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The Role of Outside Considerations in the Design of Compensation Schemes

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Abstract

We analyze a principal-agent model under incomplete information where the principal anticipates future interaction with a third party (e.g. regulators, the financial markets, or product market competitors). Knowledge of the information affects the third party's strategy in the future interaction. Consequently, output targets given the agent and the agent's compensation schedule can differ from the "no-third-party" situation; the motivation being the concealment of information when the third party's knowledge of that information is detrimental to the principal. We show that equilibrium contracts are sensitive to what aspects of the contracting game between principal and agent are observable by the third party.

JEL Classification: 026, 022

THE ROLE OF OUTSIDE CONSIDERATIONS IN THE DESIGN OF COMPENSATION SCHEMES[1]

0/ INTRODUCTION

This paper considers what happens when a principal-agent relationship reveals (or can reveal) information to a third party which is relevant to some future interactions between the *principal* and the third party. As the principal will be concerned with these future interactions (outside considerations), the design of the agent's compensation scheme will be motivated in part by the principal's desire to conceal or reveal information.[2] For example, the principal foresees going to the capital market in the future (e.g. an initial public offer, privatization of a state-run concern, etc.); he may thus wish that the value of his firm be revealed, or he may wish to have concealed how little value it has. Or, the principal is concerned about future entry into his market, in which case he may cause production to be altered to conceal that conditions are favorable for entry.

Thus our paper is a departure from standard principal-agent models which treat the principal and agent in isolation, as if they were

[1] We thank Eddie Dekel-Tabak, Franklin Fisher, Drew Fudenberg, Bob Gibbons, Oliver Hart, Bruno Jullien, Michael Katz, John Litwack, James Malcomson, Matt Rabin, and seminar participants at Berkeley, Budapest, MIT, Paris, Stanford, Tilburg, and Toulouse for helpful comments. We express special thanks to Jean Tirole for many stimulating discussions during the early stages of our project and for numerous suggestions about the presentation. Remaining errors are ours.

[2] These concerns can also be found in Gertner, Gibbons, and Scharfstein [1988] and Glazer and Israel [1987]. Those models are, however, signaling models, whereas we assume that principal is initially an *un*informed player (i.e. our model is *not* a signaling model). Moreover, unlike us, they do not analyze the role of observable negotiations and/or observable contracts.

playing alone on a desert island.[3] Given that most principal-agent relationships occur in settings more populated than that of Robinson Crusoe and Friday, this strikes us as an important departure. This is particularly true with regard to the modern corporation, a setting to which the principal-agent paradigm is often applied. The modern corporation exists in a multi-player environment, and therefore it is of value to know how the standard "desert-island" models are changed by the introduction of additional players (outside considerations). Our goal in this paper is, in part, to provide some answers to that question.

What information is revealed to a third party and how it can be controlled by the principal depends on what is observable by the third party. In what follows, we will maintain the assumption that the third party observes the physical outcome of the principal-agent relationship (e.g. revenues, sales volume, or some other performance measure). In addition, we will consider three different assumptions about what other aspects of the relationship are observable. The most informative case (from the third party's perspective) will be where the third party observes the initial negotiations between principal and agent and the terms of the contract they sign; we call this the public negotiations/ public contracts case. The least informative case will be where the

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3 We are not the first to introduce other players into the principal-agent relationship. Work by Holmstrom [1982], Hart [1983], and Scharfstein [1988] has considered the extent to which observations of other players (e.g. through product market competition) provide information to the principal which can help in the design of compensation schemes. Some authors have sought to explain the existence of principal-agent hierarchies as a consequence of interactions with third parties (Katz [1988] and Ferschtmann, Judd, and Kalai [1987]). Finally, in the literature perhaps closest to us, some authors have considered an *agent* who foresees interacting in future principal-agent relationships with other principals (Fama [1980], Holmstrom [1983], Waldman [1984], Gibbons [1986], and Ricart i Costa [1988]).

third party observes neither the initial negotiations, nor the contract signed; we call this the private negotiations/private contracts case. Finally, an intermediate case is when the third party observes the contract signed, but not the negotiations which led to it; we call this the private negotiations/public contracts case. As we will show, the design of the compensation scheme can easily depend on which of three cases we assume holds. Indeed one of the points of this paper is that in presence of outside considerations *how* contracts are entered into can be as important as what contracts are entered into.

Generally speaking, it is less costly for the principal to have information credibly revealed in the two public contract cases than in the private contract case. However when it comes to having information concealed, things are different: it is least costly for the principal in the public negotiations/public contracts case, but most costly in the private negotiations/public contracts case; the private negotiations/private contracts case is intermediate. In all cases, the equilibrium compensation scheme can be different from the one which would exist if there were no third party; and different, too, from the ones obtained under alternative observability assumptions. Consequently, a firm's actual physical production (or other publicly observable measure of performance) can be quite sensitive to the observability of negotiations and contracts. For example, when negotiations are public, information-concealing equilibria can only occur at a low production level; while when negotiations are private, information-concealing equilibria can occur at high production levels. The rent earned by good managers will differ also: it will be greater in the latter case than in the former case. Also in the latter case, if the contracts are

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As contract *negotiations* are rarely public, we feel that case is less realistic than the private negotiation cases. Consequently, much of the focus is on private negotiation. We offer no explanation for why contract negotiations are typically private -- indeed all information structures in this paper are taken to be exogenous -- however one reason could be an inability to *commit* to public negotiations: outside observers cannot be sure that the parties to the negotiations have not conversed privately or entered into other secret agreements. For similar reasons, one might also prefer the private *contracts* assumption; although here, legal requirements, such as SEC requirements, could serve to justify the assumption of public contracts.

We present the model in Section 1. In Section 2, we derive the optimal "no-third-party" compensation scheme. We treat the public/public case in Section 3. The private/private case is analyzed in Section 4. In Section 4a, we examine the conditions for separating (information-revealing) schemes to exist in equilibrium. In Section 4b, we examine the conditions for pooling (information-concealing) schemes to exist in equilibrium. In Section 4c, we briefly considered equilibria in which the principal randomizes between separating and pooling schemes. Section 5 deals with the private/public case. Applications of this analysis to entry deterrence are considered in Section 6. Empirical implications are briefly touched upon in Section 7. We finish with some concluding remarks in Section 8.

1/MODEL

This is a game with two periods and three players: a principal (P), a manager (M), and a third party (T). The principal owns the firm and hires the manager to run it in the first period. In the second period, the third party and principal interact in a game-like situation.

Revenues in the first period are $x = \theta + e$, where θ is a characteristic of the firm and e is managerial effort.[4] θ *a priori* belongs to $\{\underline{\theta},\bar{\theta}\}$ and prior beliefs are $\mathbb{P}(\theta = \bar{\theta}) = h$. Expending effort e costs the manager $\phi(e)$ in disutility. Assume

$\underline{A_1}$: $\phi(\cdot)$ is three-times differentiable on $\mathbb{R}$,

$\forall e \in \mathbb{R}_-$, $\phi(e) = 0$,

$\forall e \in \mathbb{R}_+^*$, $\phi(e) > 0$, $\phi'(e) \geq 0$, $\phi''(e) > 0$ and $\phi'''(e) \geq 0$.

The disutility of effort increases at an increasing rate with effort.[5] The assumption $\phi''' \geq 0$ ensures concavity for some of the optimization programs considered later.

M's utility function is $w - \phi(e)$, where w is first-period income. We normalize M's reservation utility level, what she would receive outside the firm, to be 0.

In the second period, P and T play a (possibly complicated) Bayesian game $\Gamma_2 = \Gamma(\mu,\nu,\theta)$, where μ is T's belief about θ and ν is P's belief about θ. We assume

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4 There is little loss of generality in this specification; we could extend the analysis to revenue functions of the form $R(x(\theta,e))$, where x is some observable measure of performance (e.g. sales volume), where R is increasing concave, and where $x_e > 0$, $x_{ee} \leq 0$, $x_\theta > 0$, and $x_{e\theta} \geq 0$ with similar results.

5 Negative effort is introduced only to make the setting as simple as possible, not for the purpose of realism. Negative effort permits us to forget boundary conditions in the maximizations performed later.

$\underline{A_2}$: Γ_2 has a unique (perfect) Bayesian equilibrium with respect to P's (expected) payoff.

Hence P's (expected) payoff in Γ_2 is a *function* $\bar{\pi}(\mu,\nu,\theta)$.

The timing of the entire game: Prior to contracting, θ is determined by nature. At this point, P has no private information. P and T share the prior h about θ. Negotiations occur in which P has all the bargaining power: he proposes a take-it-or-leave-it mechanism to the manager. If the manager rejects it, negotiations break down and the game ends.[6] If she accepts it (i.e. is hired), the mechanism is played out. Playing out the mechanism consists of M first learning θ. Only the manager learns θ at this time. After learning θ, M can quit if she wishes. Otherwise, M sends messages to P. First-period production then occurs, followed by the continuation game Γ_2.

In this paper, we limit attention to direct (truthful) revelation mechanisms of the form $(x(\hat{\theta}),w(\hat{\theta}))$, where $x(\hat{\theta})$ is the first-period revenue required of a manager who declares her type to be $\hat{\theta}$ and $w(\hat{\theta})$ is her wage *provided she obtains* $x(\hat{\theta})$ (otherwise she is paid 0). Announcing $\hat{\theta}$ is equivalent to choosing a pair (x,w) from the set $\{(x(\hat{\theta}),w(\hat{\theta}))\}_{|\hat{\theta}\in(\underline{\theta},\bar{\theta})}$. Note the set of mechanisms is a subset of $\mathbb{R}^4$. We call a pair (x,w) "chosen" in this way a contract and denote it by c.

As the mechanism is direct, P has perfect information about θ in Γ_2, i.e. $\nu = \delta_\theta$, the Dirac probability measure at θ. Define

$$\Pi(\mu,\theta) = \bar{\pi}(\mu,\delta_\theta,\theta)$$

In Appendix 1, we prove that the assumption of direct revelation mechanisms is without loss of generality for what follows, except,

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[6] Alternatively, P negotiates with another manager.

possibly, for the public negotiations and public contract case.[7] We parameterize μ as $\mu = \mathbb{P}\left\{\theta = \bar{\theta} \mid \mathcal{I}_T\right\}$; i.e. μ is T's probability assessment of the event $\theta = \bar{\theta}$ given T's information, $\mathcal{I}_T$.

We also assume

$\underline{A}_3$: $\forall\theta$, $\Pi(\mu,\theta)$ is continuous and non-decreasing in μ.[8]

In words: the better P appears in T's eyes, i.e. the more weight T puts on the event $\theta = \bar{\theta}$, the greater will be P's expected payoff in Γ_2. $\underline{A}_3$ would obtain if, for example, θ was a measure of the firm's productivity and T was a potential entrant.

We assume x, first-period revenue, is observed by T. In addition, we consider three different assumptions about T's other information:

Public Negotiations and Public Contracts: The mechanism and the contract chosen are observable by T.

Private Negotiations, but Public Contracts: The mechanism is *un*observable by T, but the chosen contract is observable by T.

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[7] The reader may also wonder about our assumption of deterministic mechanisms; i.e. what if, contrary to our assumption, M's announcement merely fixed a lottery over contracts from a set $\mathcal{C}$. In Appendix 1, we show that our assumption of deterministic mechanisms is without loss of generality, except, possibly, when contracts are public.

[8] Continuity is not necessary for much of what follows: the existence results in Sections 2-4b do not depend on it. In an earlier working paper we also considered the assumption that Π was *non-increasing* in μ, which yields similar results. Indeed, the entire analysis could be done without assuming that Π was monotonic in μ; it would suffice that Π was not constant in μ. However, such generalization adds little to the analysis, while greatly complicating the notation and discussion.

Private Negotiations

and Private Contracts: Neither the mechanism, nor the chosen contract, are observable by T.

The fourth possibility, public negotiations but private contracts, is ignored, as it is equivalent to public negotiations and public contracts. We assume in no case does T hear M's announcement; though in some cases what T observes is equivalent to hearing M's announcement.

Finally we assume the structure of the game is common knowledge.

Our solution concept will be the strong version of Perfect Bayesian Equilibrium (PBE) put forth by Fudenberg and Tirole [1989].[9] This solution concept still yields a plethora of equilibria in some cases. For the most part, we have chosen not to select from among them by employing further refinements; however, in Section 6 and Appendix 3, we show that a generalized refinement in the spirit of Farrell [1986] and Grossman and Perry [1986] could eliminate "unreasonable" equilibria in certain interesting cases. While in Section 5 we restrict attention to equilibria where out-of-equilibrium beliefs satisfy an additional "reasonableness" requirement.

As M's only strategic action is her announcement $\hat{\theta}$, we can easily solve for her optimal strategy in any PBE. Consider a mechanism m = $\left((\bar{x},\bar{w}),(\underline{x},\underline{w})\right)$, where the first contract is put in force by an announcement of $\bar{\theta}$ and the second contract by an announcement of $\underline{\theta}$. For

9 Our use of this version of PBE is motivated by our desire to rule out beliefs for T which require T to hypothesize that a deviation by P will be followed by a deviation by M. That is the uninformed player P cannot "signal" what he does not know. Were the space of mechanisms finite, then this solution concept would be equivalent to sequential equilibrium (Kreps and Wilson [1982]; see Fudenberg and Tirole [1989] for a complete discussion).

both types of M to truthfully announce their types, it must be that

$$\bar{w} - \phi(\bar{x} - \bar{\theta}) \geq \underline{w} - \phi(\underline{x} - \bar{\theta}) \tag{1a}$$

$$\underline{w} - \phi(\underline{x} - \underline{\theta}) \geq \bar{w} - \phi(\bar{x} - \underline{\theta}) \tag{1b}$$

The manager must do better by telling the truth than by lying; that is truth-telling must be *incentive compatible* (IC).

If M quits after learning θ, her utility is 0; hence M will stay only if

$$\bar{w} - \phi(\bar{x} - \bar{\theta}) \geq 0 \tag{2a}$$

$$\underline{w} - \phi(\underline{x} - \underline{\theta}) \geq 0 \tag{2b}$$

In words: it must be *individually rational* (IR) for the manager to stay. As $\left((0,0),(0,0)\right)$ is (IR), there is no loss of generality in restricting attention to (IR) mechanisms. Note that if a mechanism is (IR), M will prefer accepting it to rejecting it. A direct revelation mechanism satisfying (1) and (2) is *feasible.*

As is typical in such problems, a necessary condition for m to satisfy (1) and (2) is

$$\bar{x} \geq \underline{x} \tag{3}$$

The bad type cannot be asked to produce more than the good type; if this were not the case, no pair of wages could possibly make (1) hold. Condition (3) is also a sufficient condition given $(\bar{x},\underline{x})$ for the existence of wages $(\bar{w},\underline{w})$ such that $\left((\bar{x},\bar{w}),(\underline{x},\underline{w})\right)$ is feasible.

If $(\bar{x},\bar{w}) = (\underline{x},\underline{w})$, we call the mechanism *pooling.* If $(\bar{x},\bar{w}) \neq (\underline{x},\underline{w})$, we call the mechanism *separating.*

2/ THE MODEL WITH NO THIRD PARTY

For now, imagine there is no third party and no second period.

Define e^{FB} by $\phi'(e^{FB}) = 1$. e^{FB} is the first-best (full-information) level of effort; it is the level of effort P would induce if P knew θ (it equates the marginal product of effort with the marginal disutility of effort). Let $\bar{x}^{FB} = \bar{\theta} + e^{FB}$ and $\underline{x}^{FB} = \underline{\theta} + e^{FB}$ be the first-best revenue levels.

However P does not know θ; hence the optimal mechanism solves

$$\max_{\{\bar{w},\underline{w},\bar{x},\underline{x}\}} \ h(\bar{x} - \bar{w}) + (1-h)(\underline{x} - \underline{w}) \quad \text{s.t. (1) and (2) hold} \tag{4}$$

(2a) is slack and it is easily shown that (1a) is binding, hence (1b) is slack. The solution to (4) is thus:

$$\bar{w} = \phi(\bar{x} - \bar{\theta}) + \phi(\underline{x} - \underline{\theta}) - \phi(\underline{x} - \bar{\theta}) \tag{5a}$$

$$\underline{w} = \phi(\underline{x} - \underline{\theta}) \tag{5b}$$

$$1 - \phi'(\bar{x} - \bar{\theta}) = 0 \tag{5c}$$

$$\phi'(\underline{x}^* - \underline{\theta}) = 1 - \frac{h}{1-h}\left(\phi'(\underline{x}^* - \underline{\theta}) - \phi'(\underline{x}^* - \bar{\theta})\right) \tag{5d}$$

$\underline{A}_1$ guarantees that (5) is sufficient, as well as necessary. Let $\underline{x}^*$, $\bar{x}^*$, $\underline{w}^*$, and $\bar{w}^*$ be the solution to (5). Define $m^* = \left((\bar{x}^*,\bar{w}^*),(\underline{x}^*,\underline{w}^*)\right)$. From (5c), we see that the good type supplies her first-best level of effort (e^{FB}), for which she is paid more than under perfect information (5a). Equation (5d) entails $e(\underline{\theta}) = \underline{x}^* - \underline{\theta} < e^{FB}$; the bad type supplies less than the first-best level of effort. Note that as h increases from 0 to 1, $e(\underline{\theta})$ decreases from e^{FB} to 0. Finally note that $\bar{x}^* = \bar{x}^{FB} > \underline{x}^{FB} > \underline{x}^*$.

The solution to (4) is typical of this type of model (cf. Laffont and Tirole [1986]): the better type supplies the first-best level of effort and receives an informational rent, while the worse type supplies less than first-best level of effort and is put on her IR constraint.

Note the optimal mechanism is a separating mechanism. We have shown:

Proposition 1: *In the absence of a third party, the optimal mechanism is the separating mechanism defined by (5).*

For the purpose of later comparisons, we also calculate the optimal pooling mechanism, which is given by the program

$$\max_{\{\bar{w},\underline{w},\bar{x},\underline{x}\}} \; h(\bar{x} - \bar{w}) + (1-h)(\underline{x} - \underline{w}) \quad \text{s.t. (1) and (2) hold and s.t. } \bar{x} = \underline{x} \text{ and } \bar{w} = \underline{w} \tag{6}$$

Note (6) is (4) with the additional constraint that the mechanism be pooling. It is straightforward to show that the solution to (6) is

$$w = \phi(x - \underline{\theta}) \tag{7a}$$

$$1 - \phi'(x - \underline{\theta}) = 0 \tag{7b}$$

Let x_0 and w_0 be the solution to (7) ($x_0 = \underline{x}^{FB} < \bar{x}^*$ and $w_0 = \phi(e^{FB})$). Define $m_0 = \left[(x_0,w_0),(x_0,w_0)\right]$. The bad type supplies e^{FB} and the good type slacks (expends effort $e^{FB}-(\bar{\theta}-\underline{\theta}) < e^{FB}$). The bad type is put on her IR constraint, while the good type earns an informational rent.

For later convenience, we define the functions:

$$\bar{G}(x) = x - \phi(x-\bar{\theta})$$

$$\underline{G}(x) = x - \phi(x-\underline{\theta}) - \frac{h}{1-h}\left(\phi(x-\underline{\theta}) - \phi(x-\bar{\theta})\right) \tag{8}$$

From $\underline{A}_1$, both $\bar{G}(x)$ and $\underline{G}(x)$ are continuous, three-times differentiable, and strictly quasi-concave functions. From (5c), $\bar{x}^*$ is the solution to

$$\max_x \bar{G}(x)$$

and, from (5d), $\underline{x}^*$ is the solution to

$$\max_x \underline{G}(x) \tag{9}$$

Also, for all m which satisfy (1a) and (2b) with equality:

$$h\bar{G}(\bar{x}) + (1-h)\underline{G}(\underline{x}) = h(\bar{x}-\bar{w}) + (1-h)(\underline{x}-\underline{w})$$

Examples of such mechanisms are m^* and m_0.

$\bar{G}(x)$ is the full-information profit generated by giving a target of x to the good manager. $\underline{G}(x)$ takes into account that giving a target of x to a bad manager not only has a direct (full-information) effect on profits, but also an indirect effect on profits through the amount of rent which must be left the good manager to prevent her from lying. The relative importance of the two effects depends on the relative proportion of good types and bad types in the population.

3/ PUBLIC NEGOTIATIONS AND PUBLIC CONTRACTS

Now we consider the case where T observes the mechanism (m) proposed by P and the contract (c) chosen by M (but, recall, T is not aware of M's announcement). T's information is thus (m,c,x). If m is a separating mechanism, then observing c (or even just x) allows T to learn θ's value. However, if the mechanism is pooling, then T can draw no inferences from his observation of c (or x). His posterior on θ must equal his prior, i.e. $\mu = h$.[10]

Thus the principal knows when designing a mechanism what beliefs (μ) it will induce. P's problem is then

$$\max_{m} \quad h(\bar{x} - \bar{w}) + (1-h)(\underline{x} - \underline{w}) + h\Pi(\bar{\mu},\bar{\theta}) + (1-h)\Pi(\underline{\mu},\underline{\theta})$$

where $\bar{\mu}$ (respectively $\underline{\mu}$) represents T's beliefs upon observing that

[10] Note that regardless of the mechanism P proposes, T makes inferences by employing (trivially) Bayes' law on the equilibrium strategy of the *manager* only. This is an example of the cutting power of the strong version of PBE: under the weaker definition, we could have equilibria which are sustained by unreasonable off-the-equilibrium-path beliefs that are functions of *both* P's action and M's response; for example, we could have unreasonable equilibria in which T could hold beliefs $\mu = 0$ following any deviation by P, even if M played the response dictated by her equilibrium strategy.

$(\bar{x},\bar{w})$ (respectively $(\underline{x},\underline{w})$) has been chosen by M from m. Note $\bar{\mu} = 1$ and $\underline{\mu} = 0$, if $(\bar{x},\bar{w}) \neq (\underline{x},\underline{w})$, and otherwise $\bar{\mu} = \underline{\mu} = h$. As P's second-period profits only depend on whether the mechanism is separating or pooling, the optimal mechanism must then maximize first-period profits within one of these two classes. Hence

Proposition 2: *Under $\underline{A}_1$ and $\underline{A}_2$, and assuming public negotiations, the only PBE is*

- *Separating with P offering m^* if and only if (10) below holds.*
- *Pooling with P offering m_0 if and only if (10) strictly reversed.*
- *Indifference with P (possibly) mixing over m_0 and m^* if and only if (10) is an equality.*

$$h(\bar{x}^* - \bar{w}^*) + (1-h)(\underline{x}^* - \underline{w}^*) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(0,\underline{\theta}) >$$
$$x_0 - w_0 + h\Pi(h,\bar{\theta}) + (1-h)\Pi(h,\underline{\theta}) \qquad (10)$$

As m^* produces greater first-period profits than m_0, P will prefer m_0 only if

$$(1-h)\Big(\Pi(h,\underline{\theta}) - \Pi(0,\underline{\theta})\Big) > h\Big(\Pi(1,\bar{\theta}) - \Pi(h,\bar{\theta})\Big) \qquad (11)$$

(11) is the requirement that the expected gain from concealing $\underline{\theta}$ from T exceed the expected loss from concealing $\bar{\theta}$ from T.

Using the "G"-functions defined above, (10) can be rewritten as

$$h\Big(\bar{G}(\bar{x}^*) - \bar{G}(x_0)\Big) + (1-h)\Big(\underline{G}(\underline{x}^*) - \underline{G}(x_0)\Big)$$
$$> h\Big(\Pi(h,\bar{\theta}) - \Pi(1,\bar{\theta})\Big) + (1-h)\Big(\Pi(h,\underline{\theta}) - \Pi(0,\underline{\theta})\Big) \qquad (12)$$

If (12) holds, m^* is offered, and if (12) is reversed, m_0 is offered.

Note, finally, that in the case of public negotiations, the only possible equilibria involve P offering either the best-separating mechanism m^* or the the best-pooling mechanism m_0.

