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Los Angeles

**Longitudinally Coherent Single-Spike Radiation
from a Self-Amplified Spontaneous Emission
Free-Electron Laser**

A dissertation submitted in partial satisfaction
of the requirements for the degree
Doctor of Philosophy in Physics

by

Gabriel Andrew Marcus

2012

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ABSTRACT OF THE DISSERTATION

**Longitudinally Coherent Single-Spike Radiation
from a Self-Amplified Spontaneous Emission
Free-Electron Laser**

by

Gabriel Andrew Marcus

Doctor of Philosophy in Physics

University of California, Los Angeles, 2012

Professor James B. Rosenzweig, Chair

This work studies the production and measurement of longitudinally coherent, ultrashort pulses of light from a self-amplified spontaneous emission free-electron laser (SASE FEL) by using an energy-chirped electron beam in conjunction with a tapered undulator. This scheme effectively preserves the FEL gain only where an appropriate undulator taper compensates for the detuning experienced by an amplifying radiation spike as it slips forward in the electron beam rest frame. The simultaneous time and frequency-domain measurement of ultrashort pulses of light generated in this manner were made with an advanced transient-grating frequency-resolved optical gating (TG FROG) diagnostic, which has the potential to push ultrashort light pulse measurement at FEL facilities to shorter wavelength regimes.

The theoretical framework presented in this dissertation has two components. The FEL theory presented here includes an analysis of the coupled wave and Vlasov equations, which are linearized in the one-dimensional case, and are solved in the frequency domain by the Laplace transform technique. The exponential

gain regime for SASE FEL light is explored in detail to clearly identify concepts that are relevant to the energy-chirp and undulator tapering experiment. Some of these concepts are illustrated with fully three-dimensional, time-dependent numerical particle simulations using the FEL code GENESIS for the supportive case of ultrashort, low-charge electron beams. In addition, nonlinear optics, the foundation upon which all FROG diagnostics are built, is briefly explored using two complementary perspectives as they apply to the TG FROG geometry.

The experimental section describes in detail the first direct time-domain measurements of a single coherent radiation spike from a SASE FEL amplifier employing the energy-chirped electron beam and tapered undulator technique at the SPARC FEL test facility in Frascati, Italy. Electron beams were accelerated and compressed using the velocity bunching technique, which leaves a residual energy-chirp in the longitudinal phase space. The energy-chirp was compensated by appropriately tapering individual undulator sections. This process was optimized at a resonant wavelength of $\lambda = 530$ nm. The ultrashort light pulses that were generated had a temporal full-width at half-maximum of $\delta\tau_{FWHM}^R \sim 98$ fs and a time-bandwidth product of $TBP \sim 1.2$, indicating that the Fourier limit was nearly achieved. This experiment provides further insight into methods that can be used to shape the SASE FEL longitudinal profile and enhance coherence properties. In addition, the measurements were taken with an advanced, and relatively simple, TG FROG diagnostic that can potentially be used to measure ultrashort UV pulses at FEL facilities.

The dissertation of Gabriel Andrew Marcus is approved.

Michael McNitt-Gray

Jianwei (John) Miao

Pietro Musumeci

James B. Rosenzweig, Committee Chair

University of California, Los Angeles

2012

To my family

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CHAPTER 1

Introduction

Increasing the understanding of, and ultimately the control over, matter at the level of atoms, electrons, and spins has been one of the great challenges presented to the scientific community. Concurrently, the increase in knowledge in this regard should serve the entire gamut of scientific disciplines ranging from advanced materials to the life sciences. Reaching these goals requires new tools that will enable different communities to probe deeper into regions of higher spatial, temporal, and energy resolution. These tools, presumably, will allow for the investigation of matter at the time and length scales of atomic and electronic motion. The time and length scales of interest, however, span many orders of magnitude; from the micron (10^{-6} m) to the Ångström (10^{-10} m) length scales and from the micro- to the femto- (10^{-15}), or even the attosecond, (10^{-18} s) time scales. To this end, electron beam (e-beam) accelerator-based light sources have proven to be an invaluable tool for the production of radiation possessing many of the above characteristics. In particular, free-electron lasers, or FELs, are a source of coherent and tunable radiation and are capable of producing extremely short pulses from the far infrared down to the x-ray wavelength regime[1, 2]. FELs operate on the principle that a coherent electromagnetic wave can be emitted and amplified many orders of magnitude by the collective beam-radiation instability that microbunches a relativistic e-beam as it propagates through a periodic static magnetic undulator. First proposed in 1971 by Madey[3], this fourth-generation

light source has pushed the spatial and temporal boundaries of ultrashort light pulses to the Ångstrom and femtosecond frontier and was permitted by the congruent technological advances that have been made in the accelerator community.

The recent presence and further development of advanced fourth generation light sources has emboldened the scientific community at large to push for the production of radiation that possesses ever more complex spatial and temporal characteristics. One recurring theme that has been reverberating around the various light source user communities is the desire for the production of longitudinally coherent, ultrashort x-ray pulses. Self-amplified spontaneous emission, or SASE, is the method and operating mode by which FELs currently reach the x-ray wavelength regime. SASE FELs are only partially coherent, however, as the emitted light is the result of the amplification of initial incoherent radiation shot noise. Thus, the temporal structure of the light pulse is a collection of coherent spikes randomly distributed over the e-beam's longitudinal profile. Improving the coherence of SASE FEL light pulses has been the subject of much study[4], with emphasis placed on producing one longitudinally coherent radiation spike [5, 6, 7].

One promising method to increase the radiation's longitudinal coherence relies on electron beam phase-space manipulations that preferentially concentrate the gain along a small longitudinal portion of the e-beam. In essence, the process uses e-beam phase-space manipulations to control the FEL output phase-space characteristics. Thus, it seems appropriate to measure, or monitor, the phase-space of the FEL light directly. Observation of the development of longitudinal coherence in a SASE FEL is not possible in the x-ray wavelength range, however, because of the lack of appropriate diagnostics. Thus one must examine, as is often the case in FEL research, the physics of this scheme at longer wavelengths. In

this regard, the frequency-resolved optical gating[8] (FROG) diagnostic and pulse reconstruction technique has proven to be effective in analyzing ultra-fast light sources in the optical and near-infrared wavelength regions. FROG measures the spectrogram of the light, a quantity that is intimately related to the longitudinal phase-space.

The subject matter of this dissertation deals with the experimental results that were obtained at the SPARC (Sorgente Pulsata Auto-amplificata di Radiazione Coerente) FEL test facility in Frascati, Italy. In particular, the first time-domain measurements of near single-spike radiation produced from a single pass SASE FEL amplifier operating under advanced e-beam phase-space manipulation techniques mentioned above are reported, by utilizing a relatively new transient-grating (TG) FROG diagnostic. This particular TG FROG diagnostic geometry was chosen and constructed while keeping in mind the specific difficulties encountered while attempting to measure FEL output. Furthermore, there is a need to advance robust longitudinal pulse measurement techniques into ever shorter wavelength regimes while minimizing diagnostic complexities. The current iteration of the TG FROG implemented for this work emphasizes both of these points for the first time on an FEL. The results obtained from the experiment are compared with fully three-dimensional time dependent numerical simulations obtained from the FEL code GENESIS[9] and show excellent overall agreement.

1.1 Outline of Dissertation

The organization of this thesis is as follows. Chapter 1 continues with a basic introduction to the various aspects of this work. It proceeds with an explanation of the fundamentals of longitudinal phase-space optics and the need to explore this

pulse description modality in order to fully understand and appreciate advanced ultrashort measurement techniques. The chapter continues with an introduction to FROG and how this particular measurement technique relies on phase-space measurements to longitudinally extract the temporal and spectral information of ultrashort pulses simultaneously. It concludes with a brief introduction to the basic operating principles of FELs and the development of ultrashort, high power pulses from this particular linear accelerator based light source. Chapter 2 continues with the investigation into FEL theory in order to more fully understand and appreciate the subtleties involved in the FEL collective instability. This chapter also introduces concepts and e-beam characteristics that can effect the FEL gain process. These concepts are vital to understanding the motivation in using e-beam phase-space manipulation techniques to shape the FEL radiation phase-space. Chapter 3 describes in detail the TG FROG geometry used in the experiment. In particular, the theory behind the diagnostic operating principles (nonlinear optics) are briefly explored to more intimately understand the subtleties in modern ultrafast pulse measurement techniques. The description of the experiment is presented in detail in Chapter 4. This lays the foundation for the experimental results and analysis which are presented in Chapter 5.

1.2 Principles of Phase-Space Optics

Ultrafast laser sources have become ubiquitous in labs around the world as the rapid development of ultrafast technology over the last decade has paved the way for the production of optical pulses in both the femto- and attosecond time scales[10]. These pulses have proven to be invaluable to the scientific community as a whole because they can be utilized as tools for investigating various chemical and physical processes that operate at temporal scales of atomic and molecular

motion[11, 12, 13]. The need for new and advanced measurement techniques for pulses with these timescales, therefore, is as compelling as ever.

In principle, it should be possible to simply use an antenna to directly measure the oscillating electric field. The fastest antennas, however, are far too slow to resolve the oscillations of optical fields, where a pulse in the visible spectrum would have a period less than three femtoseconds. Therefore, in the optical regime, the fields are measured by square-law detectors, such as photodiodes. These detectors only respond to the intensity of the field and, at best, have a temporal resolution on the order of about a picosecond (in the case of a state-of-the-art streak camera). Not only are these detectors unable to resolve sub-picosecond pulses, they fail to obtain any information regarding the phase of the field. The phase of the field, as will be seen in subsequent sections and chapters, carries a wealth of information regarding the pulse under investigation and is oftentimes considered more valuable than the pulse amplitude.

Phase-space descriptions of optical pulses, not unlike phase-space descriptions of relativistic particle beams, provide a convenient and thorough framework for visualizing and developing an intuition about how these pulses propagate through and interact with matter. In fact, it is possible to qualitatively determine complex features of these ultrashort pulses, such as the time of arrival of various spectral components of the pulse, by simply examining particular features of a given phase-space trace. In addition, as shown in section 1.3, it is possible to extract both the amplitude *and* phase of ultrafast pulses from the phase-space through iterative phase-retrieval algorithms, allowing for a complete longitudinal description of the pulse under investigation. In this section, the phase-space picture of ultrafast pulses is developed with a particular emphasis placed on measurement methods that are relatively simple and are based on square-law detectors.

1.2.1 Ultrashort Laser Pulses

The fundamental quantity of interest that describes an individual pulse of light is the *real* electric field vector $\vec{\varepsilon}(\vec{r}, t)$, which is a function of both time (t) and space ($\vec{r}$). The magnetic field does not effect the propagation of light through matter in all but the most intense pulses and can be estimated directly from the electric field. Thus, the function of any ultrafast diagnostic should be to completely characterize the electric field. This can be extremely difficult because the field can potentially have a complicated spatial dependence. For the time being, the spatial dependence of the field is neglected since the longitudinal (or temporal) structure of the pulse is of more interest. The current problem is further simplified by making the *scalar approximation* where the electric field's vectoral nature is neglected. Simultaneous measurement of two orthogonal polarizations can be combined to yield the full pulse field if necessary.

With these approximations, the electric field, which is a *real* signal, has the following temporal dependence

$$\varepsilon(t) = \sqrt{I(t)} \cos(\phi_t(t)). \quad (1.1)$$

Here, $A(t) = \sqrt{I(t)}$ and $\phi_t(t)$ are the time dependent amplitude and temporal phase of the field. The field in the frequency domain is found by applying the Fourier transform, which yields

$$\tilde{\varepsilon}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \varepsilon(t) e^{-i\omega t} dt. \quad (1.2)$$

For a real signal $\varepsilon(t)$, the frequency domain representation satisfies $\tilde{\varepsilon}(-\omega) = \tilde{\varepsilon}^*(\omega)$ and thus the energy density spectrum, or simply the spectrum, $|\tilde{\varepsilon}(\omega)|^2$, is symmetric about the origin. This description becomes problematic for a variety

of reasons. For instance, the average frequency

$$\langle \omega \rangle = \int_{-\infty}^{\infty} \omega |\tilde{\varepsilon}(\omega)|^2 d\omega \quad (1.3)$$

is always zero. Also, the spread in frequencies σ_ω , where

$$\sigma_\omega^2 = \int_{-\infty}^{\infty} (\omega - \langle \omega \rangle)^2 |\tilde{\varepsilon}(\omega)|^2 d\omega, \quad (1.4)$$

will be roughly the distance between the positive and negative components of the spectrum. There is also the question of uniqueness. An infinite number of amplitudes and phases exist that can be combined to yield the real signal $\varepsilon(t)$ given by equation 1.1. It is beneficial, therefore, to define a complex field that unambiguously specifies these quantities [14]. To this end, the *analytic signal* approximation[15] is made by defining the new quantity

$$E(t) = \sqrt{I(t)} e^{-i\phi_t(t)} = \frac{1}{\sqrt{2\pi}} \int_0^\infty \tilde{\varepsilon}(\omega) e^{i\omega t} d\omega \quad (1.5)$$

where $I(t) = |E(t)|^2$ is the signal intensity. The field in the frequency domain can be found in this approximation to be:

$$\begin{aligned} \tilde{E}(\omega) &= \sqrt{S(\omega)} e^{i\phi_\omega(\omega)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(t) e^{-i\omega t} dt \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_0^\infty \tilde{\varepsilon}(\omega') e^{i\omega' t} d\omega' e^{i\omega t} dt \\ &= \frac{1}{2\pi} \int_0^\infty \int_{-\infty}^{\infty} e^{i(\omega - \omega')t} dt \tilde{\varepsilon}(\omega') d\omega' \\ &= \int_0^\infty \delta(\omega - \omega') \tilde{\varepsilon}(\omega') d\omega' \\ &= \begin{cases} \tilde{\varepsilon}(\omega) & \omega > 0 \\ 0 & \omega < 0 \end{cases} \end{aligned} \quad (1.6)$$

where $S(\omega)$ is the spectrum and $\phi_\omega(\omega)$ is the spectral phase. The positive components of the spectrum are automatically selected. The real field can be recovered by taking twice the real part of the complex analytic field: $\varepsilon(t) = 2 \times \text{Re}[E(t)]$. This can more clearly be seen by substituting equation 1.2 into equation 1.5:

$$\begin{aligned}
E(t) &= \frac{1}{\sqrt{2\pi}} \int_0^\infty \tilde{\varepsilon}(\omega) e^{i\omega t} d\omega \\
&= \frac{1}{2\pi} \int_0^\infty \int_{-\infty}^\infty \varepsilon(t') e^{-i\omega t'} e^{i\omega t} dt' d\omega \\
&= \frac{1}{2\pi} \int_{-\infty}^\infty \varepsilon(t') \int_0^\infty e^{i\omega(t-t')} d\omega dt' \\
&= \frac{1}{2\pi} \int_{-\infty}^\infty \varepsilon(t') \left[\pi \delta(t-t') + \frac{i}{t-t'} \right] dt' \\
&= \frac{1}{2} \varepsilon(t) + \frac{i}{2\pi} \int_{-\infty}^\infty \frac{\varepsilon(t')}{t-t'} dt'
\end{aligned} \tag{1.7}$$

This result also justifies the representation of the signal field in the first relationship in equation 1.5. Since equation 1.7 produces a field that has a real and imaginary part, it can always be represented in polar form using an amplitude and phase that are *unambiguous*. This allows for a clear definition of the amplitude and phase of the pulse. There is much literature on the analytic signal approximation (for an excellent summary see [16]) and it has been shown to accurately represent the real signal in the case of small bandwidth pulses, which is primarily the subject of this thesis.

