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# Absorption of Near Fields Generated by a Two-Dimensional Array of Dipoles Above a Hyperbolic Metamaterial

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***Abstract* – We investigate the power emitted by a two-dimensional array of impressed dipoles above a hyperbolic metamaterial (HM). Transmission line formalism of TM/TE spectral plane waves is used to model the scattering from the HM substrate. We show that the HM enhances the amount of emitted power which is mostly absorbed by the HM.**

## I. INTRODUCTION

Hyperbolic metamaterials (HMs) are a subcategory of artificial uniaxial anisotropic materials named after the hyperbolic iso-frequency wave-vector dispersion diagrams of the waves supported therein. HMs allow for redistribution of power in a (relatively) wide spatial spectrum compared to common dielectrics, leading to novel phenomena such as enhancing the power scattered by nanoparticles [1] or emitted power by imposed dipoles [1]-[3] located on HM surfaces. Interestingly, this power is mostly directed to HM where it can be absorbed. Other HM applications include focusing with extreme subwavelength resolution [4]-[5], absorption control [6]-[8], enhancement of spontaneous emission and engineering of decay rate [9]-[12]. HMs can be fabricated using metal-dielectric multilayers or metallic pillars embedded in dielectric substrates at infrared and optical frequencies. The emergence of hyperbolic dispersion in multilayered HMs does not rely on any resonant behavior and thus occurs in a very wide frequency band. In this study, we investigate the enhancement of the power radiated by a two-dimensional (2D) array of impressed dipoles above a HM substrate, motivated by earlier work in which small scatterers on top of HMs (which can be modeled using single dipole approximation) or roughness on HM surfaces are shown to realize unprecedented absorption of plane waves [7]-[8].

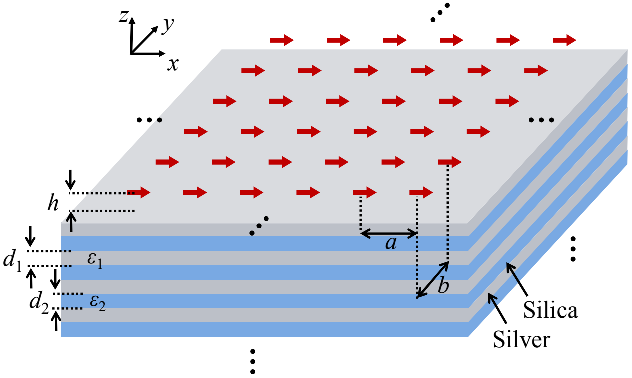

Fig. 1. Illustration of an infinitely 2D periodic array of dipolar emitters, at a distance *h* from the HM substrate composed of alternating silver and silica layers.

## II. FORMULATION EMPLOYING TRANSMISSION LINE FORMALISM FOR TE AND TM WAVES

We utilize the transmission line formalism for TE and TM plane wave spectra (based on [13], adapted to periodic structures) for modeling the power emitted by a 2D periodic array of impressed dipoles located above an infinitely extended HM. Assume that the 2D periodic array of dipoles as in Fig. 1 is above the HM substrate at a distance *h*, and each dipole is located at position $\mathbf{r}_{mn} = \mathbf{r}_{00} + ma\hat{\mathbf{x}} + nb\hat{\mathbf{y}}$ ($m, n = 0, \pm1, \pm2, \ldots$), where $a$ and $b$ are the periods along the *x* and *y* directions, respectively. Being in a periodic arrangement, a dipole at $\mathbf{r}_{mn}$ has a dipole moment $\mathbf{p}_{mn} = \mathbf{p}_{00} \exp\left[i\mathbf{k}_t \cdot (\mathbf{r}_{mn} - \mathbf{r}_{00})\right]$ where $\mathbf{k}_t = k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}}$ is the wavevector defining the progressive phasing of the dipoles and $\mathbf{r}_{00} = x_{00}\hat{\mathbf{x}} + y_{00}\hat{\mathbf{y}} + z_{00}\hat{\mathbf{z}}$ is a reference location. We are interested in the power emitted in +/– $z$ direction, in the following denoted by superscripts "up" and "down", respectively. As an initial study we, here, assume $\mathbf{p}_{00} = p_x\hat{\mathbf{x}} + p_y\hat{\mathbf{y}}$, but the formulation can be easily extended to any $\mathbf{p}_{00}$, not given here for brevity. The power directed up/down per unit cell area is given as a sum of TM and TE contributions as