4/ PRIVATE NEGOTIATIONS AND PRIVATE CONTRACTS

Now assume that T no longer observes m or c (though he continues to observe x). Thus T's beliefs must be defined for all possible observations; i.e. there exists a $\mu[x] \in [0,1]$ $\forall x \in \mathbb{R}_+$.

As T sees neither m nor c, but only x, P is free to set wages to minimize costs. Cost minimization requires that (1a) and (2b) hold as equalities. It follows that we can denote feasible mechanisms as $(\bar{x},\underline{x})$, since $\bar{w}$ and $\underline{w}$ will then be determined by the equalities (1a) and (2b). Also, we can denote first-period profits using the "G"-functions: the mechanism $(\bar{x},\underline{x})$ yields first-period expected profits $h\bar{G}(\bar{x}) + (1-h)\underline{G}(\underline{x})$.

4a/ Separating Equilibria

Here we consider separating equilibria: equilibria in which P proposes a separating mechanism $(\bar{x},\underline{x})$, $\bar{x} > \underline{x}$. As the mechanism is separating, consistent beliefs require $\mu[\bar{x}] = 1$ and $\mu[\underline{x}] = 0$. Thus, in equilibrium, the expected profits from offering $(\bar{x},\underline{x})$ are

$$h\bar{G}(\bar{x}) + (1-h)\underline{G}(\underline{x}) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(0,\underline{\theta}) \tag{13}$$

Our first result is

Lemma 1: *Under $\underline{A}_1$, $\underline{A}_2$, and $\underline{A}_3$, if a feasible mechanism $(\bar{x},\underline{x})$ is offered in a separating PBE, then $\underline{x} = \underline{x}^* < \bar{x}$.*

Proof: Suppose $\bar{x} \le \underline{x}^*$ (hence $\underline{x} < \underline{x}^*$). Consider the feasible deviation $(\bar{x},\bar{x})$, which generates profits

$$h\bar{G}(\bar{x}) + (1-h)\underline{G}(\bar{x}) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(1,\underline{\theta}) \tag{14}$$

As $\Pi(\mu,\theta)$ is non-decreasing in μ and $\underline{G}$ is strictly quasi-concave in x with a maximum at $\underline{x}^*$, (14) is strictly greater than (13). As $(\bar{x},\bar{x})$ is a profitable deviation, $(\bar{x},\underline{x})$ cannot be an equilibrium for $\bar{x} \le \underline{x}^*$; hence $\bar{x} > \underline{x}^*$. Suppose $\underline{x} \neq \underline{x}^*$ and consider the feasible deviation $(\bar{x},\underline{x}^*)$, which generates profits

$$h\bar{G}(\bar{x}) + (1-h)\underline{G}(\underline{x}^*) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(\mu[\underline{x}^*],\underline{\theta}) \tag{15}$$

As $\underline{G}(\underline{x}^*) > \underline{G}(\underline{x})$ and $\mu[\underline{x}^*] \geq 0$, (15) is strictly greater than (13); hence it cannot be that $\underline{x} \neq \underline{x}^*$ in a separating equilibrium.

■

The reasoning behind Lemma 1: in equilibrium the type of each manager is revealed and being revealed minimizes $\Pi(\mu,\underline{\theta})$, thus there is nothing to be lost by inducing the $\underline{\theta}$-type to produce as efficient as possible first-period level of x, and much to be gained.

We can now prove the main existence result of this sub-section:

Proposition 3: *Under $\underline{A}_1$-$\underline{A}_3$, there exists a separating PBE in which the feasible mechanism $m = (\bar{x},\underline{x})$ is offered if and only if*

$$\bar{G}(\bar{x}^*) - \bar{G}(\bar{x}) \leq \Pi(1,\bar{\theta}) - \Pi(0,\bar{\theta}) \tag{16}$$

$$\underline{G}(\underline{x}^*) - \underline{G}(\bar{x}) \geq \Pi(1,\underline{\theta}) - \Pi(0,\underline{\theta}) \tag{17}$$

and

$$\underline{x} = \underline{x}^* \tag{18}$$

Proof: We first prove necessity, then sufficiency.

Necessity. The necessity of (18) has been shown by Lemma 1. Profits under m are

$$h\bar{G}(\bar{x}) + (1-h)\underline{G}(\underline{x}^*) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(0,\underline{\theta}) \tag{19}$$

The necessity of (16): consider the deviation $(\bar{x}^*,\underline{x}^*)$, which yields profits

$$h\bar{G}(\bar{x}^*) + (1-h)\underline{G}(\underline{x}^*) + h\Pi(\mu[\bar{x}^*],\bar{\theta}) + (1-h)\Pi(0,\underline{\theta}) \tag{20}$$

(16) is the necessary condition for there to exist beliefs $\mu[\bar{x}^*]$ such that (20) does not exceed (19). The necessity of (17): consider the deviation $(\bar{x},\bar{x})$, which yields profits

$$h\bar{G}(\bar{x}) + (1-h)\underline{G}(\bar{x}) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(1,\underline{\theta}) \tag{21}$$

(17) is the necessary condition for (21) not to exceed (19).

Sufficiency. Given (16), (17), and (18), the following is a separating PBE: P proposes m and T holds beliefs $\mu[\bar{x}] = 1$ and $\mu[x] = 0$ $\forall x \neq \bar{x}$. Clearly T's beliefs satisfy Bayesian consistency. P's profits under m are given by (25). Consider a deviation $(\bar{x}', \underline{x}')$ where $\bar{x}' \neq \bar{x}$ *or* $\underline{x}' \neq \underline{x}^*$ *or* both; such a deviation yields profits of

$$h\bar{G}(\bar{x}') + (1-h)\underline{G}(\underline{x}') + h\Pi(\mu[\bar{x}'],\bar{\theta}) + (1-h)\Pi(\mu[\underline{x}'],\underline{\theta}) \tag{22}$$

Subtracting (22) from (19) yields

$$h\left[\left(\bar{G}(\bar{x}) - \bar{G}(\bar{x}')\right) + \left(\Pi(1,\bar{\theta}) - \Pi(\mu[\bar{x}'],\bar{\theta})\right)\right]$$
$$+ (1-h)\left[\left(\underline{G}(\underline{x}^*) - \underline{G}(\underline{x}')\right) + \left(\Pi(0,\underline{\theta}) - \Pi(\mu[\underline{x}'],\underline{\theta})\right)\right] \tag{23}$$

By (16), the top line of (23) is minimized by $\bar{x}' = \bar{x}$. By (17) and (9), the bottom line of (23) is minimized by $\underline{x}' = \underline{x}^*$. Hence no deviation results in strictly greater profits than m; m is a best response.

■

There are two potentially desirable deviations from $(\bar{x}, \underline{x}^*)$. One is to give up the additional second-period profits when the $\bar{\theta}$-type is revealed in exchange for maximizing first-period profits (we call this the *best-separating deviation*). Best-separating is ruled out by condition (16). The second deviation, which we call *secret pooling*, is to have the $\underline{\theta}$-type mimic the $\bar{\theta}$-type. The benefit from this deviation is that expected second-period profits are increased, while the cost is that the $\underline{\theta}$-type produces a less profitable level of x (less profitable in part because the $\underline{\theta}$-type may produce an inefficient level of x and in part because it raises the informational rent earned by the $\bar{\theta}$-type; see (5)). Secret pooling is ruled out by condition (17).

The set of mechanisms that can be part of a separating equilibrium can easily be described through conditions (16) and (17):

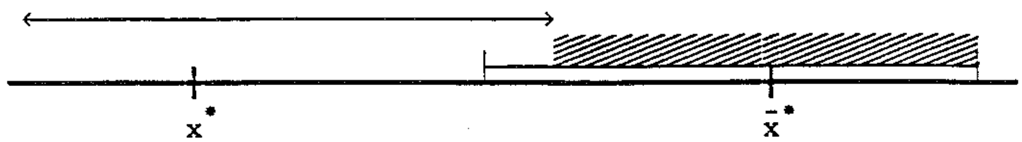
$$\bar{\mathfrak{X}} \equiv \{x \mid \bar{G}(\bar{x}^*) - \bar{G}(x) \leq \Pi(1,\bar{\theta}) - \Pi(0,\bar{\theta}) \text{ and } x > \underline{x}^*\}$$

$$\underline{\mathcal{X}} \equiv \{x \mid \underline{G}(\underline{x}^*) - \underline{G}(x) \le \Pi(1,\underline{\theta}) - \Pi(0,\underline{\theta})\}$$

$\bar{\mathcal{X}}$ is the set of $\bar{x}$ satisfying (16) and $\underline{\mathcal{X}}$ is the set of $\bar{x}$ failing (17).

Proposition 3 is illustrated by Figure 1: the shaded interval (▨) illustrates the set of $\bar{x}$ that can be part of a mechanism offered in some separating equilibrium, the interval (⟷) represents $\underline{\mathcal{X}}$, and the interval (├─┤) represents $\bar{\mathcal{X}}$.

Figure 1



A corollary to Proposition 3 is

Corollary 1: *If $\bar{\mathcal{X}} \subset \underline{\mathcal{X}}$, then no separating equilibrium exists.*

Corollary 1 pertains when beliefs are very important for second-period profits when $\theta = \underline{\theta}$ but not when $\theta = \bar{\theta}$. In such a situation $\bar{\mathcal{X}}$ would be a small interval around $\bar{x}^*$: no matter what beliefs are induced, there is little to be lost from best-separating and, provided $\bar{x}$ is not already near $\bar{x}^*$, much to be gained. $\underline{\mathcal{X}}$, on the other hand, would be a large interval: except for extreme $\bar{x}$, secret pooling is a worthwhile deviation. In such a case, it is possible, that any $\bar{x}$ close enough to $\bar{x}^*$ to make best-separating a losing deviation would also be close enough to $\underline{x}^*$ to make secret pooling a worthwhile deviation.

If beliefs are not important for second-period profits when $\theta = \underline{\theta}$, then $\underline{\mathcal{X}}$ is a small neighborhood around $\underline{x}^*$ and a separating equilibrium exists. In that case, there is little incentive to deviate from the efficient revenue target ($\underline{x}^*$); so provided $\bar{x}$ is both large enough and efficient enough (i.e. near $\bar{x}^*$), separation becomes credible. A summary sufficient condition for existence of a separating PBE is

$$\underline{G}(\underline{x}^*) - \underline{G}(\bar{x}^*) \geq \Pi(1,\underline{\theta}) - \Pi(0,\underline{\theta}) \tag{24}$$

Using (24), we do comparative statics with respect to h:

Corollary 2: *$\bar{\mathfrak{X}}\backslash\underline{\mathfrak{X}}$ is increasing (with respect to the inclusion order) as h increases and $\exists h_s < 1$ such that $\forall h \in (h_s,1]$ $\bar{\mathfrak{X}}\backslash\underline{\mathfrak{X}} \neq \emptyset$.*

Proof: As (16) and (18) are independent of h, to show that $\bar{\mathfrak{X}}\backslash\underline{\mathfrak{X}}$ is increasing, we need only show that $\underline{\mathfrak{X}}$ is decreasing as h is increasing:

$$\frac{d}{dh}\left[\underline{G}(\underline{x}^*) - \underline{G}(\bar{x})\right] = \frac{1}{(1-h)^2}\left[\left(\phi(\bar{x}-\underline{\theta}) - \phi(\bar{x}-\bar{\theta})\right) - \left(\phi(\underline{x}^*-\underline{\theta}) - \phi(\underline{x}^*-\bar{\theta})\right)\right] \tag{25}$$

by the envelope theorem. By $\underline{A}_1$, (25) is positive; hence $\underline{\mathfrak{X}}$ is decreasing. Now take $\bar{x} = \bar{x}^*$. From (5d), $\underline{G}(\underline{x}^*) \to \underline{\theta}$ as $h \to 1$, while $\underline{G}(\bar{x}^*) \to -\infty$ as $h \to 1$. Thus, as the righthand side of (24) is independent of h, there either must exist an $h_s > 0$ such that (24) is an equality or (24) must be met for all h. The rest of the result follows from (25).

∎

Corollary 2 states that for h sufficiently large, separating equilibria must exist. When h is large, there is little to be gained in terms of expected second-period profits by secret pooling (i.e. the gain $(1-h)\left(\Pi(1,\underline{\theta}) - \Pi(0,\underline{\theta})\right)$ is small). Furthermore, there is much to be lost, as almost the entire effect of raising $\underline{x}$ is to increase the $\bar{\theta}$-type's informational rent.

We conclude this sub-section by comparing the results obtained here with those obtained in Section 3. Unlike Section 3, m^* is *not* the only mechanism that can be part of a separating equilibrium. In Section 3, because T saw m, as well as x, it was possible for P to insure what beliefs T would hold in the second period. Here, as T sees only x, it is no longer possible to fix T's beliefs. Thus T is free to hold any beliefs he wishes about $\bar{x}^*$ if it is *not* offered in equilibrium; in particular, T can hold "bad" beliefs about out-of-equilibrium x's (e.g.

$\mu[\bar{x}^*] = 0)$.[11] Consequently, deviating to m^* (best-separating) may be unwise for P because, through T's beliefs, it will result in lower second-period profits.

Furthermore, there may be no separating equilibrium in which m^* is offered, yet there can exist m around which separating equilibria can be constructed (if $\bar{x}^* \in \underline{\mathfrak{X}}$ but $\bar{\mathfrak{X}} \setminus \underline{\mathfrak{X}} \neq \emptyset$). This situation can arise because, unlike in Section 3, P cannot commit not to secretly pool. In comparison, in Section 3, merely offering m^* is proof that P is not inducing pooling.

4b/ Pooling Equilibria

Now we concentrate on pooling equilibria: P proposes a feasible pooling mechanism $m = \left((x_p,w_p),(x_p,w_p)\right)$. As (2b) is binding, we simply denote such a mechanism as x_p.

As the mechanism is pooling, consistent beliefs mean $\mu[x_p] = h$, the prior. Thus, in equilibrium, the expected profit from offering x_p is

$$h\bar{G}(x_p) + (1-h)\underline{G}(x_p) + h\Pi(h,\bar{\theta}) + (1-h)\Pi(h,\underline{\theta}) \qquad (26)$$

In determining whether x_p can be supported as part of some pooling equilibrium, we need to distinguish three cases: $x_p < \underline{x}^*$, $\underline{x}^* \le x_p \le \bar{x}^*$, and $\bar{x}^* < x_p$.

Our first existence result is

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11 Admittedly, some "bad" beliefs could be considered unreasonable. For example, one could consider it unreasonable that $\mu[\bar{x}] = 1$, $\bar{x} < \bar{x}^*$, for $\bar{x}$ on the equilibrium path, while $\mu[\bar{x}^*] = 0$ off the equilibrium path. However to eliminate such "unreasonable" beliefs (and thereby the equilibria they support), one needs to employ a refinement. We suggest one such refinement in Appendix 3 (see also discussion in Section 6b).

Proposition 4: *Under $\underline{A}_1$-$\underline{A}_3$, let $m = x_p$, where $\underline{x}^* \leq x_p \leq \bar{x}^*$. There exists a pooling PBE in which m is offered if and only if*

$$\bar{G}(\bar{x}^*) - \bar{G}(x_p) \leq \Pi(h,\bar{\theta}) - \Pi(0,\bar{\theta}) \tag{27}$$

and

$$\underline{G}(\underline{x}^*) - \underline{G}(x_p) \leq \Pi(h,\underline{\theta}) - \Pi(0,\underline{\theta}) \tag{28}$$

Proof: *Necessity.* The necessity of (27): consider the deviation $(\bar{x}^*, x_p)$, which yields profits

$$h\bar{G}(\bar{x}^*) + (1-h)\underline{G}(x_p) + h\Pi(\mu[\bar{x}^*],\bar{\theta}) + (1-h)\Pi(h,\underline{\theta}) \tag{29}$$

(27) is the necessary condition for there to exist beliefs $\mu[\bar{x}^*]$ such that (29) does not exceed (26). The necessity of (28): consider the deviation $(x_p, \underline{x}^*)$, which yields profits

$$h\bar{G}(x_p) + (1-h)\underline{G}(\underline{x}^*) + h\Pi(h,\bar{\theta}) + (1-h)\Pi(\mu[\underline{x}^*],\underline{\theta}) \tag{30}$$

(28) is the necessary condition for there to exist beliefs $\mu[\underline{x}^*]$ such that (30) does not exceed (26).

Sufficiency. Given (27) and (28), the following is a pooling PBE: P proposes x_p and T holds beliefs $\mu[x_p] = h$ and $\mu[x] = 0 \ \forall x \neq x_p$. Clearly T's beliefs satisfy Bayesian consistency. P's profits under x_p are given by (26). Consider a deviation $(\bar{x}', \underline{x}')$ where $\bar{x}' \neq x_p$ *or* $\underline{x}' \neq x_p$ *or* both; such a deviation yields profits:

$$h\bar{G}(\bar{x}') + (1-h)\underline{G}(\underline{x}') + h\Pi(\mu[\bar{x}'],\bar{\theta}) + (1-h)\Pi(\mu[\underline{x}'],\underline{\theta}) \tag{31}$$

Subtracting (31) from (26) yields

$$h\left[\left(\bar{G}(x_p) - \bar{G}(\bar{x}')\right) + \left(\Pi(h,\bar{\theta}) - \Pi(\mu[\bar{x}'],\bar{\theta})\right)\right]$$
$$+ (1-h)\left[\left(\underline{G}(x_p) - \underline{G}(\underline{x}')\right) + \left(\Pi(h,\underline{\theta}) - \Pi(\mu[\underline{x}'],\underline{\theta})\right)\right] \tag{32}$$

Given μ and (27), the top line of (32) is minimized by $\bar{x}' = x_p$, while given μ and (28), the bottom line of (32) is minimized by $\underline{x}' = x_p$. Thus x_p is a best response.

∎

Potentially the most desirable deviations from x_p involve *secret separating*. P risks lower second-period profits in exchange for having $\bar{\theta}$ and/or $\underline{\theta}$ produce her (their) first-period profit-maximizing revenue(s). Conditions (27) and (28) establish that such deviations (or combinations of such deviations) are losing deviations.

Our last two existence results:

Proposition 5: *Under* $\underline{A}_1$-$\underline{A}_3$, *let* $m = x_p$, *where* $x_p < \underline{x}^*$. *There exists a pooling PBE in which m is offered if and only if (27) holds and*

$$h\left(\bar{G}(\bar{x}^*) - \bar{G}(x_p)\right) + (1-h)\left(\underline{G}(\underline{x}^*) - \underline{G}(x_p)\right) \le \mathbb{E}_\theta\left\{\Pi(h,\theta) - \Pi(0,\theta)\right\} \quad (33)$$

Proof (sketch): *Necessity.* The necessity of (27) was shown in the proof of Proposition 4. The necessity of (33): consider the deviation $(\bar{x}^*,\underline{x}^*)$; (33) guarantees that there exist beliefs $\mu[\bar{x}^*]$ and $\mu[\underline{x}^*]$ which make that deviation unprofitable.

Sufficiency. Follows the argument used in the proof of Proposition 4; the only difference is to note (a) that deviations of the form $(x_p,\underline{x})$, $\underline{x} < x_p$, are losing deviations since $\underline{x} < x_p < \underline{x}^*$ and hence $\underline{G}(\underline{x}) < \underline{G}(x_p)$; and (b) that if the $\underline{\theta}$-type is required to produce $x > x_p$, then, by (IC), the $\bar{\theta}$-type must also be required to produce $x > x_p$; however as the best such deviation $((\bar{x}^*,\underline{x}^*))$ produces lower profits than m, no such deviation will be made.

■

Proposition 6: *Under* $\underline{A}_1$-$\underline{A}_3$, *let* $m = x_p$, *where* $x_p > \bar{x}^*$. *There exists a pooling PBE in which m is offered if and only if (28) and (33) hold.*

Proof: As the proof is similar to those of Propositions 4 and 5, we leave it to the reader.

■

As a consequence of incentive compatibility, when $x_p < \underline{x}^*$ or $x_p > \bar{x}^*$, it is not always possible to have a given type of manager deviate by producing her starred revenue while the other type continues to produce x_p. For example, if $x_p < \underline{x}^*$, then deviations of the form $(x_p, \underline{x})$, $\underline{x} > x_p$, are impossible by incentive compatibility. Thus "joint" deviations must be considered: both types produce levels of output other than x_p. As the best such "joint" deviation is $(\bar{x}^*, \underline{x}^*)$ (at least when $\mu[x] = 0$ for $x \neq x_p$), we must compare the gain in first-period profits from that deviation with the loss in second-period profits.

It is worth noting that (27) and (28) imply (33). Consequently the sufficiency part of Proposition 4 can be extended to cover $x \in \mathbb{R}_+$:

Corollary 3: *Under $\underline{A}_1$-$\underline{A}_3$, (27) and (28) are sufficient conditions for there to exist a pooling equilibrium in which x_p is offered. Moreover, a pooling PBE exists if*

$$\bar{G}(\bar{x}^*) - \bar{G}(\underline{x}^*) \le \Pi(h,\bar{\theta}) - \Pi(0,\bar{\theta})$$

or if

$$\underline{G}(\underline{x}^*) - \underline{G}(\bar{x}^*) \le \Pi(h,\underline{\theta}) - \Pi(0,\underline{\theta})$$

Proof: Follows from the argument in the text and Propositions 4-6.

■

Define

$$\bar{\mathcal{P}} = \{x \mid \bar{G}(\bar{x}^*) - \bar{G}(x) \le \Pi(h,\bar{\theta}) - \Pi(0,\bar{\theta})\}$$

$$\underline{\mathcal{P}} = \{x \mid \underline{G}(\underline{x}^*) - \underline{G}(x) \le \Pi(h,\underline{\theta}) - \Pi(0,\underline{\theta})\}$$

$$\mathcal{P} = \bar{\mathcal{P}} \cap \underline{\mathcal{P}}$$

$\bar{\mathcal{P}}$ is the set of x satisfying (27), $\underline{\mathcal{P}}$ is the set of x satisfying (28), and $\mathcal{P}$ is the set satisfying both. Using this new notation we have

Proposition 7: Under $\underline{A}_1$-$\underline{A}_3$, the set of pooling mechanisms sustainable in a pooling PBE is convex, equal to $\mathcal{P}$ when $\mathcal{P} \subset (\underline{x}^*, \bar{x}^*)$, and empty if and only if $\mathcal{P} = \emptyset$.

Proposition 7 is proved in Appendix 2 and illustrated in Figures 2a and 2b respectively. In Figure 2a, the shaded interval (▨) illustrates the set of x which can be offered in some pooling equilibrium. Note, as drawn, the shaded area equals $\mathcal{P}$. Figure 2b illustrates a case with no pooling equilibria. In both figures the interval (⟷) represents $\underline{\mathcal{P}}$ and the interval (├┤) represents $\bar{\mathcal{P}}$.

Figure 2a

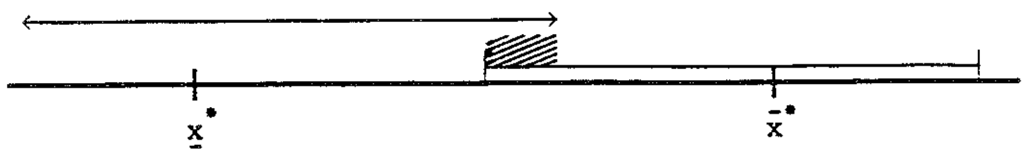
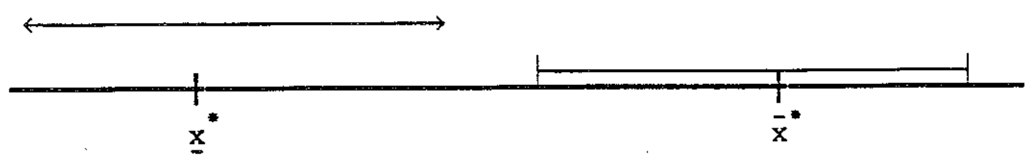


Figure 2b



A relationship between pooling equilibria and separating equilibria exists from the following:

$$\bar{\mathcal{P}} \subset \bar{\mathcal{X}} \text{ and } \underline{\mathcal{P}} \subset \underline{\mathcal{X}}.$$

Consequently

Proposition 8: *Under $\underline{A}_1$-$\underline{A}_3$, if $(\bar{x}, \underline{x}^*)$ can be offered in a separating equilibrium, $(\bar{x}, \bar{x})$ cannot be offered in a pooling equilibrium; if (x,x) can be offered in a pooling equilibrium, then $(x, \underline{x}^*)$ cannot be offered in a separating equilibrium.*

Proof: If $(\bar{x},\underline{x}^*)$ can be offered in a separating equilibrium, $\bar{x} \notin \underline{\mathfrak{X}}$. Hence $\bar{x} \notin \underline{\mathcal{P}}$. The second part of the proposition is just the contrapositive of the first part.

