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Abstract

Currently living people might reduce carbon emissions to protect themselves, their wealth, or future generations from climate damage. An overlapping generations climate model with endogenous asset price and investment levels disentangles these incentives. Asset markets capitalize the future effects of policy, regardless of people's concern for future generations. These markets can lead self-interested agents to undertake significant abatement. A small climate policy that raises the price of capital increases welfare of old agents and also increases welfare of young agents with a high intertemporal elasticity of substitution. Climate policy can also have subtle distributional effects across the currently living generations.

Keywords: Climate externality, overlapping generations, climate policy, generational conflict, dynamic bargaining, Markov perfection, adjustment costs.
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1 Introduction

The standard climate policy narrative emphasizes that climate policy requires current generations to sacrifice to protect the climate for future generations. This narrative usually ignores asset price effects or discusses them only in the context of "stranded assets", whose value is reduced by climate policy. Fossil fuel companies, the common example of stranded assets, constitute a large asset class, but a modest fraction of total world financial assets.[1] It is misleading to focus on specific asset classes harmed by climate policy.

We study the relation between climate policy and the value of assets writ large. Climate policy that protects the environment can increase future capital productivity, and asset markets transfer some of those future benefits to currently living asset owners by changing asset prices. Markets potentially induce non-altruistic agents to undertake substantial abatement.

Integrated Assessment Models (IAMs) are the standard tool for evaluating climate policy (Stern, 2006; Nordhaus, 2008). Prominent IAMs (e.g. DICE) use an Infinitely Lived Representative Agent (ILRA) setting and assume a perfectly fungible composite commodity that can be consumed or invested. The price of investment (and the end-of-period price of capital) equals the number of units of the consumption good that can be exchanged in the market for one unit of investment (or capital). The fungibility assumption, together with a choice of units, fixes this price at 1. Here, policy affects the quantity of investment, but not its price.

To examine the possible relation between climate policy and asset prices, we need a non-trivial asset market. This market requires both an endogenous end-of-period price of capital and distinct buyers and sellers of capital. We replace the linear consumption-investment production possibility frontier, implied by the assumption of a perfectly fungible composite commodity, with a strictly concave relation. The production point determines the price of investment, equal to the endogenous price of capital. To have both buyers and sellers of capital, we replace the ILRA setting with a Diamond (1965) Overlapping Generations (OLG) model. Agents live for two periods; young agents buy capital and old agents sell it. In this setting, climate policy can affect both the price of capital and the level of investment. Non-trivial asset markets alter selfish agents' incentives to undertake climate policy.

[1] Fossil fuel companies are worth about \$5 trn (trillion) (Bullard, 2014). The world stock market capitalization exceeds \$69 trn, and the value of total financial assets exceeds \$284 trn (Witkowski, 2015).

Currently living people might undertake climate policy in order: to decrease climate damage experienced by the current young agents during their life-time; to increase the current old generation's wealth by increasing the asset price; and to benefit people who are not yet born.[2] Climate policy can also have subtle distributional effects across the currently living young and old generations. Three scenarios disentangle these incentives.[3]

In two political economy scenarios, currently living selfish agents choose policy, thus eliminating the intergenerational altruism incentive. In these scenarios, the young and the old agents can make transfers with one another; they choose abatement to maximize a convex combination of their lifetime welfare. They cannot make transfers across future generations, e.g. by means of public debt or social security. In both scenarios agents understand the relation between climate policy and factor returns; in one scenario they take the asset price as fixed, and in the second they recognize that it is endogenous. By comparing equilibrium abatement in these two scenarios, we identify the role of the recognition of asset markets. A third scenario provides a benchmark. Here, a standard discounted utilitarian chooses a policy trajectory to maximize the discounted sum of the utility of aggregate consumption across all generations. In this setting, the concern for unborn generations is central and asset price changes are incidental.

We use a numerical model with the climate component calibrated to DICE to examine equilibrium outcomes and to assess the practical significance of asset markets. For our baseline calibration, the recognition of the endogenous asset price leads to a significant increase in abatement, reaching almost half the level chosen by the discounted utilitarian.

The results also show that in the short run climate policy may have negligible effect on the price of a broad portfolio of assets, even if the attempt to influence the asset price has a large effect on policy. For example, a higher incentive to abate likely has the same qualitative effect on current

[2] Recent polls show that generations differ only modestly in their opinion about the reality of climate change and the value of policies to address it. Almost 99% of Americans state that protecting future generations is an important reason for protecting the environment (Feldman et al. 2010, Jones et al. 2014). The absence of altruism in our model is not descriptive, but it shows that limited environmental protection may not require altruism.

[3] Climate policy can benefit currently living agents via other mechanisms. For example, policies that reduce greenhouse gasses can also reduce local pollutants, creating near-term health benefits. In addition, rebalancing society's investment portfolio, by reducing saving of man-made capital and increasing saving of environmental capital, can benefit all generations (Foley, 2009; Rezai et al., 2012).

and future policy. The direct effect of higher future abatement lowers future factor returns, tending to lower the current asset price. The indirect effect, via the reduction in stocks of atmospheric carbon, increases those returns, increasing the current asset price. Climate policy also reduces the current wage and changes the demand for savings. In the near term all these effects nearly balance, leading to a negligible change in asset prices. Over a couple of centuries, the higher abatement leads to much higher asset prices and increased investment. Asset markets potentially create a significant incentive to undertake climate policy, even if climate policy leads to no significant short term change in asset prices.

We also use analytic methods to investigate the effect of a small exogenous increase in current abatement. This perturbation increases the old agent's welfare if and only if the abatement increases the asset price. With a higher price, the young agent's welfare increases if and only if her elasticity of intertemporal substitution is finite and exceeds 1. In this situation, both agents benefit from a small level of abatement that increases the price of capital, even if they have no concern for their successors' welfare.

The temptation to shift costs to future generations and to currently living people in other countries creates both an intergenerational and a cross-country free-riding problem. In common with many IAMs, we ignore the cross-country free-riding problem. Asset markets potentially attenuate intergenerational free riding but do nothing to reduce cross-country free riding.

2 Related literature

Oates (1972) noted that asset markets transfer some of the future benefits of environmental policy to current asset owners, giving them a stake in environmental policy. Recent empirical evidence links environmental outcomes to asset prices; e.g., reduced air pollution increases some real estate prices (Chay and Greenstone, 2005; Bushnell, Chong, and Mansur, 2011). In a series of papers, Bansal and coauthors estimate consumption-based asset pricing models, providing evidence that increases in temperature and temperature fluctuations lower the value of most US equity portfolios; they also find that impacts of climate change on asset values have recently increased (Bansal and Ochoa, 2011, Bansal, Kiku and Ochoa, 2015 and 2016). Dietz et al. (2016) use DICE to estimate the "climate value at risk" at 1.8% of global financial assets, with much higher tail risks. Balvers et al. (2012) estimate the cost of past warming, as captured in asset prices, at 4.18% of wealth.

These empirical papers do not consider capital accumulation or the effect of asset markets on incentives to mitigate. We incorporate asset markets into a model of climate policy, building on three strands of research: (i) adjustment costs can explain the endogeneity of the price of capital when investment is endogenous, (ii) an overlapping generations model contains a market for a productive asset and captures intergenerational conflict, and (iii) a political economy model imbedded in a dynamic game determines policy in the absence of a social planner.

Adjustment costs: In the Ramsey growth model with a composite commodity, investment is endogenous but the price of capital is fixed at the price of the numeraire (Ramsey, 1928). In the Lucas tree model, the capital stock is exogenous but the asset price endogenous (Lucas, 1978). Adjustment costs produce a simple model with both endogenous capital and an endogenous asset price. These costs lead to a state-dependent price of capital in an OLG economy with productive assets (Huberman, 1984; Huffman, 1985, 1986; Labadie, 1986). Shapiro (1986) and Hall (2001, 2004) provide reduced form estimates of adjustment costs; Mumtaz and Zanetti (2015) provide structural estimates. Bond and van Reenen (2007) survey the literature.

Environmental OLG models: In overlapping generations models at least two generations are alive in each period, creating a link across time, despite finite lifetimes. Bovenberg and Heijdra (1998) show that the issuance of public debt can support Pareto-improving environmental policy by transferring across time the cost of policy to those who benefit from it. We exclude public debt, social security, and other means of intergenerational transfers from the future to the present. The rich welfare structure of OLG models has been studied in many resource-based OLG models (Howarth and Norgaard, 1992; John and Pecchenino, 1994; Gerlagh and Keyzer, 2001; Schneider et al. 2011, Williams et al., 2015; Karp, 2017; and Iverson and Karp, 2017). Karp and Rezai (2014) use a Lucas tree model to show how the asset market affects incentives to conserve a generic resource.[4] There, all adjustment occurs via prices, possibly exaggerating the role of asset markets, and that paper assumes an infinite elasticity of intertemporal substitution, where only the old

[4]That paper uses a model with two consumption goods, creating general equilibrium complications that we avoid by assuming a single consumption good. It emphasizes the distribution of revenue generated by the tax used to protect the resource. Here we sidestep that complication, by assuming that a command and control policy supports the equilibrium level of abatement.

generation cares about the asset price. Our model here includes the standard Ramsey and the Lucas tree models as limiting cases, neither assuming away nor giving undue prominence to the role of asset markets. We also show that the elasticity of intertemporal substitution is critical to the relation between asset prices and welfare.

Determination of equilibrium policy: Selfish agents alive today, not a benevolent, infinitely-lived social planner, choose current abatement in our political economy settings. The probabilistic voting model (Lindbeck and Weibull, 1987; Persson and Tabellini, 2000) provides microfoundations for this political economy structure and rationalizes our use of a policy objective function equal to the convex combination of the lifetime welfare of currently living young and old agents. These agents choose current policy (abatement in our setting) but are influenced by their expectations of future policy. Young and old generations in each period are part of a sequence of pairs of generations in a dynamic game; policy is a Markov perfect equilibrium (MPE) to this game. Hassler et al. (2003) and Conde-Ruiz and Galaso (2005) use this type of political economy model to determine intergenerational redistribution and the provision of a public good. In our model, the public good is Earth's capacity to absorb carbon emissions.

3 The Model

This section describes the young agent's savings decision and then discusses the economy-wide production function. A model of adjustment costs with congestion generates a tractable model with a strictly concave production possibility frontier (PPF). Without strict concavity, the asset price is fixed at the price of the numeraire. To emphasize the role of asset markets, we rule out other types of transfers across time, such as social security and public debt. The final subsection discusses the decentralized equilibrium.

3.1 The savings decision

In each period, a cohort of constant size, $l \equiv 1$ is born. Agents live two periods and maximize their lifetime welfare, Ω. Lifetime welfare is the discounted sum of utility, $U(\cdot)$, derived from consumption while young, c^y, and old, c^o: $\Omega_t^y = U(c_t^y) + \rho\, U(c_{t+1}^o)$ with ρ the constant utility discount factor and superscripts y and o denoting the young and old generation. Except in Section 4.2,

$U(\cdot)$ satisfies the Inada conditions. The young agent receives labor income, w_t, but no inheritance, and spends c_t^y on consumption. With a depreciation rate δ, the amount of capital remaining at the end of period t is $(1-\delta)\,k_t$. Newly produced capital, i_t, and undepreciated old capital are equally productive; therefore, in equilibrium they have the same price, p_t. The young agent buys s_t shares of the old capital stock and i_t units of new capital at the cost $p_t\,[s_t(1-\delta)k_t + i_t]$. The rental rate on capital is r_t. When old in period $t+1$, the agent earns the factor payment $r_{t+1}(s_t(1-\delta)k_t + i_t)$, and obtains revenue from selling the end-of-period stock, $s_{t+1}p_{t+1}(1-\delta)(s_t(1-\delta)k_t + i_t)$. Agents are selfish, so the old agent consumes all her income.

Agents take prices w_t and p_t as given and have rational point expectations of r_{t+1} and p_{t+1}. The young agent's maximization problem is

$$\max_{i_t,\ s_t,\ c_t^y,\ c_{t+1}^o} \quad U(c_t^y) + \rho\, U(c_{t+1}^o) \text{ subject to}$$

$$\begin{aligned} c_t^y &\leq w_t - p_t\,[s_t(1-\delta)k_t + i_t] \\ c_{t+1}^o &\leq (r_{t+1} + s_{t+1}p_{t+1}\,(1-\delta))\,(s_t(1-\delta)k_t + i_t)\,. \end{aligned} \tag{1}$$

The optimal decision to buy shares in existing capital, s_t, satisfies

$$\psi_t \equiv \frac{U'(c_t^y)}{\rho\, U'(c_{t+1}^o)} = \frac{(r_{t+1} + s_{t+1}p_{t+1}(1-\delta))}{p_t}, \tag{2}$$

which states that in equilibrium the marginal rate of intertemporal substitution equals the marginal rate of transformation. The right side in equation (2) gives the number of consumption units a young agent obtains in the next period by reducing consumption by 1 unit today and investing in capital instead. This ratio equals the marginal rate of intertemporal substitution, ψ_t, or 1 plus the endogenous interest rate between period t and $t+1$. In equilibrium, $s_t \equiv 1\ \forall t$, because the old generation has inelastic supply of undepreciated capital. Provided that $i_t > 0$ (as we hereafter assume), the optimality condition for i_t is identical to equation (2).

We rearrange the optimality condition (2) as an asset price equation:

$$p_t = \frac{r_{t+1} + p_{t+1}(1-\delta)}{\psi_t} \text{ for } t < H, \tag{3}$$

where $H < \infty$ is the last period. The young generation in H only lives one period, so it does not accumulate capital, implying the asset price is zero, $p_H = 0$. (Section 5 discusses the finite horizon assumption.)

