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Authors

Barkoczi, Daniel
Analytis, Pantelis P.
Wu, Charley M.

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Collective search on rugged landscapes: A cross-environmental analysis

Daniel Barkoczi (barkoczi@mpib-berlin.mpg.de) Pantelis P. Analytis (analytis@mpib-berlin.mpg.de)
Charley M. Wu (cwu@mpib-berlin.mpg.de)

Center for Adaptive Behavior and Cognition (ABC), Max Planck Institute for Human Development

Abstract

In groups and organizations, agents use both individual and social learning to solve problems. The balance between these two activities can lead collectives to very different levels of performance. We model collective search as a combination of simple learning strategies to conduct the first large-scale comparative study, across fifteen challenging environments and two different network structures. In line with previous findings in the social learning literature, collectives using a hybrid of individual and social learning perform much better than specialists using only one or the other. Importantly, we find that collective performance varies considerably across different task environments, and that different types of network structures can be superior, depending on the environment. These results suggest that recent contradictions in the social learning literature may be due to methodological differences between two separate research traditions, studying disjoint sets of environments that lead to divergent findings.

Keywords: Social learning; communication networks; collective behavior; search; rugged landscapes.

Introduction

There are two paths to the acquisition of knowledge. Organisms can search for new solutions through trial-and-error, testing various courses of action in isolation from others, or alternatively, they can copy existing solutions by imitating other individuals (March, 1991; Rendell et al., 2010). Imitation is a cognitively simple strategy that can lead to exceptionally good outcomes (Boyd & Richerson, 1985; Rendell et al., 2010; Laland, 2004; Miller & Dollard, 1941), however, it cannot produce new information by itself. Thus, collective learning in groups can be seen as the outcome of both individual learning and the spread of social information through imitation (Tarde, 1903). A different balance of these two strategies can lead to very different levels of individual and group performance (Rogers, 1988).

In this paper we examine how collectives using individual and/or social learning perform across a wide range of environments embedded in different network structures, governing the communication of social information. Models of social exploration-exploitation dynamics regularly use the analogy of search on a fitness landscape where nearby (similar) solutions can have quite different payoffs, forming a "rugged" multi-peaked landscape (Levinthal, 1997; Lazer & Friedman, 2007; Mason & Watts, 2012). Real-world examples of rugged landscapes may include combinatorial problems, technological innovation, or the fitness of an organism as a function of its genome. A key feature of these environments is that they can be dominated by multiple local maxima, where the best

solution in a given neighborhood may be far from the best global solution (see Figure 1 for an illustration).

Behavioral scientists have conducted a handful of experimental studies on collective learning using one or two dimensional functions (Mason, Jones, & Goldstone, 2008; Mesoudi, 2008; Mason & Watts, 2012), or simple combinatorial problems (Wisdom, Song, & Goldstone, 2013). In contrast, management and organization scientists have studied search using simulations and experiments based on the NK model inspired by evolutionary biology (Kauffman & Levin, 1987; Levinthal, 1997; Gavetti & Levinthal, 2000; Rivkin, 2000; Billinger, Stieglitz, & Schumacher, 2013), where the environment is completely determined by two parameters, the N number of components making up a solution, and the K level of interdependence between components. This type of formalism has the advantage that the difficulty of the problem can be directly set by the parameter K , but also the drawback that the problem is specifically tailored to a single type of complexity (i.e., interdependence).

Both communities have investigated the influence of communication networks on collective performance, but have found contradictory results (Derex & Boyd, 2016; Fang, Lee, & Schilling, 2010; Lazer & Friedman, 2007; Mason & Watts, 2012; Mason et al., 2008; Wisdom et al., 2013). Lazer and Friedman (2007) found that less connected (*inefficient*) networks lead to better collective performance in the NK landscape, while Mason and Watts (2012) reached the opposite conclusion, finding support for the superiority of well-connected (*efficient*) networks in a 2-dimensional landscape.

In these studies, the choice of fitness landscape was held constant, with results often derived from a single type of environment. Here we explore a promising explanation for these contradictory results, namely that these studies investigated learning on disjoint sets of environments, differently suited to specific types of collective search behavior. There are also other possible explanations for these contradictory results, such as the use of different social learning strategies (Barkoczi & Galesic, 2016) or the nature of the task that the collective has to perform (Shore, Bernstein, & Lazer, 2015).

