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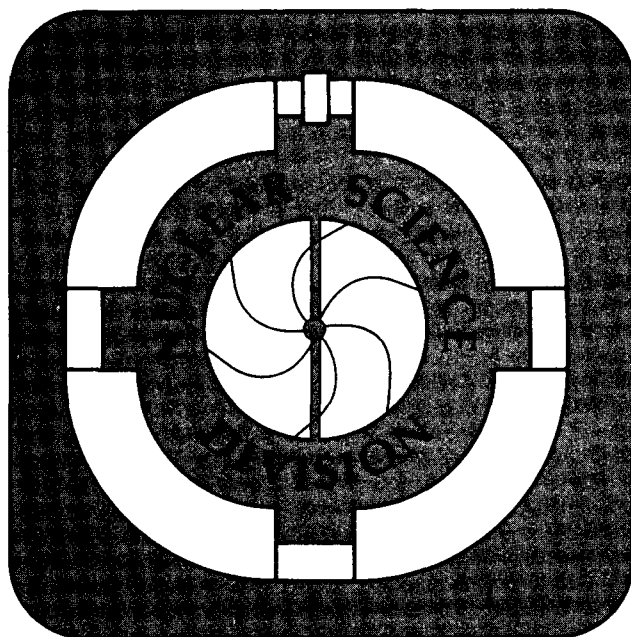
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Symmetries in Neutrino-Electron Scattering [†]

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SYMMETRIES IN NEUTRINO-ELECTRON SCATTERING

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ABSTRACT

The symmetries of the neutrino-electron scattering kernel are exposed. The kernel must explicitly conserve particle number at each vertex and vanish in local thermodynamic equilibrium. I show that the latter requirement must be met by the combination of detailed balance between in and out beams and a symmetry property of the kernel which until now has gone unnoticed. Exploitation of this extra symmetry approximately doubles the computational efficiency of the algorithm.

1 Introduction

The formulation of the neutrino-electron scattering (NES) opacities presented in this paper closely follows the appendices of (Bruenn 1985). When the approach of this seminal work is followed exactly, particle number is not conserved, although in NES the number and species of particles before and after the interaction remain unchanged.

Furthermore, the NES kernel must vanish in local thermodynamic equilibrium (lte). In Bruenn's formulation this is achieved by construction through the relation between in and out beams, c.f. eq. (2.5) in section 2 of this paper and eq. C9 in (Bruenn 1985).

In the next section the symmetries of the NES kernel and their consequences are discussed. In section 3 the kernel is expanded in Legendre polynomials to approximate its angular dependence. The in and out kernels each separately possess a symmetry which hitherto has gone unnoticed. This higher symmetry of the kernel has two consequences. First, in combination with detailed balance it makes the kernel vanish in lte. Second, its exploitation approximately doubles the computational efficiency of the NES algorithm.

In the appendix a practitioner's guide to the implementation of NES is given. The structure of the NES kernel and the way it is incorporated into spectral Eddington moment neutrino transport are summarized.

2 Constraints on The Scattering Kernel

The neutrino-electron scattering part of the collision kernel of the Boltzmann equations is a functional of the neutrino distribution function $f_\nu(x, q)$,

$$B_{NES}[f_\nu(q)] = 2C \int \frac{d^3 p'}{(2\pi)^3} \frac{d^3 p}{(2\pi)^3} \frac{d^3 q'}{(2\pi)^3} [f'_\nu f'_e \ r(p'q'|pq) \ (1 - f_e)(1 - f_\nu) - f_\nu f_e \ r(pq|p'q') \ (1 - f'_e)(1 - f'_\nu)] \ , \quad (2.1)$$

where $C = 1/(c(2\pi\hbar c)^3)$ is a constant. The distribution is a function of space-time x and four-momentum q . The four-momenta p and p' refer to the electron four-momentum before and after collision, q and q' are the in and out beam neutrino four-momenta. The electrons are always in local thermodynamic equilibrium with the rest of the matter at temperature T and chemical potential μ_e with distribution f_e . The transition rate

$$r(pq|p'q') = \frac{G^2(2\pi)^4}{\omega\omega'p_0p'_0} \delta^4(p+q-p'-q') [\alpha_1 p \cdot q \ p' \cdot q' + \alpha_2 p' \cdot q \ p \cdot q' - m_e \alpha_3 q \cdot q'] \ , \quad (2.2)$$

is invariant under the simultaneous exchange of in and out momenta $p \leftrightarrow p', q \leftrightarrow q'$, so that $r(pq|p'q') = r(p'q'|pq)$. The constants G^2 and $\alpha_i, i = 1, \dots, 3$ are functions of coupling constants, m_e is the electron mass.

