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J. Treiner, W.D. Myers, W.J. Swiatecki,
and M.S. Weiss

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Hartree-Fock, Thomas-Fermi and Droplet Model Calculations*

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Abstract: Hartree-Fock calculations for hypothetical spherical nuclei with $N = Z$ are carried out without the inclusion of Coulomb interactions, spin-orbit terms or pairing in order to study one simple property of such systems: the bulk compression due to surface tension. We find that the behavior expected from Thomas-Fermi and Droplet calculations manifests itself for medium and heavy nuclei, and the agreement between microscopic and macroscopic descriptions is further illustrated by carrying the calculations to extremely large mass numbers. Deviations of the lighter nuclei from the simplest form of the macroscopic theory are discussed and various improvements are considered.

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1. Introduction

A number of recent studies have been devoted to the comparison between density distributions predicted by the Droplet Model of atomic nuclei and those predicted by Hartree-Fock calculations¹⁻²). Some of these studies raised questions about the validity of the Droplet Model approach for treating the neutron skin in neutron-rich nuclei and, in particular, about an elementary prediction of the Droplet Model concerning the squeezing of nuclei by the surface tension. While existing Hartree-Fock calculations were not sufficiently comprehensive to be able to settle these questions, certain features of the numerical results raised concerns over whether or not the Droplet Model was correctly formulated. The relation of the Droplet Model to Thomas-Fermi (rather than Hartree-Fock) calculations had been thoroughly investigated in the original Droplet Model paper³) and perfect agreement had been found as regards the neutron skin as well as the surface tension squeezing in the appropriate limit of sufficiently large nuclei, in which the Droplet Model should formally be valid. However, two questions still remained. First, does the change from a Thomas-Fermi to a Hartree-Fock treatment, in addition to bringing in the expected oscillations in nuclear properties associated with shell effects, also modify in some unsuspected way the shell-averaged behaviour, thus invalidating the Droplet Model predictions of average nuclear properties? Second, even if the Droplet Model predictions were found to be formally valid for sufficiently large hypothetical nuclei, are they quantitatively useful for actual nuclei throughout the periodic table? Again, comparisons with Thomas-Fermi calculations in ref. ³) suggested that this might be the case but, as regards the question of the surface tension squeezing, there was already a hint of quantitative

limitations in Fig. 30 of that reference, which showed that the Droplet Model prediction of a squeezing that increased monotonically with increasing surface to volume ratio ceased to be valid below about mass number $A \approx 45$. The Thomas-Fermi calculations showed that for lighter nuclei the central density of the model nucleus decreased with decreasing A , a behaviour qualitatively different from the Droplet Model prediction.

2. The Droplet Model Squeezing Formula

The Droplet Model prediction for the density distribution inside a nucleus may be found in ref. ⁴). We recall here the derivation for the simplified situation to be studied in this paper corresponding to a fictitious, spherical, uncharged nucleus with equal numbers of neutrons and protons ($N = Z$). The density in the bulk region is assumed to be sufficiently uniform so that a single quantity, the "central density" ρ_c , can be used to characterize the bulk conditions. The equivalent sharp radius R is related to ρ_c by

$$\frac{4}{3} \pi R^3 \rho_c = A \quad . \quad (1)$$

For finite nuclei, ρ_c will, in general, be different from the nuclear matter density ρ_0 . Let ϵ be a measure of this deviation, defined by

$$\epsilon = -\frac{1}{3} [(\rho_c - \rho_0)/\rho_0] \quad .$$

In the Droplet Model the bulk energy is written as

$$-a_1 A + \frac{1}{2} K A \epsilon^2 \quad , \quad (3)$$

where a_1 is the volume energy coefficient and K is the incompressibility coefficient. With the surface energy per unit area written as σ , the surface energy of a nucleus is $4\pi R^2\sigma$ which, to lowest order in ϵ , may be written as

$$a_2 A^{2/3} (1 + 2\epsilon) \quad , \quad (4)$$

where $a_2 = 4\pi r_0^2 \sigma$, r_0 is the nuclear radius constant defined by

$$r_0 = \frac{4}{3} \pi \rho_0^{-1/3} \quad (5)$$

and a_2 is the surface energy coefficient. Minimizing the sum of eqs. (3) and (4) with respect to ϵ we find the Droplet Model relation for the relative compression,

$$\epsilon = -(2a_2/K)A^{-1/3} \quad , \quad (6)$$

which leads to the expression,

$$\frac{\Delta\rho}{\rho_0} = (6a_2/K)A^{-1/3} \quad , \quad (7)$$

where $\Delta\rho = \rho_c - \rho_0$.