3.2 Gross world product

We follow DICE and other IAMs in modeling gross world product (GWP, or "output") as a function of capital, k_t, labor, l, (normalized to 1), the stock of atmospheric carbon in excess of pre-industrial levels, e_t, and the abatement rate, $\mu_t \in [0,1]$. The endogenously changing stocks, k and e, are the state variables, and the abatement rate is the policy variable. Absent abatement and climate change, GWP equals $F(k_t, l)$, an increasing, concave, constant returns to scale function. Excess CO_2 creates climate change, causing output to equal $D(e)F(k_t, l)$, with $D(0) = 1$, $D' < 0$ and $D'' < 0$ for $e_t > 0$. Under Business as Usual (BAU, defined as the absence of climate regulation), firms choose emissions to minimize their production costs, leading to emissions $\zeta F(k, l)$; ζ is the constant BAU carbon intensity of output. Emissions raise future levels of e_t, lowering future productivity of both capital and labor.

Environmental policy obliges firms to abate the fraction $\mu_t \in [0,1]$ of emissions $\zeta F(k_t, l)$ in period t. Abatement reduces GWP by the factor $\Lambda(\mu_t)$. By definition, zero abatement minimizes costs, so $\Lambda(0) = \Lambda'(0) = 0$. With the usual monotonicity and convexity assumptions, $\Lambda' > 0$ and $\Lambda'' > 0$ for $\mu_t > 0$. Output, y_t, and emissions, z_t, equal

$$y_t = (1 - \Lambda(\mu_t))D(e_t)F(k_t, l) \quad \text{and} \quad z_t = (1 - \mu_t)\zeta F(k_t, l).$$

All markets are competitive and the representative firm hires labor and capital to equate a factor's wage or rental rate and its marginal product:

$$w_t = \frac{\partial y_t}{\partial l} \quad \text{and} \quad r_t = \frac{\partial y_t}{\partial k_t}. \tag{4}$$

With constant decay rates δ for capital and ϵ for atmospheric carbon, the transition equations for the stock of atmospheric carbon and capital are

$$e_{t+1} = (1 - \epsilon)e_t + z_t \text{ and } k_{t+1} = (1 - \delta)k_t + i_t. \tag{5}$$

The initial stocks e_0 and k_0 are given.

3.3 The production possibility frontier

Adjustment costs can explain firms' sluggish response to exogenous changes or to deviations from long run equilibria; many studies of adjustment costs focus on randomness. We use a model of adjustment costs to obtain a tractable and empirically plausible foundation for a strictly concave PPF between the consumption and the investment good. Output can be directly consumed, and it can be transformed into the investment good. Investment i requires $A(i)\, i$ units of the composite commodity, so adjustment costs equal $(A(i) - 1)\, i$. For $A(i) \equiv 1$, adjustment costs are 0 and the PPF between the consumption and the investment good is a line with slope -1, as in standard IAMs. For $A'(i) > 0$, adjustment costs are positive and the PPF is strictly concave.

With the consumption good as the numeraire, the nominal factor prices in equation (4) equal the real returns. Moreover, just as in standard IAMs, the relative factor price, w/r, does not depend on the carbon stock or abatement. Although potentially interesting, the relation between the climate and relative factor returns is not closely related to our focus on asset prices. Introducing climate-related relative factor returns would make it difficult to identify the effect of asset prices on incentives to reduce emissions.

We assume that firms producing (or transforming the composite good into) capital create congestion in their sector. Each firm takes its marginal production cost as constant at $A_t \equiv A(i_t)$ but this cost increases with aggregate i, unless adjustment costs are 0. The price of new capital equals average instead of marginal aggregate production costs ($A(i_t)$, not $A(i_t) + A'(i_t)\, i_t$), creating a static market failure.[5] Old and new capital are equally productive, so their prices are equal when investment is positive:

$$i_t > 0 \Rightarrow p_t = A(i_t). \tag{6}$$

To further motivate our choice of adjustment costs (+ congestion) as a means of introducing a concave PPF, we briefly consider alternatives:

- The Hecksher-Ohlin-Samuelson and Ricardo-Viner models also give rise to a concave PPF, but neither is analytically tractable. Furthermore, for both of these models, a change in climate policy or the carbon stock changes relative factor returns, distracting attention from our focus on asset markets. The Ricardo-Viner model also requires asset prices for the sector-specific factors.

[5] An adjustment cost model in which the price of the investment good equals marginal costs requires a sector-specific factor in the investment sector, and accompanying rent. The additional factor of production and its asset price make the model analytically intractable. The congestion assumption, implying that price equals average cost, is widespread in economics; for example, it is the basis for much of the literature on endogenous growth.

- Consumption equals $c = y\left(b - a\,(i/y)^{(1+\sigma)/\sigma}\right)^{\sigma/(1+\sigma)}$ under a constant elasticity of transformation between the consumption and investment goods, σ; a and b are parameters (Powell and Gruen, 1968). This formulation retains the independence between relative factor returns and climate-related variables, but it makes the price of investment depend on i/y, instead of merely on i as in our adjustment cost model. This complication eliminates the analytic results in Section 4.[6]

3.4 Equilibrium

Section 5 presents the political economy setting that determines climate policy, the sequence of emissions standards, $\left\{\mu_{t+h}\right\}_{h=0}^{H-t}$. For the time being, we take this policy sequence as given, and define the conditional equilibrium under the assumption of positive investment in every period:

Definition 1 *A competitive equilibrium at t, with initial condition k_t and e_t, conditional on $\left\{\mu_{t+h}\right\}_{h=0}^{H-t}$, is a sequence of the carbon and capital stocks and asset price, $\{e_{t+h}, k_{t+h}, p_{t+h}\}_{h=0}^{H-t}$, satisfying the asset market equilibrium (3) implied by the young agents' savings decision, the factor market conditions (4), the transition equations (5), and the no-arbitrage condition (6).*

We assume that preferences are constant elasticity, with $U(c) = \frac{c^{1-\eta}-1}{1-\eta}$ and $\eta \geq 0$ (so $U(c) = \ln c$ for $\eta = 1$); η is the inverse of the elasticity of intertemporal substitution (IES).

Lemma 1 *For a given climate policy and $\eta \neq 1$, equilibrium lifetime welfare in period t is Ω_t^y for the young and Ω_t^o for the old agent, with*

$$\Omega_t^y \equiv U(c_t^y) + \rho\, U(c_{t+1}^o) = \frac{(c_t^y)^{-\eta}}{1-\eta} w_t - \frac{1}{1-\eta}(1-\rho) \tag{7}$$

and

$$\Omega_t^o \equiv U(c_t^o) = \frac{\left[(r_t + (1-\delta)p_t)k_t\right]^{1-\eta} - 1}{1-\eta}. \tag{8}$$

The Appendix contains proofs; Section 4.1 discusses the case $\eta = 1$.

[6] Either of these alternatives can be interpreted as an adjustment cost model. For example, in the CES model we obtain the linear PPF as $\sigma \to \infty$. We can define adjustment cost as

$$A\left(\frac{i}{y};\sigma\right) \equiv y\left[\left(b - a\,(i/y)^{(1+\sigma)/\sigma}\right)^{\sigma/(1+\sigma)} - \left(b - a\,(i/y)\right)\right].$$

4 Climate Policy, Asset Price and Welfare

Climate policy diverts resources from consumption or savings to mitigation, imposing a cost on those who implement the policy. In an ILRA economy, today's climate investment benefits agents in the future. In an OLG economy with linear PPF and no bequest motive, the current old generation has no reason to abate. The current young generation benefits from lower climate-related damages when they are old. With concave PPF, the endogenous asset price has the potential to transfer some future benefits to the current period, creating an incentive for climate policy.

This section provides intuition for the interaction between climate policy, asset prices, and equilibrium welfare for a small level of current abatement ($\mu > 0,\ \mu \approx 0$). We approximate the welfare effect in the usual manner, using the first-order term of the Taylor expansion of welfare around $\mu = 0$.[7] Section 5 endogenizes the abatement decision, recognizing that future policy responds to current policy via changes in the state variable; here we treat future abatement levels as fixed. Because abatement costs are minimized at zero abatement ($\Lambda'(0) = 0$), the first unit of abatement has a zero first-order effect on current output and factor returns:

$$\left.\frac{\partial y_t}{d\mu_t}\right|_{\mu=0} = \left.\frac{\partial w_t}{d\mu_t}\right|_{\mu=0} = \left.\frac{\partial r_t}{d\mu_t}\right|_{\mu=0} = 0.$$

The second-order effect of abatement on output and factor returns is negative.

The old generation's consumption equals its income from renting capital and from selling undepreciated capital. Although the policy perturbation has zero first-order effect on rental income, it can change the old generation's welfare by altering the asset price:

Proposition 1 *For $\delta < 1$ and predetermined stocks of atmospheric carbon and capital and fixed future abatement levels, a small level of current abatement ($\mu > 0,\ \mu \approx 0$) increases the old generation's welfare if and only if this policy raises the asset price:*

$$\left.\frac{d\Omega_t^o}{d\mu}\right|_{\mu=0} > 0 \Leftrightarrow \left.\frac{dp_t}{d\mu}\right|_{\mu=0} > 0.$$

[7] Our results do not change if, instead of considering the perturbation of only current policy, we consider the perturbation of a sequence of abatement levels. In that case, we begin with $\bar{\boldsymbol{\mu}} \in \mathbb{R}^{H-t}$, $\bar{\mu}_h \geq 0$ and denote the sequence of climate policy as $\varepsilon\bar{\boldsymbol{\mu}}$. The comparative statics is then with respect to ε, evaluated at $\varepsilon = 0$.

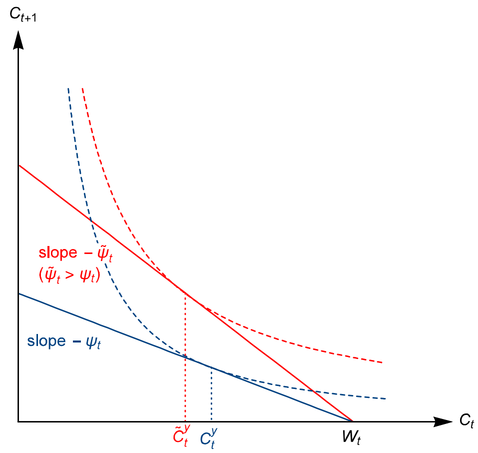

Figure 1: A higher asset price that also increases the young agent's welfare must rotate this agent's budget constraint clockwise. Market clearing implies that this change must reduce c_t^y. A fall in c_t^y following a steeper (outwardly rotated) budget constraint occurs if and only if the substitution effect is greater than the income effect, which (for constant IES) requires $\eta < 1$.

For $\delta = 1$, where the old generation has no remaining capital at the end of the period, and therefore sells nothing, climate policy has zero first-order effect on the old generation's welfare.

Proposition 2 states that a policy that increases the asset price benefits the young agent if and only if $\eta \in (0, 1)$. To provide intuition for this result, suppose that a policy increases p_t and also increases the young agent's welfare. The young's current consumption is $c_t^y = w_t - p_t\, k_{t+1}$ and next period consumption is $c_{t+1}^o = \psi_t\, p_t\, k_{t+1}$, so the budget constraint is $c_{t+1}^o = \psi_t\, (w_t - c_t^y)$, shown as solid lines in figure 1. The hypothesis that the young benefit from the policy-induced increase in p_t implies that the budget constraint rotates clockwise, increasing ψ_t to $\widetilde{\psi}_t$.[8]

The increase to $\widetilde{\psi}_t$ creates an income effect that encourages higher consumption when young, because lifetime wealth increases, and a substitution effect that encourages lower consumption when young, because the return

[8]The policy has only a second-order effect on w_t. If the policy causes ψ_t to fall, the budget constraint pivots counter-clockwise around the c_t^y intercept (equal to w_t), necessarily lowering the young agent's welfare, contrary to our hypothesis.

on savings increases. However, the small abatement policy creates only a second order reduction in aggregate output at t, while it leads to a first-order increase in the old agent's consumption (by Proposition 1), and also increases equilibrium investment. Market clearing requires that the policy lowers the young agent's consumption (from c_t^y to $\tilde{c}_t^y$). Consequently, for welfare to rise, the substitution effect must dominate the income effect. For iso-elastic utility, the substitution effect dominates the income effect if and only if $\eta < 1$.

Proposition 2 *Assume that investment is positive and that the consumption-investment PPF is strictly concave.*

(i) If $\eta \in (0,1)$, a small level of abatement increases welfare of the current young if and only if the policy causes the asset price to rise:

$$\left.\frac{d\Omega_t^y}{d\mu}\right|_{\mu=0} > 0 \Leftrightarrow \left.\frac{dp_t}{d\mu}\right|_{\mu=0} > 0.$$

(ii) If $\eta > 1$, a small level of abatement increases welfare of the current young if and only if the policy causes the asset price to fall:

$$\left.\frac{d\Omega_t^y}{d\mu}\right|_{\mu=0} > 0 \Leftrightarrow \left.\frac{dp_t}{d\mu}\right|_{\mu=0} < 0.$$

(iii) Climate policy has the same qualitative effect on welfare of the two generations alive in the first period if $\eta \in (0,1)$; climate policy has opposite effects on their welfare if $\eta > 1$.

Today's equilibrium asset price depends on future prices, making it difficult to sign $\frac{dp_t}{d\mu}$. However, for logarithmic utility ($\eta = 1$) and for a linear model with $\eta = 0$, a small increase in period-t abatement strictly increases the welfare of one of the two agents alive in that period, has a 0 first-order effect on the other agent's welfare, and strictly increases welfare for the agent born in the next period. Here, the asset market gives agents an incentive to impose climate policy, despite their lack of altruism.

4.1 Logarithmic utility

If utility is logarithmic ($\eta = 1$), income and substitution effects cancel: Equilibrium saving is a constant fraction of income and the asset price is an increasing function of the wage.

