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TIME-DEPENDENT BEHAVIOR OF REINFORCED CONCRETE COLUMNS INCLUDING EFFECTS OF SHRINKAGE, CREEP AND CRACKING

by

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ABSTRACT

An analytical model was developed for predicting the time-dependent behavior of plain and reinforced concrete columns. The model was formulated to account for the effects of non-uniform drying shrinkage, creep and cracking in concrete.

To facilitate development of a model, the study was limited to unloaded or axially loaded square concrete columns. No eccentric loading or bending was considered.

The computer program which was developed for evaluating internal stresses and deformations of a column is based on a numerical method of analysis involving discretization of both cross-sectional geometry and time. The analytical results were found to be in reasonably good agreement with available experimental data.

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1. INTRODUCTION

1.1 Nature of Problem

The effects of drying shrinkage, creep and cracking must be given proper consideration in the design of reinforced concrete structures. These three factors, along with loading and environmental history, can significantly affect the redistribution of stresses with time in a concrete structure. The effects of shrinkage and creep become particularly significant in columns of high-rise buildings due to the cumulative nature of these time-dependent deformations. Differential inelastic shortening between adjacent columns may occur, producing moments in the connecting beams and slabs along with a subsequent transfer of load to the column that undergoes less shortening. In some cases, the time-dependent deformations of a concrete column caused by creep and shrinkage may be several times greater than the elastic deformations.

The mathematical model reported herein was developed to enable structural engineers to predict the stresses and deformations which will occur in a concrete column.

Shrinkage throughout the section of a concrete member is non-uniform, particularly in the case of large concrete members, although the distribution of shrinkage strains is often considered to be uniform [2]. The shrinkage diffusion model adopted in the analysis is based on a study by Pickett [3] who proposed a solution which considers the non-uniformity of shrinkage in concrete prisms. Pickett assumed, however, that concrete behaves only elastically and neglected the effects of creep and cracking. Because creep and cracking can have a significant bearing on the stresses and deformations resulting from non-uniform shrinkage, their effects must be included in developing an analytical model.

The stress-strain relationship for concrete used in the analysis is a mathematical model proposed by Selna [4,5], which was developed to fit laboratory data and therefore allows for the aging characteristics of concrete. The principle of superposition was used in conjunction with this stress-strain model in determining creep strains.

In the analysis, concrete was allowed to crack whenever the tensile stress in an element exceeded the tensile strength of concrete at that age. Cracking of concrete was considered to be an irreversible process, i.e., a cracked concrete element could not become uncracked, but it could carry compressive stress if the crack should close.

Because shrinkage, creep and cracking interact in a complex manner with changes in environmental conditions to produce stresses and deformations in concrete, it is difficult to analyze their combined effect. A thermodynamic model for determining concrete deformations at variable temperature and humidity has been proposed by Bazant [6], but was not adopted in the present analysis due to lack of experimental data and the complexity of the model. The model reported in this study has, therefore, been simplified by assuming that temperature and humidity are constant.

1.2 Objective

The objective of this investigation was to develop an analytical model which accounted for shrinkage, creep and cracking of concrete in determining the internal stresses and deformations of both unloaded and axially loaded reinforced concrete columns. The study was restricted to square columns having symmetrical reinforcement.

The results were compared with earlier studies which did not consider cracking of concrete. A study was also made to determine how the resultant column deformations were affected by varying the length of the time interval used in the analysis. Finally, to check the accuracy of the model, a comparison was made with experimental results.

2. FORMULATION OF ANALYTICAL MODEL

2.1 Method of Analysis

A numerical method of analysis was used for evaluating the time-dependent behavior of concrete members. The method is based on the discretization of both structural geometry and time, and accounts for the material behavior laws of reinforced concrete [7,8]. Thus, the cross-section of a prism was subdivided into elements of concrete and reinforcing steel, and the stresses and deformations of these elements were determined at the end of selected time intervals.

The analysis involved releasing all constraints on the elements for the duration of a particular time interval, and then reimposing these restraints at the end of that interval. During any time step, the elements were allowed to undergo free deformation due to shrinkage and creep, completely unrestrained by either the steel reinforcement or adjacent concrete elements. A 7-day time step was selected for use in the analysis, up to a maximum of 385 days. In some cases, as will be explained later, a 2-day time step was used.

The present study was limited to square concrete prisms having symmetrically placed reinforcement. The resulting cross-section has two planes of symmetry, and it was therefore possible to analyze just one quadrant alone in predicting the behavior of the entire column. The quadrant was represented as consisting of square concrete elements, and it was assumed that the area of each segment was concentrated at its centroid. For the case of reinforced concrete, one-fourth of the total steel area was concentrated at the centroid of the quadrant.

A previous study by Bresler et al. [9] considered the effects of axial loading, shrinkage and creep, but neglected cracking. In that study, the quadrant was subdivided into 25 square elements regardless of column size. A model which did permit the cracking of concrete elements was then developed by R. Polivka [10] using the results of the earlier study. Polivka's results were obtained by dividing the quadrant into 1-inch square elements.

In the present study, the effect of element size on shrinkage and therefore on cracking was determined. Preliminary analyses were made by successively dividing the quadrant of a plain 4 x 4-in. prism into 4, 16, 64, 100 and 256 square elements. The shrinkage strains obtained after a 28-day drying period using a one-day time step were found to vary significantly with element size.

By increasing the number of elements in the quadrant from 4 to 16 and finally to 64, it was found that shrinkage strains decreased. With a further increase in the number of elements to 100 and then to 256, however, the shrinkage values slightly increased. For the purpose of this study, all quadrants were subdivided into 100 square elements as shown in Fig. 1.

2.1.1 Conditions and Assumptions

At the end of each time step, the following conditions were to be satisfied:

1. Compatibility: Plane sections remain plane, and therefore no slippage can occur either between the steel and the

concrete or between adjacent concrete elements.

2. Equilibrium: The sum of the internal concrete and steel forces throughout the section must equal the externally applied force.

For this study, the following assumptions were made concerning material behavior and environmental conditions:

1. All elements were maintained at a constant temperature throughout the analysis.
2. A curing period of 14 days at 100% relative humidity was used for the concrete, during which time no shrinkage or swelling was assumed to have occurred. Thereafter, the concrete was stored in a constant humidity environment.
3. Steel reinforcement behaves as a linear-elastic material.
4. Concrete behaves as a viscoelastic material whose material properties change with time (aging).

In order to use the analytical model in obtaining stresses and deformations for a particular concrete, it was necessary to determine the shrinkage characteristics of the concrete as well as the time-dependent stress-strain and tensile strength behavior. These properties were obtained by conducting short-range laboratory tests on the concrete.

2.2 Shrinkage

The shrinkage diffusion model used in the analysis was based on the theories of Carlson [11] and Pickett [3]. They proposed that the flow and distribution of moisture in a concrete prism subjected to drying could be predicted using an analogous technique for determining thermal gradients in an ideal body during cooling. Furthermore, Pickett proposed that the amount of free shrinkage existing at a point at any given time could be determined if the following factors were known: the shrinkage diffusivity coefficient of the material, the final value of unrestrained shrinkage, the shape of the body under consideration, the boundary conditions as defined by a surface factor and the initial conditions. Free shrinkage is the shrinkage that would occur in each element assuming it to be completely unrestrained by neighboring elements.

Such an arbitrary distribution of deformation is, of course, not possible from a physical standpoint, but the free shrinkage visualization is useful as a tool in predicting resultant stresses. Free shrinkage, therefore, is not the same as the apparent shrinkage one measures in the laboratory on so-called unrestrained specimens. This apparent shrinkage is the one most commonly referred to as shrinkage.

For a square concrete prism exposed to drying on all four faces but not on the ends, the shrinkage diffusion equation is:

$$K \cdot \left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} \right) = \frac{\partial S}{\partial t} \qquad (2.1)$$

where

K - shrinkage diffusivity coefficient, sq. in./day

S - free, unrestrained shrinkage strain under constant ambient conditions, in./in.

x,y - rectangular coordinates, in.

t - time, days

If the exposed faces of the prism are the planes $x = \pm b$ and $y = \pm b$ (Fig. 1), the boundary conditions will be:

$$\frac{\partial S}{\partial x} = \pm \frac{f}{K}(S_\infty - S) \quad @\ x = \pm b$$

$$\frac{\partial S}{\partial y} = \pm \frac{f}{K}(S_\infty - S) \quad @\ y = \pm b \tag{2.2}$$

where

S_∞ - final unrestrained shrinkage strain under constant ambient conditions, i.e. the value of S when $t = \infty$

f - surface factor, characteristic of the material and boundary conditions, in./day

The solution satisfying Eqs. (2.1) and (2.2) is:

$$\frac{S}{S_\infty} = \Phi_x + \Phi_y - \Phi_x \Phi_y \tag{2.3}$$

where

$$\Phi_x = 1 - \sum_{n=1}^{\infty} e^{-T\beta_n^2} \left\{ F_n \frac{\cos(\beta_n \frac{x}{b})}{\cos \beta_n} \right\}$$

$$\Phi_y = 1 - \sum_{n=1}^{\infty} e^{-T\beta_n^2} \left\{ F_n \frac{\cos(\beta_n \frac{y}{b})}{\cos \beta_n} \right\}$$

and

$$T = \frac{Kt}{b^2} \tag{2.4}$$

x,y - coordinates of point where S/S_∞ is to be determined.

Defining:

$$B = \frac{fb}{K} \tag{2.5}$$

then

$$\beta_n = n^{th} \text{ root of } \beta \tan \beta = B \tag{2.6}$$

$$F_n = \frac{2B}{B^2 + B + \beta_n^2} \tag{2.7}$$

The parameters T, B, β_n and F_n are all non-dimensional, and the solution of Eq. (2.3) satisfies the conditions of S = 0 at t = 0 and S = ∞ at t = ∞. Physically, the term Φ_x represents the value of S/S_∞ if only the surfaces $x = \pm b$ were exposed to drying, and Φ_y represents the value of S/S_∞ if only the surfaces $y = \pm b$ were exposed.

The number of terms used to evaluate S/S_∞ in Eq. (2.3) depends upon the accuracy desired. For a given degree of precision, the number of terms required is primarily controlled by the parameter T in Eq. (2.4), and to a lesser extent by y/b and the parameter B in Eq. (2.5). If T is greater than 0.2, very little error is introduced by neglecting all the terms in the series except the first; but if T is less than 0.2, additional terms are needed to achieve the desired precision. Generally, as the value of T becomes smaller, more terms will have to be used.

The Newton-Raphson method was used to evaluate the roots of Eq. (2.6). For this study, twenty-four roots were deemed to be sufficient to produce accurate results for S/S_∞. The values of these roots are listed in Table 1 for 6, 10, 15 and 20-inch columns.

As drying proceeds, the value of the shrinkage diffusivity coefficient K decreases. Pickett used the following expression for K, which was found to be in good agreement with his experimental results:

$$K = 0.10\sqrt{\frac{2}{2+t}} \quad \text{sq. in./day} \tag{2.8}$$

In this expression, t was taken as the time (days) after the end of the curing period.

The shrinkage-time relation used in this study was based on experimental shrinkage data obtained at the University of California, Berkeley on cylindrical 6 x 18-in. plain concrete specimens [12,13]. Pickett's expression for shrinkage diffusivity in Eq. (2.8) was used to approximate these experiments.

Pickett assumed that the surface factor f would vary with time in a manner similar to that of K, so that the ratio f/K would remain constant. This assumption greatly simplified the solution of the diffusion equation (Eq. 2.3) and was justified as its use provided good agreement with his experimental data. The following expression for f from Pickett's work was used in this study:

$$f = 1.67K \quad \text{in./day} \tag{2.9}$$

Equation (2.5) will now be reduced to:

$$B = 1.67b \tag{2.10}$$

Again, it should be emphasized that the shrinkage value S resulting from Eq. (2.3) represents free shrinkage, which is not the same as the apparent or restrained shrinkage measured on an unloaded concrete shrinkage specimen. This difference arises because the interior concrete in a prism shrinks at a slower rate than the exterior, thereby restraining the prism from undergoing completely free shrinkage.

For given ambient conditions, the observed value of laboratory shrinkage will depend on the variation of the shrinkage diffusivity coefficient K with time (Eq. 2.8), the surface factor f (Eq. 2.9), the size of the specimen (Eq. 2.10), its elastic and creep characteristics, and the limiting value of unrestrained shrinkage S_∞. A value of S_∞ = 900 micro-in./in. was selected for this work because it resulted in close agreement with the experimental shrinkage data [12,13] as shown in Fig. 2.

2.3 Elastic and Creep Response

Experimental investigations have shown that within the working stress range, concrete in compression exhibits essentially linear load-deformation characteristics with respect to both instantaneous and creep deformations. However, during the initial period (6 to 9 months) of service, these characteristics, particularly that of creep, are strongly dependent on aging. After loading, the rate of creep is initially high and then diminishes with time. The amount of creep strain will depend mainly on the age of the concrete at loading, and the intensity and duration of the sustained load. Experimental studies have shown that creep of concrete in compression is linear with stress up to

40% of the ultimate strength. This linear relationship greatly simplifies the analysis by allowing creep to be expressed as specific creep (creep per unit stress).

Specific creep compliance c is defined as:

$$c = \frac{\varepsilon}{\sigma} = \frac{\varepsilon_i + \varepsilon_c}{\sigma} \tag{2.11}$$

where

ε_i - instantaneous strain due to stress

ε_c - creep strain due to stress

σ - sustained stress

The value of specific creep is strongly dependent on age, which includes the age of the concrete at loading and the age of the concrete at which the creep is observed. Therefore, in predicting the amount of creep, it is necessary to know not only the total stress on the prism, but also the loading history of incremental stresses which led to the total stress.

A number of methods have been developed to determine the creep deformation of concrete subjected to a variable stress history. The three main methods are the effective modulus method and rate of creep method proposed by Ross [14], and McHenry's method of superposition [15]. Each method has advantages under certain conditions, depending on the nature of the stress variation, the extent of available creep data and the accuracy required.

The effective modulus method is simple to use, but is of little value for the case of varying stress. The method predicts an ultimate creep strain based only on the total stress applied, and does not account for the stress history leading up to this total stress. The rate of creep method attempts to avoid this particular weakness of the effective modulus method by taking into account the stress history. It does not provide, however, for time-dependent recovery upon removal of load.

The method of superposition allows for the variation of applied stress of any magnitude, either tension or compression. In contrast to the rate of creep method, however, it also provides for some creep recovery upon load removal.

For concretes subjected to abrupt changes of stress, the method of superposition was found to be superior to the others in predicting creep strains, and was the method selected for use in this study.

The principle of superposition as postulated by McHenry [15] states that: "The strains produced in concrete at any time t_n by a stress increment applied at any time t_i are independent of the effects of any stress applied either earlier or later than t_i." The stress increment may be either tension or compression, and it is assumed that the creep of concrete in tension or compression is the same for equal stresses. Because only limited research data are presently available on tensile creep, this is the best assumption that can be made at this time.

The results of a study on tensile creep and stress relaxation in concrete conducted at the University of California at Berkeley by Akatsuka, Chang, and Polivka [16] indicated that the difference between the magnitudes of tensile creep and compressive creep for equal stresses is very small, tensile creep being slightly higher than compressive creep. Therefore, the error introduced by the assumption that tensile creep and compressive creep are equal will be negligible.

A mathematical model of the principle of superposition is usually represented by an integral equation of the following form:

$$\varepsilon(t) = \int_{t_0}^{t} C(t,\tau) \frac{\partial\sigma(\tau)}{\partial\tau} d\tau \tag{2.12}$$

where

- $\varepsilon(t)$ - total strain $(\varepsilon_i + \varepsilon_c)$ observed at time t due to a variable stress $\sigma(\tau)$
- $C(t,\tau)$ - specific compliance function, characteristic of the material and independent of loading
- t_0 - time of initial loading
- τ - variable of integration

Creep tests were conducted at the University of California at Berkeley on cylindrical 6 x 18-in. concrete specimens having a 28-day compressive strength of 5600 psi [12,13]. The specimens were fog-cured for 7 days, and then stored in an environment of 70°F and 50% relative humidity. The cylinders were stressed to 900 psi at different ages and provided the data from which the following good fit was obtained for use in the present study:

$$C(T,\tau) = \frac{0.33 \times 10^{-3}}{\tau} - \frac{0.306 \times 10^{-4}}{\sqrt{\tau}} + 0.221 \times 10^{-3} \qquad (2.13)$$

$$+ \left[0.1096 - \frac{0.1715}{\tau^{0.85}} + \frac{34.4316}{\tau^{1.7}} - \frac{433.9611}{\tau^{2.55}} + \frac{1260.859}{\tau^{3.40}}\right]$$

$$\cdot \left\{0.99 \left[1-e^{-.2(T-\tau)}\right] + 2.06 \left[1-e^{-.02(T-\tau)}\right] + 1.125 \left[1-e^{-.002(T-\tau)}\right]\right\} \times 10^{-3}$$

where

$C(T,\tau)$ - specific compliance, in./in. per ksi

τ - age at loading, days

T - age at observation, days

The specific creep curves in Fig. 3 were plotted using Eq.(2.13) and illustrate how the rate of creep decreases with age at loading.

2.4 Cracking

In the analysis, a concrete element was permitted to crack whenever its tensile stress exceeded the tensile strength of concrete at that age. The following time-varying tensile strength relationship was used:

$$f'_t = 0.639 - \frac{1.39}{t} \qquad (2.14)$$

where

f'_t = splitting tensile strength, ksi

t = age, days

This empirical expression was derived by Selna [4,5] to approximate actual experimental data [12,13]. The splitting tension tests were conducted on 6 x 12-in. cylinders made of the same concrete and subjected to the same curing conditions as were the creep specimens used in obtaining Eq. (2.13).

2.5 Shrinkage and Creep Analysis Routine

In order to adopt this general method for the analysis of a particular concrete prism, the following must either be known, assumed or accepted:

a. Specific creep compliance properties of the particular concrete to be analyzed. An empirical relation such as Eq. (2.13) could be obtained by fitting actual experimental creep data.

b. Time-varying tensile strength of the particular concrete. A relation similar to Eq. (2.14) could be developed from experimental splitting tension test data.

c. Reinforcing steel properties. The steel used in this analysis had a modulus E_s = 29,000 ksi and a yield stress f_y = 60 ksi.

d. The method of superposition was used in predicting creep strains.

e. Creep of concrete is the same in tension or compression for equal stresses. Tensile creep was assumed to differ from compressive creep in sign only.

f. Constraints on all of the elements were released for the duration of a time step, and then reimposed at the end of each time interval by recognizing that compatibility and equilibrium must be satisfied.

The period of time considered in the analysis was broken down into equal intervals which shall be referred to as time steps. The analysis routine will initially be explained for unloaded columns having a curing period of $\bar{t}_c$ time steps. Cracking of concrete will be neglected for now, but its influence on the analysis will be discussed later.

Initially, a stress-free plane exists at the end of the curing period. This condition is based on the assumption that only a negligible amount of shrinkage occurs during curing.

During the first time step after curing ($\bar{t}_c + 1$), as well as in all subsequent time steps, each element in the quadrant undergoes a certain amount of free shrinkage deformation. The total amount of unrestrained shrinkage strain S that would exist at the centroid of an element, assuming that it were free to deform throughout the entire observation period, can be determined using the shrinkage diffusion relation given in Eq. (2.3).

During a given time step $\bar{t}$, each element 'k' will undergo the following amount of free shrinkage:

$$\Delta\varepsilon_{sh,k,\bar{t}} = (S_{\bar{t}} - S_{\bar{t}-1})_k \qquad (2.15)$$

where

$\Delta\varepsilon_{sh,k,\bar{t}}$ - free shrinkage of element 'k' during time step $\bar{t}$

$S_{\bar{t}}$ - total unrestrained shrinkage of element 'k' up to time step $\bar{t}$, evaluated using computed value of shrinkage diffusivity K from Eq. (2.8) at time step $(\bar{t} - \bar{t}_c)$

$S_{\bar{t}-1}$ - total unrestrained shrinkage of element 'k' up to time step $\bar{t}$-1, evaluated using computed value of shrinkage diffusivity K from Eq. (2.8) at time step $(\bar{t} - \bar{t}_c) - 1$.

By releasing all constraints on the elements during the first time step after curing, each element violates the plane strain condition as it shrinks downwards an amount $\Delta\varepsilon_{sh,k,\bar{t}_c+1}$ completely unrestrained by either the steel reinforcement or by neighboring elements. Because the initial stress on all of the concrete elements was zero at the beginning of the time step, no creep deformation will occur during this interval.

The incompatibility existing across the section just prior to the end of the first time step after curing is illustrated in Fig. 4. The interior concrete elements exhibit less shrinkage than the exterior elements, and the steel does not shrink at all. The corner element experiences the most drying shrinkage, as two of its four faces are exposed.

At the end of the time step, the constraints which were previously released must be reinstated. Steel does not creep during this time step, and in order to satisfy compatibility, shrinkage correction stresses of sufficient magnitude are applied to pull each element back up to the level of the steel. The elevation view of the quadrant shown in Sect. A-A of Fig. 4 illustrates how compatibility and equilibrium are satisfied.

The <u>shrinkage correction stress</u> applied to each element 'k' at the end of any time step $\bar{t}$ is:

$$\Delta f_{sh,k,\bar{t}} = \Delta\varepsilon_{sh,k,\bar{t}} \cdot E_{c,\bar{t}} \tag{2.16}$$

where

$\Delta f_{sh,k,\bar{t}}$ - shrinkage correction stress applied to element 'k' at the end of time step $\bar{t}$

$E_{c,\bar{t}}$ - modulus of elasticity of concrete at the end of time step $\bar{t}$

The modulus of elasticity of concrete at t days is nothing more than the inverse of the specific creep compliance value obtained from Eq. (2.13) for $t = \tau$.

Compatibility is now satisfied as all the elements lie in the same plane as the steel reinforcement. By satisfying compatibility, however, a force unbalance has been created across the section.

Equilibrium requires that the internal forces must balance the externally applied force. For the case where no axial load exists on the column, the sum of the internal forces across the section must be zero.

The total force unbalance at the end of the first time step after curing, $\Delta F_{sh,k,\bar{t}_c+1}$, is obtained by summing all the elemental shrinkage correction forces. Static equilibrium is then maintained by applying a shrinkage equilibrium correction force, $\Delta F_{sh,k,\bar{t}_c+1}$, to the total transformed area of the quadrant. This force is simply the reverse of the total shrinkage correction force.

The <u>shrinkage equilibrium correction force</u> applied to the quadrant at the end of any time step $\bar{t}$ is:

$$\Delta F_{sh,\bar{t}} = \sum_k \{\Delta f_{sh,k,\bar{t}} \cdot A_k\} \tag{2.17}$$

where

$\Delta F_{sh,\bar{t}}$ = shrinkage equilibrium correction force applied at end of time step $\bar{t}$

A_k = area of concrete in element 'k'

Applying this force in compression to the total transformed area of the quadrant gives rise to a "mean stress" on the section.

The <u>shrinkage equilibrium correction stress</u> resulting at the end of any time step $\bar{t}$ is:

$$\Delta\bar{\sigma}_{sh,\bar{t}} = \frac{\Delta F_{sh,\bar{t}}}{A_t} \tag{2.18}$$

where

$\Delta\bar{\sigma}_{sh,\bar{t}}$ - shrinkage equilibrium correction stress at the end of time step $\bar{t}$

A_t - transformed area of quadrant = $\sum_k A_k + (n_{\bar{t}} - 1)\cdot A_s$

$n_{\bar{t}}$ - modular ratio at time step $\bar{t}$ = $\frac{E_s}{E_{c,\bar{t}}} = E_s \cdot C(\bar{t},\bar{t})$

A_s - area of steel in quadrant

The strain corresponding to this correction stress at the end of any time step $\bar{t}$ would be:

$$\Delta\bar{\varepsilon}_{sh,\bar{t}} = \frac{\Delta\bar{\sigma}_{sh,\bar{t}}}{E_{c,\bar{t}}} \tag{2.19}$$

This value represents the increment of shrinkage strain created during the time step $\bar{t}$, and is the same as the apparent or restrained strain increment one would observe in the laboratory.

The additional amount of stress created in the steel during any time step $\bar{t}$ due to shrinkage would be $\Delta\bar{\varepsilon}_{sh,\bar{t}} \cdot E_s$.

At the end of the first time step after curing, although compatibility and equilibrium are once again satisfied, a stress-free plane no longer exists.

The increment of shrinkage stress created in an element 'k' during any time step $\bar{t}$ is:

$$\begin{aligned} \Delta\sigma_{sh,k,\bar{t}} &= \Delta\bar{\sigma}_{sh,\bar{t}} - \Delta f_{sh,k,\bar{t}} \\ &= (\Delta\bar{\varepsilon}_{sh,\bar{t}} - \Delta\varepsilon_{sh,k,\bar{t}}) \cdot E_{c,\bar{t}} \end{aligned} \tag{2.20}$$

In the analysis it is assumed that the operations involved in satisfying equilibrium and compatibility at the end of the time step occur instantaneously. The analysis procedure that has been outlined for the first time step after curing is carried out at the end of the subsequent time steps, with one important difference. The shrinkage stresses created in the concrete elements at the end of a time step will act as sustained loadings to produce increments of creep strain in subsequent time steps. The increment of creep strain produced in a particular time step $\bar{t}$ is therefore added to the shrinkage strain, $\Delta\varepsilon_{sh,k,\bar{t}}$, in determining the total deformation of an element during the time step. To satisfy compatibility in subsequent time steps, a correction stress for creep, $\Delta f_{cr,k,\bar{t}}$, has to be included in addition to the shrinkage correction stress, $\Delta f_{sh,k,\bar{t}}$, in order to pull each segment back up to the level of the steel, which did not undergo either shrinkage or creep deformation.

A typical stress history for an element 'k' is shown in Fig. 5, along with the resulting creep strain, assuming the element to be free to deform throughout the entire observation period. The total amount of creep strain an element will undergo during a particular time step $\bar{t}$ is simply the summation of all of the incremental values of creep strain that would be produced in that time step by the additional stresses created in the element during previous time steps.

In order to determine the increment of creep strain that will occur in an element during a particular time interval, the empirical relation for specific creep compliance in Eq. (2.13) is used. For example, the creep strain that would occur in element 'k' during the time step $\bar{t}$ as a result of the additional stress due to shrinkage and creep created in the concrete during time step $\bar{\tau}$ would be:

$$\Delta\varepsilon'_{cr,k,\bar{t}} = \{C(\bar{t},\bar{\tau}) - C(\bar{t}-1,\bar{\tau})\} \cdot \Delta\sigma_{k,\bar{\tau}} \qquad (2.21)$$

The <u>total amount of creep strain</u> that will occur in an element 'k' during any time step $\bar{t}$ is determined by summing up the contributions from all previous additional stresses created in that element:

$$\Delta\varepsilon_{cr,k,\bar{t}} = \sum_{\bar{\tau}=\bar{t}_c+1}^{\bar{t}-1} \{C(\bar{t},\bar{\tau}) - C(\bar{t}-1,\bar{\tau})\} \cdot \Delta\sigma_{k,\bar{\tau}} \qquad (2.22)$$

where

$\bar{t}_c$ - number of time steps the concrete is cured

This general creep-strain relation (Eq. 2.22) can now be combined with the one earlier derived for shrinkage alone (Eq. 2.15), to obtain a general expression for both shrinkage and creep.

The total shrinkage and creep deformation of an unrestrained element 'k' that occurs during a particular time step $\bar{t}$ is:

$$\Delta\varepsilon_{k,\bar{t}} = \Delta\varepsilon_{sh,k,\bar{t}} + \Delta\varepsilon_{cr,k,\bar{t}} \tag{2.23}$$

$$= (S_{\bar{t}} - S_{\bar{t}-1})_k + \sum_{\bar{\tau}=\bar{t}_c+1}^{\bar{t}-1} \{C(\bar{t},\bar{\tau}) - C(\bar{t}-1,\bar{\tau})\}\cdot\Delta\sigma_{k,\bar{\tau}}$$

The shrinkage and creep compatibility correction stress to be applied at the end of time step $\bar{t}$ is:

$$\Delta f_{k,\bar{t}} = \Delta\varepsilon_{k,\bar{t}} \cdot E_{c,\bar{t}} \tag{2.24}$$

The total equilibrium correction force applied to the transformed area of the quadrant at the end of time step $\bar{t}$ will be:

$$\Delta F_{\bar{t}} = \sum_k \{\Delta f_{k,\bar{t}} \cdot A_k\} \tag{2.25}$$

The corresponding equilibrium correction stress on the section is:

$$\Delta\bar{\sigma}_{\bar{t}} = \frac{\Delta F_{\bar{t}}}{A_t} \tag{2.26}$$

The incremental effective column strain created during time step $\bar{t}$ will be:

$$\Delta\bar{\varepsilon}_{\bar{t}} = \frac{\Delta\bar{\sigma}_{\bar{t}}}{E_{c,\bar{t}}} \tag{2.27}$$

Finally, the additional stress created in a concrete element during any time step $\bar{t}$ due to shrinkage and creep is:

$$\Delta\sigma_{k,\bar{t}} = \Delta\bar{\sigma}_{\bar{t}} - \Delta f_{k,\bar{t}}$$

$$= (\Delta\bar{\varepsilon}_{\bar{t}} - \Delta\varepsilon_{k,\bar{t}}) \cdot E_{c,\bar{t}} \qquad (2.28)$$

The additional stress created in the steel during time step $\bar{t}$ due to shrinkage and creep would be $\Delta\bar{\varepsilon}_{\bar{t}} \cdot E_s$.

The procedure that has been outlined can be carried out for each successive time step. A complete solution is obtained for stresses in the elements and effective column strain at the end of each time step by summing up the contributions from all previous time steps.

