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Abstract

When two users transmit signals to a common receiver, one can design precoders to cancel the interference for each user, if each user knows all the channel information perfectly. Also the diversity for each user is full. However, in practice, perfect channel information is not available. In this paper, we design precoders for two users with two transmit antennas and one receiver with two receive antennas using quantized feedback. We propose to construct codebook using Grassmannian line packing. By choosing precoders from the codebook properly, our proposed scheme can cancel the interference for each user. Also we analytically prove that our system can achieve full diversity for each user. Then we extend our scheme to any number of transmit and receive antennas. Simulation results confirm our analytical proof and show that our scheme can serve as a bridge between a system with no feedback and a system with perfect feedback.

Index Terms

Multi-user detection, multiple antennas, interference cancellation, precoder, quantized feedback, Grassmannian line packing

I. Introduction

In multiple access channels, when different users with multiple antennas transmit signals to a common receiver at the same time using the same frequency band, users cause interference to each other. In this paper, we investigate how to cancel the co-channel interference for users with multiple antennas

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in multiple access channels without any frequency division or spread spectrum technology. We utilize multiple antennas and channel information feedback to cancel the interference for each user.

The topic of canceling interference using multiple antennas is not new. Different schemes have been proposed with different assumptions about the channel information available at transmitters. If no channel information is available at transmitters, [1] shows how to cancel the interference using NJ receive antennas for J users equipped with N transmit antennas. In order to reduce the number of required receive antennas to J, i.e., as many as the number of users, [2] proposes a scheme for users with two transmit antennas based on the properties of orthogonal space-time block codes (OSTBCs) [3]. The work was extended to a higher number of transmit antennas but only for $J = 2$ users in [4]. Unfortunately, because of the limitations of OSTBCs, these methods cannot be used for a general case of complex constellations, $N > 2$ transmit antennas, and $J > 2$ users [5]. To solve this problem, [5] suggests a method based on quasi-orthogonal space-time block codes (QOSTBCs) [6]. The diversity of each user in such a system with $M \geq J$ receive antennas is equal to NM using maximum-likelihood detection and $N(M - J + 1)$ using low-complexity array-processing schemes [7]. However, in general, we cannot achieve full diversity if we cancel the interference using low-complexity array-processing schemes. In order to cancel the interference with low-complexity and achieve full diversity, we proposed a scheme to design precoders using the channel information feedback at transmitters [8]. As a result, interference can be canceled while achieving full diversity for each user.

Although the performance of the scheme in [8] is better than that of the previously proposed schemes, perfect channel information is needed at transmitters. It is not practical in reality. In this paper, we investigate the use of limited feedback to achieve interference cancellation as well as full diversity. Limited feedback has been used extensively in the case of the single-user MIMO systems. It has been shown that the capacity and performance of the point-to-point MIMO systems can be increased significantly using limited feedback [9]–[15]. There are few examples of multi-antenna multi-user systems with limited feedback in the literature. In [16], post-processing SNR is maximized for a given linear receiver by selecting the QOSTBC with the minimum quaternionic angle as well as realizing interference cancellation. In [17], limited feedback is utilized to adapt the phase of a transmitted signal and improve the performance of the system. However, to the best of our knowledge, there is no result showing how to achieve full diversity and interference cancellation for each user using limited feedback. A naive way is to quantize the result in [8] directly. But this will not work because the scheme in [8] relies on the perfect channel information and thus perfect orthogonality between the signal vectors of the two users. Simply quantizing the results will destroy the perfect orthogonality and thus cannot achieve full diversity. In this paper we investigate how to use quantized feedback to achieve full diversity as well as interference cancellation. Our results show that even with quantized feedback, full diversity and interference cancellation are possible by using our proposed scheme. Also our decoding complexity is the lowest to our best knowledge. By increasing the number of feedback bits, the performance of our proposed scheme will approach the performance of the scheme with perfect feedback in [8]. So our proposed scheme can serve as a bridge between the schemes with no feedback and perfect feedback.

The outline of the paper follows next. In Section II, we introduce our system model and the motivation of this work. In Section III, we present our precoding and decoding method. We show that our scheme can achieve interference cancellation for each user as long as our codebooks satisfy some conditions. In Section IV, we propose our feedback scheme and prove that each user can achieve full diversity if our codebooks can satisfy some further conditions. In Section V, we finalize our codebook design by maximizing the coding gain. In Section VI, we compare the performance of our system with two existing schemes. We show that our scheme can be extended to 2 users each with any number of transmit antennas and one receiver with any number of receive antennas in Section VII. Section VIII provides simulation results to validate our theoretical analysis and Section IX concludes the paper.

Notation: We use boldface letters to denote matrices and vectors, super-scripts $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^\dagger$ to denote transpose, conjugate and transpose conjugate, respectively. $||\mathbf{Z}||_F$ represents the Frobenius norm of matrix $\mathbf{Z}$. We denote the element in the ith row and the jth column of matrix $\mathbf{Z}$ by $Z(i,j)$. Also, we denote the jth column of a matrix $\mathbf{Z}$ by $\mathbf{Z}(j)$. The real and imaginary parts of a matrix $\mathbf{Z}$ are denoted by Real$\{\mathbf{Z}\}$ and Imag$\{\mathbf{Z}\}$, respectively.

II. Channel Model

In this paper, we assume a quasi-static flat Rayleigh fading channel. The path gains are independent complex Gaussian random variables and fixed during the transmission of one block. There are two users each with two transmit antennas and one receiver with two receive antennas.

We assume that the receiver knows the channel information perfectly but only quantized feedback is available at the transmitter. We want to design a scheme to achieve the following two goals using quantized feedback: (i) Canceling the interference at the receiver, i.e., obtaining the interference-free signals for each user at the receiver, (ii) providing full diversity for each user.

In order to achieve these two goals, we propose the following scheme in time slot 1 as shown in Figure 1: First, we assume that Users 1 and 2 transmit codewords $\mathbf{C}$ and $\mathbf{S}$, respectively. And each user can receive K bits of feedback from the receiver. Second, we design a codebook Υ_1 which contains $L_1 = 2^K$

different precoding matrices for User 1 and a codebook Υ_2 which contains $L_2 = 2^K$ different precoding matrices for User 2. Each codebook is shared by its transmitter and the receiver. Also we let $\Upsilon_i[j]$ denote the jth matrix in Codebook Υ_i. Third, the receiver sends back an index ℓ_1 to User 1 using K bits of feedback and an index ℓ_2 to User 2 using another K bits of feedback. Finally, User 1 chooses $\Upsilon_1[\ell_1]$ as its precoder $\mathbf{A}^1$ and transmits the pre-coded signals to the receiver. Also User 2 chooses $\Upsilon_2[\ell_2]$ as its precoder $\mathbf{B}^1$ and transmits the pre-coded signals to the receiver. After receiving the signals from Users 1 and 2, the receiver decodes the signals for each user separately using an array processing method.

In time slot 2, the scheme will be exactly the same as that at time slot 1. But the designed codebooks $\Upsilon_1^{'}$ for User 1 and $\Upsilon_2^{'}$ for User 2 in time slot 2 may be different from the codebooks Υ_1 and Υ_2 in time slot 1. Also the feedback indices $\ell_1^{'}$ and $\ell_2^{'}$ in time slot 2 may be different from ℓ_1 and ℓ_2 in time slot 1. As a result, the precoders $\mathbf{A}^2$ for User 1 and $\mathbf{B}^2$ for User 2 in time slot 2 may be different from $\mathbf{A}^1$ and $\mathbf{B}^1$ in time slot 1.

Now, we demonstrate the input-output relationship of our system. At the first two time slots, the channel matrices for Users 1 and 2 are

$$\mathbf{H} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \tag{1}$$

respectively, where h_{ij} and g_{ij} are i.i.d. $CN(0, 1)$. For backward compatibility with the case of no feedback in [5], Users 1 and 2 transmit Alamouti codes

$$\mathbf{C} = \begin{pmatrix} c_1 & -c_2^* \\ c_2 & c_1^* \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{pmatrix} \tag{2}$$

respectively. In order to maximize the diversity and coding gain, we add unitary rotations $\mathbf{R}_1$ and $\mathbf{R}_2$ for codewords of User 1 and User 2, respectively, such that

$$\mathbf{R}_1 \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} \widetilde{c}_1 \\ \widetilde{c}_2 \end{pmatrix}, \quad \mathbf{R}_2 \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} \widetilde{s}_1 \\ \widetilde{s}_2 \end{pmatrix}. \tag{3}$$

So the codewords in (2) become

$$\widetilde{\mathbf{C}} = \begin{pmatrix} \widetilde{c}_1 & -\widetilde{c}_2^* \\ \widetilde{c}_2 & \widetilde{c}_1^* \end{pmatrix}, \quad \widetilde{\mathbf{S}} = \begin{pmatrix} \widetilde{s}_1 & -\widetilde{s}_2^* \\ \widetilde{s}_2 & \widetilde{s}_1^* \end{pmatrix}. \tag{4}$$

Let

$$\mathbf{A}^1 = \begin{pmatrix} a_{11}^1 & a_{12}^1 \\ a_{21}^1 & a_{22}^1 \end{pmatrix}, \quad \mathbf{A}^2 = \begin{pmatrix} a_{11}^2 & a_{12}^2 \\ a_{21}^2 & a_{22}^2 \end{pmatrix} \tag{5}$$

denote the precoders of User 1 in time slots 1 and 2, respectively. Also,

$$\mathbf{B}^1 = \begin{pmatrix} b_{11}^1 & b_{12}^1 \\ b_{21}^1 & b_{22}^1 \end{pmatrix}, \quad \mathbf{B}^2 = \begin{pmatrix} b_{11}^2 & b_{12}^2 \\ b_{21}^2 & b_{22}^2 \end{pmatrix} \tag{6}$$

denote the precoders of User 2 in time slots 1 and 2, respectively. Here $||\mathbf{A}^i||_F^2 = ||\mathbf{B}^i||_F^2 = 1$, $i = 1, 2$, in order to satisfy the normalization conditions [11].