In this study, we present simulations of collective search across a wide range of multi-peak environments, which include both two dimensional (Mason & Watts, 2012) and NK environments (Lazer & Friedman, 2007). Our approach uses a cross-environmental analysis to resolve inconsistencies regarding the influence of network structure on collective performance.

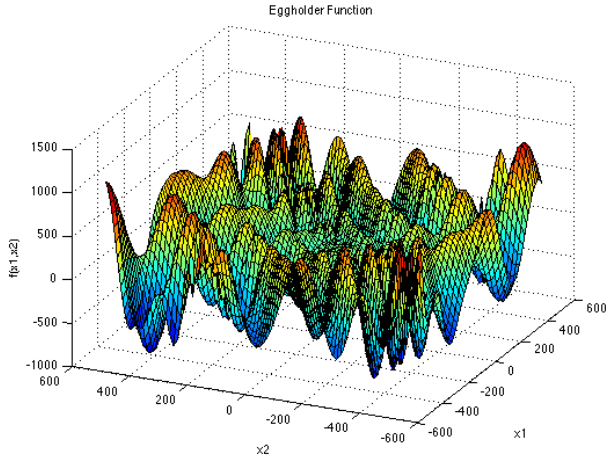


Figure 1: The Eggholder function is a typical example of a bivariate function that generates a rugged payoff landscape. The exact formula can be found at the virtual library of simulation experiments <http://www.sfu.ca/~ssurjano/optimization.html>.

Methods

Landscapes

In total we studied 15 different multi-peaked landscapes (Table 1). Environments 1-12 are bivariate functions obtained from the virtual library of simulation experiments (Surjanovic & Bingham, n.d.), which are regularly used in operations research and the field of global optimization to study how different optimization algorithms perform (e.g., Hu, Fu, & Marcus, 2007). These environments have been designed and investigated specifically because they pose challenges to adaptive optimization algorithms, covering a wide range of possible environmental structures with regards to the variability of high quality solutions, the ruggedness of the landscape, and the average pay-off. We also study the environment used in Mason and Watts (2012) and two different NK landscapes ($K=5$ and $K=10$) of the type used in Lazer and Friedman (2007).

For each environment we normalized the payoff scales to obtain relative payoffs between 0 and 1, with the global maximum set to 1 and the global minimum set to 0. Following Lazer and Friedman (2007) we re-scaled these normalized payoffs by raising each value to the power of 8. This monotonic transformation creates larger differences in the upper range of payoffs but leaves other features of the landscape unchanged.

Function-based Landscapes. Environments 1-12 are generated from bivariate functions, which translate any two values x and y (evaluated on a specific range of real numbers) into a corresponding payoff between 0 and 1. For our purposes, we first transformed the continuous environments into discrete units by dividing each axis into 1001 equally-spaced regions. Since all the environments were two dimensional, this resulted in a total of $1001 \cdot 1001$ possible locations that an agent could explore. This harmonized the different x-axis and y-axis scales of the individual environments, resulting in the same number of possible solutions across environments.

Most of these environments were initially designed as min-

imization problems. To be consistent with research on collective behavior in the behavioral and social sciences we inverted the payoffs and turned them into maximization problems.

Mason & Watts and NK Environments. We replicated the environment used in Mason and Watts (2012) and NK environments from Lazer and Friedman (2007). To retain consistency with the function-based environments described above, we enlarged the Mason and Watts (2012) environment to have $1001 \cdot 1001$ possible locations using bicubic interpolation. For the NK environments, we use $N = 20$, resulting in a similar number of possible solutions (2^{20}).

| # | Environments | μ | σ | modality |
|----------------|----------------------|-------|----------|----------|
| 1 | Ackley | 0.001 | 0.01 | 4489 |
| 2 | Cross-in-Tray | 0.14 | 0.17 | 64 |
| 3 | Drop-wave | 0.003 | 0.03 | 4391 |
| 4 | Eggholder | 0.03 | 0.07 | 353 |
| 5 | Griewank | 0.17 | 0.24 | 50861 |
| 6 | Holder table | 0.006 | 0.06 | 56 |
| 7 | Langermann | 0.02 | 0.07 | 2461 |
| 8 | Rastrigin | 0.05 | 0.1 | 121 |
| 9 | Schaffer n.2 | 0.007 | 0.03 | 88457 |
| 10 | Schaffer n.4 | 0.001 | 0.02 | 87737 |
| 11 | Schwefel | 0.03 | 0.07 | 64 |
| 12 | Shubert | 0.005 | 0.03 | 761 |
| 13 | Mason & Watts (2012) | 0.001 | 0.01 | 1090 |
| 14 | $N = 20, K = 5$ | 0.04 | 0.04 | 1143 |
| 15 | $N = 20, K = 10$ | 0.04 | 0.04 | 7131 |
| <i>Average</i> | | 0.036 | 0.07 | 16612 |