∎

Note Proposition 8 does *not* say that for any x we can construct either a pooling equilibrium in which (x,x) is offered or a separating equilibrium in which $(x,\underline{x}^*)$ is offered. Indeed, it is possible that neither separating, nor pooling, equilibria exist: if the second-period profits from inducing beliefs h are not sufficient to prevent secret-separating deviations, but if, for the $\underline{\theta}$-type, the second-period profits from inducing beliefs 1 are not small enough to prevent secret pooling, then neither pooling nor separating equilibria will exist.

The next proposition presents comparative statics results on h. The proof can be found in Appendix 2.

Proposition 9: *Under $\underline{A}_1$-$\underline{A}_3$, there exists an $\underline{h}_p > 0$ such that for any h $\in (0,\underline{h}_p)$ no pooling equilibrium exists. If (34) below holds, then there exists an $\bar{h}_p \in (0,1)$ such that for any $h \in (\bar{h}_p,1)$ no pooling exists.*

$$\bar{G}(\bar{x}^*) - \bar{G}(\underline{\theta}) > \Pi(1,\bar{\theta}) - \Pi(0,\bar{\theta}) \tag{34}$$

If (34) is reversed (strictly), then for all intervals (H,1), H ≥ 0, ∃h ∈ (H,1) such that a pooling equilibrium exists.

The first part of Proposition 9 formalizes the intuition that when bad types are very likely, there is little to be gained from appearing average rather than bad. Consequently pooling equilibria cannot exist: restoring first-period efficiency more than compensates for the small loss in second-period profits. The second part of Proposition 9 pertains when $\theta = \underline{\theta}$ is very unlikely. In that case, for any mechanism

$(\bar{x},\underline{x})$, raising $\underline{x}$ essentially just raises the informational rent earned by the good type. Hence pooling at x_P much greater than $\underline{\theta}$ is not sustainable for large h, as P would secretly separate the bad type. Thus the only candidates for pooling equilibria for large h are x_P near $\underline{\theta}$ and they will be sustainable in a pooling equilibrium if and only if they are in $\bar{\mathcal{P}}$; which is to say only if (34) is reversed.

We conclude this section by comparing the pooling equilibria under the private negotiation and private contract assumption with the pooling equilibrium under the public negotiation and public contract assumption (Section 3). Under the public/public assumption, the only possible pooling equilibrium involved P offering m_0. Here, there may exist a multitude of pooling equilibria. Furthermore m_0 may not be one of the pooling equilibria (e.g. if $\underline{x}^{FB} \notin \bar{\mathcal{P}}$). As was the case with separating equilibria, the difference between the two sets of results follows from P's inability to communicate (credibly) information to T. As T is free to hold any out-of-equilibrium beliefs, deviating with m_0 need not induce beliefs μ = h. Furthermore, as P cannot commit not to secretly separate, beliefs $\mu[x_0]$ = h can easily not be equilibrium beliefs.

4c/ Hybrid Equilibria

We noted in the previous sub-section that it was possible to have neither separating, nor pooling, equilibria exist. In such a situation, the equilibria will be *hybrid equilibria*: P plays a mixed strategy over offering pooling mechanisms and offering separating mechanisms.

For the sake of brevity, we merely show existence of a hybrid equilibrium when neither separating, nor pooling, equilibria exist.

In equilibrium, if P offers $(\bar{x},\underline{x}^*)$ with probability q and $(\bar{x},\bar{x})$

with probability 1-q, then $\mu[\underline{x}^*] = 0$ and

$$\mu[\bar{x}] = \frac{h}{(1-q)(1-h) + h} \tag{35}$$

by Bayes' Law. Note $\mu[\bar{x}] \in [h,1]$. Solving (35) for q:

$$q = \frac{\mu-h}{\mu(1-h)} \tag{36}$$

Our existence result:[12]

Proposition 10: *Under $\underline{A}_1$-$\underline{A}_3$, if no pooling nor separating equilibria exist, then there exists a hybrid equilibrium in which both $(\bar{x}^*,\underline{x}^*)$ (= m^*) and $(\bar{x}^*,\bar{x}^*)$ are offered with positive probability by P.*

Proof: Clearly $\bar{x}^* \in \bar{\mathfrak{X}}$ and $\bar{x}^* \in \bar{\mathcal{P}}$. Consequently, from Proposition 7, $\bar{x}^* \notin \underline{\mathcal{P}}$ and, from Proposition 3, $\bar{x}^* \in \underline{\mathfrak{X}}$. Hence

$$\underline{G}(\underline{x}^*) - \underline{G}(\bar{x}^*) > \Pi(h,\underline{\theta}) - \Pi(0,\underline{\theta})$$

and

$$\underline{G}(\underline{x}^*) - \underline{G}(\bar{x}^*) < \Pi(1,\underline{\theta}) - \Pi(0,\underline{\theta})$$

Therefore, $\exists\mu^*$ such that

$$\underline{G}(\underline{x}^*) - \underline{G}(\bar{x}^*) = \Pi(\mu^*,\underline{\theta}) - \Pi(0,\underline{\theta}) \tag{37}$$

Let q^* be derived from μ^* by (36). As $\mu^* \in (h,1)$, $q^* \in (0,1)$. Now the following is a PBE: P proposes m^* with probability q^* and $(\bar{x}^*,\bar{x}^*)$ with probability $(1-q^*)$ and T holds beliefs $\mu[\bar{x}^*] = \mu^*$ and $\mu[x] = 0$ $\forall x \neq \bar{x}^*$. Clearly T's beliefs satisfy Bayesian consistency. P's expected profits under m^* are

$$h\bar{G}(\bar{x}^*) + (1-h)\underline{G}(\underline{x}^*) + h\Pi(\mu^*,\bar{\theta}) + (1-h)\Pi(0,\underline{\theta}) \tag{38}$$

P's expected profits under $(\bar{x}^*,\bar{x}^*)$ are

$$h\bar{G}(\bar{x}^*) + (1-h)\underline{G}(\bar{x}^*) + h\Pi(\mu^*,\bar{\theta}) + (1-h)\Pi(\mu^*,\underline{\theta}) \tag{39}$$

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[12] This is only an existence result, not a uniqueness result. In general there may exist a multiplicity of hybrid equilibria.

From (37), (38) equals (39), so P is willing to play a mixed strategy. From (38), any deviation $(\bar{x},\underline{x})$ ($\bar{x} \neq \bar{x}^*$, $\underline{x} \neq \underline{x}^*$, or $\underline{x} \neq \bar{x}^*$) results in both lower first-period profits and, given T's beliefs, lower second-period profits than m^*. Thus P's strategy is a best response.

∎

5/ PRIVATE NEGOTIATIONS BUT PUBLIC CONTRACTS

Now assume T does not observe the mechanism offered by P during negotiations. However T does observe the contract actually signed between P and M, i.e. (x,w).

As in the case of private contracts, T's inference problem is not trivial. T's beliefs must be defined for each $(x,w) \in \mathbb{R}^2_+$. It will generally be true for contracts (x,w) not part of an equilibrium mechanism that Bayes' Law cannot be applied, i.e. $\mu[(x,w)]$ is not fixed by the equilibrium strategies. For example, T can ignore his observation of the wage (i.e. $\forall w,w'$ $\mu[(x,w)] = \mu[(x,w')]$); thus, for a given set of parameters, all the equilibria found in Section 4 can be supported as equilibria in this case as well. Moreover, the freedom to specify beliefs as a function of w, as well as x, expands the set of equilibria. A complete characterization of this set would add little to the analysis, and we have therefore chosen not to include it.

We instead focus on the situation in which T's beliefs satisfy the following condition:

$\underline{A}_4$: $\mu[(x,w)] = 1$, if

$$w - \phi(x - \bar{\theta}) \geq 0 > w - \phi(x - \underline{\theta}) \tag{40}$$

The justification for this restriction on beliefs is as follows: choosing a contract satisfying (40) would be a mistake for the $\underline{\theta}$-type, as she would have done better simply by quitting. Thus, on-the-equilibrium path $\mu[(x,w)] = 1$. Off-the-equilibrium path, observation of (x,w) is evidence that P deviated (or made a mistake), thus for T to hold beliefs $\mu[(x,w)] \neq 1$, would require T to hypothesize that deviations (mistakes) by P will be followed by mistakes by the $\underline{\theta}$-type. We find it difficult to justify requiring T to hypothesize sequential mistakes by P *and* the $\underline{\theta}$-type. Certainly such a requirement violates the spirit (though not, it should be made clear, the letter) of our solution concept.

We devote the rest of this section to studying the existence of separating and pooling equilibria under $\underline{A}_4$. As we will show, $\underline{A}_4$ yields strong predictions. For instance, consider separating equilibria:

Proposition 11: *Under $\underline{A}_1$-$\underline{A}_4$, there exists one and only one separating PBE (in terms of the path) and $\left[(\bar{x}^*,\bar{w}^*),(\underline{x}^*,\underline{w}^*)\right]$ is the mechanism offered in that PBE.*

Proof: *Existence.* Suppose P offers $\left[(\bar{x}^*,\bar{w}^*),(\underline{x}^*,\underline{w}^*)\right]$ and T holds beliefs $\mu[(x,w)] = 1$, if (x,w) satisfies (40), and $\mu[(x,w)] = 0$ otherwise. T's beliefs satisfy Bayesian consistency. Given T's beliefs and the fact that $\left[(\bar{x}^*,\bar{w}^*),(\underline{x}^*,\underline{w}^*)\right]$ maximizes first-period profits, there is no profitable deviation.

Uniqueness. Let $\left[(\bar{x},\bar{w}),(\underline{x},\underline{w})\right]$ be another separating mechanism offered in equilibrium. In this PBE, P's expected payoff is

$$h(\bar{x} - \bar{w}) + (1-h)(\underline{x} - \underline{w}) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(0,\underline{\theta})$$

Consider the deviation $\left[(\bar{x}^*,\bar{w}^*),(\underline{x}^*,\underline{w}^*)\right]$: As $(\bar{x}^*,\bar{w}^*)$ satisfies (40), $\mu[(\bar{x}^*,\bar{w}^*)] = 1$, so the expected profits from deviating are

$$h(\bar{x}^* - \bar{w}^*) + (1-h)(\underline{x}^* - \underline{w}^*) + h\Pi(1,\bar{\theta}) + (1-h)\Pi\left(\mu[(\underline{x}^*,\underline{w}^*)],\underline{\theta}\right)$$

Clearly, this a profitable deviation; by contradiction we have proved uniqueness.

∎

$\underline{A}_4$ fixes beliefs at $(\bar{x}^*,\bar{w}^*)$ to be 1, thus, as in the public/public case, offering m^* generates the best beliefs within the class of separating mechanisms. Therefore, as m^* maximizes first-period profits, only m^* is sustainable as a separating equilibrium.

We now consider pooling equilibria. Profits in a pooling equilibrium in which P offers $m_p = \left((x_p,w_p),(x_p,w_p)\right) \equiv (c_p,c_p)$ are

$$x_p - w_p + h\Pi(h,\bar{\theta}) + (1-h)\Pi(h,\underline{\theta})$$

There are four possible deviations to consider: 1) offering m^* (*best separating*); 2) offering $(\bar{c},c_p)$ where $\bar{c}$ and c_p satisfy (1a) and $\bar{c}$ fails (40) (*covert separation of the* $\bar{\theta}$*-type*); 3) offering $(c_p,\underline{c})$ where c_p and $\underline{c}$ satisfy (1b) (*covert separation of the* $\underline{\theta}$*-type*); and 4) offering $(\bar{c},c_p)$ where $\bar{c}$ and c_p satisfy (1a) and $\bar{c}$ satisfies (40) (*overt separation of the* $\bar{\theta}$*-type*). As the intuition behind the first three deviations is similar to that previously given (there is a close relationship between these deviations and conditions (33), (27), and (28) respectively), we will omit further discussion of those deviations here. Instead we focus on the fourth deviation. Intuition for what follows can be gained from

Lemma 2: *If* $\Pi(1,\bar{\theta}) > \Pi(\mu,\bar{\theta})$ $\forall\mu < 1$, *then no pooling mechanism, in which*

$$w_p - \phi(x_p - \underline{\theta}) = 0$$

is offered with positive probability in equilibrium.

Proof: Suppose not; P's expected equilibrium payoff is

$$x_p - w_p + h\Pi(\mu[c_p],\bar{\theta}) + (1-h)\Pi(\mu[c_p],\underline{\theta}) \tag{41}$$

where $c_p = (x_p,w_p)$ (and $m_p = (c_p,c_p)$). As m_p is offered with positive probability, $\mu[c_p] < 1$. [As c_p may be part of another mechanism offered with positive probability, $\mu[c_p]$ need not equal h (the prior).] Consider other mechanisms of the form $m(\varepsilon) = (\bar{c}(\varepsilon), c_p)$, where

$$\bar{c}(\varepsilon) = \left(x_p + \varepsilon,\ w_p + \phi(x_p+\varepsilon-\bar{\theta}) - \phi(x_p-\bar{\theta})\right)$$

$(\varepsilon > 0)$. Note that $m(\varepsilon)$ is feasible and that $\bar{c}(\varepsilon)$ satisfies (40). Thus P's expected payoff under $m(\varepsilon)$ is

$$x_p - w_p + h\left(\varepsilon - \phi(x_p+\varepsilon-\bar{\theta}) + \phi(x_p-\bar{\theta})\right) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(\mu[c_p],\underline{\theta}) \qquad (42)$$

As $\phi(\cdot)$ is continuous and $\mu[c_p] < 1$, there exists an ε such that (42) is strictly greater than (41). Thus m_p cannot be offered with positive probability in equilibrium.

■

Intuitively, unless some rent is left to the *bad* type, P would deviate from m_p by offering a mechanism that induced the good type to produce slightly more than x_p, but under a contract which the bad type would never choose. Under this deviation, first-period profits would be essentially the same as under m_p; but as the good type is unambiguously identified, second-period profits are strictly greater.

If, in contrast to Lemma 2, some rent is left to the bad type, then deviations of the form $(\bar{c}, c_p)$, where $\bar{c}$ satisfies (40), become more expensive for P: $\bar{c}$ must induce very different first-period profits from the good type. If those first-period profits are sufficiently less than under m_p, P will not wish to deviate in this way. Formally, $\bar{c}$ must satisfy (40) and be feasible. Given (40), feasibility reduces to (1a). Furthermore, as we need consider only optimal deviations, we can restrict attention to $\bar{c}$ which satisfy (1a) with equality:

$$\bar{w} = \phi(\bar{x} - \bar{\theta}) + w_p - \phi(x_p - \bar{\theta}) \equiv W(x, c_p) \qquad (43)$$

Let

$$Q(m_p) = \left\{x \,\middle|\, \left(x, W(x,c_p)\right) \text{ satisfies } (40)\right\}$$

$Q(m_p)$ is the set of x on the portion of the $\bar{\theta}$-type's indifference curve through c_p which lies below the $\underline{\theta}$-type's IR constraint (see Figure 3).

Now if m_p can be offered in a pooling PBE, then it must be that

$$x_p - w_p + h\Pi(h,\bar{\theta}) + (1-h)\Pi(h,\underline{\theta}) \geq$$

$$h\left(\bar{x} - \bar{w}\right) + (1-h)\left(x_p - w_p\right) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(h,\underline{\theta}) \tag{44}$$

for any overt deviation (i.e. any $(\bar{x},\bar{w})$ satisfying (40) and (1a)). In particular, if (43) holds, condition (44) can be rewritten as

$$\forall \bar{x} \in Q(m): \quad \bar{G}(x_p) - \bar{G}(\bar{x}) \geq \Pi(1,\bar{\theta}) - \Pi(h,\bar{\theta}) \tag{45}$$

From (45), if $\bar{x}^* \in Q(m_p)$, then (44) fails: clearly if having the $\bar{\theta}$-type produce her first-best output is incentive compatible given c_p and if that identifies her as the $\bar{\theta}$-type, then m_p cannot be sustained as a pooling equilibrium. Thus a necessary condition for m_p to be offered in a pooling equilibrium is

$$\bar{x}^* \notin Q(m_p) \tag{46}$$

If (46) is met, any $\bar{x}$ in $Q(m_p)$ must be larger than $\bar{x}^*$. From $\underline{A}_1$ and the convexity of $Q(m_p)$, (45) must hold for $\bar{x} = X(c_p) \equiv \inf Q(m_p)$ by continuity. As in Lemma 2, the intuition can be understood by considering deviations $c(\varepsilon) = \left(x(\varepsilon),w(\varepsilon)\right)$, where $x(\varepsilon) = X(c_p) + \varepsilon$. Thus, if (46) is met, (45) becomes

$$\bar{G}(x_p) - \bar{G}\left(X(c_p)\right) \geq \Pi(1,\bar{\theta}) - \Pi(h,\bar{\theta}) \tag{47}$$

The losses in first-period profits from any deviation $c(\varepsilon)$ must exceed the gains in second-period profits. A necessary condition for $m_p = (c_p,c_p)$ to be offered in a pooling PBE is that (46) and (47) hold. We need only retain (47) as a necessary condition, as (47) implies (46). [Proof: if (46) were not true, then $x_p < X(c_p) < \bar{x}^*$ ($x_p < X(c_p)$, since c_p is individually rational for $\underline{\theta}$); but then $\bar{G}(x_p) < \bar{G}\left(X(c_p)\right)$, contradicting (47).]

Figure 3

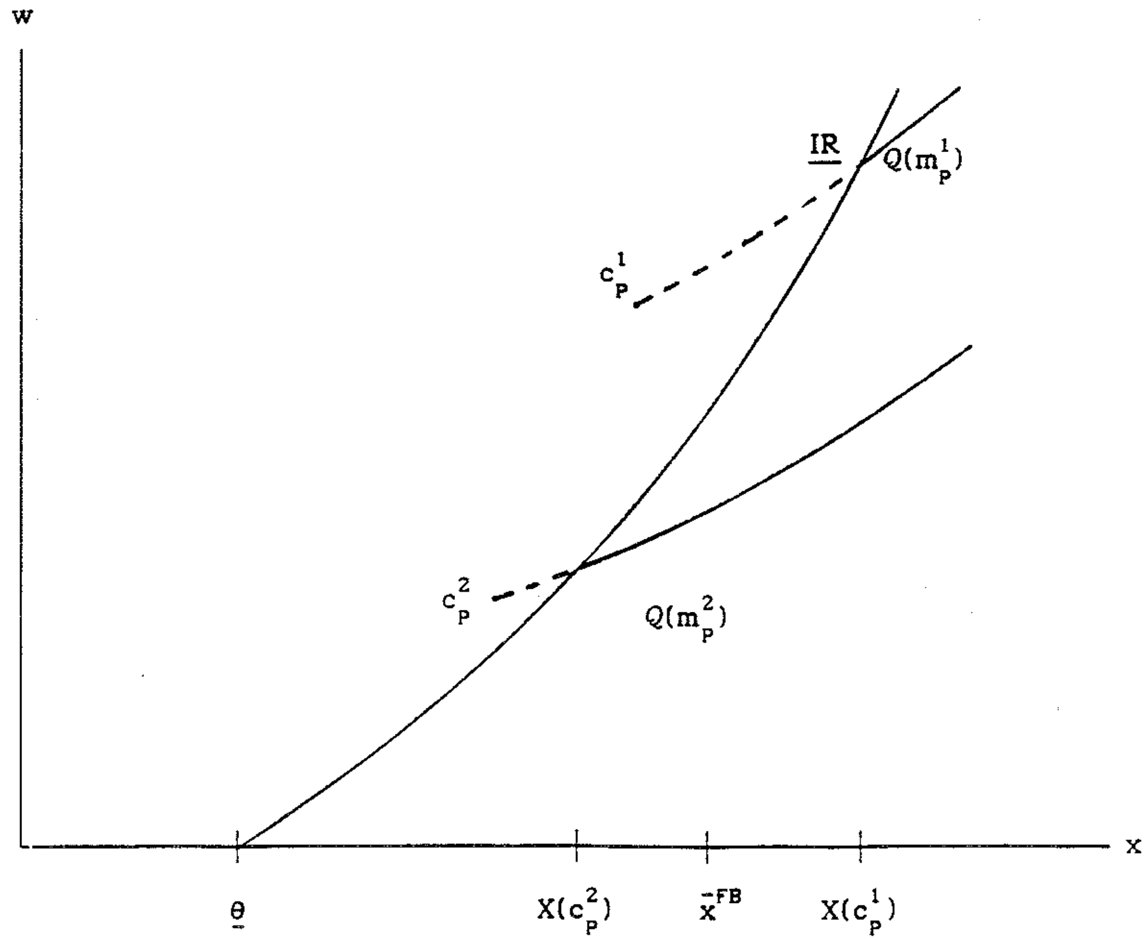


Figure 4 illustrates condition (47). Condition (47) defines a set of contracts (x_p, w_p) which are above the $\underline{\theta}$-type's IR curve and above the $\bar{\theta}$-type's indifference curve through $\left(\bar{x}', \phi(\bar{x}'-\underline{\theta})\right)$, where $\bar{x}'$ is defined by

$$\bar{G}(\bar{x}') = \bar{G}(\bar{x}^*) - \Pi(1,\bar{\theta}) + \Pi(h,\bar{\theta})$$

So the good manager's rent must be at least $\phi(\bar{x}'-\underline{\theta}) - \phi(\bar{x}'-\bar{\theta})$.