3. Hartree-Fock Calculations

In order to test eq. (7) we performed two sets of Hartree-Fock calculations, using two types of Skyrme interactions corresponding to two values of the incompressibility coefficient: $K = 355.6$ MeV for the Skyrme force S-III⁵) and $K = 216.6$ MeV for the Skyrme force SkM⁶). In each case we determined the self-consistent Hartree-Fock density distributions for all closed-shell spherical nuclei with $N = Z$ and mass number A from 16 to 3112. (There was no spin-orbit force, so the shell closures do not correspond to those for actual nuclei.)

The resulting Hartree-Fock density distributions exhibit the usual radial ripples associated with quantal effects, so the extraction of a bulk density to be compared with eq. (7) is not a trivial matter. Some kind of smoothing of the rippled distributions is necessary. The method we adopted was to make a least-squares fit of a smooth distribution to the Hartree-Fock density and to use the central value of the smooth density as a representation of the bulk density of the Hartree-Fock distribution. The smooth distribution was taken in the form of a generalized Fermi function:

$$\rho(r) = \rho_0 \left[1 + e^{\left(\frac{r-R}{a}\right)} \right]^{-\nu} , \quad (8)$$

where ρ_0 , R , a , ν are parameters adjusted to give the best fit to the Hartree-Fock density and to ensure the proper normalization of ρ . (The parameter ν was introduced to provide additional freedom in the fit, in particular to allow for a finite skewness of the density profile in the surface).

Fig. 1 shows the values of

$$(\Delta\rho/\rho_0) = [(\rho_c - \rho_0)/\rho_0] = -3\varepsilon \quad , \quad (9)$$

obtained in this way, plotted as a function of $A^{-1/3}$, where ρ_c is the value of ρ at the center, i.e. $\rho_c = \rho(r = 0)$. The two Skyrme interactions lead to qualitatively similar results. Two features are immediately apparent: most nuclei are squeezed (their bulk densities exceed nuclear matter density by several percent) but the squeezing shows very large shell-effect fluctuations. The set of nuclei with the smaller K-value (Fig. 1b) shows larger fluctuations, presumably because these nuclei are softer with respect to shell-effect perturbations. It may be verified that these fluctuations in ρ_c are related to the deviations from the average of the relative proportions of high and low angular momentum states in the nuclei. When a high angular momentum level has just been filled, there will tend to be an excess of orbitals with densities at large distances from the origin, leading to density distributions somewhat more extended than the average. When these are fitted with a generalized Fermi function, eq.(8), the resulting value of ρ_c will tend to be smaller than the average. Conversely, when low angular momentum orbitals have just been filled, the value of ρ_c will tend to be higher than the average.

As regards the absolute values of the fluctuations in Figs. 1a, b, we would expect them to be larger than in the actual nuclei. First, the spin-orbit coupling, Coulomb effect and neutron excess would reduce the degree of degeneracy of the single-particle eigenvalues in actual nuclei and thus reduce the shell effect. Second, there are indications that Hartree-Fock

calculations tend to exaggerate the ripples in the density distributions--the reason may have to do with the neglect of residual nucleonic interactions (such as pairing) or other idealizations of the Hartree-Fock theory.

Even with the probably exaggerated fluctuations of the idealized Hartree-Fock calculations in Figs. 1a and 1b it is possible to discern systematic trends, which may be compared with the Droplet Model predictions, given by the straight line. (In eq. (7) values of a_2 and K appropriate to the model in question were used. The values of a_2 were obtained from a calculation for semi-infinite matter. See Table I.) For very large systems ($A^{-1/3} < 0.15$ i.e. $A > 300$) there is clear indication that the Hartree-Fock bulk densities are converging towards the nuclear matter value (i.e. $\Delta\rho \rightarrow 0$). This removes the misgivings one might have had (by looking only at the points for $A < 300$) that even this elementary expectation was not borne out by the numerical calculations. The prediction of the Droplet Model is, roughly speaking, consistent with the trend of the Hartree-Fock calculations for $A^{-1/3} < 0.15$. (One cannot be very quantitative about this in view of the large fluctuations in the calculated points as well as the possible ambiguities associated with the extraction of bulk values from the Hartree-Fock densities.) For lighter nuclei, one can discern, despite the large fluctuations, a tendency for the average of $\Delta\rho/\rho_0$ to decrease, contrary to the Droplet Model prediction. We had already noted such a trend in the Thomas-Fermi calculations in ref. ³) and we shall find it again in the Thomas-Fermi calculations with Skyrme forces described in the next section. However, apart from shell fluctuations, there does not appear to be evidence for a fundamental disagreement between Hartree-Fock trends and Droplet Model predictions beyond those already noted in connection with Thomas-Fermi