The total stress in an element 'k' at the end of N time steps after curing will be:

$$\sigma_{k,N} = \sum_{\bar{t}=\bar{t}_c+1}^{N} \Delta\sigma_{k,\bar{t}} \qquad (2.29)$$

Similarly, the total effective column strain that will exist at the end of N time steps after curing is:

$$\bar{\varepsilon}_N = \sum_{\bar{t}=\bar{t}_c+1}^{N} \Delta\bar{\varepsilon}_{\bar{t}} \qquad (2.30)$$

This effective column strain is the same as the strain existing in the steel. Therefore, the stress in the steel after N time steps is $\bar{\varepsilon}_N \cdot E_s$.

It should be emphasized that all of the proceeding derivations apply to unloaded reinforced concrete columns, and that the influence of cracking has been neglected. The effect that external loading and cracking have on the analysis will now be discussed.

2.6 Effect of External Load

An external load $\Delta P_{\bar{t}}$ applied to a column at the end of time step $\bar{t}$ is handled in exactly the same manner as was the equilibrium correction force $\Delta F_{\bar{t}}$. Both are applied to the total transformed area of the cross-section and give rise to an effective column strain.

The external load $\Delta P_{\bar{t}}$ is simply added to the equilibrium correction force $\Delta F_{\bar{t}}$, and the total $\Delta F'_{\bar{t}}$ applied to the transformed section at the end of time step $\bar{t}$, where:

$$\Delta F'_{\bar{t}} = \Delta F_{\bar{t}} + \Delta P_{\bar{t}} \tag{2.31}$$

The force $\Delta F'_{\bar{t}}$ results in a mean stress on the cross-section, $\Delta\bar{\sigma}'_{\bar{t}}$, of:

$$\Delta\bar{\sigma}'_{\bar{t}} = \frac{\Delta F'_{\bar{t}}}{A_t} \tag{2.32}$$

This mean stress will produce an effective column strain, $\Delta\bar{\varepsilon}'_{\bar{t}}$, for time step $\bar{t}$ of:

$$\Delta\bar{\varepsilon}'_{\bar{t}} = \frac{\Delta\bar{\sigma}'_{\bar{t}}}{E_{c,\bar{t}}} \tag{2.33}$$

Finally, the additional stress created in a concrete element during time step $\bar{t}$ due to shrinkage, creep and external loading $\Delta P_{\bar{t}}$ will be:

$$\Delta\sigma'_{k,\bar{t}} = \Delta\bar{\sigma}'_{\bar{t}} - \Delta f_{k,\bar{t}} \qquad (2.34)$$

2.7 Effect of Cracking

In the analysis, the total stress in each element is determined at the end of every time step (Eq. 2.29), and compared with the tensile strength of concrete at that age (Eq. 2.14). If the tensile stress in a particular element exceeds the tensile strength of concrete at that age, the element is allowed to crack.

When an element cracks, the tensile stress in the element is immediately reduced to zero. The stress carried by this element prior to cracking is then uniformly distributed among the remaining uncracked transformed section. The tensile stress in a cracked element is reduced to zero by applying a compressive stress to that element equal to the tensile stress which existed in the element at the beginning of the time step during which it cracked. This compressive stress is equivalent to the additional stress created in the element during the time step, and is handled in a fashion similar to that used for the additional stresses created in the uncracked elements during the same time step.

A cracked element not in compression is stress-free and no longer must satisfy compatibility with the rest of the prism. It pulls away from the plane of the uncracked elements (and cracked elements in compression), developing a crack. The transformed area of the cross-section is then accordingly reduced by the area of the cracked segment.

In the analysis, therefore, when an element was found to have cracked at the end of a certain time step, it was immediately "cracked" at the beginning of the time step by applying a compressive stress equal to the tensile stress that existed in the element at the beginning of the time step. A new transformed area would then be calculated and the analysis repeated for the same time interval, but this time with the element initially cracked.

Upon application of an axial load, the elastic and creep deformations of the uncracked plane supporting the load may be sufficient to close the crack on a particular element. It was assumed in this analysis that although a cracked element can no longer support any tension, it can carry compressive stress if the crack should close. Therefore, it is necessary to determine the magnitude of crack width for all of the cracked elements at the end of each time step. The crack width at a particular time is determined by taking the difference between the effective strain of the prism and the strain in the cracked element.

Figure 6 illustrates the analytical procedure of reducing the tensile stress on a cracked element to zero by using superposition of stresses, and the subsequent determination of crack width in later time steps.

When an external load is applied to a column, the resulting elastic deformation of the uncracked plane supporting the load is compared with the minimum crack width existing at the time of load application. When the resulting elastic strain is less than the minimum existing crack width, none of the cracks will have closed and the analysis proceeds as usual. When the elastic deformation is greater than the minimum existing crack width, however, the crack will have closed and the element now aids the uncracked transformed section in supporting the load. For this case, it is necessary to determine how much of the external load will just close the smallest crack. At this point, the transformed area of the existing uncracked plane is increased by the area of the cracked segment that closed. The remaining portion of the external load to be supported is then applied to this new transformed area. The resulting elastic deformation of this plane, which now represents not only uncracked elements but also a cracked element in compression, is then compared with the next smallest crack width. When the column is able to support the remaining portion of the external load without having any additional cracks close, it will have attained its final equilibrium position for that time step. If an additional crack closes before the remaining portion of the load can be fully supported, it will again be necessary to employ a balancing procedure similar to that outlined above.

2.8 Yielding of Steel Reinforcement

Compressive yielding of steel reinforcement was not considered in developing the analytical model. If it were assumed that steel exhibits ideal elastic-plastic behavior, a check could be made at the end of each time step to determine if the steel had reached its compressive yield stress. If the above were verified at any time step, then the difference between the calculated stress and the yield stress multiplied by the area of the steel would correspond to a correction load. This load would then be applied to a new transformed area which excluded the area of the plastic steel.

The steel stress would be f_y for this time step. The steel area should be restored to the transformed area at the end of the time step so that elastic recovery could take place if load removal should occur in subsequent time steps.

No yielding of the steel reinforcement occurred in any of the columns studied in this investigation.

2.9 Formulation of Computer Program

A FORTRAN IV computer program was written for this analysis to evaluate stresses and strains in steel and concrete for square, symmetrically reinforced concrete columns subjected to time-variable axial load.

The input data consists of concrete and steel material properties, specified number of days per time step, number of time intervals over which the analysis is carried out, size of column, number of elements into which the quadrant is divided, area of steel reinforcement, load history, and duration of initial curing period.

The computer output visually displays the quadrant of the column at the end of each time interval and prints out the stresses and strains of each element in a position corresponding to the location of the element in the quadrant.

The User's Manual for the program is included in Appendix A and the required form of the input data for a sample problem is given in Appendix B. The computer program listing is contained in Appendix C.

3. CASES SELECTED FOR STUDY

3.1 Column Size and Reinforcement

The following sizes of square reinforced concrete columns were selected for this study:

(a) 6 by 6 in.

(b) 10 by 10 in.

(c) 15 by 15 in.

(d) 20 by 20 in.

Each of the above columns was analyzed using the following amounts of steel reinforcement:

(a) 0%

(b) 2%

(c) 4%

As previously discussed, all quadrants were subdivided into 100 sqare elements (Fig. 1), and one-fourth of the total steel reinforcement was symmetrically placed at the center of each quadrant.

3.2 Time Interval and Observation Period

In order to determine how the length of the time step can affect the resulting stresses and deformations, the following time intervals and observation periods were used:

(a) 7-day time intervals up to 385 days

(b) 2-day time intervals up to 190 days
(unloaded columns only)

3.3 Cracking vs. No Cracking

In order to determine the effect of cracking on the analytical results, the following two cases were studied:

(a) Cracking of concrete

(b) No cracking of concrete

The program developed for this study analyzes the no-cracking case simply by inputting a very large fictitious value for the tensile strength of the concrete. No cracking will occur because the tensile strength will always exceed the tensile stresses which develop.

3.4 Duration of Curing

A 14-day curing period was used for all of the columns studied. It was assumed that no shrinkage stresses developed during curing.

3.5 Load History

The following two loading conditions were investigated:

(a) No external load, i.e., only shrinkage and related creep deformations would occur.

(b) Axial load applied in four equal steps at the ages of 14, 49, 84 and 119 days, up to a maximum of 0.3 P_u. This loading history, as shown in Fig. 7, was selected to simulate the incremental loading conditions in a real structure.

The loading P_u represents the ultimate axial design load for a reinforced concrete column in compression. It was calculated according to the ACI Code [17] as:

$$P_u = \phi_{col} \cdot \left[0.85\ f'_c A_g + f_y A_s\right]$$

where the following values were used:

ϕ_{col} - capacity reduction factor (0.70)

f'_c - 28-day compressive strength of concrete (5000 psi)

A_g - gross area of section, sq. in.

f_y - yield strength of reinforcement (60,000 psi)

A_s - area of reinforcement, sq. in.

3.6 Material Characteristics and Environment

The concrete selected for this study had well-defined elastic, creep, shrinkage and strength properties which were experimentally determined at the University of California at Berkeley [12,13].

Shrinkage and creep tests were performed on 6 x 18-in. cylindrical concrete specimens that had been moist-cured for 7 days and then stored in an environment of 50% relative humidity and 70°F. The shrinkage and creep results were empirically represented in Eqs. (2.3) and (2.13), respectively.

Splitting tensile and compressive strength tests were conducted on 6 x 12-in. cylinders made of the same concrete and subjected to the same curing conditions as the shrinkage and creep specimens. The 28-day compressive strength (f'_c) was 5600 psi, and the tensile strength (f'_t) was 580 psi. The variation of f'_t with time was represented by Eq. (2.14).

4. RESULTS AND DISCUSSION

4.1 Shrinkage of Plain Concrete Prisms

4.1.1 Free Shrinkage

Free shrinkage strains were calculated at 7-day intervals up to 371 days after the concrete was exposed to drying at the age of 14 days. Square columns having 6, 10, 15 and 20-in. sides were investigated.

The variation of the relative free shrinkage, S/S_∞, across the inner and outer edges of the respective quadrants at selected times is shown in Figs. 8 to 11. Also, the computed values of S/S_∞ along the inner edge ($y = 0$) and the outer edge ($y = 0$) of the quadrants for the 6 and 20-in. columns are listed in Table 2.

Drying penetrated about 2 inches from the surface of all four columns after 7 days. Additionally, the pattern of the S/S_∞ variation with time for the column is similar, as would be expected. Shrinkage generally developed to 3, 4 and 5 in. from the surface after 21, 70 and 364 days of drying, respectively. These values are in good agreement with those given by Carlson [11].

4.1.2 Restrained Shrinkage

The analytical shrinkage results from a 6-in. square column are compared in Fig. 2 with those observed experimentally [12,13] on 6 x 18-in. cylindrical columns. The calculated values were obtained using 7-day time steps, and both cracking and no cracking of the concrete were considered. A value of S_∞ = 900 micro-in./in. was selected, which provided close agreement between the calculated and observed values for the case where cracking was allowed to occur. The similarity in results would seem to justify the selected values for the coefficient of diffusivity K and the surface factor f. The no-cracking shrinkage results were somewhat higher, particularly at early ages.

The analytical shrinkage results for 6, 10, 15 and 20-in. columns shown in Fig. 12 for both the cracking and no-cracking cases indicate that the size of a prism has a pronounced effect on both the rate and amount of drying shrinkage. The shrinkage values decreased significantly with increase in size up to 20 inches. For prisms larger than 20 inches, the influence of internal temperature on thermal expansion becomes a major factor at early drying times in addition to the size effect [18].

The shrinkage results shown in Fig. 12 for the case where cracking is considered are lower than those obtained for the no-cracking case, because the tension carried by a cracked section just prior to cracking is transferred to the remaining uncracked portion of the prism, resulting in a slight relaxation in the total column strain.

The progress of cracking in each of the four columns studied is shown in Fig. 13 for selected time intervals. All of the cracking occurred during the initial 7 days of drying, except in the case of the 20-in. column where an additional element cracked after 35 days of drying. All prisms suffered roughly the same amount of cracking regardless of size, i.e., after nearly one year of drying, cracking had progressed to a depth of about 1 inch. This substantiates the hypothesis that as drying shrinkage proceeds inwards from the surface of a prism, its rate is independent of prism size.

4.2 Reinforced Concrete Columns

4.2.1 Shrinkage of Unloaded Columns

The shrinkage strains of unloaded 10, 15 and 20-in. columns having 0, 2 and 4% of steel reinforcement were determined for both the cracking and no-cracking cases, as shown in Figs. 14-16, respectively. In all cases, the prisms were exposed to drying at age 14 days, and no external load was applied during the 385-day period of analysis. This condition could occur in a precast column that was moist-cured for 14 days before drying.

It is observed that increasing the percentage of reinforcement in a prism results in a subsequent decrease in the total shrinkage strain. Also, for columns of varying size but having the same steel ratio, the larger size columns exhibited less shrinkage than the smaller ones.

For reinforced concrete prisms, the shrinkage values obtained when cracking was considered were lower than those obtained for the no-cracking case. This is to be expected, and can be explained on a basis similar to that used earlier for the unreinforced prisms. The difference between the cracking and no-cracking results diminished, however, with increasing prism size.

The effect that steel reinforcement has on cracking was studied for the 10, 15 and 20-in. prisms having 0, 2 and 4% of steel, and their element cracking patterns are shown in Figs. 13, 17 and 18, respectively. The results show that the reinforcement used had little effect on the amount of cracking. In fact, the cracking patterns obtained for the 10, 15 and 20-in. prisms containing 2 or 4% of reinforcement were nearly identical to those obtained for the corresponding unreinforced prisms. Also, as was the case for the unreinforced columns, nearly all of the cracking in the reinforced prisms occurred during the first 7-day time step of drying.

4.2.2 Deformation of Loaded Columns

The deformations under load of 10, 15 and 20-in. columns containing 2 or 4% of steel reinforcement were determined for both the cracking and no-cracking cases, as shown in Figs. 19-21. These columns were cured for 14 days and then a total load equivalent to 0.3 of the ultimate axial compressive load, computed in accordance with the ACI Code [17], was applied in four equal increments at the ages of 14, 49, 84 and 119 days, as shown in Fig. 7. This type of loading was selected to simulate the actual loading conditions that would occur in a column during the construction of a multi-story building.

Upon application of the initial load increment at 14 days, larger deformations were obtained for the no-cracking case than the cracking case, but this difference in deformation diminished with each subsequent load application, becoming negligible once the total load had been applied. It may be noted that the deformations computed for the cracking and no-cracking analyses agreed much more closely for the loaded 10, 15 and 20-in. columns (Figs. 19-21) than for the corresponding unloaded columns (Figs. 14-16). The cracking and no-cracking analyses of the loaded columns gave similar results because:

1. The initial load increment, applied to the column at the end of the curing period, precompressed the column and thus

reduced the number of cracks which otherwise would have formed during the subsequent drying period.

2. Some of the cracks which did form were closed due to both the elastic deformations of subsequent load applications and creep deformations of existing loads, resulting in a larger effective cross-section of the column resisting the load.

4.2.3 Transfer of Load from Concrete to Steel

The distribution of the column load between the concrete and the steel for the 10, 15 and 20-in. columns containing 2 or 4% of reinforcement is shown in Figs. 22-24 for both the cracking and no-cracking cases. As would be expected, the results show that some of the load initially carried by the concrete is transferred with time to the steel reinforcement. This load transfer is due to the time-dependent shrinkage and creep deformations of the concrete. It can also be seen that for a given loading history, a much larger portion of the column load is transferred to the steel as the amount of reinforcement is increased from 2 to 4 percent.

The difference between the cracking and no-cracking results diminished with each successive load application, and became negligible once the total column load was applied. During the 21-day drying period following the application of the first load increment, the no-cracking results for the 10 x 10 in. column containing 4% of steel (Fig. 22) indicated that tension had developed in the concrete and the compressive axial load was being carried entirely by the steel.

4.2.4 Effect of Time Step on Shrinkage

In order to determine what effect the length of time step had on computed shrinkage strains, 2- and 7-day time steps were used in obtaining shrinkage results for 6- and 10-in. columns containing 2 and 4% of steel reinforcement. Results obtained are shown in Figs. 25 and 26, respectively.

The computed shrinkage values at the end of the 190-day observation period using a 2-day time step were about 15% higher than the results using a 7-day time step. This discrepancy was due to a difference in the total number of cracked elements. The element cracking patterns for the 10-in. column containing 2 to 4% of steel reinforcement and using a 2-day time step are shown in Fig. 27. A comparison of these patterns with similar ones obtained using a 7-day time step (Figs. 17 and 18), shows that approximately 11% fewer elements cracked using a 2-day time step.

Because nearly all cracking takes place during the first few days of drying when the shrinkage rate is quite rapid, it would be desirable to use small time steps initially, and then switch over to longer time steps at later stages.

4.2.5 Effect of Mesh Size on Shrinkage

Analytical shrinkage results for 6 x 6-in. plain prisms using 100 and 225 elements per quadrant were compared for the case where cracking was considered. Although the results shown in Fig. 28 indicated that the accuracy of the solution will be improved if a finer mesh size is used, it was apparent that the slightly greater degree of accuracy attained with the use of 225 elements per quadrant did not warrant the increased computational effort. A mesh size of 100 elements per quadrant was therefore chosen for use in this investigation.

4.3 Comparison between Analytical and Experimental Results

In order to determine the capability of the analytical model in accurately predicting the time-dependent deformations of reinforced concrete columns, a check was made with experimental results obtained at the University of California at Berkeley [19,20]. A series of tests were conducted on both plain and reinforced 5-in. cylindrical columns. The reinforced columns selected for comparison contained 1.9% of steel reinforcement, and were loaded in compression to 980 psi at 28 days. All columns were fog-cured for 21 days and then exposed to drying in an environment of 70°F and 50% relative humidity.

Analytical results were obtained for a plain and a reinforced 5-in. square column. The reinforced column also contained 1.9% of steel and was loaded to 980 psi at 28 days. A 21-day curing period was used for both columns.

A comparison between the analytical and experimental results for both shrinkage and creep deformations is shown in Fig. 29. The analytical shrinkage results for the 5-in. column containing no steel closely agreed with the experimental results using the same relationships for the diffusivity coefficient K (Eq. 2.8) and tensile strength f_t' (Eq. 2.14) used in the previous studies, and using a value of S_∞ = 875 micro-in./in. for the final unrestrained shrinkage strain. This value of S_∞ is reasonably close to the experimental results where it was found that plain concrete columns stored in air shrank up to 1000 millionths after 22 years.

The analytical shrinkage results were slightly higher during the first 150 days when compared to the experimental shrinkage values, but the difference between the results decreased and became negligible at 400 days. By making adjustments in the values of the shrinkage diffusivity function K and the tensile strength function f_t', the computed shrinkage values could be brought even closer to the experimental values.

The predicted values for the column deformation under load were very similar to those observed experimentally, being slightly higher before 100 days and then lower after 100 days. It must be recognized, however, that neither the shrinkage nor creep characteristics of the concrete used in the experiments were precisely modeled. The main purpose of this comparison is to demonstrate that the analytical model which has been developed is capable of predicting the time-dependent behavior of reinforced concrete columns under load with reasonable accuracy.

5. CONCLUSIONS

Based on the results obtained in this study, the following conclusions can be made:

1. The analytical model described in this report is capable of predicting the time-dependent behavior of plain and reinforced concrete columns.
2. The effects of shrinkage and creep are important in determining the stresses and deformations in reinforced concrete columns. Non-uniform shrinkage across the section of a prism must be considered in order to obtain a realistic evaluation of tensile stresses.
3. Reducing the time step used in the analysis from 7 to 2 days does have some effect on the long-term shrinkage strain of reinforced concrete columns. For 6- and 10-in. square columns, approximately 11% less cracking occurred using the shorter time step. Since the greater part of cracking took place during the early stages of drying, it would be desirable to use small time steps initially, and then switch over to longer steps at later ages.
4. Cracking in concrete must be considered in order to obtain a realistic evaluation of the stresses and deformations in a reinforced concrete column. Consideration of cracking is particularly important in the study of unloaded concrete columns, as significantly lower deformations were obtained as compared to the no-cracking case.

5. The accuracy of the solution improves with the fineness of mesh size used. The mesh size, however, affects only the structural solution, whereas the accuracy of the free shrinkage solution remains unaffected. It was determined that the shrinkage strains in a concrete prism could be accurately represented by using a mesh size of 100 elements per quadrant. The use of a finer mesh size of 225 elements per quadrant resulted in slightly lower shrinkage values.

6. For small columns on the order of 6 x 6 inches, it would be desirable to specify a minimum reinforcement ratio in order to control the shrinkage and creep deformations and to specify a maximum reinforcement ratio in order to control the development of tensile stresses.

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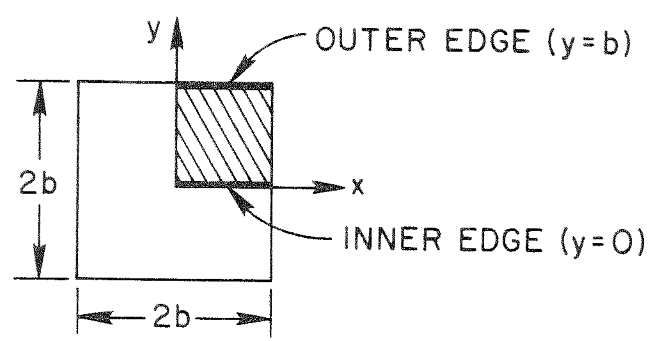
TABLE 1: VALUES OF BETA USED IN DIFFUSION EQUATION

COLUMN SIZE			
6 X 6-IN.	10 X 10-IN.	15 X 15-IN.	20 X 20-IN.
1.314251175765158	1.404188871426300	1.455136292638734	1.482269652580172
4.034438535130249	4.242313035752403	4.376247866614676	4.451868482595415
6.910483933458664	7.146117061874008	7.324802713295554	7.435112914676353
9.893525894204004	10.114884991724750	10.307015861954426	10.436996646873467
12.935876631341671	13.132710622743105	13.320981403817143	13.458827960919621
16.011217627659562	16.184275783301587	16.361326392558453	16.499403528759444
19.106003448311071	19.258663906726952	19.422321006778361	19.556334792952839
22.212980551983833	22.348711751751466	22.499112499077114	22.626962584932016
25.328024944528465	25.449771922624677	25.587944192459872	25.708822857012137
28.448653214318028	28.558784709735505	28.686008824301894	28.799816506798265
31.573292933704806	31.673687763316821	31.791229551741708	31.898228350460499
34.700905031410002	34.793055114389745	34.902066325804071	35.002684642601253
37.830777537823451	37.915876151350631	38.017367565038967	38.112093836639133
40.962407310011940	41.041418042151918	41.136262533660329	41.225589545314051
44.095429230347463	44.169138199420104	44.258084785256642	44.342482091710508
47.229572197776179	47.298627274157525	47.382317913979705	47.462219615831827
50.364630855235191	50.429571173517843	50.508556954219102	50.584357692618141
53.500446891592674	53.561725200764840	53.636480689095379	53.708535895307023
56.636896354387318	56.694896090482871	56.765831581529937	56.834459796354167
59.773880842638164	59.828929294377076	59.896401068155456	59.961887140130557
62.911321267380799	62.963699827866776	63.028018660965017	63.090617187243652
66.049153349794778	66.099105575862723	66.160543780498756	66.220482463469580
69.187324319044365	69.235062325835315	69.293859569657343	69.351342333637604
72.325790453675836	72.371500033203120	72.427868159185891	72.483077965146094

TABLE 2 VALUES OF FREE SHRINKAGE, S/S_∞, FOR 6 AND 20-IN. PRISMS

Location	Inner Edge (y=0)						Outer Edge (y=b)					
Prism Size, in.	6			20			6			20		
Days After Drying	7	70	364	7	70	364	7	70	364	7	70	364
Age, Days	21	84	378	21	84	378	21	84	378	21	84	378
(x/b)												
0.0	.002	.302	.775	0.	0.	.004	.603	.816	.943	.603	.778	.854
0.2	.005	.323	.783	0.	0.	.009	.604	.822	.945	.603	.778	.855
0.4	.023	.389	.806	0.	.001	.042	.612	.839	.951	.603	.779	.860
0.6	.092	.496	.842	0.	.026	.154	.639	.867	.960	.603	.784	.876
0.8	.273	.642	.888	.013	.211	.421	.711	.906	.972	.608	.825	.915
1.0	.603	.816	.943	.603	.778	.854	.842	.952	.986	.842	.951	.979



