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### Title

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### Permalink

<https://escholarship.org/uc/item/6dw4677c>

### Journal

Hydrogeology Journal, 27(1)

### ISSN

1431-2174

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### Publication Date

2019-02-01

### DOI

10.1007/s10040-018-1879-y

Peer reviewed

# Correction: Push-pull tests for estimating effective porosity: expanded analytical solution and in situ application

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In the recent article by Paradis et al. (2018), an error was made when developing a conceptual model and an analytical solution to describe the one-dimensional displacement of the center of mass of a non-reactive tracer during a single-well injection, drift, and extraction test (push-pull test). Paradis et al. (2018) conceptualized that the displacement of the center of mass of a tracer during the injection phase ( $r_1$ ) can be attributed to two processes, as follows: (1) radial divergent and symmetric flow about the well due to the forced-gradient injection ( $r_i$ ) and (2) one-dimensional and horizontal flow away from the well due to the natural-gradient (ambient) flow of groundwater ( $r_{a1}$ ) (Fig. 1). Conceptually, the displacement of the center of mass of a tracer due to the forced-gradient injection ( $r_i$ ) is equal to zero, i.e., the center of mass is at the exact location of the well due to radial divergent and symmetric flow about the well (Fig. 1). Therefore, the displacement of the center of mass of a tracer during the injection phase ( $r_1$ ) can be solely attributed to one-dimensional and horizontal flow away from the well due to the natural-gradient (ambient) flow of groundwater ( $r_{a1}$ ) (Fig. 1).

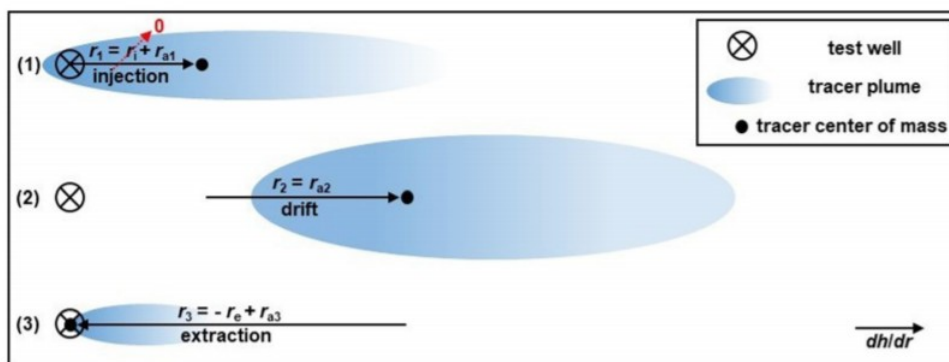


Fig. 1 Plan-view depiction of the center of mass of a tracer at the end of the injection (1), drift (2), and extraction (3) phases,  $r_i$  = displacement due to injection,  $r_a$  = displacement due to ambient groundwater flow,  $r_e$  = displacement due to extraction. The red arrow indicates correction to Fig. 1 from Paradis et al. (2018)

In Eq. (9) of Paradis et al. (2018), the displacement of the center of mass of a tracer during a push-pull test was analytically attributed to several terms, as follows:

$$\left[ \left( \frac{|Q_i|t_i}{\pi b n_e} \right)^{1/2} + v\Delta t_{a1} \right] + (v\Delta t_{a2}) + \left[ - \left( \frac{|Q_e|t_e}{\pi b n_e} \right)^{1/2} + v\Delta t_{a3} \right] = 0 \quad (C1)$$

where

$Q_i$	injection rate [L <sup>3</sup> /T]
$t_i$	total injection time [T]
$b$	saturated aquifer thickness [L]
$n_e$	effective porosity [dimensionless]
$v$	average linear groundwater velocity [L/T]
$\Delta t_{a1}$	time elapsed during injection [T]
$\Delta t_{a2}$	time elapsed during drift [T]
$Q_e$	extraction rate [L <sup>3</sup> /T]
$t_e$	total extraction time [T]
$\Delta t_{a3}$	time elapsed during extraction [T]