calculations. The rather different conclusion implied by ref. ¹⁾ may be attributed to three technical points. First, the number of nuclei considered in ref. ¹⁾ was not large enough, and the nuclei themselves were not heavy enough, to establish a trend in the Hartree-Fock calculations that could be compared with the Droplet Model. Only some 20 nuclei were analyzed in ref. ¹⁾ (and these included Coulomb and asymmetry effects) and only one symmetric case, namely ¹⁶⁴Pb, was calculated without Coulomb energy. Second, the extraction of bulk densities from the Hartree-Fock calculations was made by equating Ford-Wills moments of the densities with approximate expressions for the Ford-Wills moments of Fermi functions. This approximation may be shown to underestimate the bulk densities. Third, the use of Fermi distributions (with zero skewness) is somewhat restrictive. For example, if one fits the square of a Fermi distribution by a Fermi distribution one also underestimates the central density of the original function.

4. The (Extended) Thomas-Fermi Calculation

A Thomas-Fermi calculation based on the same (Skyrme) nucleon-nucleon interaction as a Hartree-Fock calculation is expected to have in it essentially all the physics of the latter, except for shell effects. It would seem to be an appropriate way of investigating the shell-averaged trends of a Hartree-Fock calculation.

When used with a Skyrme type interaction, the Extended Thomas-Fermi method leads to an energy-density formalism where the energy can be written as a functional of the local one-body density and its derivative^{8,9};

$$E = \int \mathcal{H}(\rho, \rho') d\vec{r} = \int [F(\rho) + G(\rho)(\text{grad } \rho)^2] d\vec{r} \quad (10)$$

Here $F(\rho)$ is the energy density of infinite, uniform nuclear matter and $G(\rho)(\text{grad } \rho)^2$ represents modifications produced by non-uniformities in the density. The Euler equation determining the density distribution has the form

$$\frac{dF}{d\rho} + \frac{dG}{d\rho} (\text{grad } \rho)^2 + 2G \Delta\rho = \lambda \quad , \quad (11)$$

where λ is the Lagrange multiplier corresponding to particle conservation and equal to the particle separation energy, viz.:

$$\lambda = \frac{\partial E}{\partial A} \quad . \quad (12)$$

We have solved eq. (11) following the method of ref. ⁸). (The Weizsäcker coefficient of the kinetic energy functional was taken to be twice the semi-classical value⁹) in order to mock up roughly the effect of the $\hbar^4$ -correction to the kinetic energy density ¹⁰.) The resulting relative excess of the central density for these Thomas-Fermi solutions is plotted in Figs. 1a,b as the dashed curve. The results are similar to the Thomas-Fermi calculation in Fig. 30, ref. ³): for very large nuclei the Thomas-Fermi calculations confirm quantitatively the Droplet Model predictions but for lighter nuclei the compression begins to decrease. For the SIII force this happens below $A \approx 37$, for the SkM force below $A \approx 100$. [The position of the maximum in the plot of $\Delta\rho/\rho_0$ vs. $A^{-1/3}$ seems to be correlated with the incompressibility coefficient K . Thus $A \approx 37$ for $K = 355.6$ MeV (SIII), $A \approx 45$ for $K = 294.8$ MeV (Seyler-Blanchard force, ref. ³)) and $A \approx 100$ for $K = 216.6$ MeV (SkM).] In a qualitative way the Thomas-Fermi calculations reproduce the average trend of the Hartree-Fock results, but there is a hint that for very light nuclei the dashed curves in Figs. 1a,b lie somewhat

higher than the average of the Hartree-Fock points. This difference may or may not be within the ambiguities of the smoothing prescription used to extract the bulk density of the rippled Hartree-Fock densities.