Table 1: The 15 landscapes that we examined. We report the mean (μ) and variability (σ) of pay-offs, and the modality (i.e., number of peaks) of the landscape. Environments 1-12 are function-based and the environmental statistics remain constant. Statistics for Mason and Watts (2012) were averaged over 10,000 replications of the environment generation function, while the NK environments were averaged over the 100 pre-generated landscapes used in simulations.

Learning processes and behavioral strategies

We considered a population of 100 agents simultaneously learning about the environment using different strategies. In separate conditions, we tested strategies that relied solely on individual learning, solely on social learning, or on a combination of both (McElreath et al., 2008). We first describe the rules associated with each strategy.

1. *Individual learning:* We studied two types of individual learning, where agents evaluated new solutions without the benefit of social information.

- **Local search.** In our simulations, local search (i.e., hill climbing) was performed by examining neighboring solutions for the largest increase in payoff. In cognitive science, local search has been studied in resource-allocation problems (Busemeyer & Myung,

1987; Rieskamp, Busemeyer, & Laine, 2003), and also corresponds to the gradient descent algorithm used in many learning systems. On the 2D landscapes (Environments 1-13), the local search strategy evaluated the 8 adjacent solutions accessible by modifying either the value of x , y , or both simultaneously by one discrete unit. If any of the explored solutions were better than the current one, it adopted the best among these solutions, otherwise, it kept the current solution. In NK landscapes, local search was implemented by modifying a single randomly chosen digit of the N -dimensional solution (Levinthal, 1997; Lazer & Friedman, 2007; Rivkin, 2000)¹.

- **Random search.** Random search was performed by randomly evaluating a new location in the fitness landscape and moving to that position if it had a higher payoff (i.e., long-jump). Random search relates to the idea of blind variation in psychology (Campbell, 1960), but also corresponds to the random search assumption made by optimal stopping models in economics and statistics (e.g. DeGroot, 2005; Analytis, Stojic, & Moussaïd, 2015).
2. *Social learning:* Social learners sampled the solutions from a number of other individuals ($n=k$) within the connected population (see Network structures), and adopted the solution with the highest observed payoff if it was better than the agent's existing solution (Lazer & Friedman, 2007). In this paper we report results for $n=3$. We refer to this this strategy as **imitate-the-best**.
 3. *Hybrid social/individual learning:* Hybrid agents used a combination of social and individual learning (Enquist, Eriksson, & Ghirlanda, 2007; McElreath et al., 2008). At each time step, agents followed a sequential strategy selection process (Gigerenzer, 2008), in which they first attempted to find a better solution through social learning (described above), but switched to individual learning, using either local search or random search, if unsuccessful. In this paper we report two variants of this strategy, one with no option to perform random search ($rs = 0$) and one where 20% of individual learning actions resulted in random search ($rs = .2$), with the remaining 80% of actions performing local search.

Network structures

To account for different channels of communication, we examined how hybrid agents performed when they were embedded in a fully connected network, allowing communication with all other agents, or a locally connected lattice in which each agent was connected with four other agents from the population (Figure 2). In both networks the agents sampled $n=3$ individuals at random from their connections. In the locally connected lattice, access to the full population was

¹The differences in the implementation of local search in 2D and high dimensional NK landscapes are due to irreconcilable differences in the number of adjacent solutions as a function of the dimensionality of the problem space. We follow standard implementations from the respective literatures.