Lemma 2 *With $\eta = 1$,*

$$(i)\quad p_t k_{t+1} = \frac{\rho}{1+\rho} w_t \text{ and } (ii)\quad p_t = f(w_t) \text{ with } f' > 0.$$

Lemma 2.i is a standard result: with logarithmic utility, an agent saves a constant fraction of her income. An increase in the wage shifts out demand for savings, increasing the price of capital due to equation 6.

Because climate policy has a zero first-order effect on the current wage, Lemma 2.ii implies that policy has zero first-order effect on the current asset price, and a zero first-order effect on the old agent's welfare. The policy has a zero first-order effect on the young agent's saving, but it creates a first-order reduction in next-period pollution stock, raising the next-period wage and asset price.[9] The higher asset price in $t = 1$ increases consumption of the old agent in period 1, thus increasing lifetime welfare of the agent who is young at $t = 0$. The higher wage in $t = 1$ also increases consumption and welfare of the young agent in $t = 1$, because of the constant saving rule. The rule also implies higher saving for the richer young agent at $t = 1$.

Proposition 3 *With $\eta = 1$, a small level of abatement: (i) has a zero first-order effect on the welfare of the old agent in period $t = 0$, (ii) increases welfare of the agent born in $t = 0$, and (iii) increases welfare of the agent born in the next period, $t = 1$.*

Agents born in subsequent periods inherit different stocks of both capital and pollution (relative to the zero-abatement equilibrium). Without imposing further structure, we cannot determine the changes in their welfare.

4.2 Linear utility

If utility is linear ($\eta = 0$), the interest rate equals the pure rate of time preference ($\psi_t = \frac{1}{\rho}$). The young generation's lifetime welfare is

$$\Omega_t^y|_{\eta=0} = w_t - 1 + \rho, \tag{9}$$

by equation (7). Here, the young's welfare does not depend on the asset price. A small level of abatement has only a second-order effect on the same-period

[9] The statement that there is a first-order change in p_1 and only a second-order change in p_0 is consistent with equation 3. The change in ψ_0 offsets the change in p_1.

wage, and therefore creates a zero first-order effect on welfare for the young agent. A non-negligible level of abatement lowers the wage, lowering the young agent's welfare. (Section 5.4 returns to this issue.) If policy increases the asset price, it raises the old agent's welfare (by Proposition 1). With a higher asset price and associated increase in investment, the young agent born at $t = 1$ inherits a larger stock of capital and a smaller carbon stock. Both of these changes increase the wage at $t = 1$, benefitting the agent born in that period. The higher k_{t+1} increases emissions in $t+1$, so the effect of the perturbation on agents' welfare at $t \geq 2$ is ambiguous.

The above chain of reasoning assumes that the perturbation increases the current asset price. To illustrate a situation where this assumption is correct, we adopt the following linear structure:

Assumption 1 *(Linearity) (i) Utility, the production technology, and average adjustment costs are linear:* $U(c_t) = c_t$, $F(k_t, l) = \alpha\ k_t + \varpi\ l$, *and* $A(i) = \phi i/2$ *with* $\alpha, \varpi, \theta, \phi > 0$. *Output is* $(1 - \Lambda(\mu_t))((\alpha - \xi e_t) k_t + (\varpi - \vartheta e_t) l)$. *(ii) Future policy is constant (*$\mu_t = \bar{\mu}$, *for* $t \geq 1$*). (iii) The stocks of capital and atmospheric carbon remain finite as* $H \to \infty$.

Assumption 1.i allows us to express the equilibrium asset price as a linear function of the state variables, where μ_0 enters parametrically. Assumption 1.ii (with $H = \infty$) makes the solution stationary, and Assumption 1.iii provides a transversality condition as $H \to \infty$.

Proposition 4 *Under Assumption 1, a small level of abatement: (i) has a zero first-order effect on the current (*$t = 0$*) young agent's welfare, (ii) increases the current asset price and (for* $\delta < 1$*) increases the current old agent's welfare, and (iii) increases the next-period young agent's welfare.*

5 Political Economy Equilibria

To move beyond the perturbation analysis (small climate policy), we must recognize that abatement is endogenous. To that end, we imbed the model above in a political economy setting. We then assess the role of asset markets in creating incentives for selfish agents to internalize future climate damages. As a benchmark, we also calculate abatement under a discounted utilitarian.

In the probabilistic voting model, voters care about their consumption-related welfare and about ideology. Political parties propose climate policies to maximize the probability of their election. The parties can attract swing voters (those more interested in consumption than ideology) by proposing an abatement level that increases those voter's consumption-related welfare. In equilibrium, parties support the same climate policy, the abatement level that maximizes a convex combination of young and old agents' consumption-related welfare, $\xi\Omega_t^y + (1-\xi)\Omega_t^o$, with $0 \leq \xi \leq 1$. The weight on each generation increases with the number of swing voters in that generation. We refer to the agent who implements the political economy equilibrium policy (maximizes this function) as the *planner*. The *discounted utilitarian*, in contrast, chooses a sequence of abatement levels to maximize the present discounted stream of utility from aggregate consumption.

Selfish agents might want to abate in order to reduce climate damage that the current young suffer when they become old, and/or to alter the asset price. To isolate these two types of incentives, we consider two assumptions regarding agents' beliefs in the political economy. "Unsophisticated" agents take the asset price as given. However, they recognize that current abatement lowers current factor returns and increases next-period factor returns by reducing climate-related damages. "Sophisticated" agents additionally understand that current climate policy alters the current asset price via the change in the future returns to capital induced by the change in the carbon stock. The asset market exists in both scenarios, but only the sophisticated agents recognize it. By comparing equilibrium abatement across scenarios, we can assess the effect of agent's *recognition* of asset markets on the incentive to reduce emissions. The discounted utilitarian benchmark shows how altruism affects incentives to abate.

To identify the role of asset markets as clearly as possible, we ignore growth in technology or population, and we use a single climate state variable. Due to these simplifications, the equilibrium abatement levels are not prescriptive even for the discounted utilitarian. We are primarily interested in the differences, across settings, of abatement levels, rather than their level in a particular setting. To obtain a stationary solution while avoiding the multiplicity arising from an incomplete transversality condition, we study the limit equilibrium (as $H \to \infty$).[10]

[10] The use of a limit equilibrium does not guarantee uniqueness; for example there might be multiple solutions to the dynamic programming equations that determine the equilibrium. No evidence of multiplicity arises in our numerical analysis.

5.1 The political economy setting

We do not want the climate tail to wag the investment dog. Therefore, we assume that in choosing climate policy, the political parties (and the planner who implements their platform) take the level of investment as given; climate policy is not used to influence investment:

Assumption 2 *(Nash) Planners take the current investment decision as given in choosing current abatement. Agents take prices and the current abatement policy as given in choosing investment.*

The planner in the probabilistic voting model maximizes the convex combination of currently living agents' welfare. Current and future abatement alter the trajectory of atmospheric carbon, thus affecting the current asset price and current welfare. Current and future planners play a sequential game; we consider a stationary Markov Perfect Equilibrium (MPE) to this game. The payoff-relevant state variable is the pair (k_t, e_t). The investment and abatement decisions jointly determine current consumption, utility levels, and the next period state variable, (k_{t+1}, e_{t+1}). A MPE is a mapping from the state variable in period t to the planner's abatement policy and young agents' saving policy in that period. Denote the abatement policy function as $\mu_t = M(k_t, e_t)$. The Markov-perfect condition to the game across periods requires that given agents' belief that subsequent generations will follow the equilibrium decision rule, $\mu_{t+j} = M(k_{t+j}, e_{t+j})$ for $j > 0$, the political economy equilibrium that determines the current abatement is $\mu_t = M(k_t, e_t)$.

Given a policy function $\mu_t = M(k_t, e_t)$, the current wage and rental rates are functions of the state variables, $W(k_t, e_t)$ and $R(k_t, e_t)$. Asset owners receive rent and revenue from asset sales. Via the dependence of factor prices on $M(\cdot)$, the equilibrium asset price, $p_t = \Psi(k_t, e_t)$, is a functional in $M(\cdot)$ satisfying equation (3). The political economy equilibrium in period t maximizes the convex combination of the lifetime welfare of agents at t:

$$\max_{\mu_t} \xi\ \Omega_t^y + (1-\xi)\ \Omega_t^o = \max_{\mu_t} \frac{\xi\left([w_t - p_t k_{t+1}]^{1-\eta} + \rho[(r_{t+1} + (1-\delta)p_{t+1})k_{t+1}]^{1-\eta}\right) + (1-\xi)[(r_t + (1-\delta)p_t)k_t]^{1-\eta}}{1-\eta} \tag{10}$$

subject to equations (5) and (1). For $\eta > 0$ and $0 < \xi < 1$, the choice of climate policy that maximizes $\xi\ \Omega_t^y + (1-\xi)\ \Omega_t^o$ involves a "redistribution motive" (Section 5.4).

Unsophisticated planners take asset prices p_t and p_{t+1}, as given, but understand that abatement affects factor returns w_t, r_t, r_{t+1}. Climate policy reduces current factor returns but increases the next-period rental rate. Sophisticated planners additionally understand that the policy-induced change in e_{t+1} alters the next-period asset price, $p_{t+1} = \Psi(k_{t+1}, e_{t+1})$, thereby changing the current price via equation (3). Both types of planner take the level of investment as given (the Nash Assumption 2). We close the model using the no-arbitrage condition, equation (6), so the investment rule, for $i > 0$, is

$$i_t = A^{-1}(\Psi(k_t, e_t)). \tag{11}$$

We substitute the primitives of the model, $F(k, l)$, $D(e)$, and $A(i)$, and the equilibrium asset prices, $W(k_t, e_t)$ and $R(k_t, e_t)$, into equation (10) to obtain the planner's problem. We obtain numerical solutions to both equilibrium problems using the collocation method and Chebyshev polynomials (Judd, 1998; Miranda and Fackler, 2002).

Example: linear utility The case $\eta = 0$ (linear utility) shows how the young and old agents' incentives differ, and the importance of their beliefs about the asset market. Table 1 presents the maximand and the first order conditions for the different problems, imposing the non-negativity constraint, $\mu \geq 0$, using the Nash assumption, and assuming concavity of the maximand.

	old choose policy ($\xi = 0$)	young choose policy ($\xi = 1$)
maximand	$(r_t + (1-\delta)p_t)k_t$	$(w_t - p_t k_{t+1}) + \rho(r_{t+1} + (1-\delta)p_{t+1})k_{t+1}$
FOC Unsophisticated	$\frac{\partial r_t}{\partial \mu_t} \leq 0$	$\frac{\partial w_t}{\partial \mu_t} + \rho \frac{\partial r_{t+1}}{\partial \mu_t} k_{t+1} = 0$
FOC Sophisticated[11]	$\frac{\partial r_t}{\partial \mu_t} + (1-\delta)\frac{\partial p_t}{\partial \mu_t} = 0$	$\frac{\partial w_t}{\partial \mu_t} \leq 0$

Table 1. Maximand and first-order conditions for the unsophisticated and the sophisticated planners in the political equilibria, for $\eta = 0$, with different weights, ξ. In the first order conditions, "=" indicates an interior optimum, $\mu > 0$; "$\leq$" indicates a boundary optimum, $\mu = 0$.

[11] The sophisticated planner recognizes that $p_t = \rho(r_{t+1} + (1-\delta)p_{t+1})$ holds in equilibrium and reduces young agents' welfare to w_t.

When the old have all of the political power ($\xi = 0$), the unsophisticated planner considers only the fact that current abatement lowers the return to capital. This planner sets $\mu_t = 0$. The sophisticated planner understands that the asset price is endogenous. Provided that $\frac{\partial p_t}{\partial \mu_t} > 0$, as is the case for our calibration, this planner chooses a positive level of abatement.

If the young have all the political power ($\xi = 1$), the unsophisticated planner uses a positive level of abatement, because abatement has a first order positive effect on the next-period return to capital, and only a second order effect on the current wage. The sophisticated planner, in contrast, recognizes that the higher current asset price exactly offsets the higher next-period return to capital. This planner chooses zero abatement, understanding that the only effect of abatement on the young agent is to lower the wage.

5.2 The discounted utilitarian

The discounted utilitarian (DU) chooses investment and abatement to maximize the discounted sum of the welfare of aggregate consumption, using agents' pure rate of time preference: $\sum_{s=0}^{\infty} \rho^s U\left(c_t^y + c_t^o\right)$, with $c_t^y + c_t^0 = y_t - A\left(i_t\right) i_t$.[12] The dynamic programming equation is

$$J\left(k_t, e_t\right) = \max_{i_t, \mu_t} \left[U\left(c_t^y + c_t^0\right) + \rho J\left(k_{t+1}, e_{t+1}\right)\right] \tag{12}$$

subject to transition equations (5). Just as in the standard ILRA IAM, the resulting decision rules, $\mu_t = M\left(k_t, e_t\right)$ and $i_t = I\left(k_t, e_t\right)$, are first-best. The only difference here is that the PPF is concave, just as in the political economy setting.

5.3 Functional forms and calibration

Assumption 3 *(Functional forms) Production and abatement functions are iso-elastic:* $F(k,l) = \alpha l^\beta k^{1-\beta}$, $\Lambda\left(\mu\right) = \theta\,\mu^\nu$, *with* $0 < \beta < 1$, $\theta > 0$, $\nu > 1$. *Aggregate investment costs are* $A(i)i = i + \frac{\phi}{2} i^2$ *with* $\phi > 0$, *and climate damages are* $D\left(e_t\right) = \left(1 + \iota\, e_t^2\right)^{-1}$ *with* $\iota > 0$.

