物 理 探 査　第 51 巻　第 5 号　518–521 頁
BUTSURI-TANSA　Vol. 51　No. 5 (1998)　p. 518–521

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物 理 探 査 学 会

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BUTSURI-TANSA
Vol. 51 No. 5 (1998)
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Exponential Extrapolation of Fourier Transformed Potentials in 2.5–D dc Resistivity Modeling

Hee Joon KIM*, Yoonho SONG** and Ki Ha LEE*3

ABSTRACT

In 2.5–D dc resistivity modeling which allows for subsurface current and potential electrodes, numerical errors depend on the coarseness of discretization and increase with spatial wavenumbers. In this regard, the exponential extrapolation of Fourier transformed potentials is useful for the evaluation of inverse Fourier transform integral in the outmost range of wavenumbers. This paper presents an accurate integral scheme in the outmost range using an exponential function of which coefficient is represented by a ratio between the modified Bessel functions of order 1 and 0. The effectiveness of this scheme has been confirmed using a homogenous half-space model and a vertical fault model.

Key words: 2.5–D dc resistivity modeling, discretization error, spatial wavenumber

1. INTRODUCTION

The dc resistivity response in the 2–D earth due to a 3–D (point) current source is described by Poisson's equation:

$$-\nabla\cdot[\sigma(x,z)\nabla\phi(x,y,z)]=i_s(x,y,z), \quad (1)$$

where $\sigma(x, z)$ is the electrical conductivity, $\phi(x, y, z)$ the electrical potential, and $i_s(x, y, z)$ the source current distribution. By taking the Fourier transform of equation (1) with respect to the y coordinate, one obtains

$$-\nabla\cdot[\sigma(x,z)\nabla\Phi(x,\lambda,z)] +\lambda^2\sigma(x,z)\Phi(x,\lambda,z)=I_s(x,\lambda,z), \quad (2)$$

where λ is the Fourier transform variable (spatial wavenumber).

If the Fourier transformed potentials $\phi(x, y, z)$ are obtained for several values of λ, then the electrical potential $\phi(x, y, z)$ can be evaluated using the inverse Fourier transform. When both current and potential electrodes are located along $y=0$, the inverse Fourier transform becomes (Dey and Morrison, 1979)

$$\phi(x,0,z)=\frac{2}{\pi}\int_0^\infty \Phi(x,\lambda,z)\,d\lambda. \quad (3)$$

Dey and Morrison (1979) performed this integration by fitting the envelope of $\Phi(\lambda)$ in each subsection $[\lambda_i, \lambda_{i+1}]$ by an exponential function and using the analytic form

Manuscript received February 19, 1998; Accepted June 23, 1998.
* Pukyong National University, 599–1 Taeyeon-dong, Nam-ku, Pusan 608–737, Korea
** Korea Institute of Geology, Mining and Materials, 30 Kajung-dong, Yusung-ku, Taejon 305–350, Korea
*3 Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, CA 94720, U.S.A.

$$\int_{\lambda_i}^{\lambda_{i+1}} \Phi(\lambda)\, d\lambda \approx \int_{\lambda_i}^{\lambda_{i+1}} c \exp(-a\lambda)\, d\lambda$$

$$= \frac{\Phi(\lambda_i) - \Phi(\lambda_{i+1})}{a}, \tag{4}$$

where

$$a = \frac{\ln(\Phi(\lambda_{i+1})/\Phi(\lambda_i))}{\lambda_i - \lambda_{i+1}}, \tag{5}$$

and c is a constant

To reduce errors associated with the Fourier integral (3), Fujisaki *et al.* (1992) suggested a logarithmic interpolation of transformed potentials for small values of λr, where r is the distance between current and potential electrodes, and an exponential interpolation for large values. The optimum value of λr to switch from the logarithmic function to the exponential function was given as about 0.8 by Kim *et al.* (1995). In the innermost interval $[0, \lambda_1]$, a logarithmic extrapolation is useful to minimize the error associated with the singularity at $\lambda=0$ (Fujisaki *et al.*, 1992; Haryu, 1996). In the outermost interval $[\lambda_{N-1}, \lambda_N]$, on the other hand, an exponential extrapolation is effective to reduce a truncation error associated with a finite value of λ_N (Kim *et al.*, 1995; Haryu, 1996).