The integrations over the electron four-momenta can be separated into the kernels

$$R^{\{out\}in}(q, q') = 2C \int \frac{d^3 p'}{(2\pi)^3} \frac{d^3 p}{(2\pi)^3} \begin{cases} (1 - f_e) f'_e r(p'q'|pq) \\ (1 - f'_e) f_e r(pq|p'q') \end{cases} \quad (2.3)$$

With the in-out invariance of the transition rate (2.2) there follows that

$$R^{in}(q, q') = R^{out}(q', q) \quad , \quad (2.4)$$

which guarantees particle number conservation, $\int \frac{d^3 q}{(2\pi)^3} B_{NES}[f_\nu(q)] = 0$. Neutrino-electron scattering (NES) leaves the electron concentration Y_e unchanged and does not directly contribute to the deleptonisation of the system. Chemical equilibration can only be attained through other processes such as neutrino absorption and emission. However, NES contributes to the exchange of energy $\int \frac{d^3 q}{(2\pi)^3} q^0 B_{NES}[f_\nu(q)] \neq 0$, thermalizing the neutrino-matter system, and momentum exchange, $\int \frac{d^3 q}{(2\pi)^3} \mathbf{q} B_{NES}[f_\nu(q)] \neq 0$, forcing the neutrino distribution $f_\nu(\omega, \Omega)$ towards isotropy, i.e. independence of the direction of the neutrino momentum¹ Ω , with $\omega = |\mathbf{q}| = q^0$.

The NES kernel must vanish in local thermodynamic equilibrium (lte), when the neutrino distribution function equals the lte distribution $f_\nu(\omega, \Omega) = f_\nu^0(\omega) = 1/[e^{(\omega - \mu_\nu)/T} + 1]$ with $\mu_\nu = \mu_p - \mu_n + \mu_e$ the equilibrium neutrino chemical potential. It does this if

$$R^{\{in\}}(q, q') = e^{-(\omega - \omega')/T} R^{\{out\}}(q, q') \quad . \quad (2.5)$$

Combined with the in-out symmetry (2.4) this constraint implies the existence of transpositional symmetry :

$$R^{\{out\}}(q', q) = e^{-(\omega - \omega')/T} R^{\{out\}}(q, q') \quad , \quad R^{\{in\}}(q, q') = e^{-(\omega - \omega')/T} R^{\{in\}}(q', q) \quad , \quad (2.6)$$

which maps $R^{\{out\}}$ and $R^{\{in\}}$ under $q \leftrightarrow q'$ reversal onto themselves. In numerical applications the neutrino distribution is sampled as a discrete set of N energy bins, in which case $R^{\{out\}in}(q_i, q'_j)$, $i, j = 1 \dots N$ may be represented as $N \times N$ matrices. Transpositional symmetry relates elements on either side of the main diagonal of these matrices, which therefore have each $N(N + 1)/2$ instead of N^2 independent elements.

¹We assume that the neutrinos are masses.

3 NES Algorithm

The scattering kernel (2.3) depends on the direction of the neutrino momentum through the inner product of $\mathbf{q} \cdot \mathbf{q}' = \omega\omega'\Omega \cdot \Omega'$. This angular dependence is approximated by expanding $R^{\{out\}}$ in a Legendre series and retaining the first two terms,

$$R^{\{out\}}(\omega, \omega', \Omega \cdot \Omega') \approx (1/2)\phi_0^{out}(\omega, \omega') + (3/2)\phi_1^{out}(\omega, \omega') \Omega \cdot \Omega' . \quad (3.1)$$