5. Possible Droplet Model Refinements

The standard Droplet Model is based on an expression for the nuclear energy that includes terms up to a sufficient order in the small quantity ϵ (itself of order $A^{-1/3}$) so that the final binding energy formula (obtained by minimizing with respect to ϵ) will contain all terms of order A , $A^{2/3}$ and $A^{1/3}$. The squeezing formula linear in $A^{-1/3}$ (eq. (6)) is the result of working in this standard Droplet-Model approximation. In order to go beyond the linear squeezing formula, in particular in trying to account for the down-turn of $\Delta\rho/\rho_0$ as a function of $A^{-1/3}$, one is tempted to extend the Droplet Model by including higher powers of $A^{-1/3}$ in the energy expression. We shall now show that, under rather general assumptions, such an extension can never lead to a down-turn of $\Delta\rho/\rho_0$ no matter how many powers of $A^{-1/3}$ are included. We shall go on to argue that the down-turn is a feature associated with terms that are non-analytic in $A^{-1/3}$, in particular of the type $\exp(-cA^{1/3})$, which do not possess a Taylor expansion in $A^{-1/3}$.

To prove the "no down-turn" theorem we start with the Euler equation (11) and apply it to the center of a spherical nucleus, where the density is ρ_C , the derivative ρ' is zero and the Laplacian $\Delta\rho$ is three times ρ_C'' , the second derivative of ρ evaluated at the center¹¹):

$$\lambda = f(\rho_C) + 6G(\rho_C)\rho_C'' \quad , \quad (13)$$

where $f \equiv \frac{dF}{d\rho}$.

For a leptodermous (thin-skinned) system, the value of ρ'' tends rapidly to zero as one leaves the surface region and approaches the center. In Thomas-Fermi type calculations, the value of ρ'' decreases exponentially with distance from the surface, the range of the exponential being a quantity of the general order of the surface width. (More precisely, the range is characteristic of the inward part of the surface diffuseness, whose value is inversely proportional to the incompressibility coefficient K .) In any case, the value ρ_c'' will be proportional to an exponential term of the form

$$\begin{aligned} \rho_c'' &\propto e^{-(R/\text{constant})} \\ &\propto e^{-(\text{constant})A^{1/3}} \end{aligned} \quad (14)$$

The separation energy λ in eq. (13) may, in general, consist of an expansion in $A^{-1/3}$, modified by an exponential similar to eq. (14). Thus if the total energy of the system is written as

$$E = a_1 A + a_2 A^{2/3} + a_3 A^{1/3} + a_4 + \text{terms in } e^{-cA^{1/3}} \quad (15)$$

the separation energy λ will be of the form

$$\begin{aligned} \lambda = \frac{\partial E}{\partial A} &= a_1 + \frac{2}{3} a_2 A^{-1/3} + \frac{1}{3} a_3 A^{-2/3} + 0 \cdot A^{-1} + \text{higher powers of } A^{-1/3} \\ &+ \text{terms in } e^{-cA^{1/3}} \end{aligned} \quad (16)$$

For sufficiently large A the exponential terms will always become negligible compared to the polynomial terms. Eq. (13) reduces then to what may be called the Extended Polynomial Droplet Model Euler Equation

$$a_1 + \frac{2}{3} a_2 A^{-1/3} + \frac{1}{3} a_3 A^{-2/3} + \dots = f(\rho_c) \quad (17)$$

For a given $F(\rho)$ the right hand side is a known function of ρ_c , so that eq. (17) defines $\rho_c(A^{-1/3})$, the functional dependence of ρ_c on $A^{-1/3}$. From the known or assumed qualitative features of $F(\rho)$ and of the separation energy we shall now deduce certain general properties of this function $\rho_c(A^{-1/3})$. This is done graphically in Fig. 2.

In part (a) of this figure we display a typical energy per particle, $e(\rho)$, for infinite nuclear matter. It is positive for small ρ (this is not visible in Fig. 2a), becomes zero as the nuclear interactions cancel the kinetic energy, has a minimum with value $e = a_1$ at $\rho = \rho_0$, crosses zero again and is assumed to increase monotonically thereafter. The energy density $F(\rho)$ is obtained by multiplying $e(\rho)$ by ρ and the derivative $f(\rho)$ is displayed in Fig. 2b. It is clear by construction that $f(\rho)$ has a minimum at some density $\rho = \rho_1 < \rho_0$ and that it becomes zero at some density $\rho = \rho_2 > \rho_0$. At $\rho = \rho_0$ the value of f is