The analytic signal approximation also leads to an important quantity known as the *instantaneous frequency*. This quantity is important when discussing exotic pulses that exhibit a time-dependent frequency; a quantity otherwise known as *chirp*. The average frequency, calculated using the analytic signal approximation,

is found to be

$$\begin{aligned}
\langle \omega \rangle &= \int_{-\infty}^{\infty} \omega |\tilde{E}(\omega)|^2 d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \omega E^*(t) E(t') e^{i\omega(t-t')} d\omega dt dt' \\
&= \frac{1}{2\pi i} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E^*(t) E(t') \frac{\partial}{\partial t} e^{i\omega(t-t')} d\omega dt dt' \\
&= \frac{1}{i} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E^*(t) \frac{\partial}{\partial t} \delta(t-t') E(t') dt dt' \\
&= \frac{1}{i} \int_{-\infty}^{\infty} E^*(t) \frac{\partial}{\partial t} E(t) dt
\end{aligned} \tag{1.8}$$

For $E(t)$ given by 1.5, $\langle \omega \rangle$ can be recast in the form:

$$\langle \omega \rangle = \int_{-\infty}^{\infty} \phi'_t(t) |E(t)|^2 dt. \tag{1.9}$$

The average frequency is obtained by integrating the instantaneous frequency with the density over all time. The instantaneous frequency is identified as the time derivative of the temporal phase, $\omega(t) = \phi'_t(t)$. The instantaneous frequency is an intuitive results that is true for complex, rather than real, signals, and lends further credence to the use of the analytic signal approximation.

1.2.2 Historical Measurement Techniques

In order to fully characterize an ultrashort pulse, or any pulse for that matter, it is sufficient to measure the intensity and phase in either the temporal or spectral domains. The only measurements that have been available, until recently, have been either of the spectrum the autocorrelation (field or intensity). These measurement techniques, as will seen shortly, fail to fully characterize the pulse under investigation because they are one-dimensional quantities, i.e. functions of either time or frequency, and fail to measure the pulse phase. It is therefore sufficient to analyze only one measurement modality as the enumerated failures

are analogous across the diagnostic platforms.

The most popular time-domain measurement is perhaps the intensity autocorrelation, which will be discussed in a slightly different context in chapter 3. The schematic for this type of measurement is shown in figure 1.1. The pulse

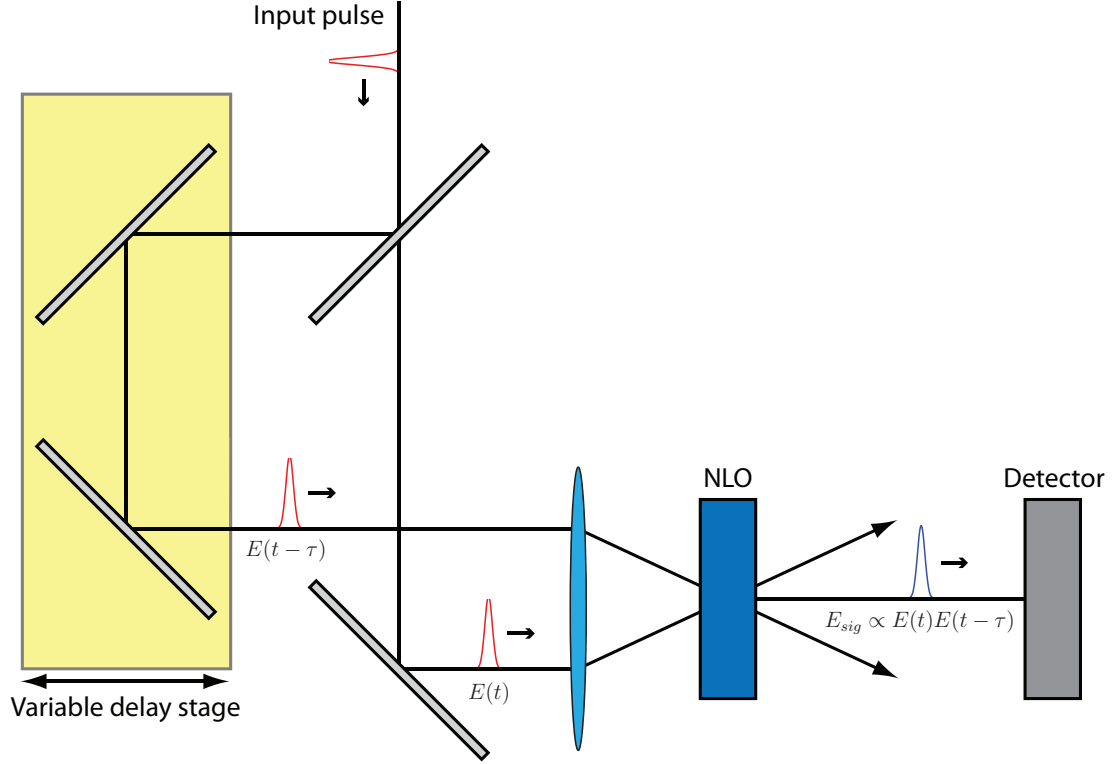


Figure 1.1: Experimental layout of a basic intensity autocorrelator using a second harmonic generation nonlinear optical process.

to be measured is split in two by a beam splitter. The path length difference between the two pulses is varied by translating a delay stage along one arm of the diagnostic. The two pulses are then recombined and interfered in a second order nonlinear optical medium, which produces a signal field at twice the input frequency. This process is referred to as second harmonic generation (SHG)[17]. A more complete description of various nonlinear optical processes are discussed

in more detail in section 3.3. This field is proportional to the amplitudes of the two pulses, and is given by

$$E_{sig}(t, \tau) \propto E(t)E(t - \tau). \quad (1.10)$$

Square-law detectors measure the signal intensity. Furthermore, these detectors measure the time integral of the signal intensity because they lack very fine temporal resolution:

$$A(\tau) = \int_{-\infty}^{\infty} I(t)I(t - \tau)dt. \quad (1.11)$$

A scan of the delay over the entire initial pulse longitudinal profile yields the full autocorrelation signal.

There are a few qualitative features of the pulse that can be extracted from the autocorrelation. The width of the intensity autocorrelation can be related to the width of the pulse intensity profile. For a gaussian pulse, as in figure 1.2 (a), the width of the intensity autocorrelation in figure 1.2 (b) is $\sqrt{2}$ times longer than the width of the pulse intensity. This numerical factor, sometimes referred to as a *deconvolution factor*, is dependent on the pulse shape. For a pulse with a hyperbolic secant squared (*sech*²) time profile, the deconvolution factor is ~ 1.54 . To obtain accurate pulse width information from an intensity autocorrelation, therefore, it is necessary to know the pulse shape *a priori*. This information is not available for most ultrafast optical pulses, especially those that are generated using exotic processes. This is certainly the case for FEL pulses, as will be seen in chapter 2. In fact, as the pulse under consideration becomes more complicated, more information is lost by the intensity autocorrelation. This can clearly be seen in figure 1.2 (c) and (d). These plots show the intensity vs. time and intensity autocorrelation vs. time of a gaussian pulse with third order spectral phase. The

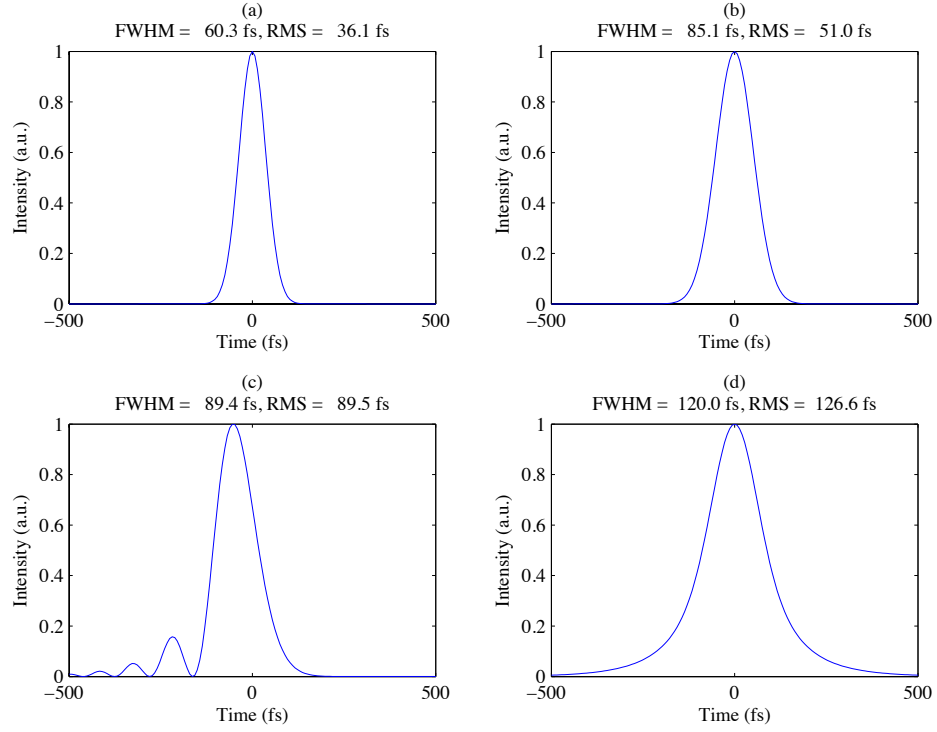


Figure 1.2: Examples of theoretical pulse intensities and their intensity autocorrelations. Intensity vs. time and intensity autocorrelation vs. time for a Fourier limited gaussian pulse, (a) and (b), and for a gaussian pulse with third order spectral phase, (c) and (d). The autocorrelation in (d) misses the details of the pulse with third order spectral phase in (c). As a result, the two very different pulses in (a) and (c) have very similar autocorrelations.

intensity autocorrelation clearly misses the details in the leading edge of the pulse and is quite similar to the intensity autocorrelation of a gaussian pulse.

The attribute that renders the intensity autocorrelation ineffective the most is that the phase information is lost. This can more clearly be seen by taking the Fourier transform of 1.11:

$$A(\omega) = \mathcal{F}\{A(\tau)\} = |\tilde{I}(\omega)|^2. \quad (1.12)$$

Therefore, if the intensity autocorrelation of a pulse is measured, the magnitude, but not the phase, of the Fourier transform of the quantity of interest, $I(t)$, is found. What remains to be seen, however, is if the phase can somehow be retrieved from the intensity autocorrelation.

1.2.3 One-dimensional Phase Retrieval

Most ultrafast diagnostics that measure pulse quantities and are a function of a single variable (t or ω) return either the magnitude squared of the quantity or the magnitude squared of its Fourier transform. This was seen above for the intensity autocorrelation measurement. There exists additional information, however, that can be used in an attempt to constrain the measurement further. Most pulses, for example, have a finite duration; the amplitude goes to zero outside a given finite time interval. The pulse is then said to have *finite support* in the temporal domain. An unresolved issue is whether additional information can be used to extract the pulse phase from these ultrafast measurements. This type of problem is dubbed *the one-dimensional phase-retrieval problem*. It has been shown that in almost all cases it is impossible to extract the phase of the pulse, regardless of the nature of the additional information.

The one-dimensional phase retrieval problem is illuminated by considering a discrete set of data (numbers), $\{x_1, x_2, \dots, x_N\}$, that is a function of a single variable, for example, time. Consider, now, a diagnostic (spectrometer) that measures the absolute value of the Fourier transform of this set, $\{|X_1|, |X_2|, \dots, |X_N|\}$. These two data sets are related via the discrete Fourier transform:

$$X_k = \sum_{n=1}^N x_n e^{-i2\pi \frac{k}{N}n} \quad (1.13)$$

where n is the time variable. Writing $y = e^{-i2\pi \frac{k}{N}}$, the expression becomes:

$$X_k = \sum_{n=1}^N x_n y^n. \quad (1.14)$$

This expression is simply a polynomial in the variable y with coefficients given by x_n . The Fundamental Theorem of Algebra states that, in so many words, every non-zero single-variable polynomial with complex coefficients has exactly as many complex roots as its degree, if each root is counted up to its multiplicity. Therefore, any single-variable polynomial can be factored into its roots. X_k then takes the form:

$$X_k = a(y - y_1)(y - y_2) \dots (y - y_N) \quad (1.15)$$

where y_i are the zeros of the polynomial in equation 1.14 and a is a coefficient of factorization. The diagnostic only measures the magnitude of this quantity,

$$|X_k| = |a(y - y_1)(y - y_2) \dots (y - y_N)|, \quad (1.16)$$

and the failure of phase-retrieval in one-dimension is immediately apparent. The measured quantity, $|X_k|$, remains unchanged under the complex conjugation of any of the factors $(y - y_i)$. There are many functions of n that can produce the measured value $|X_k|$, and therefore the set $\{x_1, x_2, \dots, x_N\}$ cannot be uniquely recovered. It turns out that the Fundamental Theorem of Algebra *fails* for polynomials of two variables. A two-dimensional picture of ultrafast pulses is therefore needed if full pulse reconstruction is to be obtained using square-law based detectors.

1.2.4 Phase-Space Descriptions

The complimentary nature of the description of ultrashort laser pulses in both the time and frequency space suggests that these pulses can be well described by joint time-frequency distributions in the two-dimensional (t, ω) phase-space. The distributions are central to the characterization of ultrashort pulses because, as stated previously, the waveform is not currently directly measurable by today's diagnostics. The phase-space description also provides an excellent framework for describing optical pulses that are measured by standard square-law detectors.