In time slots 1 and 2, the received signals are respectively denoted by

$$\mathbf{y}^1 = \begin{pmatrix} y_1^1 \\ y_2^1 \end{pmatrix}, \quad \mathbf{y}^2 = \begin{pmatrix} y_1^2 \\ y_2^2 \end{pmatrix}. \tag{7}$$

Then, in time slot 1, the signal model can be written as

$$\mathbf{y}^1 = \sqrt{E_s}\mathbf{H}\mathbf{A}^1 \begin{pmatrix} \widetilde{c}_1 \\ \widetilde{c}_2 \end{pmatrix} + \sqrt{E_s}\mathbf{G}\mathbf{B}^1 \begin{pmatrix} \widetilde{s}_1 \\ \widetilde{s}_2 \end{pmatrix} + \mathbf{W}^1. \tag{8}$$

In time slot 2, we have

$$\mathbf{y}^2 = \sqrt{E_s}\mathbf{H}\mathbf{A}^2 \begin{pmatrix} -\widetilde{c}_2^* \\ \widetilde{c}_1^* \end{pmatrix} + \sqrt{E_s}\mathbf{G}\mathbf{B}^2 \begin{pmatrix} -\widetilde{s}_2^* \\ \widetilde{s}_1^* \end{pmatrix} + \mathbf{W}^2 \tag{9}$$

where E_s denotes the total transmit energy of each user and $\mathbf{W}^1 = \begin{pmatrix} n_1^1 \\ n_2^1 \end{pmatrix}$, $\mathbf{W}^2 = \begin{pmatrix} n_1^2 \\ n_2^2 \end{pmatrix}$ denote the noise at the receiver in time slots 1 and 2, respectively. We assume that n_1^1, n_2^1, n_1^2, n_2^2 are i.i.d complex Gaussian noises with mean 0 and variance 1. In order to simplify the notation, we let

$$\widehat{\mathbf{H}}^i = \mathbf{H}\mathbf{A}^i, \quad \text{i.e.,} \begin{pmatrix} \widehat{h}_{11}^i & \widehat{h}_{12}^i \\ \widehat{h}_{21}^i & \widehat{h}_{22}^i \end{pmatrix} = \begin{pmatrix} h_{11}a_{11}^i + h_{12}a_{21}^i & h_{11}a_{12}^i + h_{12}a_{22}^i \\ h_{21}a_{11}^i + h_{22}a_{21}^i & h_{21}a_{12}^i + h_{22}a_{22}^i \end{pmatrix}, \quad i = 1, 2, \tag{10}$$

$$\widehat{\mathbf{G}}^i = \mathbf{G}\mathbf{B}^i, \quad \text{i.e.,} \begin{pmatrix} \widehat{g}_{11}^i & \widehat{g}_{12}^i \\ \widehat{g}_{21}^i & \widehat{g}_{22}^i \end{pmatrix} = \begin{pmatrix} g_{11}b_{11}^i + g_{12}b_{21}^i & g_{11}b_{12}^i + g_{12}b_{22}^i \\ g_{21}b_{11}^i + g_{22}b_{21}^i & g_{21}b_{12}^i + g_{22}b_{22}^i \end{pmatrix}, \quad i = 1, 2. \tag{11}$$

With these new notations, after applying some simple algebra to Equations (8) and (9), we have

$$\begin{pmatrix} y_1^1 \\ y_2^1 \end{pmatrix} = \sqrt{E_s} \begin{pmatrix} \widehat{h}_{11}^1 & \widehat{h}_{12}^1 & \widehat{g}_{11}^1 & \widehat{g}_{12}^1 \\ \widehat{h}_{21}^1 & \widehat{h}_{22}^1 & \widehat{g}_{21}^1 & \widehat{g}_{22}^1 \end{pmatrix} \begin{pmatrix} \widetilde{c}_1 \\ \widetilde{c}_2 \\ \widetilde{s}_1 \\ \widetilde{s}_2 \end{pmatrix} + \begin{pmatrix} n_1^1 \\ n_2^1 \end{pmatrix}, \tag{12}$$

$$\begin{pmatrix} (y_1^2)^* \\ (y_2^2)^* \end{pmatrix} = \sqrt{E_s} \begin{pmatrix} (\widehat{h}_{12}^2)^* & -(\widehat{h}_{11}^2)^* & (\widehat{g}_{12}^2)^* & -(\widehat{g}_{11}^2)^* \\ (\widehat{h}_{22}^2)^* & -(\widehat{h}_{21}^2)^* & (\widehat{g}_{22}^2)^* & -(\widehat{g}_{21}^2)^* \end{pmatrix} \begin{pmatrix} \widetilde{c}_1 \\ \widetilde{c}_2 \\ \widetilde{s}_1 \\ \widetilde{s}_2 \end{pmatrix} + \begin{pmatrix} (n_1^2)^* \\ (n_2^2)^* \end{pmatrix}. \tag{13}$$

Equations (12) and (13) are the input-output relationship of our system at the first two time slots.

III. Interference Cancellation Precoding and Decoding

In this section, we will show the property that our codebooks should possess in order to achieve our first goal, i.e., interference cancellation.

A. Precoding

First, in time slot 1, by Equation (12), $\widetilde{c}_1$, $\widetilde{c}_2$, $\widetilde{s}_1$, $\widetilde{s}_2$ are transmitted along four equivalent channel vectors $\widehat{\mathbf{H}}^1(1), \widehat{\mathbf{H}}^1(2), \widehat{\mathbf{G}}^1(1), \widehat{\mathbf{G}}^1(2)$, respectively. Suppose that we want to remove the signals of User 2, we can find a 2-by-1 complex vector $\mathbf{g}$ satisfying $\mathbf{g}^\dagger\widehat{\mathbf{G}}^1(1) = \mathbf{g}^\dagger\widehat{\mathbf{G}}^1(2) = 0$. Then by simply multiplying both sides of Equation (12) by $\mathbf{g}^\dagger$, we can remove the signals of User 2. This is our basic idea to achieve the interference cancellation.

However, since $\widehat{\mathbf{G}}^1(1), \widehat{\mathbf{G}}^1(2)$ are 2-by-1 complex vectors, a non-zero complex vector $\mathbf{g}_{2\times 1}$ that satisfies $\mathbf{g}^\dagger\widehat{\mathbf{G}}^1(1) = \mathbf{g}^\dagger\widehat{\mathbf{G}}^1(2) = 0$ does not exist unless $\widehat{\mathbf{G}}^1(1) = \alpha\widehat{\mathbf{G}}^1(2)$, where α is a constant. Therefore, in order to cancel the interference from User 2, we need $\widehat{\mathbf{G}}^1(1) = \alpha\widehat{\mathbf{G}}^1(2)$. To make $\widehat{\mathbf{G}}^1(1) = \alpha\widehat{\mathbf{G}}^1(2)$, our precoders $\mathbf{A}^1$ and $\mathbf{B}^1$ should have the following properties:

$$\mathbf{A}^1(1) = \mathbf{A}^1(2), \quad \mathbf{B}^1(1) = \mathbf{B}^1(2), \tag{14}$$

i.e.,

$$\begin{pmatrix} a_{11}^1 \\ a_{21}^1 \end{pmatrix} = \begin{pmatrix} a_{12}^1 \\ a_{22}^1 \end{pmatrix}, \quad \begin{pmatrix} b_{11}^1 \\ b_{21}^1 \end{pmatrix} = \begin{pmatrix} b_{12}^1 \\ b_{22}^1 \end{pmatrix}. \tag{15}$$

Since we choose a matrix in the codebook $\boldsymbol{\Upsilon_1}$ as the precoder for User 1 and a matrix in the codebook $\boldsymbol{\Upsilon_2}$ as the precoder for User 2, Equation (14) results in:

$$\boldsymbol{\Upsilon_1}[\mathbf{i}](\mathbf{1}) = \boldsymbol{\Upsilon_1}[\mathbf{i}](\mathbf{2}), \quad \boldsymbol{\Upsilon_2}[\mathbf{j}](\mathbf{1}) = \boldsymbol{\Upsilon_2}[\mathbf{j}](\mathbf{2}), \tag{16}$$

i.e., the two columns of any matrix in codebooks $\boldsymbol{\Upsilon_1}$ and $\boldsymbol{\Upsilon_2}$ should be the same. From Equations (10) and (15), it is easy to see that the resulted $\widehat{\mathbf{G}}^1(1)$, $\widehat{\mathbf{G}}^1(2)$ satisfy $\widehat{\mathbf{G}}^1(1) = \widehat{\mathbf{G}}^1(2)$, i.e.,

$$\begin{pmatrix} \widehat{h}_{11}^1 \\ \widehat{h}_{21}^1 \end{pmatrix} = \begin{pmatrix} \widehat{h}_{12}^1 \\ \widehat{h}_{22}^1 \end{pmatrix}, \quad \begin{pmatrix} \widehat{g}_{11}^1 \\ \widehat{g}_{21}^1 \end{pmatrix} = \begin{pmatrix} \widehat{g}_{12}^1 \\ \widehat{g}_{22}^1 \end{pmatrix}. \tag{17}$$

Then (12) can be written as

$$\begin{pmatrix} y_1^1 \\ y_2^1 \end{pmatrix} = \sqrt{E_s} \begin{pmatrix} \widehat{h}_{11}^1 & \widehat{h}_{11}^1 & \widehat{g}_{11}^1 & \widehat{g}_{11}^1 \\ \widehat{h}_{21}^1 & \widehat{h}_{21}^1 & \widehat{g}_{21}^1 & \widehat{g}_{21}^1 \end{pmatrix} \begin{pmatrix} \widetilde{c}_1 \\ \widetilde{c}_2 \\ \widetilde{s}_1 \\ \widetilde{s}_2 \end{pmatrix} + \begin{pmatrix} n_1^1 \\ n_2^1 \end{pmatrix}. \tag{18}$$

Based on Equation (18), it is easy to find a complex vector $\mathbf{g}$ satisfying $\mathbf{g}^\dagger \begin{pmatrix} \widehat{g}_{11}^1 \\ \widehat{g}_{21}^1 \end{pmatrix} = 0$ to remove the signals of User 2. Equation (16) represents the property that our codebooks need in order to achieve interference cancellation.

