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Discretization Limits in Multi-Beam Structured Light Synthesis

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Abstract

We model the Lemons multi-beam structured-light architecture as an angular sampler, derive a Nyquist limit $|\ell_{\max}| \approx N/2$ on its OAM capacity, and confirm this bound with Python-based numerical OAM decomposition.

1 Introduction

Structured-light beams, optical field engineering that use position dependent amplitude, phase, and polarization are now invaluable tools in science and technology. A particular interest are beams carrying orbital angular momentum (OAM) with azimuthal phase dependence $e^{i\ell\phi}$ and a corresponding twisted wavefront, provide an effectively unbounded discrete degrees of freedom for information encoding and optical torque. [2, 5].

On the information side, OAM enables high-dimensional quantum states and multiplexed free-space links that can increase channel capacity by using many orthogonal OAM modes as parallel carriers [2, 3]. On the imaging and manipulation side, OAM beams enhance edge contrast in microscopy, shape point-spread functions, and enable precise optical tweezers and torque transfer to microscopic particles [4, 5]. These capabilities reach directly into social and scientific drivers: higher-capacity communication networks, improved biomedical imaging, and scalable quantum technologies.

Lemons *et al.* [1] introduce a conceptually different scheme: an “integrated structured light architecture” in which N spatially separated ultrafast beamlines are independently controlled in phase, amplitude, temporal delay, polarization, and carrier envelope phase, then coherently recombined to produce custom structured fields. This multi-beam architecture is naturally compatible with high peak powers and broadband pulses. It resembles an almost optical phased array in the transverse plane.

From an engineering perspective, this unique architecture raises an immediate question: since the helical phase of an OAM beam is only sampled at N azimuthal positions, there must be a maximum OAM charge $|\ell|$ that can be represented without severe distortion. This intuition directly mirrors the well-known limits in antenna arrays. Traditional phased arrays can typically generate OAM beams only up to $|\ell| \lesssim N/2$ before mode purity deteriorates [6]. Yet the original Lemons paper does not quantify this limit for their optical system, nor does it connect their architecture to angular sampling and OAM spectral theory.

The goal of this paper is to analyze the Lemons structured-light architecture in this sampling framework and derive a clean, intuitive limit on its OAM capacity. We treat the N -beam system as an angular sampler of a continuous helical wavefront, derive a Nyquist bound $|\ell_{\max}| \approx N/2$ from basic sampling theory, and then verify this bound using numerical OAM decomposition. Beyond simply restating the already known antenna results in an optical context, this analysis directly ties the design of discrete multi-beam structured-light sources directly to their functional role in OAM communications, quantum information, and imaging systems. Practically, it tells a system designer: *if you want to support a given maximum OAM charge $|\ell_{\text{target}}|$, you must allocate at least $N \gtrsim 2|\ell_{\text{target}}|$ independently controlled beamlines.*

2 Methods

2.1 Continuous OAM Modes and Their Angular Spectrum

Ideal scalar OAM modes are often represented by Laguerre–Gaussian (LG) fields with radial index p and azimuthal index ℓ ,

$$E_{\ell,p}(r, \phi, z = 0) \propto r^{|\ell|} L_p^{|\ell|} \left(\frac{2r^2}{w_0^2} \right) \exp \left(-\frac{r^2}{w_0^2} \right) e^{i\ell\phi}, \quad (1)$$

where w_0 is the beam waist and $L_p^{|\ell|}$ is an associated Laguerre polynomial. For our purposes, the noteworthy feature is the azimuthal phase factor $e^{i\ell\phi}$, which says that the field along any ring $r = r_0$ is a pure angular harmonic of order ℓ . In the continuous limit, these angular harmonics make up an orthogonal basis, and different ℓ values are very clearly distinguishable.