[12] The consumption constraint, $c_t^y + c_t^0 = y_t - A\left(i_t\right) i_t$, means that the asset market does not affect the DU's optimization problem. However, we use the asset market equation (3) and the expressions for factor returns and welfare, equations (4), (7) and (8), to calculate the generations' equilibrium welfare in table 3 below.

Labor obtains the constant output share β; θ is the fractional reduction in output under zero emissions ($\mu = 1$); ν is the elasticity of abatement costs.

We use DICE-07 (Nordhaus, 2008) for calibration, making the model comparable to familiar IAMs. Our baseline uses moderate rates of capital depreciation, adjustment costs, and damages, somewhat expensive (relative to DICE-07) abatement, and it places equal weight on currently living generations' welfare. Table 2 collects parameter names and baseline values. Agents live for 70 years, so one period lasts 35 years. Our baseline elasticity of intertemporal substitution is 0.5, so $\eta = 2$, a conventional choice for IAMs with time-additive preferences. Agents discount future utility at $1\%/yr$, implying $\rho = 0.7$.

$\rho = 0.7$ discount factor	$\eta = 2$ inverse IES	$\beta = 0.6$ labor share	$\delta = 0.88$ capital depreciation
$\zeta = 0.062$ carbon intensity	$\epsilon = 0.126$ carbon decay rate	$\iota = 4 \times 10^{-7}$ damage parameter	$\phi = 0.0003$ adjustment cost parameter
$\alpha = 264$ TFP	$\xi = 0.5$ weight on young welfare	$\theta = 0.056$, abatement cost share	$\nu = 2.8$ abatement cost elasticity

Table 2: Parameter names and baseline values

We scale nominal units by 10^9 2010 USD (\$T). Year 2010 capital stock, K_0, is roughly 200 $\$T$ (Rezai et al., 2012). Yearly world output is roughly 63 $\$T$, so output during the first 35-year period is $y_0 = 35 \times 63\ \$T \cong 2200\ \$T$ (CIA, 2010). Given the initial endowments of capital and labor (normalized to 1), y_0, and $\beta = 0.6$, total factor productivity is calibrated to $\alpha = 264$. We set capital depreciation to $6\%/yr$, above the mean of $4\%/yr$ for 2010 of the Penn World Table and below the $10\%/yr$ used by Nordhaus (2008); this implies $\delta = 0.88$. The sensitivity analysis uses $4\%/yr$ depreciation.

We measure the carbon stock, e, in parts per million by volume (ppmv). In 2010 8.6 $Gt\ C$ are emitted per year (BP Statistical Review of World Energy, 2017), corresponding to an annual increase in atmospheric CO_2 of 3.92 $ppmv$. With yearly world output of 63 $\$T$, this implies a carbon dioxide emission intensity $\zeta = \frac{3.92}{63} \approx 0.062\ \frac{ppmv}{\$T}$. The actual increase in atmospheric CO2 concentration in 2010 was 2.42 $ppmv$ (NOAA, 2017), implying dissipation was

1.5 *ppmv*. The corresponding depreciation factor equals $\frac{1.5}{390} = 0.0038\ \%/yr$, implying $\epsilon = 1 - (1 - 0.0038)^{35} = 0.126$. This number is close to the mean of the (0.0025%/yr, 0.0055%/yr) range of the implied dissipation rates of carbon in DICE-07 (Rezai, 2010).

The DICE-07 abatement cost elasticity is $\nu = 2.8$. The parameter θ measures the share of GDP necessary to abate all emissions ($\Lambda(1) = \theta\ 1^{\nu} = \theta$). In DICE-07, it costs 5.4% to abate all emissions today, 0.9% in 30 decades and 0.4% in 60 decades. We set $\theta = 0.054$, a constant, to obtain a stationary model. Consequently, future abatement is more expensive in our model than in DICE, tending to raise current abatement and reduce future abatement (flattening the "policy ramp").

Damages depend on the single climate state, atmospheric carbon stock, e, with $D\left(e_t\right) = (1 + \iota\ e_t^2)^{-1}$; emissions in one period cause damages in the next period.[13] As in DICE, we assume that doubling the carbon stock relative to preindustrial levels reduces national income by 3%, implying $\iota = 4 \times 10^{-7}$.[14]

We use the empirical studies of Shapiro (1986), Hall (2004), and Mumtaz and Zanetti (2015) to calibrate adjustment costs. Aggregate investment cost, inclusive of adjustment costs, in our framework is $A(i)i = i + \frac{\phi}{2} i^2$; $\phi \geq 0$ determines the concavity of the production possibility frontier between the consumption and the investment goods. With $y = 2200\ \$T$, our baseline $\phi = 0.0003$ implies that when investment equals 20% of output, adjustment costs ($\frac{\phi}{2} i^2$) equal 1.3% of output. Shapiro's (1986) reduced form estimates imply higher adjustment costs for the same investment share (Appendix B.3). Our robustness check doubles our baseline value of ϕ, bringing our estimate of adjustment costs (as a percent of income) close to Shapiro's low estimate. Using a structural model, Mumtaz and Zanetti's (2015) estimate total adjust-

[13]Ricke and Caldeira (2014) estimate that most of the warming effect of current emissions, and thus most of the temperature-related damage, occurs within a decade. Allen et al. (2009) estimate that the maximum warming effect occurs after many decades. Economic IAMs also disagree about the lag between emissions and damage response. In Nordhaus' (2008) DICE, the maximum temperature response occurs after about 60 years. In Golosov et al (2014), the maximum damage response occurs within a decade. With a 35 year time step, and current emissions affecting next period damages, our model represents a middle ground.

[14]We can compare the damage functions in our model and in DICE-07, using the equilibrium relation between temperature, τ, and stocks, e: $\tau = 3\ \log[e/280]/\log[2]$. Our damage function is lower than in DICE for temperatures below $\tau < 2°C$ but rises more steeply beyond this point. The two damage functions differ by less than one percent for temperatures below $3.3°C$.

ment costs to equal 3.3% of output. Thus, our baseline represents empirically reasonable, but modest adjustment costs. If adjustment costs are zero, we are back in the world with a linear PPF, and a trivial asset market.

5.4 Application

Agents in the political economy equilibria reduce emissions, despite their indifference to future generations' welfare. The unsophisticated planner, who takes into account only current factor returns and next-period return to capital, initially reduces emissions by 14%, eventually increasing abatement to 40%. The sophisticated planner, who additionally takes into account the endogenous asset price, initially reduces emissions by 21%, increasing to nearly 55% after centuries. The DU initially reduces emissions by 40%, increasing to 85%. Reduced emissions lead eventually to lower trajectories of atmospheric carbon, lower damages, and higher levels of capital stock and investment. Figure 2 shows the trajectories of capital stock, atmospheric carbon, investment and abatement for the unsophisticated and sophisticated political equilibrium, the DU, and under BAU (zero abatement).

A higher abatement trajectory eventually leads to lower carbon stocks, a higher return to capital, and higher investment and asset prices. After a century, investment and asset prices diverge across the scenarios.

However, investment trajectories are nearly the same across scenarios early in the program. The asset price and investment level are linearly related, so in the short run the equilibrium asset price is nearly the same across scenarios. However, the comparison between the unsophisticated and the sophisticated political economies shows that the incentive to alter the asset price increases abatement even in the short run. The fact that the incentive to alter asset prices changes abatement without changing short run asset prices might seem paradoxical. The explanation is that future generations also abate emissions. In the short run, the lower factor returns resulting from the sophisticated planner's higher abatement approximately offset the increase in factor returns due to the (slightly) lower climate stock. This result may be important for empirical work: the lack of a statistically significant relation between climate variables and the price of a broad portfolio of assets does not imply that asset markets are unimportant to climate policy.

The capital stock initially increases in all scenarios. Under the unsophisticated planner (blue, dashed), the low abatement leads to high carbon stocks and high climate damage, reducing both income and the return to capital,

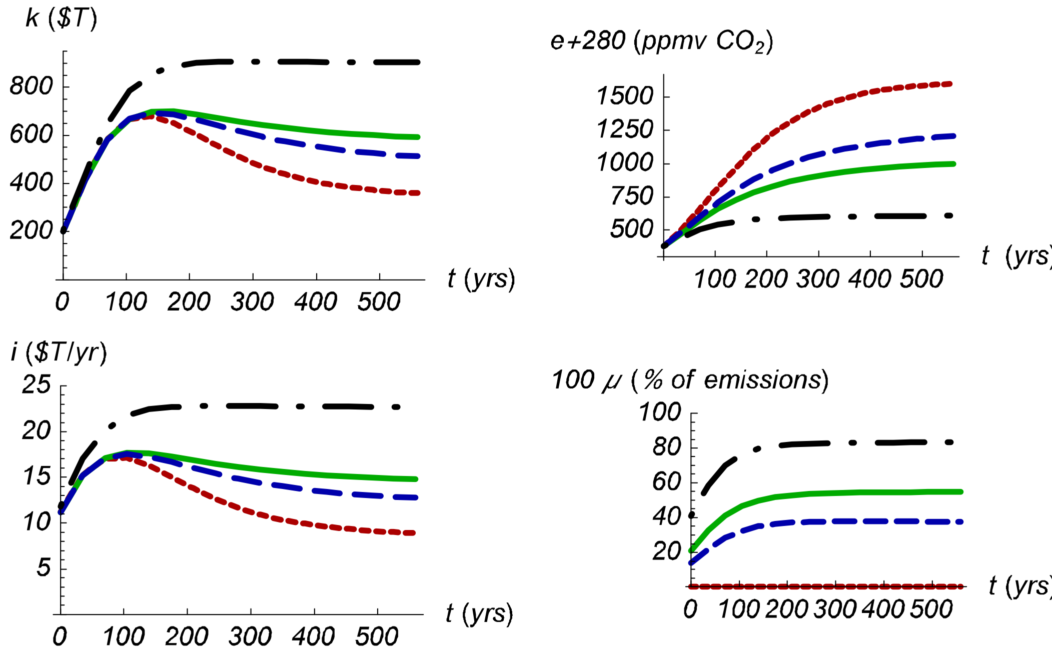

Figure 2: Equilibrium trajectories under: BAU (red dotted); unsophisticated political economy (blue dashed); sophisticated political economy (green solid); discounted utilitarian (black dot-dash). Abatement increases as planners internalize damages occurring during their lifetime (blue dashed), recognize asset price effect (green solid), or take into account future generations' welfare (black dot-dash).

lowering equilibrium investment. Eventually the stock of capital, and thus emissions, fall, slowing the growth in the carbon stock. Carbon levels stabilize at 1245 ppmv, leading to an equilibrium temperature increase of $6.4°C$ and damages of 27% of output. The eventual fall in capital stocks and rise in carbon stocks is more extreme under zero-abatement BAU (red dotted).

As noted, in the short run the trajectories of capital and investment are similar in the three scenarios with abatement, but they later diverge, lagging the divergence in the carbon trajectories. Lower levels of emissions and atmospheric carbon support a higher equilibrium capital stock and output. Under the DU, carbon levels stabilize at 610 ppmv, creating an equilibrium temperature increase of $3.4°C$ and a 4% loss of output. This temperature increase and loss in output is significantly higher than in DICE, because our model has no exogenous decrease in abatement costs or carbon intensity. Our higher abatement costs and emissions imply higher steady state carbon stocks and damages.

The DU and the unsophisticated political economy trajectories sandwich the trajectories under the sophisticated political economy (green, solid). Equilibrium abatement there is eventually about 50% greater than in the unsophisticated scenario, and 65% of the DU level. Carbon concentration in the sophisticated equilibrium stabilizes at 1025 ppmv and temperature increases to 5.6°C, resulting in climate damages of about 18% of output.

	sophisticated political economy	discounted utilitarian planner
$\Delta\Omega_t^O$	0.%	- 0.4%
$\Delta\Omega_t^y$	0.%	- 1.7%
$\Delta\Omega_{t+1}^y$	0.1%	- 1.5%
$\Delta\Omega_{t+2}^y$	0.7%	0.6%
$\Delta\Omega_{t+3}^y$	1.8%	4.1%

Table 3. Percent change in equilibrium life-time welfare ($\Delta\Omega$) of current generations and future young generations, relative to their welfare in the unsophisticated equilibrium.

Table 3 presents the current and future percent welfare changes under the sophisticated equilibrium and the DU, relative to the unsophisticated equilibrium. In the political economy setting, where generations care only for their own welfare, the recognition of asset markets increases abatement (relative to the unsophisticated scenario) but initially has negligible welfare effect.[15] The DU imposes tighter emission standards, leading to the highest welfare after a century, but causing welfare losses for generations alive today and the generation born in the next period. Only generations born in 2115 or later are best off under the DU, because of the higher capital stock and lower carbon stock there.

[15]In the sophisticated equilibrium, the generations alive at t have a small welfare loss, rounded to zero. The move from the unsophisticated to the sophisticated scenario changes future as well as current actions, and the future actions affect current welfare via the asset price. The welfare effect of the change in scenarios depends on parameter values.

Robustness and distribution Table 4 reports initial abatement levels for different parameters; results in bold correspond to our baseline. The first set of columns corresponds to abatement for $\eta = 2$ and $\eta = 0$ and three values of ξ, the welfare weight on the young in the political economy setting. This experiment sheds light on the inter-generational distributional effects of policy and on the role of η. The final column shows abatement for $\xi = 0.5$, but higher adjustment costs and lower capital depreciation.

For $\eta = 2$, current abatement tends to decrease the asset price. For our baseline calibration, the first-period young have higher marginal utility of income than the first-period old. Therefore, reducing the asset price increases the aggregate payoff for $\xi = 0.5$. This redistributive effect accounts for the increase in abatement when moving from the unsophisticated to the sophisticated equilibrium. The experiments with $\xi = 1$ and $\xi = 0$ reinforce this conclusion. The young agent wants to increase abatement, while the old agent prefers zero abatement.