Similarly, in time slot 2, our precoders should satisfy

$$\mathbf{A}^2(1) = \mathbf{A}^2(2), \quad \mathbf{B}^2(1) = \mathbf{B}^2(2). \tag{19}$$

Then using the codebook $\boldsymbol{\Upsilon}_{\mathbf{1}}^{'}$ and $\boldsymbol{\Upsilon}_{\mathbf{2}}^{'}$, for Users 1 and 2, respectively, any matrix $\boldsymbol{\Upsilon}_{\mathbf{1}}^{'}[\mathbf{i}]$ in the codebook $\boldsymbol{\Upsilon}_{\mathbf{1}}^{'}$ and any matrix $\boldsymbol{\Upsilon}_{\mathbf{2}}^{'}[\mathbf{j}]$ in the codebook $\boldsymbol{\Upsilon}_{\mathbf{2}}^{'}$ have the following properties:

$$\boldsymbol{\Upsilon}_{\mathbf{1}}^{'}[\mathbf{i}](\mathbf{1}) = \boldsymbol{\Upsilon}_{\mathbf{1}}^{'}[\mathbf{i}](\mathbf{2}), \quad \boldsymbol{\Upsilon}_{\mathbf{2}}^{'}[\mathbf{j}](\mathbf{1}) = \boldsymbol{\Upsilon}_{\mathbf{2}}^{'}[\mathbf{j}](\mathbf{2}). \tag{20}$$

Then (13) can be written as

$$\begin{pmatrix} (y_1^2)^* \\ (y_2^2)^* \end{pmatrix} = \sqrt{E_s} \begin{pmatrix} (\widehat{h}_{12}^2)^* & -(\widehat{h}_{12}^2)^* & (\widehat{g}_{12}^2)^* & -(\widehat{g}_{12}^2)^* \\ (\widehat{h}_{22}^2)^* & -(\widehat{h}_{22}^2)^* & (\widehat{g}_{22}^2)^* & -(\widehat{g}_{22}^2)^* \end{pmatrix} \begin{pmatrix} \widetilde{c}_1 \\ \widetilde{c}_2 \\ \widetilde{s}_1 \\ \widetilde{s}_2 \end{pmatrix} + \begin{pmatrix} (n_1^2)^* \\ (n_2^2)^* \end{pmatrix}. \tag{21}$$

B. Decoding

In what follows, based on Equations (18) and (21), we illustrate how to cancel the interference of User 2 and decode in detail. First, we introduce some notation to simplify the presentation. In Equations (18) and (21), we let

$$\mathbf{v}_h^1 = \begin{pmatrix} \widehat{h}_{11}^1 \\ \widehat{h}_{21}^1 \end{pmatrix}, \quad \mathbf{v}_g^1 = \begin{pmatrix} \widehat{g}_{11}^1 \\ \widehat{g}_{21}^1 \end{pmatrix}, \quad \overline{\mathbf{y}}^1 = \begin{pmatrix} y_1^1 \\ y_2^1 \end{pmatrix}, \quad \mathbf{n}^1 = \begin{pmatrix} n_1^1 \\ n_2^1 \end{pmatrix}, \tag{22}$$

$$\mathbf{v}_h^2 = \begin{pmatrix} (\widehat{h}_{12}^2)^* \\ (\widehat{h}_{22}^2)^* \end{pmatrix}, \quad \mathbf{v}_g^2 = \begin{pmatrix} (\widehat{g}_{12}^2)^* \\ (\widehat{g}_{22}^2)^* \end{pmatrix}, \quad \overline{\mathbf{y}}^2 = \begin{pmatrix} (y_1^2)^* \\ (y_2^2)^* \end{pmatrix}, \quad \mathbf{n}^2 = \begin{pmatrix} (n_1^2)^* \\ (n_2^2)^* \end{pmatrix}. \tag{23}$$

Then we introduce the following complex vectors

$$\overline{\mathbf{v}}_g^1 = \begin{pmatrix} -(\widehat{g}_{21}^1)^* \\ (\widehat{g}_{11}^1)^* \end{pmatrix}, \quad \overline{\mathbf{v}}_g^2 = \begin{pmatrix} -\widehat{g}_{22}^2 \\ \widehat{g}_{12}^2 \end{pmatrix}. \tag{24}$$

Note that $\overline{\mathbf{v}}_g^1, \overline{\mathbf{v}}_g^2$ are orthogonal to $\mathbf{v}_g^1, \mathbf{v}_g^2$ in time slots 1 and 2, respectively. In order to cancel the signals from User 2, we can multiply both sides of Equations (18) and (21) by $(\overline{\mathbf{v}}_g^1)^\dagger$ and $(\overline{\mathbf{v}}_g^2)^\dagger$. Then we have

$$(\overline{\mathbf{v}}_g^1)^\dagger \begin{pmatrix} y_1^1 \\ y_2^1 \end{pmatrix} = \sqrt{E_s}(\overline{\mathbf{v}}_g^1)^\dagger \begin{pmatrix} \widehat{h}_{11}^1 & \widehat{h}_{11}^1 \\ \widehat{h}_{21}^1 & \widehat{h}_{21}^1 \end{pmatrix} \begin{pmatrix} \widetilde{c}_1 \\ \widetilde{c}_2 \end{pmatrix} + (\overline{\mathbf{v}}_g^1)^\dagger \begin{pmatrix} n_1^1 \\ n_2^1 \end{pmatrix}, \tag{25}$$

$$(\overline{\mathbf{v}}_g^2)^\dagger \begin{pmatrix} (y_1^2)^* \\ (y_2^2)^* \end{pmatrix} = \sqrt{E_s}(\overline{\mathbf{v}}_g^2)^\dagger \begin{pmatrix} (\widehat{h}_{12}^2)^* & -(\widehat{h}_{12}^2)^* \\ (\widehat{h}_{22}^2)^* & -(\widehat{h}_{22}^2)^* \end{pmatrix} \begin{pmatrix} \widetilde{c}_1 \\ \widetilde{c}_2 \end{pmatrix} + (\overline{\mathbf{v}}_g^2)^\dagger \begin{pmatrix} (n_1^2)^* \\ (n_2^2)^* \end{pmatrix}. \tag{26}$$

Now we have removed the signals from User 2. So there is no interference for User 1. The elements of the noise vector $(\overline{\mathbf{v}}_g^1)^\dagger \begin{pmatrix} n_1^1 \\ n_2^1 \end{pmatrix}$ are correlated with covariance $|\overline{\mathbf{v}}_g^1|^2$ and the elements of the noise vector $(\overline{\mathbf{v}}_g^2)^\dagger \begin{pmatrix} (n_1^2)^* \\ (n_2^2)^* \end{pmatrix}$ are correlated with $|\overline{\mathbf{v}}_g^2|^2$. In order to detect the signals of User 1, we need to whiten the noise by multiplying both sides of Equations (25) and (26) by $|\overline{\mathbf{v}}_g^1|^{-1}$ and $|\overline{\mathbf{v}}_g^2|^{-1}$, as follows

$$\frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\begin{pmatrix} y_1^1 \\ y_2^1 \end{pmatrix} = \sqrt{E_s}\frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\begin{pmatrix} \widehat{h}_{11}^1 & \widehat{h}_{11}^1 \\ \widehat{h}_{21}^1 & \widehat{h}_{21}^1 \end{pmatrix}\begin{pmatrix} \widetilde{c}_1 \\ \widetilde{c}_2 \end{pmatrix} + \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\begin{pmatrix} n_1^1 \\ n_2^1 \end{pmatrix}, \tag{27}$$

$$\frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\begin{pmatrix} (y_1^2)^* \\ (y_2^2)^* \end{pmatrix} = \sqrt{E_s}\frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\begin{pmatrix} (\widehat{h}_{12}^2)^* & -(\widehat{h}_{12}^2)^* \\ (\widehat{h}_{22}^2)^* & -(\widehat{h}_{22}^2)^* \end{pmatrix}\begin{pmatrix} \widetilde{c}_1 \\ \widetilde{c}_2 \end{pmatrix} + \frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\begin{pmatrix} (n_1^2)^* \\ (n_2^2)^* \end{pmatrix}. \tag{28}$$

Using the notation in (22), (23) and combining Equations (27) and (28), we have

$$\begin{pmatrix} \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\overline{\mathbf{y}}^1 \\ \frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\overline{\mathbf{y}}^2 \end{pmatrix} = \sqrt{E_s}\begin{pmatrix} \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1 & \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1 \\ \frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{v}_h^2 & -\frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{v}_h^2 \end{pmatrix}\begin{pmatrix} \widetilde{c}_1 \\ \widetilde{c}_2 \end{pmatrix} + \begin{pmatrix} \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{n}^1 \\ \frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{n}^2 \end{pmatrix}. \tag{29}$$

We let $\widehat{\mathbf{H}}$ denote the equivalent channel matrix in (29) to simplify the presentation as follows

$$\widehat{\mathbf{H}} = \begin{pmatrix} \widehat{h}_{11} & \widehat{h}_{12} \\ \widehat{h}_{21} & \widehat{h}_{22} \end{pmatrix} = \begin{pmatrix} \widehat{h}_{11} & \widehat{h}_{11} \\ \widehat{h}_{21} & -\widehat{h}_{21} \end{pmatrix} = \begin{pmatrix} \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1 & \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1 \\ \frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{v}_h^2 & -\frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{v}_h^2 \end{pmatrix}. \tag{30}$$