2.2 Discrete Angular Sampling and Nyquist Limit

In the Lemons architecture, N beamlets are arranged on a ring and their phases are individually controlled. Let the emitters be located at

$$\theta_k = \frac{2\pi k}{N}, \quad k = 0, 1, \dots, N-1, \quad (2)$$

and suppose we program the k th beamlet with phase

$$\Phi_k = \ell_{\text{target}} \theta_k. \quad (3)$$

The phase increment between adjacent emitters is then

$$\Delta\Phi = \Phi_{k+1} - \Phi_k = \frac{2\pi\ell_{\text{target}}}{N}. \quad (4)$$

The discrete set of emitter phases $\{\Phi_k\}$ makes up uniform angular sampling of the continuous helical phase. In sampling theory, an angular signal with maximum spatial frequency $|\ell_{\text{max}}|$ can be reconstructed from N samples around 2π if

$$N > 2|\ell_{\text{max}}|, \quad (5)$$

or (equivalently)

$$|\ell_{\text{max}}| < \frac{N}{2}. \quad (6)$$

This is the direct angular analogue of the one-dimensional Nyquist criteria. If $|\Delta\Phi| > \pi$, adjacent emitters “jump” more than half a cycle in phase and the sampled wavefront becomes unclear and ambiguous. A different OAM mode with topological charge $\ell' = \ell_{\text{target}} - N$ produces the same sequence of phases modulo 2π aliasing.

Thus, the N -beam Lemons architecture is fundamentally limited to

$$|\ell_{\text{max}}| \approx \frac{N}{2} \quad (7)$$

for unambiguous, high-purity OAM creation, independent of radial beam shape.

2.3 Field Model of the Discretized Beam Array

To quantify how this angular limit manifests in actual beam profiles, we model the Lemons architecture as a superposition of N Gaussian beamlets:

$$E_N(x, y) = \sum_{k=0}^{N-1} \exp \left[-\frac{(x - x_k)^2 + (y - y_k)^2}{2\sigma^2} \right] e^{i\ell_{\text{target}} \theta_k}, \quad (8)$$

where (x_k, y_k) lie on a ring of radius R , each Gaussian has spot size σ , and the phase factor encodes the target OAM charge ℓ_{target} .

2.4 OAM Decomposition and Purity Metric

To make the connection between this discrete field and to the continuous OAM basis, we evaluate $E_N(r_0, \phi)$ on a ring at radius r_0 and decompose it into angular Fourier components:

$$E_N(r_0, \phi) = \sum_{\ell=-\infty}^{\infty} c_{\ell} e^{i\ell\phi}. \quad (9)$$

The coefficients

$$c_{\ell} = \frac{1}{2\pi} \int_{-\pi}^{\pi} E_N(r_0, \phi) e^{-i\ell\phi} d\phi \quad (10)$$

are computed numerically using discrete sampling in ϕ . Defining the OAM power in mode ℓ as $P_{\ell} = |c_{\ell}|^2$, the OAM *purity* of the synthesized beam with respect to the target OAM order is

$$\mathcal{P}(\ell_{\text{target}}) = \frac{|c_{\ell_{\text{target}}}|^2}{\sum_m |c_m|^2}. \quad (11)$$

2.5 Python Implementation

The numerical simulations were run in Python using NumPy and Matplotlib. A regular Cartesian grid is defined, the discretized field on top of it $E_N(x, y)$ is constructed by summing N Gaussian beamlets with the correct phase factors, and the OAM spectrum is obtained by sampling the field on a ring of radius r_0 and performing a discrete Fourier analysis. The final script produces two figures used in Sec. 3: (i) the purity curves in Fig. 1, and (ii) donut intensity snapshots for each N in Fig. 2. The full code is listed below.