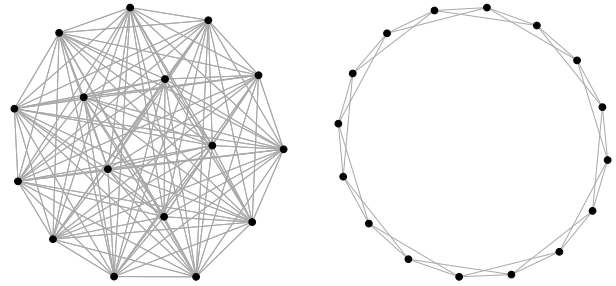


Figure 2: Examples of a fully connected (left) and locally connected lattice (right) networks.

only available indirectly, over several degrees of separation. In contrast, information flowed more freely in the fully connected network, with the possibility for each agent to imitate each other agent in the population. These two network structures were also studied by Lazer and Friedman (2007).

Simulation procedure

We assigned random starting solutions to a population of 100 agents. At each time step, agents used the strategies described above to learn about the environment. Imitation actions were performed simultaneously at the beginning of each time step to avoid sequential effects (Analytis et al., 2015; Bikhchandani, Hirshleifer, & Welch, 1992).

We repeated this process for 100 time steps and 100 replications, and recorded the average payoff achieved in the population. In total we tested 7 different strategy and network combinations; the three pure learning strategies (local search, random search, and imitate-the-best²) and four versions of the hybrid strategy (two rs values and two different network structures)³.

Results

Overall, we found that performance varied across environments, with some environments being more challenging than others (Figure 3). While the rank order remained relatively stable across environments, there were several notable differences. First, random search was always superior to local search in the 2D environments (Environments 1-13), while the opposite was the case in the NK environments (Environments 14-15). Second, imitate-the-best typically outperformed individual search strategies—by driving the population to local peaks faster than individual search—with one exception being the NK environments, where local search performed better. These two results demonstrate that local search is a highly effective strategy in the NK environment, which could be because the peaks of NK landscapes have larger basins of attraction, allowing local search to explore a larger space compared to the 2D landscapes (Kauffman & Levin, 1987).

²Note that imitate-the-best is used as a benchmark, therefore, we only study its performance in a fully connected network.

³See: https://github.com/dnlbrkc/collective_search for the simulation code.