We now state our main result concerning pooling PBE:

Proposition 12: *Given $\underline{A}_1$-$\underline{A}_4$, a pooling mechanism m_p can be offered in a pooling PBE if and only if (x_p, w_p) satisfies (47), (48), (49), and (50).*

$$x_p - w_p \geq$$
$$h\bar{G}(\bar{x}^*) + (1-h)\underline{G}(\underline{x}^*) + h\left(\Pi(1,\bar{\theta})-\Pi(h,\bar{\theta})\right) + (1-h)\left(\Pi(0,\underline{\theta})-\Pi(h,\underline{\theta})\right) \quad (48)$$

$$x_p \geq \bar{x}^* \text{ or } \bar{G}(\bar{x}^*) - \bar{G}(x_p) \leq \Pi(h,\bar{\theta}) - \Pi(0,\bar{\theta}) \quad (49)$$

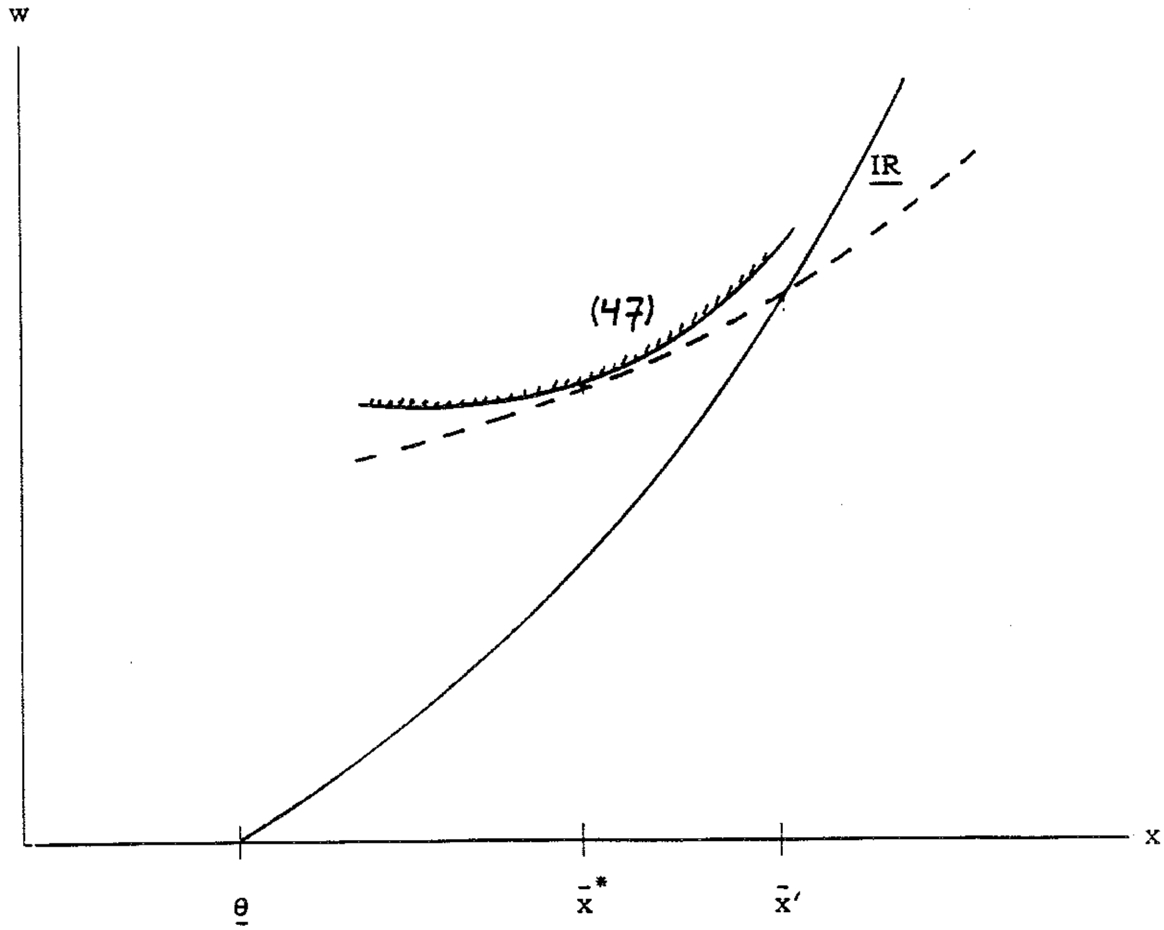
$$\underline{x}^{FB} \geq x_p \text{ or } \left(\underline{x}^{FB} - \phi(\underline{x}^{FB}-\underline{\theta})\right) - \left(x_p - \phi(x_p-\underline{\theta})\right) \leq \Pi(h,\underline{\theta}) - \Pi(0,\underline{\theta}) \quad (50)$$

Proof (sketch): The necessity of (47) has been proved in the text. (48) is necessary to rule out the best separating deviation (deviation 1), while (49) and (50) rule out the covert separating deviations (deviations 2 and 3 respectively). To prove sufficiency, we will show that it is a PBE for P to offer m_p, where (x_p, w_p) satisfies (47) - (50) and when T holds the following beliefs

$$\mu[c] = \begin{cases} h, & \text{if } c = c_p \\ 1, & \text{if } c \text{ satisfies (40)} \\ 0, & \text{if } c \neq c_p \text{ and } c \text{ fails (40)} \end{cases}$$

Clearly T's beliefs satisfy Bayesian consistency (as well as $\underline{A}_4$). Given T's beliefs, any deviation of the form $(\bar{c}, \underline{c})$, $\bar{c} \neq c_p$ and $\underline{c} \neq c_p$, is dominated by m_p from (48) ((48) establishes that m_p dominates m^* which is the best deviation in that class). Similarly, overt-separating deviations are dominated by m_p from (47); and covert-separating

Figure 4



deviations are dominated by m_P from either (49) or (50).

■

The conclusion of Proposition 12 can be summarized as follows: if P has an incentive *ex post* to reveal a good type, then pooling equilibria are necessarily very costly for P because of the rent left to $\bar{\theta}$-types *and* to $\underline{\theta}$-types. Pooling is more likely to exist if it is more important *ex post* for P to conceal a bad type than to reveal a good type. If *ex post* for some type, P does not much care how he looks to T, then pooling has to require almost efficient production from this type.

6/ EXAMPLES

In this section, we present two examples to illustrate the economic content of our results. The examples deal with entry: P owns an incumbent monopoly, M is his manager, and T is a potential entrant.

The two examples are very simple; particularly, as we assume that for one value of θ, P's second-period profits are independent of T's beliefs. We adopt this extreme assumption to facilitate intuition. In this sense, the examples are more illustrative than realistic.

6a/ Entry Deterrence with Very Efficient Firms

θ is the firm's productivity and can be complemented in the first period by managerial effort e. Second-period profits are $\pi_P^M(\theta)$ if P remains a monopolist and $\pi_P^D(\theta)$ if entry occurs. T's profits are $\pi_T(\theta) - \sigma$, if he enters, and 0 otherwise. σ is the fixed cost of entry. From P's perspective, σ is a random variable with continuous distribution F(•) on $\mathbb{R}_+$. For an efficient firm ($\theta = \bar{\theta}$), the monopoly price is less than the entrant's unit cost: $\pi_P^M(\bar{\theta}) = \pi_P^D(\bar{\theta})$ and

$\pi_T(\bar{\theta}) = 0$. For an inefficient firm, $\pi_P^M(\underline{\theta}) > \pi_P^D(\underline{\theta})$ and $\pi_T(\underline{\theta}) > 0$.

A potential entrant compares $(1-\mu)\pi_T(\underline{\theta})$ to σ; where μ is T's posterior probability assessment that $\theta = \bar{\theta}$. Thus entry occurs with probability $F\left((1-\mu)\pi_T(\underline{\theta})\right)$ (from P's perspective). We thus have

$$\Pi(\mu,\bar{\theta}) = \pi_P^M(\bar{\theta})$$

$$\Pi(\mu,\underline{\theta}) = F\left((1-\mu)\pi_T(\underline{\theta})\right)\pi_P^D(\underline{\theta}) + \left[1 - F\left((1-\mu)\pi_T(\underline{\theta})\right)\right]\pi_P^M(\underline{\theta})$$

Note $\Pi(\mu,\underline{\theta})$ is increasing in μ, so assumption $\underline{A}_3$ is satisfied.

The Public Negotiation and Public Contracts Case

A pooling equilibrium at $m_0 = (x_0,x_0)$ will exist if and only if the first-period loss from inefficient production (both types producing $\underline{x}^{FB}$) is less than the second-period loss from revealing that the firm has low productivity (thus increasing the probability of entry). Note pooling will occur at a relatively *low* level of revenue. If pooling does not exist, then the equilibrium will be separating with P offering the optimal mechanism $m^* = (\bar{x}^*,\underline{x}^*)$. If the equilibrium is separating, the entrant is fully informed when deciding whether to enter.

The Private Negotiation and Private Contracts Case

In this case, we have $\bar{\mathcal{X}} = \bar{\mathcal{P}} = \{\bar{x}^*\}$; as beliefs do not matter if $\theta = \bar{\theta}$ (as, in that case, entry does not matter), profit maximization means P must require the $\bar{\theta}$-type to produce her optimal level of revenue (provided that is incentive compatible). Thus the separating equilibrium, if it exists, is unique, with P offering the optimal mechanism m^*. This separating equilibrium exists if and only if inducing a $\underline{\theta}$-type manager to produce a high revenue $\bar{x}^*$ is more costly than the gain from fooling the entrant (making him believe $\theta = \bar{\theta}$).

The only candidates for pooling equilibria are (x,x) where x represents a *high* level of revenue ($x \geq \bar{x}^*$ and $x \in \underline{\mathcal{P}}$). Such a pooling equilibrium exists if and only if inducing a $\underline{\theta}$-type manager to produce such a high revenue generates a loss smaller than the corresponding gain from not revealing $\theta = \underline{\theta}$ (thus deterring entry).

Finally, if neither the separating equilibrium, nor the pooling equilibria exist (i.e. if $\bar{x}^* \in \underline{\mathcal{X}}$, but $\bar{x}^* \notin \underline{\mathcal{P}}$), then it is straightforward to show that there exists a unique hybrid equilibrium in which P mixes between $(\bar{x}^*,\underline{x}^*)$ and $(\bar{x}^*,\bar{x}^*)$.[13]

The Private Negotiation but Public Contracts Case

In this case, we know (Proposition 11) that there exists a unique separating equilibrium in which P proposes m^* (under $\underline{A}_4$). In such an equilibrium, the entrant has full information when deciding to enter.

We also note that Lemma 2 does not hold here, as $\Pi(\mu,\bar{\theta})$ is independent of μ. When the gains from not revealing that $\theta = \underline{\theta}$ are large, there exist pooling equilibria at *high* levels of revenue ($x_p \geq \bar{x}^*$) and high wages ($w_p \geq \phi(\bar{x}^*-\underline{\theta})$).

Summary

We summarize by noting that both pooling and separating equilibria are possible. If the equilibrium is separating, it is unique with the optimal mechanism $m^* = (\bar{x}^*,\underline{x}^*)$ being offered: as the $\underline{\theta}$ type is identified and as P does not care what beliefs he induces when $\theta = \bar{\theta}$,

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[13] Since beliefs do not matter if $\theta = \bar{\theta}$, the only separating mechanism P will offer with positive probability is $(\bar{x}^*,\underline{x}^*)$. A detailed proof is available from the authors.

both types must be required to produce their optimal first-period revenues. In the private negotiation cases, pooling equilibria involve pooling at levels of x greater than (or equal to) $\bar{x}^*$: as P does not care about beliefs when $\theta = \bar{\theta}$, pooling is sustainable only if the best separating deviation (by the $\bar{\theta}$-type) is not incentive compatible. Entry deterrence is achieved by *overprovision* of managerial effort, high first-period revenues, and high managerial rents.

6b/ Entry Deterrence with Inefficient Firms

Same notation as in the previous example. We now assume that $\pi_P^M(\underline{\theta}) < 0$ and $\pi_P^M(\bar{\theta}) > \pi_P^D(\bar{\theta}) > 0$. The assumption $\pi_P^M(\underline{\theta}) < 0$ might apply to declining industries where inefficient firms can no longer cover their fixed costs. Finally let π_T^M be the entrant's gross profits, if he captures the entire market. Now P will exit if $\theta = \underline{\theta}$ (regardless of T's action), so T's expected gross profit from entry is $(1-\mu)\pi_T^M$ and the probability of entry is $F\left((1-\mu)\pi_T^M\right)$ (without loss of generality we continue to assume $\pi_T(\bar{\theta}) = 0$). We thus have

$$\Pi(\mu,\underline{\theta}) \equiv 0$$

$$\Pi(\mu,\bar{\theta}) = F\left((1-\mu)\pi_T^M\right)\pi_P^D(\bar{\theta}) + \left[1 - F\left((1-\mu)\pi_T^M\right)\right]\pi_P^M(\bar{\theta})$$

$\Pi(\mu,\bar{\theta})$ is increasing in μ, so $\underline{A}_3$ is satisfied.

The Public Negotiation and Public Contracts Case

Condition (11) fails; the only equilibrium in this case is the separating equilibrium in which P offers m^*. As there is no gain to be had from fooling T about θ when $\theta = \underline{\theta}$, there is no benefit to pooling. Furthermore, as pooling is costly -- one, because it lowers second-period profits when $\theta = \bar{\theta}$ (relative to separating), and two, because it lowers first-period profits (again, relative to separating) -- the pooling equilibrium (offering m_0) cannot exist.

The Private Negotiation and Private Contracts Case

Now, we have $\underline{\mathcal{X}} = \underline{\mathcal{P}} = \{\underline{x}^*\}$. Hence the optimal first-period mechanism $m^* = (\bar{x}^*, \underline{x}^*)$ is sustainable as part of a separating equilibrium; however, unlike the previous example, there is no unique separating equilibrium: any $(x, \underline{x}^*)$, $x \in \bar{\mathcal{X}}$, is sustainable as part of a separating equilibrium. As regards pooling equilibria, the set of pooling equilibria is a subset of $(-\infty, \underline{x}^*] \cap \bar{\mathcal{P}}$; pooling equilibria can be constructed if and only if $(-\infty, \underline{x}^*] \cap \bar{\mathcal{P}} \neq \emptyset$. Pooling equilibria exist only if deterring entry is important for a good firm, i.e. if duopoly profits are much smaller than monopoly profits, in which case the principal would be willing to except low first-period profits in order to deter entry. Note that, unlike the previous example, pooling occurs only at *low* levels of x ($x \leq \underline{x}^*$); only if best separating by the $\underline{\theta}$-type is infeasible, can pooling occur.

Note that separating equilibria other than m^* and all pooling equilibria are sustained by T's beliefs that large revenues signal that the firm is *inefficient* (i.e. is $\underline{\theta}$). Such beliefs seem unreasonable in this situation. In Appendix 3, we present a refinement concept which eliminates them.

The Private Negotiation but Public Contracts Case

Condition (48), a necessary condition for a pooling equilibrium, fails; thus, as in the public negotiation case, the unique equilibrium is the separating equilibrium in which m^* is offered. The intuition is also the same: pooling is costly and produces no gain, whereas (overt) separating maximizes both first and second-period profits.

7/ EMPIRICAL IMPLICATIONS

A growing body of empirical work (Jensen and Murphy [1988], Gibbons and Murphy [1989], and Leonard [1989]) has found little evidence for the basic, "desert-island", principal-agent model. Regressions of compensation on performance find that performance has only a small effect on compensation. Our work offers an explanation for these results: the basic model predicts that compensation will be very responsive to performance, as the basic model implies separation. However, if economic reality is better described by the pooling equilibria which can arise due to outside considerations, then compensation can seem unresponsive to performance. Econometrically, a world of separating equilibria would appear as two clouds of data points, one around $(\underline{x}^*,\underline{w}^*)$ and another around $(\bar{x}^*,\bar{w}^*)$ (assuming the public negotiation and public contracts case for the sake of illustration). Even if there were a large amount of measurement error (i.e. each cloud is fairly disperse about (x^*,w^*)), the econometrician should still detect a strong relationship between performance and compensation. In contrast, a world of pooling equilibria would appear as one cloud of data points around (x_0,w_0) (again assuming the public negotiation and public contracts case). All the econometrician would detect is noise. Of course, we recognize that different equilibria would likely hold across different firms and different industries; yet in a world which tends to be characterized by pooling equilibria, even where pooling occurs at different points across firms/industries, the signal-to-noise ratio in a regression of compensation on performance would be lower than the signal-to-noise ratio in a world which tends to be characterized by separating equilibria (such as the "desert-island" world). If the signal-to-noise ratio is sufficiently low, then the econometrician will fail to find a strong relationship between performance and compensation.

8/ CONCLUSION

In this paper, we analyzed a principal-agent relationship under incomplete information where the principal foresees a future interaction with a third party. As the parameter of asymmetric information in the principal-agent relationship also plays a role in the interaction between the principal and the third party, the principal will be sensitive to what information is revealed through the principal-agent relationship. Consequently, the principal may alter both the output targets assigned to the agent and the agent's compensation schedule relative to the no-third-party situation; the motivation being the concealment of information detrimental to the principal. Moreover, the form of this contract is heavily dependent on the observability of the contracting game by the third party; both targets and compensation will depend on whether contract *negotiations* are observable and whether the contracts *themselves* are observable, as well as on the importance of revealing or concealing information.

In the no-third-party situation, the principal would implement a compensation scheme which fully reveals the relevant information to the outside, because such a scheme maximizes the principal's profits from the principal-agent relationship. Concealing information requires

THE ROLE OF OUTSIDE CONSIDERATIONS IN THE DESIGN OF COMPENSATION SCHEMES[1]

0/ INTRODUCTION

This paper considers what happens when a principal-agent relationship reveals (or can reveal) information to a third party which is relevant to some future interactions between the *principal* and the third party. As the principal will be concerned with these future interactions (outside considerations), the design of the agent's compensation scheme will be motivated in part by the principal's desire to conceal or reveal information.[2] For example, the principal foresees going to the capital market in the future (e.g. an initial public offer, privatization of a state-run concern, etc.); he may thus wish that the value of his firm be revealed, or he may wish to have concealed how little value it has. Or, the principal is concerned about future entry into his market, in which case he may cause production to be altered to conceal that conditions are favorable for entry.

Thus our paper is a departure from standard principal-agent models which treat the principal and agent in isolation, as if they were

[1] We thank Eddie Dekel-Tabak, Franklin Fisher, Drew Fudenberg, Bob Gibbons, Oliver Hart, Bruno Jullien, Michael Katz, John Litwack, James Malcomson, Matt Rabin, and seminar participants at Berkeley, Budapest, MIT, Paris, Stanford, Tilburg, and Toulouse for helpful comments. We express special thanks to Jean Tirole for many stimulating discussions during the early stages of our project and for numerous suggestions about the presentation. Remaining errors are ours.

[2] These concerns can also be found in Gertner, Gibbons, and Scharfstein [1988] and Glazer and Israel [1987]. Those models are, however, signaling models, whereas we assume that principal is initially an *un*informed player (i.e. our model is *not* a signaling model). Moreover, unlike us, they do not analyze the role of observable negotiations and/or observable contracts.

playing alone on a desert island.[3] Given that most principal-agent relationships occur in settings more populated than that of Robinson Crusoe and Friday, this strikes us as an important departure. This is particularly true with regard to the modern corporation, a setting to which the principal-agent paradigm is often applied. The modern corporation exists in a multi-player environment, and therefore it is of value to know how the standard "desert-island" models are changed by the introduction of additional players (outside considerations). Our goal in this paper is, in part, to provide some answers to that question.

What information is revealed to a third party and how it can be controlled by the principal depends on what is observable by the third party. In what follows, we will maintain the assumption that the third party observes the physical outcome of the principal-agent relationship (e.g. revenues, sales volume, or some other performance measure). In addition, we will consider three different assumptions about what other aspects of the relationship are observable. The most informative case (from the third party's perspective) will be where the third party observes the initial negotiations between principal and agent and the terms of the contract they sign; we call this the public negotiations/ public contracts case. The least informative case will be where the

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3 We are not the first to introduce other players into the principal-agent relationship. Work by Holmstrom [1982], Hart [1983], and Scharfstein [1988] has considered the extent to which observations of other players (e.g. through product market competition) provide information to the principal which can help in the design of compensation schemes. Some authors have sought to explain the existence of principal-agent hierarchies as a consequence of interactions with third parties (Katz [1988] and Ferschtmann, Judd, and Kalai [1987]). Finally, in the literature perhaps closest to us, some authors have considered an *agent* who foresees interacting in future principal-agent relationships with other principals (Fama [1980], Holmstrom [1983], Waldman [1984], Gibbons [1986], and Ricart i Costa [1988]).

third party observes neither the initial negotiations, nor the contract signed; we call this the private negotiations/private contracts case. Finally, an intermediate case is when the third party observes the contract signed, but not the negotiations which led to it; we call this the private negotiations/public contracts case. As we will show, the design of the compensation scheme can easily depend on which of three cases we assume holds. Indeed one of the points of this paper is that in presence of outside considerations *how* contracts are entered into can be as important as what contracts are entered into.

Generally speaking, it is less costly for the principal to have information credibly revealed in the two public contract cases than in the private contract case. However when it comes to having information concealed, things are different: it is least costly for the principal in the public negotiations/public contracts case, but most costly in the private negotiations/public contracts case; the private negotiations/ private contracts case is intermediate. In all cases, the equilibrium compensation scheme can be different from the one which would exist if there were no third party; and different, too, from the ones obtained under alternative observability assumptions. Consequently, a firm's actual physical production (or other publicly observable measure of performance) can be quite sensitive to the observability of negotiations and contracts. For example, when negotiations are public, information-concealing equilibria can only occur at a low production level; while when negotiations are private, information-concealing equilibria can occur at high production levels. The rent earned by good managers will differ also: it will be greater in the latter case than in the former case. Also in the latter case, if the contracts are public, then even the bad manager can earn a rent, though if the contracts are private, she will never earn a rent.

As contract *negotiations* are rarely public, we feel that case is less realistic than the private negotiation cases. Consequently, much of the focus is on private negotiation. We offer no explanation for why contract negotiations are typically private -- indeed all information structures in this paper are taken to be exogenous -- however one reason could be an inability to *commit* to public negotiations: outside observers cannot be sure that the parties to the negotiations have not conversed privately or entered into other secret agreements. For similar reasons, one might also prefer the private *contracts* assumption; although here, legal requirements, such as SEC requirements, could serve to justify the assumption of public contracts.

We present the model in Section 1. In Section 2, we derive the optimal "no-third-party" compensation scheme. We treat the public/public case in Section 3. The private/private case is analyzed in Section 4. In Section 4a, we examine the conditions for separating (information-revealing) schemes to exist in equilibrium. In Section 4b, we examine the conditions for pooling (information-concealing) schemes to exist in equilibrium. In Section 4c, we briefly considered equilibria in which the principal randomizes between separating and pooling schemes. Section 5 deals with the private/public case. Applications of this analysis to entry deterrence are considered in Section 6. Empirical implications are briefly touched upon in Section 7. We finish with some concluding remarks in Section 8.

1/MODEL

This is a game with two periods and three players: a principal (P), a manager (M), and a third party (T). The principal owns the firm and hires the manager to run it in the first period. In the second period, the third party and principal interact in a game-like situation.

Revenues in the first period are $x = \theta + e$, where θ is a characteristic of the firm and e is managerial effort.[4] θ *a priori* belongs to $\{\underline{\theta},\bar{\theta}\}$ and prior beliefs are $\mathbb{P}(\theta = \bar{\theta}) = h$. Expending effort e costs the manager $\phi(e)$ in disutility. Assume

$\underline{A}_1$: $\phi(\cdot)$ is three-times differentiable on $\mathbb{R}$,

$\forall e \in \mathbb{R}_-$, $\phi(e) = 0$,

$\forall e \in \mathbb{R}^*_+$, $\phi(e) > 0$, $\phi'(e) \geq 0$, $\phi''(e) > 0$ and $\phi'''(e) \geq 0$.

The disutility of effort increases at an increasing rate with effort.[5] The assumption $\phi''' \geq 0$ ensures concavity for some of the optimization programs considered later.

M's utility function is $w - \phi(e)$, where w is first-period income. We normalize M's reservation utility level, what she would receive outside the firm, to be 0.

In the second period, P and T play a (possibly complicated) Bayesian game $\Gamma_2 = \Gamma(\mu,\nu,\theta)$, where μ is T's belief about θ and ν is P's belief about θ. We assume

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4 There is little loss of generality in this specification; we could extend the analysis to revenue functions of the form $R(x(\theta,e))$, where x is some observable measure of performance (e.g. sales volume), where R is increasing concave, and where $x_e > 0$, $x_{ee} \leq 0$, $x_\theta > 0$, and $x_{e\theta} \geq 0$ with similar results.

5 Negative effort is introduced only to make the setting as simple as possible, not for the purpose of realism. Negative effort permits us to forget boundary conditions in the maximizations performed later.

$\underline{A}_2$: Γ_2 has a unique (perfect) Bayesian equilibrium with respect to P's (expected) payoff.

Hence P's (expected) payoff in Γ_2 is a *function* $\bar{\pi}(\mu,\nu,\theta)$.

The timing of the entire game: Prior to contracting, θ is determined by nature. At this point, P has no private information. P and T share the prior h about θ. Negotiations occur in which P has all the bargaining power: he proposes a take-it-or-leave-it mechanism to the manager. If the manager rejects it, negotiations break down and the game ends.[6] If she accepts it (i.e. is hired), the mechanism is played out. Playing out the mechanism consists of M first learning θ. Only the manager learns θ at this time. After learning θ, M can quit if she wishes. Otherwise, M sends messages to P. First-period production then occurs, followed by the continuation game Γ_2.

