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ABSTRACT

An error in the work of Schwartz and Zemach is corrected.

In their analysis of the scattering problem with the Bethe-Salpeter equation Schwartz and Zemach[1] showed the distorted contour (their Fig. 2) along which one must integrate the variable p_4 after performing the Wick rotation $p_0 \to ip_4$. However, after changing to polar coordinates in this Euclidean space and performing the angular integration they were left with a single radial integral (their Eq. B15) which showed no contour distortion. The integral in question contains the factor

$$S(p) = [(p^2 - \omega^2 + m^2)^2 + 4\omega^2 p^2]^{1/2}; \qquad p^2 = |\underline{p}|^2 + p_4^2.$$

This is never singular along the real axis of p and so it was earlier thought that there was no question about how to evaluate this square root. This is wrong since there was given the prescription to continue the result from the region $\omega < m$. One can easily see that as we let ω go from just below m to just above m a branch point of S moves across the real axis at $p = 0+$ and so the proper analytic continuation implies a distorted contour for the p integral as shown in the figure:

Thus the previous evaluations of these integrals must be corrected by adding the piece along the downward loop, which extends a distance $(\omega - m)$. We have made this correction and redone all the calculations reported in I. The bound state results are not affected and the corrections vanish at the elastic threshold; the numerical corrections to the phase shifts at finite energies turned out in general to be extremely small. For example at $k^2 = 0.4$ for $\lambda = 1$ the old s-wave result (see Table III of I) was $\frac{\omega}{k} \tan \delta_0 = 3.5639$ which is now corrected to be 3.4946, or 2% lower. The corrections to the data in Figs. 3, 4, 5 and Table IV of I are all too small to be seen. For the repulsive potentials the corrections to the results of Fig. 6 of I are more noticeable; the largest change is at $\lambda = -10$ at the highest energy where δ_0/π changes from 0.91 to 1.18. The corrections were also more noticeable at larger values for the mass M of the exchanged particle, but became much smaller for the higher partial waves.

Several authors[2] have reported calculations by other methods which yielded numbers in agreement with the old (wrong) Schwartz-Zemach phase shifts. Only in the case of Haymaker's work were these other calculations sufficiently accurate in themselves to warrant a recomparison with our corrected results; and we have found an improved agreement between our new numbers and Haymaker's.[3]

FOOTNOTES AND REFERENCES

* This work was performed under the auspices of the U.S. Atomic Energy Commission.

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1. C. Schwartz and C. Zemach, Phys. Rev. 141, 1454 (1966) hereafter referred to as I.

2. M. Levine, J. Tjon, and J. Wright, Phys. Rev. Letters 16, 962 (1966) and Phys. Rev. 154, 1433 (1967); R. Saenger, J. Math. Phys. 8, 2366 (1967); H. Cohen, A. Pagnamenta, and J. G. Taylor, Nuovo Cimento 50A, 796 (1967); R. Haymaker, Phys. Rev. Letters 18, 968 (1967) and Phys. Rev. 165, 1790 (1968).

3. R. Haymaker (University of California, Santa Barbara), private communication.

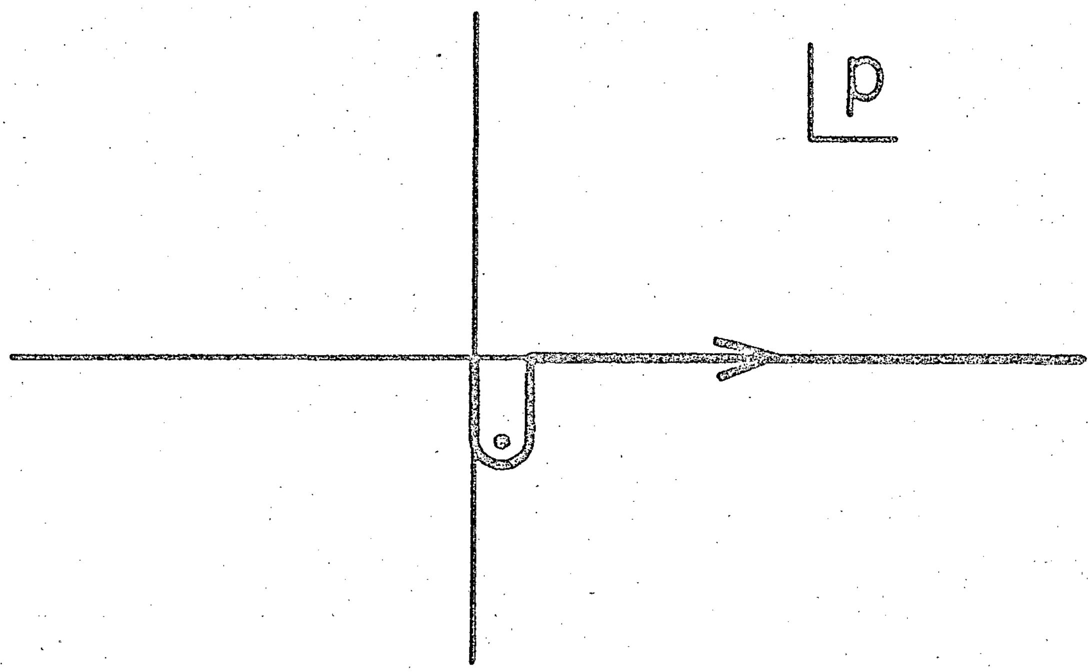

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Fig. 1. Correct contour.