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ON THE MINIMUM SIDE INFORMATION OF MIMO BROADCAST CHANNEL

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ABSTRACT

We introduce a technique to achieve dirty paper coding (DPC) capacity asymptotically with minimum feedback required reported to date in literature. Our approach called *interference management*, is based on a new multiuser diversity scheme designed for wireless cellular networks. When there are K antennas at the base station with M mobile users in the cell, the proposed technique only requires K integer numbers related to channel state information (CSI) between mobile users and base station. The encoding and decoding complexity of this scheme is the same as that of point-to-point communications. In order to guarantee fairness, a new algorithm is proposed which incorporates *interference management* into existing GSM standard.

1. INTRODUCTION

Multiuser diversity scheme [1] was introduced as an alternative to more traditional techniques like time division multiple access (TDMA) to increase the capacity of wireless cellular networks. The main idea behind this approach is that the base station (BS) selects a mobile station (MS) that has the best channel condition by taking advantage of the time varying nature of fading channels, thus maximizing the signal-to-noise ratio (SNR). This idea was later extended to ad hoc [2] and cellular networks [3].

Traditionally, fading and interference have been viewed as the two major impeding factors in increasing the capacity of wireless cellular networks. In this paper, however, we introduce a clean-slate approach to *interference management* that takes advantage of the fading in the channel to reduce the negative effects of interference.

Multiple-Input Multiple-Output (MIMO) communication gives considerable improvement in multiplexing and diversity gains. The multiplexing gain of a MIMO system increases linearly with the minimum number of transmit/receive antennas with or without knowledge of channel state information (CSI) at the transmitter [4]. Recent result [5] demonstrates that in a MIMO broadcast channel with K transmit antennas, the capacity increases linearly with K when the full CSI is known at the transmitter (CSIT). The sum-rate capacity grows only logarithmically with K [6] without CSIT. Further, for networks with large number of users $M \gg K$ exploiting opportunistic scheduling, the sum-rate capacity exhibits a double-logarithmic growth with M [6] which reflects the inherent multiuser diversity characteristic of the network.

Several schemes have been developed that achieve optimal dirty paper coding capacity by utilizing beamforming [6, 7, 8]. Most recent studies [9, 10, 11, 12, 13] have investigated the effect of partial finite-rate feedback on the capacity of MIMO broadcast channels in networks with limited number of users M which will be described in the next section.

We present an *interference management* technique for the downlink of a wireless cellular network in which d[1] ($d \leq K$) independent data streams can be broadcasted to d ($d \ll M$) mobile stations with single antenna such that these data streams do not interfere with each other. Furthermore, we demonstrate that the mean value of d, i.e. $D = \mathbf{E}[d]$, can be any number up to the maximum value of K as long as M is large enough. Therefore, *interference management* is capable of achieving the maximum multiplexing gain as long as there is a minimum number of mobile stations in the network. Surprisingly, by fully taking advantage of fading channels in the multiuser environment, the feedback requirement to transmit K independent data streams is proportional to K, and the encoding and decoding scheme is very simple and similar to that of point-to-point communications. The original multiuser diversity concept was based on searching for the best channel to communicate, while our approach shows that searching simultaneously for the best and worse channels can lead to significant capacity gains. This technique can asymptotically achieve the capacity of dirty paper coding (DPC) when M is very large. Our proposed scheme does not require mobile stations to cooperate for synchronization during transmission. It achieves optimal K maximum multiplexing gain in the downlink of cellular systems as long as $K \ll M$.

The remaining of this paper is organized as follows. Section 2 presents an overview of related work. Section 3 introduces the scheduling protocol and the model used in our analysis. Sections 4 and 5 present theoretical analysis and numerical results respectively. Fairness issues and practical considerations are discussed in Section 6 and the paper is concluded in Section 7.

2. RELATED WORK

Knopp and Humblet [1] derived the optimum capacity for the uplink of a wireless cellular network taking advantage of multiuser diversity. They proved that if the "best" channel (i.e. the channel with the highest SNR in the network) is selected, then all of the power should be allocated to the user with the "best channel" instead of using a water-filling power control technique. Tse extended this result into the downlink (broadcast) case of a wireless cellular network [14]. Furthermore, Viswanath et al. [3] used a similar idea for the downlink channel and employed the so-called "dumb antennas" by taking advantage of opportunistic beamforming. Grossglauser et al. [2] extended this multiuser diversity con-

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[1]d is a random variable.

cept into mobile ad hoc networks and took advantage of the mobility of nodes to scale the network capacity. All above schemes have taken advantage of multiuser diversity to combat the two major obstacles in wireless networks, namely, fading and interference.

Interference alignment [15] is another technique to manage interference. The main idea in this approach is to use part of the degrees of freedom available at a node to transmit the information signal and the remaining part to transmit the interference. For example, they consider $K \times M$ MIMO interference channel and demonstrate that the number of achievable degrees of freedom is $\frac{KM}{K+M-1}$. The drawback of interference alignment is that the system requires full knowledge of the CSI. This condition is very difficult to implement in practice, and feedback of CSI is MK complex numbers in a $K \times M$ interference channel. The advantage of interference alignment is that there is no minimum number of users required to implement this technique.