Note that $\widehat{\mathbf{H}}$ has the following Single Value Decomposition [18]

$$\widehat{\mathbf{H}} = \mathbf{U}_{\widehat{\mathbf{H}}}\mathbf{\Sigma}_{\widehat{\mathbf{H}}}\mathbf{V}_{\widehat{\mathbf{H}}} = \mathbf{U}_{\widehat{\mathbf{H}}}\mathbf{\Sigma}_{\widehat{\mathbf{H}}}\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix} \tag{31}$$

where $\mathbf{U}_{\widehat{\mathbf{H}}}$ is a complex matrix and $\mathbf{\Sigma}_{\widehat{\mathbf{H}}}$, $\mathbf{V}_{\widehat{\mathbf{H}}}$ are all real matrices. Then we can multiply both sides of Equation (29) by $\mathbf{U}_{\widehat{\mathbf{H}}}^\dagger$ as follows

$$\mathbf{U}_{\widehat{\mathbf{H}}}^\dagger\begin{pmatrix} \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\overline{\mathbf{y}}^1 \\ \frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\overline{\mathbf{y}}^2 \end{pmatrix} = \sqrt{E_s}\mathbf{U}_{\widehat{\mathbf{H}}}^\dagger\begin{pmatrix} \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1 & \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1 \\ \frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{v}_h^2 & -\frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{v}_h^2 \end{pmatrix}\begin{pmatrix} \widetilde{c}_1 \\ \widetilde{c}_2 \end{pmatrix} + \mathbf{U}_{\widehat{\mathbf{H}}}^\dagger\begin{pmatrix} \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{n}^1 \\ \frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{n}^2 \end{pmatrix}. \tag{32}$$

In the above equation, $\mathbf{U}_{\widehat{\mathbf{H}}}^\dagger\begin{pmatrix} \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{n}^1 \\ \frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{n}^2 \end{pmatrix}$ is still white noise and $\mathbf{U}_{\widehat{\mathbf{H}}}^\dagger\begin{pmatrix} \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1 & \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1 \\ \frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{v}_h^2 & -\frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{v}_h^2 \end{pmatrix}$ is real matrix. So if QAM is used, then we have

$$\text{Real}\left\{\mathbf{U}_{\widehat{\mathbf{H}}}^\dagger\begin{pmatrix} \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\overline{\mathbf{y}}^1 \\ \frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\overline{\mathbf{y}}^2 \end{pmatrix}\right\} = \sqrt{E_s}\mathbf{U}_{\widehat{\mathbf{H}}}^\dagger\begin{pmatrix} \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1 & \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1 \\ \frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{v}_h^2 & -\frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{v}_h^2 \end{pmatrix}\text{Real}\left\{\begin{pmatrix} \widetilde{c}_1 \\ \widetilde{c}_2 \end{pmatrix}\right\} + \text{Real}\left\{\mathbf{U}_{\widehat{\mathbf{H}}}^\dagger\begin{pmatrix} \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{n}^1 \\ \frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{n}^2 \end{pmatrix}\right\}, \tag{33}$$

$$\text{Imag}\left\{\mathbf{U}_{\widehat{\mathbf{H}}}^\dagger\begin{pmatrix} \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\overline{\mathbf{y}}^1 \\ \frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\overline{\mathbf{y}}^2 \end{pmatrix}\right\} = \sqrt{E_s}\mathbf{U}_{\widehat{\mathbf{H}}}^\dagger\begin{pmatrix} \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1 & \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1 \\ \frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{v}_h^2 & -\frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{v}_h^2 \end{pmatrix}\text{Imag}\left\{\begin{pmatrix} \widetilde{c}_1 \\ \widetilde{c}_2 \end{pmatrix}\right\} + \text{Imag}\left\{\mathbf{U}_{\widehat{\mathbf{H}}}^\dagger\begin{pmatrix} \frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{n}^1 \\ \frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{n}^2 \end{pmatrix}\right\}. \tag{34}$$

Therefore, we can use the Maximum-Likelihood method to decode the real parts and imaginary parts of $\widetilde{c}_1$, $\widetilde{c}_2$ separately. For example, when we detect the real parts of $\widetilde{c}_1$, $\widetilde{c}_2$, we have

$$\text{Real}\{\widehat{c}_1, \widehat{c}_2\} = \arg \min_{\text{Real}\{c_1, c_2\}} \left\| \text{Real}\left\{ \mathbf{U}_{\widehat{\mathbf{H}}}^{\dagger} \begin{pmatrix} \frac{(\overline{\mathbf{v}}_g^1)^{\dagger}}{|\overline{\mathbf{v}}_g^1|} \overline{\mathbf{y}}^1 \\ \frac{(\overline{\mathbf{v}}_g^2)^{\dagger}}{|\overline{\mathbf{v}}_g^2|} \overline{\mathbf{y}}^2 \end{pmatrix} \right\} - \sqrt{E_s} \mathbf{U}_{\widehat{\mathbf{H}}}^{\dagger} \begin{pmatrix} \frac{(\overline{\mathbf{v}}_g^1)^{\dagger}}{|\overline{\mathbf{v}}_g^1|} \mathbf{v}_h^1 & \frac{(\overline{\mathbf{v}}_g^1)^{\dagger}}{|\overline{\mathbf{v}}_g^1|} \mathbf{v}_h^1 \\ \frac{(\overline{\mathbf{v}}_g^2)^{\dagger}}{|\overline{\mathbf{v}}_g^2|} \mathbf{v}_h^2 & -\frac{(\overline{\mathbf{v}}_g^2)^{\dagger}}{|\overline{\mathbf{v}}_g^2|} \mathbf{v}_h^2 \end{pmatrix} \text{Real}\left\{ \begin{pmatrix} \widetilde{c}_1 \\ \widetilde{c}_2 \end{pmatrix} \right\} \right\|_F^2. \tag{35}$$

Similarly, we can decode the imaginary parts of $\widetilde{c}_1$, $\widetilde{c}_2$, and the signals of User 2. Note that the decoding complexity is symbol-by-symbol.

Till now, we have presented our precoding, decoding methods, and some necessary properties needed by our codebooks to cancel interference for each user. Note that in order to achieve interference cancellation, the only properties needed by our codebooks are (16) and (20). The remaining degrees of freedom will be used to maximize diversity and coding gain as discussed in the next two sections.

IV. Feedback Design and Diversity Analysis

In this section, we first propose our feedback scheme, i.e., how to choose an index l_i and send it back to User i. Then we prove that our feedback scheme can achieve full diversity when our codebooks satisfy some conditions.

A. Feedback Design

First, as illustrated in Figure 2, we define

$$\cos\theta_{hg}^1 = <\mathbf{v}_h^1, \overline{\mathbf{v}}_g^1> = \frac{|(\overline{\mathbf{v}}_g^1)^{\dagger}\mathbf{v}_h^1|}{|\overline{\mathbf{v}}_g^1| \cdot |\mathbf{v}_h^1|}, \quad \cos\theta_{hg}^2 = <\mathbf{v}_h^2, \overline{\mathbf{v}}_g^2> = \frac{|(\overline{\mathbf{v}}_g^2)^{\dagger}\mathbf{v}_h^2|}{|\overline{\mathbf{v}}_g^2| \cdot |\mathbf{v}_h^2|}. \tag{36}$$

Note that the maximum value of $\cos\theta_{hg}^i$ is 1 and the corresponding $\theta_{hg}^i = 0$, which means $\mathbf{v}_h^i$ and $\mathbf{v}_g^i$ are orthogonal to each other.

Now we introduce our feedback scheme with the assumption that User 1 has already got a codebook $\boldsymbol{\Upsilon_1}$ in time slot 1 and a codebook $\boldsymbol{\Upsilon}_{\mathbf{1}}^{'}$ in time slot 2. Also User 2 has already got codebooks $\boldsymbol{\Upsilon_2}$ and $\boldsymbol{\Upsilon}_{\mathbf{2}}^{'}$ in time slots 1 and 2, respectively. All these codebooks should possess the property given by (16) and (20). In time slot 1, the receiver selects an index ℓ_1 within the range from 0 to $L_1 - 1$ and sends it back to User 1. The selection criterion is that with such an index ℓ_1, $|\mathbf{v}_h^1|$ is maximized, where $|\mathbf{v}_h^1| = |\mathbf{H}\mathbf{A}^1(1)|$ as given by (22) and $\mathbf{A}^1 = \boldsymbol{\Upsilon_1}[\ell_{\mathbf{1}}]$. Maximizing $|\mathbf{v}_h^1|$ is equivalent to maximizing the received SINR for User 1. Therefore, full diversity is also achieved, as shown later. At the same time slot, the receiver also picks an index ℓ_2 and sends it back to User 2. The selection criterion is that with such an index ℓ_2, θ_{hg}^1 is minimized, where θ_{hg}^1 is given by (36) in which $\overline{\mathbf{v}}_g^1 = \begin{pmatrix} -g_{21}^* & -g_{22}^* \\ g_{11}^* & g_{12}^* \end{pmatrix} \mathbf{B}^1(1)^*$ as given by (24), $\mathbf{B}^1 = \boldsymbol{\Upsilon_2}[\ell_{\mathbf{2}}]$. We will show that by doing so, we can also maximize coding gain within our system framework.

Similarly, in time slot 2, the receiver finds an index ℓ_2' and sends it back to User 2. The selection criterion is that with such an index ℓ_2', $|\mathbf{v}_g^2|$ is maximized. The receiver also finds an index ℓ_1' and sends it back to User 1. The selection criterion is that with such an index ℓ_1', θ_{hg}^2 is minimized.