$$\begin{aligned} P^{\text{up/down}} = & \frac{\omega^2}{2ab} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{\left|\mathbf{p}_{00} \cdot \mathbf{k}_{t,pq}\right|^2}{k_{t,pq}^2} \frac{\operatorname{Re}\left(Y_{pq}^{\text{TM,up/down}}(z_{00})\right)}{\left|Y_{pq}^{\text{TM,up}}(z_{00}) + Y_{pq}^{\text{TM,down}}(z_{00})\right|^2} \\ & + \frac{\omega^2}{2ab} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{\left|\mathbf{p}_{00} \cdot \left(\mathbf{k}_{t,pq} \times \hat{\mathbf{z}}\right)\right|^2}{k_{t,pq}^2} \frac{\operatorname{Re}\left(Y_{pq}^{\text{TE,up/down}}(z_{00})\right)}{\left|Y_{pq}^{\text{TE,up}}(z_{00}) + Y_{pq}^{\text{TE,down}}(z_{00})\right|^2} \end{aligned} . \quad (1)$$

Here $Y_{pq}^{\text{TM/TE,up/down}}$ represents the wave admittance seen by TE and TM waves toward +/– $z$ directions (up and down) referred to $z_{00}$. The formulation accounts for the power in a plane wave spectrum of waves with the transverse wavevector $\mathbf{k}_{t,pq} = \mathbf{k}_t + (2\pi p / a)\hat{\mathbf{x}} + (2\pi q / b)\hat{\mathbf{y}}$. The wave admittances toward –$z$ direction can be straightforwardly evaluated using transfer matrix method and the Bloch impedance of the unit cell of HM [1]. Wave propagation in HM impacts dramatically the power splitting toward up and down. HMs with uniaxial anisotropy in permittivity host an unusually wide spatial spectrum of TM waves (extraordinary). In the case studied in Fig. 1, TM waves with high $pq$ indices can propagate inside the HM. Such waves with $\left|\mathbf{k}_{t,pq}\right| > k_0$ are normally evanescent (assuming the upper half-space to be vacuum and $k_0$ is the wavenumber therein). Note that when $a, b < \lambda_0 / 2$ only the $p = q = 0$ Floquet wave, i.e., the fundamental harmonic, propagates and all the higher harmonics decay exponentially from the array plane. High-$p,q$ waves in + $z$ direction are represented by purely imaginary $Y_{pq\neq 00}^{\text{TM/TE,up}}$ [ $\operatorname{Re}\left(Y_{pq\neq 00}^{\text{TM/TE,up}}\right) = 0$ ], on the other hand the waves propagating in –$z$ direction will evanescently couple to the HM substrate where numerous high-$p,q$ Floquet waves propagate with mainly real $Y_{pq}^{\text{TM/TE,down}}$. Therefore, the HM will substantially impact the amount of power emitted by the array, and how the emitted power is distributed. We deal here with localized dipolar sources, although realistic sources will have a finite spectral distribution that can be still taken into account (not reported). Note however that the dipolar source approximation is accurate in general because the power spectrum coupled to HM is mainly limited by the evanescent decay along the distance $h$ between the dipoles and the substrate.

