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Authors
Kishimoto, Chad T
Fuller, George M
Smith, Christel J

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Coherent Active-Sterile Neutrino Flavor Transformation in the Early Universe

Chad T. Kishimoto,* George M. Fuller,† and Christel J. Smith‡
Department of Physics, University of California–San Diego, La Jolla, California 92093-0319, USA
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We solve the problem of coherent Mikheyev-Smirnov-Wolfenstein resonant active-to-sterile neutrino flavor conversion driven by an initial lepton number in the early Universe. We find incomplete destruction of the lepton number in this process and a sterile neutrino energy distribution with a distinctive cusp and high energy tail. These features imply alteration of the nonzero lepton number primordial nucleosynthesis paradigm when there exist sterile neutrinos with rest masses $m_s \sim 1$ eV. This could result in better light element probes of (constraints on) these particles.

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Recent advances in observational cosmology and in experimental neutrino physics promise a well constrained picture for the evolution of the early Universe. The existence of a light sterile neutrino (ν_s) presents an immediate problem: How do sterile neutrinos affect primordial seas of active neutrinos ν_α or $\bar{\nu}_\alpha$ ($\alpha = e, \mu, \tau$) and consequentially affect the standard big bang paradigm? In this Letter, we study the lepton number-driven transformation of active neutrinos to sterile neutrinos in the epoch of the early Universe after weak decoupling, when neutrinos propagate coherently. This process could leave both the active neutrinos and sterile neutrinos with distorted, nonthermal energy spectra [1]. A nonthermal ν_e or $\bar{\nu}_e$ spectrum could lead to significant modification in the relationship between the lepton number and big bang nucleosynthesis (BBN) ^{4}He abundance yield [1,2]. Concomitantly, a distorted ν_s distribution function changes closure mass constraints on light sterile neutrinos [1,3], allowing rest masses and vacuum mixing angles for these species in the range (0.4 eV $< m_s <$ 5 eV) suggested by the Liquid Scintillator Neutrino Detector experiment [4,5] and currently being probed by the mini-BooNE experiment [6].

Active neutrinos propagating in the homogeneous early Universe experience a potential stemming from forward scattering $V = 2\sqrt{2}\zeta(3)\pi^{-2}G_F T^3 \mathcal{L}_\alpha - r_\alpha G_F^2 E_\nu T^4$, where T is the photon-plasma temperature, E_ν is the neutrino energy, r_α is a numerical coefficient which depends on the number of relativistic charged lepton degrees of freedom and can be found in Refs. [1,7], G_F is the Fermi constant, and $\zeta(3) \approx 1.202\,06$. Here the potential lepton number is $\mathcal{L}_\alpha \equiv 2L_{\nu_\alpha} + \sum_{\beta\neq\alpha} L_{\nu_\beta}$, where the individual lepton numbers are given in terms of the neutrino, antineutrino, and photon proper number densities by $L_{\nu_\alpha} \equiv (n_{\nu_\alpha} - n_{\bar{\nu}_\alpha})/n_\gamma$. Current observational bounds on these are $|L_{\nu_\alpha}| < 0.1$ [8–10] and could be slightly weaker if there are additional sources of energy density in the early Universe [11,12]. We have neglected contributions to V from neutrino-baryon or electron scattering since we consider relatively large lepton numbers with $\mathcal{L} \gg \eta$, where the baryon-to-photon ratio is $\eta \equiv n_b/n_\gamma$ (see Refs. [1,7]). The second term in V is negligible for the temperatures characteristic of the post-weak decoupling era $T < 3$ MeV.