In this paper, we limit attention to direct (truthful) revelation mechanisms of the form $(x(\hat{\theta}),w(\hat{\theta}))$, where $x(\hat{\theta})$ is the first-period revenue required of a manager who declares her type to be $\hat{\theta}$ and $w(\hat{\theta})$ is her wage *provided she obtains* $x(\hat{\theta})$ (otherwise she is paid 0). Announcing $\hat{\theta}$ is equivalent to choosing a pair (x,w) from the set $\{(x(\hat{\theta}),w(\hat{\theta}))\}_{|\hat{\theta}\in\{\underline{\theta},\bar{\theta}\}}$. Note the set of mechanisms is a subset of $\mathbb{R}^4$. We call a pair (x,w) "chosen" in this way a contract and denote it by c.

As the mechanism is direct, P has perfect information about θ in Γ_2, i.e. $\nu = \delta_\theta$, the Dirac probability measure at θ. Define

$$\Pi(\mu,\theta) = \bar{\pi}(\mu,\delta_\theta,\theta)$$

In Appendix 1, we prove that the assumption of direct revelation mechanisms is without loss of generality for what follows, except,

[6] Alternatively, P negotiates with another manager.

possibly, for the public negotiations and public contract case.[7] We parameterize μ as $\mu = \mathbb{P}\left\{\theta = \bar{\theta} \mid \mathcal{I}_T\right\}$; i.e. μ is T's probability assessment of the event $\theta = \bar{\theta}$ given T's information, $\mathcal{I}_T$.

We also assume

$\underline{A}_3$: $\forall\theta$, $\Pi(\mu,\theta)$ is continuous and non-decreasing in μ.[8]

In words: the better P appears in T's eyes, i.e. the more weight T puts on the event $\theta = \bar{\theta}$, the greater will be P's expected payoff in Γ_2. $\underline{A}_3$ would obtain if, for example, θ was a measure of the firm's productivity and T was a potential entrant.

We assume x, first-period revenue, is observed by T. In addition, we consider three different assumptions about T's other information:

Public Negotiations and Public Contracts: The mechanism and the contract chosen are observable by T.

Private Negotiations, but Public Contracts: The mechanism is *un*observable by T, but the chosen contract is observable by T.

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[7] The reader may also wonder about our assumption of deterministic mechanisms; i.e. what if, contrary to our assumption, M's announcement merely fixed a lottery over contracts from a set $\mathcal{C}$. In Appendix 1, we show that our assumption of deterministic mechanisms is without loss of generality, except, possibly, when contracts are public.

[8] Continuity is not necessary for much of what follows: the existence results in Sections 2-4b do not depend on it. In an earlier working paper we also considered the assumption that Π was *non-increasing* in μ, which yields similar results. Indeed, the entire analysis could be done without assuming that Π was monotonic in μ; it would suffice that Π was not constant in μ. However, such generalization adds little to the analysis, while greatly complicating the notation and discussion.

Private Negotiations

and Private Contracts: Neither the mechanism, nor the chosen contract, are observable by T.

The fourth possibility, public negotiations but private contracts, is ignored, as it is equivalent to public negotiations and public contracts. We assume in no case does T hear M's announcement; though in some cases what T observes is equivalent to hearing M's announcement.

Finally we assume the structure of the game is common knowledge.

Our solution concept will be the strong version of Perfect Bayesian Equilibrium (PBE) put forth by Fudenberg and Tirole [1989].[9] This solution concept still yields a plethora of equilibria in some cases. For the most part, we have chosen not to select from among them by employing further refinements; however, in Section 6 and Appendix 3, we show that a generalized refinement in the spirit of Farrell [1986] and Grossman and Perry [1986] could eliminate "unreasonable" equilibria in certain interesting cases. While in Section 5 we restrict attention to equilibria where out-of-equilibrium beliefs satisfy an additional "reasonableness" requirement.

As M's only strategic action is her announcement $\hat{\theta}$, we can easily solve for her optimal strategy in any PBE. Consider a mechanism m = $\left((\bar{x},\bar{w}),(\underline{x},\underline{w})\right)$, where the first contract is put in force by an announcement of $\bar{\theta}$ and the second contract by an announcement of $\underline{\theta}$. For

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[9] Our use of this version of PBE is motivated by our desire to rule out beliefs for T which require T to hypothesize that a deviation by P will be followed by a deviation by M. That is the uninformed player P cannot "signal" what he does not know. Were the space of mechanisms finite, then this solution concept would be equivalent to sequential equilibrium (Kreps and Wilson [1982]; see Fudenberg and Tirole [1989] for a complete discussion).

both types of M to truthfully announce their types, it must be that

$$\bar{w} - \phi(\bar{x} - \bar{\theta}) \geq \underline{w} - \phi(\underline{x} - \bar{\theta}) \tag{1a}$$

$$\underline{w} - \phi(\underline{x} - \underline{\theta}) \geq \bar{w} - \phi(\bar{x} - \underline{\theta}) \tag{1b}$$

The manager must do better by telling the truth than by lying; that is truth-telling must be *incentive compatible* (IC).

If M quits after learning θ, her utility is 0; hence M will stay only if

$$\bar{w} - \phi(\bar{x} - \bar{\theta}) \geq 0 \tag{2a}$$

$$\underline{w} - \phi(\underline{x} - \underline{\theta}) \geq 0 \tag{2b}$$

In words: it must be *individually rational* (IR) for the manager to stay. As $\left((0,0),(0,0)\right)$ is (IR), there is no loss of generality in restricting attention to (IR) mechanisms. Note that if a mechanism is (IR), M will prefer accepting it to rejecting it. A direct revelation mechanism satisfying (1) and (2) is *feasible.*

As is typical in such problems, a necessary condition for m to satisfy (1) and (2) is

$$\bar{x} \geq \underline{x} \tag{3}$$

The bad type cannot be asked to produce more than the good type; if this were not the case, no pair of wages could possibly make (1) hold. Condition (3) is also a sufficient condition given $(\bar{x},\underline{x})$ for the existence of wages $(\bar{w},\underline{w})$ such that $\left((\bar{x},\bar{w}),(\underline{x},\underline{w})\right)$ is feasible.

If $(\bar{x},\bar{w}) = (\underline{x},\underline{w})$, we call the mechanism *pooling*. If $(\bar{x},\bar{w}) \neq (\underline{x},\underline{w})$, we call the mechanism *separating*.

2/ THE MODEL WITH NO THIRD PARTY

For now, imagine there is no third party and no second period.

Define e^{FB} by $\phi'(e^{FB}) = 1$. e^{FB} is the first-best (full-information) level of effort; it is the level of effort P would induce if P knew θ (it equates the marginal product of effort with the marginal disutility of effort). Let $\bar{x}^{FB} = \bar{\theta} + e^{FB}$ and $\underline{x}^{FB} = \underline{\theta} + e^{FB}$ be the first-best revenue levels.

However P does not know θ; hence the optimal mechanism solves

$$\max_{\{\bar{w},\underline{w},\bar{x},\underline{x}\}} \; h(\bar{x} - \bar{w}) + (1-h)(\underline{x} - \underline{w}) \quad \text{s.t. (1) and (2) hold} \tag{4}$$

(2a) is slack and it is easily shown that (1a) is binding, hence (1b) is slack. The solution to (4) is thus:

$$\bar{w} = \phi(\bar{x} - \bar{\theta}) + \phi(\underline{x} - \underline{\theta}) - \phi(\underline{x} - \bar{\theta}) \tag{5a}$$

$$\underline{w} = \phi(\underline{x} - \underline{\theta}) \tag{5b}$$

$$1 - \phi'(\bar{x} - \bar{\theta}) = 0 \tag{5c}$$

$$\phi'(\underline{x}^* - \underline{\theta}) = 1 - \frac{h}{1-h}\left(\phi'(\underline{x}^* - \underline{\theta}) - \phi'(\underline{x}^* - \bar{\theta})\right) \tag{5d}$$

$\underline{A}_1$ guarantees that (5) is sufficient, as well as necessary. Let $\underline{x}^*$, $\bar{x}^*$, $\underline{w}^*$, and $\bar{w}^*$ be the solution to (5). Define $m^* = \left((\bar{x}^*,\bar{w}^*),(\underline{x}^*,\underline{w}^*)\right)$. From (5c), we see that the good type supplies her first-best level of effort (e^{FB}), for which she is paid more than under perfect information (5a). Equation (5d) entails $e(\underline{\theta}) = \underline{x}^* - \underline{\theta} < e^{FB}$; the bad type supplies less than the first-best level of effort. Note that as h increases from 0 to 1, $e(\underline{\theta})$ decreases from e^{FB} to 0. Finally note that $\bar{x}^* = \bar{x}^{FB} > \underline{x}^{FB} > \underline{x}^*$.

The solution to (4) is typical of this type of model (cf. Laffont and Tirole [1986]): the better type supplies the first-best level of effort and receives an informational rent, while the worse type supplies less than first-best level of effort and is put on her IR constraint.

Note the optimal mechanism is a separating mechanism. We have shown:

Proposition 1: *In the absence of a third party, the optimal mechanism is the separating mechanism defined by (5).*

For the purpose of later comparisons, we also calculate the optimal pooling mechanism, which is given by the program

$$\max_{\{\bar{w},\underline{w},\bar{x},\underline{x}\}} \; h(\bar{x} - \bar{w}) + (1-h)(\underline{x} - \underline{w}) \quad \text{s.t. (1) and (2) hold and s.t. } \bar{x} = \underline{x} \text{ and } \bar{w} = \underline{w} \tag{6}$$

Note (6) is (4) with the additional constraint that the mechanism be pooling. It is straightforward to show that the solution to (6) is

$$w = \phi(x - \underline{\theta}) \tag{7a}$$

$$1 - \phi'(x - \underline{\theta}) = 0 \tag{7b}$$

Let x_0 and w_0 be the solution to (7) ($x_0 = \underline{x}^{FB} < \bar{x}^{*}$ and $w_0 = \phi(e^{FB})$). Define $m_0 = \Big((x_0,w_0),(x_0,w_0)\Big)$. The bad type supplies e^{FB} and the good type slacks (expends effort $e^{FB}-(\bar{\theta}-\underline{\theta}) < e^{FB}$). The bad type is put on her IR constraint, while the good type earns an informational rent.

For later convenience, we define the functions:

$$\bar{G}(x) = x - \phi(x-\bar{\theta})$$

$$\underline{G}(x) = x - \phi(x-\underline{\theta}) - \frac{h}{1-h}\Big(\phi(x-\underline{\theta}) - \phi(x-\bar{\theta})\Big) \tag{8}$$

From $\underline{A}_1$, both $\bar{G}(x)$ and $\underline{G}(x)$ are continuous, three-times differentiable, and strictly quasi-concave functions. From (5c), $\bar{x}^{*}$ is the solution to

$$\max_{x} \bar{G}(x)$$

and, from (5d), $\underline{x}^{*}$ is the solution to

$$\max_{x} \underline{G}(x) \tag{9}$$

Also, for all m which satisfy (1a) and (2b) with equality:

$$h\bar{G}(\bar{x}) + (1-h)\underline{G}(\underline{x}) = h(\bar{x}-\bar{w}) + (1-h)(\underline{x}-\underline{w})$$

Examples of such mechanisms are m^* and m_0.

$\bar{G}(x)$ is the full-information profit generated by giving a target of x to the good manager. $\underline{G}(x)$ takes into account that giving a target of x to a bad manager not only has a direct (full-information) effect on profits, but also an indirect effect on profits through the amount of rent which must be left the good manager to prevent her from lying. The relative importance of the two effects depends on the relative proportion of good types and bad types in the population.

3/ PUBLIC NEGOTIATIONS AND PUBLIC CONTRACTS

Now we consider the case where T observes the mechanism (m) proposed by P and the contract (c) chosen by M (but, recall, T is not aware of M's announcement). T's information is thus (m,c,x). If m is a separating mechanism, then observing c (or even just x) allows T to learn θ's value. However, if the mechanism is pooling, then T can draw no inferences from his observation of c (or x). His posterior on θ must equal his prior, i.e. $\mu = h$.[10]

Thus the principal knows when designing a mechanism what beliefs (μ) it will induce. P's problem is then

$$\max_{m} \quad h(\bar{x} - \bar{w}) + (1-h)(\underline{x} - \underline{w}) + h\Pi(\bar{\mu},\bar{\theta}) + (1-h)\Pi(\underline{\mu},\underline{\theta})$$

where $\bar{\mu}$ (respectively $\underline{\mu}$) represents T's beliefs upon observing that

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[10] Note that regardless of the mechanism P proposes, T makes inferences by employing (trivially) Bayes' law on the equilibrium strategy of the *manager* only. This is an example of the cutting power of the strong version of PBE: under the weaker definition, we could have equilibria which are sustained by unreasonable off-the-equilibrium-path beliefs that are functions of *both* P's action and M's response; for example, we could have unreasonable equilibria in which T could hold beliefs $\mu = 0$ following any deviation by P, even if M played the response dictated by her equilibrium strategy.

$(\bar{x},\bar{w})$ (respectively $(\underline{x},\underline{w})$) has been chosen by M from m. Note $\bar{\mu} = 1$ and $\underline{\mu} = 0$, if $(\bar{x},\bar{w}) \neq (\underline{x},\underline{w})$, and otherwise $\bar{\mu} = \underline{\mu} = h$. As P's second-period profits only depend on whether the mechanism is separating or pooling, the optimal mechanism must then maximize first-period profits within one of these two classes. Hence

Proposition 2: *Under $\underline{A}_1$ and $\underline{A}_2$, and assuming public negotiations, the only PBE is*

- *Separating with P offering m^* if and only if (10) below holds.*
- *Pooling with P offering m_0 if and only if (10) strictly reversed.*
- *Indifference with P (possibly) mixing over m_0 and m^* if and only if (10) is an equality.*

$$h(\bar{x}^* - \bar{w}^*) + (1-h)(\underline{x}^* - \underline{w}^*) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(0,\underline{\theta}) >$$
$$x_0 - w_0 + h\Pi(h,\bar{\theta}) + (1-h)\Pi(h,\underline{\theta}) \quad (10)$$

As m^* produces greater first-period profits than m_0, P will prefer m_0 only if

$$(1-h)\Big(\Pi(h,\underline{\theta}) - \Pi(0,\underline{\theta})\Big) > h\Big(\Pi(1,\bar{\theta}) - \Pi(h,\bar{\theta})\Big) \quad (11)$$

(11) is the requirement that the expected gain from concealing $\underline{\theta}$ from T exceed the expected loss from concealing $\bar{\theta}$ from T.

Using the "G"-functions defined above, (10) can be rewritten as

$$h\Big(\bar{G}(\bar{x}^*) - \bar{G}(x_0)\Big) + (1-h)\Big(\underline{G}(\underline{x}^*) - \underline{G}(x_0)\Big)$$
$$> h\Big(\Pi(h,\bar{\theta}) - \Pi(1,\bar{\theta})\Big) + (1-h)\Big(\Pi(h,\underline{\theta}) - \Pi(0,\underline{\theta})\Big) \quad (12)$$

If (12) holds, m^* is offered, and if (12) is reversed, m_0 is offered.

Note, finally, that in the case of public negotiations, the only possible equilibria involve P offering either the best-separating mechanism m^* or the the best-pooling mechanism m_0.

4/ PRIVATE NEGOTIATIONS AND PRIVATE CONTRACTS

Now assume that T no longer observes m or c (though he continues to observe x). Thus T's beliefs must be defined for all possible observations; i.e. there exists a $\mu[x] \in [0,1]\ \forall x \in \mathbb{R}_+$.

As T sees neither m nor c, but only x, P is free to set wages to minimize costs. Cost minimization requires that (1a) and (2b) hold as equalities. It follows that we can denote feasible mechanisms as $(\bar{x},\underline{x})$, since $\bar{w}$ and $\underline{w}$ will then be determined by the equalities (1a) and (2b). Also, we can denote first-period profits using the "G"-functions: the mechanism $(\bar{x},\underline{x})$ yields first-period expected profits $h\bar{G}(\bar{x}) + (1-h)\underline{G}(\underline{x})$.

4a/ Separating Equilibria

Here we consider separating equilibria: equilibria in which P proposes a separating mechanism $(\bar{x},\underline{x})$, $\bar{x} > \underline{x}$. As the mechanism is separating, consistent beliefs require $\mu[\bar{x}] = 1$ and $\mu[\underline{x}] = 0$. Thus, in equilibrium, the expected profits from offering $(\bar{x},\underline{x})$ are

$$h\bar{G}(\bar{x}) + (1-h)\underline{G}(\underline{x}) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(0,\underline{\theta}) \tag{13}$$

Our first result is

Lemma 1: *Under $\underline{A}_1$, $\underline{A}_2$, and $\underline{A}_3$, if a feasible mechanism $(\bar{x},\underline{x})$ is offered in a separating PBE, then $\underline{x} = \underline{x}^* < \bar{x}$.*

Proof: Suppose $\bar{x} \le \underline{x}^*$ (hence $\underline{x} < \underline{x}^*$). Consider the feasible deviation $(\bar{x},\bar{x})$, which generates profits

$$h\bar{G}(\bar{x}) + (1-h)\underline{G}(\bar{x}) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(1,\underline{\theta}) \tag{14}$$

As $\Pi(\mu,\theta)$ is non-decreasing in μ and $\underline{G}$ is strictly quasi-concave in x with a maximum at $\underline{x}^*$, (14) is strictly greater than (13). As $(\bar{x},\bar{x})$ is a profitable deviation, $(\bar{x},\underline{x})$ cannot be an equilibrium for $\bar{x} \le \underline{x}^*$; hence $\bar{x} > \underline{x}^*$. Suppose $\underline{x} \neq \underline{x}^*$ and consider the feasible deviation $(\bar{x},\underline{x}^*)$, which generates profits

$$h\bar{G}(\bar{x}) + (1-h)\underline{G}(\underline{x}^*) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(\mu[\underline{x}^*],\underline{\theta}) \tag{15}$$

As $\underline{G}(\underline{x}^*) > \underline{G}(\underline{x})$ and $\mu[\underline{x}^*] \geq 0$, (15) is strictly greater than (13); hence it cannot be that $\underline{x} \neq \underline{x}^*$ in a separating equilibrium.

■

The reasoning behind Lemma 1: in equilibrium the type of each manager is revealed and being revealed minimizes $\Pi(\mu,\underline{\theta})$, thus there is nothing to be lost by inducing the $\underline{\theta}$-type to produce as efficient as possible first-period level of x, and much to be gained.

We can now prove the main existence result of this sub-section:

Proposition 3: *Under $\underline{A}_1$-$\underline{A}_3$, there exists a separating PBE in which the feasible mechanism $m = (\bar{x},\underline{x})$ is offered if and only if*

$$\bar{G}(\bar{x}^*) - \bar{G}(\bar{x}) \leq \Pi(1,\bar{\theta}) - \Pi(0,\bar{\theta}) \tag{16}$$

$$\underline{G}(\underline{x}^*) - \underline{G}(\bar{x}) \geq \Pi(1,\underline{\theta}) - \Pi(0,\underline{\theta}) \tag{17}$$

and

$$\underline{x} = \underline{x}^* \tag{18}$$

Proof: We first prove necessity, then sufficiency.

Necessity. The necessity of (18) has been shown by Lemma 1. Profits under m are

$$h\bar{G}(\bar{x}) + (1-h)\underline{G}(\underline{x}^*) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(0,\underline{\theta}) \tag{19}$$

The necessity of (16): consider the deviation $(\bar{x}^*,\underline{x}^*)$, which yields profits

$$h\bar{G}(\bar{x}^*) + (1-h)\underline{G}(\underline{x}^*) + h\Pi(\mu[\bar{x}^*],\bar{\theta}) + (1-h)\Pi(0,\underline{\theta}) \tag{20}$$

(16) is the necessary condition for there to exist beliefs $\mu[\bar{x}^*]$ such that (20) does not exceed (19). The necessity of (17): consider the deviation $(\bar{x},\bar{x})$, which yields profits

$$h\bar{G}(\bar{x}) + (1-h)\underline{G}(\bar{x}) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(1,\underline{\theta}) \tag{21}$$

(17) is the necessary condition for (21) not to exceed (19).

Sufficiency. Given (16), (17), and (18), the following is a separating PBE: P proposes m and T holds beliefs $\mu[\bar{x}] = 1$ and $\mu[x] = 0\ \forall x \neq \bar{x}$. Clearly T's beliefs satisfy Bayesian consistency. P's profits under m are given by (25). Consider a deviation $(\bar{x}', \underline{x}')$ where $\bar{x}' \neq \bar{x}$ *or* $\underline{x}' \neq \underline{x}^*$ *or* both; such a deviation yields profits of

$$h\bar{G}(\bar{x}') + (1-h)\underline{G}(\underline{x}') + h\Pi(\mu[\bar{x}'],\bar{\theta}) + (1-h)\Pi(\mu[\underline{x}'],\underline{\theta}) \tag{22}$$

Subtracting (22) from (19) yields

$$h\left[\left(\bar{G}(\bar{x}) - \bar{G}(\bar{x}')\right) + \left(\Pi(1,\bar{\theta}) - \Pi(\mu[\bar{x}'],\bar{\theta})\right)\right]$$
$$+ (1-h)\left[\left(\underline{G}(\underline{x}^*) - \underline{G}(\underline{x}')\right) + \left(\Pi(0,\underline{\theta}) - \Pi(\mu[\underline{x}'],\underline{\theta})\right)\right] \tag{23}$$

By (16), the top line of (23) is minimized by $\bar{x}' = \bar{x}$. By (17) and (9), the bottom line of (23) is minimized by $\underline{x}' = \underline{x}^*$. Hence no deviation results in strictly greater profits than m; m is a best response.

■

There are two potentially desirable deviations from $(\bar{x}, \underline{x}^*)$. One is to give up the additional second-period profits when the $\bar{\theta}$-type is revealed in exchange for maximizing first-period profits (we call this the *best-separating deviation*). Best-separating is ruled out by condition (16). The second deviation, which we call *secret pooling*, is to have the $\underline{\theta}$-type mimic the $\bar{\theta}$-type. The benefit from this deviation is that expected second-period profits are increased, while the cost is that the $\underline{\theta}$-type produces a less profitable level of x (less profitable in part because the $\underline{\theta}$-type may produce an inefficient level of x and in part because it raises the informational rent earned by the $\bar{\theta}$-type; see (5)). Secret pooling is ruled out by condition (17).

The set of mechanisms that can be part of a separating equilibrium can easily be described through conditions (16) and (17):

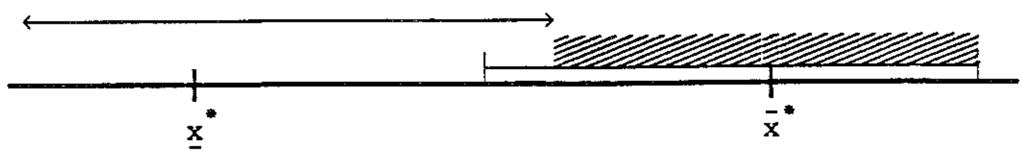
$$\bar{\mathfrak{X}} \equiv \{x \mid \bar{G}(\bar{x}^*) - \bar{G}(x) \leq \Pi(1,\bar{\theta}) - \Pi(0,\bar{\theta}) \text{ and } x > \underline{x}^*\}$$

$$\underline{\mathcal{X}} \equiv \{x \mid \underline{G}(\underline{x}^*) - \underline{G}(x) \le \Pi(1,\underline{\theta}) - \Pi(0,\underline{\theta})\}$$

$\bar{\mathcal{X}}$ is the set of $\bar{x}$ satisfying (16) and $\underline{\mathcal{X}}$ is the set of $\bar{x}$ failing (17).

Proposition 3 is illustrated by Figure 1: the shaded interval (▨) illustrates the set of $\bar{x}$ that can be part of a mechanism offered in some separating equilibrium, the interval (⟷) represents $\underline{\mathcal{X}}$, and the interval (├┤) represents $\bar{\mathcal{X}}$.