$$f(\rho_0) = \rho_0 e'(\rho_0) + e(\rho_0) = a_1 \quad (18)$$

Between $\rho = \rho_0$ and $\rho = \rho_2$, $f(\rho)$ increases monotonically with ρ . These are the properties of the right-hand side of eq. (17). The left-hand side is displayed in Fig. 2d, where the dashed curve gives λ as a function of $A^{-1/3}$ (the $A^{-1/3}$ axis points from right to left). The value of λ is a_1 at $A^{-1/3} = 0$

and increases monotonically with $A^{-1/3}$. (The separation energy eventually becomes zero for a system so small that the particle interactions are unable to ensure the binding of the most energetic particles present.) The graphical construction that results in the solution $\rho_c(A^{-1/3})$ of eq. (17) consists now of entering Fig. 2d with a given value of $A^{-1/3}$, erecting a vertical until it intersects the curve λ , extending from this point a horizontal line until it intersects the $f(\rho)$ curve in Fig. 2b and reading off the value ρ_c corresponding to this intersections. It is now clear that as $A^{-1/3}$ varies from zero to its maximum possible value (where λ would be zero) the value of ρ_c will increase monotonically from $\rho_c = \rho_0$ to $\rho_c = \rho_2$. Under the assumptions implicit in the construction of Fig. 2 (i.e., $f(\rho)$ increasing monotonically with ρ beyond ρ_0 and λ increasing monotonically with $A^{-1/3}$), there is no way in which ρ_c could go through a maximum when plotted against $A^{-1/3}$. Thus, within a Droplet Model scheme (without exponential terms) decreasing the nuclear size can only result in increased squeezing of the central density.

The standard Droplet Model squeezing formula is obtained by keeping only linear terms in the expansion of λ about $A^{-1/3} = 0$ and of $f(\rho)$ about ρ_0 :

$$\lambda \approx a_1 + \frac{2}{3} a_2 A^{-1/3} \quad , \quad (19)$$

$$\begin{aligned} f(\rho_c) &\approx f(\rho_0) + f'(\rho_0)(\rho_c - \rho_0) \\ &= a_1 + [\rho_0 e''(\rho_0) + 2e'(\rho_0)](\rho_c - \rho_0) \\ &= a_1 + \rho_0 e''(\rho_0)(\rho_c - \rho_0) \\ &= a_1 + \rho_0^2 \left. \frac{d^2 e}{d\rho^2} \right|_{\rho_0} \frac{\Delta\rho}{\rho_0} \\ &= a_1 + \frac{1}{9} K \frac{\Delta\rho}{\rho_0} \quad . \end{aligned} \quad (20)$$

Inserting in eq. (17) we find

$$(\Delta\rho/\rho_0) = (6a_2/K)A^{-1/3} \quad (21)$$

$$\text{or } \epsilon = -(2a_2/K)A^{-1/3}$$

in agreement with eq. (2).

To obtain an expression for $\Delta\rho/\rho_0$ to two more orders in $A^{-1/3}$ beyond eq. (21) we expand $f(\rho)$ to higher powers in $\rho_c - \rho_0$ and retain higher powers of $A^{-1/3}$ in λ . The result may be written as

$$(\Delta\rho/\rho_0) = (6a_2/K)A^{-1/3}(1 + \beta A^{-1/3} + \theta A^{-2/3}) \quad (22)$$

where

$$\beta = \frac{a_c}{2a_2} - \frac{10a_2}{K} + \frac{a_2 K_1}{K^2} \quad (23)$$

$$\theta = -\frac{9a_c}{K} + \frac{180a_2^2}{K^2} + \frac{K_1 a_c}{K^2} - \frac{30K_1 a_2^2}{K^3} - \frac{2}{3} \frac{K_2 a_2^2}{K^3} + \frac{2K_1^2 a_2^2}{K^4} \quad (24)$$

In the above, a_c is the curvature energy coefficient (see Table I) and K_1 and K_2 are higher-order compressibility coefficients defined in terms of an expansion of e in powers of ϵ , viz.:

$$e = e_1 + \frac{1}{2} K \epsilon^2 + \frac{1}{6} K_1 \epsilon^3 + \frac{1}{24} K_2 \epsilon^4 \quad (25)$$

This higher order (polynomial) Droplet Model approximation to $\Delta\rho/\rho_0$ is shown in Figs. 1a and 1b as a dot-dashed line. As expected, there is no indication of a tendency to produce a down-turn in $\Delta\rho/\rho_0$.