There exist many examples of these joint phase-space distributions[18], the most common of which is the Wigner distribution:

$$\begin{aligned} W(t, \omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} E^*(t - \frac{\tau}{2}) E(t + \frac{\tau}{2}) e^{-i\tau\omega} d\tau \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{E}^*(\omega + \frac{\theta}{2}) \tilde{E}(\omega - \frac{\theta}{2}) e^{-it\theta} d\theta. \end{aligned} \quad (1.17)$$

The Wigner distribution is considered to be part of a class of *bilinear* distributions because the signal enters twice in its formulation. There are many features that make this distribution attractive for representing short optical pulses. The Wigner distribution is always real, even if the signal is complex. This can be verified by taking the complex conjugate of equation 1.17:

$$\begin{aligned} W^*(t, \omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} E(t - \frac{\tau}{2}) E^*(t + \frac{\tau}{2}) e^{i\tau\omega} d\tau \\ &= -\frac{1}{2\pi} \int_{\infty}^{-\infty} E(t + \frac{\tau}{2}) E^*(t - \frac{\tau}{2}) e^{-i\tau\omega} d\tau \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} E(t + \frac{\tau}{2}) E^*(t - \frac{\tau}{2}) e^{-i\tau\omega} d\tau \\ &= W(t, \omega) \end{aligned} \quad (1.18)$$

The projection of the Wigner distribution onto one coordinate axis by integration over the conjugate coordinate yields either the pulse temporal or spectral intensity and is known as a *marginal*. The time marginal is found by integrating over the frequency

$$\begin{aligned}
I(t) &= \int_{-\infty}^{\infty} W(t, \omega) d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E^*(t - \frac{\tau}{2}) E(t + \frac{\tau}{2}) e^{-i\tau\omega} d\tau d\omega \\
&= \int_{-\infty}^{\infty} E^*(t - \frac{\tau}{2}) E(t + \frac{\tau}{2}) \delta(\tau) d\tau \\
&= |E(t)|^2.
\end{aligned} \tag{1.19}$$

Similarly, the frequency marginal is found by integrating the distribution over the time coordinate

$$\begin{aligned}
S(\omega) &= \int_{-\infty}^{\infty} W(t, \omega) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{E}^*(\omega + \frac{\theta}{2}) \tilde{E}(\omega - \frac{\theta}{2}) e^{-it\theta} d\theta dt \\
&= \int_{-\infty}^{\infty} \tilde{E}^*(\omega + \frac{\theta}{2}) \tilde{E}(\omega - \frac{\theta}{2}) \delta(\theta) d\theta \\
&= |\tilde{E}(\omega)|^2.
\end{aligned} \tag{1.20}$$

The Wigner function also encompasses the intuitive result that the instantaneous frequency is the temporal derivative of the temporal phase, which can clearly be seen by evaluating

$$\Omega(t) = \langle \omega \rangle = \frac{\int_{-\infty}^{\infty} \omega W(t, \omega) d\omega}{\int_{-\infty}^{\infty} W(t, \omega) d\omega} = \phi'_t(t). \tag{1.21}$$

The Wigner distribution also has some less intuitive, and often detrimental, features. The largest issue arising from the Wigner distribution description is that it *isnot* positive definite. This can be seen in figure 1.3 (b) and (d). Figure 1.3 (b) shows the plot of two identical Fourier limited gaussian pulses separated in time. The central region of the plot, which alternates between positive (red) and

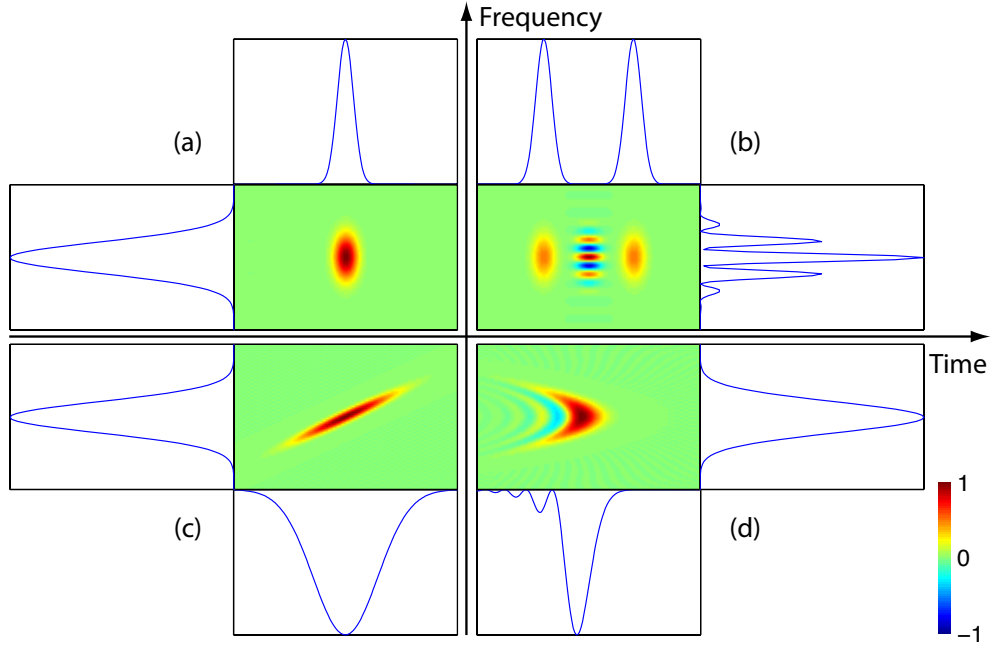


Figure 1.3: Wigner distribution (normalized to unity) of: (a) a Fourier limited gaussian pulse, (b) two identical Fourier limited gaussian pulses separated in time, (c) a gaussian pulse with second order spectral phase, (d) a gaussian pulse with third order spectral phase. The pulse marginals are also shown for each distribution.

negative (blue) values of the distribution, indicates interference between the two pulses. The spectrum, or frequency marginal, is still entirely positive, however, because the positive regions of the distribution corresponding to the single pulses cancel out the negative values from the interference region. A more detailed picture of this phenomenon is found in figure 1.4. Similar features can be seen in 1.3 (d) where a third order spectral phase causes the distribution to contain negative regions.

It turns out that the Wigner function is only one distribution belonging to a class of bilinear distributions that can be obtained from the general *Cohen class*:

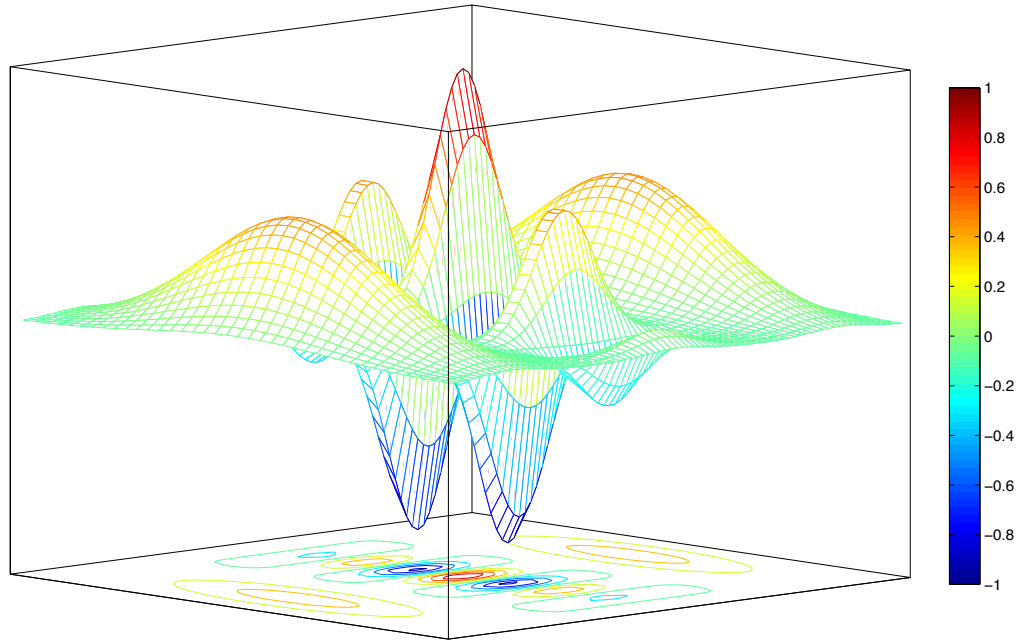


Figure 1.4: Wigner distribution of two identical Fourier limited gaussian pulses separated in time clearly exhibiting the negative components of the distribution. The contour plot is a projection onto the (t, ω) plane and is identical to figure 1.3 (b).

$$C(t, \omega) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E^*(u - \frac{\tau}{2}) E(u + \frac{\tau}{2}) K(\theta, \tau) e^{-i\theta t - i\tau\omega + i\theta u} du d\tau d\theta, \quad (1.22)$$

where $K(\theta, \tau)$ is a two dimensional function called the kernel[18]. The Wigner distribution, is recovered when $K(\theta, \tau) = 1$. As mentioned previously, square-law based detectors measure the intensity or energy of the field, which gives rise to a photocurrent or charge, making the measurement inherently positive. It would be beneficial, therefore, to find a phase-space distribution that is positive definite and that reflects the measured values. If the kernel, K , in equation 1.22 is taken to be the ambiguity function of an ancillary signal, g ,

$$A(\theta, \tau) = \int_{-\infty}^{\infty} g^*(t - \frac{\tau}{2}) g(t + \frac{\tau}{2}) e^{i\theta t} dt \quad (1.23)$$

with the exception that $K(\theta, \tau) = A(-\theta, \tau)$, the following distribution is obtained:

$$\begin{aligned} C(t, \omega) &= \frac{1}{4\pi^2} \int \int \int E^*(u - \frac{\tau}{2}) E(u + \frac{\tau}{2}) \int g^*(t' - \frac{\tau}{2}) g(t' + \frac{\tau}{2}) e^{-i\theta t'} dt' \\ &\quad \times e^{-i\theta t - i\tau\omega + i\theta u} du d\tau d\theta \\ &= \frac{1}{2\pi} \int \int \int E^*(u - \frac{\tau}{2}) E(u + \frac{\tau}{2}) g^*(t' - \frac{\tau}{2}) g(t' + \frac{\tau}{2}) e^{-i\tau\omega} \\ &\quad \times \delta(u - t - t') du d\tau dt' \\ &= \frac{1}{2\pi} \int \int E^*\left(u - \frac{\tau}{2}\right) E(u + \frac{\tau}{2}) g^*(u - t - \frac{\tau}{2}) g(u - t + \frac{\tau}{2}) e^{-i\tau\omega} du d\tau. \end{aligned} \quad (1.24)$$

Here, the limits of integration are understood to be $(-\infty, \infty)$. Making the substitutions $x' = u - \frac{\tau}{2}$, $x = u + \frac{\tau}{2} \Rightarrow u = \frac{1}{2}(x + x')$, $\tau = x - x'$ with the knowledge

that the Jacobian determinant,

$$J = \begin{vmatrix} \frac{\partial u}{\partial x'} & \frac{\partial u}{\partial x} \\ \frac{\partial \tau}{\partial x'} & \frac{\partial \tau}{\partial x} \end{vmatrix} = 1, \quad (1.25)$$

relates the differential area element $du d\tau = |J| dx dx'$, the expression for $C(t, \omega)$ can be recast as:

$$\begin{aligned} C(t, \omega) &= \frac{1}{2\pi} \int \int E^*(x') g^*(x' - t) E(x) g(x - t) e^{-i(x-x')\omega} dx dx' \\ &= \left| \frac{1}{\sqrt{2\pi}} \int E(x) g(x - t) e^{-ix\omega} dx \right|^2 = S(t, \omega). \end{aligned} \quad (1.26)$$

$S(t, \omega)$ is known as the *Gabor spectrogram*, or simply the spectrogram. The spectrogram can be interpreted as the spectrum of the field, E , after gating by the function g . A different spectrum is obtained for each delay, t , between the gate and the field. The totality of these spectra is the time-frequency distribution, $S(t, \omega)$. It is represented as a function of the frequency, ω , and the time between the gate and pulse fields, t .

The spectrogram in equation 1.26 is extremely useful as a two-dimensional phase-space distribution for a variety of reasons. It is in a form that can be measured by standard square-law detectors and is inherently positive. Some qualitative features of the pulse can often be extracted from the distribution. Figure 1.5 shows the spectrograms for different Gaussian pulses that have been gated by a separate Gaussian pulse. The spectrogram on the right clearly indicates a pulse with a positive chirp while the spectrogram in the center is clearly the distribution of a double pulse structure. These qualitative features, however, are more difficult to distinguish as the test pulse and the gate pulse become more complex.

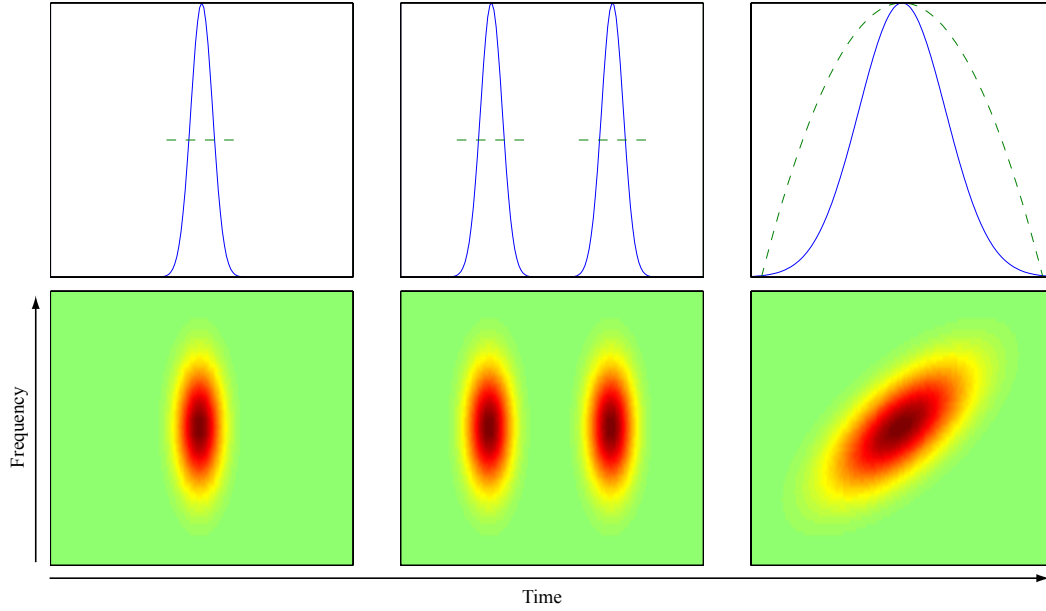


Figure 1.5: Different Gaussian pulses (top) and their Spectrograms (bottom) for a Gaussian gate pulse. There is a notable difference between the Gaussian pulse with chirp (right) and the Gaussian pulse with a flat spectral phase (left). Notice that the spectrogram does not have the interference region between the two pulses that was seen in the Wigner distribution of a similar pulse (center).

There are also some less desirable features of the spectrogram. The marginals, for instance, do not yield the pulse intensity or spectrum. The temporal marginal yields the convolution of the intensity of the pulse, E , with the intensity of the gate, g :

$$\begin{aligned}
 \int_{-\infty}^{\infty} S(t, \omega) d\omega &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E^*(\tau') g^*(\tau' - t) E(\tau) g(\tau - t) e^{-i(\tau - \tau')\omega} d\tau d\tau' d\omega \\
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E^*(\tau') g^*(\tau' - t) E(\tau) g(\tau - t) \delta(\tau - \tau') d\tau d\tau' \\
 &= \int_{-\infty}^{\infty} |E(\tau)|^2 |g(\tau - t)|^2 d\tau.
 \end{aligned} \tag{1.27}$$

A similar expression exists for the frequency marginal:

$$\int_{-\infty}^{\infty} S(t, \omega) dt = \int_{-\infty}^{\infty} |\tilde{E}(\omega')|^2 |\tilde{E}(\omega' - \omega)|^2 d\omega'. \quad (1.28)$$

Equation 1.26 can also be recast in the form[19]:

$$S(t, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} W_E(u, \Omega) W_g(u - t, \omega - \Omega) du d\Omega. \quad (1.29)$$

The spectrogram in this representation is a convolution of the Wigner function of the pulse with the Wigner function of the gate. The Wigner function of the pulse is smoothed out by the Wigner function of the gate. The spectrogram, therefore, is a smoothed version of the Wigner distribution of the pulse that is positive definite. Most measurement strategies that determine a particular phase-space distribution that ensures positivity can often be recast in this form. All of these measurements, therefore, are intimately related to the Wigner distribution.

1.3 Principles of Frequency-Resolved Optical Gating

A physical interpretation of the spectrogram can be determined by examining equation 1.26 more closely and with dependent variables that more accurately describe the temporal nature of a light pulse's longitudinal profile:

$$\begin{aligned} S(\tau, \omega) &= \left| \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(t) g(t - \tau) e^{-i\omega t} dt \right|^2 \\ &= \left| \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E_{sig}(t, \tau) e^{-i\omega t} dt \right|^2. \end{aligned} \quad (1.30)$$

It is clear that the spectrogram is simply the magnitude squared of the Fourier transform of a signal field that takes the form $E_{sig}(t, \tau) = E(t)g(t - \tau)$. For this

reason, the spectrogram is often referred to as the short-time Fourier transform, because the gate, g , presumably emphasizes the field, E , for a short time interval around the time $t = \tau$, and suppresses it at all other times. Determining the spectrum of the signal field is the mathematical equivalent of taking its Fourier transform. This is the operating principle behind frequency-resolved optical gating, which was first proposed by Trebino, *et al.*[8]. A FROG diagnostic measures the signal *spectrum vs. delay* as opposed to the signal energy vs. delay. Measuring the signal spectrum for all delays, by scanning the gate pulse over the unknown pulse longitudinal profile, results in the two-dimensional spectrogram, otherwise known as a FROG trace.