Figure 1



A corollary to Proposition 3 is

Corollary 1: *If $\bar{\mathcal{X}} \subset \underline{\mathcal{X}}$, then no separating equilibrium exists.*

Corollary 1 pertains when beliefs are very important for second-period profits when $\theta = \underline{\theta}$ but not when $\theta = \bar{\theta}$. In such a situation $\bar{\mathcal{X}}$ would be a small interval around $\bar{x}^*$: no matter what beliefs are induced, there is little to be lost from best-separating and, provided $\bar{x}$ is not already near $\bar{x}^*$, much to be gained. $\underline{\mathcal{X}}$, on the other hand, would be a large interval: except for extreme $\bar{x}$, secret pooling is a worthwhile deviation. In such a case, it is possible, that any $\bar{x}$ close enough to $\bar{x}^*$ to make best-separating a losing deviation would also be close enough to $\underline{x}^*$ to make secret pooling a worthwhile deviation.

If beliefs are not important for second-period profits when $\theta = \underline{\theta}$, then $\underline{\mathcal{X}}$ is a small neighborhood around $\underline{x}^*$ and a separating equilibrium exists. In that case, there is little incentive to deviate from the efficient revenue target ($\underline{x}^*$); so provided $\bar{x}$ is both large enough and efficient enough (i.e. near $\bar{x}^*$), separation becomes credible. A summary sufficient condition for existence of a separating PBE is

$$\underline{G}(\underline{x}^*) - \underline{G}(\bar{x}^*) \geq \Pi(1,\underline{\theta}) - \Pi(0,\underline{\theta}) \tag{24}$$

Using (24), we do comparative statics with respect to h:

Corollary 2: $\bar{\mathcal{X}}\backslash\underline{\mathcal{X}}$ *is increasing (with respect to the inclusion order) as h increases and* $\exists h_s < 1$ *such that* $\forall h \in (h_s,1]$ $\bar{\mathcal{X}}\backslash\underline{\mathcal{X}} \neq \emptyset$.

Proof: As (16) and (18) are independent of h, to show that $\bar{\mathcal{X}}\backslash\underline{\mathcal{X}}$ is increasing, we need only show that $\underline{\mathcal{X}}$ is decreasing as h is increasing:

$$\frac{d}{dh}\left[\underline{G}(\underline{x}^*) - \underline{G}(\bar{x})\right] = \frac{1}{(1-h)^2}\left[\left(\phi(\bar{x}-\underline{\theta}) - \phi(\bar{x}-\bar{\theta})\right) - \left(\phi(\underline{x}^*-\underline{\theta}) - \phi(\underline{x}^*-\bar{\theta})\right)\right] \tag{25}$$

by the envelope theorem. By $\underline{A}_1$, (25) is positive; hence $\underline{\mathcal{X}}$ is decreasing. Now take $\bar{x} = \bar{x}^*$. From (5d), $\underline{G}(\underline{x}^*) \to \underline{\theta}$ as $h \to 1$, while $\underline{G}(\bar{x}^*) \to -\infty$ as $h \to 1$. Thus, as the righthand side of (24) is independent of h, there either must exist an $h_s > 0$ such that (24) is an equality or (24) must be met for all h. The rest of the result follows from (25).

■

Corollary 2 states that for h sufficiently large, separating equilibria must exist. When h is large, there is little to be gained in terms of expected second-period profits by secret pooling (i.e. the gain $(1-h)\left(\Pi(1,\underline{\theta}) - \Pi(0,\underline{\theta})\right)$ is small). Furthermore, there is much to be lost, as almost the entire effect of raising $\underline{x}$ is to increase the $\bar{\theta}$-type's informational rent.

We conclude this sub-section by comparing the results obtained here with those obtained in Section 3. Unlike Section 3, m^* is *not* the only mechanism that can be part of a separating equilibrium. In Section 3, because T saw m, as well as x, it was possible for P to insure what beliefs T would hold in the second period. Here, as T sees only x, it is no longer possible to fix T's beliefs. Thus T is free to hold any beliefs he wishes about $\bar{x}^*$ if it is *not* offered in equilibrium; in particular, T can hold "bad" beliefs about out-of-equilibrium x's (e.g.

$\mu[\bar{x}^*] = 0$).[11] Consequently, deviating to m^* (best-separating) may be unwise for P because, through T's beliefs, it will result in lower second-period profits.

Furthermore, there may be no separating equilibrium in which m^* is offered, yet there can exist m around which separating equilibria can be constructed (if $\bar{x}^* \in \underline{\mathcal{X}}$ but $\bar{\mathcal{X}}\setminus\underline{\mathcal{X}} \neq \emptyset$). This situation can arise because, unlike in Section 3, P cannot commit not to secretly pool. In comparison, in Section 3, merely offering m^* is proof that P is not inducing pooling.

4b/ Pooling Equilibria

Now we concentrate on pooling equilibria: P proposes a feasible pooling mechanism $m = \Big((x_p,w_p),(x_p,w_p)\Big)$. As (2b) is binding, we simply denote such a mechanism as x_p.

As the mechanism is pooling, consistent beliefs mean $\mu[x_p] = h$, the prior. Thus, in equilibrium, the expected profit from offering x_p is

$$h\bar{G}(x_p) + (1-h)\underline{G}(x_p) + h\Pi(h,\bar{\theta}) + (1-h)\Pi(h,\underline{\theta}) \tag{26}$$

In determining whether x_p can be supported as part of some pooling equilibrium, we need to distinguish three cases: $x_p < \underline{x}^*$, $\underline{x}^* \le x_p \le \bar{x}^*$, and $\bar{x}^* < x_p$.

Our first existence result is

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[11] Admittedly, some "bad" beliefs could be considered unreasonable. For example, one could consider it unreasonable that $\mu[\bar{x}] = 1$, $\bar{x} < \bar{x}^*$, for $\bar{x}$ on the equilibrium path, while $\mu[\bar{x}^*] = 0$ off the equilibrium path. However to eliminate such "unreasonable" beliefs (and thereby the equilibria they support), one needs to employ a refinement. We suggest one such refinement in Appendix 3 (see also discussion in Section 6b).

Proposition 4: *Under $\underline{A}_1$-$\underline{A}_3$, let $m = x_p$, where $\underline{x}^* \le x_p \le \bar{x}^*$. There exists a pooling PBE in which m is offered if and only if*

$$\bar{G}(\bar{x}^*) - \bar{G}(x_p) \le \Pi(h,\bar{\theta}) - \Pi(0,\bar{\theta}) \tag{27}$$

and

$$\underline{G}(\underline{x}^*) - \underline{G}(x_p) \le \Pi(h,\underline{\theta}) - \Pi(0,\underline{\theta}) \tag{28}$$

Proof: *Necessity.* The necessity of (27): consider the deviation $(\bar{x}^*,x_p)$, which yields profits

$$h\bar{G}(\bar{x}^*) + (1-h)\underline{G}(x_p) + h\Pi(\mu[\bar{x}^*],\bar{\theta}) + (1-h)\Pi(h,\underline{\theta}) \tag{29}$$

(27) is the necessary condition for there to exist beliefs $\mu[\bar{x}^*]$ such that (29) does not exceed (26). The necessity of (28): consider the deviation $(x_p,\underline{x}^*)$, which yields profits

$$h\bar{G}(x_p) + (1-h)\underline{G}(\underline{x}^*) + h\Pi(h,\bar{\theta}) + (1-h)\Pi(\mu[\underline{x}^*],\underline{\theta}) \tag{30}$$

(28) is the necessary condition for there to exist beliefs $\mu[\underline{x}^*]$ such that (30) does not exceed (26).

Sufficiency. Given (27) and (28), the following is a pooling PBE: P proposes x_p and T holds beliefs $\mu[x_p] = h$ and $\mu[x] = 0\ \forall x \ne x_p$. Clearly T's beliefs satisfy Bayesian consistency. P's profits under x_p are given by (26). Consider a deviation $(\bar{x}',\underline{x}')$ where $\bar{x}' \ne x_p$ *or* $\underline{x}' \ne x_p$ *or* both; such a deviation yields profits:

$$h\bar{G}(\bar{x}') + (1-h)\underline{G}(\underline{x}') + h\Pi(\mu[\bar{x}'],\bar{\theta}) + (1-h)\Pi(\mu[\underline{x}'],\underline{\theta}) \tag{31}$$

Subtracting (31) from (26) yields

$$\begin{aligned} &h\left[\left(\bar{G}(x_p) - \bar{G}(\bar{x}')\right) + \left(\Pi(h,\bar{\theta}) - \Pi(\mu[\bar{x}'],\bar{\theta})\right)\right] \\ &\quad + (1-h)\left[\left(\underline{G}(x_p) - \underline{G}(\underline{x}')\right) + \left(\Pi(h,\underline{\theta}) - \Pi(\mu[\underline{x}'],\underline{\theta})\right)\right] \end{aligned} \tag{32}$$

Given μ and (27), the top line of (32) is minimized by $\bar{x}' = x_p$, while given μ and (28), the bottom line of (32) is minimized by $\underline{x}' = x_p$. Thus x_p is a best response.

∎

Potentially the most desirable deviations from x_p involve *secret separating.* P risks lower second-period profits in exchange for having $\bar{\theta}$ and/or $\underline{\theta}$ produce her (their) first-period profit-maximizing revenue(s). Conditions (27) and (28) establish that such deviations (or combinations of such deviations) are losing deviations.

Our last two existence results:

Proposition 5: *Under* $\underline{A}_1$-$\underline{A}_3$, *let* $m = x_p$, *where* $x_p < \underline{x}^*$. *There exists a pooling PBE in which m is offered if and only if (27) holds and*

$$h\left(\bar{G}(\bar{x}^*) - \bar{G}(x_p)\right) + (1-h)\left(\underline{G}(\underline{x}^*) - \underline{G}(x_p)\right) \le \mathbb{E}_\theta\left\{\Pi(h,\theta) - \Pi(0,\theta)\right\} \quad (33)$$

Proof (sketch): *Necessity.* The necessity of (27) was shown in the proof of Proposition 4. The necessity of (33): consider the deviation $(\bar{x}^*,\underline{x}^*)$; (33) guarantees that there exist beliefs $\mu[\bar{x}^*]$ and $\mu[\underline{x}^*]$ which make that deviation unprofitable.

Sufficiency. Follows the argument used in the proof of Proposition 4; the only difference is to note (a) that deviations of the form $(x_p,\underline{x})$, $\underline{x} < x_p$, are losing deviations since $\underline{x} < x_p < \underline{x}^*$ and hence $\underline{G}(\underline{x}) < \underline{G}(x_p)$; and (b) that if the $\underline{\theta}$-type is required to produce $x > x_p$, then, by (IC), the $\bar{\theta}$-type must also be required to produce $x > x_p$; however as the best such deviation $((\bar{x}^*,\underline{x}^*))$ produces lower profits than m, no such deviation will be made.

■

Proposition 6: *Under* $\underline{A}_1$-$\underline{A}_3$, *let* $m = x_p$, *where* $x_p > \bar{x}^*$. *There exists a pooling PBE in which m is offered if and only if (28) and (33) hold.*

Proof: As the proof is similar to those of Propositions 4 and 5, we leave it to the reader.

■

As a consequence of incentive compatibility, when $x_p < \underline{x}^*$ or $x_p > \bar{x}^*$, it is not always possible to have a given type of manager deviate by producing her starred revenue while the other type continues to produce x_p. For example, if $x_p < \underline{x}^*$, then deviations of the form $(x_p,\underline{x})$, $\underline{x} > x_p$, are impossible by incentive compatibility. Thus "joint" deviations must be considered: both types produce levels of output other than x_p. As the best such "joint" deviation is $(\bar{x}^*,\underline{x}^*)$ (at least when $\mu[x] = 0$ for $x \neq x_p$), we must compare the gain in first-period profits from that deviation with the loss in second-period profits.

It is worth noting that (27) and (28) imply (33). Consequently the sufficiency part of Proposition 4 can be extended to cover $x \in \mathbb{R}_+$:

Corollary 3: *Under $\underline{A}_1$-$\underline{A}_3$, (27) and (28) are sufficient conditions for there to exist a pooling equilibrium in which x_p is offered. Moreover, a pooling PBE exists if*

$$\bar{G}(\bar{x}^*) - \bar{G}(\underline{x}^*) \le \Pi(h,\bar{\theta}) - \Pi(0,\bar{\theta})$$

or if

$$\underline{G}(\underline{x}^*) - \underline{G}(\bar{x}^*) \le \Pi(h,\underline{\theta}) - \Pi(0,\underline{\theta})$$

Proof: Follows from the argument in the text and Propositions 4-6.

■

Define

$$\bar{\mathcal{P}} = \{x \mid \bar{G}(\bar{x}^*) - \bar{G}(x) \le \Pi(h,\bar{\theta}) - \Pi(0,\bar{\theta})\}$$

$$\underline{\mathcal{P}} = \{x \mid \underline{G}(\underline{x}^*) - \underline{G}(x) \le \Pi(h,\underline{\theta}) - \Pi(0,\underline{\theta})\}$$

$$\mathcal{P} = \bar{\mathcal{P}} \cap \underline{\mathcal{P}}$$

$\bar{\mathcal{P}}$ is the set of x satisfying (27), $\underline{\mathcal{P}}$ is the set of x satisfying (28), and $\mathcal{P}$ is the set satisfying both. Using this new notation we have

Proposition 7: Under $\underline{A}_1$-$\underline{A}_3$, the set of pooling mechanisms sustainable in a pooling PBE is convex, equal to $\mathcal{P}$ when $\mathcal{P} \subset (\underline{x}^*, \bar{x}^*)$, and empty if and only if $\mathcal{P} = \emptyset$.

Proposition 7 is proved in Appendix 2 and illustrated in Figures 2a and 2b respectively. In Figure 2a, the shaded interval (▨) illustrates the set of x which can be offered in some pooling equilibrium. Note, as drawn, the shaded area equals $\mathcal{P}$. Figure 2b illustrates a case with no pooling equilibria. In both figures the interval (⟷) represents $\underline{\mathcal{P}}$ and the interval (|—|) represents $\bar{\mathcal{P}}$.

Figure 2a

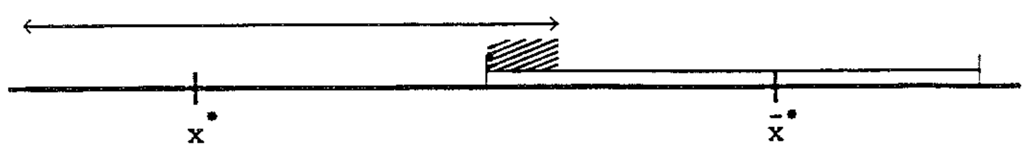
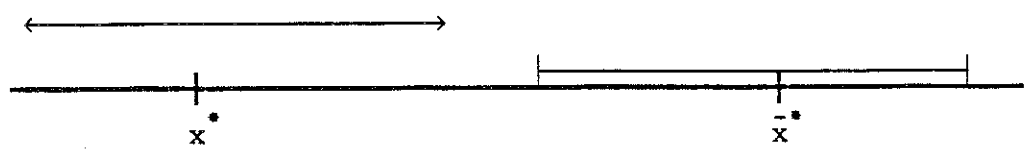


Figure 2b



A relationship between pooling equilibria and separating equilibria exists from the following:

$$\bar{\mathcal{P}} \subset \bar{\mathcal{X}} \text{ and } \underline{\mathcal{P}} \subset \underline{\mathcal{X}}.$$

Consequently

Proposition 8: *Under $\underline{A}_1$-$\underline{A}_3$, if $(\bar{x}, \underline{x}^*)$ can be offered in a separating equilibrium, $(\bar{x}, \bar{x})$ cannot be offered in a pooling equilibrium; if (x,x) can be offered in a pooling equilibrium, then $(x, \underline{x}^*)$ cannot be offered in a separating equilibrium.*

Proof: If $(\bar{x},\underline{x}^*)$ can be offered in a separating equilibrium, $\bar{x} \notin \underline{\mathfrak{X}}$. Hence $\bar{x} \notin \underline{\mathcal{P}}$. The second part of the proposition is just the contrapositive of the first part.

■

Note Proposition 8 does *not* say that for any x we can construct either a pooling equilibrium in which (x,x) is offered or a separating equilibrium in which $(x,\underline{x}^*)$ is offered. Indeed, it is possible that neither separating, nor pooling, equilibria exist: if the second-period profits from inducing beliefs h are not sufficient to prevent secret-separating deviations, but if, for the $\underline{\theta}$-type, the second-period profits from inducing beliefs 1 are not small enough to prevent secret pooling, then neither pooling nor separating equilibria will exist.

The next proposition presents comparative statics results on h. The proof can be found in Appendix 2.

Proposition 9: *Under $\underline{A}_1$-$\underline{A}_3$, there exists an $\underline{h}_p > 0$ such that for any h $\in (0,\underline{h}_p)$ no pooling equilibrium exists. If (34) below holds, then there exists an $\bar{h}_p \in (0,1)$ such that for any $h \in (\bar{h}_p,1)$ no pooling exists.*

$$\bar{G}(\vec{x}^*) - \bar{G}(\underline{\theta}) > \Pi(1,\bar{\theta}) - \Pi(0,\bar{\theta}) \tag{34}$$

If (34) is reversed (strictly), then for all intervals (H,1), H ≥ 0, ∃h ∈ (H,1) such that a pooling equilibrium exists.

The first part of Proposition 9 formalizes the intuition that when bad types are very likely, there is little to be gained from appearing average rather than bad. Consequently pooling equilibria cannot exist: restoring first-period efficiency more than compensates for the small loss in second-period profits. The second part of Proposition 9 pertains when $\theta = \underline{\theta}$ is very unlikely. In that case, for any mechanism

$(\bar{x},\underline{x})$, raising $\underline{x}$ essentially just raises the informational rent earned by the good type. Hence pooling at x_p much greater than $\underline{\theta}$ is not sustainable for large h, as P would secretly separate the bad type. Thus the only candidates for pooling equilibria for large h are x_p near $\underline{\theta}$ and they will be sustainable in a pooling equilibrium if and only if they are in $\bar{\mathcal{P}}$; which is to say only if (34) is reversed.

We conclude this section by comparing the pooling equilibria under the private negotiation and private contract assumption with the pooling equilibrium under the public negotiation and public contract assumption (Section 3). Under the public/public assumption, the only possible pooling equilibrium involved P offering m_0. Here, there may exist a multitude of pooling equilibria. Furthermore m_0 may not be one of the pooling equilibria (e.g. if $\underline{x}^{FB} \notin \bar{\mathcal{P}}$). As was the case with separating equilibria, the difference between the two sets of results follows from P's inability to communicate (credibly) information to T. As T is free to hold any out-of-equilibrium beliefs, deviating with m_0 need not induce beliefs $\mu = h$. Furthermore, as P cannot commit not to secretly separate, beliefs $\mu[x_0] = h$ can easily not be equilibrium beliefs.

4c/ Hybrid Equilibria

We noted in the previous sub-section that it was possible to have neither separating, nor pooling, equilibria exist. In such a situation, the equilibria will be *hybrid equilibria*: P plays a mixed strategy over offering pooling mechanisms and offering separating mechanisms.

For the sake of brevity, we merely show existence of a hybrid equilibrium when neither separating, nor pooling, equilibria exist.

In equilibrium, if P offers $(\bar{x},\underline{x}^*)$ with probability q and $(\bar{x},\bar{x})$

with probability 1-q, then $\mu[\underline{x}^*] = 0$ and

$$\mu[\bar{x}] = \frac{h}{(1-q)(1-h) + h} \tag{35}$$

by Bayes' Law. Note $\mu[\bar{x}] \in [h,1]$. Solving (35) for q:

$$q = \frac{\mu-h}{\mu(1-h)} \tag{36}$$

Our existence result:[12]

Proposition 10: *Under $\underline{A}_1$-$\underline{A}_3$, if no pooling nor separating equilibria exist, then there exists a hybrid equilibrium in which both $(\bar{x}^*, \underline{x}^*)$ (= m^*) and $(\bar{x}^*, \bar{x}^*)$ are offered with positive probability by P.*

Proof: Clearly $\bar{x}^* \in \bar{\mathcal{X}}$ and $\bar{x}^* \in \bar{\mathcal{P}}$. Consequently, from Proposition 7, $\bar{x}^* \notin \underline{\mathcal{P}}$ and, from Proposition 3, $\bar{x}^* \in \underline{\mathcal{X}}$. Hence

$$\underline{G}(\underline{x}^*) - \underline{G}(\bar{x}^*) > \Pi(h,\underline{\theta}) - \Pi(0,\underline{\theta})$$

and

$$\underline{G}(\underline{x}^*) - \underline{G}(\bar{x}^*) < \Pi(1,\underline{\theta}) - \Pi(0,\underline{\theta})$$

Therefore, $\exists \mu^*$ such that

$$\underline{G}(\underline{x}^*) - \underline{G}(\bar{x}^*) = \Pi(\mu^*,\underline{\theta}) - \Pi(0,\underline{\theta}) \tag{37}$$

Let q^* be derived from μ^* by (36). As $\mu^* \in (h,1)$, $q^* \in (0,1)$. Now the following is a PBE: P proposes m^* with probability q^* and $(\bar{x}^*,\bar{x}^*)$ with probability $(1-q^*)$ and T holds beliefs $\mu[\bar{x}^*] = \mu^*$ and $\mu[x] = 0$ $\forall x \neq \bar{x}^*$. Clearly T's beliefs satisfy Bayesian consistency. P's expected profits under m^* are

$$h\bar{G}(\bar{x}^*) + (1-h)\underline{G}(\underline{x}^*) + h\Pi(\mu^*,\bar{\theta}) + (1-h)\Pi(0,\underline{\theta}) \tag{38}$$

P's expected profits under $(\bar{x}^*,\bar{x}^*)$ are

$$h\bar{G}(\bar{x}^*) + (1-h)\underline{G}(\bar{x}^*) + h\Pi(\mu^*,\bar{\theta}) + (1-h)\Pi(\mu^*,\underline{\theta}) \tag{39}$$

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12 This is only an existence result, not a uniqueness result. In general there may exist a multiplicity of hybrid equilibria.

From (37), (38) equals (39), so P is willing to play a mixed strategy. From (38), any deviation $(\bar{x},\underline{x})$ $(\bar{x} \neq \bar{x}^*$, $\underline{x} \neq \underline{x}^*$, or $\underline{x} \neq \bar{x}^*)$ results in both lower first-period profits and, given T's beliefs, lower second-period profits than m^*. Thus P's strategy is a best response.

■

5/ PRIVATE NEGOTIATIONS BUT PUBLIC CONTRACTS

Now assume T does not observe the mechanism offered by P during negotiations. However T does observe the contract actually signed between P and M, i.e. (x,w).

As in the case of private contracts, T's inference problem is not trivial. T's beliefs must be defined for each $(x,w) \in \mathbb{R}^2_+$. It will generally be true for contracts (x,w) not part of an equilibrium mechanism that Bayes' Law cannot be applied, i.e. $\mu[(x,w)]$ is not fixed by the equilibrium strategies. For example, T can ignore his observation of the wage (i.e. $\forall w,w'$ $\mu[(x,w)] = \mu[(x,w')]$); thus, for a given set of parameters, all the equilibria found in Section 4 can be supported as equilibria in this case as well. Moreover, the freedom to specify beliefs as a function of w, as well as x, expands the set of equilibria. A complete characterization of this set would add little to the analysis, and we have therefore chosen not to include it.

We instead focus on the situation in which T's beliefs satisfy the following condition:

$\underline{A}_4$: $\mu[(x,w)] = 1$, if

$$w - \phi(x - \bar{\theta}) \geq 0 > w - \phi(x - \underline{\theta}) \qquad (40)$$

The justification for this restriction on beliefs is as follows: choosing a contract satisfying (40) would be a mistake for the $\underline{\theta}$-type, as she would have done better simply by quitting. Thus, on-the-equilibrium path $\mu[(x,w)] = 1$. Off-the-equilibrium path, observation of (x,w) is evidence that P deviated (or made a mistake), thus for T to hold beliefs $\mu[(x,w)] \neq 1$, would require T to hypothesize that deviations (mistakes) by P will be followed by mistakes by the $\underline{\theta}$-type. We find it difficult to justify requiring T to hypothesize sequential mistakes by P *and* the $\underline{\theta}$-type. Certainly such a requirement violates the spirit (though not, it should be made clear, the letter) of our solution concept.