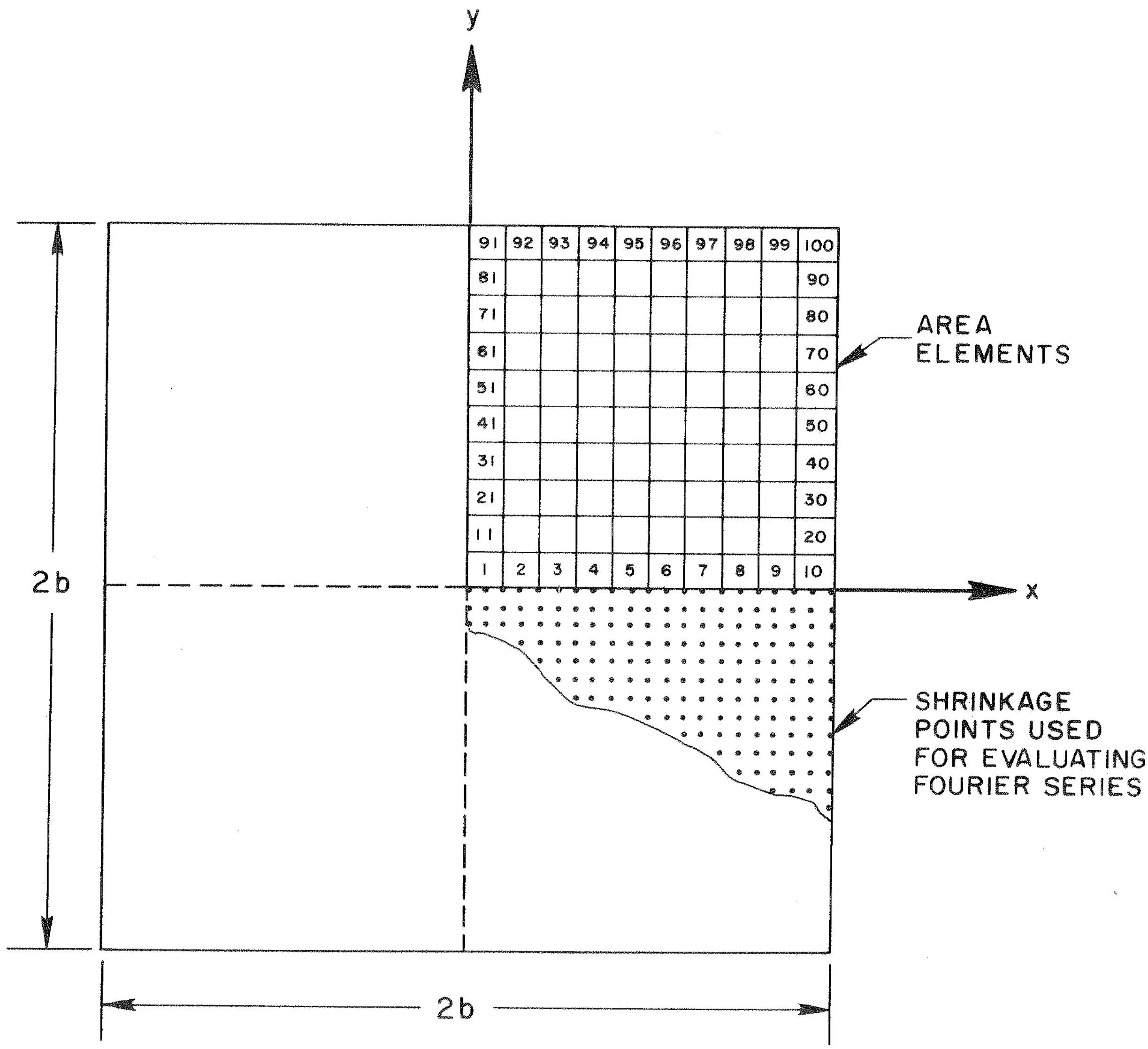


FIG. I SUBDIVISION OF PRISM CROSS-SECTION FOR ANALYSIS

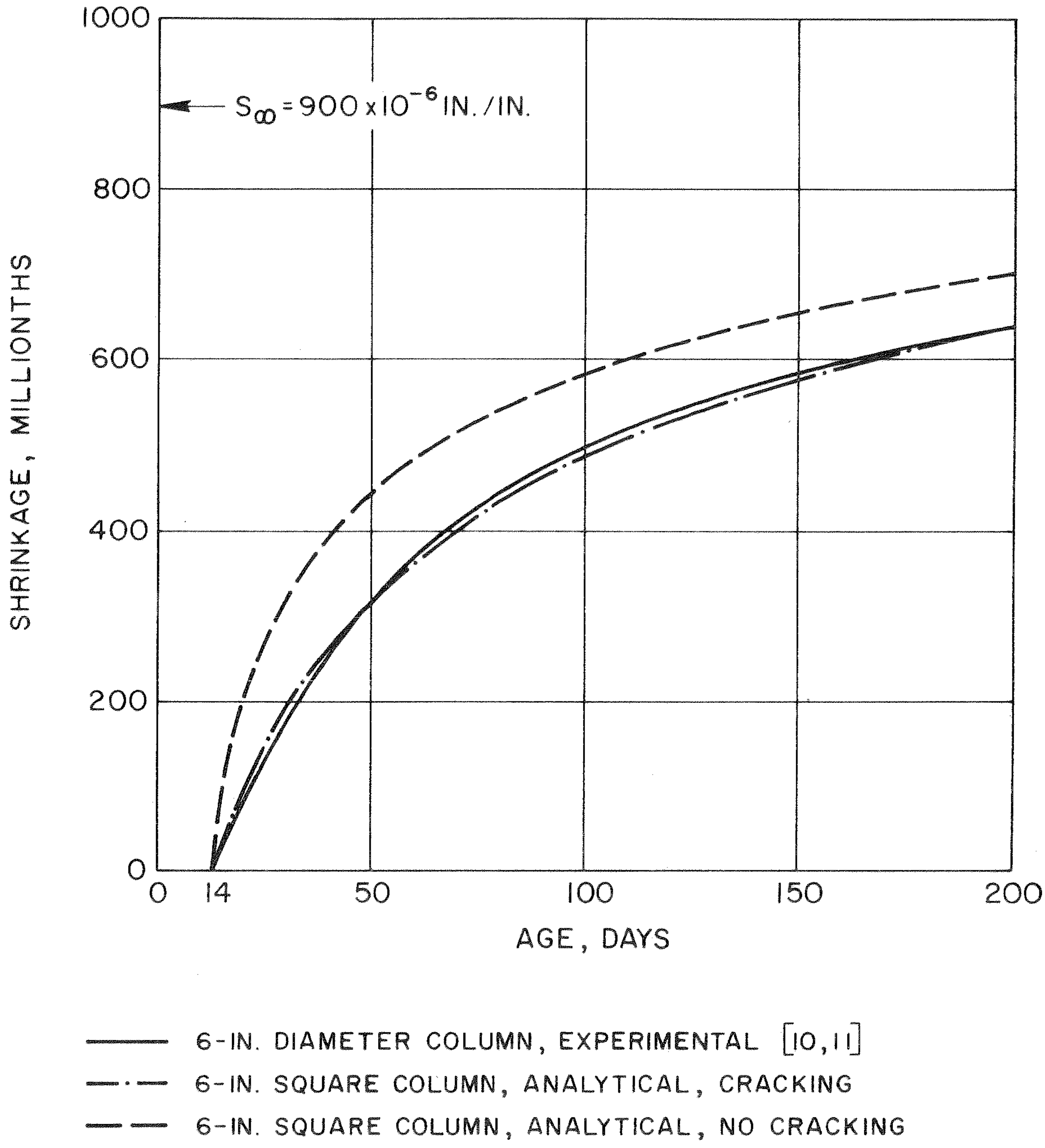


FIG. 2 COMPARISON OF SHRINKAGE RESULTS FOR PLAIN 6-IN. COLUMNS

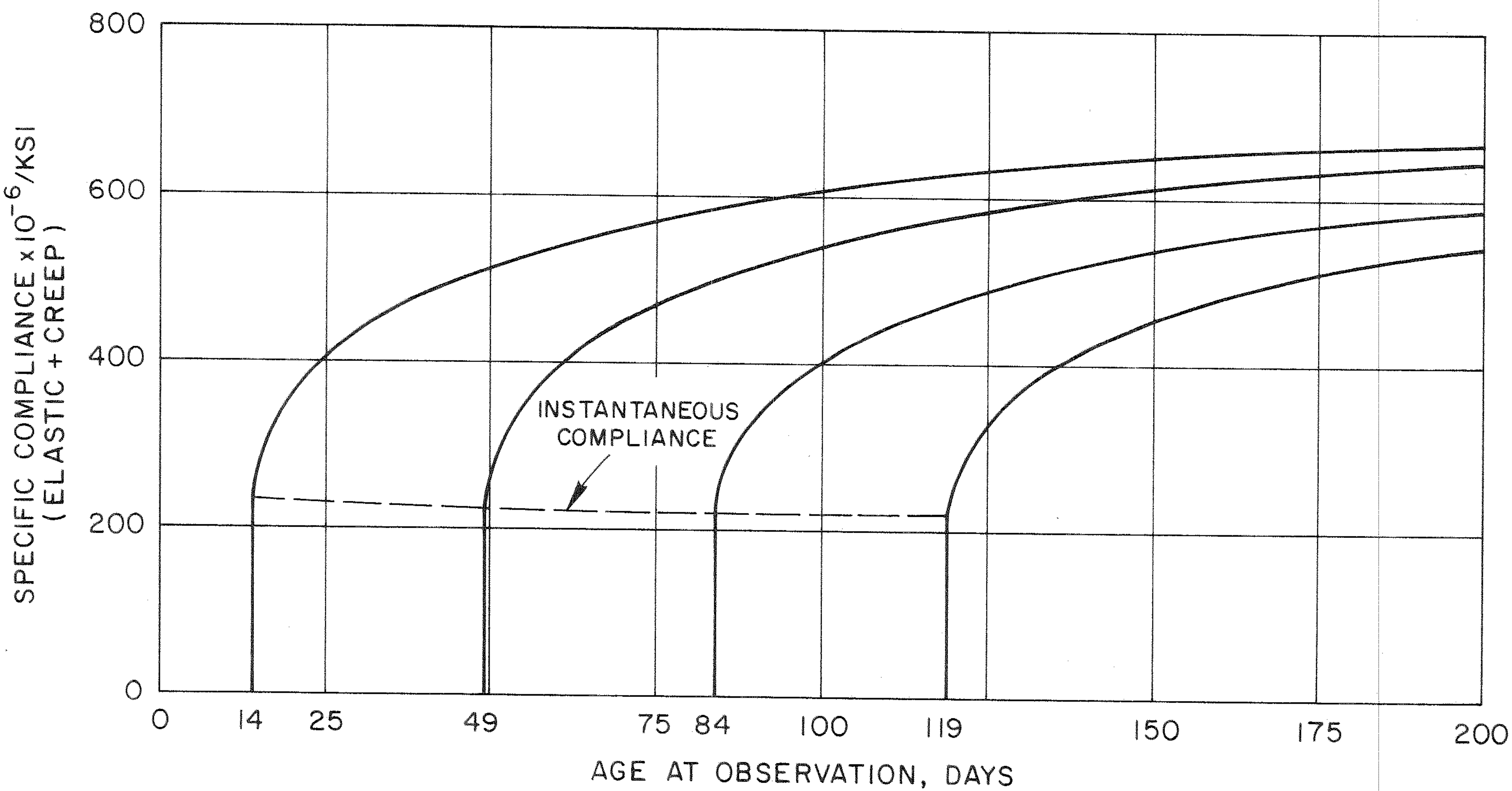
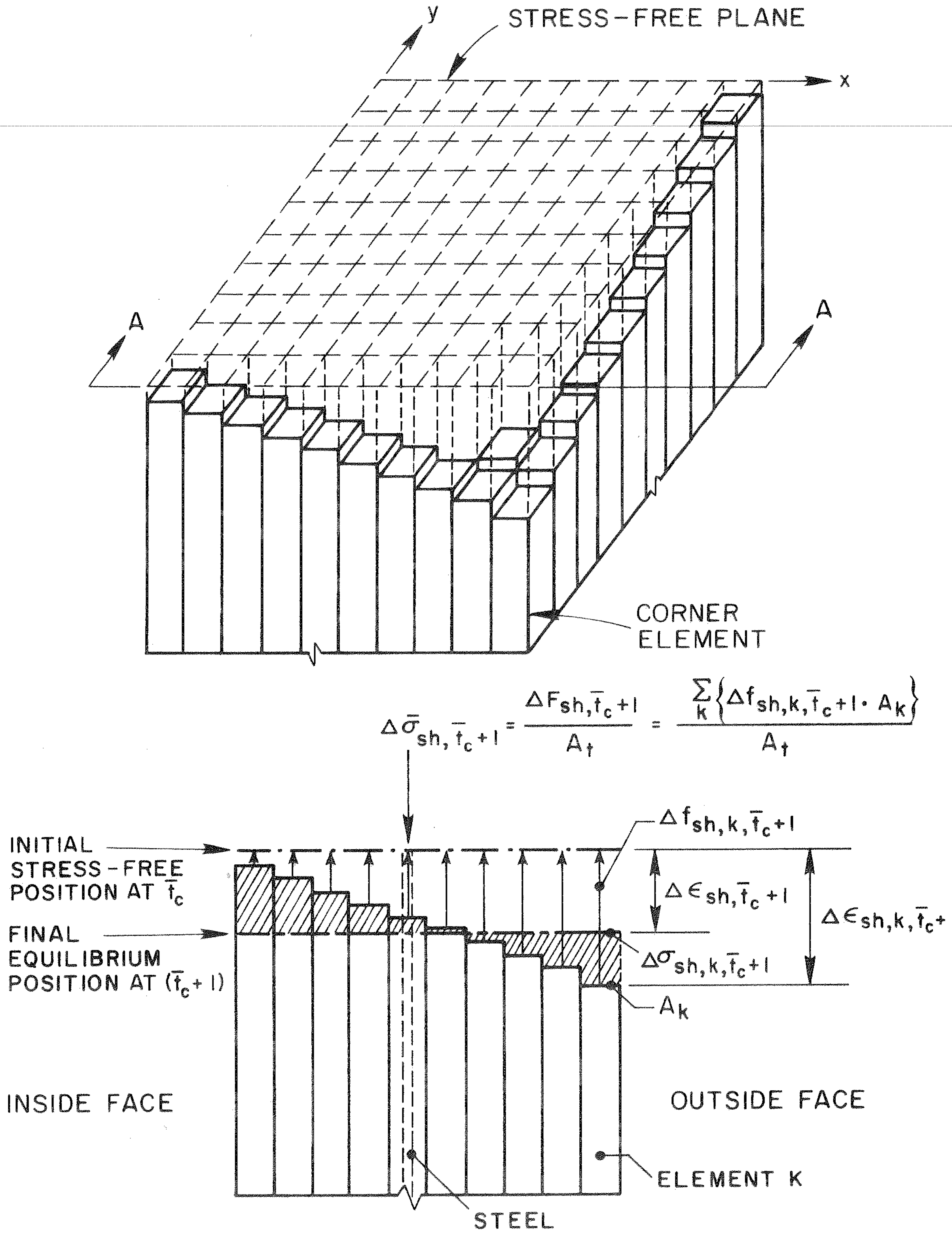
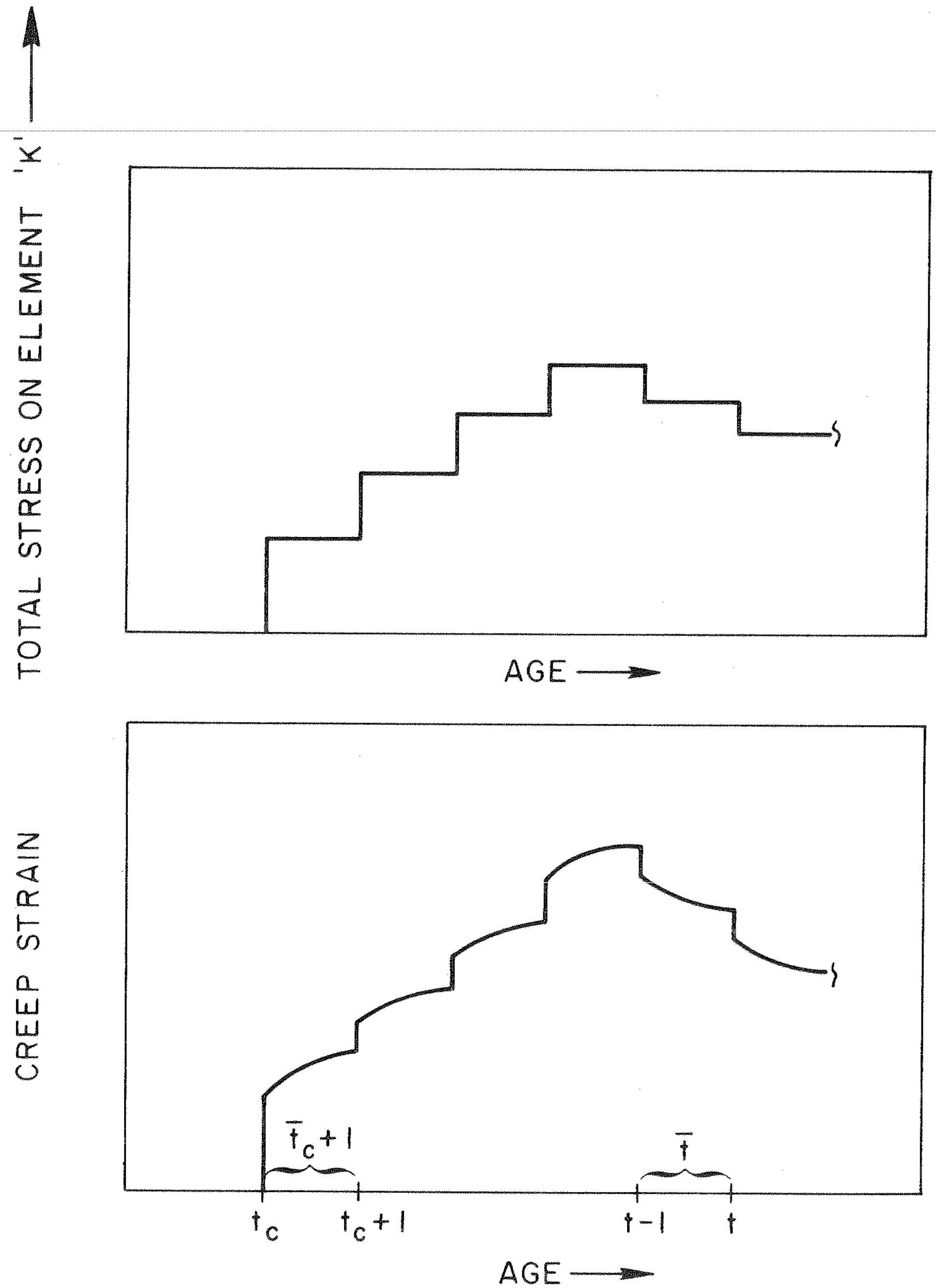


FIG. 3 SPECIFIC CREEP COMPLIANCE CURVES USED IN THE ANALYSIS



SECT. A-A: ELEVATION OF COLUMN QUADRANT

FIG. 4 SHRINKAGE ANALYSIS ROUTINE FOR QUADRANT DURING FIRST TIME STEP AFTER CURING



NOTE: $\bar{t}_c$ = NO. OF TIME STEPS CONCRETE IS CURED.

FIG. 5 STRESS-STRAIN HISTORY FOR A TYPICAL UNRESTRAINED ELEMENT 'K'