Sharif and Hassibi introduced a technique [8, 6] based on random beamforming concept to search for the best SINR in the network. Their approach requires M complex numbers for feedback instead of complete CSI information, and achieves the same capacity of $K \log \log M$ similar to DPC when M goes to infinity. There are major differences between our approach and the design in [8, 6]. First, our approach does not require beamforming, while the techniques proposed in [8, 6] take advantage of random beamforming. Second, the feedback requirement in our scheme is proportional to the maximum of K integers while this value is proportional to M complex numbers in [8, 6]. When M grows, the feedback information in [8, 6] grows linearly, while this complexity is constant with the number of antennas at the base station in our scheme. Our approach achieves DPC asymptotic capacity of $K \log \log M$ with minimum feedback requirement.

DPC provides the optimal $K \log \log M$ sum-rate capacity which is the maximum multiplexing and multiuser diversity gains. These gains are achieved at the expense of full CSI requirement and infinite-rate feedback M when M tends to infinity. In this paper, we present a new scheduling scheme which requires only minimum finite-rate feedback K and yet retains the optimal multiplexing and multiuser diversity gains achievable by dirty paper coding.

To the best of our knowledge, [7] and [9] are the only two publications with some similarities to our approach. Diaz et al. [7] proposed "1-bit" feedback from the mobile users instead of CSI information to the base station with the total feedback still proportional to M. While Tajer et al. [9] scheduling scheme is asymptotically optimal, it also exhibits a good performance for practical network sizes. They also showed [9] that by appropriate design of the feedback mechanism, they can refrain the aggregate feedback from increasing with the number of mobile users and for asymptotically large networks, the total number of feedback is bounded by $K \log K$ bits.

Our previous work [16] is the first paper which proposed *interference management* idea to consider both "good" and "bad" channels. In this paper, we present new approaches to reduce the minimum required number of mobile users M to achieve DPC capacity while maintaining the same feedback requirement of K (or equivalently $K \log K$ bits). Our approach is fundamentally different from random beamforming approach [8] while they both achieve the same asymptotic capacity. It is noteworthy to mention that our approach can be easily extended to distributed systems such as ad hoc networks while random beamforming approaches cannot be extended to distributed systems. Finally, we propose a practical technique to incorporate this scheme to existing cellular networks. There are other schemes in literature [10, 11, 12, 13] that achieve DPC capacity or close to that capacity with feedback requirement that is proportional to M.

3. INTERFERENCE MANAGEMENT

3.1 Network Model

We investigate the problem of optimal transmission in the downlink of a cellular network when the base station has independent messages for the mobile stations in the network. Clearly if the base station has only K antennas, it can transmit at most K independent data streams at any given time. We assume that all mobile stations have a single antenna for communication but later relax this constraint. The channel between the base station and mobile stations $\mathbf{H}$ is a $M \times K$ matrix with elements h_{ji}, where $i \in [1, 2, \ldots, K]$ is the antenna index of the base station and $j \in [1, 2, \ldots, M]$ is the mobile user index. We consider block fading model where the channel coefficients are constant during coherence interval of T. Then the received signal $\mathbf{Y}^{M \times 1}$ is expressed as

$$\mathbf{Y} = \mathbf{Hx} + \mathbf{n}, \qquad (1)$$

where $\mathbf{x}$ is the transmit $K \times 1$ signal vector and $\mathbf{n}$ is the $M \times 1$ noise vector. The noise at each of the receive antennas is i.i.d. with $\mathscr{CN}(0, \sigma_n^2)$ distribution.

3.2 The scheduling protocol

During the first phase of communication, the base-station antennas sequentially transmit K pilot signals. In this period, all the mobile stations listen to these known messages. After the last pilot signal is transmitted, mobile stations evaluate the SNR for each antenna. If the SNR satisfies any one of the following two cases, that particular mobile station will select that particular antenna(s) at the base station.

Strong Channel Case: SNR for only one transmit antenna is greater than a pre-determined threshold $\mathrm{SNR_{tr}}$ and below another pre-determined threshold of $\mathrm{INR_{tr}}$ for the remaining $K-1$ antennas.

Interference Cancelation Case: SNR for two transmit antennas is greater than a pre-determined threshold $\mathrm{SNR_{tr}}$ while one is at least G dB greater than the other one and is below another pre-determined threshold of $\mathrm{INR_{tr}}$ for the remaining $K-2$ antennas.

In the second phase of communication, the mobile stations that satisfy one of these two case will notify the base station that they have the required criterion to receive packets during the remaining time period of T. We will not discuss the channel access protocol required for these mobile stations to contact the base station or the case when two mobile stations satisfy interference management condition for the same base station antenna. We assume that this will be resolved by some handshake between the mobile stations and the base station. Note that, if we choose appropriate values for $\mathrm{SNR_{tr}}$ and $\mathrm{INR_{tr}}$ such that $\mathrm{SNR_{tr}} \gg \mathrm{INR_{tr}}$, then the base station can simultaneously transmit different packets from its antennas to different mobile stations. The mobile stations only receive their respective packets with a strong signal and can treat the

rest of the packets as noise. The value of SNR_{tr} (or INR_{tr}) can be selected as high (or low) as required for a given system, as long as M is large enough. Note that G will depend on the interference cancelation technique.