The scattering-induced decoherence production [13–18] of seas of ν_s and $\bar{\nu}_s$, with rest mass $m_s \sim 1$ eV, could be avoided if these species are massless for $T > 3$ MeV, inflation has a low reheat temperature [19], or there exists a preexisting lepton number $|L_{\nu_\alpha}| > 10^{-3}$ [1,20]. However, a lepton number could subsequently, after weak decoupling, drive [1] coherent medium-enhanced Mikheyev-Smirnov-Wolfenstein (MSW) [21,22] resonant conversion $\nu_\alpha \to \nu_s$ or $\bar{\nu}_\alpha \to \bar{\nu}_s$, depending on the sign of the lepton number. (Resonant decoherence production of sterile neutrinos with $m_s \sim 1$ keV with accompanying ν_s spectral distortion was considered in Refs. [7,23].) The MSW condition for the resonant scaled neutrino energy $\epsilon = E_\nu^{\rm res}/T$ is $\delta m^2 \cos 2\theta = 2\epsilon T V$, or

$$\epsilon \mathcal{L} = \left(\frac{\delta m^2 \cos 2\theta}{4\sqrt{2}\zeta(3)\pi^{-2}G_F}\right) T^{-4}, \tag{1}$$

where $\delta m^2 \equiv m_2^2 - m_1^2$ is the difference of the squares of the vacuum neutrino mass eigenvalues. For illustrative purposes, we consider 2×2 vacuum mixing with a one-parameter (vacuum mixing angle θ) unitary transformation between weak interaction eigenstates $|\nu_\alpha\rangle$, $|\nu_s\rangle$ and energy-mass eigenstates:

$$\begin{aligned} |\nu_\alpha\rangle &= \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle; \\ |\nu_s\rangle &= -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle. \end{aligned} \tag{2}$$

As the Universe expands, the temperature falls, causing the resonance to sweep from low to higher values of the scaled neutrino energy ϵ. This resonance sweep converts active neutrinos into sterile neutrinos, reducing $\mathcal{L}$, which accelerates the resonance sweep rate.

The evolution of $\mathcal{L}$ is dictated by the resonance sweep rate and the dimensionless adiabaticity parameter. The adiabaticity parameter γ is proportional to the ratio of the width of the MSW resonance $\delta t = |1/V dV/dt|^{-1} \tan 2\theta$ and the neutrino oscillation length at resonance $L_{\rm osc} = 4\pi E_\nu/(\delta m^2 \sin 2\theta)$. Combining the expansion rate of the Universe in the radiation-dominated epoch with the conservation of comoving entropy density and the forward scattering potential V, the adiabaticity parameter is

$$\gamma \approx \frac{\sqrt{5}\zeta^{3/4}(3)}{2^{1/8}\pi^3}\frac{(\delta m^2)^{1/4} m_{\rm pl} G_F^{3/4}}{g^{1/2}} \times \frac{\sin^2 2\theta}{\cos^{7/4} 2\theta}\mathcal{L}^{3/4}\epsilon^{-1/4}\left|1 + \frac{\dot{g}/g}{3H} - \frac{\dot{\mathcal{L}}/\mathcal{L}}{3H}\right|^{-1}, \quad (3)$$

where $m_{\rm pl}$ is the Planck mass, g is the total statistical weight for relativistic species in the early Universe, and $H \approx (4\pi^3/45)^{1/2} g^{1/2} T^2/m_{\rm pl}$ is the local Hubble expansion rate. If the onset of resonant flavor conversion occurs in the epoch between weak decoupling and weak freeze-out, then initially $\gamma \gg 1$ for the active-sterile mixing parameters of interest [1]. However, when the fractional time rate of change of $\mathcal{L}$ becomes larger than the expansion rate of the Universe, the evolution of the system can be nonadiabatic with $\gamma < 1$.

Large values of γ result when many oscillation lengths fit within the resonance width. In this case, there will be a small probability of jumping from the high mass eigenstate to the low mass eigenstate. In turn, this implies efficient flavor transformation at the MSW resonance. Alternatively, a small value of γ means that the resonance width is much smaller than an oscillation length, and the neutrino jumps between the two mass eigenstates, resulting in virtually no flavor transformation. To describe intermediate cases, we use the Landau-Zener jump probability $P_{\rm LZ} = \exp(-\pi\gamma/2)$ [24,25], which gives the likelihood for a neutrino at resonance to make the jump between mass eigenstates. It is valid in the limit where the change in V across the resonance width δt can be regarded as linear. This is a good approximation in part because the resonance width is small compared to the causal horizon length for the values of θ and the conditions in the early Universe considered here.