For $\eta = 0$, current abatement increases the asset price (as in Proposition 4) and both agents have the same constant marginal utility. The discussion below equation 9 notes that for $\eta = 0$, a non-negligible level of abatement harms the young, whose utility equals their wage. Therefore, the young agents who choose policy and recognize the endogeneity of asset prices prefer zero abatement. If instead they treat the asset price as fixed, they prefer substantial abatement, in order to increase the next-period return on capital. These incentives are reversed for the old agent. Taking the asset price as fixed, this agent thinks that abatement only lowers current returns, and therefore prefers zero abatement. The old agent who understands that asset prices are endogenous wants modest abatement, to raise the asset price.

The move from $\eta = 2$ to $\eta = 0$ leads to a large increase in abatement for the DU, a familiar result from the Ramsey formula for the social discount rate. With positive consumption growth, the consumption discount rate increases with η. For low η, the DU is willing to incur higher contemporaneous sacrifices to increase future consumption. The same force operates in the unsophisticated political equilibrium. With high elasticity of intertemporal substitution, and taking the asset price as exogenous, the young agent is willing to make current sacrifices to obtain a cleaner environment in the future. Therefore, if $\xi = 0.5$ or $\xi = 1$, equilibrium abatement is higher in the political economy under $\eta = 0$, compared to $\eta = 2$. As above, the old agent who takes the asset price as exogenous has no interest in a cleaner future environment, so abatement remains at zero if $\xi = 0$. Moving to the sophisticated equilibrium reverses these incentives, because abatement increases the asset price. The old agent therefore favors abatement and the young agent opposes it.

Table 4 shows that the recognition of endogenous asset prices can significantly change the incentives to abate, altering equilibrium abatement. The direction of these incentives, and direction of the asset price change resulting from changed abatement, is sensitive to the intertemporal elasticity of substitution and to the distributional weights on the two generations.

		baseline			alternate scenario	
		$\xi = 0.5$	$\xi = 1$	$\xi = 0$	$\xi = 0.5$	
$\eta = 2$	soph.	**21%**	37%	0%	soph.	20%
	unsoph.	**14%**	18%	0%	unsoph.	14%
	DU	**41%**	N/A	N/A	DU.	41%
		$\xi = 0.5$	$\xi = 1$	$\xi = 0$	$\xi = 0.5$	
$\eta = 0$	soph.	3%	0%	5%	soph.	5%
	unsoph.	24%	32%	0%	unsoph.	24%
	DU	64%	N/A	N/A	DU	63%

Table 4: Robustness. Percentage first period abatement for "soph." (the sophisticated political economy), "unsoph." (the unsophisticated political economy), and the DU with: equal welfare weights ($\xi = 0.5$); the young generation chooses policy ($\xi = 1$); and the old generation chooses policy ($\xi = 0$). "N/A" means "not applicable"; the DU cares about the utility of aggregate consumption. The baseline uses values from Table 3; the alternate scenario uses baseline values except for $\phi = 0.0006$, $\delta = 0.76$.

The final column of Table 5 reports results for $\xi = 0.5$ with higher adjustment costs ($\phi = 0.0006$ instead of 0.0003) and a 4% annual depreciation rate for capital (instead of the baseline of 6%) implying $\delta = 0.76$. The higher abatement cost increases the concavity of the PPF, making the asset price more sensitive to the level of investment. The slower depreciation rate means that the old agent has more capital at the end of a period, increasing the distributional importance of the asset price. Thus, both of these changes move our baseline further from the standard IAM with linear PPF. The equilibrium levels of abatement are similar to the baseline levels, suggesting that the results are robust to our choice of parameter values.

6 Conclusion

In the popular discussion, the primary rationale for climate policy is to benefit people born in the future. This discussion recognizes that current abatement might benefit the current young late in their life. However, it ignores the possibility that asset markets transfer some of the future benefits of climate policy to current asset owners. This omission potentially understates currently living generations' benefit from climate policy, and it obscures potentially important distributional effects. We bring asset markets into focus by constructing a climate model with a strictly concave production possibility frontier between a consumption and an investment good. With overlapping generations, there are distinct buyers and sellers of capital. These two features lead to a non-trivial asset market.

A marginal level of abatement that increases the asset price always benefits the old agent, and also benefits the young agent if and only if the elasticity of intertemporal substitution exceeds 1 (the substitution effect exceeds the income effect). We use numerical methods to study non-marginal, equilibrium climate policy, and to determine the equilibrium effect of abatement on the asset price.

In our political economy setting, currently living selfish generations choose climate policy to maximize a convex combination of their lifetime welfare. In our baseline calibration, with the commonly used value for the intertemporal elasticity of substitution of 0.5 ($\eta = 2$), there is an incentive to abate in order to benefit the current young late in their life. When the two generations have equal political influence, the recognition that the asset price is endogenous increases near-term abatement by 50%, reaching over half the level a (standard) discounted utilitarian chooses. Climate policy increases joint welfare, but in this case tends to lower the asset price, harming the old agent and benefiting the young agent. An increase in the young agent's political influence increases equilibrium abatement; increasing the old agent's influence decreases abatement. These conclusions flip if the elasticity of intertemporal substitution is infinite; here, abatement tends to increase the asset price.

When the relation between climate policy and asset values is discussed, it is usually in the context of stranded assets, those likely to be harmed by climate policy. Fossil fuels companies are worth about \$5 trillion, a large number but a small fraction of the world's financial wealth. An integrated assessment model with a single stock of capital having an endogenous price enables us to examine the relation between climate policy and an aggregate measure of assets. The desire to influence the asset price significantly affects the incentive to abate, but it has negligible effect on the *short run* equilibrium asset price. For example, a higher level of abatement and the resulting lower return on capital approximately offsets the increased return on capital due to the induced reduction in carbon stocks. However, the increased incentive to abate arising from the recognition of endogenous asset prices significantly increases investment and asset prices in the long run.

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A Appendix: Proofs

Proof. (Lemma 1) The old generation consumes all of its income, so $c_t^o = (r_t + (1-\delta)\, p_t)\, k_t$, which implies equation 8. To obtain equation 7 we begin with the definition of the young agent's lifetime welfare

$$\Omega_t^y \equiv U(c_t^y) + \rho\, U(c_{t+1}^o).$$

With isoelastic utility, $\psi_t = \frac{1}{\rho}\left(\frac{c_{t+1}^o}{c_t^y}\right)^\eta$. Using this result in equation 3 gives

$$p_t = \rho\left(\frac{c_{t+1}^o}{c_t^y}\right)^{-\eta}(r_{t+1} + (1-\delta)p_{t+1})\frac{k_{t+1}}{k_{t+1}} = \rho\frac{\left(c_{t+1}^o\right)^{1-\eta}}{\left(c_t^y\right)^{-\eta} k_{t+1}},$$

where the second equality follows from the second constraint in equation 1. Rearranging this equation gives

$$\left(c_{t+1}^o\right)^{1-\eta} = p_t\frac{\left(c_t^y\right)^{-\eta} k_{t+1}}{\rho}.$$

Using this expression for the utility of the agent who is old in period $t+1$, we write the lifetime welfare for the agent who is young in period t as

$$\begin{aligned}
\Omega_t^y &= \frac{1}{1-\eta}\left(\left(c_t^y\right)^{1-\eta} - 1 + \rho\left(\left(p_t\frac{\left(c_t^y\right)^{-\eta} k_{t+1}}{\rho} - 1\right)\right)\right) \\
&= \frac{\left(c_t^y\right)^{-\eta}}{1-\eta}\left(c_t^y + p_t k_{t+1}\right) - \frac{1}{1-\eta}(1-\rho) \\
&= \frac{\left(c_t^y\right)^{-\eta}}{1-\eta}w_t - \frac{1}{1-\eta}(1-\rho),
\end{aligned}$$

where the last equality follows from the first constraint in equation 1. ∎

Proof. (Proposition 1) Both claims follow from inspection of equation 8, together with the facts that k_t is predetermined and the policy has only a second order effect on the current return to capital. For $\delta = 1$, the old agent has nothing to sell, so the price of capital does not affect her welfare. ∎

Proof. (Proposition 2) We hold future policy levels fixed, and consider the effect of a small current policy, μ_0, at the initial time, $t = 0$. Using equation 7 (and ignoring the constant) we write the young agent's lifetime welfare as

$$\begin{aligned}
\Omega_0^y(\mu_0) &= \frac{\left(c_0^y\right)^{-\eta}}{1-\eta}w_0 \Rightarrow \\
\frac{d\Omega_0^y(\mu_0)}{d\mu_0} &= \frac{1}{1-\eta}\left(\frac{\partial w_0}{\partial \mu_0}\left(c_0^y\right)^{-\eta} - \eta w_0\left(c_0^y\right)^{-\eta-1}\frac{dc_0^y}{d\mu}\right). \qquad (13)
\end{aligned}$$

The young agent's budget constraint is

$$\begin{aligned} c_0^y &= w_0 - p_0\left(\left(1-\delta\right)k_0 + i_0\left(p_0\right)\right) \Rightarrow \\ \frac{dc_0^y}{d\mu} &= \frac{\partial w_0}{\partial \mu} - \left(p_0 \frac{\partial i_0}{\partial p_0} + \left(\left(1-\delta\right)k_0 + i_0\right)\right)\frac{dp_0}{d\mu}. \end{aligned} \tag{14}$$

A small climate policy ($\mu_0 \approx 0$) has zero first-order effect on output and on factor prices (because $\Lambda'(0) = 0$). This fact and equations 13 and 14 imply

$$\left.\frac{d\Omega_0^y\left(\mu_0\right)}{d\mu_0}\right|_{\mu_0=0} = \frac{1}{1-\eta}\eta w_0 \left(c_0^y\right)^{-\eta-1}\left(p_0\frac{\partial i_0}{\partial p_0} + \left(1-\delta\right)k_0 + i_0\right)\frac{dp_0}{d\mu}. \tag{15}$$

Using $\frac{\partial i_t}{\partial p_t} = (A^{-1})' > 0$, we conclude:

$$\begin{aligned} \text{sign}\left(\left.\frac{d\Omega_0^y(\mu)}{d\mu}\right|_{\mu=0}\right) &= \text{sign}\left(\frac{dp_0}{d\mu_0}\right) \qquad \text{for } 0 < \eta < 1 \\ \text{sign}\left(\left.\frac{d\Omega_0^y(\mu)}{d\mu}\right|_{\mu=0}\right) &= -\text{sign}\left(\frac{dp_0}{d\mu_0}\right) \qquad \text{for } \eta > 1. \end{aligned}$$

Proposition 2.iii follows from the first two statements and from Proposition 1. ■

Proof. (Lemma 2) The budget constraint for the agent who is old in period $t+1$ requires $\frac{c_{t+1}^o}{k_{t+1}} = r_{t+1} + (1-\delta)p_{t+1}$. Using this relation and the fact that $\psi_t = \frac{c_{t+1}^o}{\rho c_t^y}$ under logarithmic utility, equation 3 becomes

$$p_t = \rho\frac{c_t^y}{c_{t+1}^o}\left(r_{t+1} + (1-\delta)p_{t+1}\right) = \rho\frac{c_t^y}{k_{t+1}} \Longrightarrow$$
$$c_t^y = \tfrac{p_t k_{t+1}}{\rho}$$

Using the last equation and the young agent's budget constraint at time t, we have

$$c_t^y = w_t - p_t k_{t+1} = \frac{p_t k_{t+1}}{\rho} \Rightarrow p_t k_{t+1} = \frac{\rho}{1+\rho}w_t \Rightarrow c_t^y = \frac{1}{1+\rho}w_t.$$

The last equality states that the agent spends a constant fraction $\frac{1}{1+\rho}$ of her wage on first period consumption and saves the rest. Using the fact that

$p_t = A(i_t)$, or $i_t = A^{-1}(p_t)$, we obtain an implicit function for p_t. This function, $p_t = f(w_t)$, solves

$$p_t k_{t+1} = p_t \left[(1-\delta)k_t + A^{-1}(p_t)\right] = \frac{\rho}{1+\rho} w_t. \tag{16}$$

Replacing p_t with $f(w_t)$, using the fact that A^{-1} is an increasing function, and totally differentiating the last equation in 16 implies that $f'(w) > 0$: an exogenous increase in the current wage causes an increase in the equilibrium price of capital. ■

Proof. (Proposition 3) (i) A small policy at $t = 0$ has zero first-order effect on the wage at $t = 0$. By Lemma 2.ii, this policy therefore has zero first-order effect on the asset price at $t = 0$. By Proposition 1, the policy has zero first-order effect on the old agent's welfare at $t = 0$.

(ii) By Lemma 2.i and the fact that the policy has a zero first-order effect on wage at $t = 0$, the policy has zero first-order effect on the $t = 0$ young agent's consumption, and thus on her savings, k_1. However, the policy creates a first-order reduction in the $t = 1$ pollution stock, leading to a first-order increase in both r_1 and w_1, and (by Lemma 2.ii) on the equilibrium value of p_1. The policy therefore leads to a first-order increase in her period $t = 1$ consumption, $c_1^o = (r_1 + (1-\delta)p_1)\, k_1$, and thus in her lifetime welfare.

(iii) The period-0 policy increases w_1 by reducing the period-1 pollution stock. By Lemma 2.i, the period-0 policy therefore increases the consumption, c_1^y, of the agent born at $t = 1$. This agent's consumption in the subsequent period equals $c_2^o = (r_2 + p_2(1-\delta))\, k_2$. Because the policy increases p_1, it also increases investment, i_1, and therefore increases k_2. The period-0 policy lowers e_1 without altering k_1, so the policy lowers the subsequent pollution stock, e_2. The reduction in e_2 and the increase in k_2 both increase the equilibrium wage, w_2. By Lemma 2.ii, the policy therefore increases p_2, and thus increases $p_2 k_2$. The equilibrium condition for the rental rate implies

$$r_2 k_2 = (1 - D(e_2))\, \Lambda(\mu_2)\, (1-\beta)\, \alpha k_2^{1-\beta}.$$

Because the policy has increased k_2 and reduced e_2, it increases $r_2 k_2$. Therefore, the policy also increases consumption, at $t = 2$, of the agent who was born at $t = 1$. Consequently, the policy increases this agent's welfare. ■

Proof. (Proposition 4). We invoke the turnpike theorem in letting $H \to \infty$.