In general, there is a relationship between average number of antennas with interference management condition, $D = \mathbf{E}(d)$, and number of mobile stations, M. Clearly, *interference management* decreases the encoding and decoding complexity of MIMO broadcasting channel significantly[2] at the expense of the presence of large number of mobile stations. Fig. 1 demonstrates the system that is used here. Without loss of generality, we assume that the user i for $i \in [1,2,\ldots,d]$ is assigned to antenna i in the base station. In this figure, solid and dotted lines represent strong and weak channels between an antenna at the base station and a mobile station respectively. Note that if there is no line between the base station and mobile stations, then it means the channel is a random parameter based on the channel probability distribution function. For simplicity, Fig. 1 only illustrates the strong channel case.
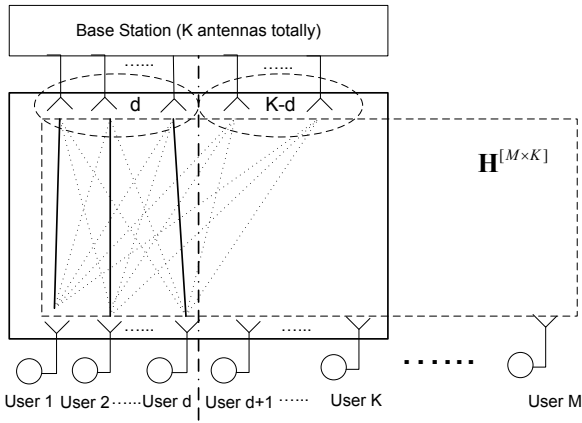

Figure 1: Interference management model in wireless cellular network

4. THEORETICAL ANALYSIS

Let's define SNR_{ji} as the signal-to-noise ratio when antenna j is transmitting packet to mobile station i in the downlink. Further denote INR_{ji} as the interference-to-noise ratio between transmit antenna j and receiver i. The objective of *interference management* is to identify d mobile stations out of M choices to satisfy either one of the following two criteria.

Strong Channel Case A_1:

$$\text{SNR}_{ii} \geq \text{SNR}_{\text{tr}}, 1 \leq i \leq d,$$
$$\text{INR}_{ji} \leq \text{INR}_{\text{tr}}, 1 \leq j \leq K, 1 \leq i \leq d, j \neq i \quad (2)$$

Interference Cancelation Case A_2:

$$\text{SNR}_{ti} \geq \text{SNR}_{ii} + G, 1 \leq i \leq d, i \neq t$$
$$\text{SNR}_{ii} \geq \text{SNR}_{\text{tr}}, 1 \leq i \leq d,$$
$$\text{INR}_{ji} \leq \text{INR}_{\text{tr}}, 1 \leq j \leq K, 1 \leq i \leq d, j \neq i, t \quad (3)$$

---

[2]For interference management technique, the encoding and decoding of multiple antennas reduces to simple point-to-point communication because the channels are decoupled from each other and no longer interfere with each other.

where G is a positive constant.

The above condition (2) states that each one of the d mobile station has a very good channel to a single antenna of the base station and strong fading to the other $K-1$ antennas of base station as shown in Fig. 1 and condition (3) states that there are two strong channels between two base station antennas and a mobile station and interference cancelation can be utilized for one of these two channels. Note that the interference cancelation criterion depends on the parameter G. After all the mobile users with interference management condition return their feedback to the base station, then the base station will select those mobile stations to participate in the communication phase such that the maximum multiplexing gain is achieved. Note that it is possible that two mobile users satisfy interference management condition for the same base station antenna.

The sum rate capacity in the downlink can be written as

$$
\begin{aligned}
R_{\text{proposed}} &= \sum_{i=1}^{d} \log\left(1 + \text{SINR}_{ii}\right) \\
&= \sum_{i=1}^{d} \log\left(1 + \frac{\text{SNR}_{ii}}{\sum_{j=1, j \neq i}^{d-1} \text{INR}_{ji} + 1}\right) \\
&\geq d \log\left(1 + \frac{\text{SNR}_{\text{tr}}}{(K-1)\text{INR}_{\text{tr}} + 1}\right) \\
&= d \log(1 + \text{SINR}_{\text{tr}}) \quad (4)
\end{aligned}
$$

where SINR_{ii} and SINR_{tr} are defined as

$$\text{SINR}_{ii} = \frac{\text{SNR}_{ii}}{\sum_{j=1, j \neq i}^{d-1} \text{INR}_{ji} + 1}, \forall i = 1, 2, \cdots, d \quad (5)$$

and

$$\text{SINR}_{\text{tr}} = \frac{\text{SNR}_{\text{tr}}}{(K-1)\text{INR}_{\text{tr}} + 1} \quad (6)$$

respectively.

First, the mean value of multiplexing gain d is derived. Then, we will prove that for any value of SINR_{tr}, there exists a minimum value of M that satisfies Eq. (4). Finally, we prove that our approach achieves the optimum capacity of DPC asymptotically.

For the rest of paper, the channel distribution is considered to be Rayleigh fading but similar result can be derived for other time-varying channel distributions. Note that for a Rayleigh fading channel $\mathbf{H}$, the probability distribution function (pdf) of SNR (or INR) is given by

$$p(\text{z}) = \begin{cases} \dfrac{1}{\sigma} \exp\left(-\dfrac{z}{\sigma}\right), & z > 0 \\ 0, & z \leq 0 \end{cases} \quad (7)$$

where z is the SNR (or INR) value and $\sigma = E_{\mathbf{H}}(z)$.