We devote the rest of this section to studying the existence of separating and pooling equilibria under $\underline{A}_4$. As we will show, $\underline{A}_4$ yields strong predictions. For instance, consider separating equilibria:

Proposition 11: *Under $\underline{A}_1$-$\underline{A}_4$, there exists one and only one separating PBE (in terms of the path) and $\left((\bar{x}^*,\bar{w}^*),(\underline{x}^*,\underline{w}^*)\right)$ is the mechanism offered in that PBE.*

Proof: *Existence.* Suppose P offers $\left((\bar{x}^*,\bar{w}^*),(\underline{x}^*,\underline{w}^*)\right)$ and T holds beliefs $\mu[(x,w)] = 1$, if (x,w) satisfies (40), and $\mu[(x,w)] = 0$ otherwise. T's beliefs satisfy Bayesian consistency. Given T's beliefs and the fact that $\left((\bar{x}^*,\bar{w}^*),(\underline{x}^*,\underline{w}^*)\right)$ maximizes first-period profits, there is no profitable deviation.

Uniqueness. Let $\left((\bar{x},\bar{w}),(\underline{x},\underline{w})\right)$ be another separating mechanism offered in equilibrium. In this PBE, P's expected payoff is

$$h(\bar{x} - \bar{w}) + (1-h)(\underline{x} - \underline{w}) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(0,\underline{\theta})$$

Consider the deviation $\left((\bar{x}^*,\bar{w}^*),(\underline{x}^*,\underline{w}^*)\right)$: As $(\bar{x}^*,\bar{w}^*)$ satisfies (40), $\mu[(\bar{x}^*,\bar{w}^*)] = 1$, so the expected profits from deviating are

$$h(\bar{x}^* - \bar{w}^*) + (1-h)(\underline{x}^* - \underline{w}^*) + h\Pi(1,\bar{\theta}) + (1-h)\Pi\left(\mu[(\underline{x}^*,\underline{w}^*)],\underline{\theta}\right)$$

Clearly, this a profitable deviation; by contradiction we have proved uniqueness.

■

$\underline{A}_4$ fixes beliefs at $(\bar{x}^*,\bar{w}^*)$ to be 1, thus, as in the public/public case, offering m^* generates the best beliefs within the class of separating mechanisms. Therefore, as m^* maximizes first-period profits, only m^* is sustainable as a separating equilibrium.

We now consider pooling equilibria. Profits in a pooling equilibrium in which P offers $m_p = \left((x_p,w_p),(x_p,w_p)\right) \equiv (c_p,c_p)$ are

$$x_p - w_p + h\Pi(h,\bar{\theta}) + (1-h)\Pi(h,\underline{\theta})$$

There are four possible deviations to consider: 1) offering m^* (*best separating*); 2) offering $(\bar{c},c_p)$ where $\bar{c}$ and c_p satisfy (1a) and $\bar{c}$ fails (40) (*covert separation of the $\bar{\theta}$-type*); 3) offering $(c_p,\underline{c})$ where c_p and $\underline{c}$ satisfy (1b) (*covert separation of the $\underline{\theta}$-type*); and 4) offering $(\bar{c},c_p)$ where $\bar{c}$ and c_p satisfy (1a) and $\bar{c}$ satisfies (40) (*overt separation of the $\bar{\theta}$-type*). As the intuition behind the first three deviations is similar to that previously given (there is a close relationship between these deviations and conditions (33), (27), and (28) respectively), we will omit further discussion of those deviations here. Instead we focus on the fourth deviation. Intuition for what follows can be gained from

Lemma 2: *If $\Pi(1,\bar{\theta}) > \Pi(\mu,\bar{\theta})$ $\forall\mu < 1$, then no pooling mechanism, in which*

$$w_p - \phi(x_p - \underline{\theta}) = 0$$

is offered with positive probability in equilibrium.

Proof: Suppose not; P's expected equilibrium payoff is

$$x_p - w_p + h\Pi(\mu[c_p],\bar{\theta}) + (1-h)\Pi(\mu[c_p],\underline{\theta}) \tag{41}$$

where $c_p = (x_p,w_p)$ (and $m_p = (c_p,c_p)$). As m_p is offered with positive probability, $\mu[c_p] < 1$. [As c_p may be part of another mechanism offered

with positive probability, $\mu[c_P]$ need not equal h (the prior).]
Consider other mechanisms of the form $m(\varepsilon) = (\bar{c}(\varepsilon), c_P)$, where

$$\bar{c}(\varepsilon) = \left(x_P + \varepsilon,\ w_P + \phi(x_P+\varepsilon-\bar{\theta}) - \phi(x_P-\bar{\theta})\right)$$

$(\varepsilon > 0)$. Note that $m(\varepsilon)$ is feasible and that $\bar{c}(\varepsilon)$ satisfies (40). Thus P's expected payoff under $m(\varepsilon)$ is

$$x_P - w_P + h\left(\varepsilon - \phi(x_P+\varepsilon-\bar{\theta}) + \phi(x_P-\bar{\theta})\right) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(\mu[c_P],\underline{\theta}) \tag{42}$$

As $\phi(\cdot)$ is continuous and $\mu[c_P] < 1$, there exists an ε such that (42) is strictly greater than (41). Thus m_P cannot be offered with positive probability in equilibrium.

■

Intuitively, unless some rent is left to the *bad* type, P would deviate from m_P by offering a mechanism that induced the good type to produce slightly more than x_P, but under a contract which the bad type would never choose. Under this deviation, first-period profits would be essentially the same as under m_P; but as the good type is unambiguously identified, second-period profits are strictly greater.

If, in contrast to Lemma 2, some rent is left to the bad type, then deviations of the form $(\bar{c}, c_P)$, where $\bar{c}$ satisfies (40), become more expensive for P: $\bar{c}$ must induce very different first-period profits from the good type. If those first-period profits are sufficiently less than under m_P, P will not wish to deviate in this way. Formally, $\bar{c}$ must satisfy (40) and be feasible. Given (40), feasibility reduces to (1a). Furthermore, as we need consider only optimal deviations, we can restrict attention to $\bar{c}$ which satisfy (1a) with equality:

$$\bar{w} = \phi(\bar{x} - \bar{\theta}) + w_P - \phi(x_P - \bar{\theta}) \equiv W(x, c_P) \tag{43}$$

Let

$$Q(m_P) = \left\{x \mid \left(x, W(x,c_P)\right) \text{ satisfies (40)}\right\}$$

$Q(m_p)$ is the set of x on the portion of the $\bar{\theta}$-type's indifference curve through c_p which lies below the $\underline{\theta}$-type's IR constraint (see Figure 3).

Now if m_p can be offered in a pooling PBE, then it must be that

$$x_p - w_p + h\Pi(h,\bar{\theta}) + (1-h)\Pi(h,\underline{\theta}) \geq$$

$$h\left(\bar{x} - \bar{w}\right) + (1-h)\left(x_p - w_p\right) + h\Pi(1,\bar{\theta}) + (1-h)\Pi(h,\underline{\theta}) \quad (44)$$

for any overt deviation (i.e. any $(\bar{x},\bar{w})$ satisfying (40) and (1a)). In particular, if (43) holds, condition (44) can be rewritten as

$$\forall\bar{x} \in Q(m): \quad \bar{G}(x_p) - \bar{G}(\bar{x}) \geq \Pi(1,\bar{\theta}) - \Pi(h,\bar{\theta}) \quad (45)$$

From (45), if $\bar{x}^* \in Q(m_p)$, then (44) fails: clearly if having the $\bar{\theta}$-type produce her first-best output is incentive compatible given c_p and if that identifies her as the $\bar{\theta}$-type, then m_p cannot be sustained as a pooling equilibrium. Thus a necessary condition for m_p to be offered in a pooling equilibrium is

$$\bar{x}^* \notin Q(m_p) \quad (46)$$

If (46) is met, any $\bar{x}$ in $Q(m_p)$ must be larger than $\bar{x}^*$. From $\underline{A}_1$ and the convexity of $Q(m_p)$, (45) must hold for $\bar{x} = X(c_p) \equiv \inf Q(m_p)$ by continuity. As in Lemma 2, the intuition can be understood by considering deviations $c(\varepsilon) = \left(x(\varepsilon),w(\varepsilon)\right)$, where $x(\varepsilon) = X(c_p) + \varepsilon$. Thus, if (46) is met, (45) becomes

$$\bar{G}(x_p) - \bar{G}\left(X(c_p)\right) \geq \Pi(1,\bar{\theta}) - \Pi(h,\bar{\theta}) \quad (47)$$

The losses in first-period profits from any deviation $c(\varepsilon)$ must exceed the gains in second-period profits. A necessary condition for $m_p = (c_p,c_p)$ to be offered in a pooling PBE is that (46) and (47) hold. We need only retain (47) as a necessary condition, as (47) implies (46). [Proof: if (46) were not true, then $x_p < X(c_p) < \bar{x}^*$ ($x_p < X(c_p)$, since c_p is individually rational for $\underline{\theta}$); but then $\bar{G}(x_p) < \bar{G}\left(X(c_p)\right)$, contradicting (47).]

Figure 3

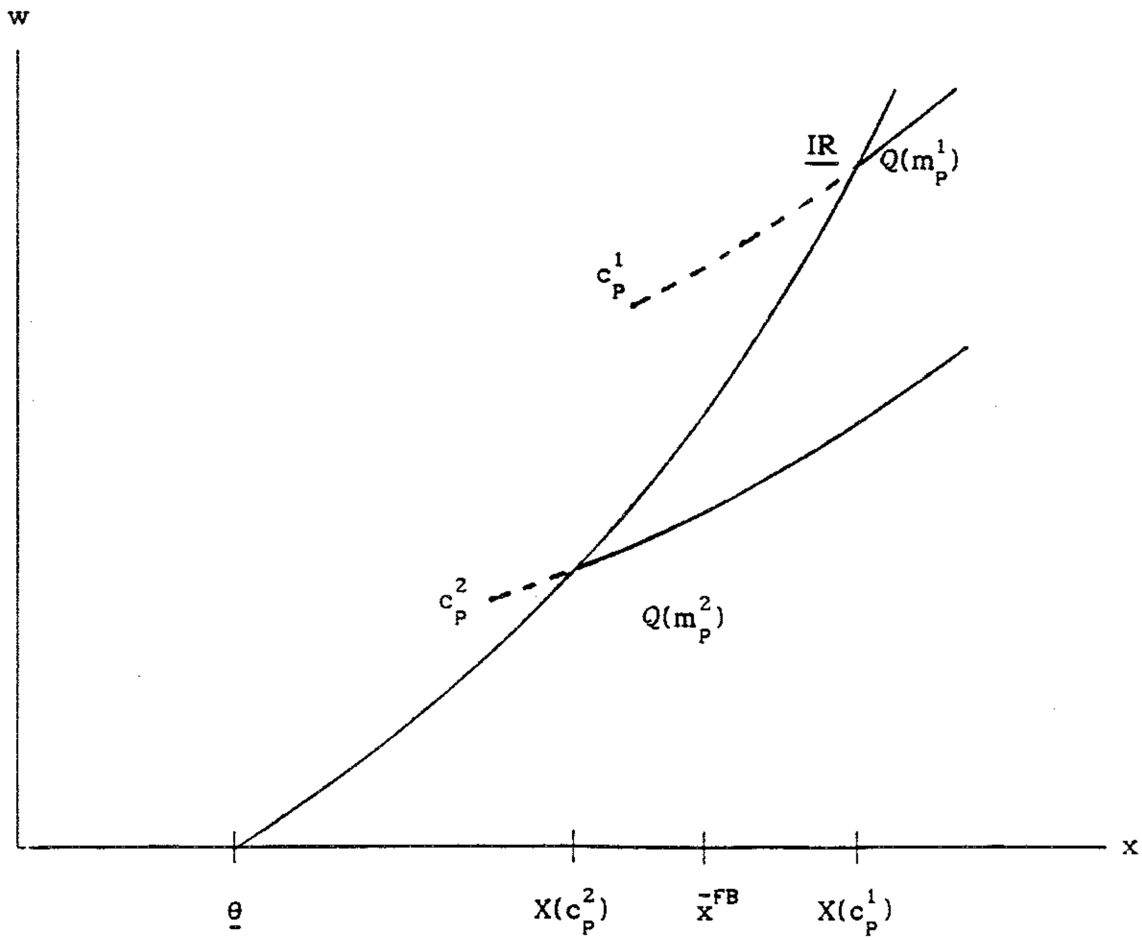


Figure 4 illustrates condition (47). Condition (47) defines a set of contracts (x_p, w_p) which are above the $\underline{\theta}$-type's IR curve and above the $\bar{\theta}$-type's indifference curve through $\left(\bar{x}', \phi(\bar{x}'-\underline{\theta})\right)$, where $\bar{x}'$ is defined by

$$\bar{G}(\bar{x}') = \bar{G}(\bar{x}^*) - \Pi(1,\bar{\theta}) + \Pi(h,\bar{\theta})$$

So the good manager's rent must be at least $\phi(\bar{x}'-\underline{\theta}) - \phi(\bar{x}'-\bar{\theta})$.

We now state our main result concerning pooling PBE:

Proposition 12: *Given $\underline{A}_1$-$\underline{A}_4$, a pooling mechanism m_p can be offered in a pooling PBE if and only if (x_p, w_p) satisfies (47), (48), (49), and (50).*

$$x_p - w_p \geq$$
$$h\bar{G}(\bar{x}^*) + (1-h)\underline{G}(\underline{x}^*) + h\left(\Pi(1,\bar{\theta})-\Pi(h,\bar{\theta})\right) + (1-h)\left(\Pi(0,\underline{\theta})-\Pi(h,\underline{\theta})\right) \qquad (48)$$

$$x_p \geq \bar{x}^* \text{ or } \bar{G}(\bar{x}^*) - \bar{G}(x_p) \leq \Pi(h,\bar{\theta}) - \Pi(0,\bar{\theta}) \qquad (49)$$

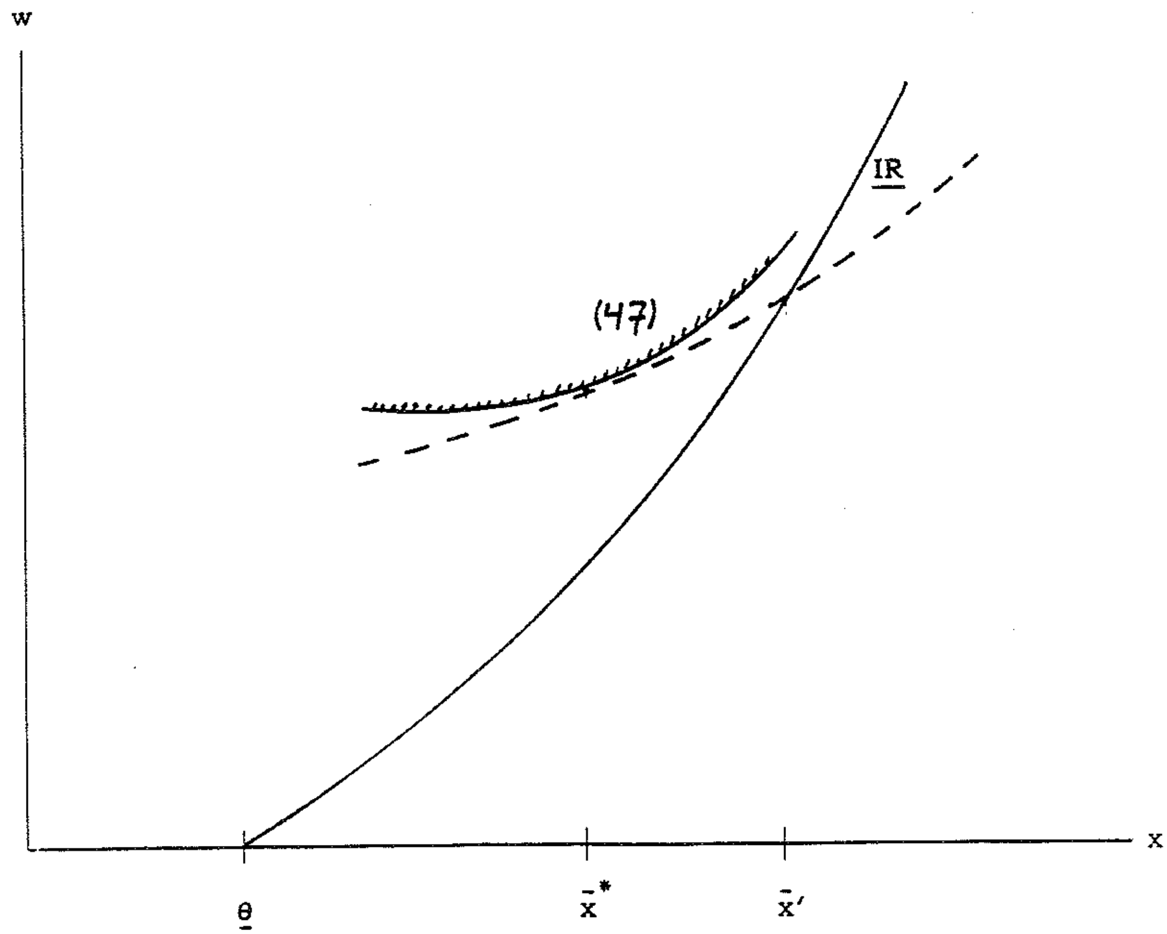
$$\underline{x}^{FB} \geq x_p \text{ or } \left(\underline{x}^{FB} - \phi(\underline{x}^{FB}-\underline{\theta})\right) - \left(x_p - \phi(x_p-\underline{\theta})\right) \leq \Pi(h,\underline{\theta}) - \Pi(0,\underline{\theta}) \qquad (50)$$

Proof (sketch): The necessity of (47) has been proved in the text. (48) is necessary to rule out the best separating deviation (deviation 1), while (49) and (50) rule out the covert separating deviations (deviations 2 and 3 respectively). To prove sufficiency, we will show that it is a PBE for P to offer m_p, where (x_p, w_p) satisfies (47) - (50) and when T holds the following beliefs

$$\mu[c] = \begin{cases} h, & \text{if } c = c_p \\ 1, & \text{if } c \text{ satisfies (40)} \\ 0, & \text{if } c \neq c_p \text{ and } c \text{ fails (40)} \end{cases}$$

Clearly T's beliefs satisfy Bayesian consistency (as well as $\underline{A}_4$). Given T's beliefs, any deviation of the form $(\bar{c}, \underline{c})$, $\bar{c} \neq c_p$ and $\underline{c} \neq c_p$, is dominated by m_p from (48) ((48) establishes that m_p dominates m^* which is the best deviation in that class). Similarly, overt-separating deviations are dominated by m_p from (47); and covert-separating

Figure 4



deviations are dominated by m_P from either (49) or (50).

∎

The conclusion of Proposition 12 can be summarized as follows: if P has an incentive *ex post* to reveal a good type, then pooling equilibria are necessarily very costly for P because of the rent left to $\bar{\theta}$-types *and* to $\underline{\theta}$-types. Pooling is more likely to exist if it is more important *ex post* for P to conceal a bad type than to reveal a good type. If *ex post* for some type, P does not much care how he looks to T, then pooling has to require almost efficient production from this type.

6/ EXAMPLES

In this section, we present two examples to illustrate the economic content of our results. The examples deal with entry: P owns an incumbent monopoly, M is his manager, and T is a potential entrant.

The two examples are very simple; particularly, as we assume that for one value of θ, P's second-period profits are independent of T's beliefs. We adopt this extreme assumption to facilitate intuition. In this sense, the examples are more illustrative than realistic.

6a/ Entry Deterrence with Very Efficient Firms

θ is the firm's productivity and can be complemented in the first period by managerial effort e. Second-period profits are $\pi_P^M(\theta)$ if P remains a monopolist and $\pi_P^D(\theta)$ if entry occurs. T's profits are $\pi_T(\theta) - \sigma$, if he enters, and 0 otherwise. σ is the fixed cost of entry. From P's perspective, σ is a random variable with continuous distribution F(•) on $\mathbb{R}_+$. For an efficient firm ($\theta = \bar{\theta}$), the monopoly price is less than the entrant's unit cost: $\pi_P^M(\bar{\theta}) = \pi_P^D(\bar{\theta})$ and

$\pi_T(\bar{\theta}) = 0$. For an inefficient firm, $\pi_P^M(\underline{\theta}) > \pi_P^D(\underline{\theta})$ and $\pi_T(\underline{\theta}) > 0$.

A potential entrant compares $(1-\mu)\pi_T(\underline{\theta})$ to σ; where μ is T's posterior probability assessment that $\theta = \bar{\theta}$. Thus entry occurs with probability $F\left((1-\mu)\pi_T(\underline{\theta})\right)$ (from P's perspective). We thus have

$$\Pi(\mu,\bar{\theta}) = \pi_P^M(\bar{\theta})$$

$$\Pi(\mu,\underline{\theta}) = F\left((1-\mu)\pi_T(\underline{\theta})\right)\pi_P^D(\underline{\theta}) + \left[1 - F\left((1-\mu)\pi_T(\underline{\theta})\right)\right]\pi_P^M(\underline{\theta})$$

Note $\Pi(\mu,\underline{\theta})$ is increasing in μ, so assumption $\underline{A}_3$ is satisfied.

The Public Negotiation and Public Contracts Case

A pooling equilibrium at $m_0 = (x_0,x_0)$ will exist if and only if the first-period loss from inefficient production (both types producing $\underline{x}^{FB}$) is less than the second-period loss from revealing that the firm has low productivity (thus increasing the probability of entry). Note pooling will occur at a relatively *low* level of revenue. If pooling does not exist, then the equilibrium will be separating with P offering the optimal mechanism $m^* = (\bar{x}^*,\underline{x}^*)$. If the equilibrium is separating, the entrant is fully informed when deciding whether to enter.

The Private Negotiation and Private Contracts Case

In this case, we have $\bar{\mathcal{X}} = \bar{\mathcal{P}} = \{\bar{x}^*\}$; as beliefs do not matter if $\theta = \bar{\theta}$ (as, in that case, entry does not matter), profit maximization means P must require the $\bar{\theta}$-type to produce her optimal level of revenue (provided that is incentive compatible). Thus the separating equilibrium, if it exists, is unique, with P offering the optimal mechanism m^*. This separating equilibrium exists if and only if inducing a $\underline{\theta}$-type manager to produce a high revenue $\bar{x}^*$ is more costly than the gain from fooling the entrant (making him believe $\theta = \bar{\theta}$).

The only candidates for pooling equilibria are (x,x) where x represents a *high* level of revenue ($x \geq \bar{x}^*$ and $x \in \underline{\mathcal{P}}$). Such a pooling equilibrium exists if and only if inducing a $\underline{\theta}$-type manager to produce such a high revenue generates a loss smaller than the corresponding gain from not revealing $\theta = \underline{\theta}$ (thus deterring entry).

Finally, if neither the separating equilibrium, nor the pooling equilibria exist (i.e. if $\bar{x}^* \in \underline{\mathcal{X}}$, but $\bar{x}^* \notin \underline{\mathcal{P}}$), then it is straightforward to show that there exists a unique hybrid equilibrium in which P mixes between $(\bar{x}^*,\underline{x}^*)$ and $(\bar{x}^*,\bar{x}^*)$.[13]

The Private Negotiation but Public Contracts Case

In this case, we know (Proposition 11) that there exists a unique separating equilibrium in which P proposes m^* (under $\underline{A}_4$). In such an equilibrium, the entrant has full information when deciding to enter.