1.3.1 Choosing the Gate Pulse

The expression for the signal field in the mathematical representation of the spectrogram contains the field under investigation, $E(t)$, as well as a gate function, $g(t)$. Naively, one might assume that the best pulse resolution, and thus the most information, would be obtained if an infinitely short gate pulse was available. Substituting $g(t - \tau) = \delta(t - \tau)$ into equation 1.30 yields:

$$\begin{aligned} S(\tau, \omega) &= \left| \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(t) \delta(t - \tau) e^{-i\omega t} dt \right|^2 \\ &= |E(\tau) e^{-i\omega \tau}|^2 \\ &= I(\tau). \end{aligned} \tag{1.31}$$

Here, the spectrogram reduces exactly to the pulse intensity. While it may be useful to extract the pulse intensity with infinite temporal resolution, all phase vs. time information, as well as all frequency information, is lost. This result is intimately related to the uncertainty principle, which relates the allowable

temporal and spectral resolutions for any measurement. The spectrogram is useful because it provides temporal and spectral information *simultaneously*, and the gate pulse width determines the shape of the phase-space distribution. It turns out that for a purely frequency modulated signal, the ideal width of the gate pulse, given by T_g , is[14]

$$T_g \sim \frac{1}{2|\phi_t''(t)|}. \quad (1.32)$$

Therefore, a gate pulse that is too short loses all phase information, while a gate pulse that is too long can't resolve the fastest phase variations in a pulse. The above relation stated in equation 1.32 is the compromise. For general pulse measurement, however, a gate pulse that has a width less than or equal to the width of the pulse under investigation is sufficient[20]. To this end, autocorrelation provides an excellent framework for pulse measurement using the FROG formalism.

A FROG diagnostic makes an autocorrelation-type measurement in which the autocorrelation signal field is spectrally resolved. The autocorrelation signal field is generated by interfering a pulse with its own copy within a nonlinear optical medium (SHG intensity autocorrelation is shown in figure 1.1). The gate pulse in the expression for the spectrogram is therefore a function of the pulse which is being measured, and depends on which type of nonlinear optical process is used to generate the autocorrelation signal field. It has been seen in equation 1.10 that an SHG autocorrelation yields a pulse with the appropriate form for the spectrogram signal field: $E_{sig}(t, \tau) = E(t)E(t - \tau)$. Here, the gate pulse is equal to the pulse under investigation. An experimental layout of a SHG FROG diagnostic can be found in figure 1.6. The spectrum of the signal field can be measured by a simple spectrometer, making sure that the signal field spectrum

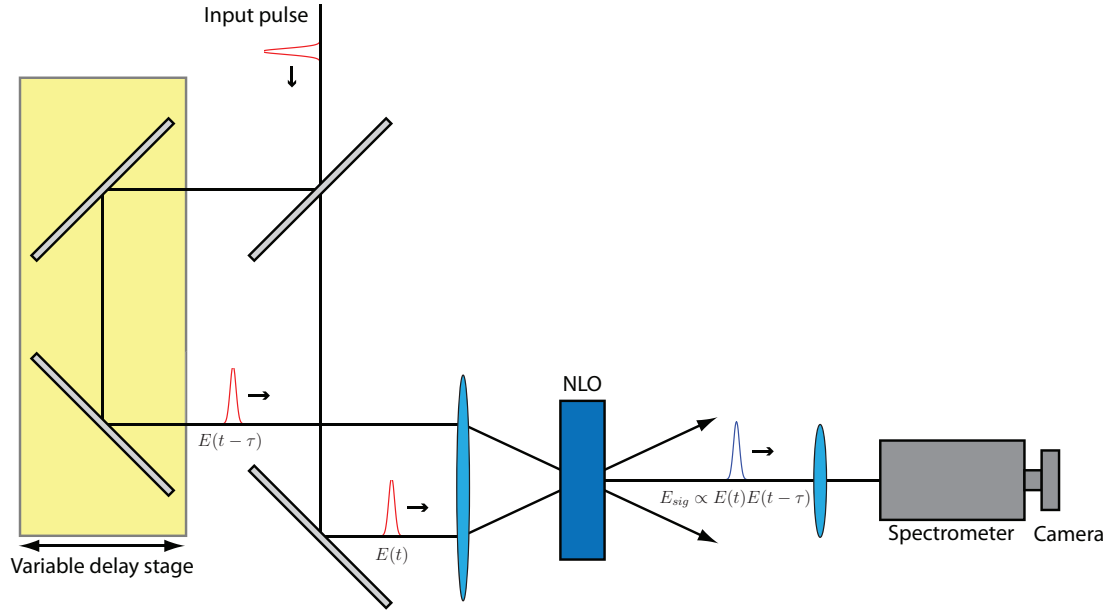


Figure 1.6: General experimental layout of a second harmonic generation FROG diagnostic.

is entirely contained within the diagnostic operating range. The full (t, ω) phase-space is explored by scanning the delay such that the gate pulse sweeps over the full longitudinal profile of the pulse under investigation. It is interesting to note that gating a complex pulse (a pulse that has a nonzero phase) with another complex pulse, as in the case of SHG FROG, does not restrict the inversion of the trace. It does, however, make the visual interpretation of the trace more difficult as the gate adds nonzero phase to the autocorrelation signal field. Gating with a real function, however, adds no phase information and yields the most intuitive traces. An example of this can be found in figure 1.7.

1.3.2 Phase Retrieval and the FROG Algorithm

Unfortunately, there is no closed-form solution for inverting a FROG trace. It was shown in section 1.2.3, however, that the Fundamental Theorem of Algebra

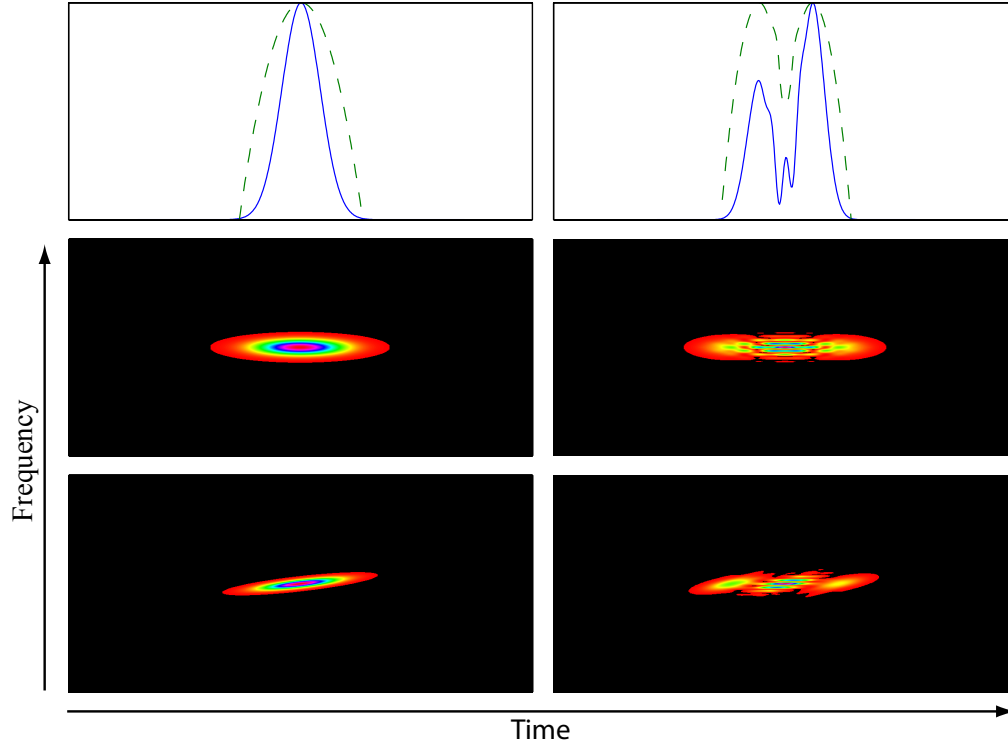


Figure 1.7: FROG traces using an SHG (middle row) and TG or PG (bottom row) nonlinear interaction for pulses of different complexity. The pulse on the left is a simple Gaussian with quadratic spectral phase while the pulse on the right has three distinct subpulses with varying degrees of quadratic phase. One can see that the chirp is better represented on the FROG trace from the bottom row. Also, as the pulse becomes more complex in nature (right column), it becomes more difficult to extract qualitative features from the FROG trace.

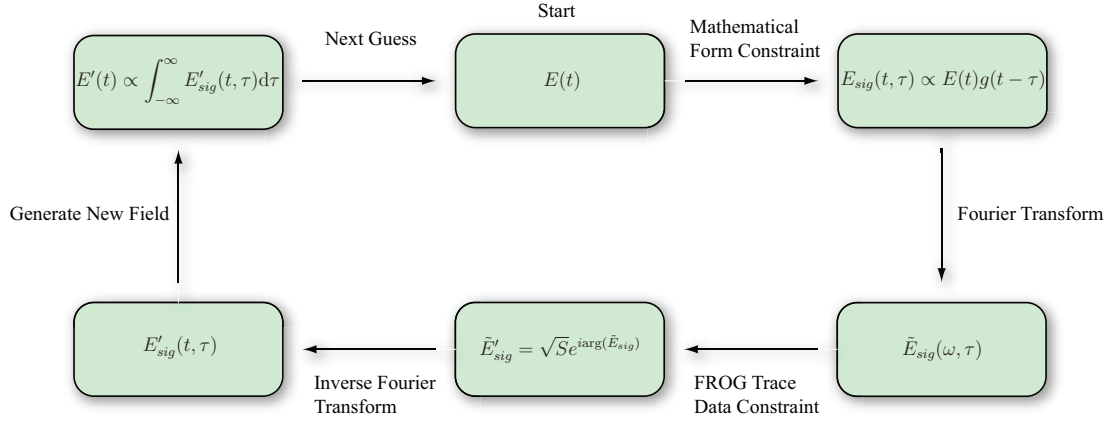


Figure 1.8: Diagram outlining the general algorithm for inverting a FROG trace.

fails for two-dimensional functions. This allows for two-dimensional iterative algorithms to extract phase information, permitting full pulse reconstruction of the field, $E(t)$. The majority of the existing FROG algorithms are based off of modified two-dimensional phase retrieval algorithms. All of these algorithms apply alternating constraints between the time and frequency domains. This is known as making “generalized projections” because the field is being projected onto solution sets that satisfy these constraints. This is schematically shown in figure 1.8. The algorithm proceeds in the following manner: Starting with an initial guess for the field $E(t)$, a signal field is generated according to $E_{sig}(t, \tau) = E(t)g(t - \tau)$, where the gate function, $g(t - \tau)$, takes a form that is dependent on the nonlinear process used in the autocorrelation. This is known as the mathematical form constraint. This field is then Fourier transformed into the frequency domain to obtain $\tilde{E}_{sig}(\omega, \tau)$. One can immediately see that taking the modulus squared of this quantity yields the appropriate form for a FROG trace. At this point, the first projection, in the frequency domain, is made by replacing the magnitude of $\tilde{E}_{sig}(\omega, \tau)$ with the square root of the experimentally measured FROG trace, $S_{FROG}(\omega, \tau)$. In this way, the phase information is kept while the amplitude

is replaced by experimentally measured values. This new field is then Fourier transformed back into the time domain. The second projection, this time in the time domain, is applied by using the new field to generate the field for the next guess. In the basic implementation of this algorithm, this simply means that the modified signal field is integrated over the delay coordinate, which results in the next iteration being proportional to the real field. This can more clearly be seen by evaluating the integral at the k^{th} step:

$$\begin{aligned}
E^{k+1}(t) &= \int_{-\infty}^{\infty} E_{sig}^k(t, \tau) d\tau = \int_{-\infty}^{\infty} E'^k(t) g'^k(t - \tau) d\tau \\
&= E'^k(t) \int_{-\infty}^{\infty} g'^k(t - \tau) d\tau \\
&\propto E'^k(t).
\end{aligned} \tag{1.33}$$

The algorithm then begins anew. After each iteration, the procedure produces a field that is a better initial guess, and eventually converges to the correct complex electric field.

There are many additional features of FROG that make it superior to other pulse measurement techniques. There are a few, however, that are worth pointing out. The feedback provided by the FROG can be used to validate the pulse retrieval. This involves comparing the marginals of the FROG trace to an independently measured spectrum and/or autocorrelation. This feedback mechanism can aid in the detection and correction of systematic errors in the measurement apparatus. Also, while there is no formal proof that the pulse reconstruction from a FROG diagnostic is essentially unique, the result is probabilistically true. This stems from the fact that the FROG trace is an $N \times N$ array of data points that is used to determine $2N$ data points (N phase points and N intensity points). Therefore, there is significant overdetermination of the phase and amplitude of

the pulse, which results from many more degrees of freedom in the trace rather than in the pulse. Another consequence of this is that it is *highly unlikely* that the FROG algorithm will converge for a trace consisting of randomly generated points, or for a trace that has been contaminated by systematic error. Likewise, convergence of the FROG algorithm that produces a trace which agrees well with the measured trace is highly indicative of a successful reconstruction for a measured trace free of systematic error. There are, however, certain quantities that a FROG diagnostic and reconstruction process cannot measure. It cannot measure the absolute phase of the pulse. This is not very important as one is usually only concerned with relative phase, or the existence of chirp. Since FROG uses the pulse, or some function of the pulse, to gate itself, there is no absolute time reference. Hence, it cannot measure the time of arrival of the pulse. These are known as *trivial ambiguities* because most scientists are unconcerned with their measurement.

1.4 Basic Principles of Free Electron Laser Operation

The details of high-gain FEL theory are explored in depth in Chapter 2. This section will review the basic ideas and physical processes behind the operation of a FEL by considering the motion of a charged particle in a planar magnetic undulator.