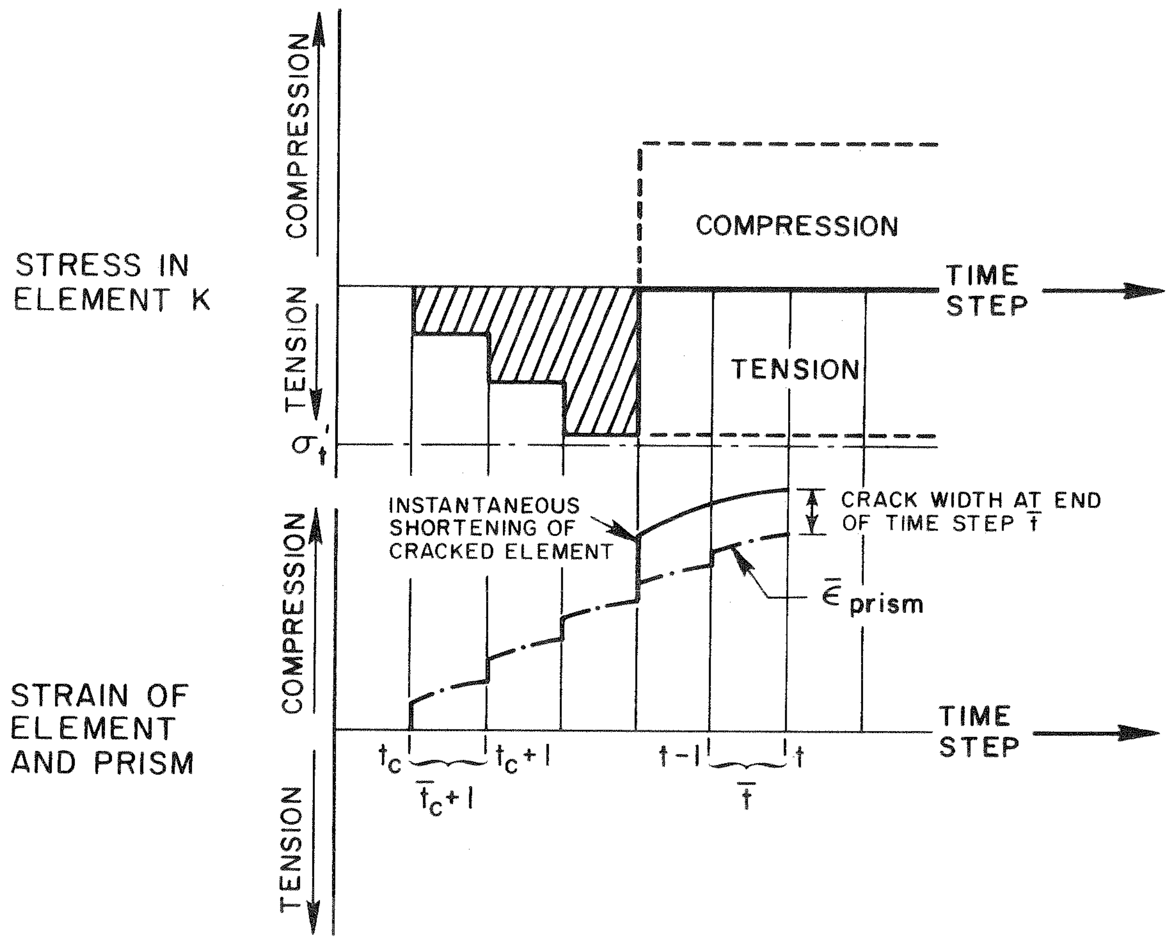


FIG. 6 DETERMINATION OF CRACK WIDTH

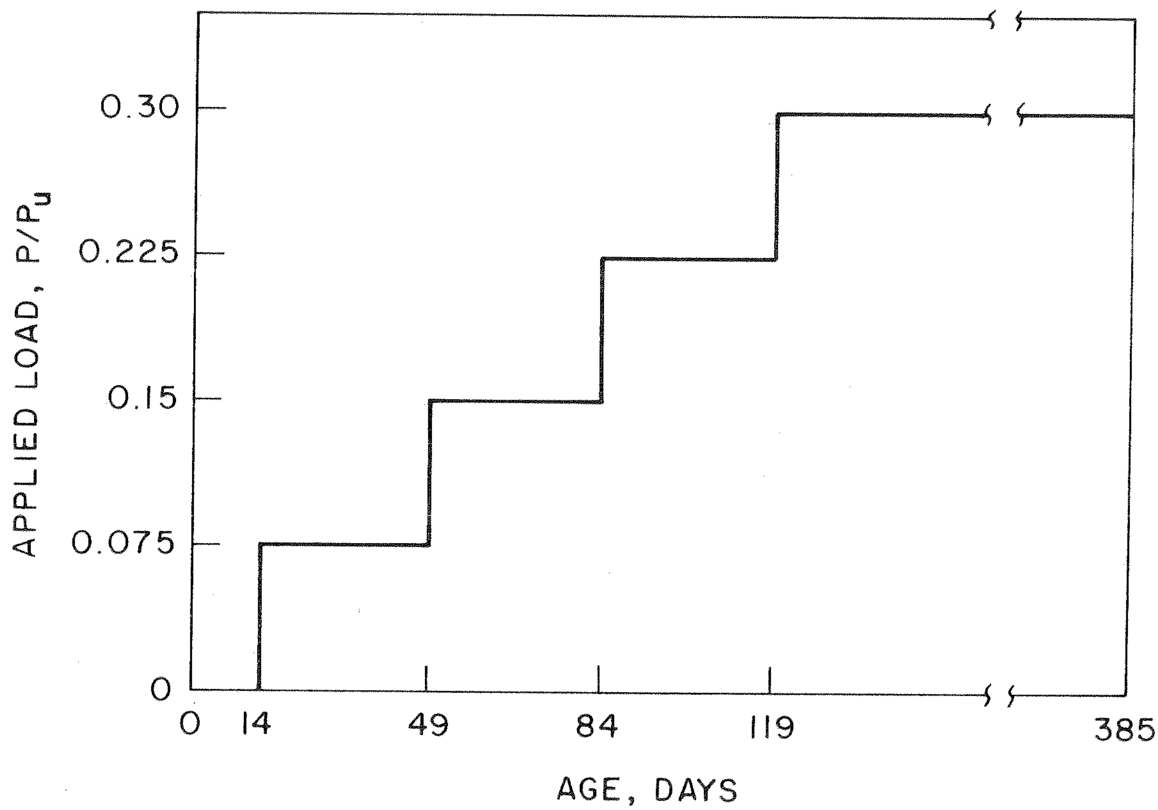


FIG. 7 LOAD HISTORY FOR REINFORCED CONCRETE COLUMNS USED IN THE ANALYSIS

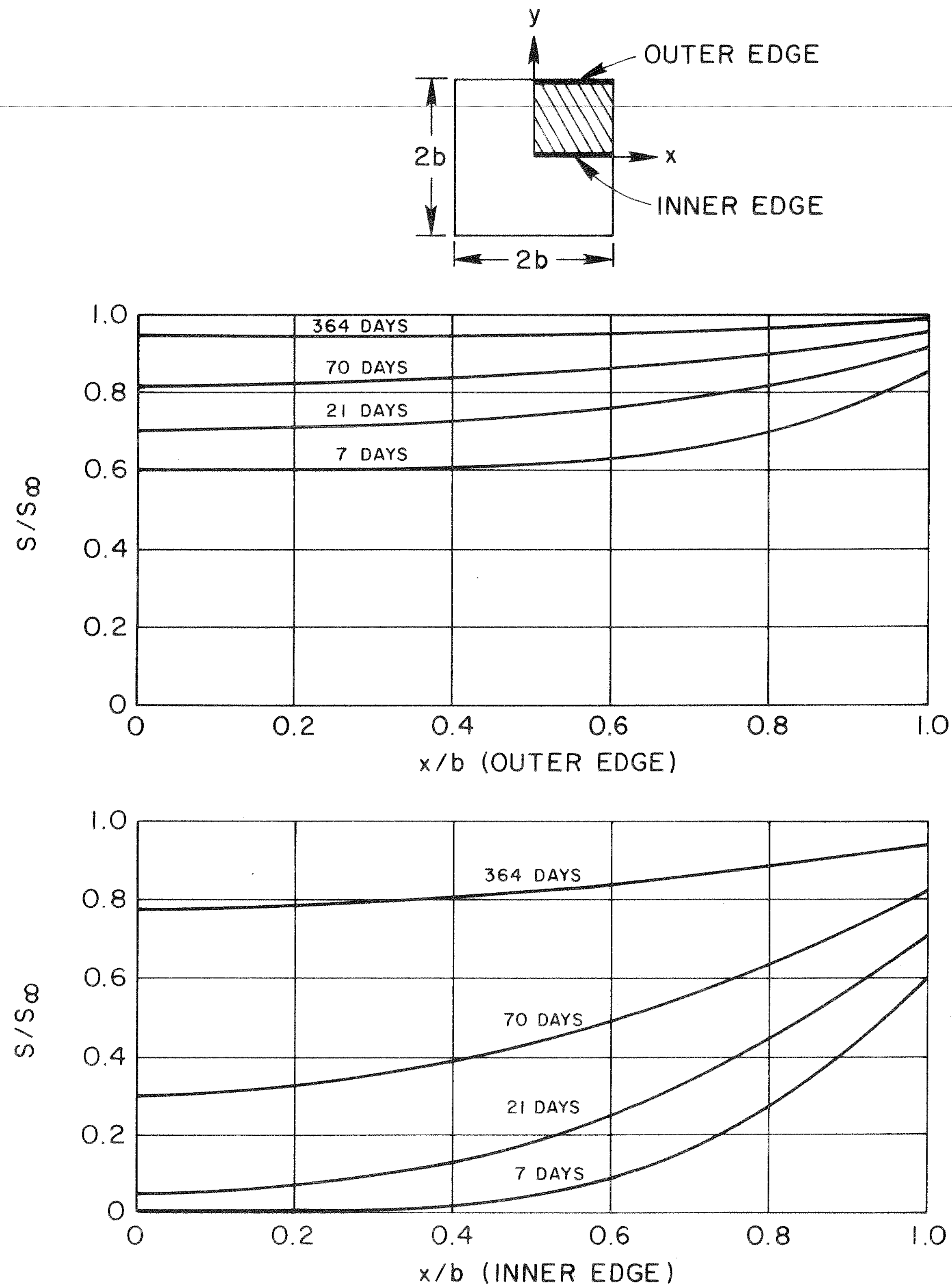


FIG. 8 VARIATION OF FREE SHRINKAGE WITHIN A 6 BY 6-IN. PRISM

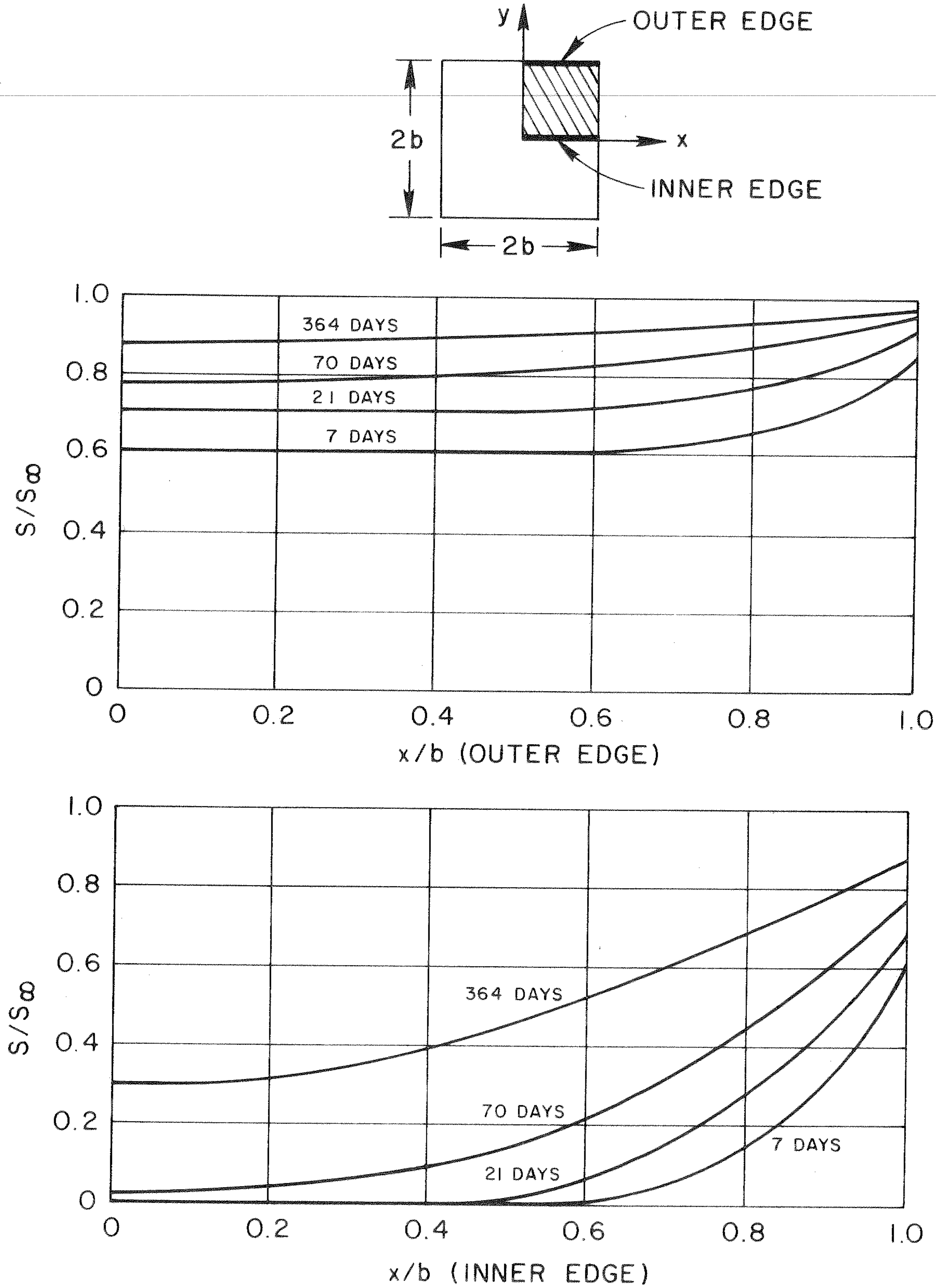


FIG. 9 VARIATION OF FREE SHRINKAGE WITHIN A 10 BY 10-IN. PRISM

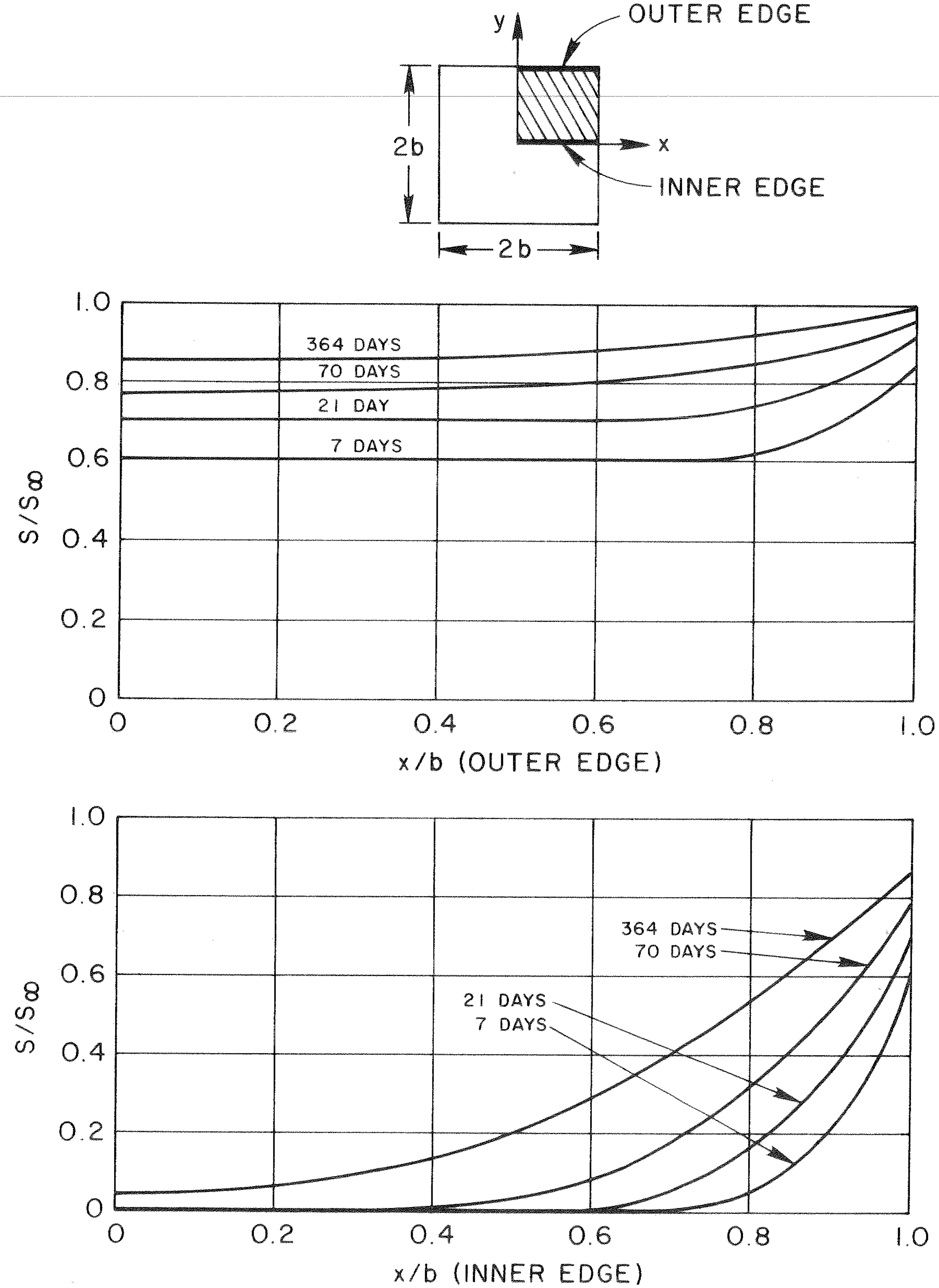


FIG. 10 VARIATION OF FREE SHRINKAGE WITHIN A 15 BY 15-IN. PRISM

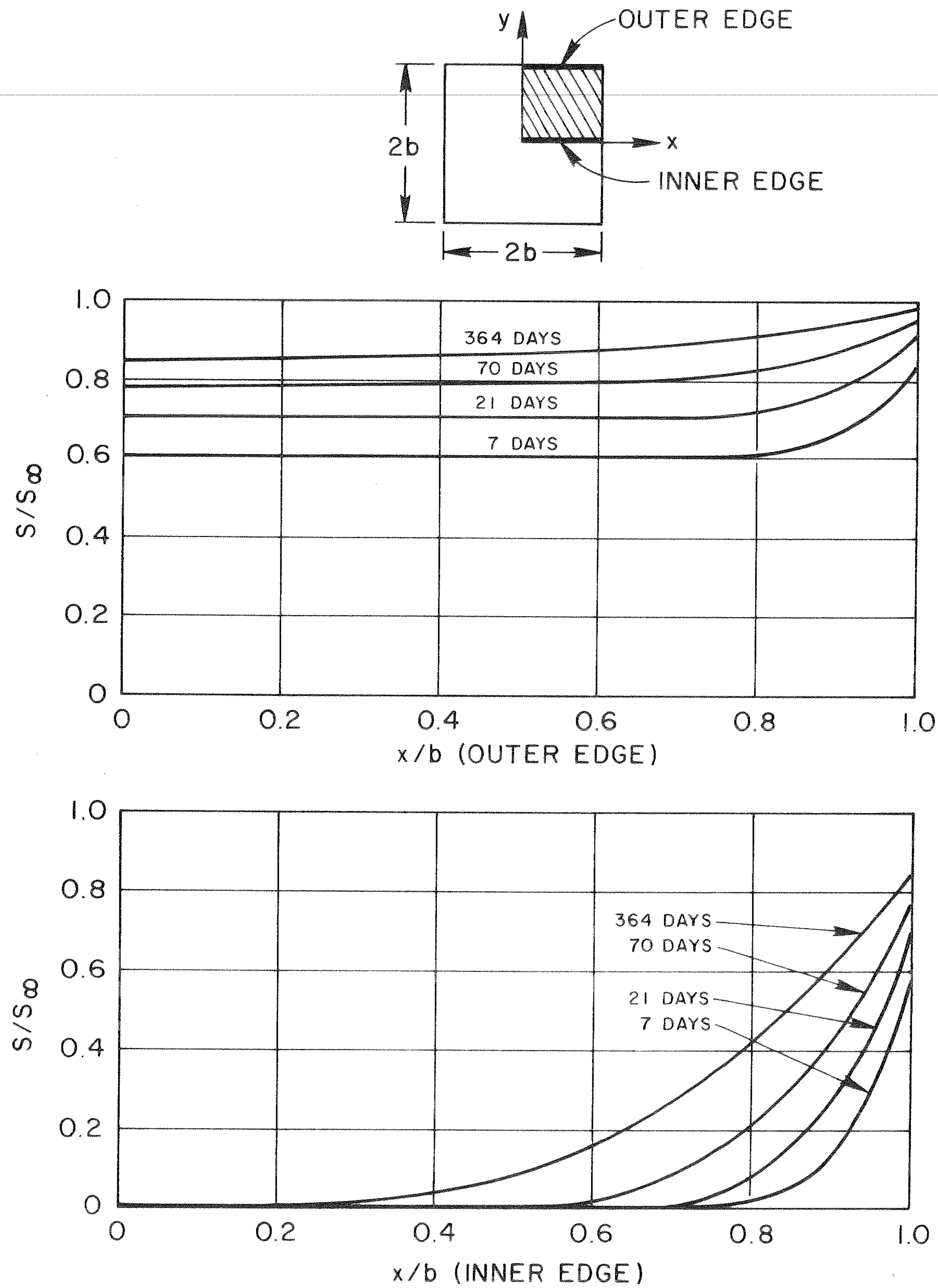


FIG. 11 VARIATION OF FREE SHRINKAGE WITHIN A 20 BY 20-IN. PRISM

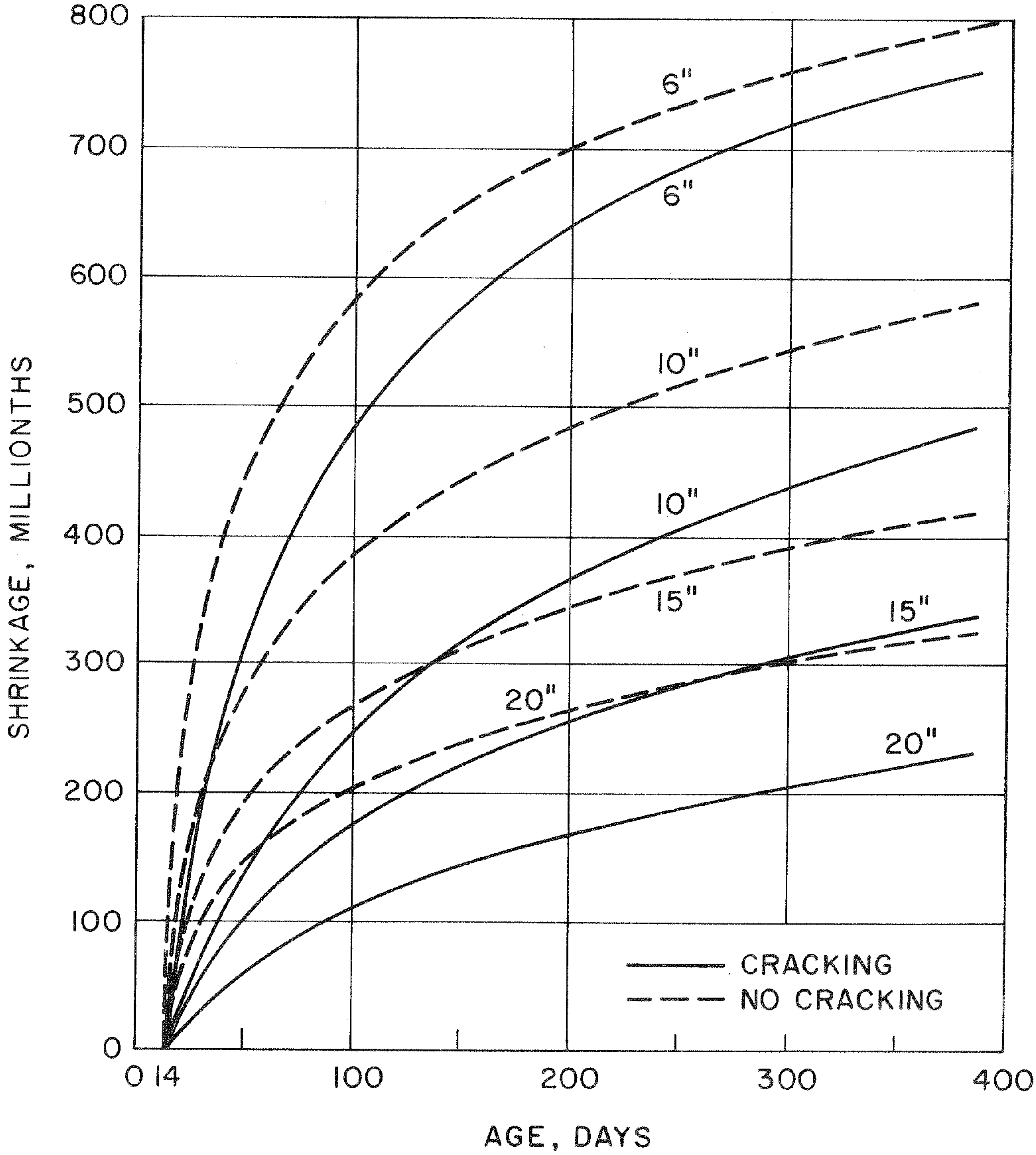
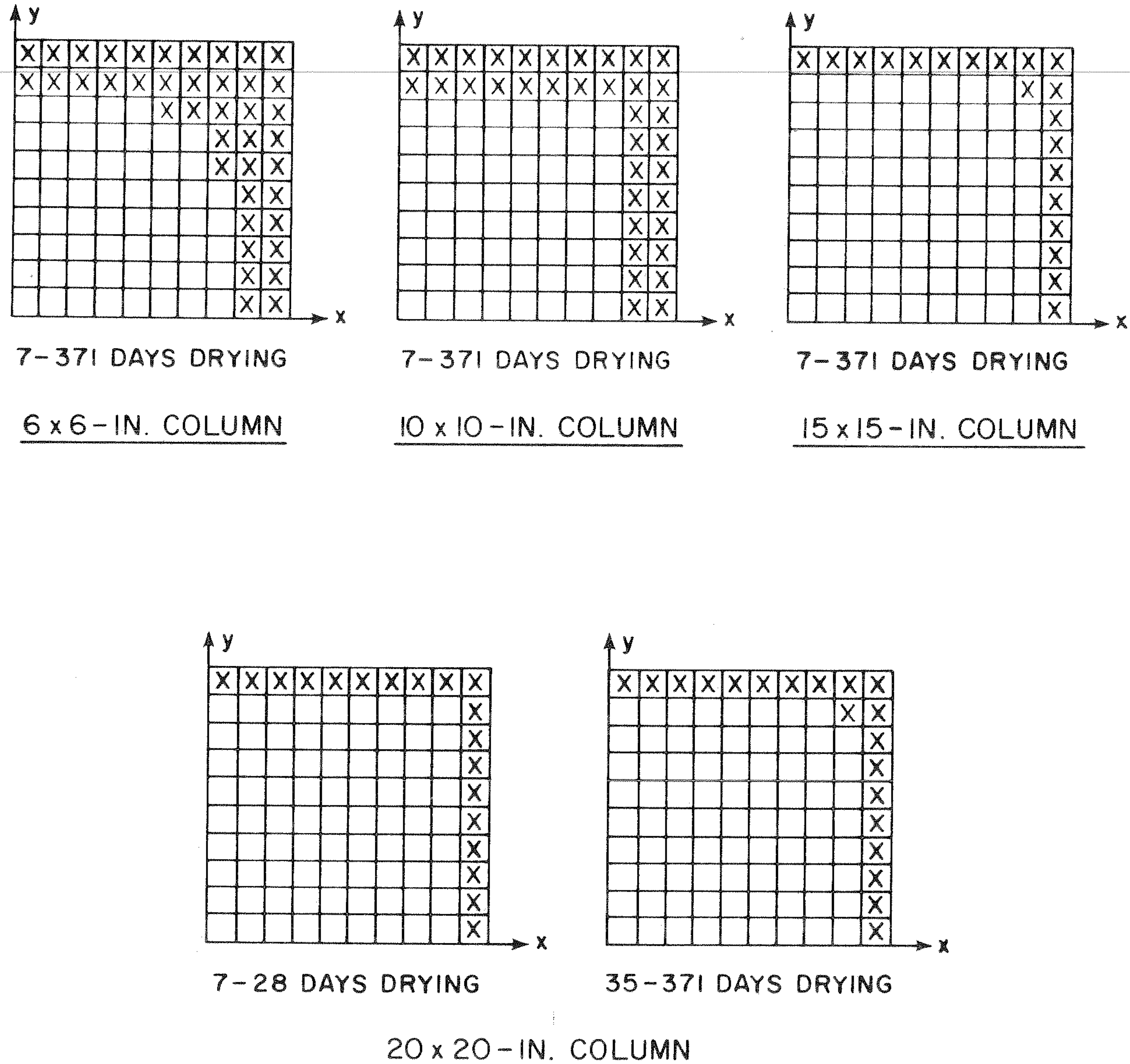


FIG. 12 SHRINKAGE RESULTS FOR UNREINFORCED PRISMS USING PICKETT'S DIFFUSION EQUATION



NOTE: ONE QUADRANT SHOWN.
14-DAYS CURING.
X=CRACKED ELEMENT.

FIG. 13 ELEMENT CRACKING PATTERNS IN UNLOADED 6, 10, 15 AND 20-IN. PLAIN CONCRETE PRISMS

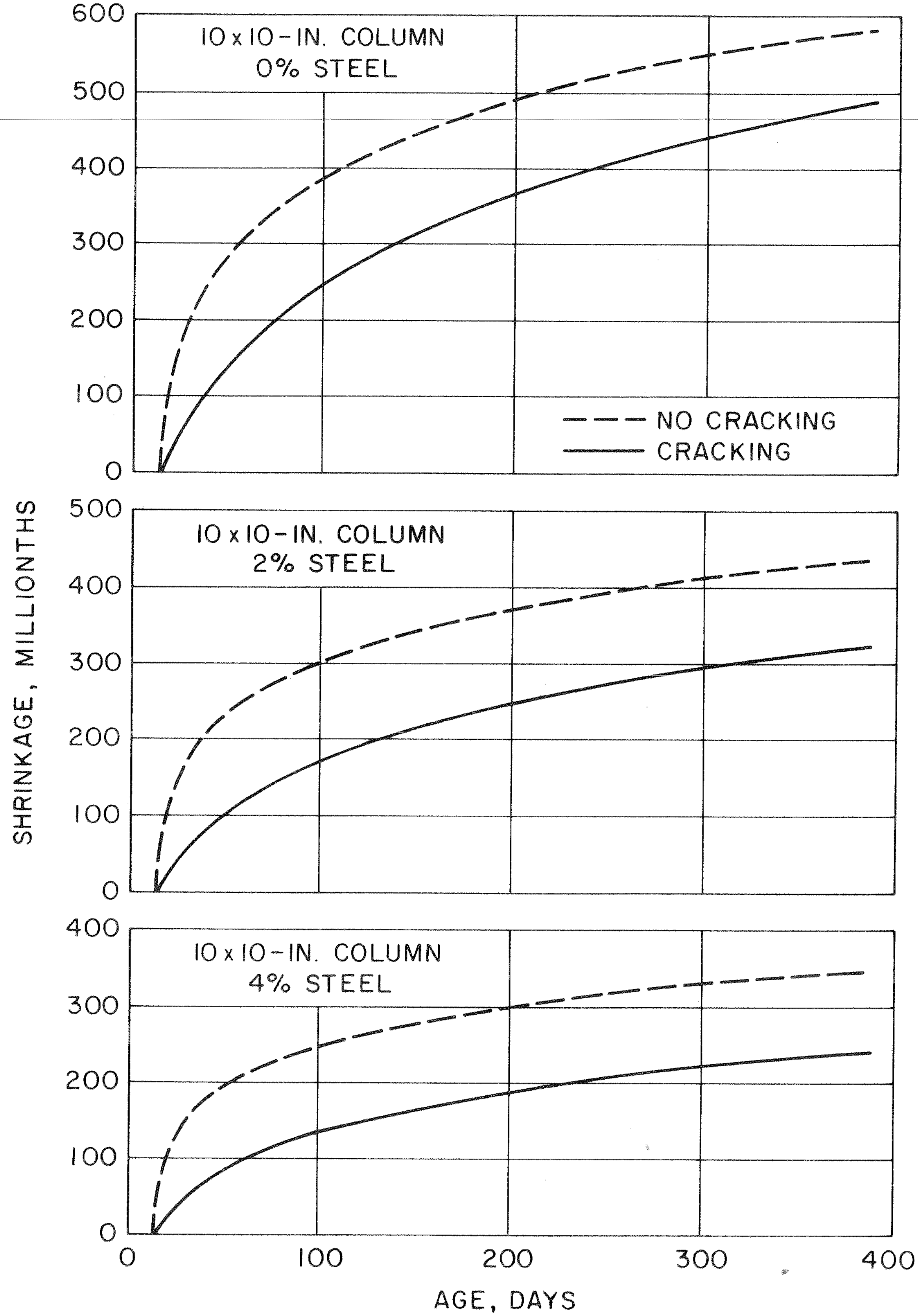


FIG. 14 SHRINKAGE RESULTS FOR UNLOADED 10 BY 10-IN. CONCRETE PRISMS

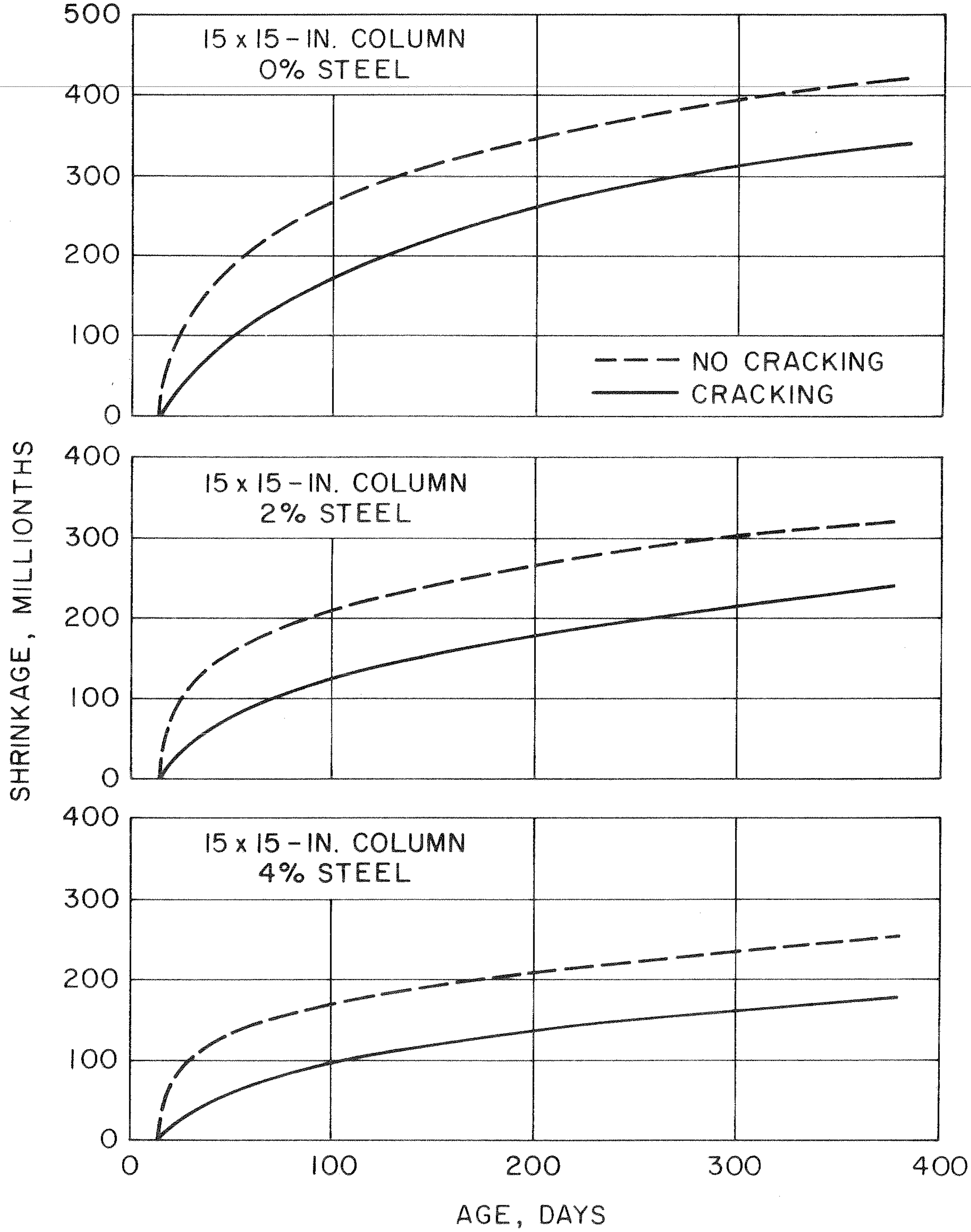


FIG. 15 SHRINKAGE RESULTS FOR UNLOADED 15 BY 15-IN. CONCRETE PRISMS

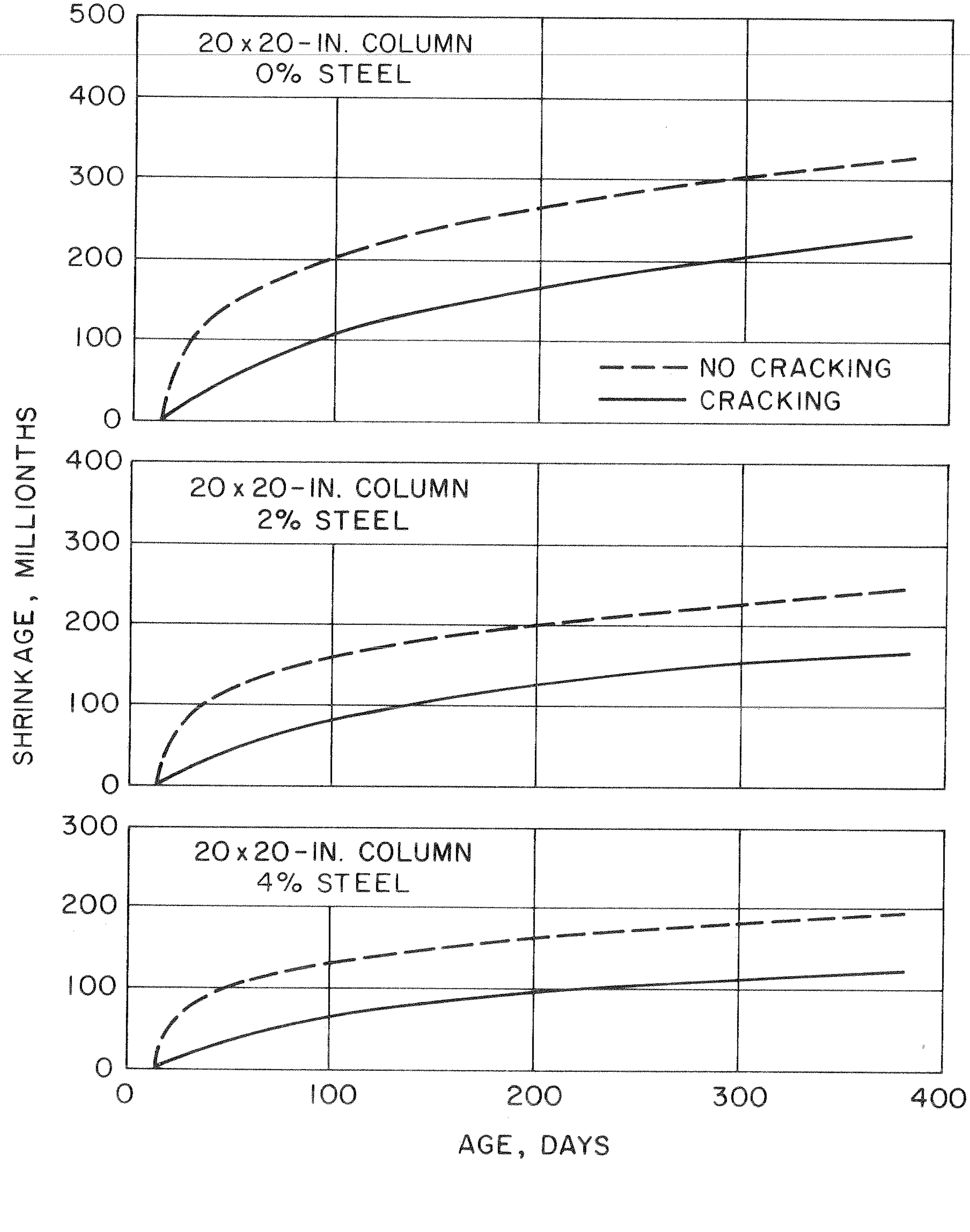
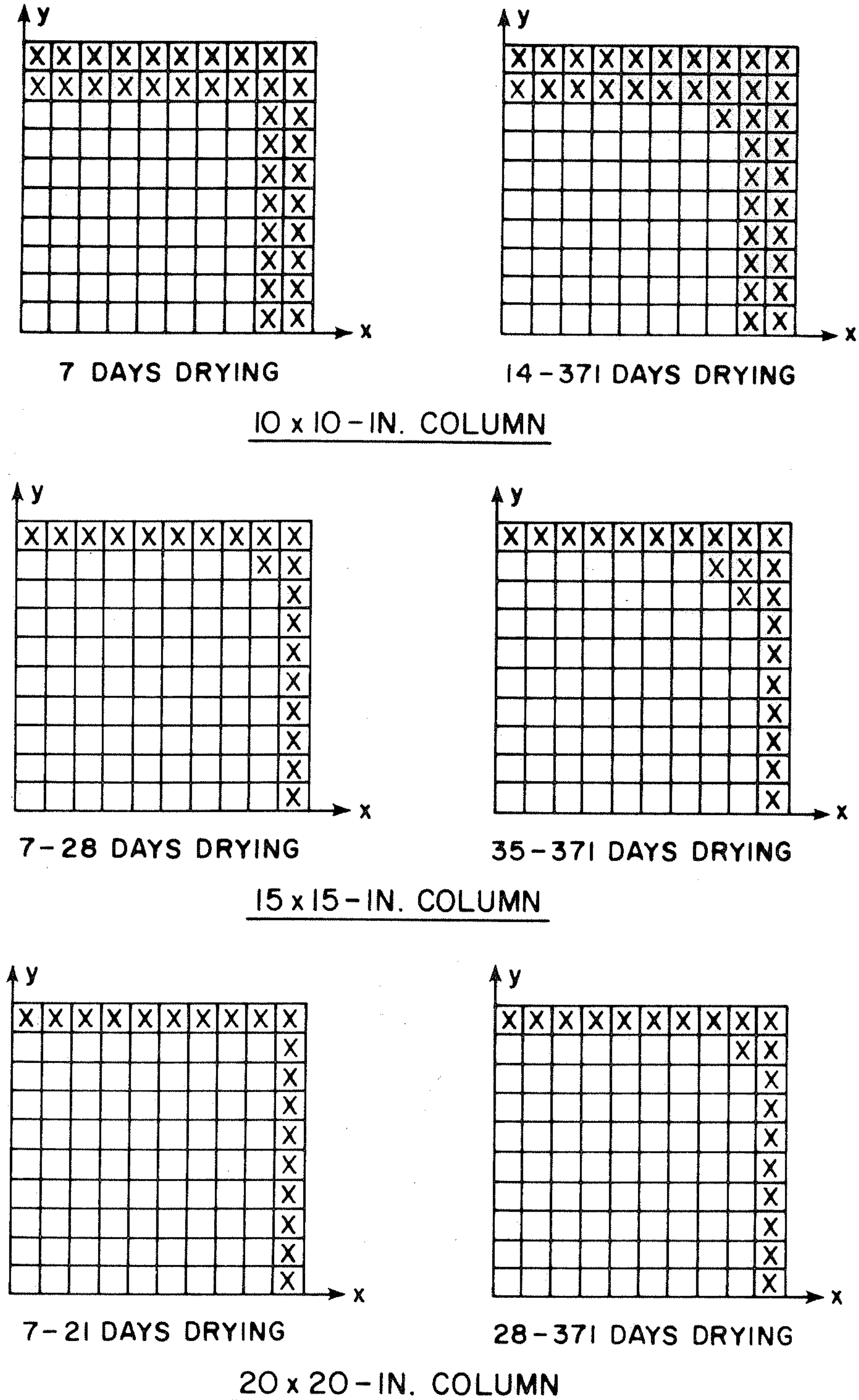
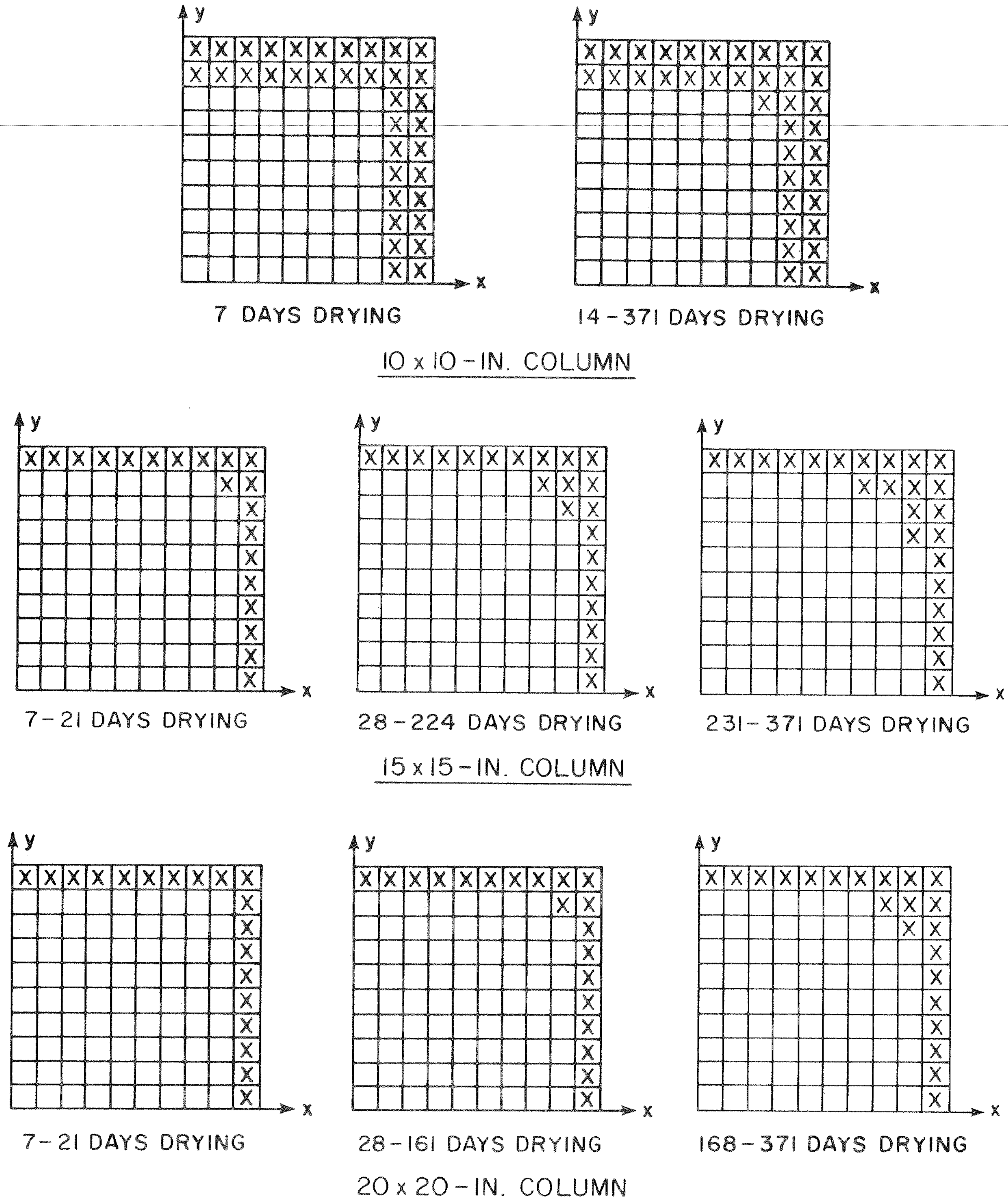


FIG. 16 SHRINKAGE RESULTS FOR UNLOADED 20 BY 20-IN. CONCRETE PRISMS



NOTE: ONE QUADRANT SHOWN.
14-DAYS CURING.
X = CRACKED ELEMENT.

