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Cognitive correlates of performance in advanced mathematics

Wei Wei[1], Hongbo Yuan[1], Chuansheng Chen[2] and Xinlin Zhou[1]*

[1]State Key Laboratory of Cognitive Neuroscience and Learning, Beijing Normal University, China

[2]Department of Psychology and Social Behavior, University of California, Irvine, USA

Background. Much research has been devoted to understanding cognitive correlates of elementary mathematics performance, but little such research has been done for advanced mathematics (e.g., modern algebra, statistics, and mathematical logic).

Aims. To promote mathematical knowledge among college students, it is necessary to understand what factors (including cognitive factors) are important for acquiring advanced mathematics.

Samples. We recruited 80 undergraduates from four universities in Beijing.

Methods. The current study investigated the associations between students' performance on a test of advanced mathematics and a battery of 17 cognitive tasks on basic numerical processing, complex numerical processing, spatial abilities, language abilities, and general cognitive processing.

Results. The results showed that spatial abilities were significantly correlated with performance in advanced mathematics after controlling for other factors. In addition, certain language abilities (i.e., comprehension of words and sentences) also made unique contributions. In contrast, basic numerical processing and computation were generally not correlated with performance in advanced mathematics.

Conclusions. Results suggest that spatial abilities and language comprehension, but not basic numerical processing, may play an important role in advanced mathematics. These results are discussed in terms of their theoretical significance and practical implications.

Much research has been conducted to investigate cognitive correlates of mathematics performance. Among the many cognitive factors, basic numerical processing (e.g., non-symbolic quantity processing, counting, and symbolic quantity processing) has been shown as a major factor in mathematics performance (e.g., Halberda, Mazzocco, & Feigenson, 2008; Landerl, Bevan, & Butterworth, 2004; Landerl & Moll, 2010; Rousselle &

*Correspondence should be addressed to Xinlin Zhou, State Key Laboratory of Cognitive Neuroscience and Learning, Beijing Normal University, Beijing 100875, China (e-mail: zhou_xinlin@bnu.edu.cn).

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Noël, 2007). In addition, previous studies have also examined the importance of general cognitive processing or general intelligence (e.g., Deary, Strand, Smith, & Fernandes, 2007; Spinath, Spinath, Harlaar, & Plomin, 2006), working memory (e.g., Berg, 2008; Passolunghi, Vercelloni, & Schadee, 2007; Swanson & Kim, 2007), spatial processing (e.g., Guay & McDaniel, 1977; Jones & Burnett, 2008), and language abilities (e.g., Lee, Ng, Ng, & Lim, 2004; Koponen, Aunola, Ahonen, & Nurmi, 2007). In the following paragraphs, we review relevant studies.

Basic numerical processing and mathematics performance

Several studies have found significant relations between non-symbolic numerical processing and mathematics performance (e.g., Halberda *et al*., 2008; Mundy & Gilmore, 2009). For example, Halberda *et al*. (2008) used dots arrays to measure 14-year-old children's approximation ability. They found that approximation ability was significantly correlated with these children's earlier third-grade mathematics ability as measured by the TEMA-2 (Test of Early Mathematics Ability-2, Ginsburg & Baroody, 1990) and the computation subtest of the Woodcock–Johnson test (revised) (Woodcock, 1989), even when spatial processing ability, working memory, IQ, executive function, and language abilities were controlled for. Similarly, some studies have found that the students with disabilities in mathematics have poorer performance on the non-symbolic quantity task than normal participants (Iuculano, Tang, Hall, & Butterworth, 2008; Landerl *et al*., 2004; Mussolin, Mejias, & Noël, 2010; Piazza *et al*., 2010).

Other studies, however, found that performance on non-symbolic quantity tasks was not associated with mathematics performance. For example, Rousselle and Noël (2007) found children with disabilities in mathematics showed normal performance on a non-symbolic quantity task (comparing two sets of dots). Kovas *et al*. (2009) also found that children with high and low mathematical ability showed equal behavioural performance on the dots-comparison task.

In addition to non-symbolic quantity processing, researchers have also examined symbolic quantity processing and its importance for elementary mathematics (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Clarke & Shinn, 2004; Jordan, Kaplan, Locuniak, & Ramineni, 2007; Koponen *et al*., 2007; Landerl *et al*., 2004). For example, Landerl *et al*. (2004) found that, compared to normal children, those who suffered from dyscalculia had poorer ability in exact verbal counting (from 1 to 10). Passolunghi *et al*. (2007) also showed that counting ability and working memory were two of the most discriminating and efficient precursors of early mathematics achievement. It has also been found that, compared to normal controls, children with disabilities in mathematics took longer to perform the number comparison task (e.g., which of '3' and '7' is larger in magnitude?) (Landerl *et al*., 2004; Rousselle & Noël, 2007).

Intelligence and mathematics performance

Previous studies have consistently found a strong association between intelligence and mathematics performance (Deary *et al*., 2007; Glutting, Watkins, Konold, & McDermott, 2006; Kuncel, Hezlett, & Ones, 2004; Lynn & Mikk, 2009; Naglieri & Bornstein, 2003; Watkins, Lei, & Canivez, 2007). For example, Deary *et al*. (2007) found that intelligence measured at age 11 with the Cognitive Ability Test (Thorndike, Hagen, & France, 1986) accounted for 58.6% of the variance of mathematics performance measured at age 16 with the General Certificate of Secondary Education. Similarly, Spinath *et al*. (2006)

found that general cognitive ability (or intelligence) explained more than 50% of the variance of mathematics performance.

Language abilities

Several aspects of language abilities have been found to be associated with mathematics performance (e.g., Alloway *et al*., 2005; Berg, 2008; Hecht, Torgesen, Wagner, & Rashotte, 2001; Koponen *et al*., 2007; Lee *et al*., 2004). First, reading ability is correlated with mathematics performance. For example, Lee *et al*. (2004) found a correlation of .59 between 10-year-old children's literacy (reading, spelling, comprehension, and vocabulary) and mathematics performance. Koponen *et al*. (2007) found that text reading ability was significantly correlated with computation fluency for fifth graders ($r = .49$).

