Title
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FORWARD HADRONIC pp AND $\bar{p}p$ ELASTIC SCATTERING AMPLITUDES: ANALYSIS OF EXISTING DATA AND EXTRAPOLATIONS TO COLLIDER ENERGIES **

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A B S T R A C T

An analysis of total cross-sections and ratios of real to imaginary parts of the forward hadronic scattering amplitude for pp and $\bar{p}p$ is given. The data above $s = 25$ GeV^2, including the latest $\bar{p}p$ results from the ISR, are simultaneously fit for both real and imaginary parts using proper analytic forms. The resulting fits provide an excellent interpolation of the data using five parameters. Extrapolations are done to collider energies. Limits are placed on the magnitude of odd amplitudes with unconventional energy dependence.

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The operation of the ISR with antiprotons has dramatically extended the domain of measurements for the total cross-section of $\bar{p}p$ and for ρ, the ratio of the real to the imaginary part of the forward hadronic scattering amplitude[1]. Another such extension will occur soon with the $\bar{p}p$ collider at CERN, followed by the Tevatron collider at Fermilab. Measurement of the pp cross-section at higher energies, though, does not seem likely this decade. It is thus an appropriate moment to analyze carefully the existing data for both pp and $\bar{p}p$ in an attempt to answer three questions:

1. How can the pp and $\bar{p}p$ data for forward elastic (hadronic) scattering (that is, the total cross-sections and ρ values) be parametrized concisely and precisely ?
2. On the basis of such a parametrization, what does an extrapolation to collider energies predict ?
3. Do the pp and $\bar{p}p$ forward elastic scattering amplitudes become equal at high energies ? How well can we exclude the possibility, for example, of a constant cross-section difference ?

These questions are not new. Indeed, the asymptotic behaviour of cross-sections and cross-section differences has been one of the most central concerns of high energy physics[3]. What is new are the data which enable us to give much more complete answers to these questions than was previously possible.

Essential to the analysis is the analyticity of the forward scattering amplitude. Analyticity is traditionally expressed in terms of dispersion relations or so-called differential dispersion relations. We are concerned with high energy data which is (so far as one knows) a smooth function of the centre-of-mass energy. As a result, analyticity can be exploited in a much more direct fashion, simply by writing down amplitudes which have the proper analyticity structure and extracting directly their real and imaginary parts. In fact, it suffices to use very simple forms[4].

It is convenient to define the even and odd amplitudes

$$\mathcal{M}_{\pm} = \frac{1}{2}\left[\mathcal{M}_{\bar{p}p} \pm \mathcal{M}_{pp}\right] , \qquad (1)$$

and the normalization

$$\sigma = - \mathcal{I}m\, \mathcal{M}/s \qquad (2)$$

where we have neglected the proton mass squared, m_p^2, relative to the centre-of-mass energy squared, s, in this high energy analysis. We parametrize the even amplitude in terms of real constants as[5)]

$$\mathcal{M}_+ = -is\left[A + \frac{B\left(\ln s/s_0 - \frac{i\pi}{2}\right)^2}{1 + a\left(\ln s/s_0 - \frac{i\pi}{2}\right)^2}\right] + C\,, \tag{3}$$

and first consider the case $a = 0$. Then, from Eqs. (2) and (3)

$$\sigma_+ = A + B\left(\ln^2 s/s_0 - \pi^2/4\right)\,, \tag{4}$$

which saturates the Froissart bound form, and which has often been used in fitting the pp cross-section data. Permitting the parameter a to take on small positive values allows for a deviation from the Froissart bound form. Indeed, asymptotically the form gives a constant cross-section, $\sigma_+(\infty) = A + B/a$. The constant C is permitted by the requirements of analyticity for the even amplitude and corresponds to a subtraction constant in the usual dispersion relation treatment. We shall show that C is unimportant in the region of interest as we might expect, since it lacks the factor of s present in the dominant terms. We shall also see that very fine fits are obtained with $a = 0$. Thus just three parameters, A (in mb), B (in mb) and s_0 (in GeV^2), are needed to parametrize the even amplitude[6)]. The parameter a is useful, however, for it will provide a means of estimating our uncertainty when we try to extrapolate our fit to higher energies.

The odd amplitude is known to be dominated by a piece with the approximate behaviour $s^{1/2}$ (that is, $\sigma_{\bar{p}p} - \sigma_{pp} \propto s^{-1/2}$). We take the power, α, and the magnitude, D, of the amplitude as parameters and write

$$\mathcal{M}_- = D\, s^{\alpha}\, e^{\frac{i\pi}{2}(1-\alpha)}\,. \tag{5}$$

Later, we shall consider odd amplitudes with unconventional asymptotic behaviour in an attempt to establish limits on the presence of such terms. For the purpose of finding an adequate fit to the present data, they are unnecessary.