FIG. 17 ELEMENT CRACKING PATTERNS IN UNLOADED 10, 15 AND 20-IN. CONCRETE PRISMS CONTAINING 2 PERCENT STEEL REINFORCEMENT



NOTE: ONE QUADRANT SHOWN.
14-DAYS CURING.
X = CRACKED ELEMENT.

FIG. 18 ELEMENT CRACKING PATTERNS IN UNLOADED 10, 15 AND 20-IN. CONCRETE PRISMS CONTAINING 4 PERCENT STEEL REINFORCEMENT

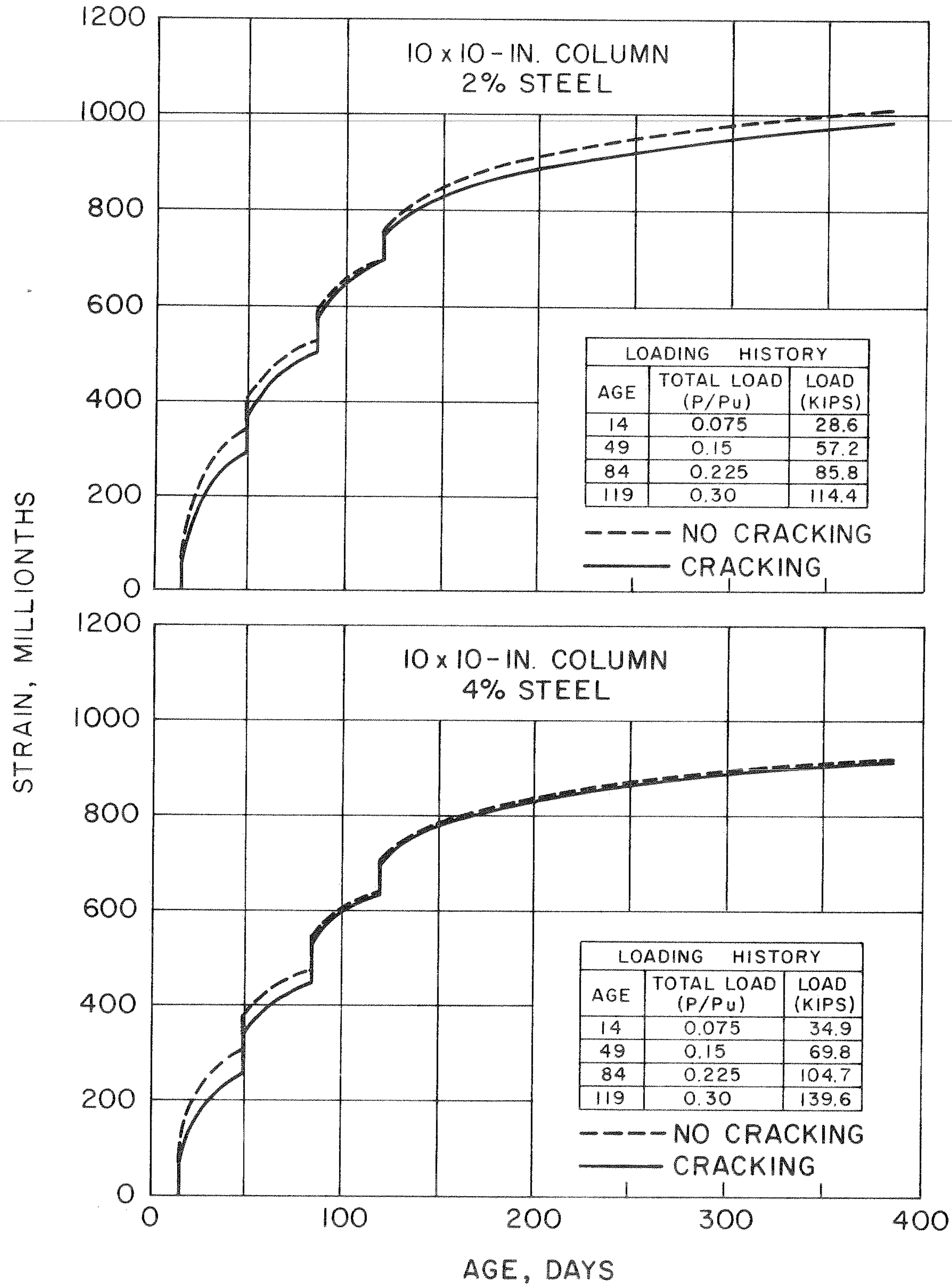


FIG. 19 DEFORMATION OF 10 BY 10-IN. CONCRETE COLUMNS LOADED IN FOUR EQUAL INCREMENTS TO 0.3 P_u

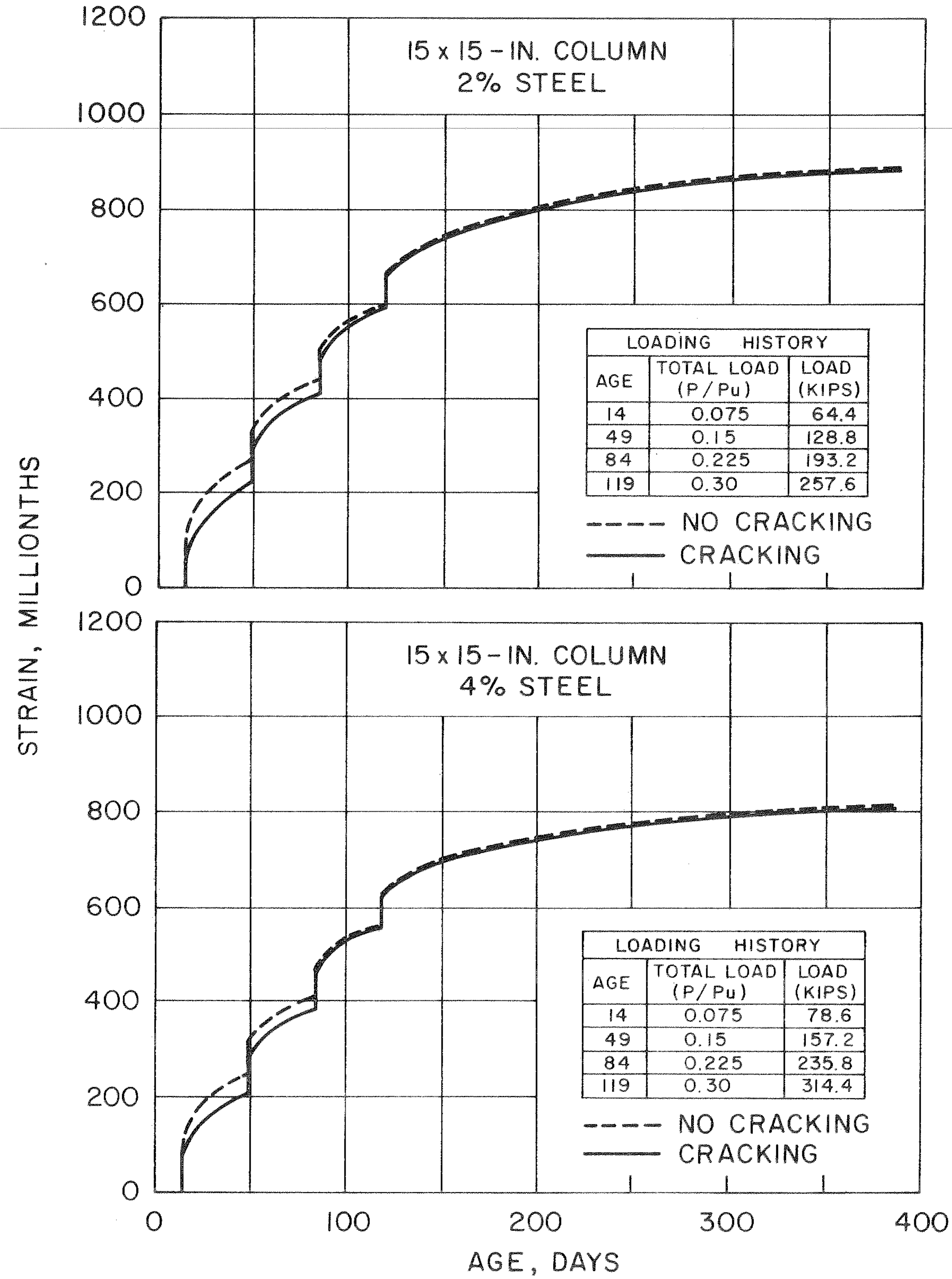


FIG. 20 DEFORMATION OF 15 BY 15-IN. CONCRETE COLUMNS LOADED IN FOUR EQUAL INCREMENTS TO $0.3\,P_u$

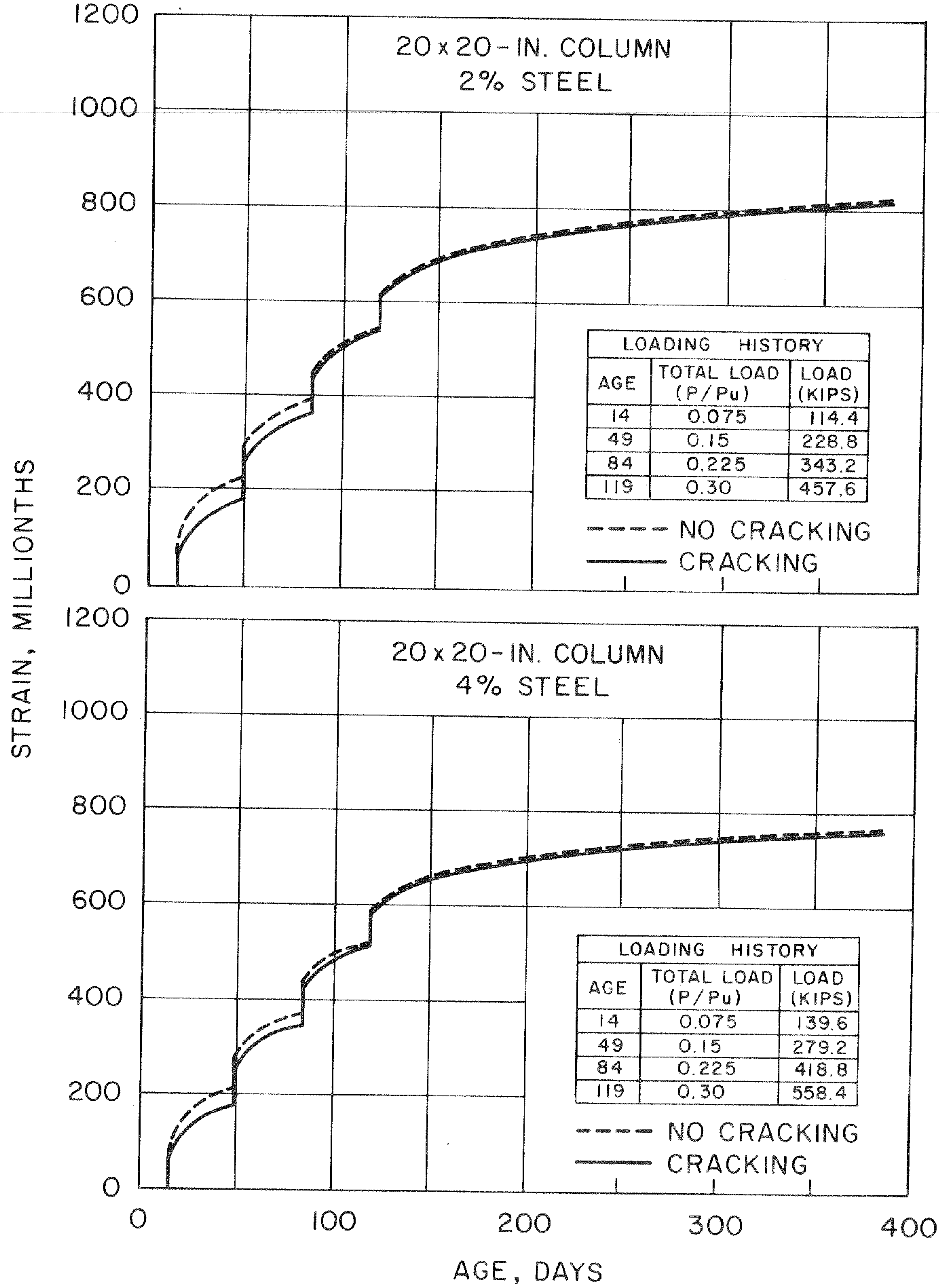


FIG. 21 DEFORMATION OF 20 BY 20-IN. CONCRETE COLUMNS LOADED IN FOUR EQUAL INCREMENTS TO 0.3 P_u

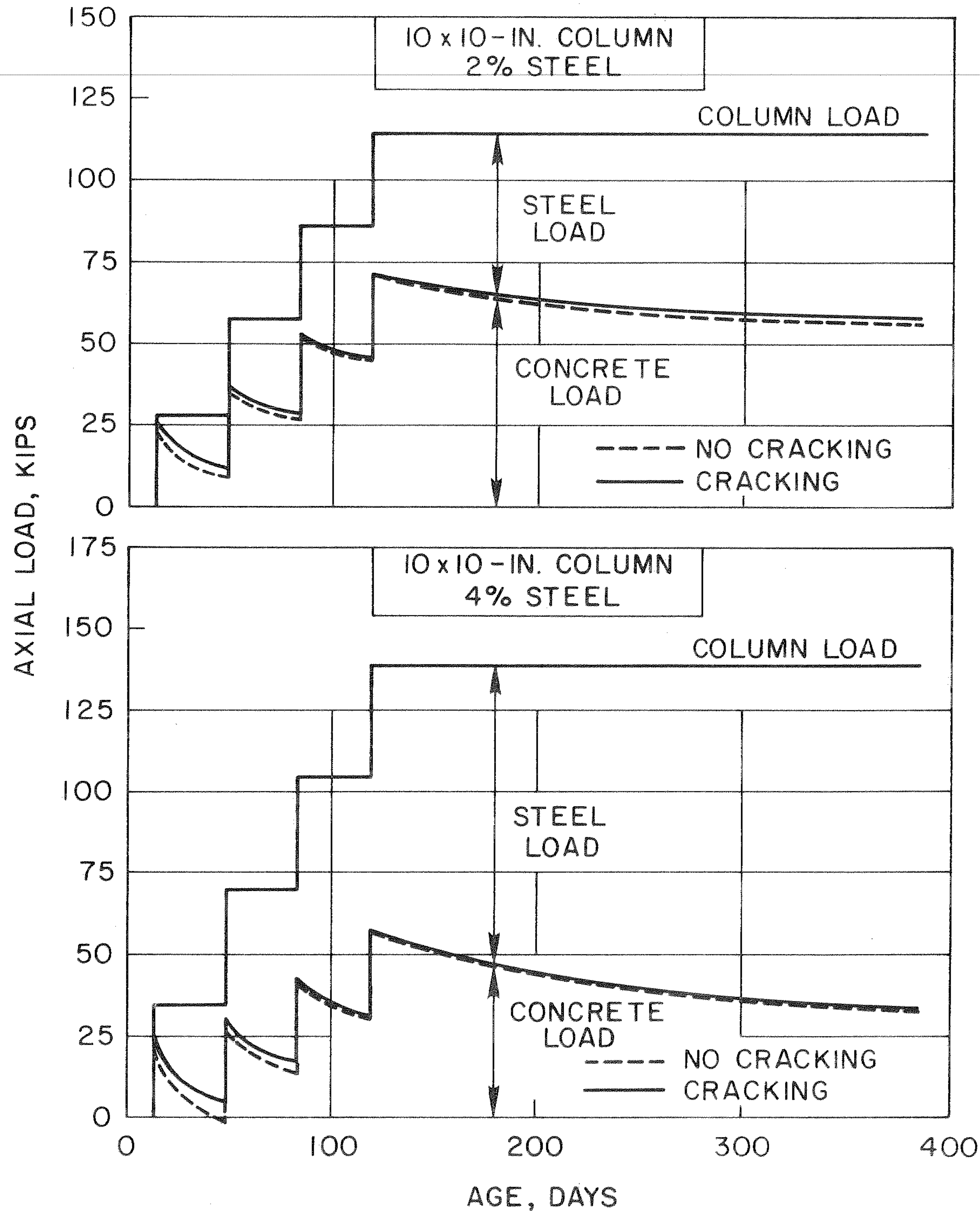


FIG. 22 TRANSFER OF LOAD FROM CONCRETE TO STEEL IN 10 BY 10-IN. REINFORCED CONCRETE COLUMNS

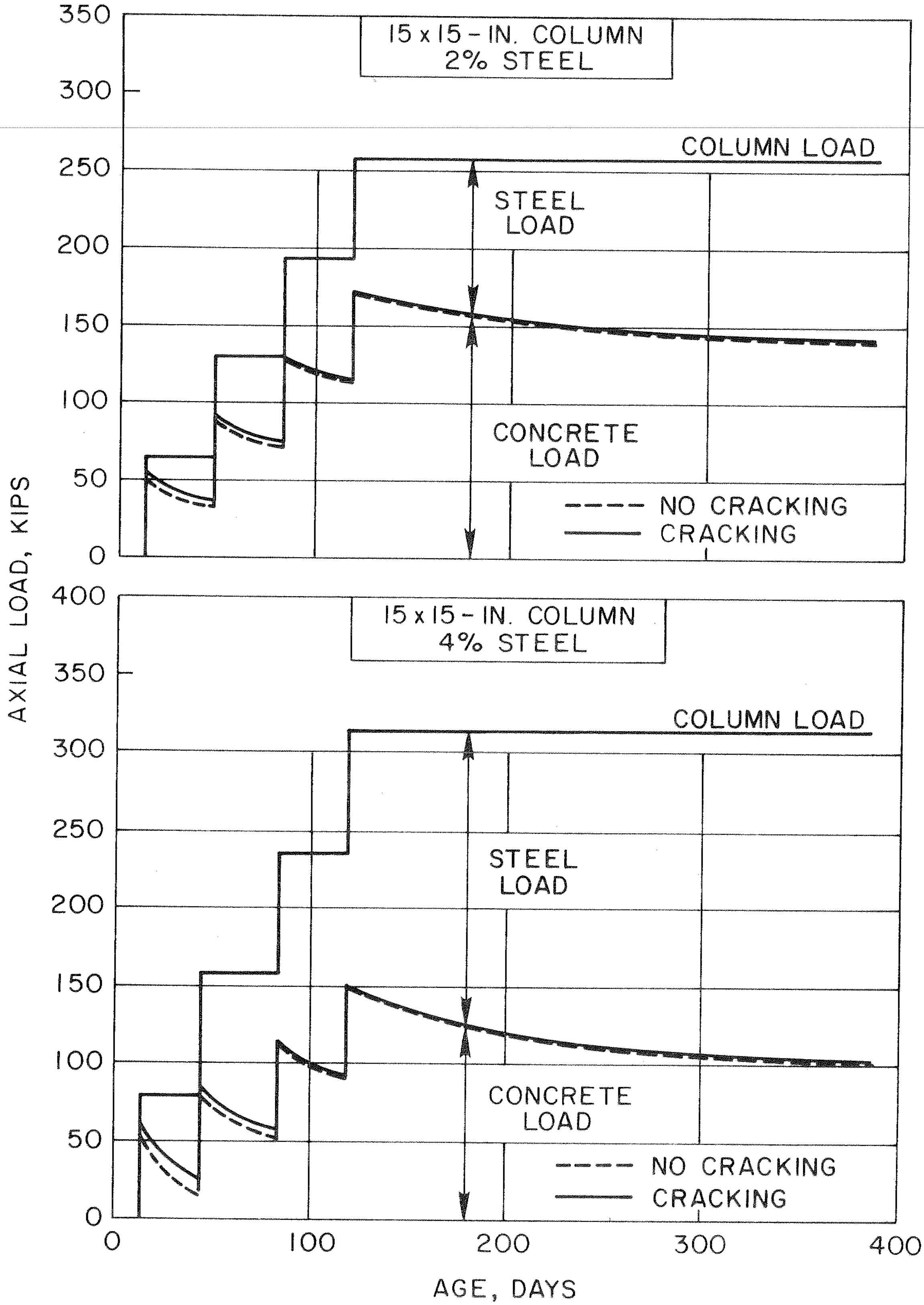


FIG. 23 TRANSFER OF LOAD FROM CONCRETE TO STEEL IN 15 BY 15-IN. REINFORCED CONCRETE COLUMNS

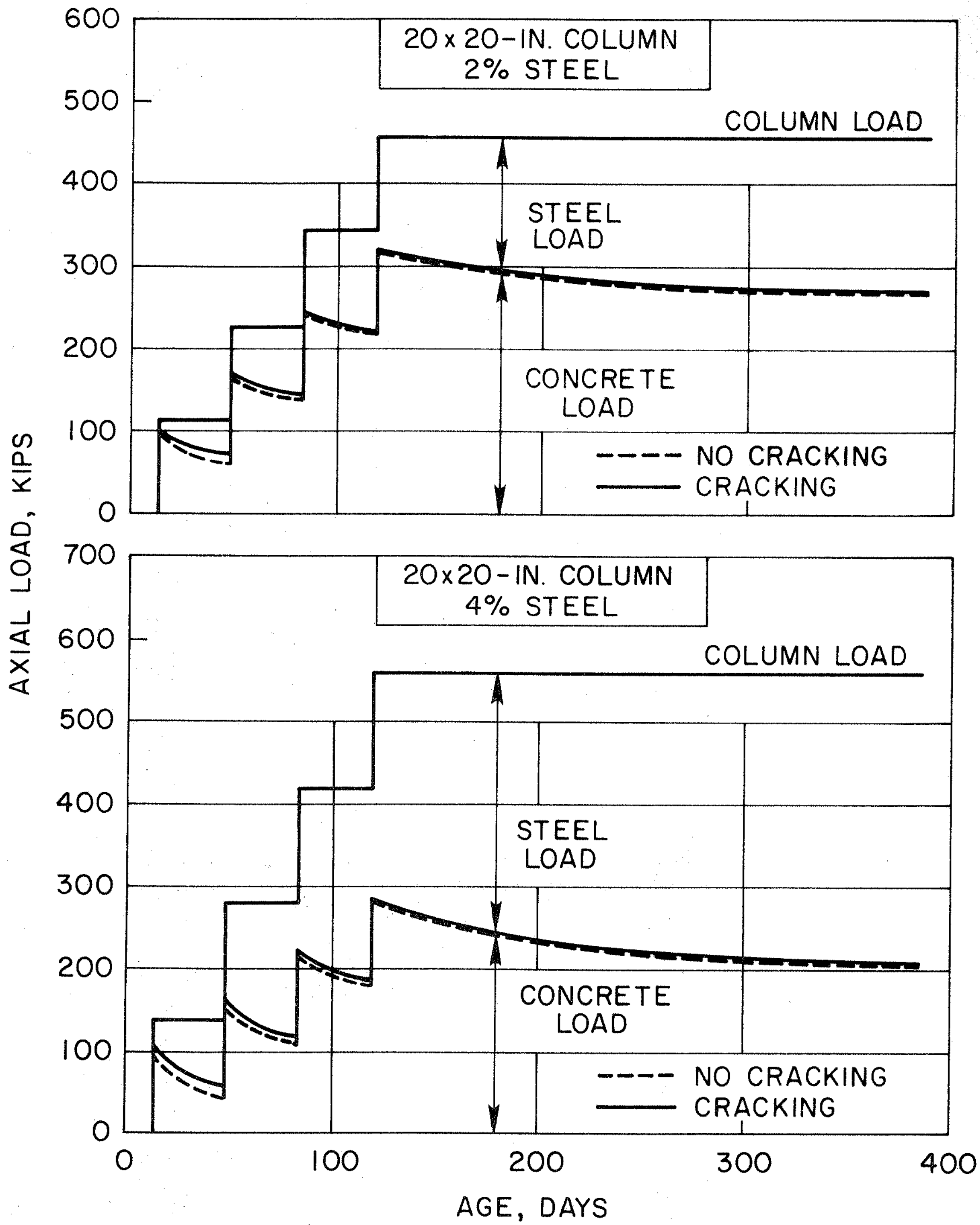


FIG. 24 TRANSFER OF LOAD FROM CONCRETE TO STEEL IN 20 BY 20-IN. REINFORCED CONCRETE COLUMNS

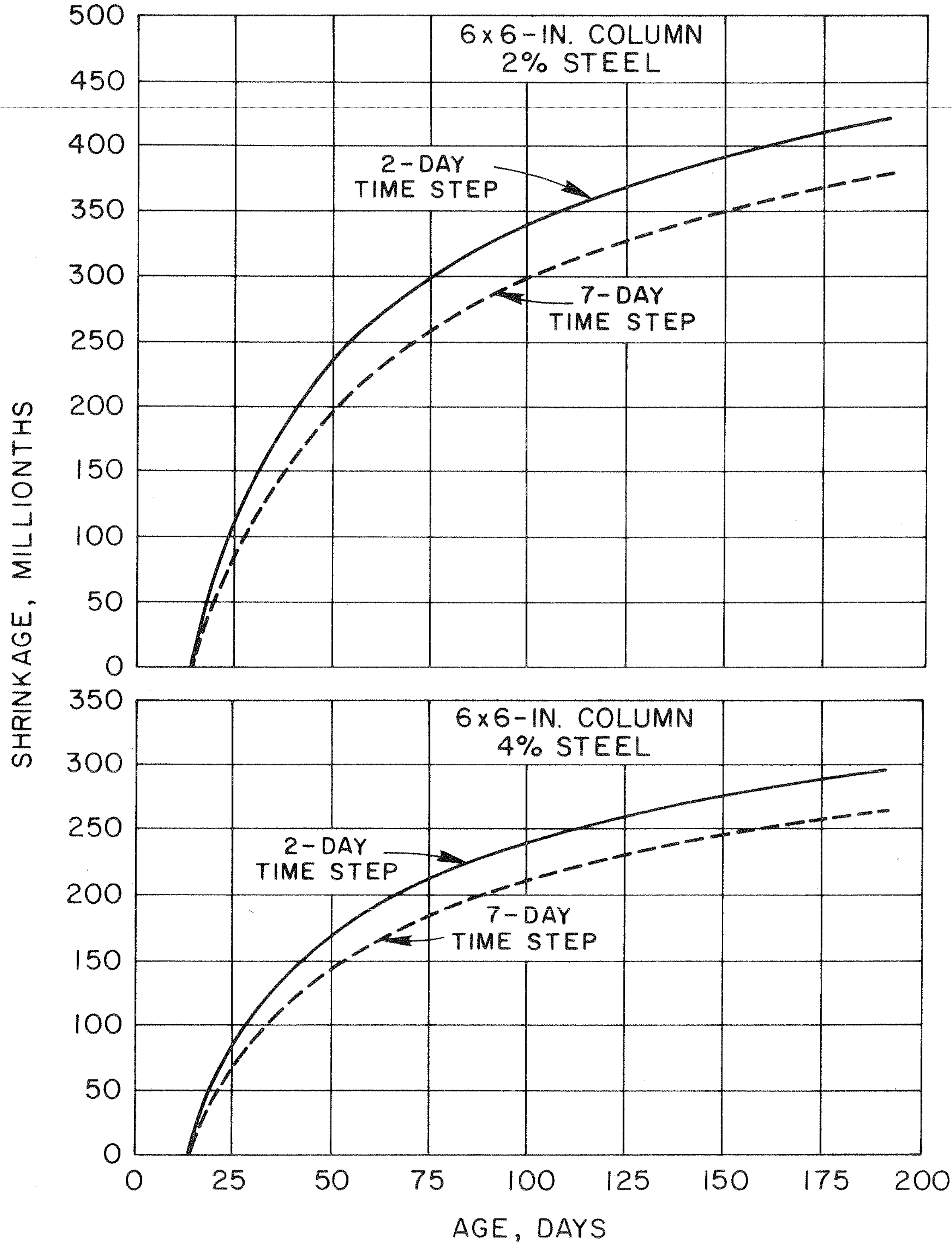


FIG. 25 EFFECT OF TIME STEP ON THE SHRINKAGE OF 6 BY 6-IN. REINFORCED CONCRETE COLUMNS

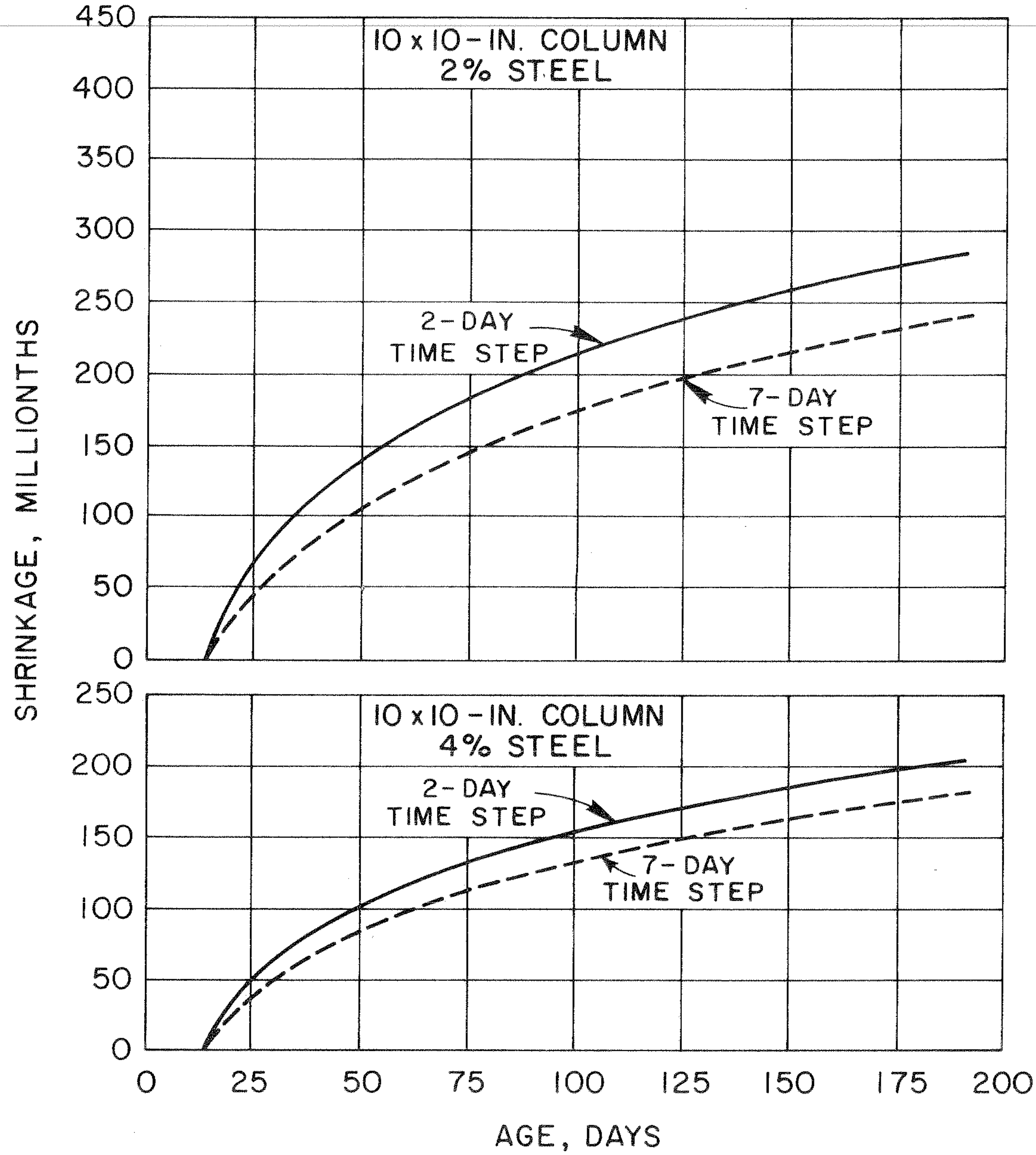
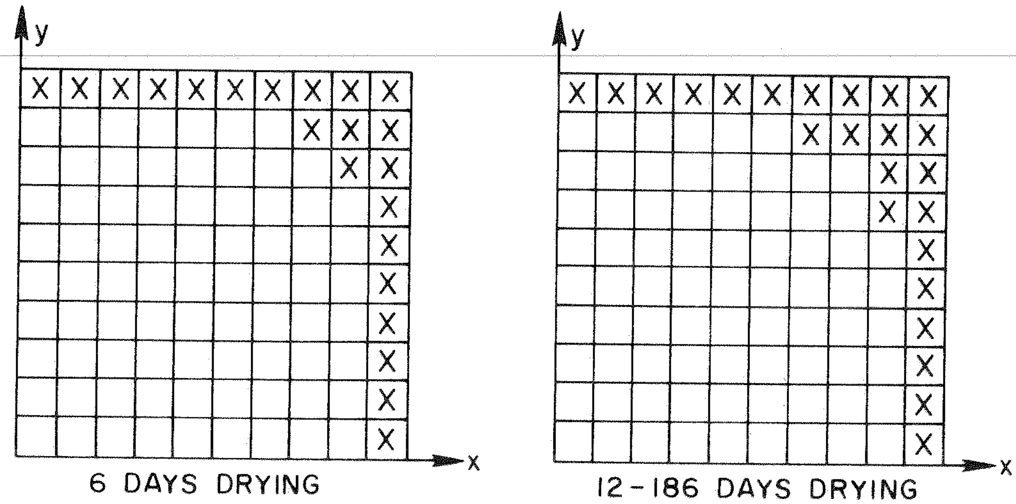
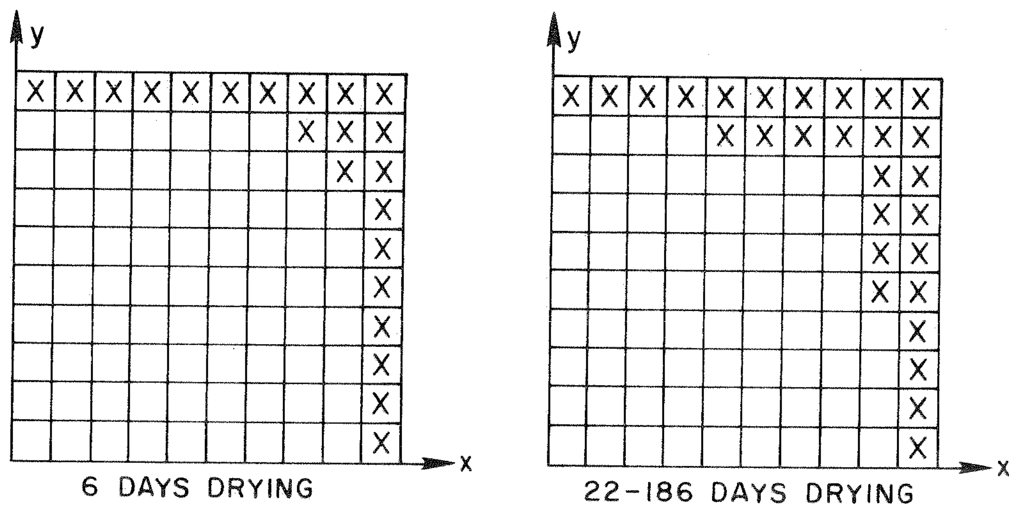


FIG. 26 EFFECT OF TIME STEP ON THE SHRINKAGE OF 10 BY 10-IN. REINFORCED CONCRETE COLUMNS



10 x 10-IN. COLUMN
2% STEEL



10 x 10-IN. COLUMN
4% STEEL

NOTE: ONE QUADRANT SHOWN.
14-DAYS CURING
X = CRACKED ELEMENT.

FIG. 27 ELEMENT CRACKING PATTERNS IN UNLOADED 10 BY 10-IN. REINFORCED CONCRETE PRISMS USING A 2-DAY TIME STEP

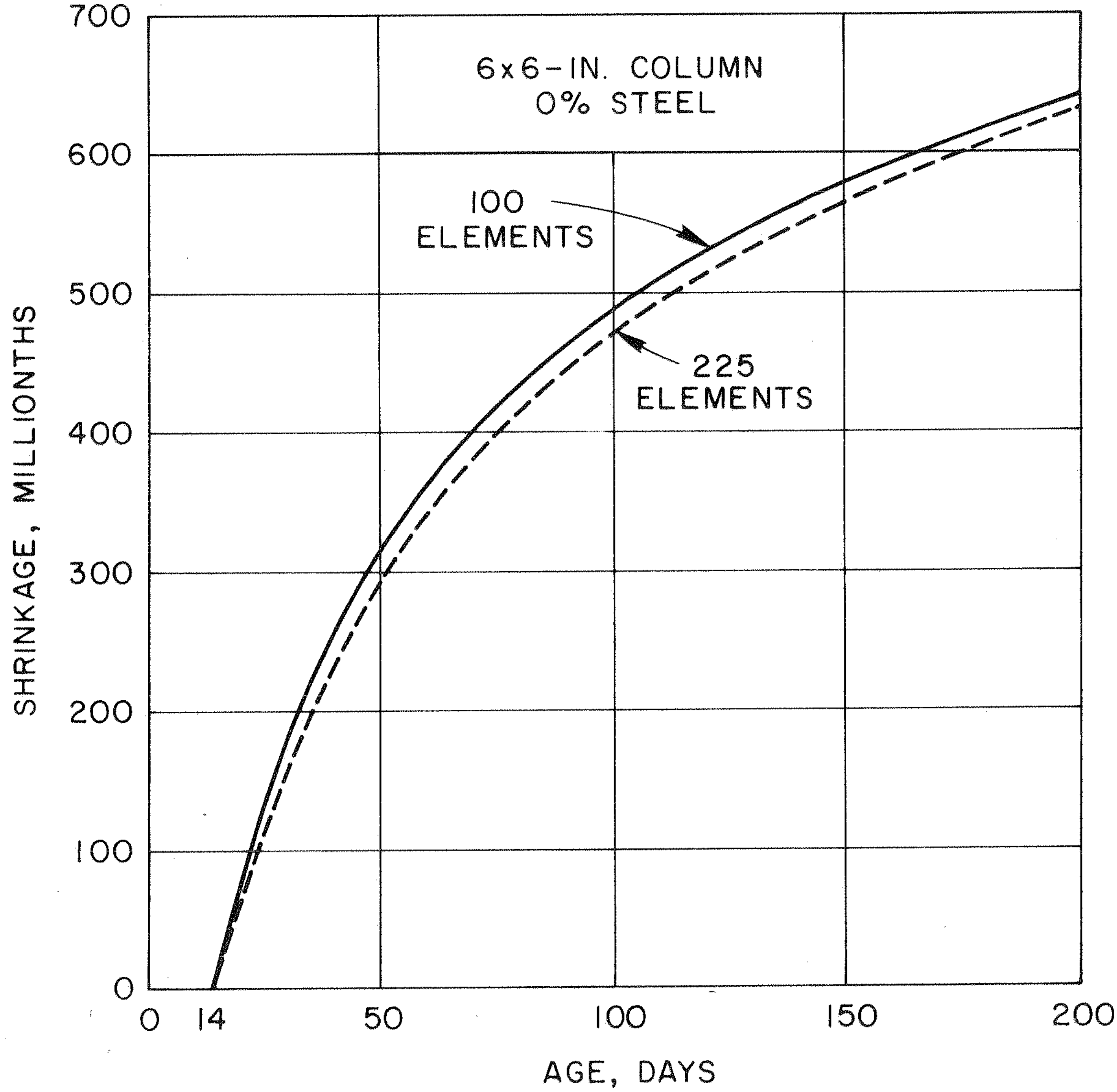


FIG. 28 EFFECT OF MESH SIZE ON SHRINKAGE RESULTS FOR A 6 BY 6-IN. PLAIN PRISM

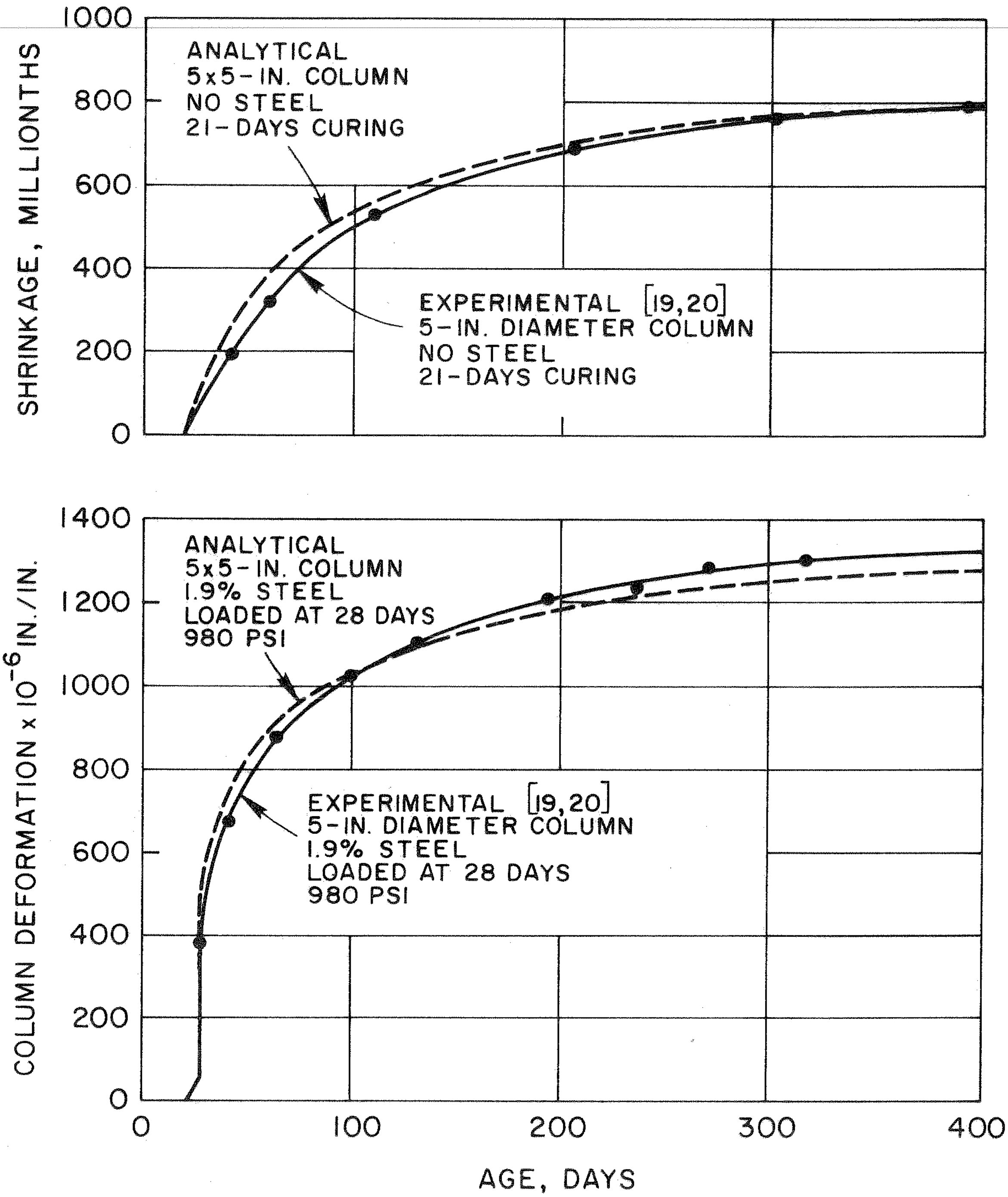


FIG. 29 COMPARISON OF SHRINKAGE AND CREEP DEFORMATIONS FOR 5-IN. COLUMNS HAVING 0% AND 1.9% OF STEEL REINFORCEMENT

APPENDIX A

COMPUTER PROGRAM FOR ANALYSIS OF CREEP AND SHRINKAGE IN REINFORCED CONCRETE COLUMNS

APPENDIX A

COMPUTER PROGRAM FOR ANALYSIS OF CREEP AND SHRINKAGE IN REINFORCED CONCRETE COLUMNS

A.1 Identification

COLUMN - Stress and strain analysis of square reinforced concrete columns using discrete time steps. Programmed by R. Polivka [10], University of California, Berkeley.

A.2 Purpose

The program provides a rapid solution determining the variation of stresses and strains in steel and concrete for square reinforced concrete columns, taking into account non-uniform shrinkage, creep and cracking of concrete as well as loading history.

A.3 Restrictions

1. The program can only be used for either plain or symmetrically reinforced concrete columns. Only square columns may be analyzed.
2. Only unloaded or axially loaded columns may be analyzed.

A.4 Description

The computer solution is based on the method of analysis which has been outlined in this report. It was coded in FORTRAN IV language and consists of a main program and ten subroutines.

The quadrant cross-section is subdivided into square elements

(100 maximum) of equal area, and each element assigned a reference number as shown in Fig. 1. The free or unrestrained shrinkage strain of the elements is determined at the end of each time step using the diffusion equation for shrinkage (Eq. 2.3). The shrinkage diffusivity coefficient K is determined using the expression in Eq. (2.8). The final unrestrained shrinkage strain for the concrete, S_∞, is a variable which must be input.

An empirical specific creep compliance expression similar to the one used in Eq. (2.13) must be determined for the particular concrete to be modeled and input in a similar form. Additionally, a tensile strength relation must be determined in a form similar to that in Eq. (2.14).

The length of each time step, as well as the total number of time steps used, should be individually selected for the particular column to be analyzed.

A.5 Sign Convention

Shortening strain is (+)

Compressive stress is (+)

A.6 General Input Data

All decimal numbers are read in using the format Fw.0, where w is the width of the data field in which the number must appear. Unless the decimal point is explicitly punched, it will be assumed to be at the right of the field w.

Integer numbers and alphabetic characters are read in using the formats Iw and Aw, respectively, and both must be right justified in the field w.

All units must be in kips and inches.

I. SPECIFIC CREEP COMPLIANCE FUNCTION

<u>Card 1</u> (4I10)

columns	variable	entry
1 - 10	NPOT	Number of polynomial fractions in Kelvin multiplier GE. 1 and LE. 4
11 - 20	NEKP	Number of exponential terms in compliance function GE. 1 and LE. 4
21 - 30	NELT	Number of elastic fractions in compliance function GE. 1 and LE. 4