The coefficients $\phi_l^{out}(\omega, \omega')$, $l = 0, 1$ may be written as

$$\phi_l^{out}(\omega, \omega') = \alpha_k A_l^k(\omega, \omega') , \quad (3.2)$$

with a summation over $k = I, II$ implied. The constants α_k are combinations of coupling constants that depend on the neutrino species

$$\begin{aligned} \alpha_I(\nu_e) &= \alpha_{II}(\bar{\nu}_e) &= (C_v + C_A)^2 , \\ \alpha_I(\bar{\nu}_e, \bar{\nu}_{\mu, \tau}) &= \alpha_{II}(\nu_e, \nu_{\mu, \tau}) &= (C_v - C_A)^2 , \\ \alpha_I(\nu_{\mu, \tau}) &= \alpha_{II}(\bar{\nu}_{\mu, \tau}) &= (C_v + C_A - 2)^2 , \end{aligned}$$

where $C_v = 1/2 + 2 \sin^2 \theta_W$ and $C_A = 1/2$, with the Weinberg mixing angle $\sin^2 \theta_W = 0.23$.

The coefficients $A_l^k(\omega, \omega')$ are defined as

$$A_l^k(\omega, \omega') = \frac{C}{\omega^2 \omega'^2} \int_{\max(0, \omega' - \omega)}^{\infty} dE f_e(E) (1 - f_e(E + \omega - \omega')) H_l^k((\omega, \omega', E)) , \quad (3.3)$$

with $E = p_0$. The functions H_l^k $l = 0, 1$, $k = I, II$ are as given in (Yueh and Buchler 1977) with the modifications indicated below equation C50 in (Bruenn 1985). The dimensional constant $C = (4G_w^2)/((\hbar c)^4(2\pi\hbar c)^3) = 1.1104 \cdot 10^{-13} \text{MeV}^{-5} \text{cm}^{-1}$, with the Fermi constant $G_w = 8.957 \cdot 10^{-44} \text{MeV cm}^3$.

Numerical evaluation of expression (3.3) shows that

$$A_l^k(\omega', \omega) = e^{-(\omega - \omega')/T} A_l^k(\omega, \omega') . \quad (3.4)$$

Because it is merely a linear superposition of these coefficients, the kernel $R^{\{out\}}$ is positionally symmetric as well, c.f. eq. (2.6).

The in kernel $R^{\{in\}}(\omega, \omega', \Omega \cdot \Omega') = R^{\{out\}}(\omega', \omega, \Omega' \cdot \Omega)$ follows from the out kernel with the in-out symmetry (2.4).

4 Conclusion

The NES kernel is more symmetric than was previously implied. The transpositional symmetry (3.4) is a property which relates off-diagonal elements of the kernel matrix. The in-out symmetry (2.4) ensures conservation of particle number. Together they make the kernel vanish in lte.

The addendum to Bruenn (1985) is that the in-out, eq. (2.4), instead of the convoluted lte-symmetry eq.(2.5) should be used to relate in and out beams.

With the existence of transpositional symmetry the coefficients $A_l^k(\omega, \omega')$ need not be calculated for all combinations ω, ω' but only for $\omega \leq \omega'$, or $\omega' \leq \omega$. Since performing the integrals (3.3) is computationally the most intensive part of the algorithm, exploitation of this property nearly doubles the computational efficiency.

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A NES Source Terms

The NES kernel can be rearranged into four terms

$$B_{NES}[f_\nu(\omega, \Omega)] = \kappa_{\nu e}^0(\omega) - f_\nu \kappa_{\nu e}^e(\omega) - f_\nu \Omega \cdot \kappa_{\nu e}^f(\omega) + \Omega \cdot \tilde{\kappa}_{\nu e}(\omega) \quad , \quad (\text{A.1})$$

with

$$\kappa_{\nu e}^0(\omega) = (1/2) \int d\omega' \omega'^2 \phi_0^{\text{in}}(\omega, \omega') e(\omega') \quad (\text{A.2})$$

$$\kappa_{\nu e}^e(\omega) = (1/2) \int d\omega' \omega'^2 \phi_0^{\text{out}}(\omega, \omega') + [\phi_0^{\text{in}}(\omega, \omega') - \phi_0^{\text{out}}(\omega, \omega')] e(\omega') \quad (\text{A.3})$$