B. Diversity Analysis

In what follows, we show that by the above proposed scheme, the diversity for each user is full as long as our codebooks satisfy some conditions. The diversity is defined as

$$d = -\lim_{\rho\to\infty} \frac{\log P_e}{\log \rho} \tag{37}$$

where ρ denotes the SNR and P_e represents the probability of error. We first consider Equation (29) to analyze the diversity for User 1. We know $\begin{pmatrix}\widetilde{c}_1\\ \widetilde{c}_2\end{pmatrix} = \mathbf{R}_1 \left(\begin{smallmatrix}c_1\\ c_2\end{smallmatrix}\right)$ and we define the error matrix $\boldsymbol{\epsilon} = \left(\begin{smallmatrix}c_1\\ c_2\end{smallmatrix}\right) - \begin{pmatrix}\widehat{c}_1\\ \widehat{c}_2\end{pmatrix}$. By (29), the pairwise error probability (PEP) can be given by the Gaussian tail function as [19]

$$P(\mathbf{d}\to\overline{\mathbf{d}}|\widehat{\mathbf{H}}) = Q\left(\sqrt{\frac{\rho||\widehat{\mathbf{H}}\mathbf{R}_1\boldsymbol{\epsilon}||_F^2}{4}}\right) = Q\left(\sqrt{\frac{\rho\boldsymbol{\epsilon}^\dagger\mathbf{R}_1^\dagger(\widehat{\mathbf{H}})^\dagger\widehat{\mathbf{H}}\mathbf{R}_1\boldsymbol{\epsilon}}{4}}\right) \le \exp\left(-\frac{\rho\boldsymbol{\epsilon}^\dagger\mathbf{R}_1^\dagger(\widehat{\mathbf{H}})^\dagger\widehat{\mathbf{H}}\mathbf{R}_1\boldsymbol{\epsilon}}{8}\right) \tag{38}$$

where we have used the inequality $Q(x) \le \exp(-\frac{x^2}{2})$. Now we assume $\mathbf{R}_1\boldsymbol{\epsilon} = \left(\begin{smallmatrix}\gamma_1\\ \gamma_2\end{smallmatrix}\right)$. Substituting $\mathbf{R}_1\boldsymbol{\epsilon}$ and $\widehat{\mathbf{H}}$ from Equation (30) in (38), we have

$$\begin{aligned} P(\mathbf{d}\to\overline{\mathbf{d}}|\widehat{\mathbf{H}}) &\le \exp\left(-\frac{\rho(|\widehat{h}_{11}|^2|\gamma_1+\gamma_2|^2 + |\widehat{h}_{21}|^2|\gamma_1-\gamma_2|^2)}{8}\right) \\ &= \exp\left(-\frac{\rho(\left|\frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1\right|^2|\gamma_1+\gamma_2|^2 + \left|\frac{(\overline{\mathbf{v}}_g^2)^\dagger}{|\overline{\mathbf{v}}_g^2|}\mathbf{v}_h^2\right|^2|\gamma_1-\gamma_2|^2)}{8}\right) \\ &\le \exp\left(-\frac{\rho(\left|\frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1\right|^2|\gamma_1+\gamma_2|^2)}{8}\right). \end{aligned} \tag{39}$$

Let us define

$$\Delta = \left|\frac{(\overline{\mathbf{v}}_g^1)^\dagger}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1\right|^2 = \frac{|(\overline{\mathbf{v}}_g^1)^\dagger\mathbf{v}_h^1|^2}{|\overline{\mathbf{v}}_g^1|^2}. \tag{40}$$

Using (36), we can rewrite Δ as

$$\Delta = |\cos\theta_{hg}^1|^2 \cdot |\mathbf{v}_h^1|^2. \tag{41}$$

Substituting (41) in (39), we have

$$P(\mathbf{d} \to \overline{\mathbf{d}}|\widehat{\mathbf{H}}) \leq \exp\left(-\frac{\rho(|\cos\theta_{hg}^1|^2 \cdot |\mathbf{v}_h^1|^2|\gamma_1+\gamma_2|^2)}{8}\right). \tag{42}$$

Since we choose our precoder $\mathbf{A}^1$ from the codebook $\mathbf{\Upsilon_1}$ such that $|\mathbf{v}_h^1|^2$ is maximized, it is easy to see

$$|\mathbf{v}_h^1|^2 = |\mathbf{H}\mathbf{A}^1(1)|^2 \geq \frac{|\mathbf{H}\overline{\mathbf{\Upsilon}}_\mathbf{1}|^2}{L} \tag{43}$$

where $\overline{\mathbf{\Upsilon}}_\mathbf{1}$ is a matrix satisfying $\overline{\mathbf{\Upsilon}}_\mathbf{1}(\mathbf{i}) = \mathbf{\Upsilon_1}[\mathbf{i}](\mathbf{1})$, $i = 1, \ldots, L$, i.e., the ith column of matrix $\overline{\mathbf{\Upsilon}}_\mathbf{1}$ is the same as the first column of the ith matrix in the codebook $\mathbf{\Upsilon_1}$. We assume $\overline{\mathbf{\Upsilon}}_\mathbf{1}$ has the following Singular Value Decomposition

$$\overline{\mathbf{\Upsilon}}_\mathbf{1} = \mathbf{U}_{\overline{\mathbf{\Upsilon}}_1}\mathbf{\Sigma}_{\overline{\mathbf{\Upsilon}}_1}\mathbf{V}^\dagger_{\overline{\mathbf{\Upsilon}}_1} = \mathbf{U}_{\overline{\mathbf{\Upsilon}}_1}\begin{pmatrix} \lambda_1^{\overline{\mathbf{\Upsilon}}_1} & 0 \\ 0 & \lambda_2^{\overline{\mathbf{\Upsilon}}_1}\end{pmatrix}\mathbf{V}^\dagger_{\overline{\mathbf{\Upsilon}}_1}. \tag{44}$$

Then (43) becomes

$$|\mathbf{v}_h^1|^2 \geq \frac{|\mathbf{H}\mathbf{U}_{\overline{\mathbf{\Upsilon}}_1}\mathbf{\Sigma}_{\overline{\mathbf{\Upsilon}}_1}\mathbf{V}^\dagger_{\overline{\mathbf{\Upsilon}}_1}|^2}{L} = \frac{|\lambda_1^{\overline{\mathbf{\Upsilon}}_1}|^2(|h'_{11}|^2+|h'_{21}|^2)+|\lambda_2^{\overline{\mathbf{\Upsilon}}_1}|^2(|h'_{12}|^2+|h'_{22}|^2)}{L} \tag{45}$$

where

$$\mathbf{H}\mathbf{U}_{\overline{\mathbf{\Upsilon}}_1} = \begin{pmatrix} h'_{11} & h'_{12} \\ h'_{21} & h'_{22}\end{pmatrix}. \tag{46}$$

Since the unitary matrix $\mathbf{U}_{\overline{\mathbf{\Upsilon}}_1}$ does not change the distribution of $\mathbf{H}$, each element of $\mathbf{H}\mathbf{U}_{\overline{\mathbf{\Upsilon}}_1}$, i.e., h'_{ij}, is also a Gaussian distributed random variable with mean 0 and variance 1. As a result, (42) can be written as

$$P(\mathbf{d} \to \overline{\mathbf{d}}|\widehat{\mathbf{H}}) \leq \exp\left(-\frac{\rho(|\cos\theta_{hg}^1|^2 \cdot (|\lambda_1^{\overline{\mathbf{\Upsilon}}_1}|^2(|h'_{11}|^2+|h'_{21}|^2)+|\lambda_2^{\overline{\mathbf{\Upsilon}}_1}|^2(|h'_{12}|^2+|h'_{22}|^2))\cdot|\gamma_1+\gamma_2|^2)}{4L}\right). \tag{47}$$

Further, we have

$$P(\mathbf{d} \to \overline{\mathbf{d}}) \leq E\left[\exp\left(-\frac{\rho(|\cos\theta_{hg}^1|^2 \cdot (|\lambda_1^{\overline{\mathbf{\Upsilon}}_1}|^2(|h'_{11}|^2+|h'_{21}|^2)+|\lambda_2^{\overline{\mathbf{\Upsilon}}_1}|^2(|h'_{12}|^2+|h'_{22}|^2))\cdot|\gamma_1+\gamma_2|^2)}{4L}\right)\right]$$

$$= E\left[E\left[\exp\left(-\frac{\rho(|\cos\theta_{hg}^1|^2 \cdot (|\lambda_1^{\overline{\mathbf{\Upsilon}}_1}|^2(|h'_{11}|^2+|h'_{21}|^2)+|\lambda_2^{\overline{\mathbf{\Upsilon}}_1}|^2(|h'_{12}|^2+|h'_{22}|^2))\cdot|\gamma_1+\gamma_2|^2)}{4L}\right)\middle|\theta_{hg}^1\right]\right]$$

$$\leq E\left[\frac{1}{\prod_{j=1}^{2}[1+(\rho|\cos\theta_{hg}^{1}|^2|\lambda_j^{\overline{\mathbf{\Upsilon}}_1}|^2|\gamma_1+\gamma_2|^2/8L)]^2}\right]. \tag{48}$$

At high SNRs, one can neglect the one in the denominator and get

$$P(\mathbf{d}\to\overline{\mathbf{d}})\leq\left(\frac{\rho}{8L}\right)^{-4}\prod_{j=1}^{2}(|\lambda_j^{\overline{\mathbf{\Upsilon}}_1}|\cdot|\gamma_1+\gamma_2|)^{-4}E\left[\frac{1}{|\cos\theta_{hg}^{1}|^8}\right]. \tag{49}$$

From (49), it is easy to see the diversity for User 1 is 4, full diversity, as long as $\lambda_j^{\overline{\mathbf{\Upsilon}}_1}\neq 0$. Note that matrix $\overline{\mathbf{\Upsilon}}_\mathbf{1}$ is a 2-by-L matrix, where L is the number of matrices in codebook $\mathbf{\Upsilon_1}$. So in order to make $\lambda_j^{\overline{\mathbf{\Upsilon}}_1}\neq 0$, we need

1) $L\geq 2$, where L is the number of matrices in our codebook.
2) The rank of matrix $\overline{\mathbf{\Upsilon}}_\mathbf{1}$ is 2.