Second, phonological awareness is significantly correlated with elementary mathematics. Phonological awareness can be assessed with a wide range of tasks associated with word phonology, such as, rhyme categorization (de Jong & van der Leij, 1999), phoneme deletion (Durand, Hulme, Larkin, & Snowling, 2005), and first or last sound matching (Fuchs *et al*., 2005). In a study of 4- or 5-year-old children in United Kingdom, Alloway *et al*. (2005) found a correlation of .49 between phonological awareness and mathematical competencies. In their longitudinal study, Koponen *et al*. (2007) found that phonological awareness measured in kindergarten was correlated ($r = .42$) with computation fluency measured in grade 4. Another longitudinal study (Hecht *et al*., 2001) revealed correlations from .59 to .63 between reading ability at grade 2 and mathematics performance at grade 5 and correlations from .47 to .56 between phonological awareness at grade 2 and mathematics performance at grade 5.

Not all results are consistent, however. De Jong and van der Leij (1999) found that phonological awareness measured in kindergarten could only predict reading ability, but not mathematical ability, measured in grade 2. Durand *et al*. (2005) also found that phonological awareness of 7-year-old children did not predict mathematical skills measured 3 years later.

Working memory and short-term memory

Studies of both normal children and those with mathematical disabilities have found that working memory plays an important role in mathematics performance. In their study of children with mathematical disabilities, Siegel and his colleagues found that these children showed a deficiency in short-term memory or number-related working memory (Siegel & Linder, 1984; Siegel & Ryan, 1989). Examining different types of working memory and other cognitive factors, Berg (2008) found that children's (aged from 98 to 145 months) arithmetic computation performance was correlated highly (r's $> .50$) with processing speed, short-term memory, verbal working memory, and visual–spatial working memory. Similarly, Swanson and Kim (2007) found that short-term memory, working memory, and naming speed were correlated with children's mathematics performance, r's $= .60, .81, .71$, respectively. A number of other studies have also confirmed the association between working memory or short-term memory and mathematical achievement (Adams & Hitch, 1997; Bull, Espy, & Wiebe, 2008; Bull & Scerif, 2001; Furst & Hitch, 2000; Gathercole & Pickering, 2000a, 2000b; Geary, Hamson, & Hoard, 2000; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Hecht *et al*., 2001; Landerl *et al*., 2004; Swan & Goswami, 1997; Swanson, 2004; Swanson

& Sachse-Lee, 2001). There is a minority view, however, that short-term memory may not be as important for mathematics performance as others claimed. For example, Bull and Johnston (1997) found that, when reading ability was controlled for, arithmetic ability was best predicted by processing speed, with short-term memory accounting for no further unique variance. Hitch and McAuley (1991) also attributed mathematical difficulties to limited counting span, not to limited working memory.

Spatial abilities

Studies have shown close relations between spatial abilities and mathematics performance. For example, Krajewski and Schneider (2009) found a strong correlation (.41) between the spatial sketchpad of working memory (measured with the matrix task [Wilson, Scott, & Power, 1987] and the Corsi block task [Milner, 1971]) and mathematics performance (measured by German Mathematics Test for second graders [Krajewski, Liehm, & Schneider, 2004]). Rohde and Thompson (2007) found that spatial ability as well as processing speed made independent contributions to the mathematical portion of the SAT while controlling for the general cognitive ability measured by the Raven's Advanced Progressive Matrices and the Mill Hill Vocabulary Scales. Significant correlations between spatial processing tasks and mathematics performance were also revealed by other studies (Berg, 2008; Bull *et al*., 2008; Swanson & Kim, 2007). In fact, the numeral system has been shown to have spatial attributes. The spatial layout of numbers is called the mental number line (Dehaene, Bossini, & Giraux, 1993), with small numbers on the left side and large numbers on the right side. Evidence for the existence of a mental number line has come from research on the SNARC effect (Calabria & Rossetti, 2005; Dehaene, Dupoux, & Mehler, 1990; Dehaene *et al*., 1993; Fias, 1996; Nuerk, Wood, & Willmes, 2005). Neuropsychological research with patients also has shown a close relationship between numbers and space. Zorzi, Priftis, and Umiltà (2002) found that, when hemispatial neglect patients were asked to perform a number-bisection task, they tended to locate the mid-point to one side as compared to the mid-point shown by normal controls. After reviewing relevant literature, Hubbard proposed that both number processing and spatial cognition rely on the parietal lobe, especially the intraparietal sulcus (IPS) region (Hubbard, Piazza, Pinel, & Dehaene, 2005).

However, several studies failed to find a significant association between spatial abilities and mathematical achievement. For example, Lee *et al*. (2004) reported that the spatial sketchpad did not contribute to abilities in solving algebraic word problems. Bull, Johnston, and Roy (1999) tested children of 7 years old in central executive and visuo-spatial skills using the Wisconsin Card Sorting Test and Corsi span tests. Children of high and low mathematics ability as measured by a standardized assessment test did not differ in their visuo-spatial skills, although they did differ in their central executive skills. A longitudinal study by Mazzocco and Thompson (2005) found that spatial abilities did not predict later mathematics performance after controlling for prior number skills.

The current study

Although the literature is extensive on the correlates or predictors of mathematics performance, most of the studies (including those reviewed above) focused on elementary mathematics performance (e.g., memory of arithmetic facts, computation ability). Thus far, only a few studies have investigated the cognitive correlates of advanced mathematics (Dowker, 1992, 1996; Healy & Hoyles, 2000; Hermelin & O'Connor, 1986;

Kruteckij, 1976; Lewis, 1981; Tall, 1991; Tall & Mejia-Ramos, 2006). For example, in a study of mathematically talented children, Kruteckij (1976) found that these children possessed cognitive characteristics such as superior abilities in logical reasoning involving quantitative and spatial relationships, numbers, mathematical operations, and letter symbols. Similarly, Hermelin and O'Connor (1986) found that mathematically gifted adolescents performed better on spatial tasks than their non-gifted peers. In a study of adults, Dowker (1992, 1996) found that, compared to non-experts, mathematicians were more accurate and used a wider variety of strategies to do computational estimation. Finally, Jones and Burnett (2007) explored the role of spatial ability in computer programming ability (a task similar to advanced mathematics), and found a significant correlation between mental rotation ability and performance in programming. Although these few studies have contributed to our understanding of individuals with superior mathematical abilities (across different age groups), they are limited in several ways. First, these studies have focused on children and young adolescents. Second, most of these studies focused on mathematically gifted subjects with little attention paid to explaining the variations within the large group of subjects in the normal range. Finally and perhaps most importantly, all of these studies took a static approach (i.e., comparing cognitive correlates between the subjects who already had acquired advanced mathematical knowledge and their non-gifted peers). None of them took a dynamic approach (i.e., examining cognitive correlates that predicted the efficiency in learning advanced mathematics).