The accuracy of numerical model depends on the coarseness of discretization (Kim *et al.*, 1995; Haryu, 1996). In 2.5-D dc resistivity modeling, the coarseness of discretization is essentially varied with spatial wavenumbers. Haryu (1996) suggested that the mesh size could be decided based on the Nyquist wavenumber. If we do not change the mesh size for all wavenumbers, Fourier transformed potentials $\Phi(\lambda)$ for large values of λ contain significant errors because the mesh size may be too large to simulate the potentials (see Figure 2 in Kim *et al.* (1995)). Since this discretization error increases with increasing λ and is most obvious in the outmost interval $[\lambda_{N-1}, \lambda_N]$, Kim *et al.* (1995) used the following extrapolation fromula:

$$\int_{\lambda_{N-1}}^{\infty} c \exp(-a\lambda)\, d\lambda = \frac{\Phi(\lambda_{N-1})}{a}. \tag{6}$$

Here, it is assumed that potentials $\Phi(\lambda)$ outside of λ_{N-1} are beyond an acceptable lavel of the discretization error. The coefficient a can be approximated using known $\Phi(\lambda_{N-1})$ and $\Phi(\lambda_{N-2})$ as

$$a = \frac{\ln(\Phi(\lambda_{N-1})/\Phi(\lambda_{N-1}))}{\lambda_{N-1} - \lambda_{N-2}}. \tag{7}$$

The coefficient a controls the shape of exponential function, and the error of the integration in equation (6) may be small when λ_{N-2} is sampled near λ_{N-1}. However, to suppress the amount of computations, Fujisaki *et al.* (1992) and Kim *et al.* (1995) used a relatively wide samplings: $\lambda_{N-2}=0.32$ and $\lambda_{N-1}=0.64$. Thus there is a room for improvement in determining a if we adopt their sampling scheme for an efficient and accurate evaluation of the Fourier integral.

2. EXPONENTIAL EXTRAPOLATION

When λ approaches to λ_{N-1}, the mesh size would become enormously large to simulate Fourier transformed potentials $\Phi(\lambda)$, which largely depend on the resistivity distribution surrounding an observation point accordingly. If the resistivity distribution near an observation point is homogeneous, the potential $\Phi(\lambda)$ for large λ may have the form

$$\Phi(\lambda) = A[K_0(\lambda r) + K_0(\lambda r_s)], \tag{8}$$

where A is a constant, K_0 the modified Bessel function of order 0, and r_s the distance between image source and observation point. Substituting equation (8) into equation (7) yields

$$a = \frac{\ln\left(\dfrac{[K_0(\lambda_{N-2} r) + K_0(\lambda_{N-2} r_s)]}{[K_0(\lambda_{N-1} r) + K_0(\lambda_{N-1} r_s)]}\right)}{\lambda_{N-1} - \lambda_{N-2}}. \tag{9}$$

If λ_{N-2} approaches to λ_{N-1} equation (9) can be rewritten as

$$a = -\left.\frac{\partial \ln(K_0(\lambda r) + K_0(\lambda r_s))}{\partial \lambda}\right|_{\lambda=\lambda_{N-1}},$$

$$= \frac{r K_1(\lambda_{N-1} r) + r_s K_1(\lambda_{N-1} r_s)}{K_0(\lambda_{N-1} r) + K_0(\lambda_{N-1} r_s)} \tag{10}$$

where K_1 is the modified Bessel function of order 1. This scheme uses only one wavenumber, λ_{N-1}, and the coefficient a is expressed by the ratio between the modified Bessel functions of order 1 and order 0.