Condition 1 requires that $K\geq 1$, where K is the number of feedback bits available to each user. Condition 2 is a constraint we need to design our codebook $\mathbf{\Upsilon_1}$. There is no other constraint on the codebook $\mathbf{\Upsilon_1}$ in order to achieve full diversity. In time slot 1, there is no further requirement on Codebook $\mathbf{\Upsilon_2}$ for User 2 other than (16). In time slot 2, by a similar proof, the codebook $\mathbf{\Upsilon}_\mathbf{2}^{'}$ for User 2 should satisfy the above two conditions and the only requirement on Codebook $\mathbf{\Upsilon}_\mathbf{1}^{'}$ for User 1 is (20). Similarly, we can prove that the diversity for User 2 is also full.

V. Coding Gain Analysis and Codebook Design

In the last two sections, we have presented some properties needed by our codebooks in order to achieve interference cancellation and full diversity. However, there are still some degrees of freedom in our codebook design. In this section, we use the remaining degrees of freedom to maximize the coding gain.

By (42), in order to maximize coding gain, we need to maximize $|\mathbf{v}_h^1|$ and $|\cos\theta_{hg}^1|$. We first analyze $\mathbf{v}_h^1$. Note that

$$\mathbf{v}_h^1=\mathbf{H}\mathbf{A}^1(1). \tag{50}$$

To maximize $|\mathbf{v}_h^1|$, the best choice for $\mathbf{A}^1(1)$ is [20]

$$\mathbf{A}^1(1)=\frac{1}{\sqrt{2}}\mathbf{V_H}(1) \tag{51}$$

where $\mathbf{V_H}$ comes from the singular value decomposition

$$\mathbf{H}=\mathbf{U_H}\mathbf{\Sigma_H}\mathbf{V}_\mathbf{H}^{\dagger}=\mathbf{U_H}\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}\mathbf{V}_\mathbf{H}^{\dagger}. \tag{52}$$

$\mathbf{V_H}(1)$ is the singular vector of $\mathbf{H}$ corresponding to the largest singular value and we assume $\lambda_1 > \lambda_2$ without loss of generality. If we have perfect feedback, we can simply choose $\mathbf{A}^1(1) = \frac{1}{\sqrt{2}}\mathbf{V_H}(1)$ and the precoder $\mathbf{A}^1 = \frac{1}{\sqrt{2}}[\mathbf{V_H}(1), \mathbf{V_H}(1)]$. Since we only have access to quantized feedback, we should design a codebook in which we can find a matrix whose column is the best approximation to $\frac{1}{\sqrt{2}}\mathbf{V_H}(1)$.

It has been shown in [21] that $\mathbf{V_H}(1)$ is an isotropically distributed unitary vector. The intuitive meaning of an isotropically distributed complex unit vector is that it is equally likely to point in any direction in complex space. Therefore, the problem to design a codebook to maximize $|\mathbf{v}_h^1|$ becomes how to pack one-dimensional subspaces of a complex space known as Grassmannian line packing [22]. In other words, it is the problem of finding a set of L_1 one-dimensional subspaces in the complex space that maximize the minimum distance between any pair of subspaces in the set.

The problem of finding optimal line packings using analytical or numerical methods is not new [22]–[25]. We utilize the existing methodologies in the literature to design a codebook for User 1 in time slot 1.

Now we summarize the procedures to construct our codebook for User 1 in time slot 1:

1) For K bits of feedback, find $L_1 = 2^K$ two-by-one unit norm complex vectors which can maximize the minimum distance between any pair of vectors in the two-dimensional complex space. We denote all these vectors as $\psi_{\mathbf{i}}$, $i = 1, \ldots, L_1$.
2) Create a codebook $\boldsymbol{\Upsilon}_1$ that contains $L_1 = 2^K$ matrices satisfying $\boldsymbol{\Upsilon}_1[i] = \frac{1}{\sqrt{2}}[\psi_{\mathbf{i}}, \psi_{\mathbf{i}}]$.

It is easy to check that the created codebook satisfies all the conditions we need. Therefore, $|\mathbf{v}_h^1|$ can be maximized if User 1 adopts the above codebook.

In what follows, we will show that if User 2 adopts the above codebook, $|\cos\theta_{hg}^1|$ will also be maximized. By (36), we know that once $|\mathbf{v}_h^1|$ and $|\cos\theta_{hg}^1|$ are maximized at the same time, the coding gain will be maximized. Therefore, the above codebook is the optimal codebook that both User 1 and User 2 should adopt in time slot 1.

First, note that in order to maximize $|\cos\theta_{hg}^1|$, by (36), we need $\bar{\mathbf{v}}_g^1 = \eta\mathbf{v}_h^1$, i.e.,

$$\begin{pmatrix} -(\widehat{g}_{21}^1)^* \\ (\widehat{g}_{11}^1)^* \end{pmatrix} = \eta \begin{pmatrix} \widehat{h}_{11}^1 \\ \widehat{h}_{21}^1 \end{pmatrix} \text{ or } \quad \eta \begin{pmatrix} (\widehat{h}_{21}^1)^* \\ -(\widehat{h}_{11}^1)^* \end{pmatrix} = \begin{pmatrix} \widehat{g}_{11}^1 \\ \widehat{g}_{21}^1 \end{pmatrix} \tag{53}$$

where η is a constant. Further, we have

$$\eta \begin{pmatrix} (\widehat{h}_{21}^1)^* \\ -(\widehat{h}_{11}^1)^* \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} b_{11}^1 \\ b_{21}^1 \end{pmatrix} \text{ or } \quad \begin{pmatrix} b_{11}^1 \\ b_{21}^1 \end{pmatrix} = \eta \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}^{-1} \begin{pmatrix} (\widehat{h}_{21}^1)^* \\ -(\widehat{h}_{11}^1)^* \end{pmatrix}. \tag{54}$$

Since the norm of $\begin{pmatrix} b_{11}^1 \\ b_{21}^1 \end{pmatrix}$ is 1, we have

$$\begin{pmatrix} b_{11}^1 \\ b_{21}^1 \end{pmatrix} = \frac{\begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}^{-1} \begin{pmatrix} (\widehat{h}_{21}^1)^* \\ -(\widehat{h}_{11}^1)^* \end{pmatrix}}{\left\| \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}^{-1} \begin{pmatrix} (\widehat{h}_{21}^1)^* \\ -(\widehat{h}_{11}^1)^* \end{pmatrix} \right\|_F}. \tag{55}$$

So we know that in order to maximize $|\cos\theta_{hg}^1|$, we can choose $\begin{pmatrix} b_{11}^1 \\ b_{21}^1 \end{pmatrix}$ as described by (55) if we have perfect feedback. Since we only have quantized feedback, we should design a codebook in which we can find a vector as close to the one described by (55) as possible. So, first, we need to determine the distribution of the optimal $\begin{pmatrix} b_{11}^1 \\ b_{21}^1 \end{pmatrix}$ in (55). Note that Equation (55) can also be written as

$$\begin{pmatrix} b_{11}^1 \\ b_{21}^1 \end{pmatrix} = \eta' \begin{pmatrix} g_{22} & -g_{12} \\ -g_{21} & g_{11} \end{pmatrix} \begin{pmatrix} (\widehat{h}_{21}^1)^* \\ -(\widehat{h}_{11}^1)^* \end{pmatrix} = \eta' \begin{pmatrix} g_{22}(\widehat{h}_{21}^1)^* + g_{12}(\widehat{h}_{11}^1)^* \\ -g_{21}(\widehat{h}_{21}^1)^* - g_{11}(\widehat{h}_{11}^1)^* \end{pmatrix} = \eta' \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \tag{56}$$

where $\eta' = \left\| \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}^{-1} \begin{pmatrix} (\widehat{h}_{21}^1)^* \\ -(\widehat{h}_{11}^1)^* \end{pmatrix} \right\|_F^{-1} |g_{11}g_{22} - g_{21}g_{12}|^{-1}$. Let us assume that the singular value decomposition of $\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$ is

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \mathbf{U}_\alpha \mathbf{\Sigma}_\alpha \mathbf{V}_\alpha^\dagger = \mathbf{U}_\alpha \begin{pmatrix} \lambda_1^\alpha \\ 0 \end{pmatrix} \cdot \mathbf{1} = \lambda_{\mathbf{1}}^\alpha \cdot \mathbf{U}_\alpha(\mathbf{1}). \tag{57}$$

Since $\widehat{h}_{11}^1$ and $\widehat{h}_{21}^1$ are independent from $\mathbf{G}$, conditioned on $\widehat{h}_{11}^1$ and $\widehat{h}_{21}^1$, elements of $\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$ are all Guassian distributed random variables with the same mean and variance, so any column of $\mathbf{U}_\alpha$ and thus $\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$ will be an isotropically distributed unitary vector [21]. Further, we can conclude that $\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$ and thus $\begin{pmatrix} b_{11}^1 \\ b_{21}^1 \end{pmatrix}$ are all isotropically distributed unitary vectors.

Therefore, in order to maximize $|\cos\theta_{hg}^1|$, the codebook for User 2 should provide the best approximation to any isotropically distributed unitary vector and the problem becomes exactly the same as the one we discussed before, i.e., to pack one-dimensional subspaces of a complex space known as Grassmannian line packing. Therefore, the resulting codebook for User 2 will be the same as the codebook $\mathbf{\Upsilon}_1$ for User 1 at time slot 1.

So far, we have shown that by using our codebook, we can maximize $|\mathbf{v}_h^1|$ and $|\cos\theta_{hg}^1|$ at the same time. From (42), it is easy to see that the coding gain is maximized.

Similarly, we can prove that in time slot 2, both User 1 and User 2 should adopt the above codebook.

VI. Comparison of Our Scheme with Two Existing Schemes

In this section, we compare our scheme with two other schemes proposed in the literature. The first scheme is the interference cancellation scheme without feedback proposed in [5], [7]. With the same system model, this scheme can provide a diversity of 2. The second scheme is the interference cancellation scheme with perfect feedback proposed in [8]. With the same system model, this scheme can provide a diversity of 4, i.e., full diversity. We show that our scheme can also provide a diversity of 2 with no feedback. With perfect feedback, our scheme provides the performance of the scheme in [8].