31 - 40	NPRT	Compliance and tensile strength function printout code EQ. 0, No compliance or tensile strength printout EQ. 1, Print compliance and tensile strength functions

<u>Card 2</u> (4F20.0)

columns	variable	entry
1 - 20	POLEXP(1)	Exponent of time in first term of Kelvin multiplier
21 - 40	POLEXP(2)	Exponent of time in second term of Kelvin multiplier
.	.	.
.	.	.
.	.	.
	POLEXP(NPOT)	Exponent of time in NPOT term of Kelvin multiplier

I. SPECIFIC CREEP COMPLIANCE FUNCTION (continued)

Card 3 (4F20.0)

columns	variable	entry
1 - 20	POLCON(1)	Numerator of first polynomial fraction in Kelvin multiplier
21 - 40	POLCON(2)	Numerator of second polynomial fraction in Kelvin multiplier
⋮	⋮	⋮
	POLCON(NPOT)	Numerator of NPOT polynomial fraction in Kelvin multiplier

Card 4 (F20.0)

columns	variable	entry
1 - 20	CONSP	Constant term in Kelvin multiplier

Card 5 (4F20.0)

columns	variable	entry
1 - 20	EXPM(1)	Exponent factor for first term in compliance function
21 - 40	EXPM(2)	Exponent factor for second term in compliance function
⋮	⋮	⋮
	EXPM(NEKP)	Exponent factor for NEKP term in compliance function

Card 6 (4F20.0)

columns	variable	entry
1 - 20	TERMM(1)	Exponential multiplier of first term in compliance function
21 - 40	TERMM(2)	Exponential multiplier of second term in compliance function
. . .	. . .	. . .
	TERMM(NEKP)	Exponential multiplier of NEKP term in compliance function

Card 7 (4F20.0)

columns	variable	entry
1 - 20	BEXP(1)	Exponent of time for first elastic fraction
21 - 40	BEXP(2)	Exponent of time for second elastic fraction
. . .	. . .	. . .
	BEXP(NELT)	Exponent of time for NELT elastic fraction

Card 8 (4F20.0)

columns	variable	entry
1 - 20	AFAC(1)	Numerator of first elastic fraction
21 - 40	AFAC(2)	Numerator of second elastic fraction
. . .	. . .	. . .
	AFAC(NELT)	Numerator of NELT elastic fraction

Card 9 (F20.0)

columns	variable	entry
1 - 20	ELCON	Constant term in elastic polynomial

II. TIME STEP DATA

Card 1 (F20.0)

columns	variable	entry
1 - 20	TIFAC	Number of days per time step GE. 1

Card 2 (I10)

columns	variable	entry
1 - 10	LT	Number of time steps GE. 1 and LE. 55

III. MATERIAL PROPERTIES DATA

Card 1 (2F20.0)

columns	variable	entry
1 - 20	TSA	Constant term in tensile strength function
21 - 40	TSB	Coefficient of time in tensile strength function

Card 2 (2F20.0)

columns	variable	entry
1 - 20	ESTL	Modulus of elasticity of steel
21 - 40	YSTRTH	Yield strength of steel

IV. SECTION PROPERTIES

Card 1 (8A10)

columns	variable	entry
1 - 80	WORD(I)	Heading information for use in labeling output

Card 2 (F20.0)

columns	variable	entry
1 - 20	B	Length of one side of quadrant

Card 3 (2I10)

columns	variable	entry
1 - 10	NSEG	Number of segments along one side of quadrant GE. 1
11 - 20	NELBAR	Number of elastic steel bars EQ. 1

Card 4 (3F20.0)

Input NELBAR sets of the following data cards:

columns	variable	entry
1 - 20	AS(I)	Area of steel in bar I
21 - 40	XS(I)	X-coordinate of steel bar I
41 - 60	YS(I)	Y-coordinate of steel bar I

V. AXIAL LOAD DATA

Card 1 (I10)

columns	variable	entry
1 - 10	NLC	Number of external load applications

Card 2 (I10, F10.0)

Input NLC sets of the following data cards.
If NLC.EQ.0, omit this section

columns	variable	entry
1 - 10	I	Time step of load application
11 - 20	P(I)	External load applied at time step I

VI. SHRINKAGE DATA

Card 1 (3I10)

columns	variable	entry
1 - 10	NBETA	Number of β terms used in Pickett's diffusion equation GE. 1 and LE. 24
11 - 20	NTC	Number of time steps curing GE. 1 and LE. LT
21 - 30	IOPT	Printing option to print either S/S_∞ or S at centroids; EQ. 0; supresses printing of S/S_∞ or S EQ. 1; prints S/S_∞ values at centroids EQ. 2; prints shrinkage values at centroids

Card 2 (4F20.0)

columns	variable	entry
1 - 20	BETA(1)	First β root used in Pickett's diffusion equation
21 - 40	BETA(2)	Second β root used in Pickett's diffusion equation
. . .	. . .	. . .
	BETA(NBETA)	NBETA β root used in Pickett's diffusion equation

Card 3 (F20.0)

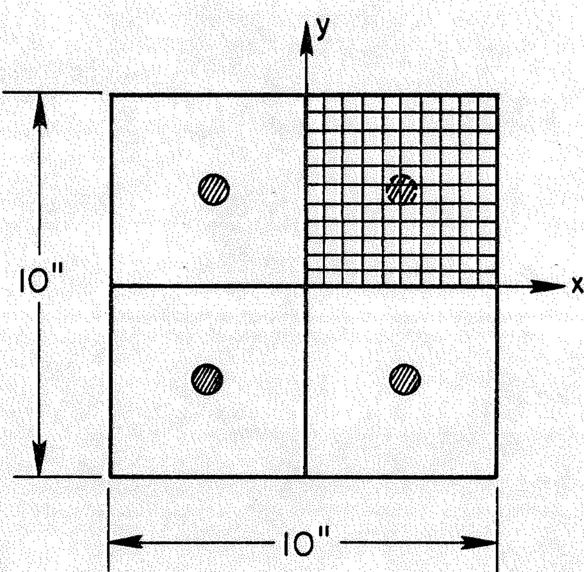
columns	variable	entry
1 - 20	SINF	Final unrestrained shrinkage strain S_∞

APPENDIX B

INPUT FOR A SAMPLE PROBLEM

APPENDIX B

INPUT FOR A SAMPLE PROBLEM



B.1 Data

Size of Column: 10 by 10 in.

Steel Ratio: 0.02

Load: Axial load of $P = 0.3P_u$ = 114.4 kips applied incrementally in four equal time steps at the ages of 14, 49, 84 and 119 days, and sustained throughout the entire observation period of 385 days. This is equivalent to 28.6 kips/quadrant.

Area of Steel: 2.0 sq. in. (Four bars of 0.50 sq. in. each)

Value of S_∞: 900 micro-in./in. or 0.000900 in./in.

Specific creep compliance (Eq. 2.13) and tensile strength (Eq. 2.14) expressions are input term-by-term. The cross-section of the column is subdivided into segments as shown in Fig. 1.

Input data for the above problem is given on page B-2.