(i) This result is an immediate consequence of equation 9, $w = (1 - \Lambda(\bar{\mu}))(\varpi - \vartheta e)$, and the fact that $\left.\frac{d\Lambda(\bar{\mu})}{d\bar{\mu}}\right|_{\bar{\mu}=0} = 0$.

(ii) (sketch; see Reviewers' Appendix B.1 for details) Using Proposition 1 we need only establish that $\left.\frac{dp_0}{d\mu_0}\right|_{\mu_0=0} > 0$. We confirm this inequality by examining the equilibrium conditions, a system of three linear difference equations in p, k, e. Price p is a forward looking variable, and the equilibrium price is linear in k, e. Using a linear trial solution and the assumption that the state variables, k, e, remain finite, yields $\left.\frac{dp_0}{d\mu_0}\right|_{\mu_0=0} > 0$.

(iii) This result follows from equation 9, $w = (1 - \Lambda(\mu))(\varpi - \vartheta e)$, $\Lambda'(0) = 0$, and $\frac{de_1}{d\mu_0} = -\zeta(\alpha k_0 + \varpi) < 0$. ■

B Reviewers' Appendix (not for publication)

This appendix provides the detailed proof of Proposition 4.ii, and it contains details of our numerical calibration and solution.

B.1 Proof of Proposition 4.ii

Using Proposition 1 we need only establish that $\left.\frac{dp_0}{d\mu_o}\right|_{\mu_0=0} > 0$. Under Assumption 1, the dynamics of the competitive equilibrium, equations 3 and 5, from periods $t = 1$ onward, reduce to the following system of linear difference equations (with $\Delta = 1 - \delta$, $E = 1 - \epsilon$, and $\varphi = 2/\phi$ to simplify notation):

$$\begin{aligned} p_t &= \rho\ \left[(1 - \Lambda(\bar{\mu}))(\alpha - \xi e_{t+1}) + \Delta\ p_{t+1}\right] \\ k_{t+1} &= \Delta\ k_t + \varphi\ (p_t - 1) \\ e_{t+1} &= E\ e_t + \zeta\ (1 - \bar{\mu})\ (\alpha\ k_t + \varpi)\,. \end{aligned} \tag{17}$$

The initial conditions for this system are k_1 and e_1. These values depend on the state variables and the policy at time 0, k_0, e_0, and μ_0. In this linear model, where at t the states k_t and e_t are predetermined, and p_t is "forward looking", the equilibrium p_t is a linear function of k_t and e_t: $p_t = X + Y\ k_t + Z\ e_t$. The coefficients X, Y, Z depend on model parameters, including the constant $\bar{\mu}$. Substituting the trial solution, $p_t = X + Y\ k_t + Z\ e_t$, into system 17 we obtain

$$p_t = \begin{pmatrix} 1 & k_t & e_t \end{pmatrix} \cdot \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \rho \begin{pmatrix} 1 & k_t & e_t \end{pmatrix} \begin{pmatrix} A_X \\ A_Y \\ A_Z \end{pmatrix} \quad \text{with}$$

$$\begin{pmatrix} A_X \\ A_Y \\ A_Z \end{pmatrix} = \begin{pmatrix} A_{x1} + A_{x2} \\ Y\ \Delta\ (\Delta + Y\ \varphi) + \alpha\ \zeta\ (1-\bar{\mu})\ (Z\ \Delta - \xi\ (1-\Lambda(\bar{\mu}))) \\ Z\ \Delta\ (E + Y\ \varphi) - E\ \xi\ (1-\Lambda(\bar{\mu})) \end{pmatrix}$$

and

$$A_{x1} = \alpha\ (1-\Lambda(\bar{\mu})) + X\ \Delta + Y\ \Delta\ \varphi\ (X-1)$$

$$A_{x2} = \varpi\ \zeta\ (1-\bar{\mu})\,(Z\ \Delta - \xi\ (1-\Lambda(\bar{\mu})))\,.$$

Equating coefficients yields

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \rho \begin{pmatrix} A_X \\ A_Y \\ A_Z \end{pmatrix} \tag{18}$$

We show that there is a unique negative value of Z that satisfies this system: the asset price declines in the pollution stock. (Other roots of the system are either complex or positive.)

The last two equations in system 18 are independent of X. Solving the last equation for Z gives

$$Z\,(Y) = -\frac{E\xi\rho(1-\Lambda(\bar{\mu}))}{1-\Delta\rho(E+Y\varphi)}. \tag{19}$$

with the implication that $Z < 0 \Leftrightarrow 1 - \Delta\rho(E+Y\varphi) > 0$. Substituting the expression for Z into the second equation of system 18, we obtain

$$0 = \Upsilon\,(Y) \equiv$$
$$Y - \rho\left[Y\ \Delta\ (\Delta + Y\ \varphi) - \alpha\ \zeta(1-\bar{\mu})\ \xi\ (1-\Lambda(\bar{\mu}))\left(1 + \tfrac{\Delta\rho E}{1-\Delta\rho(E+Y\varphi)}\right)\right].$$

Define

$$\Theta = \Upsilon \times [1 - \Delta\rho(E+Y\varphi)]\,.$$

For

$$Y \neq Y_{crit} \equiv \frac{1-\Delta\ E\ \rho}{\Delta\ \rho\ \varphi} > 0,$$
$$\Upsilon(Y) = 0 \Leftrightarrow \Theta\,(Y) = 0.$$

Θ, is a cubic and thus has one or three real roots (as does Υ). The cubic Θ can be written

$$\begin{aligned}
\Theta &= B_3 Y^3 + B_2 Y^2 + B_1 Y + B_0 \quad \text{with} \\
B_3 &\equiv \left(\Delta^2 \rho^2 \varphi^2\right) > 0 \\
B_2 &\equiv \Delta\rho\varphi\left(\Delta\rho(\Delta + E) - 2\right) < 0 \\
B_1 &\equiv (1 - \rho\Delta E)(1 - \rho\Delta^2) - \alpha\zeta\xi\varphi\Delta\rho^2(1 - \bar{\mu})(1 - \Lambda) \\
B_0 &\equiv \alpha\zeta\xi\rho\left(1 - \Lambda\right)\left(1 - \bar{\mu}\right) > 0
\end{aligned}$$

The sign pattern of $\begin{pmatrix} B_3 & B_1 & B_1 & B_0 \end{pmatrix}$ is either $\begin{pmatrix} + & - & + & + \end{pmatrix}$ or $\begin{pmatrix} + & - & - & + \end{pmatrix}$. In either case, there are two sign differences between consecutive coefficients. By Descartes' rule of signs, there exist either two or zero positive roots. (In a knife-edge case, there is a single positive root.) To apply the corollary of Descartes' rule of signs, multiply the coefficients of odd powered terms by -1. This gives the sign pattern $\begin{pmatrix} - & - & - & + \end{pmatrix}$ or $\begin{pmatrix} - & - & + & + \end{pmatrix}$. In either case, there is one change in signs. The corollary states that the number of negative roots is either the number of sign changes (one), or fewer than that number by a multiple of 2. Because there cannot be a negative number of negative roots, we conclude that there is a unique negative root. We cannot rule out the existence of up to two roots with positive values for Y.

The system is explosive for $Y \geq Y_{crit}$; therefore, if Y is positive and the state variables remain finite (as assumed), $Y \in (0, Y_{crit})$. Substituting $p_t = X + Y\ k_t + Z\ e_t$ into the last two equations in system 17 gives

$$\begin{aligned}
k_{t+1} &= \Delta\ k_t + \varphi\ \left(X + Y\ k_t + Z\ e_t - 1\right) \\
e_{t+1} &= E\ e_t + \zeta\ \left(1 - \bar{\mu}\right)\ \left(\alpha\ k_t + \varpi\right).
\end{aligned}$$

The Jacobian of this system (with respect to the state variables, k_t and e_t) is

$$J = \begin{pmatrix} \Delta + \varphi Y & \varphi Z \\ \alpha\zeta(1 - \bar{\mu}) & E \end{pmatrix}.$$

Stability requires that all eigenvalues of J lie within the unit circle. A weaker necessary condition requires the sum of the eigenvalues, or the trace of J, $Tr(J) = \Delta + E + \varphi Y$, to be less than 2. For the parameter restrictions $0 \leq \Delta \leq 1$ (capital decays) and $\rho > 0$ (positive discount factor) we have $Y \geq Y_{crit} \Longrightarrow Tr(J) > 2$:

$$Tr(J)|_{Y = Y_{crit}} = \Delta + E + \varphi \frac{1 - \Delta\ E\ \rho}{\Delta\ \rho\ \varphi} = \Delta + \frac{1}{\Delta\ \rho} > 2.$$

Because $Tr(J)$ is increasing in Y, we rule out all positive roots $Y^+ > Y_{crit}$. Therefore, any equilibrium root (which we have shown to exist) satisfies $Y < Y_{crit}$; consequently, $Z < 0$.

The next step obtains the current ($t = 0$) asset price as a function of the next period pollution stock. Using equation 3 and $p_t = X + Y\ k_t + Z\ e_t$, we have

$$p_0 = \rho\ \left[(1 - \Lambda(\mu_0))(\alpha - \xi e_1) + \Delta\ p_1\right]$$

$$= \rho\left[(1 - \Lambda(\mu_0))(\alpha - \xi e_1) + \Delta(X + Y\ k_1 + Z\ e_1)\right]$$

$$= \rho\left[(1 - \Lambda(\mu_0))\,\alpha + X\Delta\right] + \rho Y\Delta k_1 - \rho\left[(1 - \Lambda(\mu_0))\,\xi - Z\Delta\right] e_1.$$

We now use the relation $k_{t+1} = \Delta\ k_t + \varphi\ (p_t - 1)$ to eliminate k_1 and write

$$p_0 = \rho\left[(1 - \Lambda(\mu_0))\,\alpha + X\Delta\right] + \rho Y\Delta\left(\Delta\ k_0 + \varphi\ (p_0 - 1)\right)$$
$$-\rho\left[(1 - \Lambda(\mu_0))\,\xi - Z\Delta\right] e_1.$$

Solving for p_0 gives

$$p_0 = \tfrac{1}{(1-\rho Y\Delta\varphi)}\times$$
$$\left[\rho\left[(1 - \Lambda(\mu_0))\,\alpha + X\Delta\right] + \rho Y\Delta(\Delta k_0 - \varphi) - \rho\left[(1 - \Lambda(\mu_0))\,\xi - Z\Delta\right] e_1\right].$$

The assumption that abatement costs are minimized at zero abatement, $\Lambda'(0) = 0$, means that we now need only to consider the effect of μ_0 on p_0 via e_1. Using the transition equation of e_t in 17, we establish

$$\left.\frac{dp_0}{d\mu_o}\right|_{\mu_0=0} = B_4 \frac{de_1}{d\mu_0} > 0 \quad \text{with} \tag{20}$$
$$B_4 \equiv -\frac{\rho\left((1 - \Lambda(\bar{\mu}))\,\xi - Z\Delta\right)}{(1 - \rho Y\Delta\varphi)} = \frac{Z}{E} < 0 \quad \text{and}$$
$$\frac{de_1}{d\mu_0} = -\zeta\left(\alpha k_0 + \varpi\right) < 0.$$

B.2 Definition of Political Economy Equilibria

The standard cases of policy-making are either business-as-usual (with zero abatement) or first-best (socially optimal policy) chosen by the DU. Our political economy equilibria are situated between these two extremes and, depending on the degree of sophistication, are closer to BAU or the DU. (Given the absence of bequest motives, the political economy equilibria will never reach the DU outcome.) Restating equation 10, we have the political preference function

$$\xi\ \Omega_t^y + (1-\xi)\ \Omega_t^o =$$

$$\frac{\xi\left\{[w_t - p_t k_{t+1}]^{1-\eta} + \rho[(r_{t+1} + (1-\delta)p_{t+1})k_{t+1}]^{1-\eta}\right\} + (1-\xi)[(r_t + (1-\delta)p_t)k_t]^{1-\eta}}{1-\eta}.$$

Under BAU, agents do not abate. In our unsophisticated equilibrium, agents recognize that emissions affect current and next-period factor returns, but they take the asset price as given. Because the next period rental rate directly affects the currently young agents, and because the marginal cost of abating the first unit is zero, young agents want positive abatement. In the sophisticated equilibrium, agents understand the formation of asset prices following equation 3

$$p_t = \frac{r_{t+1} + p_{t+1}(1-\delta)}{\psi_t}.$$

Mitigation today reduces future levels of carbon stocks, affecting all future rental rates. These rates are capitalized in today's asset price, possibly creating an additional motive for climate policy. Mitigation also reduces investment levels, but agents take these as given by Assumption 2.

The example of linear utility, presented in Section 5.1, helps clarify the differences between the two equilibrium concepts. For linear utility ($\eta = 0$), we have the political preference function

$$\xi\,\Omega_t^y + (1-\xi)\,\Omega_t^o = (1-\xi)(r_t + (1-\delta)p_t)k_t + \xi\left[(w_t - p_t k_{t+1}) + \rho\left((r_{t+1} + (1-\delta)p_{t+1})\,k_{t+1}\right)\right]$$

Setting $\xi \in \{0, 0.5, 1\}$, we obtain the maximand for the unsophisticated planner in the political equilibria, shown in Table 1. We obtain the maximand for the sophisticated planner by using the asset price equation, $p_t = \rho\,(r_{t+1} + (1-\delta)p_{t+1})$, resulting other entries in Table 1.