All particle-based light sources rely on the principle that accelerated charges emit light. This is a consequence of a finite value for the velocity of light. Consider a coordinate system moving with a constant velocity equal to that of a relativistic particle before acceleration. In this coordinate system, the electric field emitted by the particle is purely radial. If this particle is then accelerated, it is forced to move with respect to its own coordinate system. As a consequence, parts

of the electric field cannot “catch up” with the particle and cause distortions in the radial field. These distortions manifest in radiation. This is expressed more clearly by using the Léinard-Wiechert potentials to describe the electric field[21]

$$\vec{E}(\vec{x}, t) = \frac{e}{4\pi\epsilon_0} \left[\frac{\hat{n} - \vec{\beta}}{\gamma (1 - \vec{\beta} \cdot \hat{n})^3 R^2} \right]_{ret} + \frac{e}{4\pi\epsilon_0 c} \left[\frac{\hat{n} \times \{ (\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}} \}}{(1 - \vec{\beta} \cdot \hat{n})^3 R} \right]_{ret}, \quad (1.34)$$

where e is the electric charge, the Lorentz factor, $\gamma = 1/\sqrt{1 - \beta^2}$, is the particle energy in units of the rest mass mc^2 , $\vec{\beta} = \vec{v}/c$ ($\vec{v}$ is the particle velocity and c is the speed of light), $\vec{R}(t') = R(t')\hat{n} = |\vec{x} - \vec{r}(t')|\hat{n}$ is a vector pointing from the source to the observer, the ‘.’ indicates a derivative with respect to time, and ‘ret’ means the quantities must be evaluated at the retarded time, $t = t' + [R(t')/c]$. The quantity on the left is referred to as the velocity field, which is independent of acceleration and falls off as R^{-2} , while the quantity on the right is referred to as the acceleration field, which depends linearly on $\dot{\vec{\beta}}$ and falls off as R^{-1} . Not all forms of acceleration produce equal amounts of radiated power though. This can be seen by examining the instantaneous radiated power, derived from the acceleration field, and expressed as

$$\mathcal{P} = \frac{2}{3} r_c mc \gamma^6 \left[\dot{\vec{\beta}}^2 - (\vec{\beta} \times \dot{\vec{\beta}})^2 \right], \quad (1.35)$$

where $r_c = e^2/4\pi\epsilon_0 mc^2$ is the classical electron radius. Evaluating this expression for particle acceleration both perpendicular and parallel to the particle velocity yields[22, 23]

$$\mathcal{P}_{\parallel} = \frac{2}{3} \frac{r_c}{mc} \left(\frac{dp_{\parallel}}{dt} \right)^2 \quad (1.36)$$

$$\mathcal{P}_{\perp} = \frac{2}{3} \frac{r_c}{mc} \gamma^2 \left(\frac{dp_{\perp}}{dt} \right)^2. \quad (1.37)$$

An equal accelerating force leads to a much higher radiation power, by a factor of γ^2 for relativistic particles, for transverse acceleration compared to longitudinal acceleration. Magnetic fields are often used to produce this transverse acceleration. An electron traversing such a deflecting magnetic field produces synchrotron radiation. This radiation was harnessed in second generation light sources where dedicated electron storage rings were used exclusively to produce improved photon beams for experimental uses. Unfortunately, the synchrotron radiation spectrum is extremely broadband, as can be seen in figure 1.9[24]. This

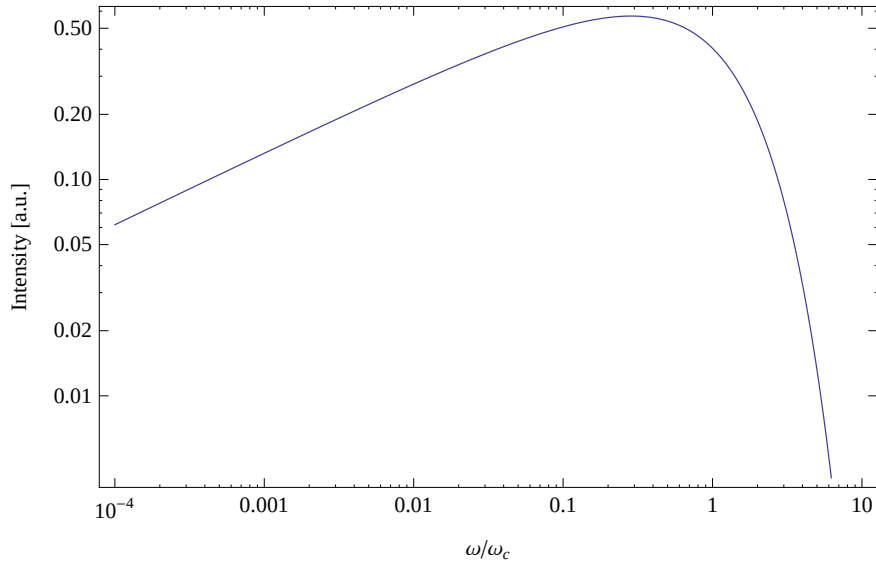


Figure 1.9: Synchrotron radiation spectral distribution.

is a consequence of the extremely short duration of a synchrotron radiation pulse in the temporal domain. Third generation light sources used insertion devices in storage rings. These insertion devices, such as wigglers and undulators, introduce periodicity in the radiating structure, which produces a repetitive pulse train in the time domain, and results in a spectral narrowing in the frequency domain. Fourth generation light sources, such as FELs, extend this principle further to

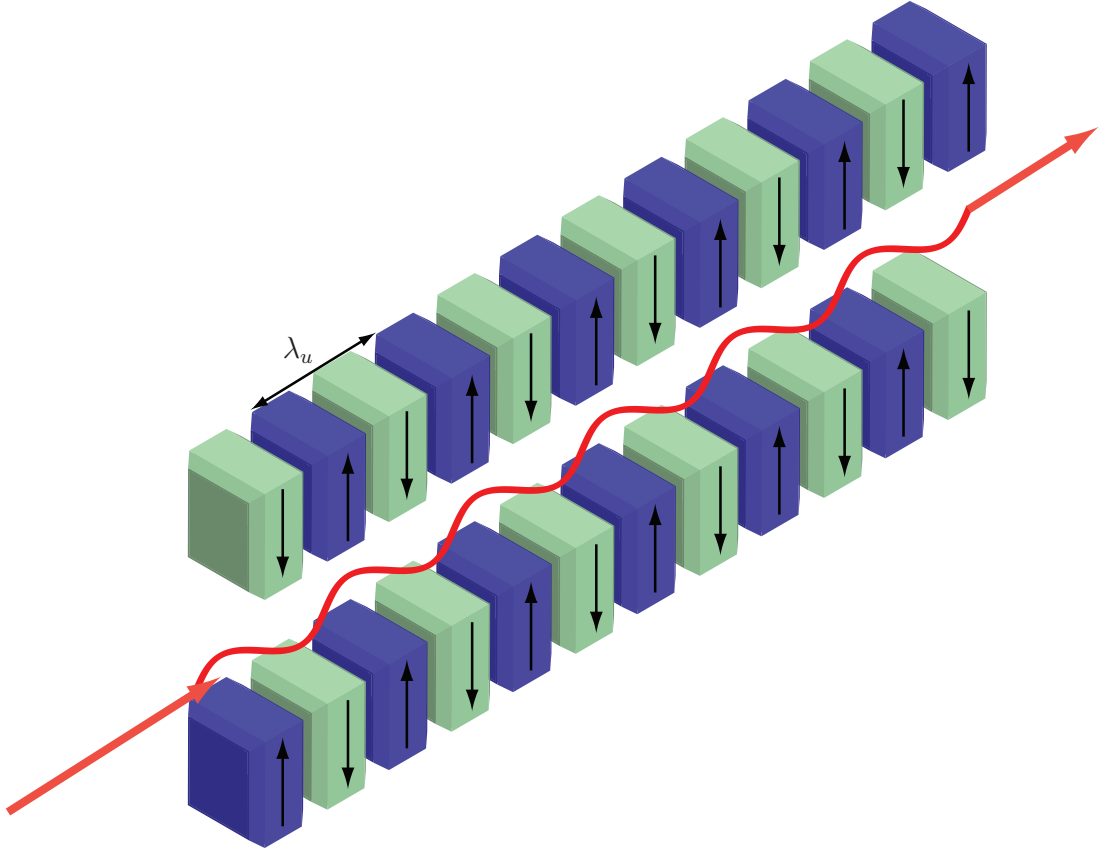


Figure 1.10: The electron undergoes sinusoidal motion in a planar magnetic undulator with period λ_u .

produce the brightest light sources currently available.

1.4.1 Undulator Radiation

The investigation into the physics of FEL operation begins with the study of electron motion through a planar magnetic undulator such as the one shown in Figure 1.10. An undulator usually consists of a series of equal dipole magnets arranged with an alternating magnetic field direction. The alternating magnet direction produces a sinusoidal magnetic field in the y direction on axis ($y = 0$),

which can be described to first order by

$$\begin{aligned} B_x(z) &= 0, \\ B_y(z) &= B_0 \sin(k_u z), \\ B_z(z) &= 0. \end{aligned} \tag{1.38}$$

Here, B_0 is the magnetic field amplitude on axis and $k_u = 2\pi/\lambda_u$ is the undulator wavenumber with undulator period λ_u . The electron motion is governed by the *Lorentz force equation*

$$\vec{F} = e \left(\vec{E} + \vec{v} \times \vec{B} \right), \tag{1.39}$$

which yields

$$\gamma m \frac{dv_x}{dt} = e (v_y B_z - v_z B_y) \approx -ecB_0 \sin(k_u z) \tag{1.40}$$

for an electron on axis in the above field. Here, the longitudinal velocity of the electron is taken to be $v_z \approx c$ in a first approximation. Since the electron energy is unchanged in a magnetic field, γ remains constant, and equation 1.40 can be integrated to obtain

$$v_x = \frac{eB_0}{\gamma m k_u} \cos(k_u z) = \frac{Kc}{\gamma} \cos(k_u z), \tag{1.41}$$

$$v_y = 0, \tag{1.42}$$

$$v_z = c \sqrt{1 - \frac{1}{\gamma^2} - \frac{v_x^2}{c^2}} \approx c \underbrace{\left(1 - \frac{1 + K^2/2}{2\gamma^2} \right)}_{=\bar{v}_z} - \frac{K^2 c}{4\gamma^2} \cos(2k_u z). \tag{1.43}$$

Here, $K = eB_0/mck_u$ is the dimensionless deflection parameter (undulator parameter), $\bar{v}_z$ is the average longitudinal velocity over an undulator period, and $\gamma \gg 1$ for relativistic electrons. At this point it is instructive to analyze the radiation from a relativistic electron undergoing oscillatory motion in an undula-

tor. Imagine that the electron emits radiation at the peak of it's deviation from the undulator axis (see Figure 1.11). For constructive interference between pho-

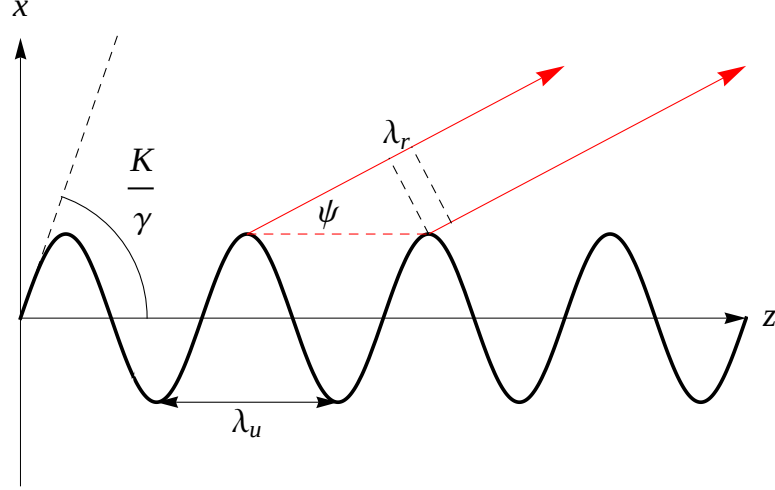


Figure 1.11: Illustration of constructive interference yielding the resonance condition in an undulator.

tons emitted at adjacent peaks, the difference in path lengths must be an integer multiple of the radiation wavelength. The distance the first photon travels is $c\tau$, where τ is the time it takes the electron to travel from peak to peak. This time is found from $\lambda_u = \beta_z c\tau$. Therefore, constructive interference requires that the radiation wavelength satisfies

$$\lambda_r = \frac{\lambda_u}{\beta_z} - \lambda_u \cos(\psi), \quad (1.44)$$

where ψ is the emission angle from the nominal beam propagation direction, z . Since the electrons are relativistic, the radiation is emitted within a cone ($\psi \propto 1/\gamma$) confined axially along the undulator. Expanding the expression for

$1/\beta_z$ using equation 1.43, the resonant coherent radiation wavelength is

$$\lambda_r = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2\psi^2 \right). \quad (1.45)$$

Equation 1.45 is known as the *resonance condition*, and can be generalized to harmonic emissions. It is interesting to note that as the observation angle, ψ , is increased from 0, the radiation is red-shifted:

$$\frac{\lambda_r(\psi) - \lambda_r(0)}{\lambda_r(0)} = \frac{\gamma^2\psi^2}{1 + K^2/2} > 0. \quad (1.46)$$

This is a spatio-temporal phenomenon often referred to as *spatial chirp*, because different spectral components of a pulse are distributed across it's transverse profile. This effect is detrimental to pulse reconstruction and will be discussed in more detail in Chapter 3.

An undulator with N_u periods, and therefore a length of $L_u = N_u\lambda_u$, will generate a wave train with N_u cycles. The spectrum of the undulator radiation, therefore, is peaked around the resonant frequency

$$\omega_r = \frac{2\pi c}{\lambda_r} = ck_u \frac{2\gamma^2}{1 + K^2/2 + \gamma^2\psi^2} \quad (1.47)$$

with an intrinsic bandwidth

$$\frac{\Delta\omega}{\omega_r} = \frac{\Delta\lambda}{\lambda_r} \sim \frac{1}{N_u}. \quad (1.48)$$

Increasing the number of periods in an undulator reduces the spectral bandwidth of the radiation. In principle, one can obtain very narrow bandwidth pulses by increasing the number of periods. The above characteristics can more clearly be seen by considering the photons radiated per unit solid angle into the spectral

bandwidth $\Delta\omega/\omega$ [23] given by

$$\begin{aligned} \frac{dN_{ph}(\omega)}{d\Omega} &= \alpha\gamma^2\bar{\beta}_z^2 N_u^2 \frac{\Delta\omega}{\omega} \sum_{k=1}^{\infty} k^2 \left[\frac{\sin(\pi N_u \Delta\omega_k/\omega_r)}{\pi N_u \Delta\omega_k/\omega_r} \right]^2 \\ &\times \frac{[2\gamma\psi\Sigma_1 \cos(\phi) - K\Sigma_2]^2 \mathbf{x}^2 + [2\gamma\psi\Sigma_1 \sin(\phi)]^2 \mathbf{y}^2}{(1 + K^2/2 + \gamma^2\psi^2)^2}, \end{aligned} \quad (1.49)$$

where

$$\frac{\Delta\omega_k}{\omega_r} = \frac{\omega}{\omega_r} - k, \quad (1.50)$$

$$\Sigma_1 = \sum_{m=-\infty}^{\infty} J_{-m}(u) J_{k-2m}(v), \quad (1.51)$$

$$\Sigma_2 = \sum_{m=-\infty}^{\infty} J_{-m}(u) [J_{k-2m-1}(v) + J_{k-2m+1}(v)], \quad (1.52)$$

$$u = \frac{\omega}{\omega_r} \frac{2K_o\bar{\beta}_z\gamma\psi \cos(\phi)}{1 + K^2/2 + \gamma^2\psi^2}, \quad (1.53)$$

$$v = \frac{\omega}{\omega_r} \frac{K^2\bar{\beta}_z}{4(1 + K^2/2 + \gamma^2\psi^2)}, \quad (1.54)$$

k indicates the harmonic number, $\alpha = e^2/\hbar c \approx 1/137$ is the fine structure constant, $J_m(x)$ are Bessel's functions, and $\mathbf{x}$ and $\mathbf{y}$ are orthogonal unit vectors indicating contributions from the σ and π mode polarizations respectively. The radiation intensity distributions are determined by the factors

$$I_{\sigma,k} = \frac{[2\gamma\psi\Sigma_1 \cos(\phi) - K\Sigma_2]^2}{(1 + K^2/2 + \gamma^2\psi^2)^2}, \quad (1.55)$$

$$I_{\pi,k} = \frac{[2\gamma\psi\Sigma_1 \sin(\phi)]^2}{(1 + K^2/2 + \gamma^2\psi^2)^2}, \quad (1.56)$$

and are seen for the first three harmonics in Figure 1.12. There is no emission in the forward direction for any harmonic from the π -mode while the σ -mode emits in the forward direction for the odd harmonics only. The second harmonic