The present study aimed to investigate the cognitive correlates of learning advanced mathematics among adults who had not had a specialized training. Five types of cognitive correlates were included: basic numerical processing (i.e., numerosity comparison, number comparison), complex numerical processing (i.e., multiple-digit calculation, number series completion, artificial arithmetic learning), visuo-spatial processing (i.e., three-dimensional mental rotation, spatial working memory, and figure analysis), language processing (i.e., word rhyming, word semantics processing, sentence syntactic processing, and word paired-associate learning), and general cognitive processing (i.e., simple reaction time, attention, and Raven's Progressive Matrices) (see the Tasks section for details).

Basic numerical processing was included because of its important role in elementary mathematics (e.g., Halberda *et al.*, 2008; Landerl *et al.*, 2004; Landerl & Moll, 2010; Rousselle & Noël, 2007). One hypothesis is that basic numerical processing continues to play a central role in advanced mathematics. However, we expected to disconfirm this 'number sense' hypothesis because advanced mathematics is far removed from basic numerical processing. With our college sample, inter-individual variations in basic number processing should be limited and should not be consequential to their learning advanced mathematics. Instead, it was expected that complex numerical processing would be more likely to be associated with advanced mathematics.

More important than either type of numerical processing, however, is spatial processing. As mentioned earlier, spatial abilities are important for both elementary and advanced mathematics. For example, advanced mathematics relies on spatial reasoning (e.g., geometry) and graphic representations (e.g., functions).

Finally, we included tasks related to language processing and general cognitive processing to explore whether they would make significant contributions to the learning of advanced mathematics. By including this large number of cognitive tests, the current study was the first to systematically examine which cognitive factors would be significantly related to the learning of advanced mathematics.

Advanced mathematics in the current study was defined as the mathematics that is acquired by college students majoring in mathematics, such as modern algebra, mathematical logic, function theory, mathematical analysis, and computational mathematics. In this study, we taught these concepts to college students who had not been exposed to such materials because they did not major in mathematics or allied disciplines (e.g., physics, statistics, engineering, and computer science).

Methods

Participants

Eighty undergraduates (40 males and 40 females) were recruited from four universities in Beijing (i.e., Beijing Normal University, Beijing University of Posts and Telecommunications, University of Science and Technology in Beijing, and Beijing Wuzi University). Their mean age was 21.9 years, ranging from 18.0 to 27.2 years. All participants were native Chinese speakers. They were right-handed, had normal or corrected-to-normal eyesight, and were free from colour blindness. After the experiment, each participant received a payment of RMB 80 yuan (about US$12).

General procedure

There were 18 tasks in the present study. Seventeen tasks were administered on the computer, and only the advanced mathematics test was conducted with paper and pencil. The 18 tasks were administered in 1 day, lasting about 4 hr (2 hr in the morning session for nine tasks and 2 hr in the afternoon session for the other nine tasks). The interval between the two sessions was about 4 hr to ensure that the participants had adequate rest. Tests were conducted in small groups of one to four participants. All participants followed the same order of tasks. Before each task, participants were given instructions and practice trials. They responded by pressing ‘P’ or ‘Q’ on the keyboard, or clicking on the right answer on the screen, or entering a numerical value into a blank. All the tasks are available on the website ‘http://www.dweipsy.com/’.

Tasks

Table 1 lists all tasks used in this study. It also includes the number of trials for practice and actual tests, the time length for the tests, indices of test performance, and the means and standard deviations of the indices. For some measures (as indicated in the table), we used Guilford correction formula ‘$CP = (KP - 1) / (K - 1)$’ (CP, corrected percent of pass; K, alternative choice; P, original percent of pass) to calculate the number of correct trials (Guilford, 1936). The split-half reliabilities for all the tasks were calculated from the data in the current study (see Table 1).

Basic numerical processing tests

Comparison of dots of two arrays

This task was adapted from TEMA-2 (Ginsburg & Baroody, 1990). Two dot arrays were presented sequentially on the screen, each lasting for 200 ms. Participants were asked to judge which dot array contained more dots by pressing ‘Q’ if the first array contained more dots or ‘P’ if the second array contained more dots. The interval between

Table 1. Basic characteristics of the advanced mathematics test and cognitive tasks

Tasks	Number of practice trials	Number of test trials	Time length	Index	Means(*SD*)	Split-half reliability
Basic numerical processing						
Comparison of dots of two arrays	8	148	8 min	RT	497 (109)	.96
Comparison of coloured dots	10	100	6 min	Percent correct	67 (8)	.77
Estimation of numerosity	8	56	7 min	Deviation	8.9 (2.2)	.83
Number comparison	10	84	3 min	RT	500 (75)	.95
Complex numerical processing						
Computation	10	100	10 min	Number of correct trials[a]	81 (28)	.93
Number series completion	2	30	8 min	Number of correct trials[a]	15 (6)	.95
Arithmetic learning	3	15	10 min	Percent correct	69 (13)	.69
Spatial processing						
3-dimensional mental rotation	12	300	6 min	Number of correct trials[a]	49 (17)	.91
Spatial working memory	3	14	7 min	Deviation	1,956 (605)	.81
Figure analysis test	3	18	4 min	Number of correct trials[a]	7.2 (3.3)	.68
Language processing						
Word rhyming	10	70	4 min	RT	1,558 (404)	.93
Word semantic analysis	6	110	7 min	Number of correct trials[a]	34 (11)	.92
Sentence syntactic analysis	5	55	8 min	Number of correct trials[a]	9.4 (6.1)	.89
Word paired-associate learning	3	15	10 min	Percent correct	63 (12)	.69
General cognitive processing						
Simple reaction time	10	40	3 min	RT	251 (32)	.84
Attention	24	192	16 min	RT	520 (53)	.97
Raven's progressive matrices	12	36	15 min	Number of correct trials[a]	20 (4)	.79
Advanced mathematics performance	2	36[b]	30 min	Number of correct trials[a]	14 (5)	.81

Note. Time length refers to the length of time for the formal test, not including time for the practice trials. min, minutes; RT, reaction time.
[a]Number of correct trials were based on Guilford correction.
[b]There were 18 concepts (two questions each) for the advanced mathematics test.

participants' response to the last dot array and the onset of next dot array was 1,000 ms. The number of dots for each dot array varied from seven to 14. The dots were spread out and occupied the same size of area regardless of the number of dots. The reaction time for correct responses was analysed because the average percentage of correct responses was 90%.

