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The Energy-Dependent Single Nucleon Potential in a Relativistic Field Theory of Nuclear Matter*

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The energy-dependent single nucleon potential of symmetric nuclear matter in a quantum field model is studied. The results are compared to experiments and predictions of the nonrelativistic mass-operator theory. Meson exchange contributions are discussed.

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The energy dependence of the single nucleon potential has been studied extensively by Jeukenne et al.[1] in the low density approximation of the mass-operator. Recently Jamion et al.[2] calculated the mean potential for scattering states in a relativistic quantum field model considering σ, ω, and ϕ mesons. J. Boguta[3] has calculated the nucleon-nucleus potential in Walecka's relativistic quantum field model.

We investigate the energy dependence of the single nucleon potential for bound and scattering states of symmetric nuclear matter in Walecka's relativistic quantum field model and compare the results to experiments and to nonrelativistic mass-operator calculations. We examine how corrections to the mean field treatment like the meson exchange contributions change the energy dependence for bound states.

In an attempt to develop a full relativistic description of the many-particle nuclear matter system and to allow explicitly for the mesonic degrees of freedom, Walecka[4] has proposed a model quantum field theory in which baryons interact with each other via a scalar and a vector meson exchange. Denoting with ψ, $\underset{\sim}{\sigma}$, and $\underset{\sim}{\omega}_\mu$ the baryon, scalar and vector-fields and with m, m_s, m_v their respective masses, the Lagrangian density is

$$L = -\bar{\psi}(\gamma_\mu \partial_\mu + m)\psi - \frac{1}{2}(\partial_\mu \underset{\sim}{\sigma})^2 - \frac{1}{2}m_s^2 \underset{\sim}{\sigma}^2 - \frac{1}{4}(\partial_\mu \underset{\sim}{\omega}_\nu - \partial_\nu \underset{\sim}{\omega}_\mu)^2$$
$$- \frac{1}{2}m_v^2 \underset{\sim}{\omega}_\mu \underset{\sim}{\omega}_\mu - g_s \bar{\psi}\psi \underset{\sim}{\sigma} + i g_v \bar{\psi}\gamma_\mu \psi \underset{\sim}{\omega}_\mu \tag{1}$$

Here g_s and g_v are Yukawa coupling constants for the scalar field coupled to the scalar density $\bar{\psi}\psi$ and the neutral vector meson field coupled to the conserved baryon current $i\bar{\psi}\gamma_\mu\psi$. It is argued in ref. 4) to replace the meson field operators $\underset{\sim}{\sigma}$ and $\underset{\sim}{\omega}_\mu$ by their expectation values σ and $\omega_\mu = i\delta_{\mu 4}\omega_o$.

Chin[5] has shown that this mean field approximation is equal to the Hartree approximation with additional vacuum fluctuation corrections. For a mean field treatment of symmetric nuclear matter the rho meson has not to be included in the Lagrangian (1). The field equations in the mean field approximation are of the simple form

$$(\gamma_\mu \partial_\mu + m^* + \gamma_0 g_v \omega_0) f_\alpha = 0 \tag{2a}$$

$$m^* - m = -\frac{g_s^2}{m_s^2} \frac{4}{(2\pi)^3} \int^{K_F} \frac{m^*}{\sqrt{K^2 + m^{*2}}} d^3K \tag{2b}$$

$$g_v \omega_0 = \frac{g_v^2}{m_v^2} \frac{2}{3\pi^2} K_F^3 \tag{2c}$$

Here f_α is a relativistic nucleon plane wave function, m^* the effective nucleon mass $m^* = m + g_s \sigma$, and K_F is the Fermi momentum of the nuclear matter system considered.