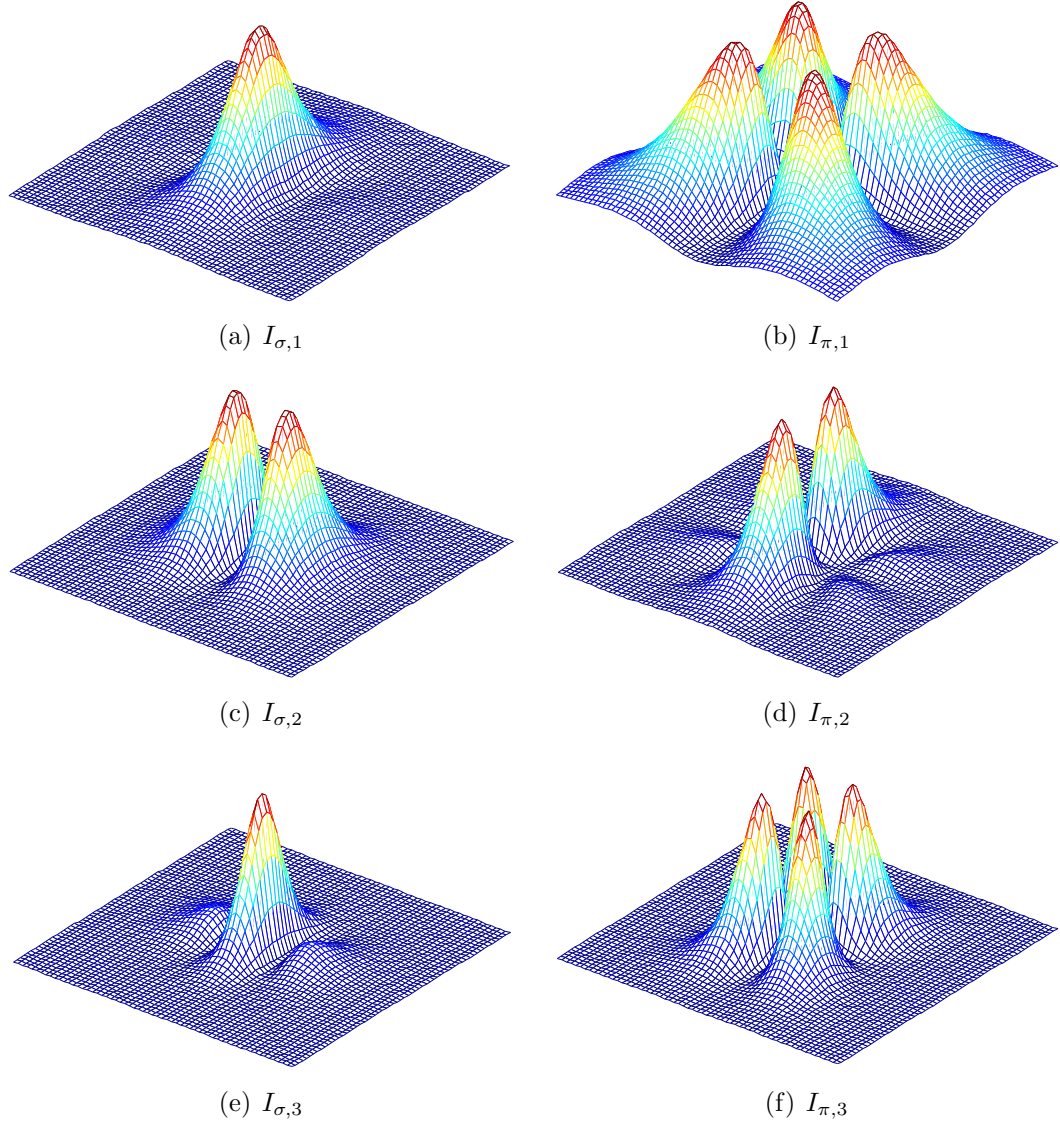


Figure 1.12: Single electron planar undulator radiation transverse profile for the first three harmonics. The left column corresponds to the σ -mode polarization from equation 1.55. The right column corresponds to the π -mode polarization from equation 1.56.

radiation therefore vanishes in the forward direction; a fact that is true for all even harmonics. An electron beam with zero emittance can be said to emit no even harmonics in the forward direction.

The function $\text{sinc}(x) = \sin(x)/x$ is peaked around $x = 0$, which gives a strong peak in the spectral angular distribution about the frequency

$$\omega = k\omega_r. \quad (1.57)$$

This is a restatement of the resonance condition in equation 1.45. One can determine the full-width at half-maximum (FWHM) of the radiation in both angle and frequency by considering that the half-maximum of $\text{sinc}^2(x)$ is found at $x \sim 1.39$. Thus, the spectral FWHM on axis is

$$\Delta\omega_{FWHM} = \frac{0.88}{N_u}\omega_r. \quad (1.58)$$

One can determine the FWHM opening angle at resonance in a similar manner, and is found to be

$$\Delta\psi_{FWHM} \approx \frac{1}{\gamma} \sqrt{\frac{0.88(1 + K^2/2)}{N_u}}. \quad (1.59)$$

It is interesting to note that an increase in K leads to an increase in the angular FWHM. This occurs because a larger value for K leads to a larger electron deflection (see equation 1.41 and figure 1.11) providing radiation downstream that sweeps out larger angles. This phenomenon is commonly referred to as the *searchlight* effect.

1.4.2 Pendulum Equations

The only subject discussed thus far is the generation of an electromagnetic (EM) wave from “free” electrons as they propagate through an undulator and emitting spontaneous radiation. This process is analogous to the spontaneous radiation from “bound” electrons in atoms from more conventional laser sources. The free electrons in the undulator, however, can interact with the EM wave, and under certain favorable conditions, can lead to significant amplification (stimulated emission). Therefore, the energy exchange between the electrons and the EM wave must be studied to fully appreciate this process.

Consider, first, a linearly polarized plane EM wave propagating co-linearly with the electron beam

$$\vec{E} = E_0 \cos(kz - \omega t + \varphi_0) \hat{x}. \quad (1.60)$$

Energy can be exchanged between the electrons and the field at a rate given by

$$\frac{dU}{dt} = e\vec{E} \cdot \vec{v} = \frac{eE_0 K c}{\gamma} \cos(k_u z) \cos(kz - \omega t + \varphi_0), \quad (1.61)$$

where $U = \gamma mc^2$ is the electron energy and the expression for $\vec{v}$ is found in equations 1.41-1.43. If $dU/dt > 0$ the electrons are accelerated and energy is taken from the field. If, however, $dU/dt < 0$ energy is taken from the electrons and the EM wave is amplified. This is the operating principle behind the FEL. In general, the interaction of an EM wave and electrons is not sustained for free-space propagation because the EM wave is always faster than the electron beam. As the EM wave slips forward in the beam frame the electrons sample various phases of the EM wave and there is no net energy exchange. In an undulator, however, the EM wave slips past the electrons by one radiation wavelength for one

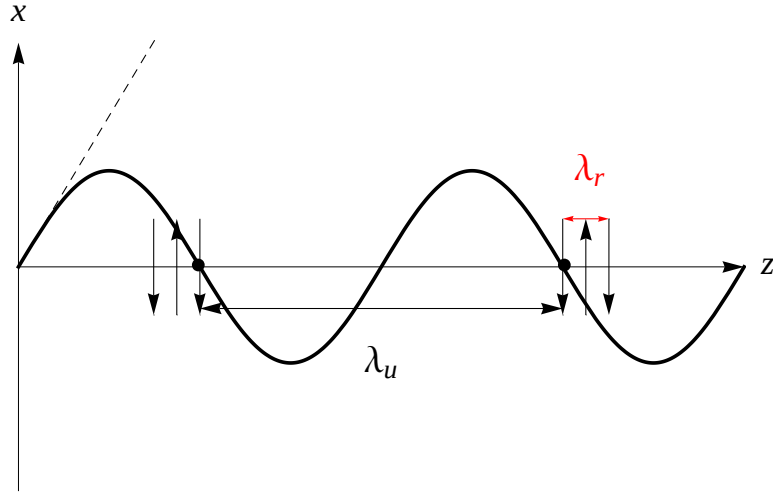


Figure 1.13: Sustained interaction between an electron and an EM wave in an undulator.

undulator period of propagation (from the resonance condition). This concept is commonly referred to as *slippage*. The electrons, therefore, sample the same phase of the EM wave as they co-propagate and the interaction can be sustained. If the interaction phase is such that the electron lose energy on average, then the EM wave gains energy and undergoes a sustained amplification. This can more clearly be seen in Figure 1.13.

The energy exchange between the electrons and radiation field is governed by

$$\begin{aligned}
 mc^2 \frac{d\gamma}{dt} &= \frac{eE_0 K c}{\gamma} \cos(k_u z) \cos(kz - \omega t + \varphi_0) \\
 &= \frac{eE_0 K c}{2\gamma} \left[\cos(\underbrace{(k + k_u)z - \omega t + \varphi_0}_{\theta + \pi/2}) + \cos((k - k_u)z - \omega t + \varphi_0) \right].
 \end{aligned} \tag{1.62}$$

To provide a sustained beam-radiation interaction the total phase in the above expression must be kept constant. This implies that the time derivative must

vanish. Analyzing the phase from the first term in 1.62 produces the condition:

$$\frac{d\theta}{dt} = (k + k_u)v_z - ck = 0. \quad (1.63)$$

Therefore, optimal energy exchange is fulfilled for an electron with a velocity

$$\beta_z = \frac{k}{k + k_u}. \quad (1.64)$$

The wavelength that satisfies this relationship can be found by substituting in the average longitudinal velocity ($\beta_z \rightarrow \bar{\beta}_z$) and evaluating the expression in the limit $k_u/k = \lambda/\lambda_u \ll 1$ to produce

$$1 - \frac{1 + K^2/2}{2\gamma^2} = \frac{1}{1 + k_u/k} \simeq 1 - \frac{k_u}{k}, \quad (1.65)$$

and thus

$$\frac{k_u}{k} = \frac{\lambda}{\lambda_u} = \frac{1 + K^2/2}{2\gamma^2}. \quad (1.66)$$

This is, once again, a restatement of the undulator resonance condition, and has now been shown to be the condition for a sustained interaction between the EM wave and the electron beam. A similar condition to equation 1.64 is found if the second term in equation 1.62 is examined under a vanishing time derivative:

$$\beta_z = \frac{k}{k - k_u}. \quad (1.67)$$

This result requires an electron to have a velocity faster than the speed of light in order for a sustained interaction to occur. This is, of course, a non-physical result. The electrons would sample all phases of the radiation as it slips over the electron beam. This term, therefore, does not contribute to the exchange in energy between the electron beam and the EM wave on average.

At this point it is appropriate to define the electron beam energy at which resonance occurs because γ is now a function of time. The resonant energy is found from the resonance condition

$$\gamma_R^2 = \frac{\lambda_u}{\lambda} \frac{1 + K^2/2}{2}. \quad (1.68)$$

During the FEL interaction only small deviations from the resonant energy are expected to first order. It is useful, therefore, to define a normalized electron energy variable

$$\eta = \frac{\gamma - \gamma_R}{\gamma_R} \ll 1, \quad (1.69)$$

so that the energy exchange rate becomes $d\gamma/dt = \gamma_R d\eta/dt$. Equations 1.62 and 1.63 become the well known *Pendulum equations*[25]

$$\frac{d\theta}{dz} = 2k_u \eta, \quad (1.70)$$

$$\frac{d\eta}{dz} = -\frac{eE_0 K}{2\gamma_R^2 mc^2} \sin(\theta), \quad (1.71)$$

where the independent variable was change from t to z by $cdt \approx dz$. It should be noted here that there is an oscillatory component to the longitudinal velocity (see equation 1.43) that has so far been neglected. When this term is properly taken into account in the phase equation with period averaging it introduces a change from $K \rightarrow K[JJ]$, where

$$[JJ] = J_0 \left(\frac{K^2}{4 + 2K^2} \right) - J_1 \left(\frac{K^2}{4 + 2K^2} \right) \quad (1.72)$$

is the well known Bessel form factor for planar undulators. The pendulum equa-

tions now take the final form

$$\frac{d\theta}{dz} = 2k_u\eta, \quad (1.73)$$

$$\frac{d\eta}{dz} = -\epsilon \sin(\theta), \quad (1.74)$$

where $\epsilon = eE_0K[JJ]/2\gamma_R^2mc^2$.

An FEL operating under these principles is often referred to as being in the *small signal gain* (SSG) regime[26]. Here, the gain in a single pass is small enough that the field can be regarded as constant. The phase term θ is often referred to as the *ponderomotive phase* and the pendulum equations describe the motion of electrons in the *ponderomotive potential*. The motion of electrons in this potential is illustrated in Figure 1.4.2, and can be described by the Hamiltonian

$$H(\theta, \eta) = k_u\eta^2 - \epsilon \cos(\theta). \quad (1.75)$$

One can clearly see that the potential forms a series of buckets in the longitudinal phase-space (θ, η) of the electrons, where the solid lines indicate contours of constant energy. The ponderomotive phase can be regarded as the longitudinal position of an electron relative to a reference electron in units of $\lambda/2\pi$. Therefore, the phase spread of a typical electron beam, which can be on the order of $\sim 0.1 - 10$ picoseconds long, is a very large number, and the longitudinal profile of the electron beam is divided into many phase buckets. An electron beam with a small energy spread (blue in figure 1.4.2) will develop an energy modulation (green in figure 1.4.2) when injected into an undulator. As this electron beam propagates through the undulator the energy modulation begins to turn into a density modulation (magenta and red in figure 1.4.2) as the energy deviations change the relative positions of the electrons. This process, known as

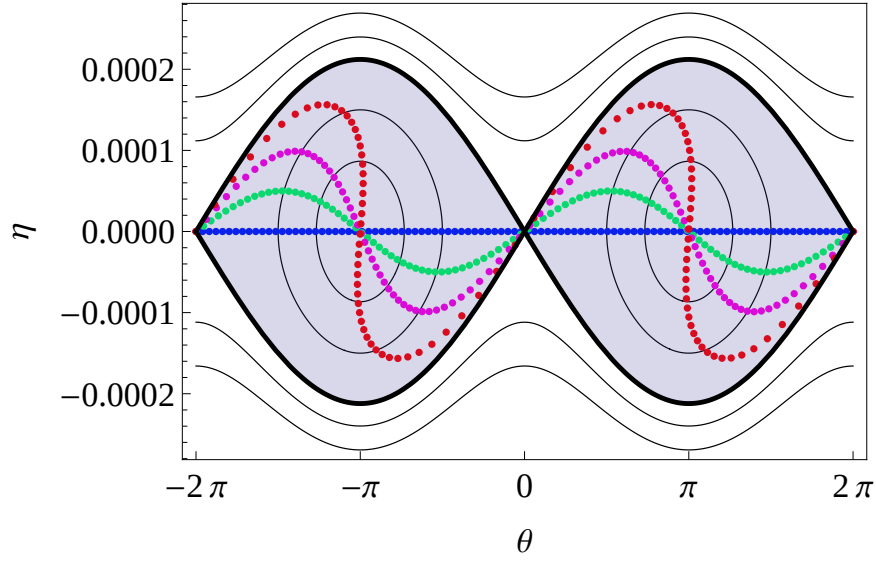


Figure 1.14: The small signal gain regime illustrating the interaction of electrons with an EM wave in the ponderomotive potential. An initially cold electron beam (blue) develops an energy modulation (green) as it interacts with an EM wave in an undulator. The energy modulation turns to a density modulation (magenta then red) as the electrons rotate in phase-space. The bold black line indicates the separatrix of the ponderomotive potential.

microbunching, bunches the electrons at the resonant wavelength. In a high-gain FEL, which is analyzed in detail in chapter 2, the microbunching has a feedback mechanism on the phase of the EM wave and effectively shifts the phase buckets to the left. The bunched electrons begin to radiate coherently and the EM radiation field grows exponentially in a process known as the FEL collective instability. When the microbunching reaches a maximum value the energy spread in the electron beam covers the bandwidth of the FEL and the process reaches saturation, where the radiation field amplitude no longer grows.