Let's define events A_1 and A_2 for any mobile station that satisfies the condition in Eqs. (2) and (3) respectively. Since the channels between the base station and the mobile stations are i.i.d., then the probabilities of these two events can be derived as

$$
\begin{aligned}
\Pr(A_1) &= \binom{K}{1} \int_{\text{SNR}_{\text{tr}}}^{\infty} p(z)dz \left(\int_{0}^{\text{INR}_{\text{tr}}} p(z)dz\right)^{K-1}. \\
&= \binom{K}{1} e^{-\frac{\text{SNR}_{\text{tr}}}{\sigma}} \left(1 - e^{-\frac{\text{INR}_{\text{tr}}}{\sigma}}\right)^{K-1} \quad (8)
\end{aligned}
$$

and

$$\begin{aligned}
\Pr(A_2) &= \binom{K}{2} \int_{\text{SNR}_{\text{tr}}}^{\infty} p(z_1) dz_1 \int_{z_1+G}^{\infty} p(z) dz \\
&\quad \cdot \left(\int_0^{\text{INR}_{\text{tr}}} p(z) dz \right)^{K-2} \\
&= \binom{K}{2} \frac{1}{2} e^{-\frac{2\text{SNR}_{\text{tr}}+G}{\sigma}} \left(1 - e^{-\frac{\text{INR}_{\text{tr}}}{\sigma}} \right)^{K-2}. \quad (9)
\end{aligned}$$

Let's define event A the condition for any mobile station that satisfies either Eq. (2) or Eq. (3). Since the events A_1 and A_2 are mutually exclusive, then

$$\Pr(A) = \Pr(A_1) + \Pr(A_2). \quad (10)$$

Our objective is to maximize this probability based on network parameters. Maximizing $\Pr(A)$ will minimize the number of required mobile stations M as will be proved later. Note that among all network parameters $K, \text{SNR}_{\text{tr}}, \text{INR}_{\text{tr}}$, and σ, the values of K and σ are really related to the physical properties of the network and are not design parameters. Further, the parameters SNR_{tr} and INR_{tr} can be replaced with a single parameter SINR_{tr} using Eq. (6).

Let $x - 2x_1$ and x_1 be the random variables related to the number of mobile stations satisfying the *interference management* condition for Eqs. (2) and (3) respectively. Note that it is possible that two mobile stations satisfy *interference management* condition for the same base-station antenna. The probability of x is computed as

$$\begin{aligned}
\text{Prob}(X=x) &= \sum_{x_1=0}^{x/2} \binom{M}{x-x_1} \binom{x-x_1}{x_1} (\Pr(A_2))^{x_1} \\
&\quad \times (\Pr(A_1))^{x-2x_1} (1-\Pr(A))^{M-x+x_1}. \quad (11)
\end{aligned}$$

We intend to solve this problem by formulating it as "bins and balls" problem. However, for the condition in Eq. (3), each mobile user is associated to two antennas at the base station, i.e., one strong channel and another even stronger channel by G dB. Therefore, for x_1 users (or balls) satisfying condition in Eq. (3), we consider that there are also x_1 virtual balls associated to these mobile users in order to allocate to two antennas in the base station. Thus, the total number of mobile stations satisfying *interference management* criteria in Eqs. (2) and (3) is $x - x_1$. The total number of balls will be x, as it can be explicitly seen later.

Note that there are x balls ($x - x_1$ real balls plus x_1 virtual balls) that satisfy the *interference management* condition. The probability distribution of x is given in Eq. (11). Let's define the conditional probability of choosing y base-station antennas (or bins) when there are x mobile stations (or balls) satisfying the *interference management* condition and denote it as $\Pr_B(d=y|X=x)$. Note that this probability includes the possibility that some of y antennas are not associated to any of x mobile stations and some correspond to more than one mobile station, i.e., some bins are empty and some bins have more than one ball in them. This conditional probability is equal to

$$\Pr_B(d=y|X=x) = \left(\frac{y}{K} \right)^x, \qquad y \le K \quad (12)$$

Let's define $\Pr_C(d=y|X=x)$ the probability that all of x mobile stations are associated to y base-station antennas

and there is no antenna in this set that is not associated to at least one of the x mobile stations. Then, this conditional probability can be derived as

$$\Pr_C(d=y|X=x) = \begin{cases} \Pr_B(d=1|X=x), & y=1 \\ \Pr_B(d=y|X=x) - \sum_{j=1}^{y-1} \binom{y}{j} \\ \quad \cdot (\Pr_C(d=j|X=x)), \\ & 1 < y \le \min(x,K) \\ 0. & y > \min(x,K) \end{cases} \quad (13)$$

This equation is derived iteratively and in order to initialize it for $y=1$, we utilize $\Pr_B(d=1|X=x)$. Since $\Pr_C(d=y|X=x)$ represents the probability of selecting a specific combination of y antennas, the total possible choices can be derived as

$$\Pr_D(d=y|X=x) = \binom{K}{y} \Pr_C(d=y|X=x). \quad (14)$$

Finally, we derive the expected value of d using law of total probability.