Comparison of coloured dots

This task was adapted from Halberda *et al*. (2008). Dot arrays including both blue and red dots were presented on the screen for 200 ms. The participants were asked to judge which colour had more dots by pressing the key 'Q' or 'P'. The ratio of the two types of coloured dots varied from 1: 2, 2: 3, 3: 4, 5: 6 to 7: 8. The total areas for each colour type were kept the same. The number of dots in a dot array varied from five to 16. The formal test had 100 trials in two sessions. The accuracy rates were analysed.

Estimation of numerosity

This task was adapted from Butterworth (Butterworth, 1999; Landerl *et al*., 2004). Participants were asked to quantify dot arrays from 11 to 99. Dot arrays were presented on the screen for 200 ms. Participants entered their estimate for each array into the computer using the number keypad. After each trial, participants received feedback whether their estimate was correct. They were asked to adjust the next estimation according to the feedback. The formal test had two sessions, each with 28 trials. The dependent variable was the deviation of estimated quantity from the correct quantity.

Number comparison task

In the number comparison task (part of the classic number Stroop task, Girelli, Lucangeli, & Butterworth, 2000; Vuilleumier, Ortigue, & Brugger, 2004; Zhou *et al*., 2007), participants were asked to judge which of two single-digit numbers, presented side-by-side on the screen, was larger in numerical magnitude, while ignoring the differences in physical size. Participant made the judgment by pressing key 'Q' if the number on the left side was larger or pressing key 'P' if the number on the right side was larger. The physical size of the numbers also varied: 'Small' numbers were 1.6° in horizontal visual angle and 2.4° in vertical visual angle, and 'large' numbers had corresponding angles of 2.0° and 2.9°. There were three conditions (or types of number pairs). For the neutral condition, both numbers were of the same physical size (either small or large); for the congruent condition, the numerically smaller number was also physically smaller; and for the incongruent condition, the numerically smaller number was physically larger. This test included 84 trials in three sessions (28 trials for each session). Only the reaction times for correct trials were analysed because the average percentage of correct responses was 95%.

Complex numerical processing tests

Multiple-digit computation

Multiple-digit addition, subtraction, multiplication, and division problems were used. For each trial, the problem was presented in the middle of the upper part of the screen, and four candidate answers were presented in the lower part of the screen, two on the left side and the other two on the right side. One of the four answers was correct. The incorrect answers were constrained to be the correct answer plus or minus 1 or 10. All four answers had the same number of digits. The addition tasks involved two two-digit numbers as addends with the sum as a three-digit number, and the subtraction tasks involved two two-digit numbers with the result as a two-digit number. The multiplication tasks had one two-digit number and one single-digit number with the product as a three-digit number, and the division tasks had one three-digit number divided by one single-digit number with the result as a two-digit number. Participants were asked to first solve the problem, and then decide whether the correct answer was one of the two candidate answers on the left side or one of the two candidate answers on the right side by pressing a left or a right key on the keyboard. The Guilford correction formula was used to calculate participants' score.

Number series completion

This task was adapted from the Cognitive Abilities Test 3 (CAT 3;Smith, Fernandes, & Strand, 2001). In this task, a series of numbers was presented in the middle of the screen, and the participants were asked to judge what the next number would be based on the pattern of the series. For example, the series of numbers '1 3 5 7 9' would have 11 as the next number. There were five alternative answers on the screen below the given series. The participants responded by clicking the correct answer. The number series and alternative answers remained on the screen until participants responded, or when 8 min had lapsed. We used the Guilford correction formula to calculate the participants' score.

Arithmetic learning

This test was based on Delazer and colleagues' study (Delazer *et al*., 2005). An artificial operation '§' was defined as '$12b - 9a + 70$'. That is, $a \S b = 12b - 9a + 70$. For example, $5 \S 3 = 61$ (because $5 \S 3 = 12 \times 3 - 9 \times 5 + 70$), Two other examples of the equations were $2\S1 = 64$, $4\S2 = 58$. Fifteen equations were created based on the artificial operation. The subjects, however, were not given the above definition of the operation. Instead, they were briefly presented an expanded form of the operation (i.e., '$a \S b$' = '$(10 - a) \times b - (b - a) \times (b - a) - (a - 3) \times (b - 1) + a \times a + b \times b - 10 \times a - b + 73$'). Because of its complexity, the participants were not able to memorize the definition of the artificial operation. Instead, they were asked to memorize the associations between pairs of operands and their answer for the 15 equations. During the learning stage, an equation was presented in the middle of the screen for 10 s. After subjects memorized all equations, they were tested. During the test stage, participants needed to judge whether a given equation (e.g., '$5 \S 3 = 61$', '$5 \S 3 = 64$') was correct or not. Half of the trials in the test stage were correct equations, and the other half was incorrect equations. Each trial was presented for 9 s. After this test stage, participants learned the equations again and were tested again. The percentage of correct answers on the second test was analysed.

Spatial processing tests

Three-dimensional mental rotation

The three-dimensional mental rotation task was based on Shepard's mental rotation task (Shepard & Metzler, 1971). For each trial, one three-dimensional image was presented in the upper part of the screen, and two others in the lower part. After mentally rotating the upper one (or one of the bottom ones), the participants needed to choose the bottom figure that would match the top one. The angles of rotation were 15°, 30°, ... , 345°, with a step of 15°. There were two blocks, one for easy trials that involved small rotating angles (i.e., 15°–60° and 300°–345°), and the other for difficult trials that involved large rotating angles (i.e., 180° and 210°). Each block was limited to 3 min. Each trial remained on the screen until participants responded by pressing either the left or the right key to indicate their choice. We used the Guilford correction formula to calculate the participants' score.