It follows that the evolution of the potential lepton number as the resonant scaled neutrino energy sweeps from 0 to ϵ is

$$\mathcal{L}(\epsilon) = \mathcal{L}^{\rm initial} - \frac{1}{2\zeta(3)}\left(\frac{T_\nu}{T}\right)^3 \int_0^\epsilon \frac{x^2(1 - e^{-\pi\gamma(x)/2})}{e^{x-\eta_{\nu_\alpha}} + 1} dx, \quad (4)$$

where T_ν is the temperature of the active neutrino distribution function with degeneracy parameter $\eta_{\nu_\alpha} \equiv \mu_{\nu_\alpha}/T_\nu$ and where μ_{ν_α} is the ν_α chemical potential.

The evolution of the active neutrino spectrum is dictated by three conspiring factors: the MSW resonance condition [Eq. (1)], the adiabaticity parameter [Eq. (3)], and the evolution of potential lepton numbers through active-sterile conversion [Eq. (4)]. We solve Eqs. (1), (3), and (4) simultaneously and self-consistently to obtain γ and $\mathcal{L}$ as continuous functions across the entire range of ϵ.

Resonant conversion of active neutrinos to sterile neutrinos begins at $\epsilon \ll 1$. The resonance sweeps to higher values of ϵ as the temperature of the Universe drops. When $\dot{\mathcal{L}}/\mathcal{L} \ll H$, we have $\gamma \gg 1$, and adiabatic conversion of active neutrinos to sterile neutrinos ensues. However, this trend cannot continue. Note that the right-hand side of Eq. (1) is a monotonically increasing function of time, while the left-hand side is a peaked function if one assumes continued adiabatic conversion of neutrino flavors. At this peak, this assumption fails. Taking the time derivative of the resonance condition [Eq. (1)] shows that the sweep rate is $\dot{\epsilon} \propto T^{-5}\dot{T}(d(\epsilon\mathcal{L})/d\epsilon)^{-1}$. At the peak, $d(\epsilon\mathcal{L})/d\epsilon = 0$, causing the sweep rate to diverge. Taking the time derivative of both sides of Eq. (4) and assuming that T_ν/T is constant, we conclude that $\dot{\mathcal{L}} \propto \dot{\epsilon}$. With this relation, it follows from Eq. (3) that the MSW resonance is no longer adiabatic. We define $\epsilon_{\rm max}$ as the particular value of ϵ at this peak, implicitly specified by

$$\frac{1}{2\zeta(3)}\frac{\epsilon_{\rm max}^3}{e^{\epsilon_{\rm max}-\eta_{\nu_\alpha}} + 1} = \mathcal{L}^{\rm initial} - \frac{1}{2\zeta(3)}\left(\frac{T_\nu}{T}\right)^3 \times \int_0^{\epsilon_{\rm max}} \frac{x^2}{e^{x-\eta_{\nu_\alpha}} + 1} dx. \quad (5)$$

Our complete continuous solution for γ shows that neutrino flavor evolution or transformation is adiabatic for $\epsilon < \epsilon_{\rm max}$ but becomes (quickly) progressively less adiabatic for $\epsilon > \epsilon_{\rm max}$. For $\epsilon \geq \epsilon_{\rm max}$, our solution yields a large, but finite resonance sweep rate and concomitant large fractional lepton number destruction rate $\dot{\mathcal{L}}/\mathcal{L} \gg H$, leading to $\gamma \lesssim 1$. This behavior continues through the heart of the active neutrino distribution until the reso-