$$\begin{aligned}
D &= \mathbf{E}(d) = \sum_{y=1}^{K} \sum_{x=1}^{2M} y \Pr_D(d=y|X=x) \Pr(X=x) \\
&= \sum_{x=1}^{2M} \mathbf{E}(d|X=x) \Pr(X=x) \\
&= \sum_{x=1}^{2M} \mathbf{E}(d|X=x) \sum_{x_1=0}^{\lfloor x/2 \rfloor} \binom{M}{x-x_1} \binom{x-x_1}{x_1} \\
&\quad (\Pr(A_2))^{x_1} (\Pr(A_1))^{x-2x_1} (1-\Pr(A))^{M-x+x_1} \quad (15)
\end{aligned}$$

where $\mathbf{E}(d|X=x)$ is defined as

$$\mathbf{E}(d|X=x) = \sum_{y=1}^{K} y \Pr_D(d|X=x) \quad (16)$$

and $\Pr(X=x)$ is computed from Eq. (11). Note that the reason for $x = 2M$ is the fact that if all the mobile users satisfy Eq. (3), then the total number of balls (real and virtual) is $2M$.

5. NUMERICAL RESULTS

Figs. 2 and 3 illustrate the minimum required value for M for $\text{SINR}_{\text{tr}} = 3$dB and $\text{SINR}_{\text{tr}} = 10$dB respectively when D varies and for $K = 3, \sigma = 100$. The solid line illustrates the minimum required value for M based on original introduction of interference management in [16] and the dashed line in these figures is based on the analysis in this paper. One of the main differences between the analysis in this paper and that of [16] is the introduction of interference cancelation in this paper. As we can see from this result, when interference cancelation technique is allowed, the number of mobile stations required to implement this technique decreases significantly (up to 20% and 30% for $\text{SINR}_{\text{tr}} = 3$dB and $\text{SINR}_{\text{tr}} = 10$dB respectively for multiplexing gain of $D = 2$). Therefore, using capacity approaching techniques such as Turbo code or Low Density Parity Check (LDPC) that requires very low SINR_{tr} (such as 3dB) will help to implement

this technique with modest number of MS users. Note that in this approach, a minimum SINR_{tr} is always guaranteed. However, in current cellular systems such as TDMA, since the channel assignment is not based on the channel condition, there is usually a minimum link margin of at least 20 dB in order for the link to operate under different channel variations such as fade. In these figures, lines represent the analytical result while the circle, square, or diamond points represent the outcome of simulation environment. The results clearly demonstrate that there is an agreement between analytical and simulation results.
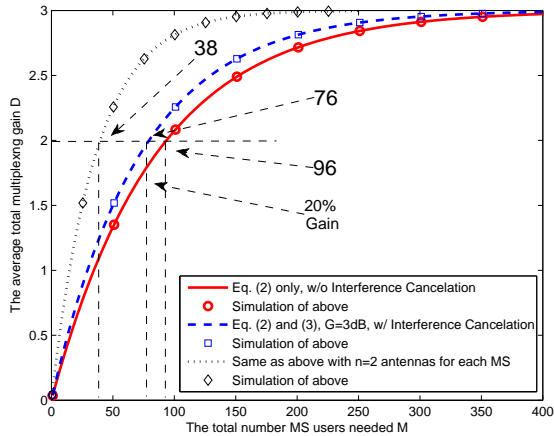
Figure 2: Simulation and analytical results demonstrate relationship between D and M for $\text{SINR}_{tr} = 3\text{dB}$
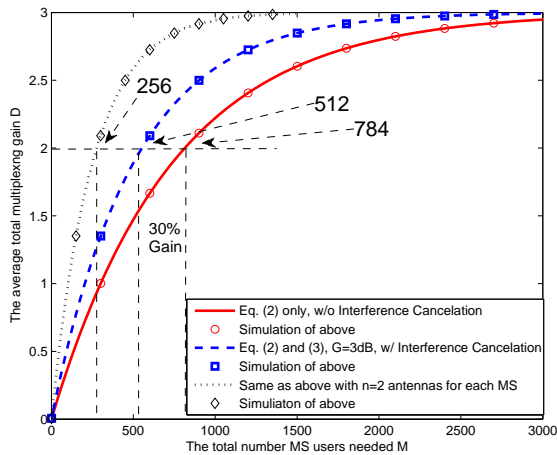

Figure 3: Simulation and analytical results demonstrate relationship between D and M for $\text{SINR}_{tr} = 10\text{dB}$

In order to reduce the minimum required number of mobile users further, we can allow each mobile user to utilize two antennas and try to select one of the antennas that satisfies interference management condition. However, such increase in the number of antennas does not require space-time encoding or decoding. From base station point of view, the additional antenna for each mobile user is equivalent of

increasing the number of mobile users twofold or equivalently, the actual minimum number of mobile users required to achieve a multiplexing gain is reduced by a factor of 2. Note that one of the advantages of this approach is the fact that the transmitter and receiver use simple point-to-point communications even if a receiver has multiple antennas. The goal is simply to identify one of the antennas that conforms with interference management constraint. The dotted line in Figs. 2 and 3 demonstrate the relationship between minimum number of mobile users and the multiplexing gain when each user has two antennas. As it can be seen from simulation, the minimum M is reduced further while the approach still achieves DPC asymptotic capacity.