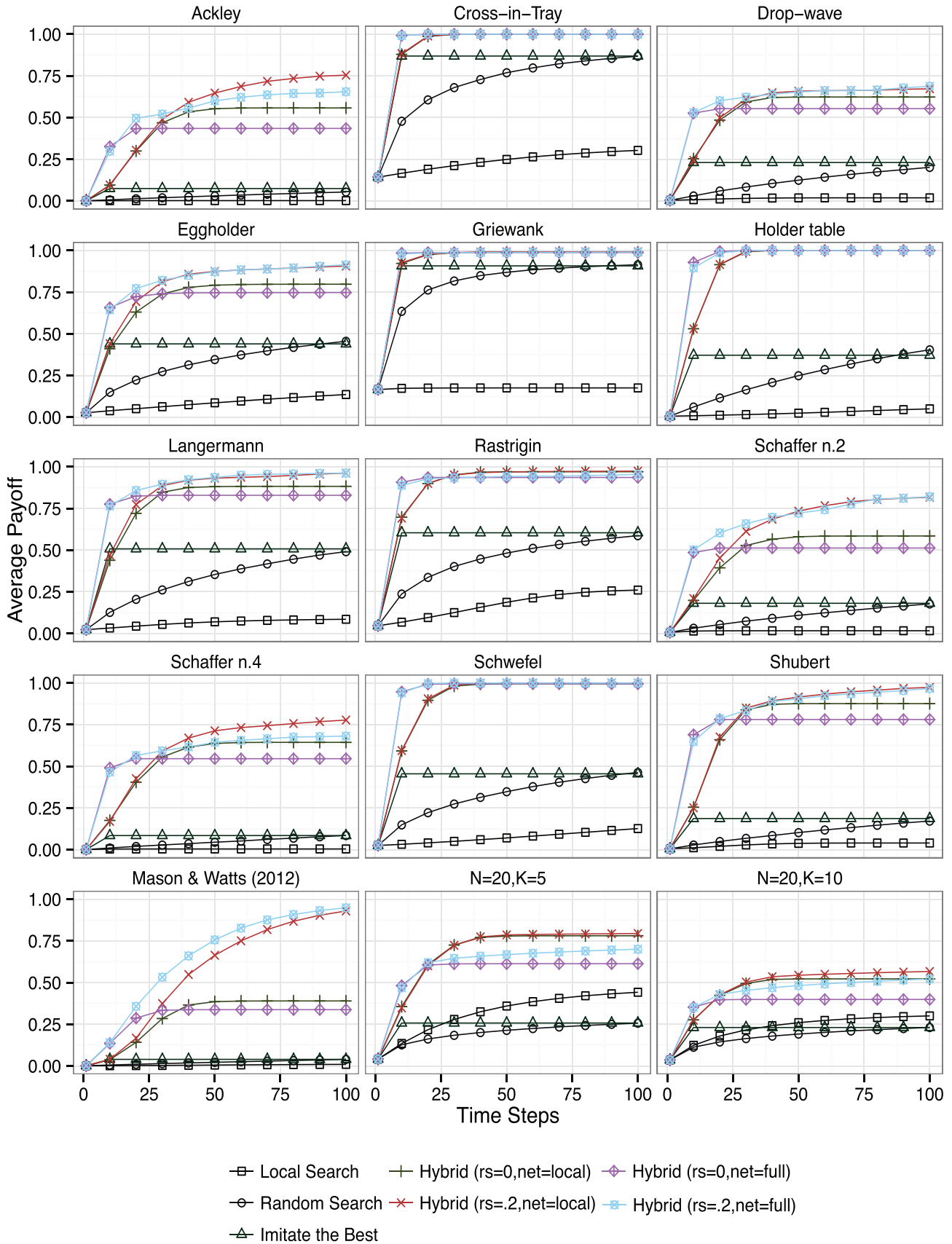


Figure 3: Learning curves for the different models across 15 environments, reporting the change in average payoff (over each population of 100 agents and over 100 replications) for 100 time steps. We studied 7 different models using combinations of simple heuristic rules. Hybrid models utilize both social and individual search strategies, where the random search parameter (rs) governs the balance of random search vs. local search, while the network parameter (net) specifies either a fully connected or a locally connected lattice network. Hybrid strategies consistently performed best with some random search ($rs = .2$); however, the superiority of different network structures depended on the structure of the environment.

Network structure and performance We replicated the results found in both Mason and Watts (2012) and Lazer and Friedman (2007), and show that the contradictory conclusions can be explained by the choice of environment they studied. Depending on the environment, network structure can lead to three different types of results. First, hybrid agents with a local lattice network can converge on a higher payoff than those with a fully connected network (particularly when $rs = 0$; e.g., Ackley, Schaffer n.4, and NK environments). This replicates the results of Lazer and Friedman (2007), who showed that local connections force the population to explore the landscape more extensively, delaying convergence on local optima. Second, fully connected networks can initially outperform the local lattice network, and reach comparable performance over time (e.g., Eggholder and Mason & Watts environments). This result is consistent with Mason and Watts (2012) and shows that in some environments, slower communication does not lead to higher performance in the long-run. Third, there can very small differences between networks (e.g., Cross-in-Tray and Griewank), which happens when the problem is relatively easy and hybrid agents quickly reach high-value optima.

The role of random exploration Allowing for a mixture of both local exploration (hill-climbing) as well as the ability to occasionally undertake random exploration (long-jumps) greatly benefited hybrid collectives, making it the overall best performing strategy ($rs=.2$) by a large margin. Random search generated new information that gradually diffused to other agents by means of imitation. In fact, heuristics for global optimization problems such as genetic algorithms (Goldberg & Holland, 1988) or simulated annealing (Kirkpatrick, Gelatt, Vecchi, et al., 1983) have explicit strategies to escape from the local optima. The ability to search randomly (via long-jumps) in our simulation played exactly the same role.

Explaining differences across environments. What are the environmental features that influence the performance of different strategies and networks? An obvious candidate is the the number of peaks (i.e., modality) in an environment. Figure 4 shows that modality can explain some of the variability in performance across environments, but not all. Higher modality is correlated with lower payoffs for both the locally connected (top; $r=-.53, p=.04$) and fully connected (bottom; $r=-.55, p=.03$) hybrid strategies (where $rs = .2$). Still, a large proportion of the variance remains unexplained. One striking difference is that while the Mason & Watts (2012) and $N=20, K=5$ environments have similar modality, the rank order of average payoffs are inverted between the two network structures. In the future we aim to identify relevant features of task environments (see Mersmann et al., 2011), that would allow us to make predictions about which cognitive strategies and network structures are best suited to a particular environment.