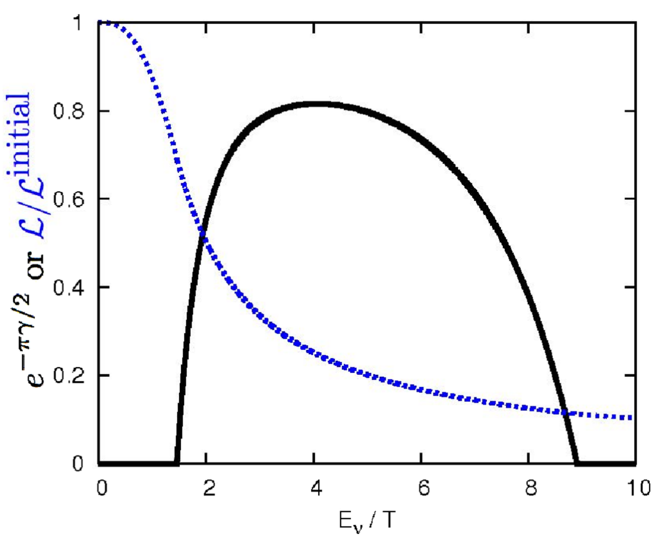

FIG. 1 (color online). Landau-Zener jump probability $e^{-\pi\gamma/2}$ (solid curve) and potential lepton number given as a fraction of its initial value (dashed curve) are shown as a function of MSW scaled resonance energy E_ν/T. Here we assume $\delta m^2 = 1$ eV^2, $\sin^2 2\theta = 10^{-3}$, and initial individual lepton numbers $L_{\nu_\mu} = L_{\nu_\tau} = 0.15$ and $L_{\nu_e} = 0.0343$.

nance sweep rate decreases to a point where $\gamma \gg 1$ again. This last transition back to adiabatic evolution occurs at $\epsilon \sim \mathcal{O}(10)$, approximately where $\mathcal{L}(\epsilon) = 1/(2\zeta(3))\epsilon^3/(e^{\epsilon-\eta_{\nu_\alpha}}+1)$. (Note that this is the same condition as for $\epsilon_{\max}$.) The resonance sweep continues to higher ϵ, adiabatically converting active neutrinos to sterile neutrinos.

The evolution of the Landau-Zener jump probability $e^{-\pi\gamma/2}$ and the history of the potential lepton number as a fraction of its initial value are both shown in Fig. 1 for the particular case where $\delta m^2 = 1\ \mathrm{eV}^2$, $\sin^2 2\theta = 10^{-3}$, and where we assume initial lepton numbers near their conventional upper limits $L_{\nu_\mu} = L_{\nu_\tau} = 0.15$ and $L_{\nu_e} = 0.0343$. For this particular case, $\epsilon_{\max} = 1.46$, and Fig. 1 shows the rather abrupt (but continuous) change to nonadiabatic evolution for $\epsilon \approx \epsilon_{\max}$. In this example, the final transition back to adiabatic evolution occurs at $\epsilon \approx 8.9$. Altogether, more than 90% of the initial potential lepton number is destroyed for this case. We find that the fractional depletion of potential lepton number is ~90% across a wide range of initial values of this parameter. This, in turn, suggests that this new solution will result in little change in the existing closure mass constraints on light sterile neutrinos [1].