Listing 1: Python code used to simulate discretized OAM synthesis, compute OAM purity, and generate Figures 1 and 2.

```
import numpy as np
import matplotlib.pyplot as plt

# ----- global parameters -----
grid_size = 256
extent = 20.0          # physical size of grid (arbitrary units)
R = 7.0                # radius of beamlet ring
sigma = 2.0            # beamlet width
l_max_analyze = 20     # OAM orders analyzed around 0

# coordinate grid
x = np.linspace(-extent/2, extent/2, grid_size)
y = np.linspace(-extent/2, extent/2, grid_size)
X, Y = np.meshgrid(x, y)
r = np.sqrt(X**2 + Y**2)
phi = np.arctan2(Y, X)

def discretized_field(N, l_target):
    E = np.zeros_like(X, dtype=complex)
    thetas = 2 * np.pi * np.arange(N) / N

    for theta in thetas:
        x0 = R * np.cos(theta)
        y0 = R * np.sin(theta)
        r2 = (X - x0)**2 + (Y - y0)**2
        envelope = np.exp(-r2 / (2 * sigma**2))
        phase = np.exp(1j * l_target * theta)
        E += envelope * phase

    # normalize total power
```

```

    E /= np.sqrt(np.sum(np.abs(E)**2))
    return E

def oam_spectrum(E, r0, n_phi_samples=720, l_max=20):
    phis = np.linspace(-np.pi, np.pi, n_phi_samples, endpoint=False)
    xs = r0 * np.cos(phis)
    ys = r0 * np.sin(phis)

    # nearest-neighbor sampling of E(x,y) on the grid
    ix = np.clip(((xs - x[0]) / (x[1] - x[0])).astype(int), 0, grid_size - 1)
    iy = np.clip(((ys - y[0]) / (y[1] - y[0])).astype(int), 0, grid_size - 1)
    E_ring = E[iy, ix]

    ls = np.arange(-l_max, l_max + 1)
    powers = []

    for l in ls:
        integrand = E_ring * np.exp(-1j * l * phis)
        # numerical approximation to  $(1/2) \int_{-\pi}^{\pi} E e^{-il\phi} d\phi$ 
        coeff = np.trapezoid(integrand, phis) / (2 * np.pi)
        powers.append(np.abs(coeff)**2)

    powers = np.array(powers)
    powers /= powers.sum() # normalize total power
    return ls, powers

# ----- Purity vs l_target for N = 7, 19, 37 -----
Ns = [7, 19, 37]
l_targets = np.arange(-10, 11)
r0 = R # sample near the ring

plt.figure(figsize=(7, 4))
for N in Ns:
    purities = []
    for l_t in l_targets:
        E = discretized_field(N, l_t)
        ls, P = oam_spectrum(E, r0, l_max=l_max_analyze)
        if l_t < ls[0] or l_t > ls[-1]:
            purities.append(0.0)
        else:
            idx = np.where(ls == l_t)[0][0]
            purities.append(P[idx])
    plt.plot(l_targets, purities, 'o-', label=f"N = {N}")

plt.xlabel(r"Target twist (OAM charge)  $\ell_{\mathrm{target}}$ ")
plt.ylabel(r"Purity  $P_{\ell_{\mathrm{target}}}$ ")
plt.title("How cleanly can different twists be generated?")
plt.grid(True, alpha=0.3)
plt.legend()
plt.tight_layout()
plt.savefig("twists.pdf", bbox_inches="tight", dpi=300)
plt.show()

# ----- Donut intensity plots for each N -----
l_donut = 3

fig, axes = plt.subplots(1, len(Ns), figsize=(10, 3))
for ax, N in zip(axes, Ns):
    E = discretized_field(N, l_donut)