****
SAMPLE INPUT DATA FOR A 10X10-INCH COLUMN
****

```
         4           3           2           1
-.85                -1.70                   -2.55                   -3.40
-.00017154434       .034431629              -.4339611               1.2608592
.00010964038
-.2                 -.02                    -.002
.991                2.06                    1.125
-1.                 -.5
.000333             -.0000306
.000221
7.
        55
.639                1.39
29000.              60.
COLUMN SIZE = 10 X 10 IN. STEEL RATIO = 0.02, LOAD = 0.3PU, CRACKING
5.
        10          1
.5                  2.5                     2.5
         4
         2  7.153125
         7  7.153125
        12  7.153125
        17  7.153125
        24          2          0
   1.404188871426300   4.242313035752403   7.146117061874008  10.114884991724750
  13.132710622743105  16.184275783301587  19.258663906726952  22.348711751751466
  25.449771922624677  28.558784709735505  31.673687763316821  34.793055114389745
  37.915876151350631  41.041418042151918  44.169138199420104  47.298627274157525
  50.429571173517843  53.561725200764840  56.694896090482871  59.828929294377076
  62.963699827866776  66.099105575862723  69.235062325835315  72.371500033203120
.000900
```

APPENDIX C

COMPUTER PROGRAM LISTING

```
      PROGRAM COLUMN (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C************************************************************************
C                                                                       *
C     PROGRAM IDENTIFICATION  - COLUMN                                  *
C     PROGRAM DESCRIPTION     - STRESS AND STRAIN ANALYSIS OF UNLOADED  *
C                               OR AXIALLY LOADED, SQUARE REINFORCED    *
C                               CONCRETE COLUMNS, ALLOWING FOR          *
C                               NON-UNIFORM SHRINKAGE, CREEP AND        *
C                               CRACKING OF THE CONCRETE.               *
C     PROGRAMMED BY           - R.M. POLIVKA                            *
C                               UNIVERSITY OF CALIFORNIA BERKELEY 1975  *
C                                                                       *
C************************************************************************
C
      COMMON A,B,LT,ESTL,NELBAR,NSEG,NTC,NTS,TIFAC,AC(100),CERN(55,55),
     .       AS(1),DSHREPS(100,55),NDIAG(15),P(55),RATIO(55),TS(55)
      COMMON/ANAL/ ADDSTRS(100),MCOMP,NC,NCOUNT(100),NCOUNT2(100),PT,TP,
     .       TOTEPSC(100,55),TOTEPSS(1,55),TOTSTRC(100,55),TOTSTRS(1,55)
      COMMON/CRAK/ DILBERT(100),SHRP
      COMMON/KERNEL/ AFAC(4),BEXP(4),CONSP,ELCON,EXPM(4),NEKP,NELT,NPOT,
     .       NPRT,POLCON(4),POLEXP(4),TERMM(4),TSA,TSB
C
      CALL RHEOL
      CALL RHCALC
      CALL SECI
      CALL EXLOAD
      CALL SHRINK
C
      DO 2 I=1,LT
      DO 1 K=1,NELBAR
      TOTSTRS(K,I) = 0.
    1 TOTEPSS(K,I) = 0.
      DO 2 K=1,NTS
      TOTSTRC(K,I) = 0.
    2 TOTEPSC(K,I) = 0.
C
      NC = 0
      PT = 0.
C
      DO 3 I=1,LT
      CALL DRYLOAD(I)
      TP = P(I)
      PT = PT+TP
      CALL STREPS(I)
      CALL RELESE(I)
    3 CONTINUE
C
      END
```

```
      SUBROUTINE RHEOL
C
      COMMON A,B,LT,ESTL,NELBAR,NSEG,NTC,NTS,TIFAC,AC(100),CERN(55,55),
     .       AS(1),DSHREPS(100,55),NDIAG(15),P(55),RATIO(55),TS(55)
      COMMON/KERNEL/ AFAC(4),BEXP(4),CONSP,ELCON,EXPM(4),NEKP,NELT,NPOT,
     .       NPRT,POLCON(4 ,POLEXP(4),TERMM(4),TSA,TSB
C
C************************************************************************
C     INPUT OF TERMS IN COMPLIANCE SURFACE FUNCTION
C             C(T,TAU) = A(TAU)B(T) + D(T)
C************************************************************************
C
      READ (5,1001) NPOT,NEKP,NELT,NPRT
      READ (5,1002) (POLEX (I), I=1,NPOT)
      READ (5,1002) (POLCON(I), I=1,NPOT)
      READ (5,1002) CONSP
      WRITE(6,2001) NPOT
      WRITE(6,2002) (I,POLEXP(I), I=1,NPOT)
      WRITE(6,2003) (I,POLCON(I), I=1,NPOT)
      WRITE(6,2004) CONSP
      READ (5,1002) (EXPM(I), I=1,NEKP)
      READ (5,1002) (TERMM(I),I=1,NEKP)
      WRITE(6,2005) NEKP
      WRITE(6,2006) (I,EXPM(I), I=1,NEKP)
      WRITE(6,2007) (I,TERMM(I),I=1,NEKP)
      READ (5,1002) (BEXP(I), I=1,NELT)
      READ (5,1002) (AFAC(I), I=1,NELT)
      READ (5,1002) ELCON
      WRITE(6,2008) NELT
      WRITE(6,2009) (I,BEXP(I), I=1,NELT)
      WRITE(6,2010) (I,AFAC(I), I=1,NELT)
      WRITE(6,2011) ELCON
      READ (5,1002) TIFAC
      READ (5,1001) LT
      WRITE(6,2012) TIFAC,LT
      READ (5,1002) TSA,TSB
      WRITE(6,2013) TSA,TSB
      READ (5,1002) ESTL,YSTRTH
      STY = YSTRTH/ESTL
      WRITE(6,2014) ESTL,YSTRTH,STY
C
      RETURN
C
C     FORMAT STATEMENTS
C
 1001 FORMAT (8I10)
 1002 FORMAT (4F20.0)
 2001 FORMAT (1H7,/////42H KERNEL FUNCTION WHICH FITS EXPERIMENTALLY
     .        30H DETERMINED COMPLIANCE SURFACE//
     .        20X,28HC(T,TAU) = A(TAU)B(T) + D(T)//
     .        41H NO. OF VARIABLE TERMS IN A(TAU)           = I3)
 2002 FORMAT (5X,16HEXPONENTS OF TAU/(10X,5HTERM I1,24X,1H= F8.4))
 2003 FORMAT (5X,19HCOEFFICIENTS OF TAU/(10X,5HTERM I1,24X,1H= F15.11))
 2004 FORMAT (5X,23HCONSTANT TERM IN A(TAU),12X,1H= F15.11//)
 2005 FORMAT (/41H NO. OF EXPONENTIALS IN B(T)              = I3)
 2006 FORMAT (5X,31HEXPONENT MULTIPLIERS OF (T-TAU)/
     .        (10X,5HTERM I1,24X,1H= F8.4))
 2007 FORMAT (5X,34HCOEFFICIENTS OF (1 - EXP C(T-TAU))/
     .        (10X,5HTERM I1,24X,1H= F8.4))
 2008 FORMAT (/// 41H NO. OF ELASTIC FRACTIONS IN D(T)         = I3)
 2009 FORMAT (5X,14HEXPONENTS OF T/ (10X,5HTERM I1,24X,1H= F8.4))
 2010 FORMAT (5X,17HCOEFFICIENTS OF T/
     .        (10X,5HTERM I1,24X,1H= F12.8))
 2011 FORMAT (5X,24HELASTIC CONSTANT IN D(T),11X,1H= F11.7)
 2012 FORMAT (/// 33H TIME STEP MULTIPLIER (DAYS/STEP),7X,1H= F5.1/
     .        21H NUMBER OF TIME STEPS,19X,1H= I3)
 2013 FORMAT (/// 51H TENSILE STRENGTH FUNCTION WHICH FITS EXPERIMENTAL
     .        4HDATA//10X,8H TS(T) = F8.5,2H - F8.5,7H/T KSI.)
 2014 FORMAT (// 26H PROPERTIES OF STEEL REBAR/
     .        5X,28HYOUNGS MODULUS (KIPS/SQ.IN.),7X,1H= F11.3/
     .        5X,26HYIELD SRESS (KIPS/SQ.IN.),9X,1H= F11.3/
     .        5X,22HYIELD STRAIN (IN./IN.),13X,1H= F16.8)
C
      END
```

```
      SUBROUTINE RHCALC
C
      COMMON A,B,LT,ESTL,NELBAR,NSEG,NTC,NTS,TIFAC,AC(100),CERN(55,55),
     .       AS(1),DSHREPS(100,55),NDIAG(15),P(55),RATIO(55),TS(55)
      COMMON/KERNEL/ AFAC(4),BEXP(4),CONSP,ELCON,EXPM(4),NEKP,NELT,NPOT,
     .       NPRT,POLCON(4),POLEXP(4),TERMM(4),TSA,TSB
C
C************************************************************************
C     EVALUATE COMPLIANCE SURFACE FUNCTION C(T,TAU)
C************************************************************************
C
      DO 5 I=1,LT
      RTI = I*TIFAC
      IP = I+1
C
```

```
C     DETERMINE TENSILE STRENGTH FUNCTION FOR EACH TIME STEP
C
      TS(I) = -(TSA-TSB/RTI)
      POLY = CONSP
      DO 2 K=1,NPOT
    2 POLY = POLY+POLCON(K)*(RTI**POLEXP(K))
C
      ELAC = ELCON
      DO 3 K=1,NELT
    3 ELAC = ELAC+AFAC(K)*(RTI**BEXP(K))
C
C     FIND MODULAR RATIO, E(STEEL)/E(CONCRETE), FOR EACH TIME STEP
C
      RATIO(I) = ESTL*ELAC
      CERN(I,I) = ELAC
      DO 5 J=IP,LT
      RTJ = J*TIFAC
      CERN(J,I) = 0.
      RT = RTJ-RTI
      DO 4 K=1,NEKP
    4 CERN(J,I) = CERN(J,I) + TERMM(K)*(1.-EXP(EXPM(K)*RT))
      CERN(J,I) = ELAC + POLY*CERN(J,I)
    5 CONTINUE
C
      IF(NPRT.EQ.0) GO TO 20
      WRITE(6,2001)
      DO 10 I=1,LT
      NTI = I*TIFAC
   10 WRITE(6,2002) NTI,TS(I)
C
      WRITE(6,2003)
      DO 15 I=1,LT
      NTI = I*TIFAC
      WRITE(6,2004)
      DO 15 J=I,LT
      NTJ = J*TIFAC
   15 WRITE(6,2005) NTI,NTJ,CERN(J,I)
C
   20 RETURN
C
C     FORMAT STATEMENTS
C
 2001 FORMAT (1H1,16X,10HAGE (DAYS),5X,22HTENSILE STRENGTH (KSI)/)
 2002 FORMAT (19X,I3,16X,F8.5)
 2003 FORMAT (1H1,12X,10HAGE LOADED,10X,12HAGE OBSERVED,10X,9HSPECIFIC
     .        10HCOMPLIANCE/2(15X,6H(DAYS)),16X,12H(STRAIN/KSI))
 2004 FORMAT (/)
 2005 FORMAT (15X,I3,18X,I3,17X,F12.10)
C
      END


      SUBROUTINE SECI
C
      COMMON A,B,LT,ESTL,NELBAR,NSEG,NTC,NTS,TIFAC,AC(100),CERN(55,55),
     .       AS(1),DSHREPS(100,55),NDIAG(15),P(55),RATIO(55),TS(55)
      DIMENSION WORD(8),XS(1),YS(1)
C
C*********************************************************************
C     INPUT SECTION PROPERTIES
C*********************************************************************
C
      READ (5,1001) (WORD(I), I=1,8)
      WRITE(6,2001) (WORD(I), I=1,8)
      READ (5,1002) B
      READ (5,1003) NSEG,NELBAR
C
C     CALCULATE AREAS AND COORDINATES OF CONCRETE SEGMENTS
C
      NTS = NSEG*NSEG
      ASEG = B*B/NTS
      DO 10 I=1,NTS
   10 AC(I) = ASEG
C
      WRITE(6,2002) NTS,NELBAR,ASEG
C
C     INPUT STEEL AREA (USE 1/4TH TOTAL STEEL AREA FOR QUADRANT)
C
      READ (5,1002) (AS(I),XS(I),YS(I), I=1,NELBAR)
      WRITE(6,2003) (I,AS(I),XS(I),YS(I), I=1,NELBAR)
C
C     COMPUTE ADDRESSES OF DIAGONALS
C
      N = 1
      DO 20 K=1,NSEG
      NDIAG(K) = N
   20 N = N + NSEG + 1
C
      RETURN
C
C     FORMAT STATEMENTS
C
 1001 FORMAT (8A10)
 1002 FORMAT (4F20.0)
 1003 FORMAT (8I10)
 2001 FORMAT (1H1,8A10,//)
 2002 FORMAT (30H NUMBER OF CONCRETE SEGMENTS = I4/
     .        30H NUMBER OF STEEL BARS         = I4/
     .        30H AREA OF EACH SEGMENT         = F9.5)
 2003 FORMAT (//50H BAR NO.  BAR AREA    X-COORDINATE      Y-COORDINATE/
     .        (3X,I2,5X,F8.5,7X,F5.2,13X,F5.2))
C
      END


      SUBROUTINE EXLOAD
C
      COMMON A,B,LT,ESTL,NELBAR,NSEG,NTC,NTS,TIFAC,AC(100),CERN(55,55),
     .       AS(1),DSHREPS(100,55),NDIAG(15),P(55),RATIO(55),TS(55)
C
C*********************************************************************
C     INPUT EXTERNAL LOAD HISTORY
C*********************************************************************
C
      DO 1 I=1,LT
    1 P(I) = 0.
      READ (5,1001) NLC
      IF(NLC.EQ.0) GO TO 11
C
C     LOAD INCREMENT OF 0.25 P(TOTAL) APPLIED TO QUADRANT
C
      WRITE(6,2001)
      PT = 0.
      DO 10 J=1,NLC
      READ (5,1001) I,P(I)
      NTI = I*TIFAC
      PT = PT+P(I)
   10 WRITE(6,2002) NTI,P(I),PT
      GO TO 12
   11 WRITE(6,2003)
C
   12 RETURN
C
C     FORMAT STATEMENTS
C
 1001 FORMAT (I10,F10.0)
 2001 FORMAT (////35X,21HEXTERNAL LOAD HISTORY,//,18X,11HTIME (DAYS),
     .        11X,14HDELTA P (KIPS),8X,8HP (KIPS)/)
 2002 FORMAT (20X,I3,16X,F10.3,9X,F10.3)
 2003 FORMAT (////35X,21HEXTERNAL LOAD HISTORY,//20H NO EXTERNAL LOAD --
     .        15H SHRINKAGE ONLY)
C
      END
```

```
      SUBROUTINE SHRINK
C
      COMMON A,B,LT,ESTL,NELBAR,NSEG,NTC,NTS,TIFAC,AC(100),CERN(55,55),
     *       AS(1),DSHREPS(100,55),NDIAG(15),P(55),RATIO(55),TS(55)
      DIMENSION BETA(24),DIFCOF(55),PHI(21),SSINF(21,21),SHCENT(100)
C
C***********************************************************************
C     DETERMINE SHRINKAGE STRAINS IN A PRISM USING DIFFUSION EQUATION
C***********************************************************************
C
      READ (5,1001) NBETA,NTC,IOPT
      READ (5,1002) (BETA(I), I=1,NBETA)
      READ (5,1002) SINF
C
C     CALCULATE DIFFUSIVITY K FOR DIFFERENT TIME STEPS
C
      WRITE(6,2000)
      DO 10 I=1,LT
      TIMET = TIFAC*I
      NDAYS = TIMET
      DIFCOF(I) = 0.10*SQRT(2./(2.+TIMET))
   10 WRITE(6,2001) (NDAYS,DIFCOF(I))
      WRITE(6,2002) SINF
      NCLSIZ = 2.*B
      WRITE(6,2003) (BETA(I), I=1,NBETA)
C
C     CALCULATE B IN PICKETTS EQUATION
C
      BIGB = 1.67*B
      FNUMER = 2.*BIGB
      FDENOM = BIGB*BIGB + BIGB
C
C     DIVIDE QUADRANT INTO NPARTS FOR CALCULATING PHI VALUES
C
      NPARTS = NSEG*2
      NODES = NPARTS+1
      RNPART = FLOAT(NPARTS)
      FRLEN = 1./RNPART
      AQUAD = B*B
C
C
      DO 50 I=1,LT
      TIMET = TIFAC*I
      BIGT = 0.4/AQUAD*(SQRT((2.+TIMET)/2.)-1.)
      NDAYS = TIMET
C
C     CALCULATE ARGUMENTS AND VALUES OF PHI
C
      DO 20 J=1,NODES
      YB = (J-1)*FRLEN
      SIGMA = 0.
      DO 15 K=1,NBETA
      F = FNUMER/(FDENOM + BETA(K)*BETA(K))
      POWER = -BIGT*BETA(K)*BETA(K)
      PHINUM = BETA(K)*YB
      ARG = EXP(POWER)*F*COS(PHINUM)/COS(BETA(K))
   15 SIGMA = SIGMA + ARG
      PHI(J) = 1.-SIGMA
      IF(PHI(J)) 16,20,20
   16 PHI(J) = 0.
   20 CONTINUE
C
C     CALCULATE S/SINF AT NODAL POINTS FOR DIFFERENT TIME STEPS
C
      DO 25 M=1,NODES
      DO 25 N=1,M
   25 SSINF(M,N) = PHI(M) + PHI(N) - PHI(M)*PHI(N)
      DO 30 M=1,NPARTS
      MM = M+1
      DO 30 N=MM,NODES
   30 SSINF(M,N) = SSINF(N,M)
C
C     PRINT S/SINF (K=1) AND/OR SHRINKAGE (K=2) AT NODES.
C     IF NO PRINTING OF SHRINKAGE RESULTS DESIRED (K=0).
C
      IF(IOPT.EQ.0) GO TO 36
      DO 35 K=1,IOPT
   35 CALL VISPRIN (SSINF,NODES,NODES,NODES,NDAYS,NCLSIZ,SINF,K)
C
C     CALCULATE FREE SHRINKAGE STRAIN
C
   36 IF(I.GT.LT-NTC) GO TO 50
      DO 40 K=1,NTS
      NN = (K-1)/NSEG +1
      N1 = 2*NN
      N2 = 2*(K-(NN-1)*NSEG)
      IF(I.EQ.1) GO TO 37
      DSHREPS(K,I+NTC) = (SSINF(N1,N2) - SHCENT(K)) * SINF
      GO TO 40
   37 DSHREPS(K,I+NTC) = SSINF(N1,N2)*SINF
      DO 38 LC=1,NTC
   38 DSHREPS(K,LC) = 0.
   40 CONTINUE
C
C     CALCULATE FREE S/SINF AT CENTROIDS
C
      DO 45 MH=1,NSEG
      MI = 2*MH
      DO 45 MV=1,NSEG
      MJ = 2*MV
      KK = (MH-1)*NSEG + MV
   45 SHCENT(KK) = SSINF(MI,MJ)
   50 CONTINUE
C
C
      RETURN
C
C     FORMAT STATEMENTS
C
 1001 FORMAT (8I10)
 1002 FORMAT (4F20.0)
 2000 FORMAT (1H1,23X,14HSHRINKAGE DATA/////,23X,15HCOEFFICIENT OF
     *       11HDIFFUSIVITY///14X,11HTIME (DAYS),10X,14HK (SQ.IN./DAY)/)
 2001 FORMAT (22X,I3,15X,F13.10)
 2002 FORMAT (//7X,6HSINF = 6PE9.2,14H MICRO-IN./IN.)
 2003 FORMAT (1H1,23X,14HVALUES OF BETA,//,(25X,F16.11))
C
      END


      SUBROUTINE VISPRIN (A,NR,NC,MAX,NDAYS,NCLSIZ,SINF,IOPT)
      DIMENSION A(MAX,1),FMT(10)
C
C***********************************************************************
C     PRINTING ROUTINE FOR SHRINKAGE VALUES AT NODAL POINTS
C     IF NOTHING IS TO BE PRINTED   IOPT = 0
C     TO PRINT S/SINF VALUES USE    IOPT = 1
C     TO PRINT SHRINKAGE STRAINS    IOPT = 2
C***********************************************************************
C
      IF(IOPT.EQ.2) GO TO 1
      WRITE(6,2000) NDAYS,NCLSIZ
      GO TO 101
    1 WRITE(6,2001) NDAYS,NCLSIZ
C
  101 DO 100 J=1,NR
      JH=NR+1-J
      IF(IOPT.EQ.1) GO TO 3
      DO 10 K=1,NC
```

```
   10 A(JH,K) = A(JH,K)*SINF*1.E+6
      NC1 = NC-1
      IF(JH.EQ.1) GO TO 2
      IF(NC.GT.11) GO TO 91
      ENCODE(100,2004,FMT) NC1
      WRITE(6,FMT) (A(JH,I), I=1,NC)
      GO TO 102
    2 IF(NC.GT.11) GO TO 92
      ENCODE(100,2005,FMT) NC1
      WRITE(6,FMT) (A(JH,I), I=1,NC)
      GO TO 102
   91 WRITE(6,2006) (A(JH,I), I=1,11)
      GO TO 100
   92 WRITE(6,2007) (A(JH,I), I=1,11)
      GO TO 100
  102 DO 11 K=1,NC
   11 A(JH,K) = A(JH,K)/(SINF*1.E+6)
      GO TO 100
    3 IF(NC.GT.11) GO TO 93
      NC1 = NC-1
      IF(JH.EQ.1) GO TO 4
      ENCODE(100,2002,FMT) NC1
      WRITE(6,FMT) (A(JH,I), I=1,NC)
      GO TO 100
    4 ENCODE(100,2003,FMT) NC1
      WRITE(6,FMT) (A(JH,I), I=1,NC)
      GO TO 100
   93 IF(JH.EQ.1) GO TO 94
      WRITE(6,2008) (A(JH,I), I=1,11)
      GO TO 100
   94 WRITE(6,2009) (A(JH,I), I=1,11)
  100 CONTINUE
C
      IF(NC.LE.11) RETURN
      NC2 = NC-12
      WRITE(6,2010)
C
      DO 103 J=1,NR
      JH = NR+1-J
      IF(IOPT.EQ.1) GO TO 95
      IF(JH.EQ.1) GO TO 96
      ENCODE(100,2011,FMT) NC2
      WRITE(6,FMT) (A(JH,I), I=12,NC)
      GO TO 97
   96 ENCODE(100,2012,FMT) NC2
      WRITE(6,FMT) (A(JH,I), I=12,NC)
   97 DO 13 K=1,NC
   13 A(JH,K) = A(JH,K)/(SINF*1.E+6)
      GO TO 103
   95 IF(JH.EQ.1) GO TO 98
      ENCODE(100,2013,FMT) NC2
      WRITE(6,FMT) (A(JH,I), I=12,NC)
      GO TO 103
   98 ENCODE(100,2014,FMT) NC2
      WRITE(6,FMT) (A(JH,I), I=12,NC)
  103 CONTINUE
C
      RETURN
C
C     FORMAT STATEMENTS
C
 2000 FORMAT (1H7,5/,6X,28HS/SINF AT NODAL POINTS AFTER I4,10H DAYS DRYI
     .        8HNG FOR A I3,12H INCH COLUMN///,
     .        2X,11HCENTER LINE/,7X,1H*,/,7X,1H*)
 2001 FORMAT (1H1,6X,48HSHRINKAGE (MICRO-IN./IN.) AT NODAL POINTS AFTER
     .        I4,19H DAYS DRYING FOR A I3,12H INCH COLUMN///
     .        2X,11HCENTER LINE/,7X,1H*,/,7X,1H*)
 2002 FORMAT (4H(3X,I2,32H(F8.7,3X),F8.7/,7X,1H*,/,7X,1H*))
 2003 FORMAT (4H(3X,I2,28H(F8.7,3H---),F8.7,7H---- CL))
 2004 FORMAT (4H(3X,I2,32H(F8.2,3X),F8.2/,7X,1H*,/,7X,1H*))
 2005 FORMAT (4H(3X,I2,28H(F8.2,3H---),F8.2,7H---- CL))
 2006 FORMAT (3X,10(F8.2,3X),F8.2,2X,1H*,/,7X,1H*)
 2007 FORMAT (3X,10(F8.2,3H---),F8.2,6H- * --)
 2008 FORMAT (3X,10(F8.7,3X),F8.7,2X,1H*,/,7X,1H*)
 2009 FORMAT (3X,10(F8.7,3H---),F8.7,6H- * --)
 2010 FORMAT (1H1,5/)
 2011 FORMAT (7H(3H * ,I2,24H(F8.2,3X),F8.2,/,1X,1H*))
 2012 FORMAT (7H(3H * ,I2,38H(F8.2,3H---),F8.2,16H---- CENTER LINE))
 2013 FORMAT (7H(4H * ,I2,24H(F8.7,3X),F8.7,/,1X,1H*))
 2014 FORMAT (7H(3H * ,I2,38H(F8.7,3H---),F8.7,16H---- CENTER LINE))
C
      END
```

```
      SUBROUTINE DRYLOAD(I)
C
      COMMON A,B,LT,ESTL,NELBAR,NSEG,NTC,NTS,TIFAC,AC(100),CERN(55,55),
     .       AS(1),DSHREPS(100,55),NDIAG(15),P(55),RATIO(55),TS(55)
      COMMON/ANAL/ ADDSTRS(100),MCOMP,NC,NCOUNT(100),NCOUNT2(100),PT,TP,
     .       TOTEPSC(100,55),TOTEPSS(1,55),TOTSTRC(100,55),TOTSTRS(1,55)
      COMMON/CRAK/ DILBERT(100),SHRP
      DIMENSION EXSTR(100),NCOUNT3(100),NCOUNT5(100)
C
C*********************************************************************
C     THIS ROUTINE CALCULATES THE EXTRA STRESS CREATED IN THE ELEMENTS
C     DURING THE TIME INTERVAL (I-1) TO (I).
C*********************************************************************
C
      EXTLOAD = 0.
      MADJ = 0
      MCOMP = 0
      MCRK = 0
      EINV = CERN(I,I)
      IM1 = I-1
C
C     CALCULATE THE TRANSFORMED AREA ASSUMING ALL ELEMENTS UNCRACKED.
C
      AT = 0.
      DO 3 K=1,NELBAR
    3 AT = AT + (RATIO(I)-1.)*AS(K)
      AT = AT + B*P
      A = AT
      IF(I.LE.NTC) GO TO 99
C
C     ASSUMING NO ELEMENTS HAVE CRACKED, CALCULATE THE STRESS REQUIRED
C     TO PULL EACH ELEMENT BACK UP TO ITS ORIGINAL LEVEL ( A DISTANCE
C     EQUAL TO THE AMOUNT OF SHRINKAGE + CREEP DURING THE I-TH PERIOD)
C
      DO 4 K=1,NTS
    4 DILBERT(K) = DSHREPS(K,I)/EINV
C
      IF(NC.EQ.0) GO TO 16
C
C     DETERMINE WHICH CRACKED ELEMENTS ARE IN COMPRESSION.
C
      MCOMP = 0
      DO 10 JJ=1,NC
      NN = NCOUNT(JJ)
      IF(TOTSTRC(NN,IM1).EQ.0.) GO TO 10
      MCOMP = MCOMP + 1
      NCOUNT2(MCOMP) = NN
   10 CONTINUE
C
C     CRACKED ELEMENTS NOT IN COMPRESSION ARE STRESS-FREE AND DO NOT
C     HAVE TO SATISFY COMPATIBILITY WITH THE REST OF THE PRISM. ALLOW
C     THESE ELEMENTS TO CRACK AT BEGINNING OF TIME STEP AND DO NOT
C     APPLY ANY SHRINKAGE CORRECTION STRESS TO THEM.
C
  100 A = AT
      DO 15 JJ=1,NC
      NN = NCOUNT(JJ)
      IF(MCOMP.EQ.0) GO TO 14
      DO 13 II=1,MCOMP
      N2 = NCOUNT2(II)
   13 IF(NN.EQ.N2) GO TO 15
C
```

```
   14 DILBERT(NN) = 0.
C
C     SUBTRACT FROM THE TRANSFORMED AREA ALL ELEMENTAL AREAS
C     WHICH HAVE CRACKED AND ARE NOT IN COMPRESSION.
C
      A = A - AC(NN)
C
   15 CONTINUE
C
C     CALCULATE TOTAL SHRINKAGE EQUILIBRIUM CORRECTION LOAD.
C
   16 SHRP = 0.
      DO 20 K=1,NTS
   20 SHRP = SHRP + AC(K)*DILBERT(K)
C
      IF(NC.EQ.0) GO TO 40
C
C     THE STRESS A NEWLY CRACKED ELEMENT OR CRACKED ELEMENT WHICH LOST
C     COMPRESSION HAD AT BEGINNING OF TIME STEP MUST NOW BE CARRIED BY
C     REMAINING ELEMENTS. THIS IS DONE BY ADDING THE STRESS TO THE
C     TOTAL SHRINKAGE CORRECTION LOAD.
C
      SHRP = SHRP + EXTLOAD
C
      IF(MCOMP.EQ.0) GO TO 40
C
C     CHECK IF A CRACKED ELEMENT PREVIOUSLY IN COMPRESSION IS STILL
C     IN COMPRESSION IN FINAL POSITION.
C
      MAJOLD=MADJ
C
      DO 25 JJ=1,MCOMP
      N2 = NCOUNT2(JJ)
      STRTEMP = TOTSTRC(N2,IM1) + SHRP/A - DILBERT(N2)
      IF(STRTEMP.GT.0.) GO TO 25
C
C     A CRACKED ELEMENT HAS LOST ITS COMPRESSION DURING THIS TIME STEP.
C     REMOVE COMPRESSIVE STRESS ELEMENT HAD AT BEGINNING OF PERIOD.
C
      MADJ = MADJ+1
      NCOUNT3(MADJ) = N2
      EXSTR(N2) = -TOTSTRC(N2,IM1)
      TOTSTRC(N2,I) = 0.
C
C     DETERMINE EXTRA LOAD WHICH MUST BE CARRIED BY REMAINING ELEMENTS.
C
      EXTLOAD = EXTLOAD + TOTSTRC(N2,IM1)*AC(N2)
C
   25 CONTINUE
C