Simple numerical experiments are now conducted to find the validity of equation (10). In the experiments we use the same sampling scheme as Fujisaki *et al.* (1992) and Kim *et al.* (1995). If one or both of source and observation points are located on the surfce of a homogeneous half-space model and the distance between them is $r=10$ m, for example, the coefficient a in the interval [0.64, 1.28], a_1 is obtained from equation (5) as

$$a_1=\frac{\ln\left(K_0(6.4)/K_0(12.8)\right)}{1.28-0.64}=10.53.$$

If we use the exponential extrapolation scheme of Kim *et al.* (1995), the coefficient a in the interval [0.32, 0.64], a_2 is given by

$$a_2=\frac{\ln\left(K_0(3.2)/K_0(6.4)\right)}{0.64-0.32}=11.03.$$

This result shows that a_2 is 4.75% greater than a_1, and thus the integrated value with a_2 will be 4.75% smaller than with a_1. On the other hand, the new algorithm yields a coefficient a_3 as

$$a_3=10\frac{K_1(6.4)}{K_0(6.4)}=10.75.$$

Because a difference between the coefficients of a_3 and a_1 is only 2.09%, the coefficient a_3 is much better approximation to a_1 than a_2.

Next example is for a vertical fault model, for which the potential $\Phi(\lambda)$ is obtained analytically, as shown in Figure 1. Two media of 10 and 100 Ω-m are horizontally contacted. A current soruce of 1 A is located on the surface 4 m away from the vertical contact. Seven observation points are horizontally positioned at 2 m in depth near the contact. It is clear that equation (8) is derived by ignoring the effect of inhomogeneity on $\Phi(\lambda)$. However, our scheme works well even in the non-uniform model. From Table 1, we can see that the coefficient a_3 is much better approximation to a_1 than a_2.

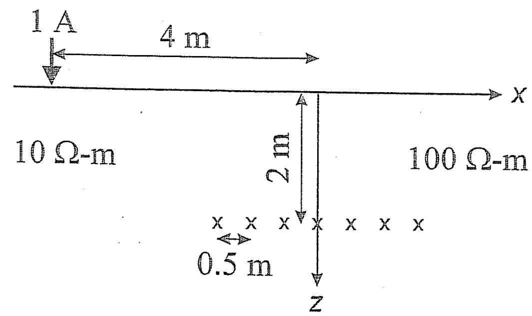

Fig. 1 A vertical fault model. A current source of 1 A is located on the surface 4 m away from the vertical contact of two media of 10 and 100 Ω-m. Seven observation points are located at 2 m in depth near the contact.

Table 1 The coefficient a_1, a_2, and a_3 determining the shape of exponential function, which are evaluated as a function of horizontal distances of observation points from the source in the vertical fault model as shown in Figure 1

x (m)	a_1	a_2	a_3
2.5	3.843	4.571	3.915
3.0	4.317	4.987	4.325
3.5	4.759	5.309	4.756
4.0	4.986	5.460	5.201
4.5	5.440	5.919	5.657
5.0	5.903	6.385	6.121
5.5	6.372	6.858	6.592

3. CONCLUSIONS

We have analyzed discretization errors of 2.5–D dc resistivity modeling that allows for subsurface current and potential electrodes. Since numerical errors depend on the coarseness of discretization and increase with increasing spatial wavenumber λ, the exponential extrapolation of Fourier transformed potentials is useful for the evaluation of inverse Fourier transform integral in the outmost range of λ. In this paper we have developed an integral scheme in the outmost range using an accurate exponential extrapolation function. The coefficient of the exponential function is represented by the ratio between the modified Bessel functions of order 1 and order 0, and its shape is much better than that used in Kim *et al.* (1995).

ACKNOWLEDGMENTS

This reseach was partly supported by the U.S. Environmental Protection Agency and the Center for Mineral Resources Research sponsored by the Korea Science and Engineering Foundation. We wish to thank an anonymous referee for valuable comments on the manuscript.

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2.5次元直流比抵抗モデリングにおける フーリエ変換されたポテンシャルの指数関数外挿

Hee Joon Kim, Yoonho Song and Ki Ha Lee

要　旨

地下に電流および電位電極が存在する場合の2.5次元直流比抵抗モデリングにおいて，数値誤差は離散化の粗さに依存し，空間波数が大きくなるにつれて増大する。このような観点から，フーリエ変換されたポテンシャルの指数関数による外挿は波数の最外側区間における逆フーリエ変換積分の評価に有効である。本論文では，最外側区間において係数がオーダー０および１の修正ベッセル関数の比によって表される指数関数を用いる精度のよい積分スキームを紹介する。この方法の有効性は均質半空間モデルおよび垂直断層モデルに対する簡単な数値実験を通して確かめられた。