```

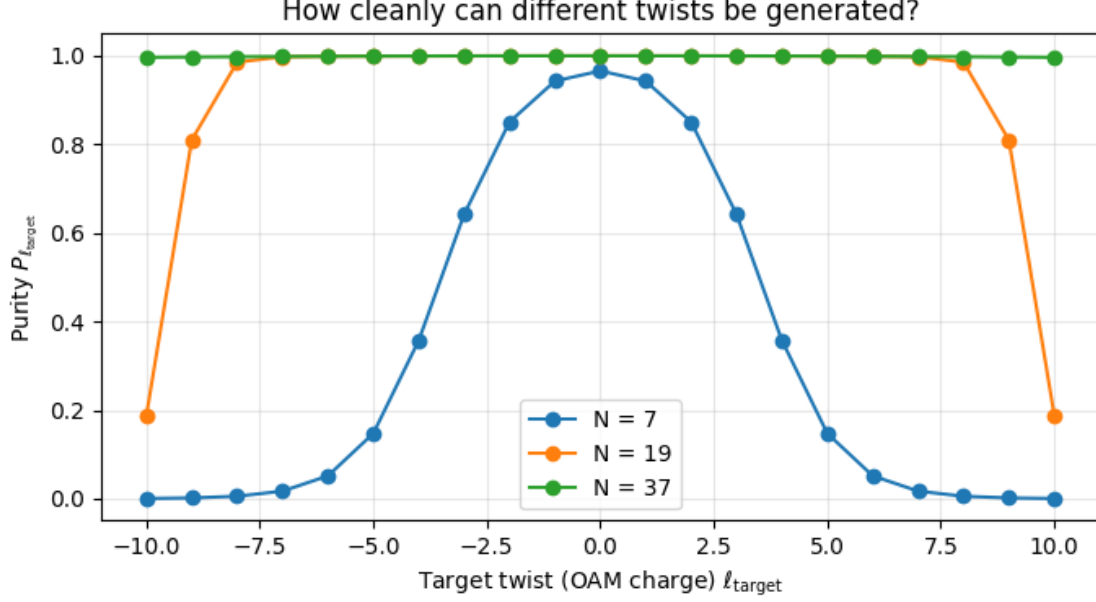


Figure 1: Mode purity $P_{\ell_{\text{target}}}$ as a function of target OAM charge for arrays with $N = 7, 19$, and 37 beamlets. Each curve collapses near $|\ell_{\text{target}}| \approx N/2$, confirming the angular Nyquist limit derived in Sec. 2.

```

intensity = np.abs(E)**2

im = ax.imshow(intensity,
                extent=[x[0], x[-1], y[0], y[-1]],
                origin='lower')
ax.set_title(f"N = {N},  $\ell_{\text{target}}={l_{\text{donut}}}$")
ax.set_xlabel("x")
ax.set_ylabel("y")
ax.set_aspect('equal')
ax.set_xticks([])
ax.set_yticks([])

fig.suptitle("Discretized donut intensity patterns for different N", y=0.92)
plt.tight_layout(rect=[0, 0, 1, 0.9])
plt.savefig("donuts.pdf", bbox_inches="tight", dpi=300)
plt.show()

```

3 Results

Across all simulations, a discretization trend appears. For small OAM charges satisfying $|\ell_{\text{target}}| \ll N/2$, the synthesized field behaves like a clean helical mode: the OAM decomposition yields a dominant component at $\ell = \ell_{\text{target}}$, and the corresponding purity $\mathcal{P} \approx 1$. As $|\ell_{\text{target}}|$ increases, the discrete phase sampling imposed by Eq. (4) becomes rough. Once the phase step between (adjacent) emitters approaches or exceeds π , the array can no longer represent a smooth helical wavefront. The result is a rapid degradation of purity, with power leaking into aliased OAM orders separated by integer multiples of N .

This behavior is shown in Fig. 1: for each (tested) array size, the purity curve remains flat and near unity until $|\ell_{\text{target}}| \approx N/2$, after which it collapses sharply. The observed cutoff points—around $\ell \approx 3$ for $N = 7$, $\ell \approx 9$ for $N = 19$, and beyond the simulated range for $N = 37$ —match precisely the angular Nyquist limit predicted in Sec. 2.