The nucleon energy spectrum resulting from Eq. (2a) is

$$\varepsilon_K = \sqrt{K^2 + m^{*2}} + g_v \omega_0 \tag{3}$$

To compare this energy spectrum to experimentally found single particle properties and to theoretical nonrelativistic many-body results, we introduce a single particle mean potential. This can simply be done by subtracting from the nucleon energy the kinetic energy of a free nucleon. The mean potential U_R is

$$U_R = \sqrt{K^2 + m^{*2}} + g_v \omega_0 - \sqrt{K^2 + m^2} \tag{4}$$

Another way of introducing a mean potential is as follows. In the Dirac equation (2a) we express the small component of f_α in terms of the large component φ_L and use the definition $\varepsilon_K = T + m$. With this we obtain a

Schrödinger equation with an energy-dependent single particle potential U.[2,3]

$$\frac{-\Delta}{2m}\varphi_L + U(T)\varphi_L = T\varphi_L \tag{5}$$

Here

$$U = g_s\sigma + g_v\omega_o + \frac{g_s^2\sigma^2 - g_v^2\omega_o^2}{2m} + \frac{g_v\omega_o}{m}T - \frac{T^2}{2m} \tag{6}$$

All expressions up to now only depend on the ratios of the Yukawa coupling constants to their respective masses. These ratios are fitted to reproduce the energy per nucleon of -15.75 MeV at a saturating Fermi momentum $K_F = 1.34\ fm^{-1}$. One finds[6)]

$$C_s = mg_s/m_s = 17.95 \text{ and } C_v = mg_v/m_v = 15.60$$

In Fig. 1 we compare the energy-dependent mean potentials U and U_R with the depth of the real part of the phenomenological optical potential[7)] for bound and scattering states. The overall energy dependence is excellently reproduced. The difference between U and U_R is only significant for scattering states at higher energies.

To see how good the predictions for the single particle mean potential at less dense nuclear matter are, a comparison with the nonrelativistic mass-operator theory is offered. There in the Brückner-Hartree-Fock approximation the real part of the mean potential U_M is determined by a complex effective nucleon-nucleon interaction[8)]. The main ingredient is the bare phenomenological nucleon-nucleon force. The many-body effects are due to the Pauli blocking and the mean potential itself in which the nucleons interact in pairs.

Figure 2 shows U_M and U_R versus K/K_F for two different values of K_F. The curve for U_M at $K_F = 1.1\ \mathrm{fm}^{-1}$ is taken from ref. 8). In the case of $K_F = 0.8\ \mathrm{fm}^{-1}$, we took the results published in ref. 9). Since the effective mass m* appears in the relativistic expression for the single particle kinetic energy the potential U_R is not simply quadratic in K but includes higher powers of K, which obviously yields the similarity with the mass-operator theory results. One should be aware that the potentials of the mass-operator theory correspond to the real part of the optical potential, and dispersion effects are therefore correctly taken into account for $K > K_F$. This is not the case in the relativistic mean field approximation. However, from nonrelativistic many-body calculations[8] one knows that the influence of the imaginary part on the real part of the potential is of minor importance in the K-domain which we consider here.

A correction to the mean field approximation is the inclusion of the scalar and vector-field exchange contributions. The scalar and vector-field exchange energy densities have been calculated previously by Bolsterli[10] and Chin[5]. They obtain

$$E_{ex}^{\,s} = \frac{g_s^2}{(2\pi)^6} \int^{K_F} \frac{d^3K\ d^3j}{E_K\ E_j}\ \frac{m^{*2} + E_K E_j - \vec{K}\vec{j}}{m_s^2 - (E_K - E_j)^2 + (\vec{K} - \vec{j})^2} \tag{7a}$$

and

$$E_{ex}^{\,v} = \frac{-2g_v^2}{(2\pi)^6} \int^{K_F} \frac{d^3K\ d^3j}{E_K\ E_j}\ \frac{2m^{*2} - E_K E_j + \vec{K}\vec{j}}{m_v^2 - (E_K - E_j)^2 + (\vec{K} - \vec{j})^2} \tag{7b}$$

here

$$E_\ell = \sqrt{\ell^2 + m^{*2}} \tag{8}$$

These expressions can be used to determine by Landau's Fermi liquid[11] prescription the exchange contributions to the single particle spectrum for bound states. One derives[5]