Figure 2 shows the original ν_e Fermi-Dirac $[f(E_\nu/T) = 1/[T_\nu^3 F_2(\eta_\nu)]E_\nu^2/(e^{E_\nu/T_\nu-\eta_\nu}+1)$, where $F_2(\eta_\nu) \equiv \int_0^\infty x^2/(e^{x-\eta_\nu}+1)dx]$ and final ν_e and the ν_s energy distribution functions resulting from the $\nu_e \to \nu_s$ resonance sweep for the example parameters in Fig. 1. Forced, adiabatic resonance sweep to $\epsilon_{\mathrm{c.o.}}$ would result in complete depletion of the initial potential lepton number. $\epsilon_{\max}$ and $\epsilon_{\mathrm{c.o.}}$ are shown for this case in Fig. 2. Forced, adiabatic resonance sweep would result in a final ν_e spectrum identical to the initial one except cut off (hence, "c.o."), with zero population, for $E_\nu/T \le \epsilon_{\mathrm{c.o.}}$. The ν_s distribution in this case would be simply the complement. By contrast, with the full resonance sweep solution presented here, we see that the actual final ν_e spectrum has a population deficit relative to the original distribution, even for $E_\nu/T > \epsilon_{\mathrm{c.o.}}$. Likewise, the actual final ν_s spectrum will now have a tail extending to higher E_ν/T. Including simultaneous active-sterile and active-active neutrino flavor transformation in a full 4×4 scheme will modify this result, but we can expect some general features of our solution to remain. In particular, although neutrino flavor evolution will start out adiabatic, the transition to nonadiabatic evolution could be altered by, e.g., active-active neutrino mixing partially "filling in" the depleted ν_e population [1].

The BBN ^{4}He yield can depend sensitively on the shape of the ν_e energy distribution function [1,26,27]. This is because the neutron-to-proton ratio n/p is a crucial determinant of the ^{4}He abundance, and, in turn, this ratio is set by the competition among the charged current weak neutron-proton interconversion processes:

$$\nu_e + n \rightleftharpoons p + e^-; \qquad \bar{\nu}_e + p \rightleftharpoons n + e^+; \qquad n \rightleftharpoons p + e^- + \bar{\nu}_e. \tag{6}$$

The net rate for the forward direction in the first of these processes will be reduced if the ν_e population is removed via $\nu_e \to \nu_s$, resulting in a larger n/p and, hence, a larger ^{4}He yield. Likewise, a negative potential lepton number-

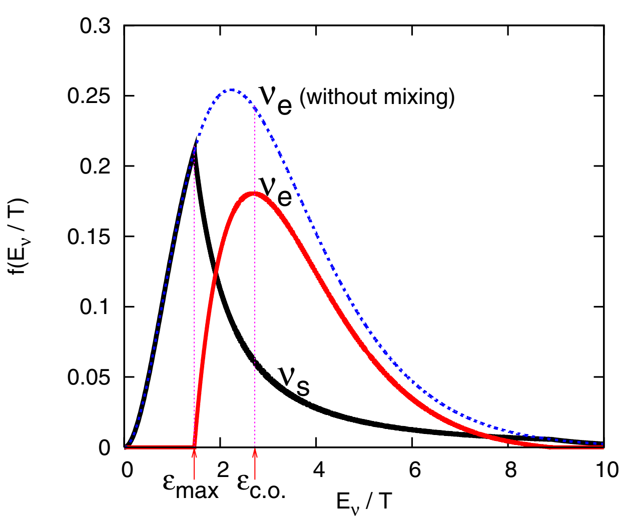

FIG. 2 (color online). Shown are the original ν_e distribution function (dashed curve), the final ν_e distribution function (lighter solid curve), and the final ν_s distribution function (heavier solid curve), all as functions of scaled neutrino energy E_ν/T for a $\nu_e \to \nu_s$ resonant, coherent flavor conversion process with $\delta m^2 = 1\ \mathrm{eV}^2$, $\sin^2 2\theta = 10^{-3}$, and individual lepton numbers as in Fig. 1. Vertical dotted lines indicate $\epsilon_{\max}$ and $\epsilon_{\mathrm{c.o.}}$.