Because we have chosen such simple analytic forms for our amplitudes, it is easy to express the physical quantities in terms of our parameters: if $a = 0$ and $C = 0$, the expressions are

$$\sigma_{pp} = A + B\left(\ln^2 s/s_0 - \pi^2/4\right) + D s^{\alpha-1} \cos\frac{\pi\alpha}{2} , \tag{6a}$$

$$\sigma_{\bar{p}p} = A + B\left(\ln^2 s/s_0 - \pi^2/4\right) - D s^{\alpha-1} \cos\frac{\pi\alpha}{2} , \tag{6b}$$

$$\rho_{pp} = \frac{\pi B}{\sigma_{pp}} \ln s/s_0 + \frac{D s^{\alpha-1}}{\sigma_{pp}} \sin\frac{\pi\alpha}{2} , \tag{6c}$$

$$\rho_{\bar{p}p} = \frac{\pi B}{\sigma_{\bar{p}p}} \ln s/s_0 - \frac{D s^{\alpha-1}}{\sigma_{\bar{p}p}} \sin\frac{\pi\alpha}{2} . \tag{6d}$$

The datum points used and their uncertainties were taken directly from Ref. 7). Only data with s greater than 25 GeV^2 have been used. The total number of datum points fitted was 78. Some available data have not been included. In most cases, these unused data have much larger quoted uncertainties than the data included in the same energy region. For the Louvain-Northwestern collaboration data at $s^{1/2} = 52.8$ GeV, the experimentally measured quantities are $\Delta\sigma = \sigma_{\bar{p}p} - \sigma_{pp}$, $\Delta\rho = \rho_{\bar{p}p} - \rho_{pp}$ and $\rho_{ave} = \frac{1}{2}(\rho_{\bar{p}p} + \rho_{pp})$ along with their associated errors. All other experimental data are for σ_{pp}, $\sigma_{\bar{p}p}$, ρ_{pp}, and $\rho_{\bar{p}p}$. The fit was made using a mimimization in these seven quantities.

The results of these fits are displayed in Table 1. We note that all three fits have a very acceptable χ^2. As an interpolation, the five parameter fit is excellent, in addition to being extremely simple [see Eqs. (6a) - (6d)], and this fit is displayed in Figs. 1 and 2. The three fits are essentially indistinguishable over this energy range. Introducing the parameter C has virtually no effect. Similarly, adding the parameter a has no influence on the fit in the energy range for which there are data.

Extrapolating the present fit to collider energies is a speculation, but it is more than just curve fitting because of the constraints imposed by analyticity. A sudden change in the cross-section just above ISR energies would have been presaged by a signal in the real part of the amplitude. Thus such an extrapolation is based both on experimental input and theoretical principles. On the other hand, a bias is introduced by our choice of the $\ln^2 s$ parametrization of the existing data. The present rise in the cross-section need not persist indefinitely. The introduction of the parameter $a > 0$ yields a cross-section which has a $\ln^2 s$ dependence near the minimum of the cross-section but which is asymptotically constant. Of course, that the data slightly prefer a small positive value for a is not necessarily an indication that the cross-sections are going to become constant asymptotically. We consider the difference between the $a = 0$ and $a \neq 0$ fits as providing an estimate of the uncertainty in our extrapolation.

In Fig. 3 we show the five parameter fit $(a = 0, C = 0)$ and the six parameter fit $(a = 0.0050, C = 0)$ extrapolated to collider energies. We remind the reader that these fits are simultaneously constrained by data for cross-sections and ρ values, for both pp and $\bar{p}p$. In Table 2, we display some values obtained in these fits, including extrapolations to collider energies. The uncertainties quoted are just those due to the uncertainties for the parameters as determined by the fits. We note that the $a \neq 0$ fit predicts a cross-section at $s^{1/2} = 540$ GeV of 66.0 mb ± 2.8 mb, while the $a = 0$ fit gives 70.9 mb ± 0.6 mb. This difference is in rough accord with the result that the best fit for a differs from zero by a little less than two standard deviations: $a = 0.0050 \pm 0.0031$.

The new $p\bar{p}$ data from the ISR are especially powerful in determining the odd amplitude. Of special interest is their relevance to Pomeranchuk theorems[8]. The original Pomeranchuk theorem stated that if pp and $\bar{p}p$ cross-sections became asymptotically constant, and if $\rho/\ln s \to 0$ as $s \to \infty$, then the difference of the cross-sections tended to zero as $s \to \infty$. The present data suggest that the cross-sections are not tending to constants, but instead increase without limit. Other versions of the theorem cover such circumstances. For example, if either the pp or the $\bar{p}p$ cross-section tends to infinity, so does the other, and their ratio tends to unity. Moreover, if the cross-sections grow as $(\ln s)^\gamma$, then the difference of the cross-sections cannot grow faster than $(\ln s)^{\gamma/2}$.