Spatial working memory

This task was similar to Corsi block task (Corsi, 1972). Dots were sequentially presented in an implicit lattice of 3 by 3 on the computer screen. Each dot was presented for 1,000 ms, and dots were presented with an interval of 1,000 ms. After the last dot was presented and disappeared, the participants clicked the positions where the dots had appeared in the same sequence as their appearance. The number of dots ranged from 4 to 9. There was no feedback to participants. The average distance between the position where the dot appeared and the position where participants clicked was calculated and treated as an index of spatial working memory. Larger average distances reflected poorer spatial working memory.

Figure analysis test

This task was adapted from the Cognitive Ability Test (level G, Lohman & Hagen, 2001). In the upper part of the screen, there were three or four pictures that showed a step-by-step procedure of folding and perforating a piece of square-shaped paper. Five candidate answers were presented in the lower part of the screen. The participants needed to decide what the paper would be like when unfolded. Four minutes were allowed for 18 items. We also used the Guilford correction formula to calculate the participants' score.

Language processing tests

Word rhyming

The task was similar to that used by Tan *et al*. (2001, 2003). Two Chinese characters were presented simultaneously on the screen. Participants needed to judge if the two characters rhyme by pressing the right or the left key. For example, the characters马(mǎ, horse) and 打 (dǎ, hit) rhyme, but 牛 (niú, cattle) and 歌 (gē, song) do not rhyme. The percentage of correct answers was high, 89%, and thus was not further analysed. Instead, participants' reaction time for correct responses was analysed.

Word semantic processing

This task was similar to the one used by Siegel and Ryan (1988) and So and Siegel (1997). Materials in the task were adapted from the college entrance examinations used in China in recent years. In the task, a sentence was presented in the centre of the computer screen with a word missing. Participants needed to select one of two candidate words presented beneath the sentence by pressing a left or a right key. The stimulus remained on the screen until the participants responded.

Sentence syntactic processing

The task was similar to the one used by Hagoort, Brown, and Groothusen (1993). Materials in the task were adapted from the college entrance examinations used in China in recent years. Participants were asked to decide if a sentence was syntactically correct (by pressing 'P') or incorrect (by pressing 'Q'). Each trial stayed on the screen until the participants responded.

Word paired-associate learning.

Paired-associate learning was invented by Calkins (1894). It contains the pairing of two items, usually words, with the first as the stimulus and the second as the response. Previous studies have shown that word paired-associate learning is related to language processing ability (e.g., Krug, Shafer, Dardick, Magalis, & Parenté, 2002; Messbauer & de Jong, 2003; Windfuhr & Snowling, 2001). We used two-character nouns to form the pairs, such as, "火花→刀口" (flare→blade of a knife). Participants firstly tried to memorize 15 word pairs, each of which was presented for 10 s in the middle of the screen in the learning stage. During the test stage, participants needed to judge if pairs were the ones that they had just learned by pressing a key. Each trial lasted 3 s. The learning and test stages were repeated. The percentage of correct answers in the second test was analysed.

General cognitive processing tests

Simple reaction time task

The simple reaction time task was adapted from Nelson Dyscalculia screener (Butterworth, 2003). Simple reaction time was measured by asking the participants to press a key in response to a black solid dot or a solid triangle. The position where the stimulus occurred on the screen was randomly determined, in a range of 15° of visual angles. The inter-stimulus interval was randomly set between 1,500 ms and 3,000 ms. This test had two sessions (each with 20 trials), one using the left hand to respond and the other using the right hand. Before the formal test, participants received 10 practice trials. Only the reaction time was analysed because the average percentage of correct responses was 97%.

Attention

The attention test was adopted from Fan *et al.*'s attention network test (Fan, McCandliss, Sommer, Raz, & Posner, 2002). It has been extensively used to assess attention (Greene *et al.*, 2008; Rueda *et al.*, 2004; Weaver, Bédard, McAuliffe, & Parkkari, 2009). Five arrows in one line were presented on the screen, and the participants needed to judge the direction of the arrow in the middle by pressing the left or the right key to be consistent with the direction of the arrow. Before the arrow line was presented, there was a cue for alerting. There were two types of middle arrows: Their direction was either the same as that of the other arrows (i.e., the congruent condition) or opposite of the direction of the other arrows (i.e., the incongruent condition). There were 192 trials presented in two blocks. Before each trial, a '+' sign was presented for a random duration between 400 ms and 1,200 ms, followed by the cue sign for 100 ms, then the '+' sign again for 400 ms, and finally by the arrow line. The arrow line was presented for 1,700 ms or until the participants pressed a key. The average percentage of correct responses was 98.2%, so we just used the reaction time on the correct trials as the dependent variable.

Raven's Progressive Matrices

The Raven's Progressive Matrices are a well-known test of abstract reasoning (Raven, Raven, & Court, 1998). It has been widely used to assess general intelligence. The participants needed to identify the missing segment of a figure according to the figure's inherent regularity. Six or eight alternate pictures were presented, and one of them was the correct one. Participants used the computer mouse to choose the correct answer.

Advanced mathematical concepts. This task was conducted with paper and pencil. Eighteen advanced mathematics concepts were introduced to the participants. The 18 concepts were selected from 10 areas in advanced mathematics, including modern algebra, statistics, mathematical logic, function theory, graph theory, geometry, mathematical analysis, computational mathematics, advanced mathematics, and set theory (See Appendix A). All of the concepts used in this study came from mathematical textbooks used by university students majoring in mathematics. Participants had not learnt these mathematical concepts based on the screening interview. Participants first read a passage that introduced a mathematical concept, and then answered two multiple-choice questions (See examples in Appendix B). This process was repeated for each concept. Participants were given 30 min to complete the whole test. We used the Guilford correction formula to calculate the participants' score.

Data analysis

We first calculated the correlation coefficients between the scores for cognitive tasks and the score for the advanced mathematics test. We then obtained partial correlations after controlling for general cognitive abilities. Because of the large number of tests and a modest sample size, we did not conduct a factor analysis of the cognitive predictors. Nevertheless, we derived a summary score for each type of cognitive factors (e.g., basic numerical processing) by averaging standardized scores of the tasks that were theoretically grouped. Scores based on the reaction times (i.e., comparison of dots of two arrays, estimation of numerosity, and number comparison) and deviations (i.e., spatial working memory) were transformed by subtracting them from 0 before standardization. Consequently, all summary scores were in the direction of higher scores indicating better cognitive abilities.