First, let us consider the case without feedback. When the number of feedback bits $K = 0$, we can not choose the best precoders according to the feedback. So our precoders are fixed: in time slot 1, both users use precoder $\binom{1}{0}$ and in time slot 2, both users use precoder $\binom{0}{1}$.

By (39), we know

$$\begin{aligned} P(\mathbf{d} \to \overline{\mathbf{d}}) &\leq E\left[\exp\left(-\frac{\rho(\left|\frac{(\overline{\mathbf{v}}_g^1)^T}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1\right|^2|\gamma_1+\gamma_2|^2 + \left|\frac{(\overline{\mathbf{v}}_g^2)^T}{|\overline{\mathbf{v}}_g^2|}\mathbf{v}_h^2\right|^2|\gamma_1-\gamma_2|^2)}{4}\right)\right] \\ &= E\left[E\left[\exp\left(-\frac{\rho(\left|\frac{(\overline{\mathbf{v}}_g^1)^T}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1\right|^2|\gamma_1+\gamma_2|^2 + \left|\frac{(\overline{\mathbf{v}}_g^2)^T}{|\overline{\mathbf{v}}_g^2|}\mathbf{v}_h^2\right|^2|\gamma_1-\gamma_2|^2)}{4}\right)|\overline{\mathbf{v}}_g^1, \overline{\mathbf{v}}_g^2\right]\right]. \end{aligned} \quad (58)$$

Since $\mathbf{v}_h^1 = \begin{pmatrix} h_{11} \\ h_{21} \end{pmatrix}, \mathbf{v}_h^2 = \begin{pmatrix} h_{12} \\ h_{22} \end{pmatrix}$, if conditioned on $\overline{\mathbf{v}}_g^1, \overline{\mathbf{v}}_g^2$, both $\frac{(\overline{\mathbf{v}}_g^1)^T}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1$ and $\frac{(\overline{\mathbf{v}}_g^2)^T}{|\overline{\mathbf{v}}_g^2|}\mathbf{v}_h^2$ are linear combination of independent Gaussian random variables with mean 0 and variance 1. In addition, if conditioned on $\overline{\mathbf{v}}_g^1, \overline{\mathbf{v}}_g^2$, then $\frac{(\overline{\mathbf{v}}_g^1)^T}{|\overline{\mathbf{v}}_g^1|}\mathbf{v}_h^1$ and $\frac{(\overline{\mathbf{v}}_g^2)^T}{|\overline{\mathbf{v}}_g^2|}\mathbf{v}_h^2$ are independent. So we have

$$P(\mathbf{d} \to \overline{\mathbf{d}}) \leq \frac{1}{(1+\rho|\gamma_1+\gamma_2|^2/8)(1+\rho|\gamma_1-\gamma_2|^2/8)}. \quad (59)$$

At high SNRs, one can neglect the one in the denominator and get

$$P(\mathbf{d} \to \overline{\mathbf{d}}) \leq (\frac{\rho}{8})^{-2}\frac{1}{|\gamma_1+\gamma_2|^2|\gamma_1-\gamma_2|^2}. \quad (60)$$

It is easy to see that the achievable diversity is 2, which is exactly the same as that of the scheme proposed in [5].

Now we consider the case with perfect feedback. Since the diversity for any $K > 0$ is always 4, obviously, in the case of $K = \infty$, perfect feedback, the diversity of our scheme is the same as that of the scheme proposed in [8].

When there are K bits of feedback, the performance of our system is given by (49). We know that as long as the number of feedback bits $K > 0$, our scheme can provide full diversity. Also with the increase of K, the interference term $E\left[\frac{1}{|\cos\theta_{hg}^1|^8}\right]$ decreases to 1. Therefore, the coding gain and the performance of our scheme will approach those of the system with perfect feedback.

VII. Extension to Any Number of Antennas

In this section, we show that our scheme can also be extended for 2 users with any number of antennas and one receiver with any number of antennas. We will consider two cases. The first one is the case in which the number of transmit antennas N is greater than or equal to the number of receive antennas M. The second one is the case in which $M > N$.

First, we assume $N \geq M$. Similar to the case in Section III, User 1 and User 2 transmit Alamouti codes $\mathbf{C}$ and $\mathbf{S}$, respectively. The channels for Users 1 and 2 are

$$\mathbf{H} = [h_{ij}]_{M\times N}, \quad \mathbf{G} = [g_{ij}]_{M\times N}. \tag{61}$$

The precoders for Users 1 and 2 are

$$\mathbf{A}^t = [a^t_{ij}]_{N\times 2}, \quad \mathbf{B}^t = [b^t_{ij}]_{N\times 2}. \tag{62}$$

Then we can use exactly the same method to design the codebook and precoders. However, when $N \geq M > 2$, with K bits of feedback, the diversity is $M \cdot \min(N, L)$, where $L = 2^K$ is the number of vectors in the codebook. To prove this, we note that in the case of $N \geq M > 2$, (43) becomes

$$|\mathbf{v}^1_h|^2 \geq \frac{|\mathbf{H}\boldsymbol{\Upsilon}_1|^2}{L} = \frac{|\mathbf{H}\mathbf{U}_{\boldsymbol{\Upsilon}_1}\Sigma_{\boldsymbol{\Upsilon}_1}\mathbf{V}^{\dagger}_{\boldsymbol{\Upsilon}_1}|^2}{L} = \frac{\sum_{j=1}^{L'}(|\lambda_j^{\boldsymbol{\Upsilon}_1}|^2\sum_{i=1}^{M}|h'_{ij}|^2)}{L} \tag{63}$$

where $L' = \min(N, L)$. It is easy to see that the number of Gaussian random variables on the right side of (63) is ML'. Therefore, when $N \geq M > 2$, following the proof presented in Section IV, the diversity of our scheme is $M \cdot \min(N, L)$ or $M \cdot \min(N, 2^K)$. In order to achieve a diversity of MN, we need

1) $L \geq N$, i.e., $K \geq \log_2 N$.
2) The rank of matrix $\overline{\boldsymbol{\Upsilon}}_1$ to be N.

Now we consider the case that $N < M$. In this case, we assume the channel matrices and precoders for Users 1 and 2 are given by (61) and (62). We can use the same method as discussed before to maximize $|\mathbf{v}^1_h|$. However, if we want to maximize $|\cos\theta^1_{hg}|^2$, like (53), we need to design precoders to make

$$\widetilde{\mathbf{H}}_{M\times 1} = \eta \cdot \mathbf{G}_{M\times N} \cdot \mathbf{B}^1(1)_{N\times 1} \tag{64}$$

which means the equivalent signal vectors of the two users are orthogonal to each other. In the above equation, we need to determine N unknown parameters by M equations. Since $N < M$, the number of equations is greater than the number of unknown parameters. Therefore, even with prefect feedback, we cannot find these unknown parameters to satisfy the equations. In other words, since we do not have enough dimensions for precoders, we cannot make $\mathbf{v}_g^i$ orthogonal to $\mathbf{v}_h^i$.

In order to make our proposed scheme extendable to the case of $M > N$, we can choose N receive antennas among all M receive antennas as follows:

In time slot 1, we can choose the N receive antennas such that $||\mathbf{H}_{\text{new}}||_F$ is maximized, where $\mathbf{H}_{\text{new}}$ is the new channel matrix with N transmit antennas and the selected N receive antennas. Once the number of receive antennas is equal to the number of transmit antennas, the same method used in Section IV can be used to determine the codebook and precoders for Users 1 and 2. At time slot 2, we choose the N receive antennas such that $||\mathbf{G}_{\text{new}}||_F$ is maximized, where $\mathbf{G}_{\text{new}}$ is the new channel matrix with N transmit antennas and the selected N receive antennas. Then we design the codebook and precoders for Users 1 and 2 using the same method in the case that $M = N$.

In order to show that we can achieve full diversity for each user using the above proposed method, we consider (43). By (43), we know

$$\begin{aligned} |\mathbf{v}_h^1|^2 &\geq \frac{|\mathbf{H}\boldsymbol{\Upsilon}_1|^2}{L} = \frac{|\mathbf{H}\mathbf{U}_{\boldsymbol{\Upsilon}_1}\boldsymbol{\Sigma}_{\boldsymbol{\Upsilon}_1}\mathbf{V}_{\boldsymbol{\Upsilon}_1}^{\dagger}|^2}{L} = \frac{\sum_{j=1}^{N}(|\lambda_j^{\boldsymbol{\Upsilon}_1}|^2\sum_{i=1}^{N}|h_{ij}^{'}|^2)}{L} \\ &\geq \frac{|\lambda_{\min}^{\boldsymbol{\Upsilon}_1}|^2\sum_{j=1}^{N}\sum_{i=1}^{N}|h_{ij}^{'}|^2}{L} = \frac{|\lambda_{\min}^{\boldsymbol{\Upsilon}_1}|^2\sum_{j=1}^{N}\sum_{i=1}^{N}|h_{ij}|^2}{L} = \frac{|\lambda_{\min}^{\boldsymbol{\Upsilon}_1}|^2||\mathbf{H}_{\text{new}}||_F^2}{L}. \end{aligned} \tag{65}$$

Since we know $||\mathbf{H}_{\text{new}}||_F^2$ is maximized, the average of the norms of all columns in matrix $\mathbf{H}_{\text{new}}$ will be no less than the average of the norms of all columns in matrix $\mathbf{H}$, i.e.,

$$\frac{||\mathbf{H}_{\text{new}}||_F^2}{N} = \frac{\sum_{i=1}^{N}|\mathbf{H}_{\text{new}}(i)|^2}{N} \geq \frac{||\mathbf{H}||_F^2}{M} = \frac{\sum_{i=1}^{M}|\mathbf{H}(i)|^2}{M}. \tag{66}$$

Substituting (66) to (65), we have

$$|\mathbf{v}_h^1|^2 \geq \frac{|\lambda_{\min}^{\boldsymbol{\Upsilon}_1}|^2||\mathbf{H}_{\text{new}}||_F^2}{L} \geq \frac{N|\lambda_{\min}^{\boldsymbol{\Upsilon}_1}|^2||\mathbf{H}||_F^2}{ML} = \frac{N|\lambda_{\min}^{\boldsymbol{\Upsilon}_1}|^2\sum_{j=1}^{N}\sum_{i=1}^{M}|h_{ij}|^2}{ML}. \tag{67}$$

Since there are MN Gaussian random variables on the right side of (67), it is easy to prove that User 1 can achieve a diversity of MN, i.e., full diversity. Similarly, it can be proved that User 2 can also achieve full diversity. When there are more than two users, there will be more interference to be dealt with. The precoding and decoding scheme will be more complex. Due to the limitation of the space, we leave the extension of the scheme to more than two users as our future work.