We shall limit our further considerations to the situation in which the cross-sections saturate the Froissart bound form, i.e., the even amplitude grows as $s(\ln s)^2$. This allows the cross-section difference, $\sigma_{\bar{p}p} - \sigma_{pp}$, which comes only from the odd amplitude, to grow as fast as $\ln s$. We introduce, ad hoc, three particularly simple possibilities[9).

$$\mathcal{M}_-^0 = E s \,, \tag{7a}$$

$$\mathcal{M}_-^1 = E s \left(\ln s/s_0 - \frac{i\pi}{2} \right), \tag{7b}$$

$$\mathcal{M}_-^2 = E s \left(\ln s/s_0 - i\frac{\pi}{2}\right)^2 , \tag{7c}$$

where E is a real constant. We shall refer to the amplitude in Eqs. (7a), (7b) and (7c) as Odderon-0, Odderon-1, and Odderon-2, respectively. The full odd amplitude is given by the sum $\mathcal{M}_-^{tot}$ of $\mathcal{M}_-$, from Eq. (5), and one of the terms from Eq. (7). Odderon-0 affects the ρ values, but not the cross-sections, being entirely real. Odderon-1 gives constant cross-section differences, while Odderon-2 gives cross-section differences growing as $\ln s$. If the scale, s_1, in Odderon-1 or Odderon-2 is too large, the magnitude of the real part of the amplitude is not an increasing function in the energy domain of interest. To prevent this distortion of the meaning of these amplitudes, we constrain s_1 to be the same as s_0.

There is a theorem, due to Fischer and co-workers[10)], which states, in part, that if above some energy the signs of $\mathrm{Im}\,\mathcal{M}_-^{tot}$ and $\mathrm{Re}\,\mathcal{M}_-^{tot}$ are the same, then the difference of the cross-sections tends to zero. Clearly, this theorem is satisfied by the amplitude $\mathcal{M}_-$ of Eq. (5) (for $0 < \alpha < 1$). The addition of an Odderon-1 or Odderon-2 amplitude can be seen to lead to opposite signs for $\mathrm{Im}\,\mathcal{M}_-^{tot}$ and $\mathrm{Re}\,\mathcal{M}_-^{tot}$ in the limit of high s. This is of course in accord with the Fischer theorem, since these terms lead to non-vanishing cross-section differences.

We have made three separate fits to the data using successively, Odderon-0, Odderon-1 and Odderon-2. The results of these fits are shown in Table 1. In all three cases, the value of E is about two standard deviations away from zero and there is thus no proven need for these amplitudes. It is of interest to examine quantitatively the limits that can be placed on their presence. For Odderon-0 an appropriate comparison is that between A and E, the coefficients of the purely imaginary odd amplitude and the purely real even amplitude with the same s dependence, respectively. The magnitude of E is less than one per cent of that of A. This is an impressive limit since this odd amplitude cannot contribute to the cross-section. The limits on the other fits are comparable. Altogether, then, we conclude that these amplitudes which are allowed by analyticity, if present at all, are less than one per cent as strong as the dominant portion of the forward scattering amplitude.

Using the values for Odderon-1 found in Table 1, we note that since D and E are both negative, $\mathrm{Re}\,\mathcal{M}_-^{tot}$ is negative for $s > s_0$. On the other hand, $\mathrm{Im}\,\mathcal{M}_-^{tot}$ is negative and dominated by $\mathcal{M}_-$ [Eq. (5)] at ISR energies. It does not change sign until $s^{1/2} \equiv 200$ GeV. Thus any attempt to invoke the Fischer theorem at present energies is premature. The corresponding sign change for Odderon-2 would be at $s^{1/2} \approx 75$ GeV. Since these sign changes occur in $\mathrm{Im}\, \mathcal{M}_{-}^{tot}$, they reflect a change in the sign of $\Delta\sigma = \sigma_{\bar{p}p} - \sigma_{pp}$. The existing data are thus compatible with such a sign change, and using Table 1 we extrapolate the Odderon-2 fit to $s^{1/2} = 540$ GeV, where we find $\sigma_{\bar{p}p} - \sigma_{pp} = -1.6 \pm 0.8$ mb. The unconventional sign of the difference is possible because the Odderon contributes oppositely to the amplitude $\mathcal{M}_{-}$, Eq. (15), which dominates the odd amplitude at lower energies. The magnitude of this difference and its uncertainty show clearly the desirability of making both pp and $\bar{p}p$ cross-section measurements at collider energies. At the same time, these numbers provide a quantitative estimate of the required precision. The presence of Odderon-0 is especially difficult to detect experimentally. Data at very high energies would not particularly improve the situation. For example, using our values for Odderon-0 from Table 1, we would predict $\Delta\rho = \rho_{\bar{p}p} - \rho_{pp}$ at $s^{1/2} = 540$ GeV to be 0.008 ± 0.003, whereas the fit without any Odderon gives $\Delta\rho = 0.0009 \pm 0.0002$ at the same s, a difference unfortunately too small to be detected experimentally.