We have proved analytically in the Appendix that under event A_1 constraint, interference management achieves DPC asymptotic capacity. However, when both A_1 and A_2 events are considered, then it is not easy to prove analytically the asymptotic capacity of this scheme. However, using the probability distribution function of variables from our analytical derivation in the previous section, we can derive the capacity numerically as follows.

$$\lim_{M \to \infty} C_{\text{IM}} = C_{\text{DPC}} = K \log \log M \qquad (17)$$

This result implies that

$$\lim_{M \to \infty} \text{SINR}_{tr} = \Theta(\log M). \qquad (18)$$

Our objective is to show, via simulation, that when SINR_{tr} grows proportional to $\Theta(\log M)$, the maximum multiplexing gain can be achieved when M tends to infinity. Let's define SINR_{tr} as

$$\text{SINR}_{tr} = \frac{\sigma}{c_0} \log \left(\left(\frac{1}{e} \right)^{K-1} M \right). \qquad (19)$$

Fig. 4 confirms that when SINR_{tr} grows logarithmically with M, this approach achieves the maximum multiplexing gain for different values of c_o based on Eq. (19). It is noteworthy
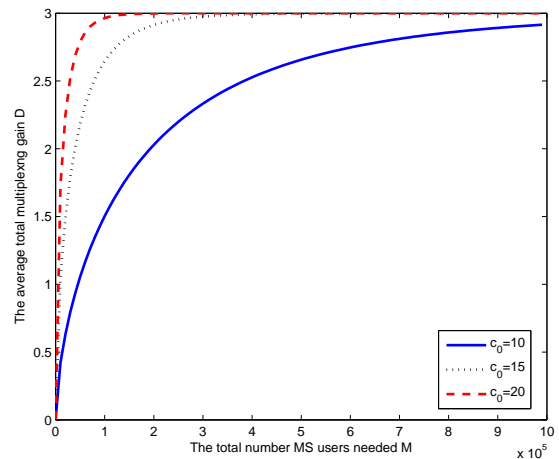

Figure 4: Simulation results demonstrate DPC capacity and maximum multiplexing gain are achieved simultaneously.

to point out that when the value of σ is small or equivalently,

if the channel fading is not strong, then interference management cannot converge to the maximum multiplexing gain of K rapidly. In the new multiuser diversity scheme that is introduced in this paper, both strong and weak channels are important. When the fading coefficient σ is stronger, then this technique performs better. Fig. 5 illustrates this importance point.
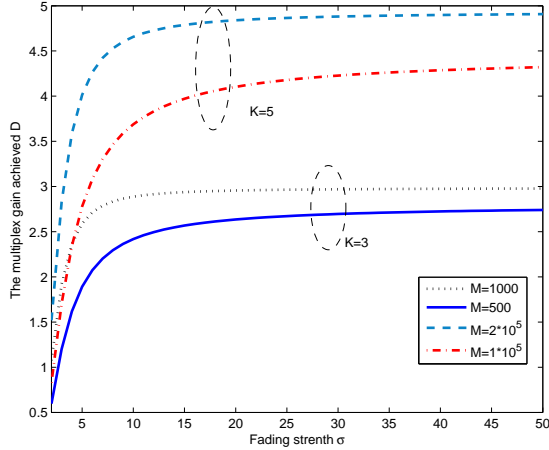

Figure 5: Simulation results demonstrate relationship between fading strength and multiplexing gain.

When $K = 1$, then our approach is similar to that of [1]. Moreover if $M \to \infty$ and $D = K$, then our scheme has the same asymptotic scaling laws capacity result as that of [8]. The cost of the proposed scheme is the need for a minimum number of mobile stations, M. In most practical cellular systems, in any given frequency and time inside a cell, there is only one assigned mobile station while this technique suggests that we can have up to the number of base-station antennas utilizing the same spectrum at the same time with no bandwidth expansion. Clearly, this approach can increase the capacity of wireless cellular networks significantly. This gain is achieved with modest feedback requirement which is proportional to the number of antennas at the base station.

It is easy to prove that the number of mobile users X (which is a random variable) with interference management constraint is always smaller than K with probability arbitrarily close to 1 with the correct choice of network parameters. More specifically, the probability that $X \leq K$ mobile users satisfy the interference management criteria denoted as η can be arbitrarily close to 1 if we select proper SINR_{tr} based on network parameters such as σ and M.

$$
\begin{aligned}
\text{Prob}(X \leq K) &= \sum_{i=0}^{2K} \sum_{x_1=0}^{\lfloor i/2 \rfloor} \binom{M}{i - x_1} \binom{i - x_1}{x_1} (\text{Pr}(A_2))^{x_1} \\
&\quad \cdot (\text{Pr}(A_1))^{i-2x_1} (1 - \text{Pr}(A))^{M-i+x_1} \\
&\geq \eta
\end{aligned} \tag{20}
$$

where $0 < \eta < 1$ can be arbitrarily close to 1 , i.e., $\eta = 99\%$.