CHAPTER 2

Coupled Maxwell-Klimontovich Solution of the High-Gain 1-D FEL Equations

2.1 Introduction

The small signal gain regime analysis presented in section 1.4 provides a good foundation for understanding some of the key concepts involved in the FEL interaction process. The concepts introduced thus far include *slippage*, where the light emitted by electrons slips forward in the beam frame. This in turn leads to the concept of *resonance*, where the electron beam is able to sustain a prolonged interaction with a specific wavelength of light. This wavelength can be tuned by changing either the electron beam energy or the undulator period or magnetic field strength. The SSG analysis also illustrates some of the longitudinal dynamics of an electron under the influence of an undulator and EM field. What the SSG fails to capture, however, is the feedback that the electrons can have on the EM field and vice versa. The EM field influences the motion of the electrons in the undulator, which, in turn, are the current source that generates the radiation field. In particular, the radiation field can grow exponentially under the appropriate conditions. This process of electron *microbunching* and coherent EM radiation amplification is known as the *collective instability*, and is discussed in detail below.

This chapter begins with the study of the electromagnetic wave equation for the transverse electric field in the frequency domain. The e-beam current distribution can act as a source of radiation and is described here by the Klimontovich distribution function, the evolution of which is governed by the Vlasov equation. The coupled wave and Vlasov equations are linearized in the one-dimensional case and are solved via the Laplace transform technique. The exponential gain regime for SASE FEL light is then explored in detail. Here, three-dimensional and other higher order effects are only discussed qualitatively. The chapter concludes with examples of SASE FELs, given by GENESIS simulations, for three specific cases which are of current interest. The analysis presented in this chapter draws heavily on the work of Kim[27, 28, 29, 30], Colson[31], and Yu[32, 33].

2.2 High-Gain FEL

The investigation into the FEL collective instability and high-gain FEL operation begins with the relevant Maxwell equations, which in turn yields the wave equation for the transverse electric field:

$$\left[\left(\frac{1}{c} \frac{\partial}{\partial t} \right)^2 - \left(\frac{\partial}{\partial z} \right)^2 - \nabla_{\perp}^2 \right] E_x = -\frac{1}{\epsilon_0 c^2} \left[\frac{\partial J_x}{\partial t} + c^2 \frac{\partial \rho_e}{\partial x} \right]. \quad (2.1)$$

Here, $\nabla_{\perp}$ is the transverse laplacian operator, ϵ_0 is the permittivity of free space, $J_x = \rho_e v_x$ is the transverse current density, ρ_e is the charge density, $E_x = E_x(\mathbf{x}, z, t)$, and $\mathbf{x}$ is the transverse position vector. The transverse charge density gradient of the electron beam is often very small in an FEL and is usually neglected in the limit $c^2 \partial \rho_e / \partial x \ll \partial J_x / \partial t$. In order to effectively analyze SASE FEL light, it becomes useful to move from the time domain to the frequency domain. This is because a continuous frequency distribution about the resonant

frequency must be explored since the spectral characteristics change as the system evolves. Also, the study of the initial growth of radiation shot noise to an intense signal is more effectively analyzed in the frequency domain where the initial electron beam distribution is entirely stochastic. To this end, the frequency domain amplitude is defined as:

$$E_x(\mathbf{x}, z, t) = \frac{1}{2} \int_{-\infty}^{\infty} d\nu A(\mathbf{x}, z, \nu) e^{i\nu k_1(z-ct)} + c.c. \quad (2.2)$$

Here, $k_1 = \omega_1/c = 2\pi/\lambda_1$, λ_1 is the resonant wavelength in equation 1.45, $\nu = \omega/\omega_1 = k/k_1$, and $c.c$ stands for complex conjugate. The plane wave approximation has also been implemented by including the factor $\exp(ikz)$ in the above expression. If $A = A(\mathbf{x}, z, \nu)$ is assumed to vary slowly with z , i.e., the amplitude does not change appreciably over the length scale of the radiation, the slowly varying envelope approximation (SVEA) can be made, which allows for the elimination of the second derivative of the amplitude with respect to z in the limit

$$\left| \frac{\partial^2 A}{\partial z^2} \right| \ll \left| 2k_1 \frac{\partial A}{\partial z} \right|. \quad (2.3)$$

The wave equation 2.2 transforms under this approximation to

$$\frac{1}{2} \int_{-\infty}^{\infty} e^{i\nu k_1(z-ct)} \left(2i\nu k_1 \frac{\partial}{\partial z} + \nabla_{\perp}^2 \right) A(\mathbf{x}, z, \nu) d\nu = \frac{1}{\epsilon_0 c^2} \frac{\partial J_x}{\partial t}. \quad (2.4)$$

Equation 2.3 can be inverted by multiplying on the left by

$$\frac{ck_1}{\pi} e^{i\nu' k_1 ct}$$

and integrating over t from $[-\infty, \infty]$. Note that this is not the inverse Fourier transform and leaves the field in the frequency domain. The left-hand side be-

comes

$$\int_{-\infty}^{\infty} d\nu \int_{-\infty}^{\infty} \underbrace{\frac{ck_1 dt}{2\pi} e^{i(\nu' - \nu)k_1 ct}}_{\delta(\nu' - \nu)} e^{i\nu k_1 z} \left(2i\nu k_1 \frac{\partial}{\partial z} + \nabla_{\perp}^2 \right) A(\mathbf{x}, z, \nu) =$$

$$e^{i\nu k_1 z} \left(2i\nu k_1 \frac{\partial}{\partial z} + \nabla_{\perp}^2 \right) A(\mathbf{x}, z, \nu). \quad (2.5)$$

The right-hand side becomes

$$\frac{1}{\epsilon_0 c^2} \int_{-\infty}^{\infty} e^{i\nu k_1 ct} \frac{\partial J_x}{\partial t} \frac{ck_1 dt}{\pi} \quad (2.6)$$

which is integrated by parts to yield

$$-\frac{eK}{\epsilon_0} \int_{-\infty}^{\infty} e^{i\nu k_1 ct} \frac{i\nu k_1}{\gamma_0} \cos(k_u z) \sum_{j=1}^{N_e} \delta(\mathbf{x} - \mathbf{x}_j) \delta(t - t_j) \frac{k_1}{\pi} dt. \quad (2.7)$$

Here, N_e is the total number of electrons in the beam. The expression for $J_x = \rho_e v_x$ uses equation 1.41 for each electron and the energy spread is assumed to be small so that $\gamma_j \approx \gamma_0$. Rearranging terms, using $\theta = (k + k_u)(z - \bar{v}_z t)$ to change variables from $t \rightarrow \theta$, and properly averaging over the fast wiggles reduces equation 2.4 to

$$\left(\frac{\partial}{\partial z} + \frac{\nabla_{\perp}^2}{2i\nu k_1} \right) A(\mathbf{x}, z, \nu) = -\frac{ek_1 K[JJ]}{4\pi\epsilon_0\gamma_0} e^{i\Delta\nu k_u z} \int_{-\infty}^{\infty} e^{-i\nu\theta} \sum_{j=1}^{N_e} \delta(\mathbf{x} - \mathbf{x}_j) \delta(\theta - \theta_j) d\theta, \quad (2.8)$$

where $\Delta\nu = \nu - 1$.

2.2.1 1-D Maxwell-Klimontovich Equations

The analysis presented thus far is a fully 3-D treatment, which has been investigated in detail using a variety of techniques[34, 35, 36, 37]. The 1-D case, however,

provides a simpler framework with which to analyze SASE FEL physics directly applicable to this thesis. Some of the key concepts considered here include start-up from noise, exponential gain, effects of electron beam energy spread, and the development of longitudinal coherence. Here, the focus is primarily on the 1-D treatment where the electron beam is considered uniform and infinite in the transverse dimension. In this limit all transverse modes become degenerate and have the same growth rate (described by μ below). Relevant 3-D effects are discussed briefly in section [2.2.4](#).

At this point it is useful to consider the Klimontovich distribution function[\[38\]](#) to describe the electron beam. This distribution takes into account the discreteness of the electrons that initiate the SASE process and is defined as:

$$F(\theta, \eta; z) = \frac{k_1}{dN_e/dz} \sum_{j=1}^{N_e} \delta(\theta - \theta_j) \delta(\eta - \eta_j). \quad (2.9)$$

Here, dN_e/dz is the line density. The distribution can be expanded to first order in perturbation theory:

$$F(\theta, \eta; z) = F_0(\eta) + F_1(\theta, \eta; z). \quad (2.10)$$

The smooth background, F_0 , is independent of θ ; an approximation that is valid for bunch lengths that are much longer than the slippage length ($N_u \lambda_1$). The normalization is such that

$$\int_{-\infty}^{\infty} F_0(\eta) d\eta = 1. \quad (2.11)$$

F_1 is a perturbation due to the FEL interaction and contains the shot-noise fluctuation in the electron beam as well as the FEL-induced microbunching. Moving

to the frequency domain, the expression for $F(\nu, \eta; z)$ becomes

$$F(\nu, \eta; z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\nu\theta} F(\theta, \eta; z) d\theta = \frac{k_1}{2\pi dN_e/dz} \sum_{j=1}^{N_e} e^{-i\nu\theta_j} \delta(\eta - \eta_j). \quad (2.12)$$

Here, $\theta_j = \theta_j(z)$ and $\eta_j = \eta_j(z)$. Eliminating the transverse dependence in equation 2.8, the 1-D wave equation in the frequency domain becomes

$$\left(\frac{\partial}{\partial z} + i\Delta\nu k_u \right) A(z, \nu) = -\kappa_2 n_0 \int_{-\infty}^{\infty} F(\nu, \eta; z) d\eta, \quad (2.13)$$

where $\kappa_2 = eK[JJ]/(2\epsilon_0\gamma_R)$, and n_0 is the electron density. The smooth background, F_0 , vanishes here in the limit of small bandwidth, i.e., $\nu \rightarrow 1$, and $A(z, \nu) \rightarrow A(z, \nu)\exp(-i\Delta\nu k_u z)$ for notational convenience.

The Klimontovich distribution is governed by the continuity equation

$$\frac{d}{dz} F(\theta, \eta; z) = \frac{\partial F}{\partial z} + \frac{\partial F}{\partial \theta} \frac{\partial \theta}{\partial z} + \frac{\partial F}{\partial \eta} \frac{\partial \eta}{\partial z} = 0. \quad (2.14)$$

It should be noted here that equation 2.14 is equivalent to the Vlasov equation, which is often used to describe the fluid limit of the plasma distribution function. Here, though, the microscopic description of the electrons is used to more accurately describe SASE radiation from the initial stochastic electron beam distribution. The terms θ' and η' , where the prime indicates a partial derivative with respect to z , are determined by the pendulum equations:

$$\theta' = 2k_u \eta, \quad (2.15)$$

$$\eta' = \kappa_1 \int_{-\infty}^{\infty} d\nu A(z, \nu) e^{i\nu\theta} + c.c., \quad (2.16)$$

where $\kappa_1 = eK[JJ]/4\gamma_R^2 mc^2$. The continuity equation 2.14 is linearized by treat-

ing both F_1 and A as first order quantities to obtain in the frequency domain

$$\frac{\partial F_1}{\partial z} + i\nu 2k_u \eta F_1 + \kappa_1 A \frac{\partial F_0}{\partial \eta} = 0, \quad (2.17)$$

where F_0, F_1 , and A are all functions of ν, η , and z . Henceforth, subscripts will be used to indicate the operational space, i.e., equation 2.17 becomes

$$\frac{\partial F_{1,\nu}}{\partial z} + i\nu 2k_u \eta F_{1,\nu} + \kappa_1 A_\nu \frac{\partial F_{0,\nu}}{\partial \eta} = 0. \quad (2.18)$$

2.2.2 Laplace Transform Solution of the Maxwell-Klimontovich Equations

The boundaries and constraints involved in the FEL interaction make it suitable to be solved by using the Laplace transform. The Laplace transform is ideal for solving initial value problems and problems that are one sided. The Laplace transform is defined for this problem to be

$$A_{\nu,\mu} = \int_0^\infty A_\nu e^{i\mu 2\rho k_u z} dz. \quad (2.19)$$

A similar expression exists for the electron distribution function:

$$F_{\nu,\mu} = \int_0^\infty F_\nu e^{i\mu 2\rho k_u z} dz. \quad (2.20)$$

Here, ρ is the universal FEL scaling parameter[39], which will be defined shortly, and is often referred to as the Pierce parameter[40]. The above transform exists and converges uniformly if there exists some $\mu = i\lambda$ for some real λ such that

$$\begin{pmatrix} A_\nu \\ F_\nu \end{pmatrix} < e^{-\lambda 2\rho k_u z}. \quad (2.21)$$

Applying the Laplace transform to equations 2.13 and 2.18 yields

$$-i\mu 2\rho k_u A_{\nu,\mu} + i\Delta\nu k_u A_{\nu,\mu} + \kappa_2 n_0 \int_{-\infty}^{\infty} F_{\nu,\mu} d\eta = A_{\nu}(0), \quad (2.22)$$

$$-i\mu 2\rho k_u F_{\nu,\mu} + i2k_u \eta \nu F_{\nu,\mu} + \kappa_1 A_{\nu,\mu} \frac{\partial F_{0,\nu}}{\partial \eta} = F_{\nu}(0). \quad (2.23)$$

Solving for $F_{\nu,\mu}$ above, while letting $\eta\nu \rightarrow \eta$ for small bandwidth pulses, yields

$$F_{\nu,\mu} = \frac{F_{\nu}(0) - \kappa_1 A_{\nu,\mu} \frac{\partial F_{0,\nu}}{\partial \eta}}{-i\mu 2\rho k_u + i2k_u \eta}. \quad (2.24)$$

Substituting equation 2.24 into equation 2.22 produces

$$\begin{aligned} A_{\nu,\mu} \left(-i\mu 2\rho k_u + i\Delta\nu k_u + \frac{\kappa_1 \kappa_2 n_0}{i2\rho k_u} \int_{-\infty}^{\infty} \frac{\frac{\partial F_{0,\nu}}{\partial \eta}}{\mu - \frac{\eta}{\rho}} d\eta \right) = \\ A_{\nu}(0) - \frac{\kappa_2 n_0}{i2\rho k_u} \int_{-\infty}^{\infty} \frac{F_{\nu}(0)}{\frac{\eta}{\rho} - \mu} d\eta. \end{aligned} \quad (2.25)$$

The second term on the right-hand side can be evaluated by substituting equation 2.12

$$F_{\nu}(0) = \frac{1}{N_{\lambda}} \sum_{j=1}^{N_e} e^{-i\nu\theta_j} \delta(\eta - \eta_j), \quad (2.26)$$

where N_{λ} is the number of electrons in a radiation wavelength. Making this substitution and applying the inverse Laplace transform produces

$$A_{\nu,\mu}(z) = \oint \frac{d\mu}{2\pi i} \frac{e^{-i\mu 2\rho k_u z}}{\Omega(\mu)} \left[A_{\nu}(0) + \frac{i\kappa_2 n_0}{2\rho k_u N_{\lambda}} \sum_{j=1}^{N_e} \frac{e^{-i\nu\theta_j}}{\frac{\eta_j}{\rho} - \mu} \right]. \quad (2.27)$$

Here, $\theta_j = \theta_j(0)$, $\eta_j = \eta_j(0)$, and

$$\Omega(\mu) = \mu - \frac{\Delta\nu}{2\rho} + \rho \int_{-\infty}^{\infty} \frac{\frac{\partial F_{0,\nu}}{\partial \eta}}{\mu - \frac{\eta}{\rho}} d\eta. \quad (2.28)$$

$\Omega(\mu)$ is known as the beam plasma dielectric function, and is given here in the 1-D limit including effects of energy spread. The parameter ρ is defined as $\rho^3 = \kappa_1 \kappa_2 n_0 / 4k_u^2$ so that

$$\rho = \left(\frac{e^2 K^2 [JJ]^2 n_0^2}{32 \gamma_R^3 m c^2 \epsilon_0 k_u^2} \right)^{1/3}. \quad (2.29)$$

The first term in the bracket in equation 2.27 refers to the amplification of an external field and is used to describe a *seeded* FEL. The second term, which contains the random phase $e^{-i\nu\theta_j}$, describes the phenomenon of self-amplified spontaneous emission and will be explored further below.