We summarize our results by answering the questions posed at the outset:

1. All the cross-sections and ρ values for pp and $\bar{p}p$ data above $s^{1/2} = 5$ GeV can be parametrized simply and very satisfactorily by Eq. (6) with $A = 41.77$ mb, $B = 0.68$ mb, $s_0 = 343$ GeV, $D = -39.0$ mb and $\alpha = 0.48$.
2. Using this simple fit in which the Froissart bound form is saturated and σ_{pp} and $\sigma_{\bar{p}p}$ approach each other at high energies, the extrapolations to collider energies give the predictions σ ($s^{1/2} = 540$ GeV) = 71 mb, σ ($s^{1/2} = 2000$ GeV) = 100 mb, ρ ($s^{1/2} = 540$ GeV) = 0.20 and ρ ($s^{1/2} = 2000$ GeV) = 0.20. A measure of our extrapolation uncertainty is provided by our alternative predictions based on a fit in which the cross-sections eventually become constant. The corresponding predictions are σ ($s^{1/2} = 540$ GeV) = 66 mb, σ ($s^{1/2} = 2000$ GeV) = 82 mb, ρ ($s^{1/2} = 540$ GeV) = 0.14 and ρ ($s^{1/2} = 2000$ GeV) = 0.12.
3. The Odderons are less than about one percent as strong as the dominant even amplitude.

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Fit type	A(mb)	B(mb)	s_0 (GeV^2)	D(mb GeV$^{2-2\alpha}$)	α	a	C(mb GeV^2)	E(mb)	χ^2/d.f.
1) Simple (a=0,C=0)	41.77 ±0.04	0.68 ±0.01	343. ±8.	-39.0 ±1.7	0.48 ±0.01	-	-	-	86.7/73
2) Constant asymptotic cross-section (a≠0,C=0)	41.74 ±0.04	0.66 ±0.02	338. ±8.	-38.7 ±1.6	0.49 ±0.01	0.0050 ±0.0031	-	-	84.0/72
3) With subtraction constant (a=0,C≠0)	41.77 ±0.04	0.68 ±0.01	344. ±8.	-39.2 ±1.8	0.48 ±0.01	-	5.0 ±10.6	-	86.5/72
4) Odderon 0	41.77 ±0.04	0.69 ±0.02	345. ±8.	-41.7 ±2.4	0.46 ±0.02	-	-	-0.26 ±0.13	82.6/72
5) Odderon 1	41.74 ±0.04	0.69 ±0.01	350. ±8.	-40.8 ±1.8	0.49 ±0.01	-	-	-0.10 ±0.04	80.1/72
6) Odderon 2	41.70 ±0.05	0.66 ±0.01	356. ±10.	-35.2 ±2.2	0.50 ±0.02	-	-	-0.04 ±0.02	81.8/72

TABLE I - Parameters for the best fits to the cross-section and ρ values for pp and $\bar{p}p$ data. The even amplitude is given in Eq. (3). The parameters a and C are set to zero except in Fits 2 and 3. The odd amplitude for the first three fits is given by Eq. (5). For the last three fits the odd amplitude is a sum of this term and one term from among the three Odderons, Eqs. (7a), (7b) and (7c).

	σ_{pp}(mb)	$\sigma_{\bar{p}p}$(mb)	ρ_{pp}	$\rho_{\bar{p}p}$
$s^{\frac{1}{2}}$ = 23.5 GeV				
Fit 1	39.2±0.03	41.3±0.07	0.00±0.001	0.05±0.003
Fit 2	39.2±0.04	41.3±0.07	0.00±0.002	0.05±0.003
$s^{\frac{1}{2}}$ = 62.5 GeV				
Fit 1	43.7±0.1	44.5±0.1	0.11±0.003	0.12±0.002
Fit 2	43.8±0.1	44.6±0.1	0.10±0.005	0.12±0.005
$s^{\frac{1}{2}}$ = 540 GeV				
Fit 1	70.9±0.6	71.0±0.6	0.20±0.002	0.20±0.002
Fit 2	66.0±2.8	66.1±2.8	0.14±0.03	0.14±0.03
$s^{\frac{1}{2}}$ = 2000 GeV				
Fit 1	99.6±1.2	99.6±1.2	0.20±0.001	0.20±0.001
Fit 2	82.3±8.0	82.3±8.0	0.12±0.03	0.12±0.03

TABLE II - Values of σ and ρ at selected energies. Fit 1 has $a = 0$. Fit 2 has $a = 0.0050$. See Table I for a complete listing of the parameters.

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3) I.Ya. Pomeranchuk - Soviet Phys. JETP 7 (1958) 499.