For any values of K, M and σ, the designer can select the appropriate value for SINR_{tr} such that with probability close to 1 the value of random variable X is less than K as shown in Fig. 6.
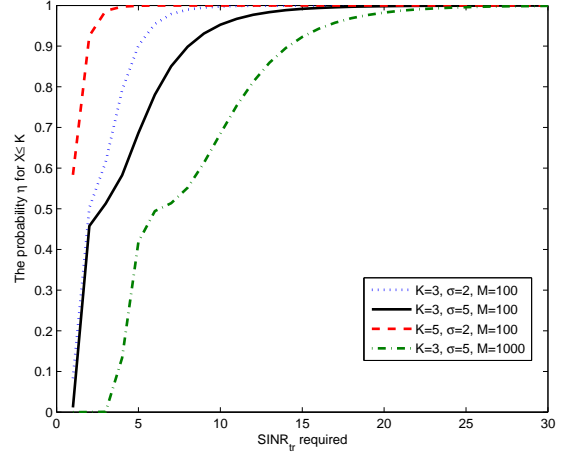

Figure 6: The feedback is at most K with probability close to 1.

In practical cellular systems, it is possible that the minimum number of mobile users may not be available in a cell. Note that it is easy to show that for any value of K, M and σ, the designer can select the appropriate value for SINR_{tr} such that the maximum multiplexing gain is achieved at the expense of reduced rate for each individual mobile user, i.e., $D \cong K$.

There are still two important issues with interference management scheme. One is the fact that in current cellular systems, the assignment of users is based on pre-determined schemes such as time-division. The other issue is the fairness problem which is important so that all users have minimum access to the channel. For example, some mobile users may be close to the base station for a long period of time with line of sight. In the following section, we provide an approach to incorporate interference management scheme into existing TDMA systems to assure fairness in terms of accessing the channel for all users. The extension of this approach to other standards such as CDMA is straightforward.

6. FAIRNESS AND PRACTICAL ISSUES

In this section, we propose one practical approach for existing GSM cellular systems to guarantee the fairness and Quality of Service (QoS) for TDMA users while allowing other users to take advantage of *interference management* scheme without interrupting the main user. For any TDMA user, the received signal vector can be written as

$$
\mathbf{R}_{\text{TDMA}}^{\text{T}} = \mathbf{S}_{\text{TDMA}}^{\text{T}} h_{\text{TDMA}} + \sum_{i=1}^{d} S_i h_i \mathbf{V}^{\text{T}} + \mathbf{n}^{\text{T}}, \tag{21}
$$

where $\mathbf{R}_{\text{TDMA}}$ and $\mathbf{S}_{\text{TDMA}}$ are the TDMA signal vectors received by a mobile user and transmitted by an antenna in the base station respectively, provided that this antenna does not participate in *interference management* scheme, i.e., $d < K$. The superscript T represents transpose of a vector, S_i and V^T are the signal transmitted by the antenna that is utilizing interference management scheme and a vector with unit weight that will be multiplied by each signal S_i respectively. $\mathbf{n}$ is the additive Gaussian noise vector with zero mean i.i.d. elements

and variance of σ_n. h_{TDMA} and h_i are the CSI between base station and mobile users that are participating in TDMA and interference management scheme respectively.

At the receiver, we multiply the received vector by a vector $\mathbf{U}$. This vector is orthonormal to $\mathbf{V}$, i.e., $\mathbf{UV}^T = 0$. Thus, the received signal will be equal to

$$
\begin{aligned}
\mathbf{UR}_{\text{TDMA}}^{\text{T}} &= \mathbf{US}_{\text{TDMA}}^{\text{T}} h_{\text{TDMA}} + \sum_{i=1}^{d} s_i h_i \mathbf{UV}^T + \mathbf{Un}^{\text{T}} \\
&= \mathbf{US}_{\text{TDMA}}^{\text{T}} h_{\text{TDMA}} + \mathbf{n}'
\end{aligned}
\tag{22}
$$

Note that the signals transmitted utilizing interference management scheme are now multiplied by this new vector $\mathbf{V}$. Even though the TDMA user does not have the interference management capability and therefore other users are interfering with this user, but when we multiply the orthogonal vector $\mathbf{U}$ by the received vector, we can get rid of these interfering signals. Further, the vector $\mathbf{V}$ does not have any relationship with CSI and we are not really using any beamforming scheme. We will later describe the criterion for selecting this vector. For block fading channel, this vector only requires to be of length 2. We notice that by the new transmission policy, we have reduced the actual rate of signals participating in interference management scheme by a factor proportional to the length of vector $\mathbf{V}$. However, the rate of TDMA signal is still one symbol per channel use.

If the wireless channel is block fading, then $\mathbf{U} = [u_1, u_2]$ and $\mathbf{V} = [v_1, v_2]$ are enough for implementation. However, for fast fading the implementation of this technique is more complicated and we omit that here. For the rest of paper, we assume that the QPSK signals are used for transmission. Since the TDMA vector signal is multiplied by $\mathbf{U}$ as shown in Eq. (22), then our criterion for designing this signal is based on the condition that the combination of multiple QPSK signals results in optimum separation of points in the two-dimensional space. This condition will help in decoding performance of the received signal. Note that again this vector is not really a function of channel matrix as it is common in beamforming techniques.