6. The Exponential Extension of the Droplet Model

Since the only approximation involved in going from the exact Euler equation (13) to the polynomial approximation, eq. (17), was the disregard of exponential terms in $\lambda + 6G(\rho_c)\rho''$, it is these exponential terms that must be responsible for the down-turn in $\Delta\rho/\rho_0$. An analysis of the separation energies in the Thomas-Fermi solutions (plotted as the solid curve in Fig. 2d) shows that λ is not responsible, so the down-turn must be due to the term $6G(\rho_c)\rho''$. (Since G is positive and ρ_c'' is negative, the sign of this term is right to cause the down-turn.) This means that the underlying reason for the down-turn is the breakdown of the leptodermous assumption, i.e. the fact that the exponential tail of the inner part of the surface diffuseness is beginning to reach into the center of the system, causing ρ_c'' to deviate appreciably from zero. It follows that, in order to have a model which is capable of describing the reversal of the bulk squeezing of nuclei below some mass number like 45 (if $K = 295$ MeV) or 100 (if $K = 217$ MeV) it is essential to go beyond a polynomial Droplet Model and include exponential terms of the type $\exp(-cA^{1/3})$. (Fig. 2d as well as Fig. 30 in ref. ³) show that if only binding energies were the focus of attention, the need for such a term would be less obvious.)

Such non-analytic, exponential terms appear explicitly in folding-type calculations, where a finite-range interaction is folded into a generating density distribution¹²). Such exponential terms can also be exhibited

in Thomas-Fermi calculations for which explicit solutions can be written down¹³⁾. (The nuclear proximity interaction belongs also to this type of term.) Recently, an attempt was made in ref. ¹⁴⁾ to include an adjustable exponential term in a semi-empirical nuclear mass formula, based on the "Finite Range Droplet Model." The new term in the binding energy was taken to be

$$-CA\epsilon e^{-\gamma A^{-1/3}}$$

where C and γ were treated as adjustable parameters and ϵ (the dilatation variable, eq. (2)) was allowed to assume its optimum value. A fit of the new formula to nuclear ground state masses and fission barriers demonstrated the practical utility of the exponential term. It was also remarkable that the resulting value of ϵ (or of $\Delta\rho/\rho_0$), showed a dependence on $A^{-1/3}$ very similar to that found in Thomas-Fermi calculations (e.g. Fig. 1). The conclusion stated in ref. ¹⁴⁾, which may also be relevant here, was that "the limit of a useful Droplet Model type of power expansion in $A^{-1/3}$ is probably reached around A^0 , and that future efforts should concentrate on a better understanding of the exponential, non-analytic terms ... which begin to dominate for small (holodermous) systems."

We hope that the present study, in addition to removing certain misgivings concerning the Droplet Model, will contribute to a better understanding of these terms.

The authors wish to acknowledge the contributions made to this work by J. M. Pearson and F. Tondeur through their continued critical interest in the Droplet Model and its limitations.

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Fig. 1a. The fractional deviation of the bulk density extracted from Hartree-Fock calculations using the force S3 is plotted against $A^{-1/3}$ for nuclei with $N = Z$ and with mass numbers from 16 to 3112. The straight line represents the lowest order Droplet Model prediction, eq. (21), the dot-dash curve the higher-order approximation, eq. (22), and the dashed curve the Thomas-Fermi result.

Fig. 1b. The same as Fig. 1a but for the force SkM.

Fig. 2. In (a) the energy per particle is shown as a function of the nuclear density. In (b) the derivative of the energy density is similarly plotted. In (d) the solid curve is the separation energy λ calculated by the Thomas-Fermi method, plotted against $A^{-1/3}$. (The dashed curve is the polynomial approximation, eq. (17).) Using a construction explained in the text, the curves in (d) and (b) can be used to generate a plot of the central density ρ_c against $A^{-1/3}$, shown in (c) as the solid curve. The down-turn depicted by the dot-dashed curve in Fig. 2c can never be generated by energy curves with the qualitative properties of Figs. 2a,b,d.

Table 1

	SkM	S III
K [MeV]	216.6	355.6
K ₁ [MeV]	386.07	- 101.38
K ₂ [MeV]	1542.8	-1148.4
r ₀ [fm]	1.142	1.180
ρ_0 [fm ⁻³]	0.1603	0.1453
σ [MeV fm ⁻³]	1.169	1.164
γ [MeV fm ⁻³]*	0.535	0.381

*related to a_c by $a_c = 8\pi r_0 \gamma$

Fig. 1a

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Fig. 1b

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Fig. 2

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