4) See, for example, R.J. Eden - High Energy Collisions of Elementary Particles, Cambridge University Press (1967).
An analysis with much the same spirit as ours is given by C. Bourrelly and J. Fischer - Nuclear Phys. B61 (1973) 513.
See also : J. Fischer - Physics Reports 73 (1981) 157.

5) We may regard $\ell n\ s/s_0 - i\pi/2$ as the limit, for $s \gg s_C$, of $f(s) = \frac{1}{2}[\ell n\ (s+s_C)/s_0 + \ell n\ (s_C-s)/s_0]$. This is a real analytic function with cuts from $-s_C$ to $-\infty$ and from s_C to ∞. It is symmetric under s-u crossing provided we ignore m_p^2 relative to s, as we shall throughout. With f(s) used instead of $[\ell n\ s/s_0 - i\pi/2]$ in Eq. (3), if $a > 0$ the denominator produces a singularity when $f(s) = \pm i/a^{1/2}$. This is not on the first Riemann sheet provided $a < 4/\pi^2$. The other odd and even functions are conventional. Thus even amplitudes vary as $s^\alpha \exp(-i\pi\alpha/2)$ and odd amplitudes as $s^\alpha \exp i\pi(1-\alpha)/2$. See, for example, R.J. Eden, Ref. 4).

6) Traditional Regge fits would include in $\mathcal{M}_+$ terms varying as $s^{1/2}$. These terms are effectively subsumed by the $\log^2 s/s_0$ piece for $s < s_0$.

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8) For a more complete discussion, see A. Martin, Ref. 2). See also R.J. Eden, Ref. 4).

9) See A. Martin, Ref. 2), for a discussion and references to the various proposals.

10) J. Fischer et al. - Phys.Rev. D18 (1978) 4271.
See also : A. Martin, Ref. 2), and J. Fischer, Ref. 4).

FIGURE CAPTIONS

Fig. 1 : Total cross-sections, in mb, for pp (solid curve) and $\bar{p}p$ (dashed curve) as a function of $S^{1/2}$ in GeV. The curves correspond to Fit 1 of Table 1. See Eqs. (3) - (6). The data are taken from sources listed in Ref. 6).

Fig. 2 : The values of ρ, the ratio of the real to the imaginary part of the forward elastic scattering amplitude, for pp (solid curve) and $\bar{p}p$ (dashed curve), as a function of $s^{1/2}$ in GeV. The curves correspond to Fit 1 of Table 1. See Eqs. (3) - (6). The data are taken from the sources listed in Ref. 6).

Fig. 3 : a) The total cross-sections, in mb, for pp and $\bar{p}p$ as a function of $s^{1/2}$ in GeV. Both Fit 1, with Froissart bound form, and Fit 2, with an asymptotically constant cross-sections are shown. For $s^{1/2}$ less than about 100 GeV the fits are nearly indistinguishable, the upper curve being $\bar{p}p$ and the lower pp. For $s^{1/2}$ greater than 100 GeV, the difference between the $\bar{p}p$ and pp total cross-sections becomes very small, and the upper curve corresponds to Fit 1 and the lower to Fit 2.

b) The values of ρ, the ratio of the real to the imaginary part of the forward elastic scattering amplitude for $\bar{p}p$ and pp as a function of $s^{1/2}$ in GeV. Both Fit 1, with Froissart bound form and Fit 2, with an asymptotically constant total cross-section are shown. For $s^{1/2}$ less than about 40 GeV the fits are nearly indistinguishable, the upper curve being $\bar{p}p$ and the lower pp. Above this energy, the fits diverge, the lower of each pair being Fit 2. At very high energy, $\rho_{\bar{p}p}$ and ρ_{pp} become equal, and the upper pair of curves come from Fit 1, and the lower pair from Fit 2.