B.3 Calibration of Adjustment Costs

For gross investment I, our estimate of marginal adjustment cost is ϕI. Shapiro (1986) estimates marginal adjustment costs of gross investment using quarterly data, and Hall (2004) estimates marginal adjustment costs of net investment using annual data. These studies use the functional forms for marginal costs:

$$\begin{aligned} \text{Shapiro } I \text{ (1986)} &: \phi_S^I \; Y \; I \\ \text{Shapiro } II \text{ (1986)} &: \phi_S^{II} \; Y \; \left(\frac{I}{\Delta K}\right) \\ \text{Hall (2004)} &: \phi_H \; \left(\frac{I^{net}}{K}\right) \end{aligned}$$

where K is capital, Y equals after tax income, $\Delta = 1 - \delta$ equals the depreciation factor, and I^{net} is net investment. We use Shapiro's estimates to calibrate our adjustment cost parameter ϕ. His point estimates are $\phi_S^I = 0.0014$ and $\phi_S^{II} = 0.25$. Hall's median estimate is $\phi_H = 0.15$ and his high value for certain industries is $\phi_H = 1$.

We begin by reviewing Hall's derivation of the relation between the parameter estimates in his and in Shapiro's formulations (Hall 2001, appendix C, http://web.stanford.edu/~rehall/SMCA-AER-Dec-2001.pdf). With quarterly data, $\Delta \approx 1$ and $I \approx I^{net}$. Using these approximations, Hall ignores the distinction between net and gross investment, and the depreciation factor. Denote as $\phi_H^{\text{[Shapiro I]}}$ the parameter that, with Hall's functional form, gives the same marginal adjustment cost as Shapiro (I). That is, $\phi_H^{\text{[Shapiro I]}} \left(\frac{I}{K}\right) \equiv \phi_S^I \; Y \; I$ or

$$\phi_H^{\text{[Shapiro I]}} \equiv \phi_S^I \; YK.$$

Similarly, $\phi_H^{\text{[Shapiro II]}}$ is the parameter that, with Hall's functional form, gives the same marginal adjustment cost as Shapiro (II): $\phi_H^{\text{[Shapiro II]}} \left(\frac{I}{K}\right) \equiv \phi_S^{II} \; Y \; \left(\frac{I}{K}\right)$, or

$$\phi_H^{\text{[Shapiro II]}} \equiv \phi_S^{II} \; Y.$$

Shapiro uses $Y = 33 \; bn$ and $K = 203 \; bn$. Using these values in the above identities, we have

$$\phi_H^{[\text{Shapiro I}]} \equiv \phi_S^I \; Y \; K = 0.0014 * 33 * 203 = 9.24 \text{ and}$$
$$\phi_H^{[\text{Shapiro II}]} \equiv \phi_S^{II} \; Y = 0.25 * 33 = 8.25.$$

To compare these converted estimates to Hall's, we divide the former by 4 (because Shapiro uses quarterly and Hall uses annual data) to obtain 2. 3 and 2. 1. These values are over twice as large as Hall's largest estimate, and 15 times as large as Hall's median estimate of 0.15. Thus, Hall's estimate of adjustment cost are much smaller than Shapiro's.

The units of both adjustment costs and investment are $\left[\frac{\text{dollars}}{\text{time}}\right]$, so marginal adjustment cost is unit-free. With quarterly data, the units of $\frac{I}{K}$ are $\left[\frac{\frac{\$}{\text{quarter}}}{\$}\right] = \left[\frac{1}{\text{quarter}}\right]$. Because marginal adjustment costs are unit-free, the units of $\phi_H^{[\text{Shapiro I}]}$ and $\phi_H^{[\text{Shapiro II}]}$ are [quarters]. Our unit of time is $35 \times 4 =$ 140 quarters, so we need to divide $\phi_H^{[\text{Shapiro i}]}$ by 140; we also need to divide by K to account for difference between the Hall/Shapiro formulations and ours. For example, if quarterly investment is I^{quarter}, investment over a 35 year period is $I^{35 \text{ year}} = 140 I^{\text{quarter}}$. In order that our estimate of marginal adjustment costs is comparable to Shapiro's, we choose our adjustment cost parameter ϕ by setting marginal cost in our formulation equal to marginal cost in Hall's conversion of Shapiro's formulation:

$$\phi^{35 \text{ years}} \times 140 \times I^{\text{quarter}} = \phi_H^{\text{Shapiro i}} \times I^{\text{quarter}} \times \frac{1}{K}$$
$$\Rightarrow \phi^{35 \text{ years}} = \phi_H^{\text{Shapiro i}} \times \frac{1}{K \times 140}$$

Using our estimate $K_0 = K = 200$ (see Section 5.3) and the two values of $\phi_H^{\text{Shapiro i}}$ we have

$$\phi^{35 \; years} = \frac{1}{140} \frac{9.24}{200} = 0.00033 \text{ and}$$
$$\phi^{35 \; years} = \frac{1}{140} \frac{8.24}{200} = 0.000295 \qquad .$$

We set our benchmark to $\phi = 0.0003$.

A simple calculation shows that at plausible levels of investment, our baseline estimate of adjustment costs, expressed as a percent of income, is much less than Shapiro's estimates, but much larger than Hall's. If investment is 20% of output, and given a 35-year output of $\$T$ 2200, our baseline estimate of adjustment costs equals $0.0003 \frac{(0.2 \times 2200)^2}{2}$. Adjustment cost as a percent of *income* is $0.0003 \frac{(0.2 \times 2200)^2}{2 \times 2200} 100\% = 1.32\%$.

Shapiro's first estimate of adjustment cost is $\frac{\phi_S^I \ Y \ I^2}{2}$, so his estimate of adjustment cost as a percent of income is $\frac{\phi_S^I \ Y \ I^2}{2Y}100$. If investment is 20% of income ($= Y = 33$ for his data), his estimate of adjustment cost as a percent of income is $0.0014\frac{(.2\times 33)^2}{2}100 = 3.05$, over twice our baseline estimate. Shapiro's two estimates of adjustment costs are equal if $\phi_S^I \ Y \ I = \phi_S^{II} \ Y \ \left(\frac{I}{\Delta K}\right)$. Using his value for capital, $K = 203$, equality of his estimates of adjustment costs requires $0.0014 = 0.25\frac{1}{203\Delta}$, or $\Delta = 0.88$. A larger value of Δ implies that Shapiro's second estimate of adjustment cost, as a percent of income, is smaller than his first estimate. An annual depreciation rate of 6% implies that $\Delta = 0.9898$. Using this value, $\phi_S^{II} = 0.25$, $I = 0.2Y$, $Y = 33$, and $K = 203$, Shapiro's second estimate of adjustment cost as a percent of income, is

$$\begin{aligned}\phi_S^{II} \ \left(\frac{I^2}{2\Delta K}\right) 100\% &= 0.25\left(\frac{.04Y^2}{2(0.9898)K}\right)100\% \\ &= 0.25\left(\frac{.04(33)^2}{2(0.9898)203}\right)100\% = 2.71\%,\end{aligned}$$

over twice the percentage implied by our baseline. These estimates are similar to Mumtaz and Zanetti's (2015); using a structural model, they estimate total adjustment costs to equal 3.3% of output.

Hall's median estimate implies negligible adjustment costs. His largest estimate implies roughly the same magnitude of adjustment costs as our baseline, but only if we set depreciation to zero. Using the definition of net investment, $I^{net} + \delta K = I$, Hall's estimate of adjustment cost, as a percent of income, is

$$\frac{\phi_H \frac{\left(I^{net}\right)^2}{2K}}{Y}100 = \frac{\phi_H \ \frac{(I-\delta K)^2}{2K}}{Y}100.$$

Evaluated at $I = 0.2Y$ and using $\phi_H = 0.15$, adjustment cost as a percent of income equals

$$\frac{0.15 \ \frac{(0.2Y-\delta K)^2}{2K}}{Y}100 = \frac{0.15 \ \frac{\left(0.2\frac{Y}{K}-\delta\right)^2 K^2}{2K}}{Y}100 = \frac{0.15\left(0.2\frac{Y}{K}-\delta\right)^2}{\frac{Y}{K}}100.$$

At an annual depreciation rate of 6% ($\delta = 0.06$) and using the estimates $Y = 63$ and $K = 200$ (Section 5.3), we obtain the estimate

$$\frac{0.15\left(0.2\frac{63}{200}-0.06\right)^2}{\frac{63}{200}}100\% = 4.286\times 10^{-4}\% \approx 0.$$

Even for $\delta = 0$, the estimate is only 0.189%. Thus, Hall's median estimate implies that adjustment costs are negligible. Using his largest estimate (1) and a 6% annual depreciation rate, his estimate of adjustment cost as a percent of income is 2.9×10^{-3}, rising to 1.26 if we set depreciation to 0.

B.4 Numerical Approximation

Multiplicity: The possibility of multiple equilibria arises for two kinds of reasons. First, for a given candidate $M^c(k_{t+1}, e_{t+1})$, there might be multiple solutions to the political economy problem at time t. Because the function $M^c(k_{t+1}, e_{t+1})$ is a polynomial approximation, we cannot rely on curvature properties to guarantee uniqueness; instead, we depend on the numerical algorithm. Our numerical work finds no evidence of this kind of multiplicity. Second, the infinite horizon (required in our stationary setting) generically raises the possibility of non-uniqueness, a standard result in dynamic games where there is an "incomplete transversality condition". However, our algorithm works backwards, beginning with a scrap function to represent the last period; we iterate until convergence, so that the algorithm selects a solution that is close to the "limit equilibrium", as the horizon goes to infinity. We confirm that the converged equilibrium is insensitive to changes in the scrap function.

The political economy equilibria: The solution algorithm is standard. For both the unsophisticated and the sophisticated equilibria, we select a grid of points in state space (k, e) and choose Chebyshev polynomials to interpolate the endogenous functions investment function, $\mu_t = M(k_t, e_t)$, and price function, $p_t = \Psi(k_t, e_t)$, at points off this grid. The solution requires finding coefficients of these polynomials such that the approximations to the endogenous functions satisfy all equilibrium conditions at points on the grid. We begin with a guess of these coefficients to obtain candidate policy and investment functions, $M^c(k_t, e_t)$ and $\Psi^c(k_t, e_t)$ and the associated investment rule, $A^{-1}(\Psi^c(k_t, e_t))$. Using these candidates, we solve the optimization problem (10) (for both the unsophisticated and sophisticated planners) at every point on the grid, thus generating values of p_t and μ_t at every point on the grid. We update the Chebyshev coefficients so that the revised candidates satisfies the equilibrium conditions at points on the grid. We continue this iteration until the estimated coefficients, and thus the endogenous functions, approximately converge.

That is, we approximate $M(k_{t+1}, e_{t+1})$ and $\Psi(k_{t+1}, e_{t+1})$ as polynomials in k_{t+1} and e_{t+1}. We find coefficients of those polynomials so that, on the grid points, the recursion is satisfied:

$$p_t = \Psi(k_t, e_t) = \rho \frac{k_{t+1}^{-\eta} \left[R(k_{t+1}, e_{t+1}) + \Psi(k_{t+1}, e_{t+1})(1-\delta)\right]^{1-\eta}}{\left[W(k_t, e_t) - \Psi(k_t, e_t) k_{t+1}\right]^{-\eta}}. \tag{21}$$

In addition, the maximization of the joint welfare

$$\max_{\mu_t} (1-\xi) \frac{\left[(r_t + (1-\delta)p_t)k_t\right]^{1-\eta} - 1}{1-\eta} + \tag{22}$$
$$\xi \quad \left\{ \frac{\left[w_t - p_t k_{t+1}\right]^{1-\eta} - 1}{1-\eta} + \rho \frac{\left[r_{t+1} + (1-\delta)p_{t+1}k_{t+1}\right]^{1-\eta} - 1}{1-\eta} \right\}$$

subject to

$$\begin{aligned} e_{t+1} &= (1-\epsilon)e_t + \zeta(1-\mu_t) F(k_t, l). \\ k_{t+1} &= (1-\delta)k_t + A^{-1}(p_t). \end{aligned} \tag{23}$$

with k_t, e_t given and $A^{-1}(.)$ the inverse function of $A(.)$, equals $M(k_t, e_t)$ on the grid points.

Equation 21 is an implicit expression in $\Psi(k_t, e_t)$. For the specific values of η used in our simulations, equation 21 can be reformulated to give explicit expressions of $\Psi(k_t, e_t)$. For $\eta = 0$, we have $p_t = \Psi(k_t, e_t) = \rho \ [R(k_{t+1}, e_{t+1}) + \Psi(k_{t+1}, e_{t+1})(1-\delta)]$ and, for $\eta = 2$,

$$\Psi(k_t, e_t) = \frac{W(k_t, e_t)}{k_{t+1}} + \frac{c_{t+1}^o}{2\ \rho} \pm \frac{\sqrt{c_{t+1}^o \left\{k_{t+1} c_{t+1}^o + 4\rho W(k_t, e_t)\right\}}}{2\ \rho\ \sqrt{k_{t+1}}}, \tag{24}$$

with $c_{t+1}^o = [R(k_{t+1}, e_{t+1}) + \Psi(k_{t+1}, e_{t+1})(1-\delta)]$. Of the two solutions for $\eta = 2$, only one is consistent with positive investment levels.

In the "unsophisticated" equilibrium, policy-makers take p_t and p_{t+1} as given. In the "sophisticated" equilibrium policy-makers understand the process under which asset prices are formed. We replace $\Psi(k_t, e_t)$ in objective (22) with its explicit expression (for $\eta = 2$ the right-hand side of (24)).

