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Author
Friedkin, Noah E

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Scale-free interpersonal influences on opinions in complex systems

Noah E. Friedkin [1]

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[1] *Center for Control, Dynamical Systems and Computation, College of Engineering, and Department of Sociology, College of Letters and Science, University of California, Santa Barbara .*

Abstract

An important side effect of the evolution of the human brain is an increased capacity to form opinions in a very large domain of issues, which become points of aggressive interpersonal disputes. Remarkably, such disputes are often no less vigorous on small differences of opinion than large differences. Opinion differences that may be measured on the real number line may not directly correspond to the subjective importance of an issue and extent of resistance to opinion change. This is a hard problem for field of opinion dynamics, a field that has become increasingly prominent as it has attracted more contributions to it from investigators in the natural and engineering sciences. The paper contributes a scale-free approach to assessing the extents to which individuals, with unknown heterogeneous resistances to influence, have been influenced by the opinions of others.

1 Introduction

Observations of opinion changes on an issue are a necessary basis of asserting the existence of some coordinative mechanism of interpersonal influence in which individuals' opinions are affecting other individuals' opinions. The classical literature on interpersonal influence was developed with experimental designs with subjects exposed to a fixed displayed opinion of one or more confederates of the experimenter [1, 2, 3]. In such designs, where exogenous disturbances on opinions are minimal, interpersonal influence is apparent when subjects' displayed opinions are altered in the direction of the displayed fixed position to which each subject has been exposed.

However, observed extents of opinion differences on the real number line, and opinion changes, depend the scaling of the opinions and the times at which they are measured,

$$\Delta(ij,t) = \alpha\big[x(i,t+k) - x(j,t)\big],\ j \in \{i,j\},\ t = 0,1,2,\dots, \tag{1}$$

where $\alpha > 0$ is the scalar value. As t and/or k increase, $\Delta(ii,t)$ typically decreases as in Sherif's [4] seminal investigation of norm formation, and the observed amount of opinion change will depend on α for all $\Delta(ii,t) > 0$. The sensitivity of $\Delta(ii,t)$ to arbitrary selection of $\{t,k\}$ temporal measurement points is ameliorated when $\mathbf{x}(t) = \mathbf{x}(0)$ are individuals' pre-process initial opinions and $\lim_{k\to\infty} \mathbf{x}(t+k) = \mathbf{x}(\infty)$ are their post-process equilibrium opinions. It is aggravated when it is assumed that interpersonal influence declines as the absolute value of $\Delta(ij,t)$ increases, or that interpersonal influence does not exist above threshold values of $\mid \Delta(ij,t) \mid$, especially when these thresholds may vary among individuals [5, 6, 7]. Here, I shall assume that individuals' quantitative differences of opinion are not informative of their levels of resistance to opinion change.

A markedly enlarged scientific community is now engaged in the study of complex systems of interpersonal influences composed of individuals with heterogeneous initial opinions (some or all of whom are responding to the changing opinions of others) and heterogeneous extents of resistance to interpersonal influence. With the influx of investigators from the natural and engineering sciences into the field of opinion dynamics, the development of mathematical models of opinion dynamics is a rapidly advancing frontier of scientific work [8, 9, 10]. The collection of observations, which was limited to small groups, now includes the behaviors and networks of communicating individuals in large-scale field settings of Internet social media. The present results contribute an approach to a scale-free derivation of individuals' heterogeneous resistances to interpersonal influences in complex systems.

Finite convex sets are important in various areas of basic and applied mathematics, and appear in linear state-space processes. One area of application is the first-order DeGroot [11] discrete-time state-space process in which the state of each point of a set n points, $\mathbf{x}_i(k+1),\ i = 1, ..., n$, is a convex combination of the immediately prior states of all points of the system $\mathbf{x}_j(k) \in \mathbb{R}^n,\ j = 1, ..., n$. With a $x(0) \in \mathbb{R}^n$ and a row-stochastic $\mathbf{W}$, the model

$$\mathbf{x}(t+1) = \mathbf{W}\mathbf{x}(t), \quad t = 0, 1, ... , \tag{2}$$

was formulated as a mechanism by which consensus might be reached among a set of individuals. It has become the benchmark model of the literature on opinion dynamics. Its precursors include the models of French [12] and Harary [13]. In addition, the model has become increasingly prominent in control theory [14, 15, 16, 17]. When the mechanism unfolds in an aperiodic irreducible network, the system converges to a single value on the real number line and, more generally for a $\mathbf{X}(k) \in \mathbb{R}^{n\times m}$, to a single location in m-dimensional space.

It will be shown that scale-free interpersonal influences are detectable for this model. Note that the DeGroot model assumes measures of $\mathbf{x}(0)$ and $\mathbf{W}$. In it, the main-diagonal values of $\mathbf{W}$ correspond to individuals' resistance to interpersonal influence. In practice, a direct measure of these main-diagonal values are rarely available, so that the measure of $\mathbf{W}$ is a matrix with (a) zeros on its main-diagonal and (b) off-diagonal values that are relative *interpersonal* weights based on the available network data. Thus, the usual implementation of the model assumes individuals with zero resistance, who adopt the weighted average of *others'* positions on an issue at each time t. But if the system is, in truth, composed of individuals with high resistances to opinion change, then the process of consensus formation may be exceedingly slow and difficult. With high resistances to opinion change, the process may be terminated prior to its convergence to consensus.

Hence, the analytical problem may formalized as follows. Let $\mathbf{C}$ be a row stochastic matrix of relative *interpersonal* weights with a zero main diagonal. Given a measure of such a matrix and measures of individuals' opinions, what are their' resistances to opinion change?