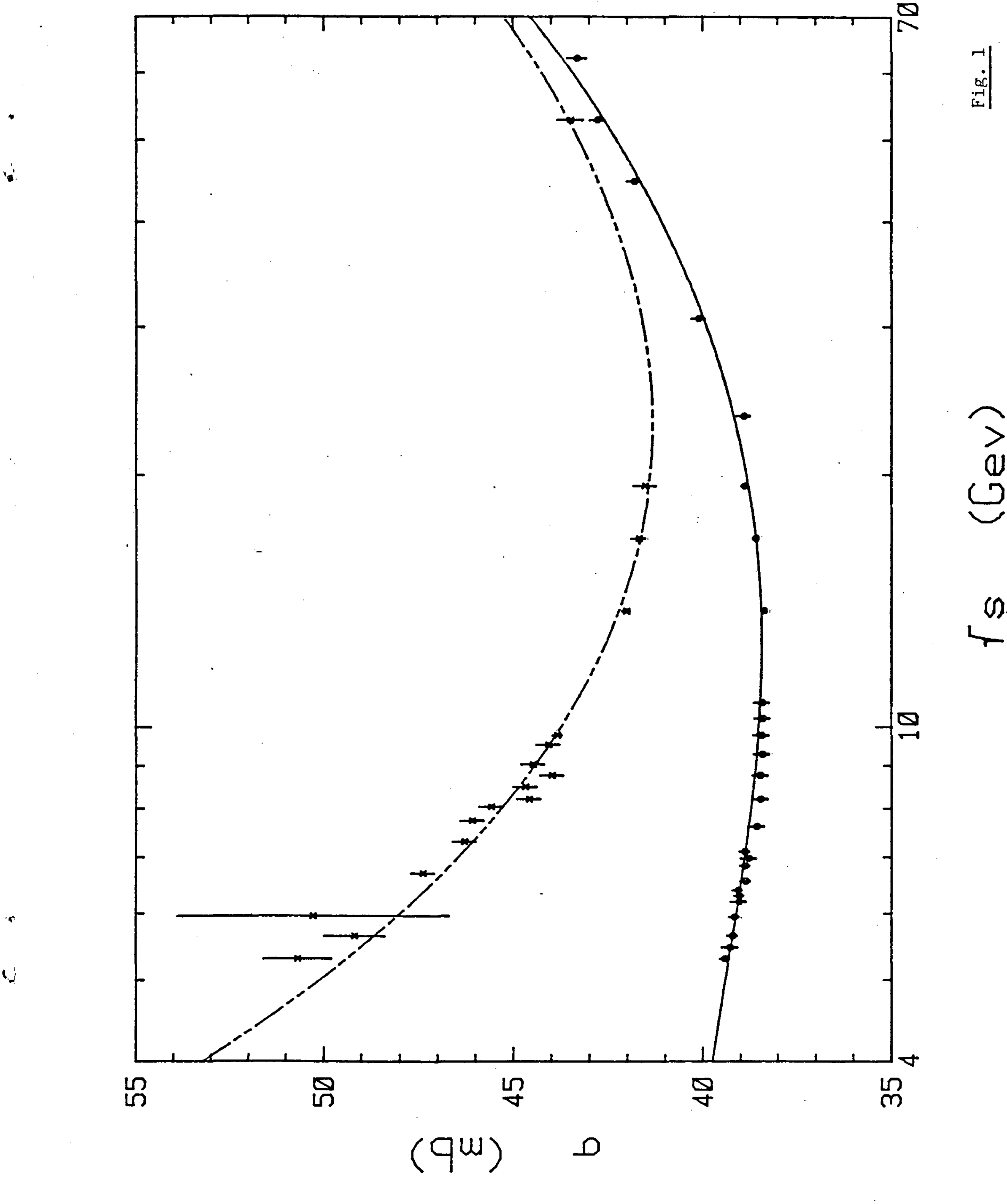
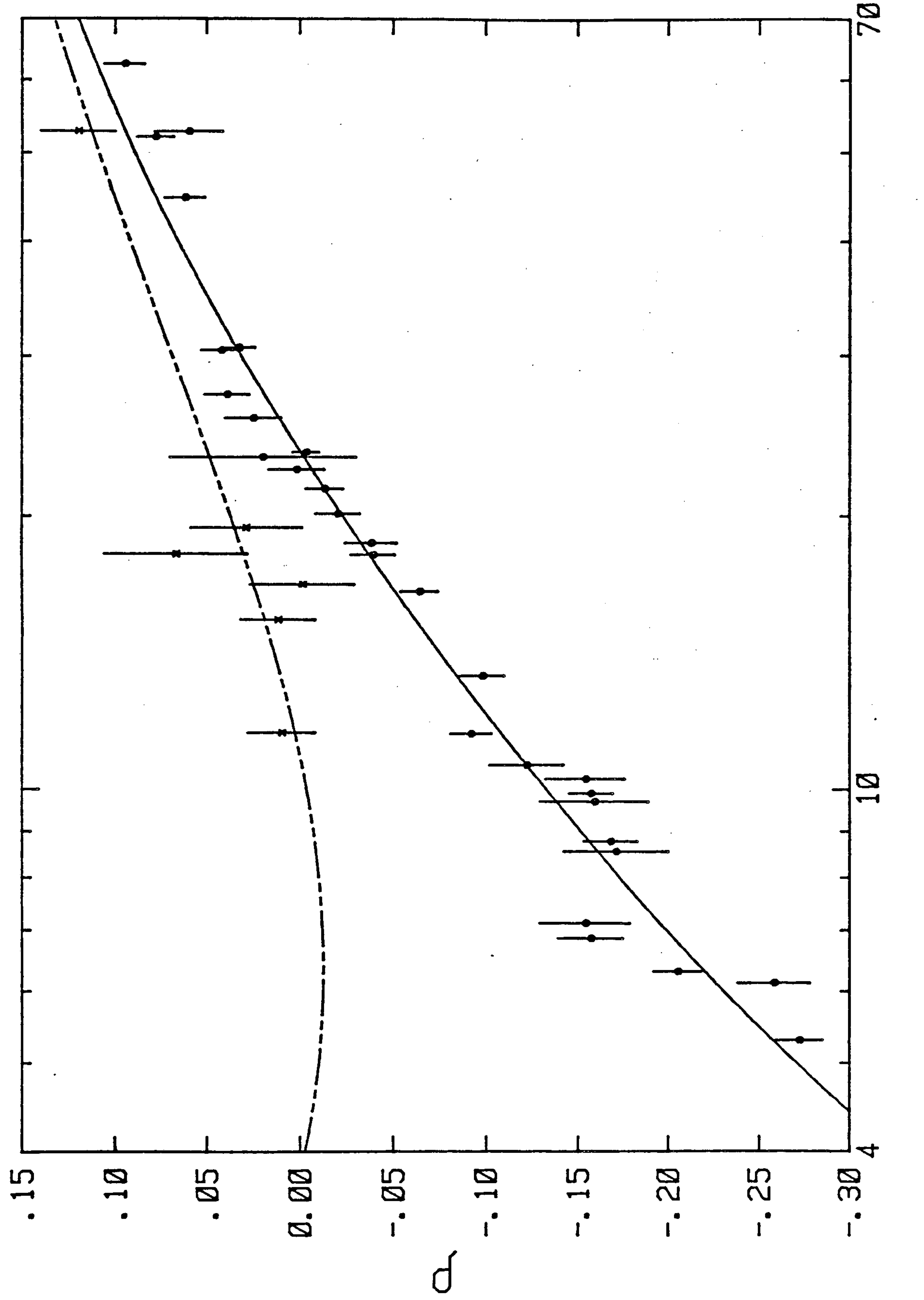