$$\varepsilon^{s}_{ex}(K) = \frac{g_s^2}{(2\pi)^2}\frac{1}{2E_K}\left\{\frac{1}{2}\left(K_F E_{K_F} - m^{*2}\ln\frac{K_F+E_{K_F}}{m^*}\right) + (2 - \omega_s/2)\frac{m^{*3}}{K}I(\omega_s,\frac{K}{m^*})\right\} \tag{9a}$$

and

$$\varepsilon^{v}_{ex}(K) = \frac{g_v^2}{(2\pi)^2}\frac{1}{2E_K}\left\{K_F E_{K_F} - m^*\ln\frac{K_F+E_{K_F}}{m^*} - (2 + \omega_v)\frac{m^{*3}}{K}I\left(\omega_v,\frac{K}{m^*}\right)\right\} \tag{9b}$$

here

$$\omega_s = (m_s/m^*)^2 \quad , \quad \omega_v = (m_v/m^*)^2 \tag{10}$$

and

$$I(\omega, \frac{K}{m^*}) = \frac{1}{4}\int_1^{\xi} du(1 - u^{-2})\,\ell n\frac{(ut-1)^2+\omega ut}{(u-t)^2+\omega ut} \tag{11}$$

where

$$\xi = \frac{K_F+E_{K_F}}{m^*} \quad , \quad t = \frac{K+E_K}{m^*} \tag{12}$$

The integral (11) has to be calculated numerically. Including meson exchange corrections the single nucleon energy spectrum is of the form

$$\varepsilon_K = \sqrt{K^2 + m^{*2}} + g_v\omega_o + \varepsilon^{s}_{ex}(K) + \varepsilon^{v}_{ex}(K) \tag{13}$$

For the mean field approximation with exchange corrections, Chin[5] has determined the Yukawa coupling constants by fitting the binding energy of nuclear matter for a saturating Fermi momentum $K_F = 1.42\ \mathrm{fm}^{-1}$.

Assuming a sigma meson mass m_s = 550 MeV and an omega meson mass m_v = 783 MeV he gets for the Yukawa coupling constants $g_s^2/4\pi = 6.75$ and $g_v^2/4\pi = 8.86$. With these parameters we calculate the single particle exchange contributions (9). Figure 3 shows that the corrections are of opposite sign and the energy dependence of the sum is rather weak. In Fig. 4 we display the results for the single particle potentials U_R and U_{RE}. Here

$$U_{RE} = \epsilon_K - \sqrt{K^2 + m^2} \tag{15}$$

The potential U_R is without meson exchange contributions. The values for C_s and C_v at $K_F = 1.42\ \mathrm{fm}^{-1}$ are taken from ref. 5). The consideration of the exchange corrections does not change the energy dependence of the potential for bound states significantly. With exchange corrections the Fermi energy is -20.1 MeV.

The conclusion of our investigation is that in the mean field approximation the energy dependence of the single particle potential agrees very well with experiment for bound and scattering states. At different nuclear matter densities it shows the same momentum dependence as computationally extensive nonrelativistic many-body calculations. At least for bound states the meson exchange corrections do not change the energy dependence significantly.

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Figure captions

Fig. 1 The single nucleon potentials U (full line) and U_R (dashed line) for symmetric nuclear matter of $K_F = 1.34\ fm^{-1}$ versus the nucleon energy T. The dots are experimental values from ref. 7).

Fig. 2 The single nucleon potentials U_R (dashed line) and the potential U_M (full line) resulting from a Brückner-Hartree-Fock calculation versus the momentum ratio K/K_F. U_M for $K_F = 1.1\ fm^{-1}$ is taken from ref. 8), in the case of $K_F = 0.8\ fm^{-1}$ from ref. 9).

Fig. 3 The scalar meson exchange contribution ε_{ex}^{s} and the vector meson exchange contribution ε_{ex}^{v} versus the nucleon energy T for bound states.

Fig. 4 The single nucleon potential U_R (dashed line) without exchange and U_{RE} (full line) with meson exchange corrections versus the nucleon energy T for $K_F = 1.42\ fm^{-1}$.