The first term on the left-hand side of Eq. (C1) describes the flow due to the forced-gradient injection. However, the center of mass of a tracer, that can be attributed solely to the forced-gradient injection, is always the center of the well, i.e., equal to zero displacement, due to radial divergent and symmetric flow about the well. Therefore, Eq. (C1) given here and Eq. (9) in Paradis et al. (2018) should be written as:

$$(v\Delta t_{a1}) + (v\Delta t_{a2}) + \left[ - \left( \frac{|Q_e|t_e}{\pi b n_e} \right)^{1/2} + v\Delta t_{a3} \right] = 0 \quad (C2)$$

As described in Paradis et al. (2018), Darcy's law can be written to include effective porosity, substituted into Eq. (C2), simplified, re-arranged to solve for effective porosity, and re-written to describe the center of mass of a tracer as given by:

$$n_e = \frac{\pi b \left(\frac{dh}{dr}\right)^2 K^2 (t_i + t_d + \tau_e)^2}{V_e} \quad (C3)$$

where

$\frac{dh}{dr}$  hydraulic gradient [L/L]  
 $K$  hydraulic conductivity [L/T]  
 $t_i$  time elapsed during injection [T]  
 $t_d$  time elapsed during drift [T]  
 $\tau_e$  time elapsed from start of extraction until one-half of tracer mass is recovered [T]  
 $V_e$  volume of water extracted until one-half of tracer mass is recovered [L<sup>3</sup>]

Equation (C3) more accurately describes effective porosity when accounting for the one-dimensional displacement of the center of mass of the tracer due to ambient groundwater flow during the injection phase ( $t_i$ ). This is in contrast to the solution by Hall et al. (1991) as given by:

$$n_e = \frac{\pi b \left(\frac{dh}{dr}\right)^2 K^2 (t_d + \tau_e)^2}{V_e} \quad (C4)$$

Equation (C4), like Eq. (C3), describes effective porosity but does not account for the one-dimensional displacement of the center of mass of the tracer due to ambient groundwater flow during the injection phase ( $t_i$ ).

One of the conclusions from Paradis et al. (2018) stated that “the failure to account for displacement during the injection phase may lead to a substantial underestimation of the magnitude of effective porosity”. This conclusion still holds true as demonstrated by values of effective porosity that are substantially greater ( $\approx 2$ -fold) as calculated from Eq. (C3) (corrected Paradis et al. (2018)) versus Eq. (C4) (Hall et al. (1991)) for the tests conducted by Paradis et al. (2018) (Table 1, which is a modification of the original Table 3 in Paradis et al. 2018).

**Table 1** Effective porosity calculated from the incorrect Paradis et al. (2018) solution (Eq. (18) in Paradis et al. (2018),  $n_{e1}$ ), the Hall et al. (1991) solution (Eq. (C4),  $n_{e2}$ ), and the corrected Paradis et al. (2018) solution (Eq. (C3),  $n_{e3}$ ) for tests from Paradis et al. (2018) (FW220-FW225), Hall et al. (1991), and Istok (2013)

Test well/study	Paradis et al. (2018) $n_{e1}$ (%)	Hall et al. (1991) $n_{e2}$ (%)	Corrected Paradis et al. (2018) $n_{e3}$ (%)
FW220	0.6	0.1	0.2
FW221	5.0	1.3	2.1
FW222	3.3	0.4	0.7
FW223	2.8	0.2	0.4
FW224	2.3	0.5	0.8
FW225	-0.1	0.1	0.1
Hall et al. (1991)	6.2	6.1	6.2
Istok (2013)	37	13	14

Moreover, all other conclusions from Paradis et al. (2018) that include: “(1) the analytical solution to describe the displacement of the center of mass of a tracer during a push-pull test can be expanded to account for displacement during the injection phase, (2) the transport of a tracer during the injection phase of a push-pull test may not be truly negligible,” and “(4) single-well push-pull tests can be readily applied to multiple wells within a study site to assess the spatial variability of effective porosity, and (5) the error-propagated uncertainty in the value of effective porosity can be mitigated to a reasonable level by careful consideration for the precise determination of the aquifer properties and the push-pull test parameters”, also hold true.

In summary, Eq. (C3) given here is the corrected version of Eq. (18) in Paradis et al. (2018) to describe effective porosity when accounting for the one-dimensional displacement of the center of mass of the tracer due to ambient groundwater flow during the injection phase ( $t_i$ ). The authors would like to sincerely thank Felix Tritschler from the Helmholtz Center for Environmental Research - UFZ for bringing the error to light and for assisting in the correction to the error.

## References

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