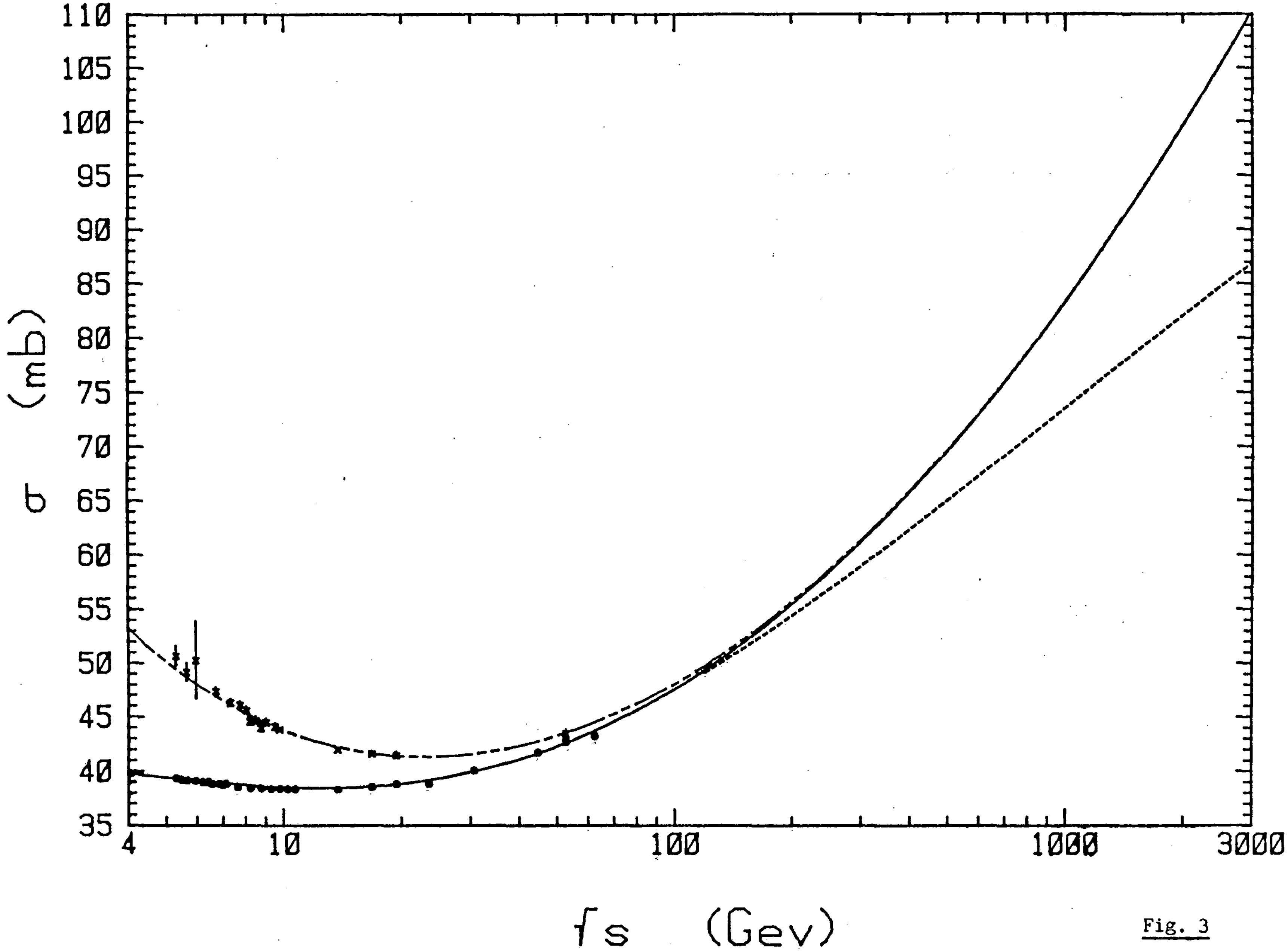