For both equilibria we use 36-degree Chebyshev polynomials evaluated at 6x6 Chebyshev nodes on the $[200, 800]$ interval for k and the $[200, 1000]$ interval for e. At each node the recursion defining $\Psi(k_t, e_t)$ is satisfied. For the political economy equilibria we have the optimality condition,

$$\frac{d}{d\mu_t}\left[\xi\frac{\left(W(k_t,e_t)-p_tk_{t+1}\right)^{-\eta}}{1-\eta}W(k_t,e_t)+(1-\xi)\,\frac{\left[(R(k_t,e_t)+(1-\delta)p_t)k_t\right]^{1-\eta}-1}{1-\eta}\right]=0, \tag{25}$$

the Nash equilibrium condition, $\mu_t = M(k_t, e_t)$, and $p_t = \Psi(k_t, e_t)$ in the "unsophisticated" and p_t equal its "forward-looking" expression (for $\eta = 2$ the right-hand side of (24)) in the "sophisticated" equilibrium. In addition, system (23) must be satisfied. The first order condition (25) takes i_t (and consequently k_{t+1}) as given, implying that when political representatives choose abatement they take current investment as given. This procedure means that current generations do not use the abatement instrument to target investment.

Starting with an initial guess for the coefficients of the approximations of $\Psi(\cdot)$ and $M(\cdot)$, we evaluate the right side of equation 21 for at each node. Using these function values, we obtain new coefficient values for the approximation of $\Psi(\cdot)$. We then use the optimality condition 25 to find the values of μ at the nodes; we use those values to update the coefficients for the approximation of $\Psi(\cdot)$. We repeat this iteration until the coefficients' relative difference between iterations falls below 10^{-6}. See chapter 6 of Miranda and Fackler (2002) for details. Figures 3 and 4, graph the asset and abatement functions, the differences (the "residuals") between the right and left sides of equations 21 and 25, and the %-change in the coefficients of the approximated function between iterations. Residuals equal 0 at the nodes because we set both the degree of the polynomial and the number of nodes equal to n. We choose $n = 36$ to ensure that residuals are at least 4 orders of magnitudes below the solution values on the approximation interval. In certain simulations it proved numerically easier to iterate on the investment rather than the asset price function with the former a linear transformation of the latter: $p_t = A(i_t)$.

The DU: For the DU's problem, we approximate $J(k_{t+1}, e_{t+1})$, $I(k_{t+1}, e_{t+1})$ and $M(k_{t+1}, e_{t+1})$ as polynomials in k_{t+1} and e_{t+1}, and find coefficients of those polynomials so that the solution to the maximization of welfare

$$\max_{\iota_t,\mu_t}\frac{\left[y_t - A(i_t)i_t\right]^{1-\eta}-1}{1-\eta}+\rho\ J(k_{t+1},e_{t+1}) \tag{26}$$

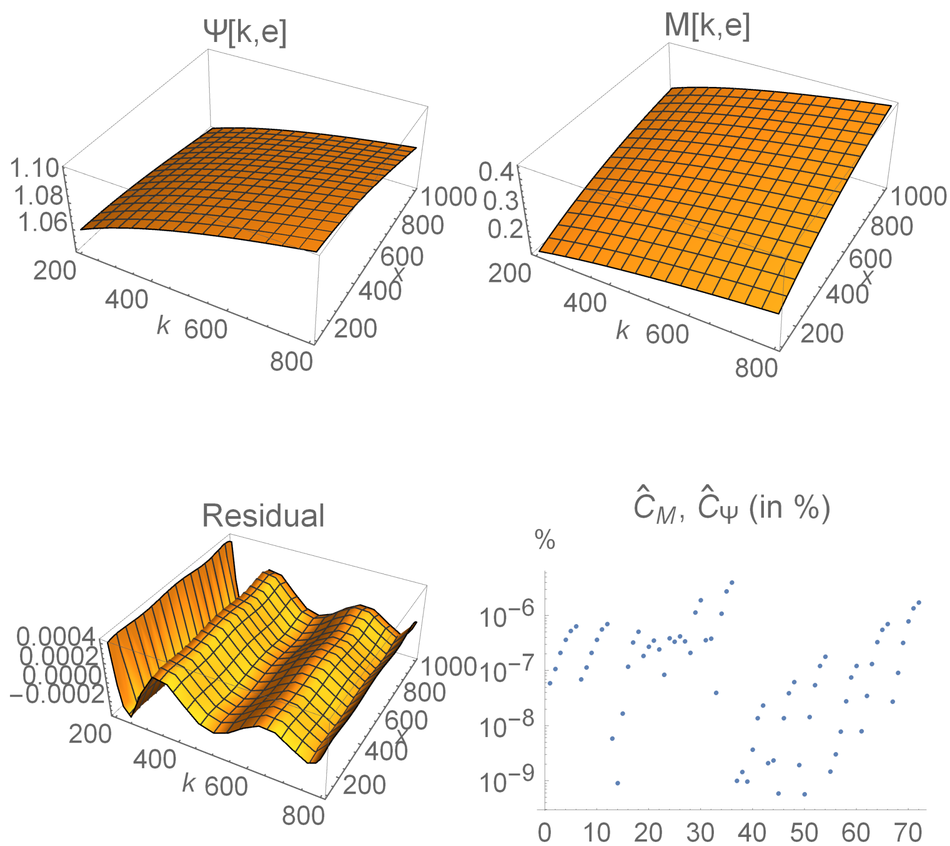

Figure 3: Benchmark "unsophisticated" equilibrium solution: *upper* Asset price, $\Psi(\cdot)$, and policy, $M(\cdot)$, function; *lower* deviation from true asset value outside of approximation nodes ("Residuals) and %-change of coefficients between iterations ("$\hat{C}$")

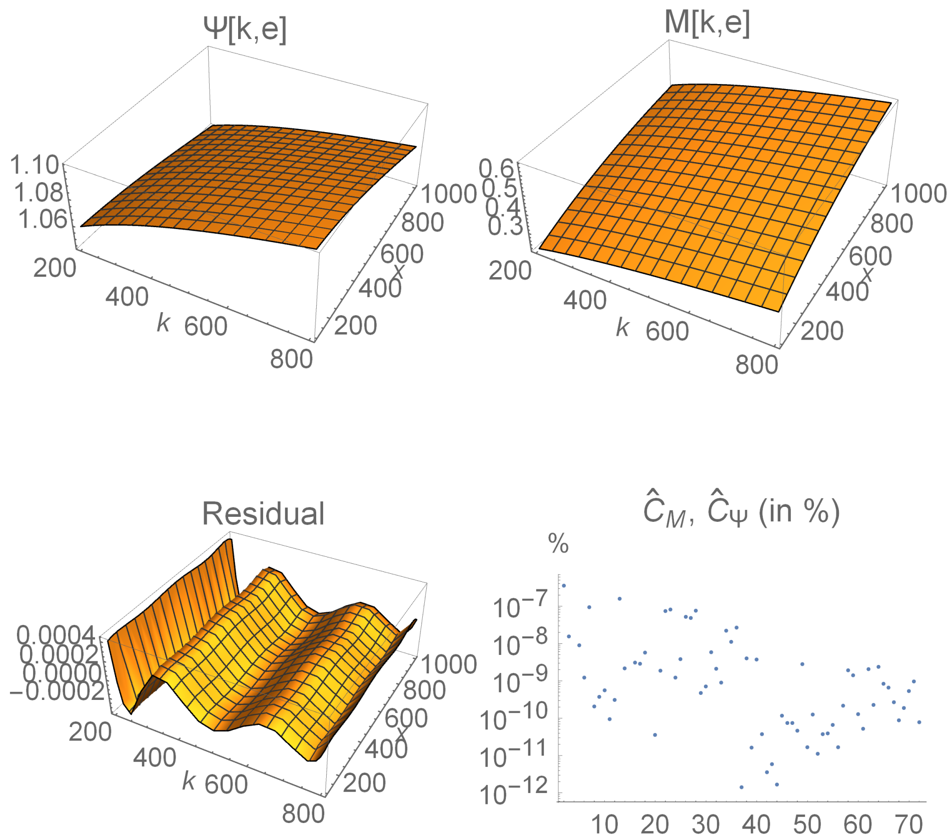

Figure 4: Benchmark "sophisticated" equilibrium solution: *upper* Asset price, $\Psi(\cdot)$, and policy, $M(\cdot)$, function; *lower* approximation from true asset value outside of approximation nodes ("Residuals") and %-change of coefficients between iterations ("$\hat{C}$")

subject to

$$\begin{aligned} e_{t+1} &= (1-\epsilon)e_t + \zeta\left(1-\mu_t\right)F\left(k_t, l\right). \\ k_{t+1} &= (1-\delta)k_t + i_t. \end{aligned} \tag{27}$$

with k_t, e_t given, approximately equals $I\left(k_t, e_t\right)$ and $M\left(k_t, e_t\right)$. At each node, the value function recursion is

$$J\left(k_t, e_t\right) = \frac{\left[y_t - A(i_t)i_t\right]^{1-\eta} - 1}{1-\eta} + \rho\ J\left(k_{t+1}, e_{t+1}\right). \tag{28}$$

the DU's optimality conditions are

$$\begin{aligned} \tfrac{d}{d\iota_t}\left[\tfrac{\left[y_t - A(i_t)i_t\right]^{1-\eta}-1}{1-\eta} + \rho\ J\left(k_{t+1}, e_{t+1}\right)\right] = 0 \\ \tfrac{d}{d\mu_t}\left[\tfrac{\left[y_t - A(i_t)i_t\right]^{1-\eta}-1}{1-\eta} + \rho\ J\left(k_{t+1}, e_{t+1}\right)\right] = 0. \end{aligned} \tag{29}$$

For the DU, we use 72-degree Chebyshev polynomials evaluated at 12x6 Chebyshev nodes on the $[200, 1000]$ interval for k and $[100, 1000]$ interval for e. Starting with an initial guess for the coefficients of the approximations of $J\left(\cdot\right)$, $I\left(\cdot\right)$ and $M\left(\cdot\right)$, we evaluate the right side of equation 28 for at each node. Using these function values, we obtain new coefficient values for the approximation of $J\left(\cdot\right)$. We then use the optimality conditions 29 to find the values of i and μ at the nodes; we use those values to update the coefficients for the approximation of $J\left(\cdot\right)$. We repeat this iteration until the coefficients' relative difference between iterations falls below 10^{-6}. Figure 5 graphs the value, investment, and abatement functions, the differences (the "residuals") between the right and left sides of equations 21 and 25, and the %-change in the coefficients of the approximated function between iterations. Residuals equal 0 at the nodes because we set both the degree of the polynomial and the number of nodes equal to n. We choose $n = 72$ to ensure that residuals are at least 5 orders of magnitudes below the solution values on the approximation interval.

The approximation of the political economy equilibria requires a dynamic programming approach. The DU problem, however, can also be solved using optimal control. As a consistency check, we also solving the social planning problem using the optimization software GAMS and found that both approaches give the same results, accounting for the finite horizon of the numerical optimal control approach.

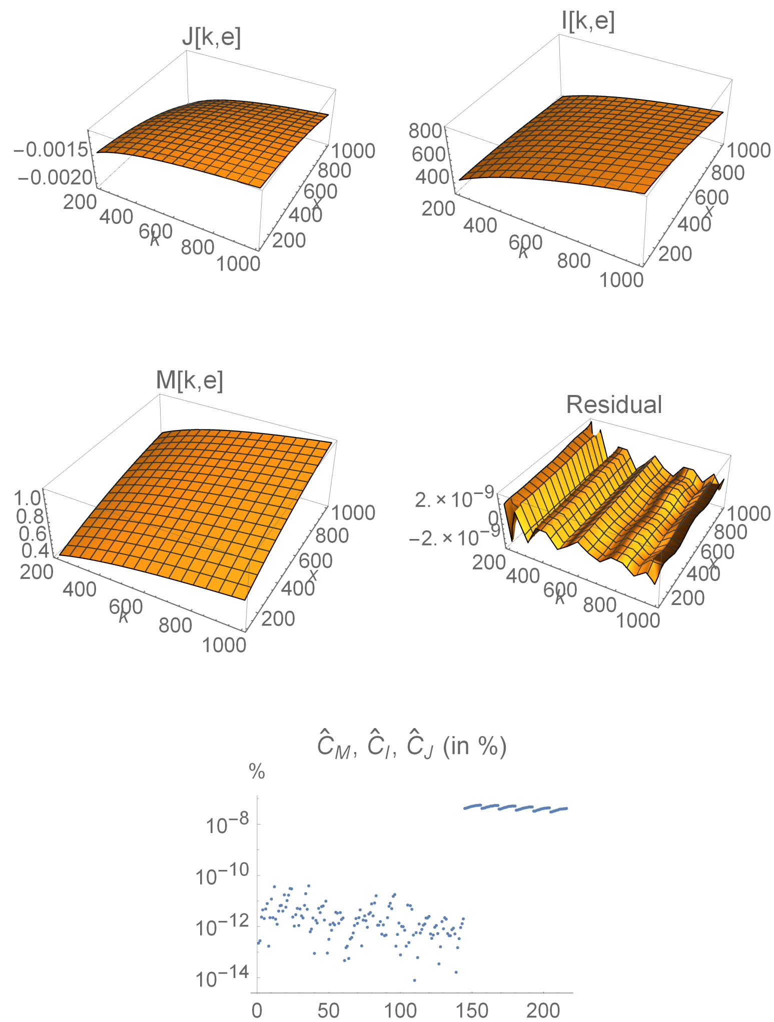

Figure 5: SP solution: *upper* Value, $J(\cdot)$, and investment function, $I(\cdot)$; *middle* mitigation policy function, $M(\cdot)$, and deviation from true value function value outside of approximation nodes ("Residuals"); *lower* %-change of coefficients between iterations ("$\hat{C}$")