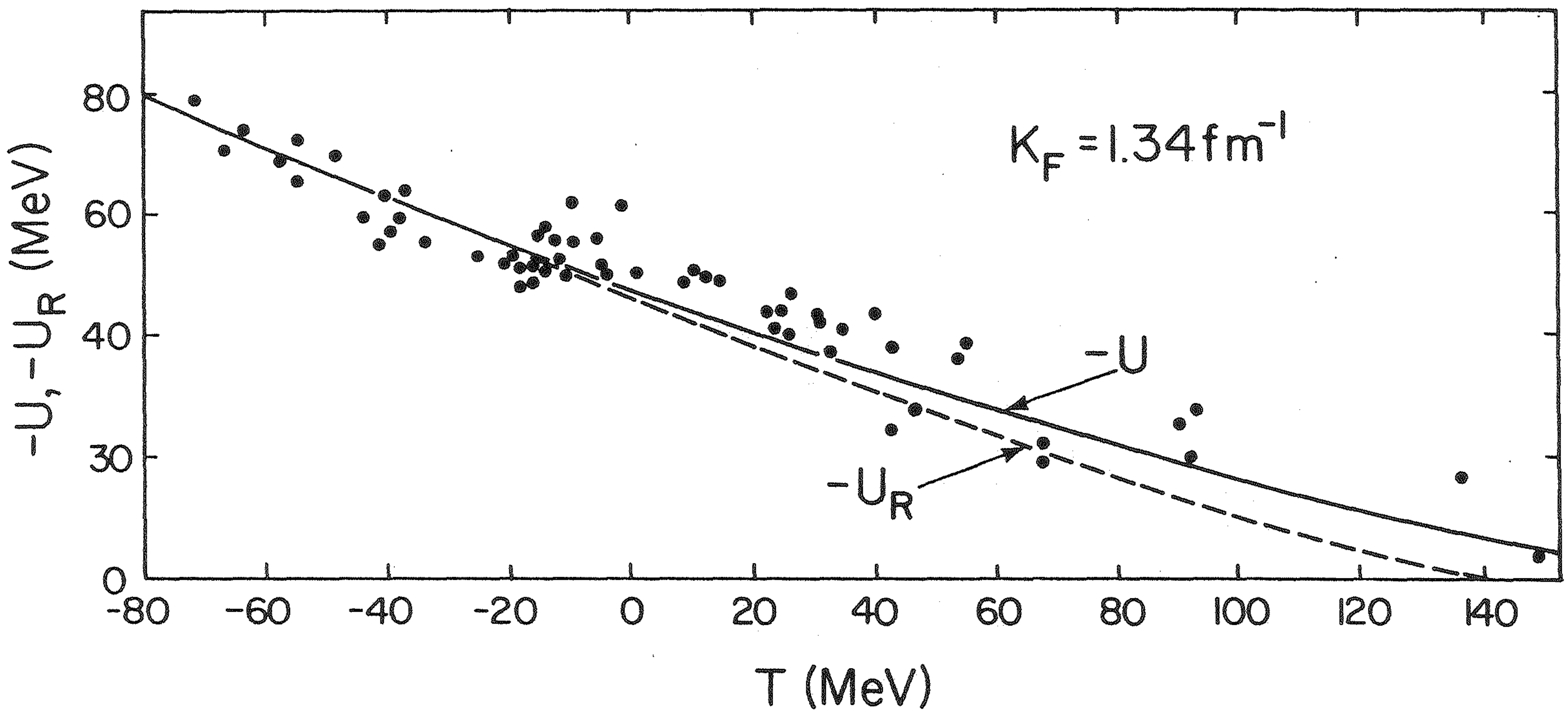

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Fig. 1

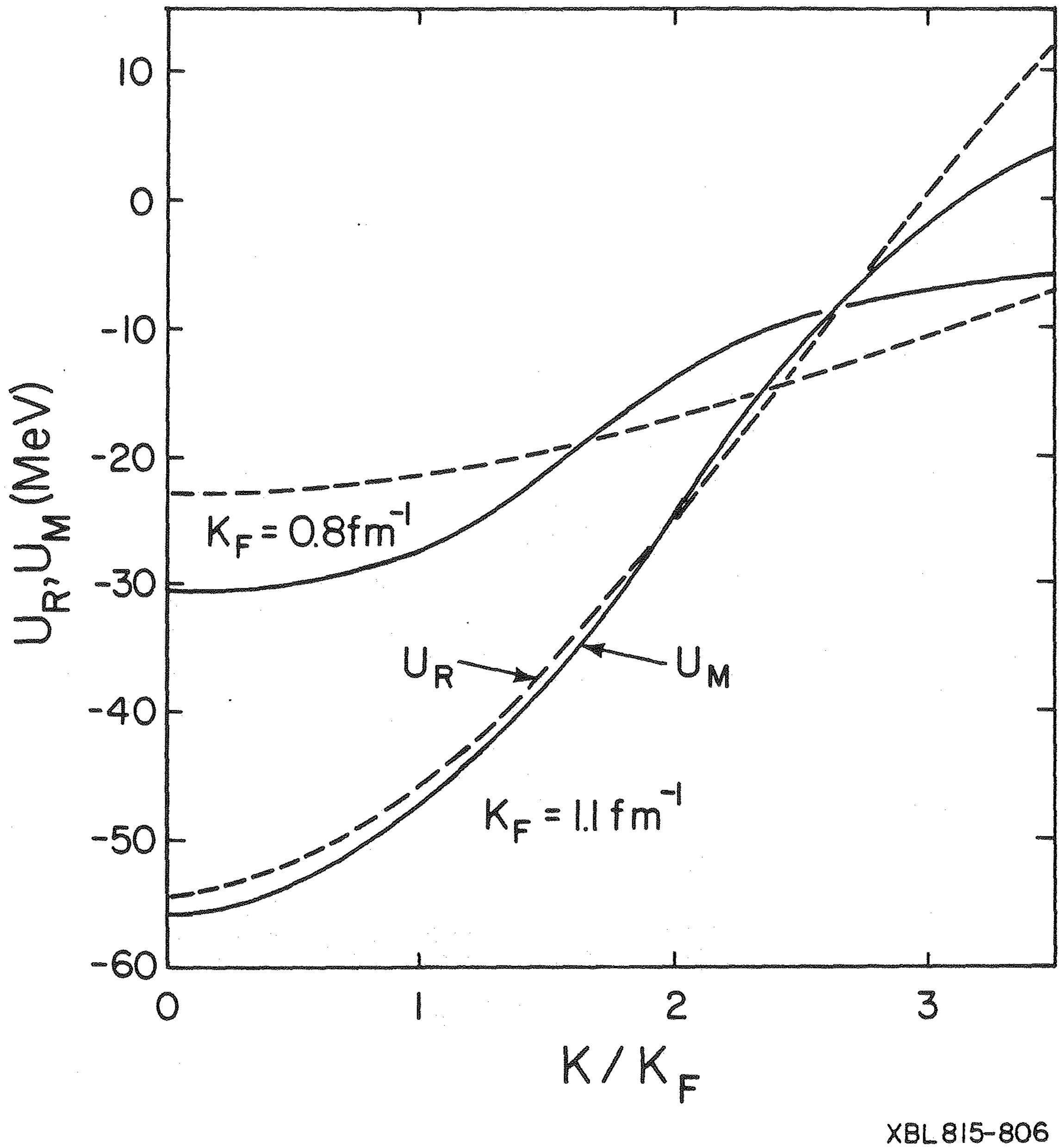
10
0
-10
-20
-30
-40
-50
-60
0
1
2
3
U_R, U_M (MeV)
K/K_F
$K_F = 0.8 fm^{-1}$
U_R
U_M
$K_F = 1.1 fm^{-1}$
XBL 815-806