Fig. 1



√s (Gev)

Fig. 2


110
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σ (mb)
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√s (Gev)

Fig. 3

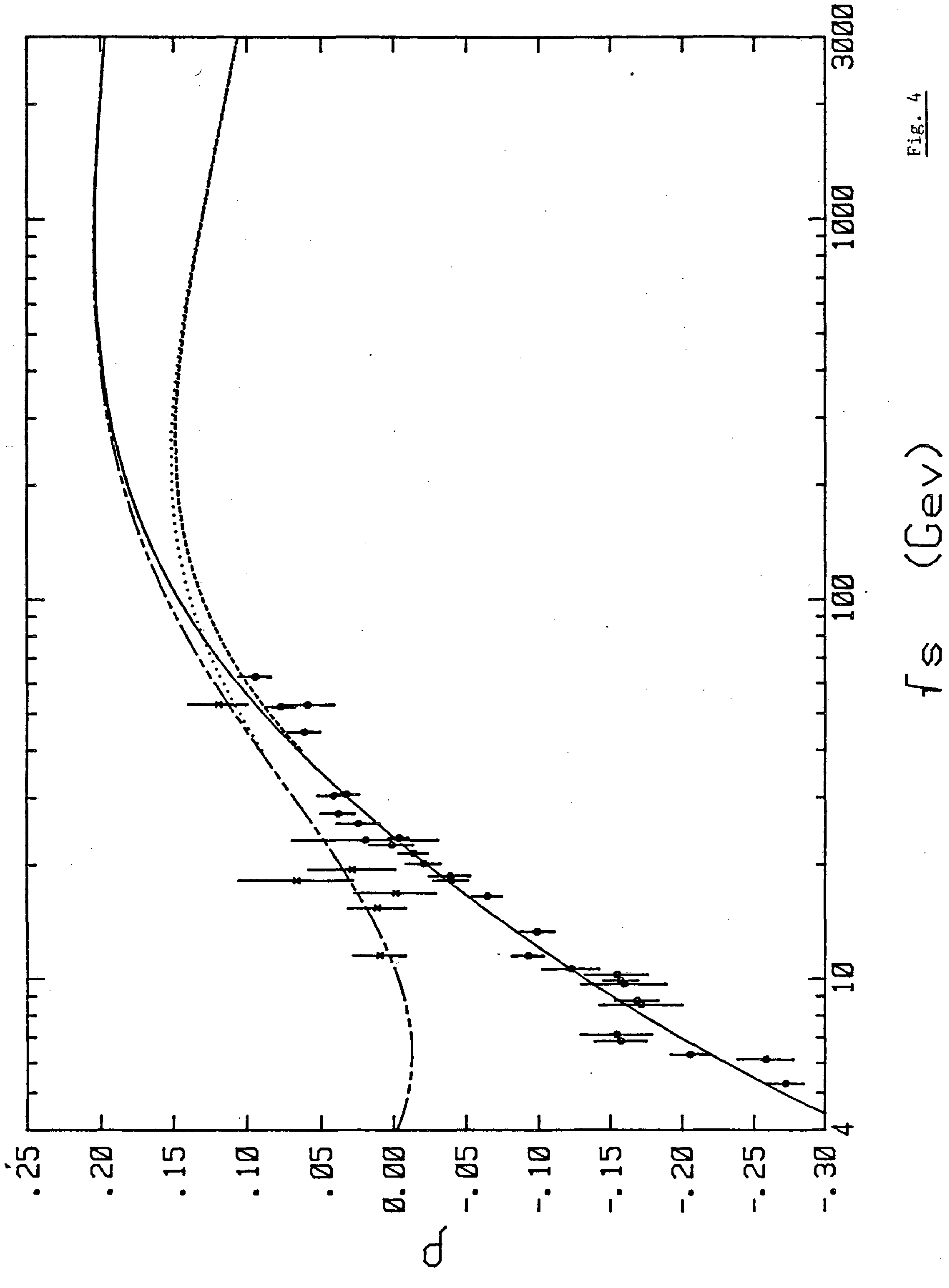

Fig. 4

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