Results

The means and standard deviations for all measures are displayed in Table 1 and their intercorrelations are presented in Table 2. For basic numerical processing, the two numerosity-related tasks (i.e., comparison of dots of two arrays, and comparison of coloured dots) did not correlate with each other, and they also did not show consistent correlations with other measures. Scores on the tests of comparison of coloured dots and number comparison were not correlated with advanced mathematics. Among the four tasks within the category of basic numerical processing, the estimation of numerosity

Table 2. The intercorrelations among all measures

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. Comparison of dots of two arrays	-																
2. Comparison of coloured dots	.12	-															
3. Estimation of numerosity	.16	−**.42**	-														
4. Number comparison	**.53**	.14	**.31**	-													
5. Computation	−.02	**.26**	−.19	−.01	-												
6. Number series completion	−.12	.11	−**.35**	−.18	.09	-											
7. Arithmetic learning	−.03	**.26**	−**.43**	−.18	−.04	**.32**	-										
8. 3-dimenstional mental rotation	−.15	**.35**	−**.25**	−.04	.05	**.37**	**.26**	-									
9. Spatial working memory	.16	−.06	.19	.08	−.17	−**.39**	−.17	−**.25**	-								
10. Figure analysis test	−.06	**.39**	−**.23**	.07	.07	**.25**	**.23**	**.58**	−**.31**	-							
11. Word rhyming	.20	−.17	**.41**	**.26**	−.16	−.20	−**.24**	−.21	**.29**	−**.22**	-						
12. Word semantic test	−.08	.08	−**.22**	−**.26**	−.08	**.35**	**.30**	**.25**	−.16	.21	−**.46**	-					
13. Sentence syntactic test	−.22	.02	−.02	−**.24**	−.05	.07	−.08	.15	.05	.13	−**.40**	**.42**	-				
14. Word paired-associate learning	−.12	.21	−**.22**	−.11	.03	.06	.22	.14	−.09	.18	−**.30**	**.23**	.05	-			
15. Simple reaction time	**.29**	−.16	**.39**	**.32**	−.04	−.03	−.14	−.01	**.27**	−.12	**.22**	−.08	.02	−**.31**	-		
16. Attention	**.29**	−.11	**.38**	**.44**	−.10	−.12	−.14	−.13	**.23**	−.12	**.45**	−**.26**	−**.23**	−.19	**.50**	-	
17. Raven's progressive matrices	−.04	**.32**	−**.29**	−.05	.13	**.37**	.18	**.39**	−**.27**	**.37**	−**.29**	.16	.02	.14	−.21	−.14	-
18. Advanced mathematics	−**.23**	.18	−**.25**	−.19	.22	**.54**	**.37**	**.45**	−**.41**	**.38**	−**.41**	**.41**	**.23**	.12	−.09	−**.28**	**.39**

Note. Correlation coefficients in bold were statistically significant.

appeared to have consistent correlations with other measures (except for computation, spatial working memory, and sentence syntactic test). It should be noted that the test of comparison of coloured dots appeared to have the floor effect.

For complex numerical processing, number series completion and arithmetic learning were correlated, but neither was correlated with computation. The three spatial tasks (i.e., three-dimension mental rotation, spatial working memory, and figure analysis test) were significantly correlated with one another. All three tasks also were correlated with advanced mathematics and number series completion, but not with the sentence syntactic test and the word paired-associate learning.

The intercorrelations among the four measures of language abilities (word rhyming, word semantics test, sentence syntactic test, and word paired-associate learning) were mostly significant, with the exception of that between the sentence syntactic test and the word paired-associate learning. Scores on the tests of word rhyming, word semantics, and sentence syntax showed significant correlations with performance in advanced mathematics.

In terms of the tests of general cognitive processing, simple reaction time and attention were significantly correlated, but neither of them was correlated with the Raven's Progressive Matrices. Both the attention test and Raven's Progressive Matrices were significantly correlated with advanced mathematics.

The above bivariate correlations showed that, among the numerical processing tasks, only the comparison of dots of two arrays and the estimation of numerosity demonstrated significant correlations with advanced mathematics, whereas all of the spatial processing tasks were significantly correlated with advanced mathematics. These analyses, however, did not exclude potential confounding factors. Thus, we tested the number sense hypothesis and the spatial processing hypothesis by using partial correlations. To test the number sense hypothesis, we controlled for general cognitive processing, spatial processing, and language processing. To test the spatial processing hypothesis, we controlled for general cognitive processing, number sense, and language processing. The results are shown in Table 3. None of the four tasks measuring number sense had a significant partial correlation with advanced mathematics. This result did not support the number sense hypothesis. Consistent with the spatial processing hypothesis, the mental rotation, spatial working memory and figure analysis tests had significant partial correlations with advanced mathematics.

Although tangential to our hypothesis-testing, Table 3 also shows the partial correlations between language processing tests and advanced mathematics. After controlling for general cognitive processing, spatial processing, and basic numerical processing, the word semantics test was still correlated with advanced mathematics. Finally, when we controlled for general cognitive processing, basic numerical processing, spatial processing, and language processing, the number series completion and arithmetic learning were still correlated with advanced mathematics, $r = .313$, $p = .011$ and $r = .254$, $p = .041$, respectively.

The final set of analyses was based on the summary scores for each type of cognitive factors. The left panel of Figure 1 displays no correlation ($r = .021$, $p = .852$) between the combined z-value for basic numerical processing (i.e., comparison of dot arrays, coloured dot comparison, estimation of numerosity, and number comparison) and advanced mathematics performance, after general cognitive processing, language processing, and spatial processing were controlled for. The right panel of Figure 1 shows a significant correlation ($r = .540$, $p = .000$) between the combined z-value for spatial processing (i.e., mental rotation, spatial working memory, figure analysis test)

Table 3. Partial correlations of basic numerical processing, spatial processing, and language processing with advanced mathematics performance

Tasks	Coefficients	Significance
Basic numerical processing		
Comparison of dots of two arrays	−.167	.168
Comparison of coloured dots	−.013	.915
Estimation of numerosity	−.015	.904
Number comparison	−.097	.427
Spatial processing		
3-dimenstional mental rotation	.254	.035
Spatial working memory	−.311	.009
Figure analysis	.244	.064
Language processing		
Word rhyming	−.188	.118
Word semantics	.282	.018
Sentence syntax	.150	.214
Word associate learning	.025	.839

Note. Partial correlation between each test and advanced mathematics was calculated after controlling for the general cognitive processing and the other two types of cognitive processing.