## III. ILLUSTRATIVE EXAMPLES

The power in (1) shows strong dependence on many parameters: the periods $a$ and $b$, the distance $h$, the phasing of the dipoles $\mathbf{k}_t$, and the direction of the dipoles $\mathbf{p}_{mn}$. In the following, we utilize a setup as in Fig. 1, with $d_1 = d_2 = 10$ nm. The dipoles are assumed to be in free space, polarized along the $x$ direction, and $\mathbf{k}_t = 0.5k_0\hat{\mathbf{x}}$. We then report in Figs. 2 and 3 the plot of the ratios $P^{\text{tot}} / P^{\text{free space}}$ and $P^{\text{down}} / P^{\text{up}}$, varying period ($a$ and $b$) and varying the distance $h$, respectively. Note that $P^{\text{tot}} = P^{\text{up}} + P^{\text{down}}$ and $P^{\text{free space}}$ is the total power emitted by the same array of dipoles in free space. The period of the array plays a significant role in the emitted power and its distribution in the +/– $z$ directions, as shown in Fig. 2. When the period is increased both $P^{\text{tot}} / P^{\text{free space}}$ and $P^{\text{down}} / P^{\text{up}}$ are enhanced. This is in good agreement with the results in [1] drawn for a single dipole. Moreover a sharp peak appears for $a = b = 300$ nm case, which is attributed to the coupling to a surface mode. In the same case, we observe higher order harmonics (appearing as the frequency increases) radiating into free space (not shown here), whose impact is manifested by the sharp decrease in $P^{\text{down}} / P^{\text{up}}$ at around 665 THz. Note that most of the power (with a ratio of around 20) is still directed toward the HM. Also $h$ dramatically affects the power distribution. For example, when $h$ is increased from 5 to 25 nm as reported in Fig. 3, the two ratios above degrade significantly, confirming the importance of evanescent coupling of the field from the dipoles to the HM, in agreement with the results drawn in [1] for a single dipole.

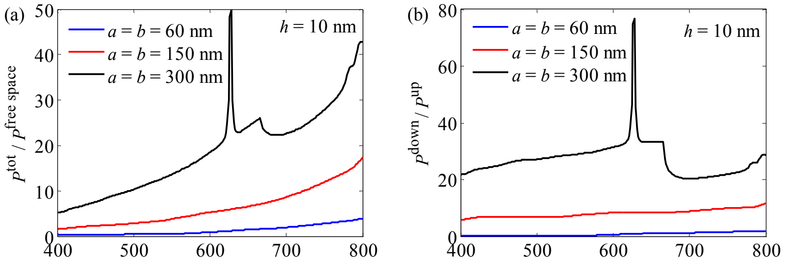

Fig. 2. Ratios (a) $P^{\text{tot}} / P^{\text{free space}}$ and (b) $P^{\text{down}} / P^{\text{up}}$ varying $a$ and $b$, for constant $h = 10$ nm .

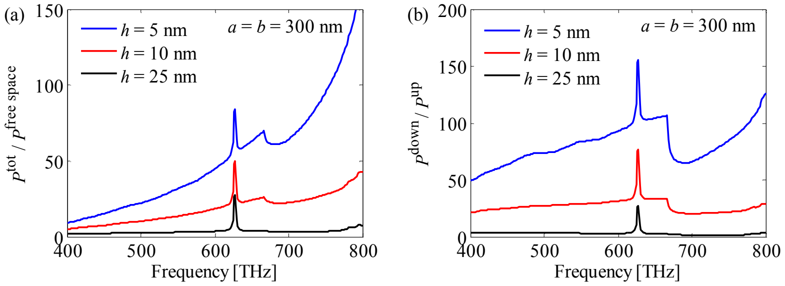

Fig. 3. As in Fig. 2, by varying $h$, for constant periods $a = b = 300$ nm .

## VI. CONCLUSION

We show novel results pertaining to power distribution from a periodic arrays of emitters located above a HM, and how physical parameters significantly impact on the enhancement of $P^{\text{tot}} / P^{\text{free space}}$ and $P^{\text{down}} / P^{\text{up}}$ . The model proposed here allows for engineering the distribution of power and absorption of near fields.

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