General discussion

In this paper we investigated how groups of individuals relying on different cognitive strategies performed across 15

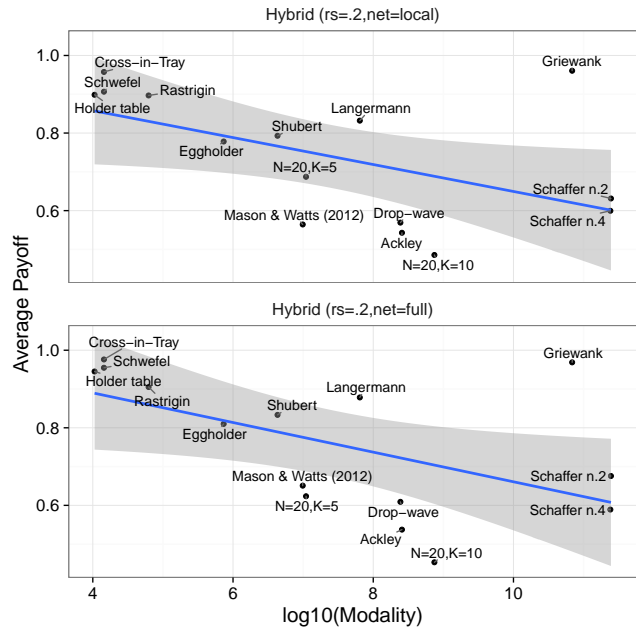


Figure 4: The number of local maxima (modality) explains some of the variability in performance across environments, but not all. Here we show average payoff as a function of the (\log_{10}) modality of the environments, for the locally connected (top) and fully connected (bottom) hybrid models (where $rs = .2$). Average payoff was derived by averaging over all individuals and time steps.

challenging search environments. As Herbert Simon illustrated with his scissors analogy, "[h]uman rational behavior (and the rational behavior of all physical symbol systems) is shaped by a [pair of] scissors whose two blades are the structure of task environments and the computational capabilities of the actor" ((1990), p. 7). In a similar vein, to understand collective performance, it is essential to study cognition in tandem with the task environment in which it operates.

A large number of studies have addressed collective problem solving; however, they have focused on only one type of task environment. This makes it (i) hard to compare results across studies, (ii) uncertain whether they generalize beyond a specific environment, and (iii) unknown whether certain strategies lead to superior collective performance when paired with specific environments. To the best of our knowledge, this is the first study in the social and behavioral sciences that analyses collective problem-solving across many different types of environments⁴.

Network and environmental structure

An open question in the study of collective learning is the influence of network structure on collective performance in groups and organizations. Derex and Boyd (2016), Lazer and Friedman (2007), Fang et al. (2010) and Mason et al. (2008) found that less efficient networks may led to higher

⁴To this end, we borrowed several environments that have been used as test-beds for search algorithms in Operations Research. Conversely, researchers from the OR community have often turned to collective cognitive systems for inspiration. Algorithms imitating the behavior of ant colonies (Bonabeau, Dorigo, & Theraulaz, 2000) and swarms (Kennedy, 2010) have proved to be particularly powerful in solving combinatorial and global optimization algorithms.

levels of collective performance. In contrast, a recent study by Mason and Watts (2012) came to the opposite conclusion, that more efficient networks are better. A crucial difference between these studies is the type of environment that they investigated, because as we have found, the performance of a type of network structure depends on the underlying task environment. We found that in a number of environments (e.g., Ackley, Schaffer n.4, and NK environments), inefficient networks eventually converged on better payoffs than efficient networks, while in others (including the Mason & Watts environment), efficient networks had an initial advantage, but reached similar performance as inefficient networks over time.

Extensions and limitations

In this study we chose to adhere closely to the designs of Mason and Watts (2012) and Lazer and Friedman (2007) to resolve contradictory results in the literature on the role of network structure in collective performance. We covered a broad range of environmental structures, but we only looked at two popular types of networks. In the future, we intend to extend our analysis to additional networks (see Mason & Watts, 2012), different behavioral rules (see Barkoczi & Galesic, 2016) and types of tasks (see Rahwan, Krasnoshtan, Shariff, & Bonnefon, 2014). However, the key challenge will be to identify features of tasks or environments that favor the usage of certain networks over others.

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