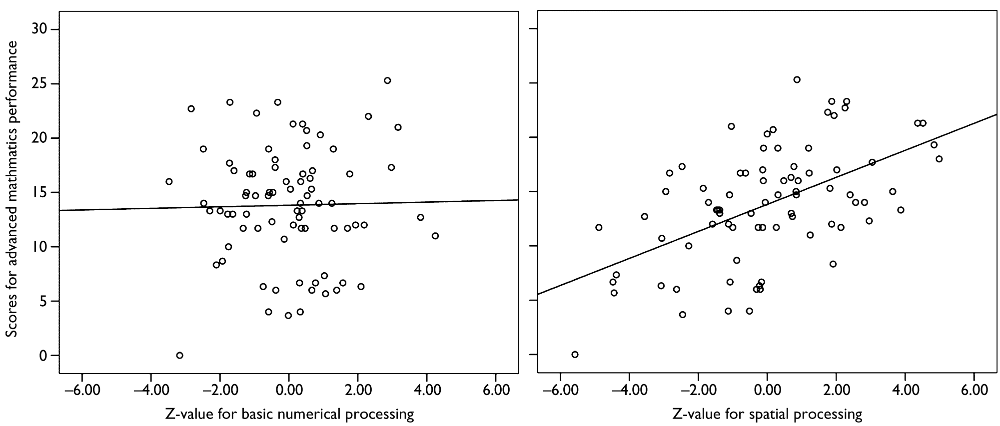

Figure 1. Scatter plots between the z-value for basic numerical processing and advanced mathematics performance (the left panel) and between the z-value for spatial processing and advanced mathematics performance (the right panel).

and advanced mathematics performance, after general cognitive processing, language processing and basic numerical processing were controlled for.

Discussion

The goal of the current study was to investigate cognitive correlates of learning advanced mathematics. The number sense hypothesis and the spatial processing hypothesis in advanced mathematics performance were tested. The results showed that basic numerical processing did not substantially contribute advanced mathematics performance, although it has been found to be significantly correlated with elementary mathematics performance. In contrast, spatial ability was correlated with advanced mathematics, even after general cognitive processing, number sense, and language processing were partialled out. These results supported the spatial processing hypothesis. Additionally, the processing of word semantics was significantly associated with the performance in advanced mathematics after the general cognitive processing, number sense, and spatial processing were partialled out. These results are discussed in terms of their theoretical significance and practical implications.

Basic numerical processing in mathematics performance

Non-symbolic and/or symbolic number sense has been found to be significantly correlated with elementary mathematics performance (e.g., Aunola *et al.*, 2004; Clarke & Shinn, 2004; Halberda *et al.*, 2008; Holloway & Ansari, 2009; Jordan *et al.*, 2007; Koponen *et al.*, 2007; Krajewski & Schneider, 2009; Landerl *et al.*, 2004; Mundy & Gilmore, 2009; Passolunghi *et al.*, 2007; Rousselle & Noël, 2007;). For example, after controlling for spatial processing ability, working memory, IQ, executive function, and language abilities, Halberda *et al.* (2008) found that visual approximation ability of numerosity was still significantly and positively correlated with children's mathematics ability. Rousselle and Noël (2007) found that children with mathematics disabilities showed deficient symbolic numerical processing. In contrast, the current study did not find any significant partial correlations between measures of basic numerical processing and advanced mathematics. One explanation for this is that elementary mathematics involves numerosity and numbers, but advanced mathematics is too far removed from basic numerical processing. After all, college students should have proficient skills in basic numerical processing. In sum, basic non-symbolic and symbolic numerical processing may be critical for elementary mathematics, but it was not a significant correlate of advanced mathematics in this study.

Spatial processing and mathematics performance

Previous studies have shown close relations between spatial abilities and mathematics performance (e.g., Berg, 2008; Bull *et al.*, 2008; Krajewski & Schneider, 2009; Rohde & Thompson, 2007; Swanson & Kim, 2007). This relation might reflect the inherent spatial attribute of mathematics performance (see a review by Hubbard *et al.*, 2005). For example, the number representation is along a mental number line with small numbers on the left side and large numbers on the right side. The mental number line exists not only for the one-digit numbers but also for the two-digit numbers (e.g., Dehaene *et al.*, 1993; Zhou, Chen, Chen, & Dong, 2008). For advanced mathematics, it is crucial to have accurate mental spatial layout of mathematical symbols. For example, the summation formula '$\sum_{k=1}^{m} k = 1 + 2 + 3 + ...+m$' used in the present study consists of operation symbols, numerals, and letters, all of which need to be spatially represented. More importantly, spatial representations are extensively used in advanced mathematics (e.g., advanced geometry, advanced algebra, analytic mathematics, and statistics). For example, analytic functions in mathematics could be transformed into graphic representations to show the relations among variables. Finally, the close relations between spatial and mathematical abilities are also reflected in their overlapping neural mechanisms. Brain-imaging studies have shown that the bilateral parietal lobes, particularly the intra-parietal sulcus (IPS), are important for both numerical processing (see a review by Dehaene, Piazza, Pinel, & Cohen, 2003) and spatial processing (e.g., Coull & Frith, 1998; Göbel, Calabria, Farn, & Rossetti, 2006; Kaufmann *et al*., 2008).

Language processing and mathematics performance

Four types of language processing tasks were used in this study, including word semantics, word rhyming, sentence syntax analysis, and word paired-associate learning. Only the processing of word semantics was associated with mathematics performance after controlling for the general cognitive processing, number sense, and spatial processing. Subjects in the current study needed to understand the 18 mathematical terminologies, just as they would with words included in the word semantics task. Therefore, semantic comprehension is an important correlate of advanced mathematics. Previous studies also consistently showed close relations between reading ability and elementary mathematics performance (e.g., Koponen *et al*., 2007; Lee *et al*., 2004).

Unlike the semantic task, the other three language processing tasks (word rhyming, sentence syntax analysis, and word paired-associate learning) do not seem to involve necessary cognitive processes relevant to advanced mathematics. For example, word rhyming is a typical phonological processing task that is not crucial for advanced mathematics. Even for elementary mathematics, the results have been inconsistent regarding the associations between phonological processing (e.g., phonological awareness) and elementary mathematics performance. Some studies found positive results (e.g., Koponen *et al*., 2007), but others did not (e.g., de Jong & van der Leij, 1999; Durand *et al*., 2005). Similarly, sentence syntax analysis and word paired-associate learning may not involve much semantic processing, resulting in no significant associations with performance in advanced mathematics.