Fig. 2

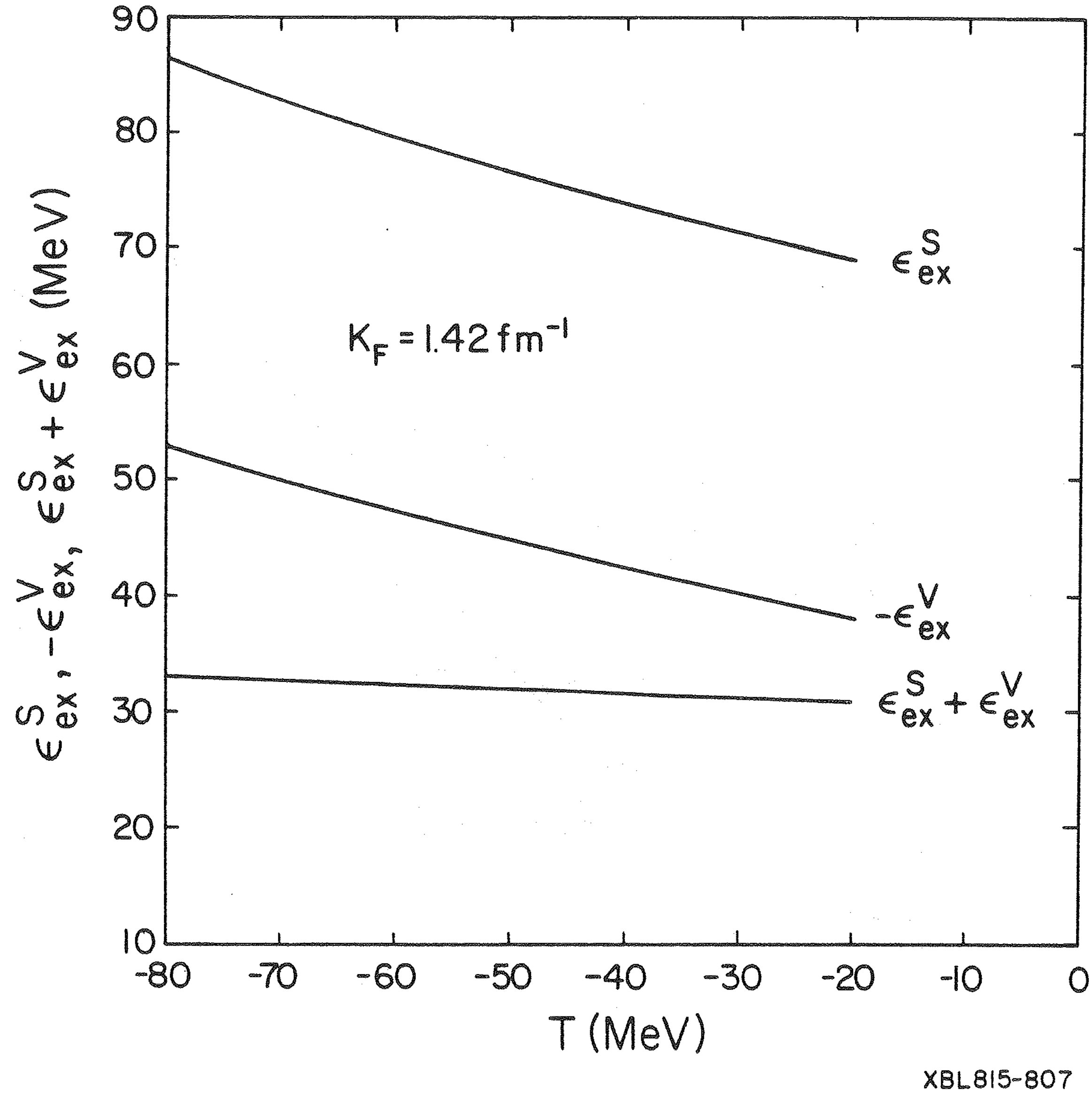
90
80
70
60
50
40
30
20
10
-80
-70
-60
-50
-40
-30
-20
-10
0
T (MeV)
$\epsilon_{ex}^S, -\epsilon_{ex}^V, \epsilon_{ex}^S + \epsilon_{ex}^V$ (MeV)
$K_F = 1.42\ fm^{-1}$
ϵ_{ex}^S
$-\epsilon_{ex}^V$
$\epsilon_{ex}^S + \epsilon_{ex}^V$
XBL815-807