VIII. Simulation Results

In this section, we provide simulation results that confirm our analysis in the previous sections. We assume a quasi-static Rayleigh fading channel. The performance of our proposed scheme is shown in

Figures 3, 4 and 5. In each figure, the curves for Users 1 and 2 are identical. In Figure 3, we consider 2 users each equipped with 2 transmit antennas and a receiver with 2 receive antennas. We compare our results using QPSK with the results in [5] for the same configuration without channel information at the transmitter and the results in [8] for the same configuration with perfect feedback. Note that if the feedback is zero in our system (no channel information), we can pick an identity matrix as our precoder and our transmitter will be the same as the transmitter in [5]. In fact, this backward compatibility is the main reason for using an Alamouti code. Otherwise, our scheme also works for other full rate space time codes and all the above derivations are still valid.

In order to illustrate the effect of the number of bits, we provide the performance with 1, 3, 6, 8 bits feedback, respectively. It can be seen that with 2 receive antennas, the multi-user detection (MUD) method proposed in [5] can cancel the interference but only provides a diversity of 2. The scheme proposed in [8] with perfect feedback can achieve interference cancellation and provide a diversity of 4, full diversity. In comparison, using the proposed scheme in this paper, we can also achieve interference cancellation as well as full diversity only with quantized feedback, even with only 1 bit of feedback. But the performance highly depends on the number of feedback bits. When the number of feedback bits is small, the performance of our scheme is close to the performance of the scheme without feedback. When the number of feedback bits increases, the performance will approach the performance of the system with perfect feedback. Therefore, our proposed scheme provides a solution to fill the performance gap between [5] and [8]. Finally, we also provide the simulation results for the time-division multiplexing (TDM) case in which the two users transmit Alamouti codes in different time slots. In this case, there will be no interference at all. In order to match the rate, each user adopts 16-QAM. From the simulation results, we can see that although the TDM scheme can achieve full diversity and the decoding complexity is low, it will lose coding gain.

In Figure 4, we provide the performance of our scheme with 8 bits of feedback for 2 users each with 4 transmit antennas and one receiver with 2 receive antennas. Also we compare the performance of our scheme with the schemes in [5] and [8]. It is easy to see that our scheme with 8 bits of feedback has achieved full diversity and has outperformed the scheme in [5]. Compared with the scheme with perfect feedback, the performance difference is about 1 dB.

In Figure 5, we present the performance of our scheme with 8 bits of feedback for 2 users each with 2 transmit antennas and one receiver with 3 receive antennas. Once again, the performance of our scheme outperforms the performance of the scheme in [5] and approaches the performance of the scheme in [8]. Simulation results show that by using only a few bits of feedback, one can approach the performance of a system with perfect feedback.

IX. Conclusions

In this paper, we investigate how to cancel the interference and achieve full diversity for two users with two transmit antennas and one receiver with two receive antennas in a multiple access channel using quantized feedback. Using quantized feedback, we propose the precoding and decoding method, the feedback scheme and the codebook design to cancel interference and achieve full diversity. Also we show that the performance of our proposed scheme is determined by the number of feedback bits. With the increase of the feedback bits, the performance of our scheme approaches that of the system with perfect feedback. Finally we extend our scheme to two users with any number of transmit antennas and one receiver with any number of receive antennas. Simulation results are provided to confirm our analytical results.

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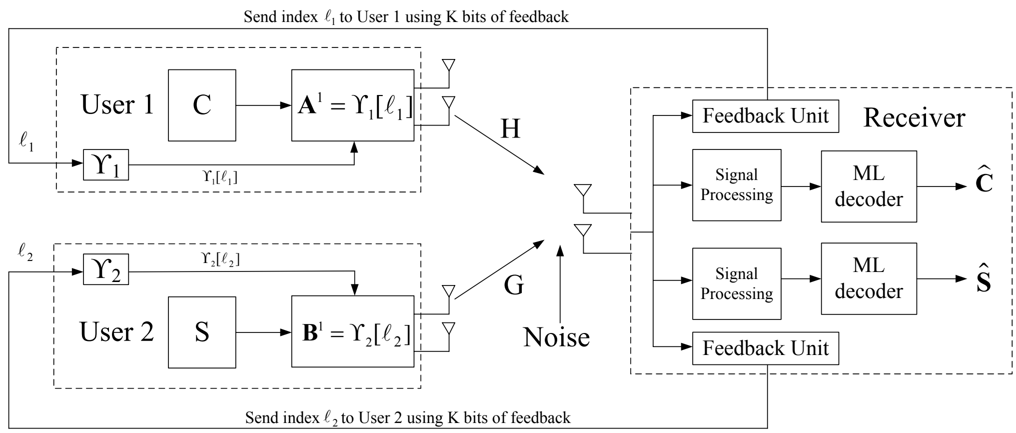

Fig. 1. Block Diagram in time slot 1

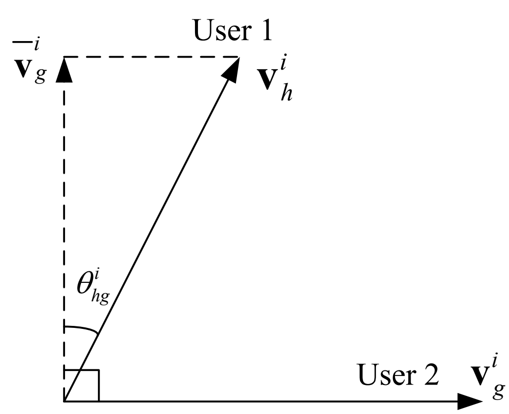

Fig. 2. Precoder design illustration

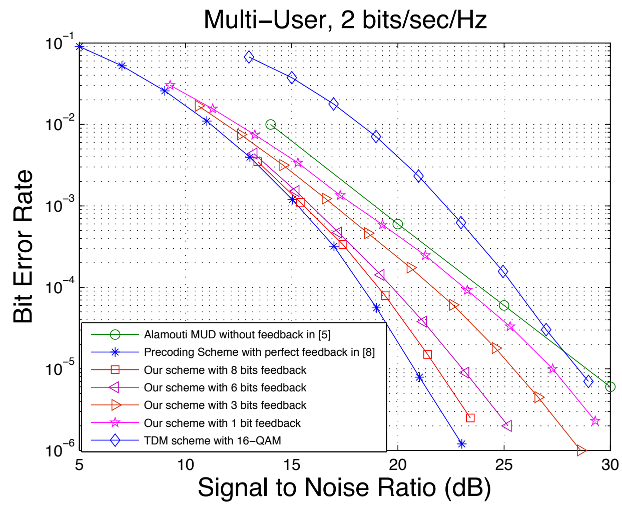

Fig. 3. Comparison of our scheme, Alamouti MUD in [5] and Precoding scheme in [8] for 2 users each with 2 transmit antennas and 1 receiver with 2 receive antennas

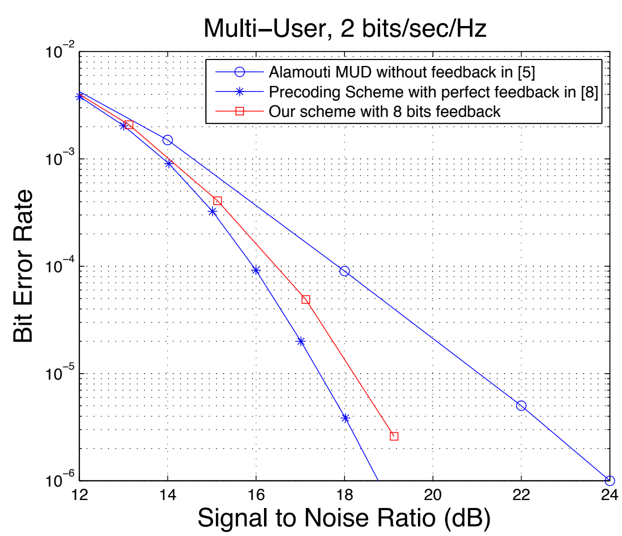

Fig. 4. Comparison of our scheme, Alamouti MUD in [5] and Precoding scheme in [8] for 2 users each with 4 transmit antennas and 1 receiver with 2 receive antennas

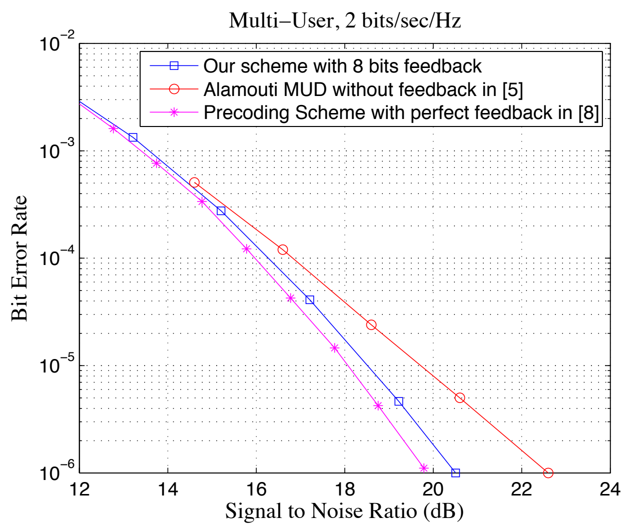

Fig. 5. Comparison of our scheme, Alamouti MUD in [5] and Precoding scheme in [8] for 2 users each with 2 transmit antennas and 1 receiver with 3 receive antennas