Practical implications

This study found that spatial processing played an important role in advanced mathematics performance. One implication of this result is that spatial strategies can be used to improve students' mathematics performance. Spatial strategies have been part of Johnson-Laird's (1983) mental models (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993; Huttenlocher, Jordan, & Levine, 1994). Previous studies have shown that the subjects who used the diagram strategy were more likely to correctly solve mathematics problems than those who used the conventional strategies (e.g., Fan & Jiannong, 1992; Hegarty & Kozhevnikov, 1999; Sherman, 1979; Zhou & Zhang, 2000). Hegarty and Kozhnevikov (1999; Van Garderen & Montague, 2003) proposed two types of visuo-spatial representations: pictorial and schematic representations, the latter of which has been shown to be more effective in mathematical problem solving.

Spatial processing skills can be enhanced through practice (e.g., Balke-Aurell, 1982; Deno, 1995; Rafi, Anuar, Samad, Hayati, & Mahadzir, 2005; Yang & Chen, 2010). For example, Deno (1995) found positive correlations between non-academic activities (e.g., model building, sketching, and assembly of parts) and spatial visualization ability. Balke-Aurell's (1982) study found that students educated in schools using a verbally oriented curriculum had more growth in verbal abilities, whereas those educated in schools using a technical curriculum showed more growth in spatial abilities. An intervention study found the spatial ability was significantly improved among the pre-service teachers after 5 weeks of training involving a web-based virtual environment (Rafi *et al*., 2005).

If spatial abilities are important for advanced mathematics, the training-induced improvement in spatial abilities may lead to better performance in learning advanced mathematics. To our knowledge, no studies have been conducted to investigate the effects of spatial ability training on mathematics performance. Such studies are needed to establish casual relations. It should be noted, however, the spatial strategy is by no means the only strategy for problem solving in advanced mathematics. Penrose and Gardner (1999) indicated that some professional mathematicians were more likely to use verbal strategies to solve mathematical problems. The current study also showed that the semantic processing of words was closely associated with advanced mathematics performance. Therefore, it is also important to enhance semantic processing ability in order to facilitate the learning of advanced mathematics.

Limitations of the current study and future directions

First, a relatively small range of high-level mathematical abilities were assessed in the current study. That is, we only investigated the ability to acquire new advanced mathematical concepts and to solve mathematical problems. Future research is needed to further validate and expand our test of advanced mathematics, perhaps even develop a more extensive subtest for each area. Second, the tests included for each type of cognitive factors can be further improved. For example, the tests for numerical processing were not highly correlated among themselves. It is not clear whether they reflect different types of number sense. In contrast, the spatial processing tests were highly intercorrelated. Third, the range of participants was also limited. The participants were adults who did not major in mathematics or allied disciplines. Future research needs to include participants with mathematical difficulties as well as those with high-level mathematical abilities.

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Appendix A

The 18 concepts in advanced mathematics used in the current study

Order	Concept in English	Concept in Chinese
1	Group	群
2	Odd/even permutation	奇/偶排列
3	Permutation group	置换群
4	Algebraic operation	代数运算
5	Properties of relation	关系的性质
6	Relation	关系
7	Systematic sampling	等距抽样
8	Complete event group	完备事件组
9	Sample space	样本空间
10	Proposition	命题
12	Sequential presentation	序列呈现
12	Bounded	有界
13	Directed graph	有向图
14	Convex figure	凸图形
15	Convex function	凸函数
16	Stack algorithm	堆栈算法
17	Summation sign ($\sum$)	求和符号$\sum$
18	Symmetric difference	对称差

Appendix B

Examples of the tasks tapping advanced mathematical concepts. Participants were given a brief introduction of each concept, followed by two multiple-choice questions.
Example of '*group*':
If the set G and operation "•" satisfy:

A. associative: $(a \bullet b) \bullet c = a \bullet (b \bullet c)$ for all a, b, c $\in G$
B. **identity elemen**t exists, i.e.: there exists an element $e \in G$ such that $e \bullet a = a \bullet e = a$ for all $a \in G$, (then e is called the identity element of G);
C. **inverse element** exists, i.e.: for every $a \in G$ there exists a $b \in G$ such that $a \bullet b = b \bullet a = e$, (then b is called the inverse element of a, denoted $b = a^{-1}$);

then the set G form a **group** under the operation "•", denoted $(G, \bullet)$.

For example: because multiplication satisfies associative, number 1 is the identity element of multiplication and the inverse element of every non-zero rational number q is $\frac{1}{q}$, the set of all non-zero rational numbers Q form a commutative group under the multiplication. However, the set of all rational numbers cannot form a group, because 0 is not reciprocal, so there is no inverse element.
Question 1: N represents integers which are greater than or equal to 0, the identity element in the group $(N, +)$ is(), the inverse element is( )

1. 0 ; only identity element 0
2. 0 ; all

3. 1 ; only identity element 1
4. 1 ; all

Question 2: In the set of all integers Z, we provide operation“$\oplus$”as follows: define $a \oplus b = a + b - 2$ for any integers a,b. Then in group$(G, \oplus)$,the inverse element of an arbitrary element m is ( )

1. $-m$
2. $\frac{1}{m}$
3. $2 - m$
4. $4 - m$

Example of ’Algebraic operation’:

We assume that an operation is an *algebraic operation* of a set, if only the result of the operation on any two elements in the set remains in the set.

For example: Let G be the set of all even numbers, then addition is an algebraic operation on G, because the sum of any two even numbers is still an even number, which must be in G. If G is the set of all odd numbers, then the addition of G is not an algebraic operation, because that the sum of two odd numbers is an even number, but there is no even number in G.

Question 1: For: A, division on the set of all integers; B, subtraction on the set of all positive numbers. Which is (are) (an) algebraic operation(s) ()

1. Both
2. Neither
3. A, not B
4. B, not A

Question 2: Which of the following is not an algebraic operation ()

1. Addition operation on the set of all irrational numbers
2. Addition operation on the set of all rational numbers
3. Square root operation on the set of all positive numbers
4. Cubic root operation on the set of all negative numbers