2 Scale-free Resistance in the DeGroot Model

With little loss of generality, let $0 < w_{ii} < 1$ for all $i = 1, ..., n$. Let $\mathbf{D} \equiv \mathrm{diag}(w_{11}, ..., w_{nn})$. Let $\mathbf{C} \equiv (\mathbf{I} - \mathbf{D})^{-1}(\mathbf{W} - \mathbf{D})$. Hence, $\mathbf{C} = [c_{ij}]$ is a matrix with $c_{ii} = 0$ for all i and $c_{ij} = w_{ij}/(1 - w_{ii})$ for all $i \neq j$. N.B. The foregoing are the theoretical definitions of constructs. The matrix $\mathbf{D}$ is the unknown values of the main-diagonal of $\mathbf{W}$, and the matrix $\mathbf{C}$ is the available measure of the relative interpersonal influences of $\mathbf{W}$.

From the definitions of these constructs, it follows that

$$\mathbf{W} = (\mathbf{I} - \mathbf{D})\mathbf{C} + \mathbf{D} \tag{3}$$

with which a solution of the resistance values of $\mathbf{D}$ is available.

The scalar equation of the DeGroot model may now be expressed in terms of $\mathbf{C}$ and $\mathbf{D}$

$$\begin{aligned} x_i(t+1) &= \sum_{j=1}^{n} w_{ij} x_j(t), \quad i = 1, 2, ..., n,\ t = 0, 1, 2, ... \\ &= (d_{ii})x_i(t) + (1 - d_{ii}) \sum_{j\neq i} c_{ij} x_j(t), \end{aligned} \tag{4}$$

whence

$$d_{ii} = 1 - \frac{x_i(t+1) - x_i(t)}{\sum_{j \neq i} c_{ij} x_j(t) - x_i(t)}, \tag{5}$$

for $\sum_{j \neq i} c_{ij} x_j(t) \neq x_i(t)$.

Under the assumption of a time-invariant $\mathbf{W}$, the derived d_{ii} values, $i = 1, 2, ..., n$, apply to all t. This assumption may be relaxed with a $\mathbf{W}(t)$. In either case, the derived values are scale-free. In applications, solutions are constrained to the $(0, 1)$ interval. Under the assumption that the model is correct, unreliable or invalid measurement models of its constructs may generate departures from the $(0, 1)$ interval, and unreliable or invalid d_{ii} values.

3 Discussion

A model-based approach has been presented for a sale-free detection of extents to which individuals' opinions are closed or open to interpersonal influences on their issue-specific opinions. Influence detection does not require full data on the network in which opinion dynamics unfold. Because only direct influences on an individual are involved in the detection, the approach may be applied to a selected set (or random sample) of individuals and their adjacent neighbors.

The class of models considered in this paper is based on the assumption that endogenous interpersonal influence is a convex combination mechanism with weights that are allocated by individuals to their own and others' displayed orientations to an object. The available empirical supports for this mechanism are inconsistent with an assumption that allows negative interpersonal influences or resistances to influences. The mechanism predicts (a) an initial consensus will be maintained and (b) that modifications of disagreeing initial opinions will be constrained to range (or convex hull) of initial opinions. These predictions are strongly supported in the observations of experimental social psychology collected on small groups assembled in laboratory settings, where individuals' pre- and post-discussion opinions have been measured.

(a) There is strong support in the literature for the prediction that an initial consensus will be maintained. Barnlund [18] reported that, in small groups assembled to solve problems of logic, an initial consensus was not questioned (the group moved on to the next problem) regardless of whether the consensus was correct or incorrect. Similarly, Thorndike [19] found that an initial consensus was rarely modified regardless of whether the consensus was correct or incorrect; in his results, an initial consensus was modified in only 3 of 725 group problem-solving trials in which the groups judgment was correct, and in 1 of 263 trials in which the groups judgment was incorrect. Consensus is either assumed to be correct (whether or not it is) or satisfactory; in either case, it is deemed conclusive.

(b) Given initial disagreement, Friedkin and Johnsen [20] find one instance of an individual post-discussion opinion outside the range of initial opinions in 1,000 cases based on 50 groups of tetrads discussing five issues (two monetary issues and three issues of acceptable risk) in sequence. Similarly, they find one instance of an individual post-discussion opinion outside the range of initial opinions in 288 cases based on 32 groups of triads discussing three issues of acceptable risk in sequence. A larger incidence of anomaly occurs in dyads, and appears to be linked to the special property of this smallest of groups in which an aperiodic influence system is approached as their members' allocations of weight to their own opinions approach 0.

An additional important property of the class of models considered in this paper is that interpersonal influence is independent of $\mid \Delta(ij, t) \mid$. The available evidence on this matter, amassed over decades, is mixed [20] and erodes the assertion of a reliable dependency. The subjective importance that individuals assign to particular opinion differences $\mid \Delta(ij, t) \mid$ appears to be unconstrained by the values of these differences, so that large resistances to opinion change may exist for small differences of opinions, and small resistances to opinion change may exist for large differences of opinions.

From a social psychological perspective, convex combination mechanisms describe an automatic heuristic "cognitive algebra" [21, 22] with which the brain integrates heterogeneous information. Thus, the influence network is a social cognition structure assembled by individuals' allocation of weights to self and particular others on a specific issue. Investigators in the discipline of experimental social psychology have been remarkably adept in their detection numerous conditions with effects on the interpersonal influence relation. The implication of this work is that the antecedents of individuals' allocated weights are complex, i.e., the weights of the convex combination mechanism are net resultants of numerous conditions of the individuals and their interpersonal relations. As such, the set of ordered pairs of weights, which form an influence network, is unlikely to be based on any single condition, e.g., opinion differences, expertise, authority, rewards, affection, or coercion.

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