We also note that Lemma 2 does not hold here, as $\Pi(\mu,\bar{\theta})$ is independent of μ. When the gains from not revealing that $\theta = \underline{\theta}$ are large, there exist pooling equilibria at *high* levels of revenue ($x_p \geq \bar{x}^*$) and high wages ($w_p \geq \phi(\bar{x}^* - \underline{\theta})$).

Summary

We summarize by noting that both pooling and separating equilibria are possible. If the equilibrium is separating, it is unique with the optimal mechanism $m^* = (\bar{x}^*,\underline{x}^*)$ being offered: as the $\underline{\theta}$ type is identified and as P does not care what beliefs he induces when $\theta = \bar{\theta}$,

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[13] Since beliefs do not matter if $\theta = \bar{\theta}$, the only separating mechanism P will offer with positive probability is $(\bar{x}^*,\underline{x}^*)$. A detailed proof is available from the authors.

both types must be required to produce their optimal first-period revenues. In the private negotiation cases, pooling equilibria involve pooling at levels of x greater than (or equal to) $\bar{x}^*$: as P does not care about beliefs when $\theta = \bar{\theta}$, pooling is sustainable only if the best separating deviation (by the $\bar{\theta}$-type) is not incentive compatible. Entry deterrence is achieved by *overprovision* of managerial effort, high first-period revenues, and high managerial rents.

6b/ Entry Deterrence with Inefficient Firms

Same notation as in the previous example. We now assume that $\pi_P^M(\underline{\theta}) < 0$ and $\pi_P^M(\bar{\theta}) > \pi_P^D(\bar{\theta}) > 0$. The assumption $\pi_P^M(\underline{\theta}) < 0$ might apply to declining industries where inefficient firms can no longer cover their fixed costs. Finally let π_T^M be the entrant's gross profits, if he captures the entire market. Now P will exit if $\theta = \underline{\theta}$ (regardless of T's action), so T's expected gross profit from entry is $(1-\mu)\pi_T^M$ and the probability of entry is $F\left((1-\mu)\pi_T^M\right)$ (without loss of generality we continue to assume $\pi_T(\bar{\theta}) = 0$). We thus have

$$\Pi(\mu,\underline{\theta}) \equiv 0$$

$$\Pi(\mu,\bar{\theta}) = F\left((1-\mu)\pi_T^M\right)\pi_P^D(\bar{\theta}) + \left[1 - F\left((1-\mu)\pi_T^M\right)\right]\pi_P^M(\bar{\theta})$$

$\Pi(\mu,\bar{\theta})$ is increasing in μ, so $\underline{A}_3$ is satisfied.

The Public Negotiation and Public Contracts Case

Condition (11) fails; the only equilibrium in this case is the separating equilibrium in which P offers m^*. As there is no gain to be had from fooling T about θ when $\theta = \underline{\theta}$, there is no benefit to pooling. Furthermore, as pooling is costly -- one, because it lowers second-period profits when $\theta = \bar{\theta}$ (relative to separating), and two, because it lowers first-period profits (again, relative to separating) -- the pooling equilibrium (offering m_0) cannot exist.

The Private Negotiation and Private Contracts Case

Now, we have $\underline{\mathfrak{X}} = \underline{\mathcal{P}} = \{\underline{x}^*\}$. Hence the optimal first-period mechanism $m^* = (\bar{x}^*, \underline{x}^*)$ is sustainable as part of a separating equilibrium; however, unlike the previous example, there is no unique separating equilibrium: any $(x, \underline{x}^*)$, $x \in \bar{\mathfrak{X}}$, is sustainable as part of a separating equilibrium. As regards pooling equilibria, the set of pooling equilibria is a subset of $(-\infty, \underline{x}^*] \cap \bar{\mathcal{P}}$; pooling equilibria can be constructed if and only if $(-\infty, \underline{x}^*] \cap \bar{\mathcal{P}} \neq \emptyset$. Pooling equilibria exist only if deterring entry is important for a good firm, i.e. if duopoly profits are much smaller than monopoly profits, in which case the principal would be willing to except low first-period profits in order to deter entry. Note that, unlike the previous example, pooling occurs only at *low* levels of x ($x \leq \underline{x}^*$); only if best separating by the $\underline{\theta}$-type is infeasible, can pooling occur.

Note that separating equilibria other than m^* and all pooling equilibria are sustained by T's beliefs that large revenues signal that the firm is *inefficient* (i.e. is $\underline{\theta}$). Such beliefs seem unreasonable in this situation. In Appendix 3, we present a refinement concept which eliminates them.

The Private Negotiation but Public Contracts Case

Condition (48), a necessary condition for a pooling equilibrium, fails; thus, as in the public negotiation case, the unique equilibrium is the separating equilibrium in which m^* is offered. The intuition is also the same: pooling is costly and produces no gain, whereas (overt) separating maximizes both first and second-period profits.

7/ EMPIRICAL IMPLICATIONS

A growing body of empirical work (Jensen and Murphy [1988], Gibbons and Murphy [1989], and Leonard [1989]) has found little evidence for the basic, "desert-island", principal-agent model. Regressions of compensation on performance find that performance has only a small effect on compensation. Our work offers an explanation for these results: the basic model predicts that compensation will be very responsive to performance, as the basic model implies separation. However, if economic reality is better described by the pooling equilibria which can arise due to outside considerations, then compensation can seem unresponsive to performance. Econometrically, a world of separating equilibria would appear as two clouds of data points, one around $(\underline{x}^*,\underline{w}^*)$ and another around $(\bar{x}^*,\bar{w}^*)$ (assuming the public negotiation and public contracts case for the sake of illustration). Even if there were a large amount of measurement error (i.e. each cloud is fairly disperse about (x^*,w^*)), the econometrician should still detect a strong relationship between performance and compensation. In contrast, a world of pooling equilibria would appear as one cloud of data points around (x_0,w_0) (again assuming the public negotiation and public contracts case). All the econometrician would detect is noise. Of course, we recognize that different equilibria would likely hold across different firms and different industries; yet in a world which tends to be characterized by pooling equilibria, even where pooling occurs at different points across firms/industries, the signal-to-noise ratio in a regression of compensation on performance would be lower than the signal-to-noise ratio in a world which tends to be characterized by separating equilibria (such as the "desert-island" world). If the signal-to-noise ratio is sufficiently low, then the econometrician will fail to find a strong relationship between performance and compensation.

8/ CONCLUSION

In this paper, we analyzed a principal-agent relationship under incomplete information where the principal foresees a future interaction with a third party. As the parameter of asymmetric information in the principal-agent relationship also plays a role in the interaction between the principal and the third party, the principal will be sensitive to what information is revealed through the principal-agent relationship. Consequently, the principal may alter both the output targets assigned to the agent and the agent's compensation schedule relative to the no-third-party situation; the motivation being the concealment of information detrimental to the principal. Moreover, the form of this contract is heavily dependent on the observability of the contracting game by the third party; both targets and compensation will depend on whether contract *negotiations* are observable and whether the contracts *themselves* are observable, as well as on the importance of revealing or concealing information.

In the no-third-party situation, the principal would implement a compensation scheme which fully reveals the relevant information to the outside, because such a scheme maximizes the principal's profits from the principal-agent relationship. Concealing information requires implementing other schemes, and consequently means lower first-period profits for the principal. Thus concealing information is costly to the principal. When there is little or no gain to concealing information, the equilibrium will, in all likelihood, consist of the principal implementing the optimal no-third-party scheme even in the presence of a third party. When there is a large gain to concealing information, then it is more likely that the equilibrium will consist of the principal implementing one of the information-concealing schemes.

We employed our general results to shed some light on entry deterrence when an incumbent monopolist (the principal) can affect his potential entrants' decision to enter by manipulating their his productivity. Unlike other approaches (e.g. Milgrom and Roberts' [1982]), our game is not a signaling (nor a signal jamming) game. Furthermore, in many interesting cases, the nature of the equilibrium, i.e. its informational content for the third party, is unique (uniqueness which is usually obtained without resorting to refinements). More importantly, the cost of separating or pooling is endogenous: it is embodied in the distortions of the contract compared to the no-third-party situation. This cost arises through a distortion in the firm's physical output and, possibly, through an increase in the informational rent earned by the manager.

In a companion paper, we apply our general conclusions to a model of takeovers. There, the third party is a potential buyer of the firm's shares, who will make a bid given his own (random) valuation for the assets of the firm. The model illustrates how the threat of takeovers affects the managerial compensation scheme and the production of the firm, according to different observability assumptions: it identifies the adverse effects, thereby suggesting new elements in the debate on the desirability of takeovers.

We conclude on a somewhat pessimistic note about principal-agent theory. As we suggested in the introduction, the predictions of simple (two-player) principal-agent models are generally not robust to the introduction of outside considerations. Furthermore the predictions of a principal-agent model with outside considerations is very sensitive to the assumptions one makes about what aspects of the principal-agent relationship are observable to the outside. As such, this point stresses the importance of informational assumptions in economic theory based on sensible empirical analysis.

APPENDIX 1: Direct Revelation Mechanisms

In this appendix we justify restricting attention to direct revelation mechanisms (DRM). We also discuss our assumption of deterministic mechanisms (see footnote 6).

For what follows, we consider any set of characteristics Θ and allow the set of potential messages $\mathcal{A}$ to be richer than the set of characteristics.

Suppose P offers the mechanism $(X(a),W(a))_{a\in\mathcal{A}}$ and let $a^*(\cdot)$ from Θ to $\mathcal{A}$ be M's strategy in the PBE. P could design a direct mechanism in the usual way $x(\theta) \equiv X(a^*(\theta))$ and $w(\theta) \equiv W(a^*(\theta))$. This DRM would yield separation of M's types via the announcements (although possibly not via the actual contract). *Under private negotiations*, T could not observe this deviation. And this deviation would be profitable, since given any beliefs T might have on P's information partition, P would be weakly better off with a finer information partition (fully separating partition). Thus, under *private negotiations*, we can extend the Revelation Principle to cover our game, which justifies restricting attention to DRMs in that case. Of course, if there were no third party, then we could also appeal to the Revelation Principle (i.e. we are justified in restricting attention to DRMs in Section 2).

The previous proof can fail in the case of public negotiations. As T observes the mechanism, T knows exactly P's information partition at the beginning of Γ_2. Therefore, through the choice of mechanism, P can commit to a specific information partition when playing Γ_2. It is easy to construct games where P would prefer to commit *not to have perfect information*. A simple example, with $h = 1/2$, is

$\underline{\Gamma_2}$

T

P

		L	R
U		2,0	0,-1
D	$\bar{\theta}$	3,0	1,2
	$\underline{\theta}$	0,0	-2,3

If P commits to no information (and thus T has no information), the unique equilibrium of Γ_2 is (U,L), which yields P an expected payoff of 2. If P does not commit, then (assuming T still has no information), the unique equilibria are (U,R) (if $\theta = \underline{\theta}$) and (D,R) (if $\theta = \bar{\theta}$), which yield P an expected payoff of 1/2. If P does not commit and T learns θ, then the unique equilibria are (U,L) (if $\theta = \underline{\theta}$) and (D,R) (if $\theta = \bar{\theta}$), which yield P an expected payoff of $1\frac{1}{2}$. Thus P loses unless he commits not to learn θ.

Note that even in the *public* negotiations case, we could appeal to the Revelation Principle if P expects to do better in Γ_2 when he is committed to know θ than when he is not.

Now we turn to the question of deterministic mechanisms.[14] For convenience, we return to our assumption that all mechanisms are DRMs. However now we consider the possibility that announcing $\hat{\theta}$ does not fix a pair $\left((x(\hat{\theta}), w(\hat{\theta})\right)$ but rather a lottery over some specified $\mathcal{E}(\hat{\theta}) \subset \mathbb{R}^2$. As P is risk-neutral, while M is income risk-neutral but effort risk-averse, P would never offer a random mechanism in a world without a third party (i.e. the world considered in Section 2). In Section 4, as

[14] We are grateful to John Litwack for suggesting a discussion of this issue.

T observes neither the mechanism, nor the chosen contract, and as P wishes to minimize costs, P would never offer random mechanisms. If P wishes to randomize (e.g. in a hybrid equilibrium), he will randomize *over* mechanisms, not offer random mechanisms.

When T observes the contract, the restriction of deterministic mechanisms could matter depending on what is observed by T: does T observe which lottery is chosen or only the outcome of the lottery? If the former, then there is no advantage to random mechanisms: they cannot affect T's beliefs differently than deterministic mechanisms and they are more costly. If the latter, then there may be a trade-off between the benefits of affecting T's beliefs and the cost of randomization. However, it is our feeling that assuming T can observe only the outcome and not the chosen lottery is not in keeping with the spirit of the public contracts assumption.

APPENDIX 2: Proofs

Proof of Proposition 7

Let $\mathcal{M}_p$ be the set of points (mechanisms) sustainable as a pooling PBE (i.e. $x \in \mathcal{M}_p$ means there exists a pooling PBE in which x is offered). We will first show that $\bar{\mathcal{P}}$ and $\underline{\mathcal{P}}$ are convex and that if $\mathcal{M}_p \subset [\underline{x}^*, \bar{x}^*]$, then $\mathcal{M}_p = \mathcal{P}$, then use that to prove $\mathcal{M}_p = \emptyset$ if and only if $\mathcal{P} = \emptyset$. We will then prove that $\mathcal{M}_p$ is convex.

$\bar{\mathcal{P}}$ and $\underline{\mathcal{P}}$ convex: The convexity of $\bar{\mathcal{P}}$ and $\underline{\mathcal{P}}$ follows because $\bar{G}(\bar{x}^*) - \bar{G}(x)$ and $\underline{G}(\underline{x}^*) - \underline{G}(x)$ are strictly quasi-convex functions of x.

If $\mathcal{M}_p \subset [\underline{x}^*, \bar{x}^*]$, then $\mathcal{M}_p = \mathcal{P}$: Follows from Proposition 4.

If $\mathcal{P} = \emptyset$, then $\mathcal{M}_p = \emptyset$: $\underline{x}^* \in \underline{\mathcal{P}}$ and $\bar{x}^* \in \bar{\mathcal{P}}$. As $\mathcal{P} = \emptyset$, $\underline{x}^* \notin \bar{\mathcal{P}}$ and $\bar{x}^* \notin \underline{\mathcal{P}}$; hence, by the convexity of $\bar{\mathcal{P}}$ and $\underline{\mathcal{P}}$, $(-\infty, \underline{x}^*] \cap \bar{\mathcal{P}} = \emptyset$ and $[\bar{x}^*, \infty) \cap \underline{\mathcal{P}} =$

$\emptyset$. So, by Propositions 5 and 6, $\mathcal{M}_p \subset [\underline{x}^*, \bar{x}^*]$; hence $\mathcal{M}_p = \mathcal{P} = \emptyset$.

If $\mathcal{M}_p = \emptyset$, *then* $\mathcal{P} = \emptyset$: $x \in \mathcal{P}$ are sustainable as pooling PBEs by Proposition 4, hence $\mathcal{P} = \emptyset$.

$\mathcal{M}_p$ *is convex:* Define $\mathcal{Z} = \{x \mid x \text{ satisfies (33)}\}$. As $h\left(\bar{G}(\bar{x}^*) - \bar{G}(x)\right) + (1-h)\left(\underline{G}(\underline{x}^*) - \underline{G}(x)\right)$ is a strictly quasi-convex function of x, $\mathcal{Z}$ is convex. From Propositions 4-6:

$$\mathcal{M}_p = \left((-\infty,\underline{x}^*] \cap \bar{\mathcal{P}} \cap \mathcal{Z}\right) \cup \left([\underline{x}^*,\bar{x}^*] \cap \bar{\mathcal{P}} \cap \underline{\mathcal{P}}\right) \cup \left([\bar{x}^*,\infty) \cap \underline{\mathcal{P}} \cap \mathcal{Z}\right)$$

That is $\mathcal{M}_p$ is the union of three convex sets. The proof is complete if we can show (a) $\left((-\infty,\underline{x}^*] \cap \bar{\mathcal{P}} \cap \mathcal{Z}\right)$ and $\left([\underline{x}^*,\bar{x}^*] \cap \bar{\mathcal{P}} \cap \underline{\mathcal{P}}\right)$ are non-disjoint or $\left((-\infty,\underline{x}^*] \cap \bar{\mathcal{P}} \cap \mathcal{Z}\right) = \emptyset$ and (b) $\left([\underline{x}^*,\bar{x}^*] \cap \bar{\mathcal{P}} \cap \underline{\mathcal{P}}\right)$ and $\left([\bar{x}^*,\infty) \cap \underline{\mathcal{P}} \cap \mathcal{Z}\right)$ are non-disjoint or $\left([\bar{x}^*,\infty) \cap \underline{\mathcal{P}} \cap \mathcal{Z}\right) = \emptyset$. Now if $\underline{x}^* \in \bar{\mathcal{P}}$, then $\underline{x}^* \in \mathcal{Z}$ (recall $\underline{x}^* \in \underline{\mathcal{P}}$) and if $\underline{x}^* \notin \bar{\mathcal{P}}$, then $(-\infty,\underline{x}^*] \cap \bar{\mathcal{P}} = \emptyset$; hence either $\left((-\infty,\underline{x}^*] \cap \bar{\mathcal{P}} \cap \mathcal{Z}\right)$ and $\left([\underline{x}^*,\bar{x}^*] \cap \bar{\mathcal{P}} \cap \underline{\mathcal{P}}\right)$ are non-disjoint (they both contain $\underline{x}^*$) or $\left((-\infty,\underline{x}^*] \cap \bar{\mathcal{P}} \cap \mathcal{Z}\right) = \emptyset$. Thus, we have shown (a). (b) is shown by a similar argument.

■

Proof of Proposition 9

We note that $\lim_{h\to 0} \underline{\mathcal{P}} = \left\{\lim_{h\to 0} \underline{x}^*\right\} = \{\underline{x}^{FB}\}$ and $\lim_{h\to 0} \bar{\mathcal{P}} = \{\bar{x}^*\}$. [Note these and the other limits considered in this proof exist as a consequence of the continuity of $\Pi(\mu,\theta)$.] As $\underline{x}^{FB} < \bar{x}^{FB} = \bar{x}^*$, $\lim_{h\to 0} \mathcal{P} = \emptyset$; which proves the first part of the proposition. For the second part of proposition, note that $\lim_{h\to 1} \underline{\mathcal{P}} \cap [\underline{x}^*,\bar{x}^*] = \left\{\lim_{h\to 1} \underline{x}^*\right\} = \{\underline{\theta}\}$, so as $h \to 1$, if $\underline{\theta} \notin \lim_{h\to 1} \bar{\mathcal{P}}$, i.e. if (34) holds, then $\lim_{h\to 1} \mathcal{P} = \emptyset$; which proves the second part of the proposition. Suppose (34) is reversed, given the continuity of the "G"-functions, $\underline{x}^*$, and $\Pi(h,\theta)$, we have $\underline{\theta} \in \lim_{h\to 1} \underline{\mathcal{P}} \cap \bar{\mathcal{P}}$. Thus for any neighborhood (H,1) there must exist an $h \in (H,1)$ such that pooling exists; which proves the third part of the proposition.

■

APPENDIX 3: Equilibrium Refinements

In this Appendix, we propose an equilibrium refinement in the context of the private negotiations/private contracts case. It is an adaptation of the ideas of Grossman-Perry [1986] and Farrell [1986].

For simplicity, we restrict attention to mechanisms that satisfy (1a) and (2b) as equalities. Given a mechanism m, let EP(m) denote the set of revenue levels x which can occur with positive probability under m (e.g. $EP(m^*) = \{\bar{x}^*, \underline{x}^*\}$ and $EP(m_0) = \{x_0\}$) and let OEP(m) denote the complement of EP(m) (i.e. the set of revenues off-the-equilibrium path if m is an equilibrium mechanism). Let $V(m,\mu)$ be expected payoffs for P from proposing m when beliefs are $\mu[\cdot]$. Finally suppose $\hat{m}$ is part of a proposed PBE, with beliefs $\hat{\mu}[\cdot]$.

For any revenue level x, define $C(x,\mu)$ as the set of mechanisms that attain x with positive probability and that yield an expected payoff for P under beliefs $\mu[\cdot]$ strictly larger than the proposed PBE payoffs $V(\hat{m},\hat{\mu})$:

$$C(x,\mu) \equiv \left\{ m \mid x \in EP(m) \text{ and } V(m,\mu) > V(\hat{m},\hat{\mu}) \right\}$$

Also define $\Theta(x,m)$ the set of types θ that choose to produce at level x when m is proposed:

$$\Theta(x,m) \equiv \left\{ \theta \in \{\underline{\theta},\bar{\theta}\} \mid x(\theta) = x \right\}$$

A revenue level $x \in OEP(\hat{m})$ is said to be *interpretable under beliefs* $\mu[\cdot]$ if $C(x,\mu)$ is not empty and for any two mechanisms m and m′ in $C(x,\mu)$, $\Theta(x,m) = \Theta(x,m')$. A deviation produces an interpretable revenue x under beliefs $\mu[\cdot]$ if, given these beliefs, all strictly profitable deviations for P lead T to the same conclusion about what types chose x.

A revenue level x in OEP($\hat{m}$) is said to be *consistently interpretable* if there exist beliefs $\mu[\cdot]$ under which x is interpretable and such that $\mu[x]$ equals the Bayesian revision of the prior conditional to $\theta \in \Theta(x,m)$, i.e. the prior restricted to $\Theta(x,m)$ (whatever m in $C(x,\mu)$) and renormalized. That is, posterior beliefs restricted to the set of types who produce the interpretable revenue must be the same as the beliefs under which the revenue is interpretable (a fixed-point requirement).

Our refinement is then the following: $(\hat{m},\hat{\mu})$ is a *reasonable PBE* if no x in OEP($\hat{m}$) is consistently interpretable.

We now illustrate how this refinement works to eliminate the pooling PBE of Section 6b. Consider a candidate pooling PBE at $\hat{x}_p \le \underline{x}^*$, with (33) holding. Consider $\bar{x}_0$ uniquely defined by: $\bar{x}_0 > \bar{x}^*$ and

$$\bar{G}(\bar{x}_0) + \Pi(1,\bar{\theta}) = \bar{G}(\hat{x}_p) + \Pi(h,\bar{\theta}) \qquad (3.1)$$

Note that (3.1) entails

$$\underline{G}(\bar{x}_0) < \underline{G}(\hat{x}_p) \qquad (3.2)$$

The mechanism $(\bar{x},\hat{x}_p)$, with $\bar{x} = \bar{x}_0-\varepsilon$ and ε not too large, would strictly dominate $(\hat{x}_p,\hat{x}_p)$ if T's beliefs were $\mu[\bar{x}] = 1$, $\mu[\hat{x}_p] = h$ and $\mu[x]=0$ $\forall x \notin \{\hat{x}_p,\bar{x}\}$; consequently $C(\bar{x},\mu)$ contains separating mechanisms. However, for ε sufficiently small, $C(\bar{x},\mu)$ does not contain pooling mechanisms:

$$h\bar{G}(\bar{x}) + (1-h)\underline{G}(\bar{x}) + h\Pi(1,\bar{\theta}) < h\bar{G}(\hat{x}_p) + (1-h)\underline{G}(\hat{x}_p) + h\Pi(h,\bar{\theta}) \qquad (3.3)$$

by (3.1) and (3.2) (recall $\Pi(\mu,\underline{\theta})$ is a constant under the assumptions of Section 6b). Expression (3.3) also proves that $C(\bar{x},\mu)$ does not contain separating mechanisms in which $\bar{x}$ is chosen by the $\underline{\theta}$-type. Hence, we have proved that for $(\bar{x},\mu)$ defined as above, $C(\bar{x},\mu)$ is not empty and $\forall m \in C(\bar{x},\mu)$, $\Theta(\bar{x},m)=\{\bar{\theta}\}$: $\bar{x}$ is interpretable under μ. Moreover consistency is actually verified by μ, so that $\bar{x}$ is consistently interpretable. Consequently, the candidate pooling PBE is not reasonable.

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