$$\kappa_{\nu e}^f(\omega) = (3/2) \int d\omega' \omega'^2 [\phi_1^{\text{in}}(\omega, \omega') - \phi_1^{\text{out}}(\omega, \omega')] \mathbf{F}(\omega') \quad (\text{A.4})$$

$$\tilde{\kappa}_{\nu e}(\omega) = (3/2) \int d\omega' \omega'^2 \phi_1^{\text{in}}(\omega, \omega') \mathbf{F}(\omega') \quad (\text{A.5})$$

The NES opacity terms contain angular moments of the distribution function defined as

$$\begin{aligned} e &\simeq (1/4\pi) \int d\Omega f_\nu(\omega, \Omega) \quad , \\ \mathbf{F} = \mathbf{f}(\omega)e &\simeq (1/4\pi) \int d\Omega \Omega f_\nu(\omega, \Omega) \quad , \\ \vec{\mathbf{P}} = \vec{\mathbf{p}}e &\simeq (1/4\pi) \int d\Omega \Omega \Omega f_\nu(\omega, \Omega) \quad , \end{aligned} \quad (\text{A.6})$$

with $\mathbf{f}(\omega)$ and $\vec{\mathbf{p}}(\omega)$ the first and second Eddington factors, corresponding to the flux and pressure-tensor respectively, and $e(\omega)$ the energy occupation density. The quantities f and p that feature in the following are the magnitude of the first Eddington factor $f = \pm|\mathbf{f}|$ and the forward component of the pressure tensor, $p \equiv |\vec{\mathbf{p}} \cdot \mathbf{f}/f|$.

In a spectral Eddington factor approach to neutrino transport, not the Boltzmann equation itself, but its angular moments are solved. The moment equations are obtained by integrating the Boltzmann equation over the direction of the neutrino momentum Ω . The energy (or particle number)-balance and momentum balance NES kernels follow from equation (A.1) by the application of the operators $(1/4\pi) \int d\Omega$, and $(1/4\pi) \int d\Omega \Omega$ respectively, and are given by

$$S_{NES}^e = \kappa_{\nu e}^0 - e \kappa_{\nu e}^e - \mathbf{F} \cdot \kappa_{\nu e}^f \quad , \quad (\text{A.7})$$

$$S_{NES}^f = -\mathbf{F} \kappa_{\nu e}^e - \vec{\mathbf{P}} \cdot \kappa_{\nu e}^f + (1/3) \tilde{\kappa}_{\nu e} \quad . \quad (\text{A.8})$$

The momentum balance equation can with the aid of the energy balance equation be trivially rewritten into the form

$$-\frac{\nabla \ln e + \kappa_{\nu e}^f}{\kappa_{\text{tot}} + (\kappa_{\nu e}^0/e - \tilde{\kappa}_{\nu e}/(3ef))} = \frac{\mathbf{f}}{p - f^2} \equiv R \quad . \quad (\text{A.9})$$

Here κ_{tot} is the total inverse mean free path due to all other (baryonic) interactions except NES, and includes artificial opacity (Cernohorsky and van Weert 1992; Dgani and Janka 1992). The NES opacity terms are shown explicitly. The interaction kernel due to pair processes $e^- + e^+ \leftrightarrow \nu + \bar{\nu}$ has the same structure as the NES kernel (A.1) and the corresponding opacity terms may be added to the NES opacities (A.2–A.5). The effective Knudsen-number R , the quotient of the mean free path and the energy scale height, is small, $R \ll 1$ in the diffusive limit, and very large, $R \gg 1$, in the free-streaming regime. The neutrino electron scattering terms enter the expression for R in the combination $S_{NES}^e/e - S_{NES}^f/(ef)$ being added to κ_{tot} . Expression (A.9) follows after some rewriting and cancellation of terms. Interestingly, $\kappa_{\nu e}^f$ enters in the numerator as an additive factor to the energy scale height, whereas $\kappa_{\nu e}^e$ cancels out entirely in the NES contribution to the inverse mean free path.

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