Discretized donut intensity patterns for different N

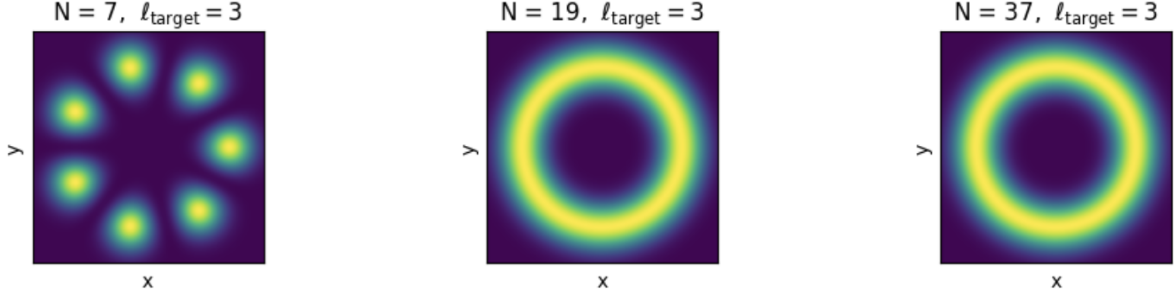


Figure 2: Simulated intensity patterns $|E_N(x, y)|^2$ for arrays with $N = 7, 19$, and 37 beamlets at fixed target charge $\ell_{\text{target}} = 3$. Larger N produces a smoother, more rotationally symmetric donut, reflecting the higher OAM purity.

Quantitatively, $N = 7$ the purity remains high until about $|\ell_{\text{target}}| \approx 3$, then collapses rapidly for $\ell_{\text{target}} = \pm 4$. For $N = 19$, high purity extends up to $|\ell_{\text{target}}| \approx 9$ with degradation at $\ell_{\text{target}} = \pm 10$. For $N = 37$, no strong degradation is observed within the tested range $|\ell_{\text{target}}| \leq 10$, but this is consistent with the fact that $N/2 \approx 18.5$ lies well beyond the simulated charges. In every case, the onset of purity loss coincides with the condition $|\Delta\Phi| \gtrsim \pi$.

3.1 Donut Intensity Patterns for Different N

To visualize how discretization affects the real-space beam profile, we also plot the intensity $|E_N(x, y)|^2$ for each array size at a fixed target charge $\ell_{\text{target}} = 3$. As shown in Fig. 2, the $N = 7$ array produces a visibly segmented ring, while $N = 19$ and $N = 37$ yield progressively smoother, more symmetric “donut” beams. This behavior is consistent with the spectral trends in Fig. 1.

3.2 Dependence on Beamlet Size and Ring Radius

While the Nyquist limit in Eq. (6) is from angular sampling, the shape of $\mathcal{P}(\ell_{\text{target}}; N)$ also depends on the radial parameters (R, σ) . If the beamlets are very broad ($\sigma \sim R$), their fields overlap strongly, smoothing out some artifacts and improving the beams purity for $|\ell_{\text{target}}|$ near the Nyquist edge. If the beamlets are tightly focused ($\sigma \ll R$), the field becomes more “patchy,” and purity can start to degrade even before $|\ell_{\text{target}}| \approx N/2$.

4 Conclusion

This paper expands on the analysis of Lemons *et al.* by treating their multi-beam structured-light architecture like an angular sampling system and deriving a simple Nyquist bound on its OAM capacity. By examining the helical phase of an OAM mode is discretely sampled at N azimuthal positions, we see that the maximum unaliased donut satisfies $|\ell_{\text{max}}| \approx N/2$. Numerical simulation validated this through OAM spectrum decomposition confirming that mode purity collapses once the phase step between emitters exceeds π , leading to aliasing into modes separated by integer multiples of N .

From a broader perspective, this discretization limit connects to the practical design of high-capacity communication links, quantum information systems, and imaging modalities that rely on structured light. It provides a clear engineering rule: to support a target OAM range $|\ell_{\text{target}}| \leq L$ with high fidelity, a discrete structured-light architecture must allocate at least $N \gtrsim 2L$ independently controlled channels.

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