Fig. 3

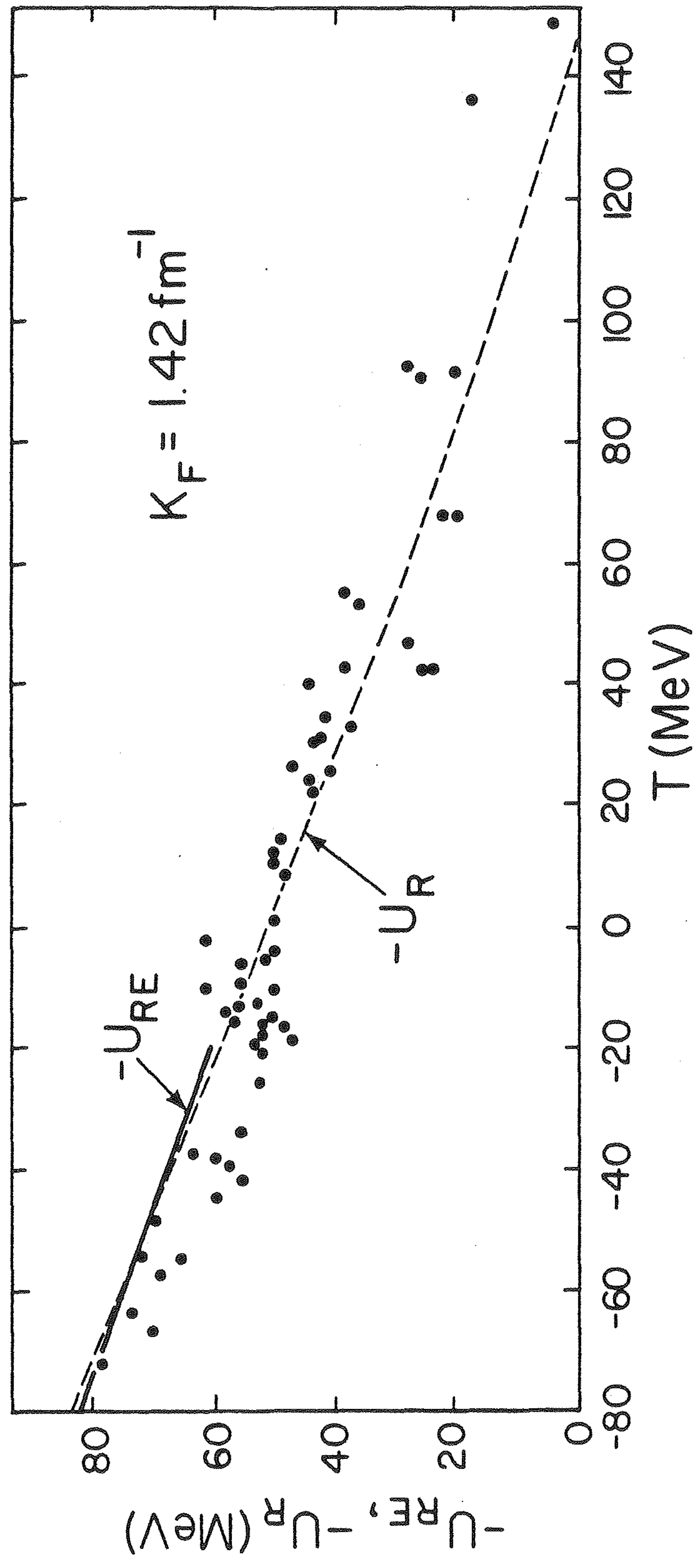

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Fig. 4