For a combination of two QPSK signals, an appropriate choice would be a 16-QAM signal. It has been shown in [17] that any combination of QPSK signals can be mapped into M-QAM signals. For the specific case of 16-QAM, we have

$$
16\text{-QAM} = \sum_{i=0}^{1} 2^i \left(\frac{\sqrt{2}}{2} \right) (j^{x_i}) \exp\left(\frac{\pi j}{4} \right)
\tag{23}
$$

where $x_i \in Z_4 = \{0, 1, 2, 3\}$. The QPSK constellation can be realized as $\text{QPSK} = j^{x_i}$. Thus, one can use shift and rotation operation to create M-QAM constellations from QPSK symbols. It is easy from Eq. (23) to show that the normalized values of vectors $\mathbf{U}$ and $\mathbf{V}$ are

$$
\mathbf{U} = \sqrt{\frac{2}{5}} \exp\left(\frac{\pi j}{4} \right) \left[\frac{\sqrt{2}}{2}, \sqrt{2} \right]
\tag{24}
$$

and

$$
\mathbf{V} = \sqrt{\frac{2}{5}} \left[\sqrt{2}, -\frac{\sqrt{2}}{2} \right]
\tag{25}
$$

respectively. Since the vector $\mathbf{U}$ is normalized, then the variance of Gaussian noise remains the same.

Note that with this signalling at the base station, the Quality of Service (QoS) and fairness for all users are guaranteed in a time-division approach while other users can utilize the spectrum taking advantage of *interference management* scheme.

7. CONCLUSION

In this paper, we proposed an *interference management* technique that asymptotically achieves DPC capacity with minimum feedback by taking advantage of the multiuser diversity and fading channel in the network to minimize the negative effects of interference in wireless cellular networks. Besides, this technique requires simple encoding and decoding for the downlink of wireless cellular networks similar to that of point-to-point communications. Furthermore, we have shown that by utilizing interference cancelation, the number of required mobile users can be reduced significantly. Finally, a practical way to guarantee the fairness in existing TDMA cellular systems is proposed.

8. APPENDIX

In this appendix, we will prove that the sum-rate of the proposed scheme under A_1 condition achieves the optimum asymptotic DPC capacity, i.e. $K \log \log M$. Let's define x as the number of mobile stations that satisfy Eq. (2). The probability that the first user associated to any of the antennas at the base station is $\Pr(A_1)$, and this probability for the second user is $\frac{K-1}{K}\Pr(A_1)$ and this probability can be similarly computed for all other users. The probability for the last (d^{th}) user to satisfy Eq. (2) is $\frac{K-x+1}{K}\Pr(A_1) > \frac{1}{K}\Pr(A_1)$. From this argument, it is clear that these probabilities are lower bounded as $\frac{1}{K}\Pr(A_1)$.

The lower bound for the expected value of d is given by

$$
D = \mathbf{E}(d) \geq \frac{M}{K} P(A_1).
\tag{26}
$$

It is noteworthy to mention again that the number of mobile stations that satisfy *interference management* condition is a random variable and D is simply the average value of this random variable. Thus,

$$
M \leq DK(P(A_1))^{-1}.
\tag{27}
$$

Note that M is upper bounded by the inverse of $P(A_1)$. Therefore, in order to minimize M, it is necessary to minimize $(\Pr(A))^{-1}$ such that the SINR_{tr} condition in Eq. (6) is satisfied.

This optimization problem was computed in [16] and the details are omitted here. Note that SINR_{tr} is usually a predetermined variable for most applications and we need to optimize this equation with respect to INR_{tr}^*. The solution for INR_{tr}^* is

$$
\text{INR}_{\text{tr}}^* = \frac{\sigma}{\text{SINR}_{\text{tr}}}.
\tag{28}
$$

Then the optimum value for $(P(A_1))^{-1}$ is given by

$$
M^* \leq DK(P^*(A_1))^{-1} = De^{\frac{\text{SINR}_{\text{tr}}}{\sigma}} (\text{SINR}_{\text{tr}}e)^{K-1}.
\tag{29}
$$

For constant values of SINR_{tr} and when $\sigma \to \infty$, then $(P(A_1))^{-1*}$ is

$$
\lim_{\sigma \to \infty} (P(A_1))^{-1*} = \frac{1}{K} (\text{SINR}_{\text{tr}}e)^{K-1}.
\tag{30}
$$

This results implies that even for very strong fading channels, there exists a minimum value of mobile stations to implement this technique.

Now we investigate the asymptotic behavior of the network(i.e. $M \to \infty$) and try to compute the maximum achievable capacity and scaling laws for this scheme. When M tends to infinity, the SINR_{tr} increases since we can select higher value for SNR_{tr} and smaller value for INR_{tr}. Under such conditions, the value of $(P(A_1))^{-1}$ is approximated as

$$\lim_{M \to \infty} (P(A_1))^{-1^*} = \frac{1}{K} e^{K-1} e^{\frac{\text{SINR}_{\text{tr}}}{\sigma}} \geq \frac{M}{DK}. \qquad (31)$$

Note that we use the property that $\lim_{x \to \infty} \frac{x^c}{e^x} = 0$, where c is a constant.

The lower bound of SINR_{tr} is asymptotically computed as

$$\lim_{M \to \infty} \text{SINR}_{\text{tr}}^{\max} \geq \sigma \log \left(\frac{1}{D} \left(\frac{1}{e} \right)^{K-1} M \right). \qquad (32)$$

Thus, the $\text{SINR}_{\text{tr}}^{\max}$ scales at least with $\log M$ so that by utilizing Eq. (4), the scaling laws of *interference management* scheme is

$$C = \Theta(K \log \log M). \qquad (33)$$

Note that when M tends to infinity, then there are always K mobile users with interference management capability, i.e., $D = \mathbf{E}(d) = K$.

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