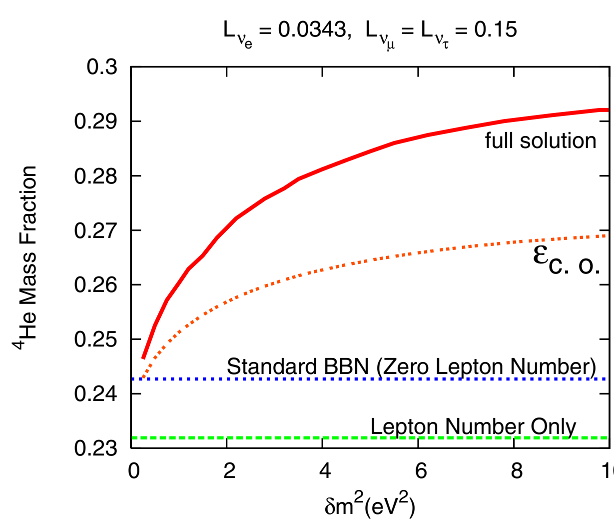

FIG. 3 (color online). Primordial nucleosynthesis (BBN) ^{4}He abundance yield as a function of δm^2 for the $\nu_e \to \nu_s$ channel and the indicated initial individual lepton numbers (same as in Fig. 1). Standard BBN (zero lepton numbers, no sterile neutrinos) is the heavy dashed horizontal line. The case for BBN with the indicated lepton number, but no active-sterile mixing, is the light dashed horizontal line. The case for the forced, adiabatic resonance sweep to $\epsilon_{\mathrm{c.o.}}$ is the light dotted line. The full nonadiabatic solution is given by the heavy solid line.

driven $\bar{\nu}_e \to \bar{\nu}_s$ scenario could result in $\bar{\nu}_e$ spectral depletion, which will result in a smaller n/p and, hence, less ^{4}He. Removing the ν_e ($\bar{\nu}_e$) population at higher E_ν/T values in the energy distribution function accentuates these effects, because the cross section for the ν_e ($\bar{\nu}_e$) capture process scales as E_ν^2 and because the Fermi-Dirac spectral peak, where neutrino populations are large, corresponds to values of neutrino energy satisfying $E_\nu/T > \epsilon_{\mathrm{c.o.}}$ for the potential lepton numbers $\mathcal{L}$ of interest here. As a consequence, our full resonance sweep scenario can result in significant alteration in the ^{4}He yield over the forced, adiabatic scenario.

We have computed the BBN ^{4}He abundance yield with a version of the Kawano-Wagoner-Fowler-Hoyle code [28,29] modified to allow for dynamic alteration or distortion in the neutrino energy distribution functions. The results of these calculations for the initial lepton numbers adopted in the example in Fig. 1 are shown in Fig. 3. The standard (zero lepton number, no sterile neutrinos) BBN ^{4}He abundance yield mass fraction is $\approx 24\%$ when we adopt neutron lifetime $\tau_n = 887.8$ s and $\eta = 6.1102 \times 10^{-10}$. The adopted value of η corresponds to the central value of the cosmic microwave background radiation acoustic peak-determined Wilkinson microwave anisotropy probe 3-year data, $\eta = (6.11 \pm 0.22) \times 10^{-10}$ [30]. The observational error in η corresponds to a $\pm 0.03\%$ range in the calculated ^{4}He abundance yield.

Alternatively, the case with the example lepton numbers but with no active-sterile neutrino mixing gives a healthy ^{4}He yield suppression. However, once the spectral distortion is included, the ^{4}He yield is larger than in standard BBN. Given that the observationally inferred helium abundance is between 23% and 26% [31] (and possibly more precisely determined [32,33]), we see that the dramatically larger ^{4}He yield in the cases with ν_e spectral distortion may allow for new constraints on a combination of lepton number and sterile neutrino masses. Our resonance sweep solution gives a larger ^{4}He yield than in previous models of active-sterile neutrino transformation which employ, e.g., forced, adiabatic resonance sweep to $\epsilon_{\mathrm{c.o.}}$ [1]. This shows the sensitivity of BBN abundance yields to sterile neutrino-induced active neutrino spectral distortion. This effect eventually may allow light element probes or constraints on the sterile neutrino sector which complement those of mini-BooNE and may extend to sterile neutrino mass or mixing parameters currently inaccessible experimentally.

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*Electronic address: ckishimo@ucsd.edu
†Electronic address: gfuller@ucsd.edu
‡Electronic address: csmith@ucsd.edu

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