities. *Cognition*, 85(3), 203–221.
Hyde, D.C. & Mou, Y. (2017). Magnitude rather than number: More evidence needed. *Behavioral and Brain Sciences*, 40, e173
Izard, V., Sann, C., Spelke, E. S. & Steri, A. (2009) Newborn infants perceive abstract numbers. *Proceedings of the National Academy of Sciences of the United States of America* 106(25):10382–85.
Jordan, K.E., & Brannon, E. M. (2006).The multisensory representation of number in infancy. *Proceedings of the National Academy of Sciences of the United States of America* 103(9): 3486–3489. doi: 10.1073/pnas.0508107103
Jordan, K,. Maclean, E., & Brannon, E. (2008). Monkeys match and tally quantities across senses. *Cognition* 108. 617-25. 10.1016/j.cognition.2008.05.006.
Kahneman, D., Treisman, A. & Gibbs, B. J. (1992) The reviewing of object files: Object-specific integration of information. *Cognitive Psychology* 24:175– 219.
Leibovich, T. & Henik, A. (2013) Magnitude processing in non-symbolic stimuli. *Frontiers in Psychology* 4:375. doi: 10.3389/fpsyg.2013.00375.
Leibovich, T., & Ansari, D. (2016). The symbol-grounding problem in numerical cognition: A review of theory, evidence, and outstanding questions. *Canadian Journal of Experimental Psychology/Revue Canadienne De Psychologie Expérimentale*, 70, 12–23. doi:10.1037/cep0000070
Leibovich, T., Katzin, N., Harel, M., and Henik, A. (2017). From 'sense of number' to 'sense of magnitude' - The role of continuous magnitudes in numerical cognition. *Behavioral and Brain Sciences*. 1-62. doi:10.1017/S0140525X16000960
Lourenco, S. F. (2015) On the relation between numerical and non-numerical magnitudes: Evidence for a general magnitude system. In: *Evolutionary origins and early development of number processing*. D. C. Geary, D. B. Berch & K. M. Koepke (Eds). (pp. 145–74.) London: Elsevier Academic Press.
Meck, W. H., & Church, R. M. (1983). A mode control model of counting and timing processes. *Journal of Experimental Psychology: Animal Behavior Processes*, 9, 320–334.
Nys, J. & Content, A. (2012) Judgement of discrete and continuous quantity in adults: Number counts! *Quarterly Journal of Experimental Psychology* 65 (4):675–90. doi: 10.1080/17470218.2011.619661.
Rips, L. J., Bloomfield, A., and Asmuth, J. (2008). From numerical concepts to concepts of number. *Behavioral and Brain Sciences*, 31, 623–642.
Simon, T.J. (1997). Reconceptualizing the origins of number knowledge: A 'non- numerical' account. *Cognitive Development*, vol. 12 : 349-372.
Soltész, F. & Szücs, D. (2014) Neural adaptation to non-symbolic number and visual shape: An electrophysiological study. *Biological Psychology* 103:203–11.doi: 10.1016/j.biopsycho.2014.09.006.
Starkey, P., Spelke, E. S., & Gelman, R. (1990). Numerical abstraction by human infants. *Cognition*, 36, 97–128.
Wynn, K. (1992). Addition and subtraction by human infants. *Nature*, 358, 749–750.