C     IF NO CRACKED ELEMENTS LOSE THEIR COMPRESSION - CONTINUE ANALYSIS.
C
      IF(MAJOLD.EQ.MADJ) GO TO 40
C
C     RE-ORDER THE LIST AND THE NUMBER OF CRACKED ELEMENTS WHICH ARE
C     IN COMPRESSION IF ONE OF THEM HAS RELEASED ITS COMPRESSION.
C
      MCOLD = MCOMP
      JJ=1
   26 N2 = NCOUNT2(JJ)
C
      DO 30 II=1,MADJ
      N3 = NCOUNT3(II)
      IF(N2.NE.N3) GO TO 30
      MCOMP = MCOMP-1
      IF(JJ.GT.MCOMP) GO TO 31
      DO 28 KK=JJ,MCOMP
   28 NCOUNT2(KK) = NCOUNT2(KK+1)
      GO TO 26
   30 CONTINUE
C
      JJ = JJ+1
      GO TO 26
C
   31 MCP = MCOMP+1
      DO 35 JJ=MCP,MCOLD
   35 NCOUNT2(JJ) = 0
C
C     THE QUADRANT MUST NOW BE RE-EXAMINED AND A NEW FORCE BALANCE MADE
C     BECAUSE SOME CRACKED ELEMENTS HAVE LOST THEIR COMPRESSION.
C
      GO TO 100
C
C-----------------------------------------------------------------------
C     CALCULATE THE TOTAL STRESS ON ALL OF THE ELEMENTS IN THEIR FINAL
C     POSITION AND CHECK IF ANY NEW ELEMENTS HAVE CRACKED.
C-----------------------------------------------------------------------
C
   40 DO 60 K=1,NTS
C
      IF(NC.EQ.0) GO TO 50
C
      DO 42 JJ=1,NC
      NN = NCOUNT(JJ)
   42 IF(K.EQ.NN) GO TO 43
      GO TO 50
C
   43 IF(MCOMP.EQ.0) GO TO 45
      DO 44 II=1,MCOMP
      N2 = NCOUNT2(II)
   44 IF(K.EQ.N2) GO TO 60
C
   45 IF(MADJ.EQ.0) GO TO 47
      DO 46 KK=1,MADJ
      N3 = NCOUNT3(KK)
   46 IF(K.EQ.N3) GO TO 60
C
C     CHECK IF A CRACKED ELEMENT (NOT ONE IN COMPRESSION OR ONE WHICH
C     HAS LOST ITS COMPRESSION) HAS ITS CRACK CLOSED UPON APPLICATION OF
C     SHRINKAGE EQUILIBRIUM CORRECTION LOAD TO UNCRACKED SECTION.
C     IF CRACK DOES NOT CLOSE, ELEMENT IS OF NO FURTHER INTEREST.
C
   47 EPSTEMP = TOTEPSC(K,IM1) + DSHEPS(K,I)
      EPSCHEK = TOTEPSC(1, M1) + SHRP/A*EINV
      IF(EPSTEMP.GT.EPSCHEK) GO TO 60
C
      CALL DRY2(I)
      RETURN
C
C     CHECK TO ASCERTAIN IF ANY MORE ELEMENTS HAVE CRACKED. IF NO NEW
C     ELEMENTS CRACK, WE REACH THE FINAL EQUILIBRIUM POSITION.
C
   50 STRTEMP = TOTSTRC(K,IM1) + SHRP/A - DILBERT(K)
      IF(STRTEMP.LE.TS(I)) GO TO 51
      GO TO 60
C
C     REMOVE THE STRESS IN A NEWLY CRACKED ELEMENT BY APPLYING A STRESS,
C     EXSTR(K), EQUAL AND OPPOSITE TO THE STRESS IT HAD AT BEGINNING OF
C     TIME STEP. THIS IS THE EXTRA STRESS CREATED DURING TIME STEP.
C
   51 DO 52 KK=1,NSEG
      NDG = NDIAG(KK)
   52 IF(K.EQ.NDG) GO TO 55
C
C     FIND SYMMETRICAL COUNTERPART TO CRACKED ELEMENT.
C
      DO 53 KK=1,NSEG
      NREF = KK*NSEG
   53 IF(K.LE.NREF) GO TO 54
   54 ND = NDIAG(KK)
      NDIF = K-ND
      KS = NDIAG(KK+NDIF) - NDIF
C
   55 NC = NC+1
      IF(NC.LT.NTS) GO TO 56
      WRITE(6,3000)
      STOP
   56 NCOUNT(NC) = K
      EXSTR(K) = -TOTSTRC(K,IM1)
C
```

```
C     KEEP ACCOUNT OF HOW MANY ELEMENTS CRACK DURING THIS STEP (=MCRK).
C
      MCRK = MCRK+1
      NCOUNT5(MCRK) = K
C
C     DETERMINE EXTRA LOAD TO BE CARRIED BY UNCRACKED ELEMENTS.
C
      EXTLOAD = EXTLOAD + TOTSTRC(K,IM1)*AC(K)
      TOTSTRC(K,I) = 0.
C
C     THE QUADRANT MUST NOW BE RE-EXAMINED AND A NEW FORCE BALANCE
C     PERFORMED IF A DIAGONAL ELEMENT CRACKED.
C
      IF(K.EQ.NDG) GO TO 100
C
C     FOLLOW SIMILAR BOOKKEEPING PROCEDURES FOR THE SYMMETRICAL
C     COUNTERPART OF OFF-DIAGONAL ELEMENT WHICH CRACKED.
C
      NC = NC+1
      NCOUNT(NC) = KS
      EXSTR(KS) = -TOTSTRC(KS,IM1)
      MCRK = MCRK+1
      NCOUNT5(MCRK) = KS
      EXTLOAD = EXTLOAD + TOTSTRC(KS,IM1)*AC(KS)
      TOTSTRC(KS,I) = 0.
      TOTSTRC(KS,IM1) = 0.
C
C     RE-EXAMINE QUADRANT AND PERFORM A NEW FORCE BALANCE.
C
      GO TO 100
C
   60 CONTINUE
C
C------------------------------------------------------------------------
C
      DO 80 K=1,NTS
C
      IF(NC.EQ.0) GO TO 75
C
      DO 62 JJ=1,NC
      NN = NCOUNT(JJ)
   62 IF(K.EQ.NN) GO TO 63
      GO TO 75
C
   63 IF(MCOMP.EQ.0) GO TO 65
C
C     CALCULATE EXTRA SHRINKAGE STRESS CREATED IN CRACKED ELEMENTS
C     WHICH ARE IN COMPRESSION.
C
      DO 64 JJ=1,MCOMP
      N2 = NCOUNT2(JJ)
   64 IF(K.EQ.N2) GO TO 75
C
   65 IF(MADJ.EQ.0) GO TO 70
C
C     EXTRA STRESS CREATED IN ELEMENTS WHICH LOST COMPRESSION DURING
C     THIS TIME STEP HAS ALREADY BEEN DETERMINED.
C
      DO 66 JJ=1,MADJ
      N3 = NCOUNT3(JJ)
   66 IF(K.EQ.N3) GO TO 76
C
C     THE EXTRA STRESS CREATED IN ELEMENTS WHICH CRACKED PREVIOUS TO
C     THIS TIME STEP AND ARE NOT IN COMPRESSION IS ZERO.
C
   70 IF(MCRK.NE.0) GO TO 72
C
      EXSTR(K) = 0.
      GO TO 76
C
C     EXTRA STRESS CREATED IN ELEMENTS WHICH CRACKED DURING
C     THIS TIME STEP HAS ALREADY BEEN DETERMINED.
C
   72 DO 74 KK=1,MCRK
      N5 = NCOUNT5(KK)
   74 IF(K.EQ.N5) GO TO 76
C
      EXSTR(K) = 0.
      GO TO 76
C
C     EXTRA SHRINKAGE STRESS CREATED IN UNCRACKED ELEMENTS OR
C     CRACKED ELEMENTS IN COMPRESSION DURING THE TIME STEP.
C
   75 EXSTR(K) = SHRP/A - DILBERT(K)
C
C     TOTAL STRESS AND STRAIN FOR UNCRACKED ELEMENTS OR CRACKED
C     ELEMENTS IN COMPRESSION, INCLUDING EFFECTS OF EXTERNAL LOADS
C     APPLIED PRIOR TO TIME STEP (I).
C
      TOTSTRC(K,I) = TOTSTRC(K,IM1) + EXSTR(K)
      TOTEPSC(K,I) = TOTEPSC(K,IM1) + SHRP/A*EINV
      GO TO 80
C
C     TOTAL STRAIN FOR: 1. NEWLY CRACKED ELEMENTS
C                       2. OLD CRACKED ELEMENTS WHICH LOST COMPRESSION
C                       3. OLD CRACKED ELEMENTS NOT IN COMPRESSION
C     THE TOTAL STRESS HAS PREVIOUSLY BEEN COMPUTED.
C
   76 TOTEPSC(K,I) = TOTEPSC(K,IM1) + DSHREPS(K,I) + EXSTR(K)*EINV
C
   80 CONTINUE
C------------------------------------------------------------------------
C
C     CALCULATE SPECIFIC CREEP COMPLIANCE AND INCREMENTAL CREEP EFFECTS
C     CREATED BY EXTRA SHRINKAGE STRESS IN EACH SUBSEQUENT TIME PERIOD.
C
      IF(I.EQ.LT) GO TO 99
      IP1 = I+1
C
      DO 90 J=IP1,LT
      JM1 = J-1
      CREEP = CERN(J,I) - CERN(JM1,I)
      DO 90 K=1,NTS
   90 DSHREPS(K,J) = DSHREPS(K,J) + CREEP*EXSTR(K)
C
   99 RETURN
C
C     FORMAT STATEMENTS
C
 3000 FORMAT (1H1,50HTHE ENTIRE COLUMN CROSS-SECTION HAS NOW CRACKED --,
     .       //,10X,28HPROGRAM EXECUTION TERMINATED)
C
      END



      SUBROUTINE DRY2(I)
C
      COMMON A,B,LT,ESTL,NELBAR,NSEG,NTC,NTS,TIFAC,AC(100),CERN(55,55),
     .       AS(1),DSHREPS(100,55),NDIAG(15),P(55),RATIO(55),TS(55)
      COMMON/ANAL/ ADDSTRS(100),MCOMP,NC,NCOUNT(100),NCOUNT2(100),PT,TP,
     .       TOTEPSC(100,55),TOTEPSS(1,55),TOTSTRC(100,55),TOTSTRS(1,55)
      COMMON/CRAK/ DILBERT(100),SHRP
      DIMENSION NCOUNT4(100)
C
C***********************************************************************
C     THIS ROUTINE CALCULATES STRESSES AND STRAINS CREATED IN ELEMENTS
C     FOR THE CASE WHERE THE EQUILIBRIUM CORRECTION LOAD HAS CAUSED
C     A PREVIOUSLY CRACKED ELEMENT TO CLOSE.
C***********************************************************************
C
      DIFF = 0.
      NCO = 0
      EPSCHEK = 0.
      EINV = CERN(I,I)
      IM1 = I-1
C
```

```
      DO 1 K=1,NTS
    1 ADDSTRS(K) = 0.
C
C     FIND WHICH CRACKED SEGMENT HAS THE SMALLEST STRAIN (CRACK WIDTH).
C
  100 NGOLLUM = 1
      NCOLD = NCO
C
      DO 20 II=1,NC
      NN = NCOUNT(II)
      IF(NCO.EQ.0) GO TO 16
C
C     DISCARD FROM THE MINIMUM FINDING PROCESS THOSE ELEMENTS THAT WERE
C     CRACKED BUT ARE NOW CARRYING COMPRESSION.
C
      DO 15 JJ=1,NCO
      NNCO = NCOUNT4(JJ)
   15 IF(NN.EQ.NNCO) GO TO 20
C
   16 IF(NGOLLUM.EQ.2) GO TO 18
      NMIN = NN
      EPSMIN = TOTEPSC(NN,IM1) + DSHREPS(NN,I)
      NGOLLUM = 2
C
   18 EPSNEW = TOTEPSC(NN,IM1) + DSHREPS(NN,I)
      IF(EPSMIN.LE.EPSNEW) GO TO 20
      NMIN = NN
      EPSMIN = EPSNEW
   20 CONTINUE
C
      IF(NCO.EQ.0) GO TO 22
      GO TO 24
C
   22 EPSCHEK = TOTEPSC(1,IM1)
      GO TO 25
   24 EPSCHEK = EPSCHEK + DIFF
   25 ADDSTR = SHRP/A
C
C     CALCULATE MINIMUM CRACK WIDTH AND ASCERTAIN IF EQUILIBRIUM
C     CORRECTION LOAD WILL CAUSE CRACK TO CLOSE.
C
      DIFF = EPSMIN - EPSCHEK
      DIFCHEK = ADDSTR*EINV
      IF(DIFCHEK.GT.DIFF) GO TO 28
C
C     NO FURTHER CRACKS CLOSE - ANALYSIS IS COMPLETE.
C
      NGOLLUM = 1
      XSTRS = ADDSTR
      GO TO 40
C
C     CALCULATE STRESS NECESSARY TO CLOSE SMALLEST CRACK.
C
   28 XSTRS = DIFF/EINV
C
C     DETERMINE AMOUNT OF EXTERNAL LOAD REMAINING TO BE SUPPORTED.
C
      SHRP = SHRP - XSTRS*A
C
C     FIND SYMMETRICAL COUNTERPART OF ELEMENT WHOSE CRACK CLOSED.
C
      DO 30 KK=1,NSEG
      NDG = NDIAG(KK)
   30 IF(NMIN.EQ.NDG) GO TO 35
C
      DO 32 KK=1,NSEG
      NREF = KK*NSEG
   32 IF(NMIN.LE.NREF) GO TO 33
   33 ND = NDIAG(KK)
      NDIF = NMIN - ND
      KS = NDIAG(KK+NDIF) - NDIF
C
   35 NCO = NCO+1
      NCOUNT4(NCO) = NMIN
C
C     INCREASE TRANSFORMED AREA BY AREA OF SEGMENT WHOSE CRACK CLOSED.
C
      A = A + AC(NMIN)
C
C     CHECK IF A DIAGONAL ELEMENT CLOSED.
C
      IF(NMIN.EQ.NDG) GO TO 40
C
C     FOLLOW SIMILAR BOOKKEEPING PROCEDURES FOR THE SYMMETRICAL
C     COUNTERPART OF OFF-DIAGONAL ELEMENTS WHOSE CRACKS CLOSED.
C
      NCO = NCO+1
      NCOUNT4(NCO) = KS
      A = A + AC(KS)
C
   40 DO 50 K=1,NTS
C
C     CRACKED ELEMENTS THAT REMAIN CRACKED HAVE NO ADDITIONAL
C     STRESS CREATED.
C
      DO 42 JJ=1,NC
      NN = NCOUNT(JJ)
   42 IF(K.EQ.NN) GO TO 43
      GO TO 48
C
C     LOCATE ALL PREVIOUSLY CRACKED ELEMENTS IN COMPRESSION.
C
   43 IF(MCOMP.EQ.0) GO TO 45
      DO 44 JJ=1,MCOMP
      N2 = NCOUNT2(JJ)
   44 IF(K.EQ.N2) GO TO 48
C
   45 IF(NCOLD.EQ.0) GO TO 50
C
C     LOCATE CRACKED ELEMENTS WHOSE CRACKS HAVE NOW CLOSED
C     DUE TO APPLICATION OF EQUILIBRIUM CORRECTION LOAD.
C
      DO 46 II=1,NCOLD
      NNCO = NCOUNT4(II)
   46 IF(K.EQ.NNCO) GO TO 48
      GO TO 50
C
C     CALCULATE ADDITIONAL STRESS CREATED IN UNCRACKED ELEMENTS
C     AND CRACKED ELEMENTS IN COMPRESSION.
C
   48 ADDSTRS(K) = ADDSTRS(K) + XSTRS
   50 CONTINUE
C
C     CHECK IF SOME OF THE LOAD STILL HAS TO BE SUPPORTED.
C
      IF(NGOLLUM.NE.1) GO TO 100
C
C     REDUCE ADDITIONAL STRESS BY COMPATIBILITY CORRECTION STRESS FOR
C     UNCRACKED ELEMENTS AND PREVIOUSLY CRACKED ELEMENTS IN COMPRESSION.
C
      DO 55 K=1,NTS
C
      DO 51 JJ=1,NC
      NN = NCOUNT(JJ)
   51 IF(K.EQ.NN) GO TO 52
      GO TO 54
C
   52 IF(MCOMP.EQ.0) GO TO 55
C
      DO 53 JJ=1,MCOMP
      N2 = NCOUNT(JJ)
   53 IF(K.EQ.N2) GO TO 54
      GO TO 55
C
   54 ADDSTRS(K) = ADDSTRS(K) - DILBERT(K)
C
   55 CONTINUE
C
C     UPDATE NUMBER OF CRACKED ELEMENTS IN COMPRESSION.
C
```

```
      MCP1 = MCOMP +1
      MCOMP = MCOMP + NCO
      N4C = 1
C
      DO 58 KK=MCP1,MCOMP
      NCOUNT2(KK) = NCOUNT4(N4C)
   58 N4C = N4C + 1
C
C     COMPUTE TOTAL STRESS AND STRAIN IN ALL CONCRETE ELEMENTS.
C
      DO 70 K=1,NTS
      DO 62 JJ=1,NC
      NN = NCOUNT(JJ)
   62 IF(K.EQ.NN) GO TO 63
      GO TO 68
C
   63 IF(MCOMP.EQ.0) GO TO 69
      DO 64 JJ=1,MCOMP
      N2 = NCOUNT2(JJ)
   64 IF(K.EQ.N2) GO TO 68
      GO TO 69
C
C     TOTAL STRESS AND STRAIN FOR UNCRACKED ELEMENTS OR CRACKED
C     ELEMENTS IN COMPRESSION.
C
   68 TOTSTRC(K,I) = TOTSTRC(K,IM1) + ADDSTRS(K)
      TOTEPSC(K,I) = EPSCHEK + DIFCHEK
      GO TO 70
C
C     TOTAL STRAIN FOR CRACKED ELEMENTS NOT IN COMPRESSION.
C
   69 TOTEPSC(K,I) = TOTEPSC(K,IM1) + DSHREPS(K,I)
   70 CONTINUE
C
C     CALCULATE SPECIFIC CREEP COMPLIANCE AND INCREMENTAL CREEP EFFECTS
C     CREATED BY ADDITIONAL STRESS IN EACH SUBSEQUENT TIME PERIOD.
C
      IF(I.EQ.LT) GO TO 99
      IP1 = I+1
C
      DO 90 J=IP1,LT
      JM1 = J-1
      CREEP = CERN(J,I) - CERN(JM1,I)
      DO 90 K=1,NTS
   90 DSHREPS(K,J) = DSHREPS(K,J) + CREEP*ADDSTRS(K)
C
   99 RETURN
      END


      SUBROUTINE STREPS(I)
C
      COMMON A,B,LT,ESTL,NELBAR,NSEG,NTC,NTS,TIFAC,AC(100),CERN(55,55),
     *       AS(1),DSHREPS(100,55),NDIAG(15),P(55),RATIO(55),TS(55)
      COMMON/ANAL/ ADDSTRS(100),MCOMP,NC,NCOUNT(100),NCOUNT2(100),PT,TP,
     *       TOTEPSC(100,55),TOTEPSS(1,55),TOTSTRC(100,55),TOTSTRS(1,55)
      DIMENSION NCOUNT4(100)
C
C*********************************************************************
C     THIS ROUTINE CALCULATES STRESSES AND STRAINS CREATED IN ELEMENTS
C     DUE TO APPLICATION OF EXTERNAL LOAD AT TIME STEP (I).
C*********************************************************************
C
      IF(TP.EQ.0.) GO TO 99
C
      DIFF = 0.
      NCO = 0
      EPSCHEK = 0.
      EINV = CERN(I,I)
C
      DO 1 K=1,NTS
    1 ADDSTRS(K) = 0.
C
      IF(NC.NE.0) GO TO 100
C
      NGOLLUM = 1
      XSTRS = TP/A
      GO TO 40
C
C     FIND WHICH CRACKED SEGMENT HAS THE SMALLEST STRAIN (CRACK WIDTH).
C
  100 NGOLLUM = 1
      NCOLD = NCO
C
      DO 20 II=1,NC
      NN = NCOUNT(II)
      IF(NCO.EQ.0) GO TO 16
C
C     DISCARD FROM THE MINIMUM FINDING PROCESS THOSE ELEMENTS THAT WERE
C     CRACKED BUT ARE NOW CARRYING COMPRESSION.
C
      DO 15 JJ=1,NCO
      NNCO = NCOUNT4(JJ)
   15 IF(NN.EQ.NNCO) GO TO 20
C
   16 IF(NGOLLUM.EQ.2) GO TO 18
      NMIN = NN
      EPSMIN = TOTEPSC(NN,I)
      NGOLLUM = 2
C
   18 IF(EPSMIN.LE.TOTEPSC(NN,I)) GO TO 20
      NMIN = NN
      EPSMIN = TOTEPSC(NN,I)
   20 CONTINUE
C
      IF(NCO.EQ.0) GO TO 22
      GO TO 24
C
   22 EPSCHEK = TOTEPSC(1,I)
      GO TO 25
   24 EPSCHEK = EPSCHEK + DIFF
   25 ADDSTR = TP/A
C
C     CALCULATE MINIMUM CRACK WIDTH AND ASCERTAIN IF EXTERNALLY APPLIED
C     LOAD WILL CAUSE CRACK TO CLOSE.
C
      DIFF = EPSMIN - EPSCHEK
      DIFCHEK = ADDSTR*EINV
      IF(DIFCHEK.GT.DIFF) GO TO 28
C
C     NO FURTHER CRACKS CLOSE - ANALYSIS IS COMPLETE.
C
      NGOLLUM = 1
      XSTRS = TP/A
      GO TO 40
C
C     CALCULATE STRESS NECESSARY TO CLOSE SMALLEST CRACK.
C
   28 XSTRS = DIFF/EINV
C
C     DETERMINE AMOUNT OF EXTERNAL LOAD REMAINING TO BE SUPPORTED.
C
      TP = TP - XSTRS*A
C
C     FIND SYMMETRICAL COUNTERPART OF ELEMENT WHOSE CRACK CLOSED.
C
      DO 30 KK=1,NSEG
      NDG = NDIAG(KK)
   30 IF(NMIN.EQ.NDG) GO TO 35
C
      DO 32 KK=1,NSEG
      NREF = KK*NSEG
   32 IF(NMIN.LE.NREF) GO TO 33
   33 ND = NDIAG(KK)
      NDIF = NMIN - ND
      KS = NDIAG(KK+NDIF) - NDIF
```

```
C
   35 NCO = NCO+1
      NCOUNT4(NCO) = NMIN
C
C     INCREASE TRANSFORMED AREA BY AREA OF SEGMENT WHOSE CRACK CLOSED.
C
      A = A + AC(NMIN)
C
C     CHECK IF A DIAGONAL ELEMENT CLOSED.
C
      IF(NMIN.EQ.NDG) GO TO 40
C
C     FOLLOW SIMILAR BOOKEEPING PROCEDURES FOR THE SYMMETRICAL
C     COUNTERPART OF OFF-DIAGONAL ELEMENTS WHOSE CRACKS CLOSED.
C
      NCO = NCO+1
      NCOUNT4(NCO) = KS
      A = A + AC(KS)
C
   40 DO 50 K=1,NTS
      IF(NC.EQ.0) GO TO 48
C
C     CRACKED ELEMENTS THAT REMAIN CRACKED HAVE NO ADDITIONAL
C     STRESS CREATED.
C
      DO 42 JJ=1,NC
      NN = NCOUNT(JJ)
   42 IF(K.EQ.NN) GO TO 43
      GO TO 49
C
C     LOCATE ALL PREVIOUSLY CRACKED ELEMENTS IN COMPRESSION.
C
   43 IF(MCOMP.EQ.0) GO TO 45
      DO 44 JJ=1,MCOMP
      N2 = NCOUNT2(JJ)
   44 IF(K.EQ.N2) GO TO 48
C
   45 IF(NCOLD.EQ.0) GO TO 50
C
C     LOCATE CRACKED ELEMENTS WHOSE CRACKS HAVE NOW CLOSED
C     DUE TO APPLICATION OF AN EXTERNAL LOAD.
C
      DO 46 II=1,NCOLD
      NNCO = NCOUNT4(II)
   46 IF(K.EQ.NNCO) GO TO 48
      GO TO 50
C
C     CALCULATE ADDITIONAL STRESS CREATED IN UNCRACKED ELEMENTS
C     AND CRACKED ELEMENTS IN COMPRESSION.
C
   48 ADDSTRS(K) = ADDSTRS(K) + XSTRS
   50 CONTINUE
C
C     CHECK IF SOME OF THE LOAD STILL HAS TO BE SUPPORTED.
C
      IF(NGOLLUM.NE.1) GO TO 100
C
C     CALCULATE SPECIFIC CREEP COMPLIANCE AND INCREMENTAL CREEP EFFECTS
C     CREATED BY ADDITIONAL STRESS IN EACH SUBSEQUENT TIME PERIOD.
C
      IF(I.EQ.LT) GO TO 70
      IP1 = I+1
C
      DO 60 J=IP1,LT
      JM1 = J-1
      CREEP = CERN(J,I) - CERN(JM1,I)
      DO 60 K=1,NTS
   60 DSHREPS(K,J) = DSHREPS(K,J) + CREEP*ADDSTRS(K)
C
C     COMPUTE TOTAL STRESS AND STRAIN IN ALL CONCRETE ELEMENTS.
C
   70 DO 80 K=1,NTS
      TOTSTRC(K,I) = TOTSTRC(K,I) + ADDSTRS(K)
      TOTEPSC(K,I) = TOTEPSC(K,I) + ADDSTRS(K)*CINV
   80 CONTINUE
C
C     COMPUTE TOTAL STRESS AND STRAIN IN STEEL.
C
   99 TOTEPSS(1,I) = TOTEPSC(1,I)
      TOTSTRS(1,I) = TOTEPSS(1,I)*ESTL
C
      RETURN
      END



      SUBROUTINE RELESE(I)
C
      COMMON A,P,LT,ESTL,NELBAR,NSEG,NTC,NTS,TIFAC,AC(100),CERN(55,55),
     .       AS(1),DSHREPS(100,55),NDIAG(15),P(55),RATIO(55),TS(55)
      COMMON/ANAL/ ADDSTRS(100),MCOMP,NC,NCOUNT(100),NCOUNT2(100),PT,TP,
     .       TOTEPSC(100,55),TOTEPSS(1,55),TOTSTRC(100,55),TOTSTRS(1,55)
      DIMENSION FMT(10)
C
C***********************************************************************
C     THIS ROUTINE VISUALLY PRINTS STRESS-STRAIN RESULTS
C     WITHIN THE QUADRANT AT THE END OF EACH TIME STEP.
C***********************************************************************
C
      NAROW = NSEG*12
      IF(NSEG.EQ.10) NAROW=NSEG*11
      NDAYS = TIFAC*I
      NCLSIZ = 2.*8
      NODES = NSEG*2 + 1
      IF(NODES.GT.11) GO TO 101
      WRITE(6,2000) NDAYS, NCLSIZ
      GO TO 102
  101 WRITE(6,2001) NDAYS, NCLSIZ
C
  102 DO 10 NUM=1,NSEG
      K = NSEG-NUM
      IF(K.EQ.0) GO TO 11
      KK = NSEG*K
      IF(NODES.GT.11) GO TO 92
      ENCODE(100,2002,FMT) NSEG
      WRITE(6,FMT) (TOTSTRC(KK+J,I), J=1,NSEG)
      DO 69 J=1,NSEG
   69 TOTEPSC(KK+J,I) = TOTEPSC(KK+J,I)*1.E+6
      ENCODE(100,2003,FMT) NSEG
      WRITE(6,FMT) (TOTEPSC(KK+J,I), J=1,NSEG)
      DO 70 J=1,NSEG
   70 TOTEPSC(KK+J,I) = TOTEPSC(KK+J,I)/(1.E+6)
      GO TO 10
   92 ENCODE(100,2002,FMT) NSEG
      WRITE(6,FMT) (TOTSTRC(KK+J,I), J=1,NSEG)
      DO 71 J=1,NSEG
   71 TOTEPSC(KK+J,I) = TOTEPSC(KK+J,I)*1.E+6
      ENCODE(100,2004,FMT) NSEG
      WRITE(6,FMT) (TOTEPSC(KK+J,I), J=1,NSEG)
      DO 72 J=1,NSEG
   72 TOTEPSC(KK+J,I) = TOTEPSC(KK+J,I)/(1.E+6)
   10 CONTINUE
C
      GO TO 12
C
   11 IF(NODES.GT.11) GO TO 91
      ENCODE(100,2002,FMT) NSEG
      WRITE(6,FMT) (TOTSTRC(J,I), J=1,NSEG)
      DO 73 J=1,NSEG
   73 TOTEPSC(J,I) = TOTEPSC(J,I)*1.E+6
      ENCODE(100,2004,FMT) NSEG
      WRITE(6,FMT) (TOTEPSC(J,I), J=1,NSEG)
      DO 74 J=1,NSEG
   74 TOTEPSC(J,I) = TOTEPSC(J,I)/(1.E+6)
      ENCODE(100,2005,FMT) NAROW
      WRITE(6,FMT)
      GO TO 12
```

```
   91 ENCODE(100,2002,FMT) NSEG
      WRITE(6,FMT) (TOTSTRC(J,I), J=1,NSEG)
      DO 75 J=1,NSEG
   75 TOTEPSC(J,I) = TOTEPSC(J,I)*1.E+6
      ENCODE(100,2006,FMT) NSEG
      WRITE(6,FMT) (TOTEPSC(J,I), J=1,NSEG)
      DO 76 J=1,NSEG
   76 TOTEPSC(J,I) = TOTEPSC(J,I)/(1.E+6)
      ENCODE(100,2007,FMT) NAROW
      WRITE(6,FMT)
   12 CONTINUE
C
      STRAIN = TOTEPSC(1,I)*1.E+6
      WRITE(6,2008) STRAIN
      ASTL = 0.
      DO 15 K=1,NELBAR
   15 ASTL = ASTL + AS(K)
C
      SRATIO = ASTL/(B*B)
      WRITE(6,2009) SRATIO
C
      DO 20 IT=1,LT
   20 IF(P(IT).NE.0.) GO TO 100
      WRITE(6,2010)
      GO TO 200
C
  100 WRITE(6,2011) PT
C
  200 WRITE(6,2012)
      FORCE = 0.
      DO 30 K=1,NELBAR
      FORCES = TOTSTRS(K,I)*AS(K)
      FORCE = FORCE + FORCES
      FORCEC = PT - FORCE
   30 WRITE(6,2013) K, TOTSTRS(K,I), FORCES, FORCEC
C
      IF(TP.EQ.0.) RETURN
C
      ADSTRC = ADDSTRS(1)
      ADEPSC = ADSTRC*CERN(I,I)*1.E+6
      DSTRAIN = STRAIN - ADEPSC
      ADSTRS = ADSTRC*CERN(I,I)*ESTL
      ADFRCS = ADSTRS*AS(1)
      DFORCES = FORCE - ADFRCS
      PL = PT - P(I)
      DFORCEC = PL - DFORCES
C
      WRITE(6,2014) ADEPSC,DSTRAIN,ADFRCS,DFORCES,DFORCEC
C
      RETURN
C
C     FORMAT STATEMENTS
C
 2000 FORMAT (1H1,6X,31HSTRESS (KSI) AT CENTROIDS AFTER I4,
     .        11H DAYS FOR A I3,12H INCH COLUMN/////
     .        2X,11HCENTER LINE,2(/,7X,1H↑))
 2001 FORMAT (1H1,6X,31HSTRESS (KSI) AT CENTROIDS AFTER I4,10H DAYS FOR
     .        1HA I3,12H INCH COLUMN////2X,11HCENTER LINE,3(/,7X,1H↑))
 2002 FORMAT (8H(7X,1H↑,I2,10H(3X,F8.5)))
 2003 FORMAT (8H(7X,1H↑,I2,22H(3X,F8.2),4(/,7X,1H↑)))
 2004 FORMAT (8H(7X,1H↑,I2,19H(3X,F8.2),/,7X,1H↑))
 2005 FORMAT (8H(7X,1H↑),I3,22H(1H←),12H CENTER LINE))
 2006 FORMAT (8H(7X,1H↑,I2,10H(3X,F8.2)))
 2007 FORMAT (8H(7X,1H↑,I3,12H(1H←),3H CL))
 2008 FORMAT (/,7X,8HSTRAIN = F8.2,14H MICRO-IN./IN.)
 2009 FORMAT (/,7X,13HSTEEL RATIO = F6.4)
 2010 FORMAT (7X,24HNO AXIAL LOAD IS APPLIED)
 2011 FORMAT (/,7X,28HTOTAL AXIAL LOAD ON COLUMN = F7.2,5H KIPS)
 2012 FORMAT (/,7X,7HBAR NO.,12X,12HSTRESS (KSI),10X,12HSTEEL FORCE
     .        6H(KIPS),10X,21HCONCRETE FORCE (KIPS))
 2013 FORMAT (8X,I3,16X,F8.5,15X,F10.5,19X,F10.5)
 2014 FORMAT(/,7X,6HADEPSC,4X,7HDSTRAIN,5X,6HADFRCS,4X,7HDFORCES,4X,
     .        7HDFORCEC/2X,5F11.3)
C
      END
```