The contour integral in equation 2.27 must contain all singularities in order to invert the Laplace transform. The dynamics of the system, therefore, are controlled by both the singularities of kinematic origin (from the denominator within the summation) and the singularity from the solution of the dispersion relation

$$\Omega(\mu) = 0. \quad (2.30)$$

An exponentially growing mode results if there exists a solution to this equation, μ_k , that has a positive imaginary part. The resulting field will grow according to

$$A_\nu(z) = \frac{e^{-i\mu_k 2\rho k_u z}}{\Omega'(\mu)} \left[A_\nu(0) + \frac{i\kappa_2 n_0}{2\rho k_u N_\lambda} \sum_{j=1}^{N_e} \frac{e^{-i\nu\theta_j}}{\frac{\eta_j}{\rho} - \mu_k} \right], \quad (2.31)$$

where

$$\Omega'(\mu) = \frac{d\Omega}{d\mu} = 1 - 2 \int_{-\infty}^{\infty} \frac{F_{0,\nu}}{\left(\frac{\eta}{\rho} - \mu\right)^3} d\eta. \quad (2.32)$$

Once the amplitude is found the power spectrum can be computed using the formula

$$\frac{dP}{d\omega} = \frac{\epsilon_0 c}{4\pi} \lambda^2 \frac{\Sigma_A}{T} \langle |E_\nu(z)|^2 \rangle, \quad (2.33)$$

where Σ_A is the beam cross sectional area, T is the pulse duration, and $\langle \rangle$ indicates an ensemble average over the e-beam distribution. At this point it is useful to consider a few examples that will illustrate basic properties of SASE FEL's in the exponential gain regime.

2.2.3 Exponential Gain Regime

Many useful properties can be found by considering an initially cold (zero energy spread) e-beam where the distribution in energy is given by $F_{0,\nu}(\eta) = \delta(\eta)$. For this case the dispersion relation reduces to that of the well known cubic equation generalized to include the effects of detuning[\[41\]](#)

$$\Omega(\mu) = \mu - \frac{\Delta\nu}{2\rho} - \frac{1}{\mu^2} = 0. \quad (2.34)$$

Solving the dispersion relation produces three roots, the second of which is:

$$\begin{aligned} \mu_2 = & \frac{\frac{\Delta\nu}{2\rho}}{3} - \frac{(1 + i\sqrt{3}) \frac{\Delta\nu^2}{2\rho}}{3 \times 2^{2/3} \left(27 + 2\frac{\Delta\nu^3}{2\rho} + 3\sqrt{3}\sqrt{27 + 4\frac{\Delta\nu^3}{2\rho}} \right)^{1/3}} - \dots \\ & \dots - \frac{(1 - i\sqrt{3}) \left(27 + 2\frac{\Delta\nu^3}{2\rho} + 3\sqrt{3}\sqrt{27 + 4\frac{\Delta\nu^3}{2\rho}} \right)^{1/3}}{6 \times 2^{1/3}}, \end{aligned} \quad (2.35)$$

The positive imaginary part of μ_2 ($\mu_{2,I}$) yields a dominant exponentially growing solution. The growth rate has a complicated dependance on the detuning as can be seen in figure [2.1](#). It is useful to expand the growth rate $\mu_{2,I}$ about the maximum value ($\Delta\nu_m$) as a function of the detuning to second order. For a cold e-beam one finds that the growth rate becomes

$$\mu_{2,I} = \frac{\sqrt{3}}{2} - \frac{\Delta\nu^2}{24\rho^2\sqrt{3}}. \quad (2.36)$$

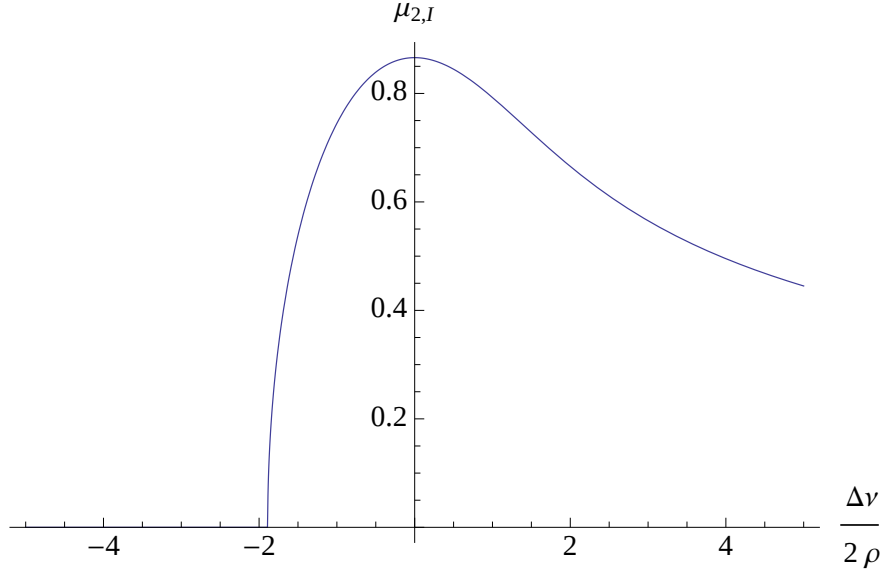


Figure 2.1: Detuning curve for a 1-D cold beam.

The FEL power, therefore, grows as

$$e^{4\mu_{2,I}\rho k_u z} = e^{\frac{z}{L_g}} e^{-\frac{1}{2}\left(\frac{\omega - \omega_m}{\omega_m \sigma_\nu}\right)^2} \quad (2.37)$$

where

$$L_g = \frac{\lambda_u}{4\pi\sqrt{3}\rho}, \quad (2.38)$$

$$\sigma_\nu = \sigma_{\Delta\omega/\omega} = \sqrt{\frac{3\sqrt{3}}{k_u z}} \rho. \quad (2.39)$$

Here, L_g is the power gain length and σ_ν is the rms relative bandwidth. These results hold for the very specific example of a 1-D cold e-beam while other significant effects due to e-beam emittance and space-charge are neglected. The FEL growth rate has a complicated dependence on these parameters. For instance, the detuning curve quickly becomes more complicated as an energy spread is

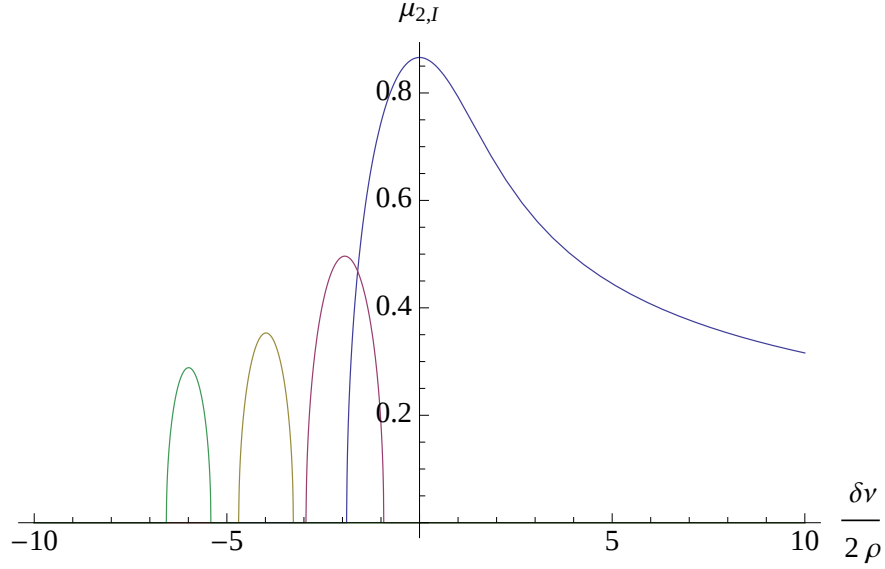


Figure 2.2: Detuning curves for a 1-D e-beam with a flat top energy distribution. The energy distribution has full widths of $\Delta\eta = 0$ (blue), 4ρ (red), 8ρ (yellow) and 12ρ (green).

introduced to the e-beam description. Figure 2.2 shows detuning curves for an e-beam with a flat top energy distribution of varying widths. The growth rate decreases as the energy spread increases and the optimal gain no longer occurs for an e-beam injected on resonance. Figure 2.3 shows the detuning curves for an e-beam with a Lorentzian distribution of varying widths of the form

$$F_{0,\nu}(\eta) = \frac{1}{2\pi} \frac{\delta\eta}{\eta^2 + \left(\frac{\delta\eta}{2}\right)^2}. \quad (2.40)$$

As one can see, the FEL growth rate exhibits a complicated dependence on the e-beam energy spread even for the simple cases considered in this chapter. This has obvious implications for the investigation of a FEL operating with a longitudinal e-beam energy chirp. The effects of a correlated energy spread on FEL performance will be discussed in detail in chapters 4 and 5. Other parameters

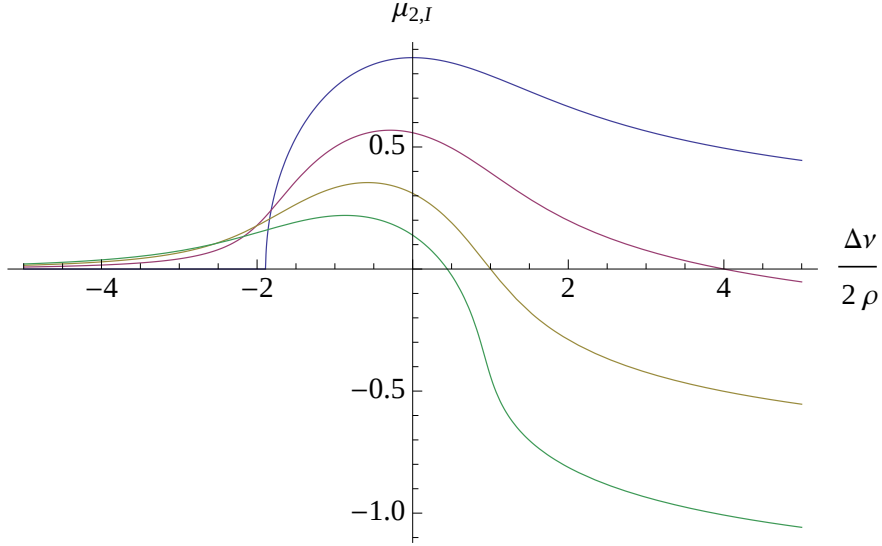


Figure 2.3: Detuning curves for a 1-D e-beam with a Lorentzian energy distribution. The energy distribution has widths of $\delta\eta = 0$ (blue), ρ (red), 2ρ (yellow) and 3ρ (green).

that have an effect on FEL performance are discussed qualitatively in section 2.2.4 below.

Moving back to the time domain by applying equation 2.2 to equation 2.31 for a cold e-beam while expanding the growth rate, μ_2 , to second order an approximate result for the temporal profile of SASE FEL light is obtained as[42]

$$E(z, t) \propto \frac{e^{\sqrt{3}\rho k_u z}}{\sqrt{z}} \sum_{j=1}^{N_e} \exp \left[i\omega_1 \left(\frac{z}{c} (1 + \rho\Delta\beta) - t - t_j \right) \right] \quad (2.41)$$

$$\times \exp \left[-\frac{\left(t - \frac{z}{c} (1 + 2\Delta\beta/3) - t_j \right)^2}{4\sigma_\tau^2} \left(1 + \frac{i}{\sqrt{3}} \right) \right]. \quad (2.42)$$

Here, $\sigma_\tau = 1/2\sigma_\omega$ is the coherence time and $\Delta\beta = 1 - \beta_z \approx k_u/k$. This equation describes a sum of N_e wavepackets of rms pulse length σ_τ propagating with a

group velocity

$$v_g = \frac{c}{1 + \frac{2}{3}\Delta\beta} = \frac{\omega}{k + \frac{2}{3}k_u}, \quad (2.43)$$

which is slightly less than the speed of light but slightly faster than the electrons. This type of time domain picture was first emphasized by Bonifacio[43]. It is also interesting to note that there exists an intrinsic frequency chirp within each SASE spike due to the quadratic dependence on the time in the imaginary part of the electric field's phase. This chirp is easily calculated and can be expressed as

$$b = \frac{d\phi}{dt} = \frac{1}{2\sqrt{3}\sigma_\tau^2}. \quad (2.44)$$

The frequency dependent propagation velocity and the intrinsic frequency chirp for each of the individual spikes have a large influence on the longitudinal characteristics of SASE FEL light and are discussed in greater detail in the following chapters.

As mentioned previously, ρ is a scaling parameter that determines many characteristics of high-gain FEL systems. A few important characteristics, which will be discussed in later chapters as they pertain to the e-beam energy-chirp and undulator experiments and results, are given below.

- Saturation length: $L_s \sim \frac{\lambda_u}{\rho}$
- Saturation power: $P_s \sim \rho I_{beam} E_{beam}$
- Frequency bandwidth: $\frac{\Delta\omega}{\omega} \sim \rho$
- Power gain length: $L_g = \frac{\lambda_u}{4\pi\sqrt{3}\rho}$
- Cooperation length: $L_c = \frac{\lambda_r}{4\pi\sqrt{3}\rho}$

2.2.4 3-D and Other Electron Beam Effects

Effects other than the e-beam energy detuning can have a strong influence on the performance and characteristics of FEL systems, many of which come from a fully three-dimensional treatment. The easiest way to see this is by taking the logarithmic derivative of the resonance condition, equation 1.45, and evaluating the expression about the forward propagation direction ($\psi = 0$), which produces

$$\frac{\Delta\lambda}{\lambda} = -\frac{2\Delta\gamma}{\gamma} + \frac{K\Delta K}{1 + K^2/2} + \frac{\gamma^2 (\Delta\psi)^2}{1 + K^2/2}. \quad (2.45)$$

The first term on the right, $\Delta\gamma/\gamma$, represents the broadening of the spectrum due to an energy spread in the e-beam. The second term, $K\Delta K/(1 + K^2/2)$, represents the effects due to undulator field or undulator period errors. The third term, $2\gamma^2(\Delta\psi)^2/(1 + K^2/2)$, includes the effects of an angular spread in the electron e-beam, which is usually described by the e-beam *emittance*[44]. Some of these effects are discussed below.

It has already been shown that the relative bandwidth at saturation for a SASE FEL scales as $\Delta\omega/\omega = \Delta\lambda/\lambda \sim \rho$. The energy spread in the e-beam at saturation, therefore, must also scale by a factor

$$\frac{\Delta\gamma}{\gamma} \sim \rho. \quad (2.46)$$

This places a constraint on the maximum initial energy spread an e-beam can have at injection to the FEL. It has been shown above that an initial energy spread can shift the detuning curves and can change the e-beam injection energy responsible for optimal gain. Figures 2.2 and 2.3 also illustrate that an increasing energy spread reduces the maximum growth rate and increases the gain length. The saturation length is also reduced when compared to the ideal case of a cold

e-beam. This is because the energy spread in the e-beam at saturation covers the FEL bandwidth. If the e-beam injected into the FEL has an initial energy spread it occupies a larger phase-space area than a cold beam and can more quickly cover the bandwidth. An obvious constraint, therefore, is that the initial rms energy spread in the e-beam must be much less than the FEL bandwidth:

$$\sigma_\gamma \ll \sigma_\omega \sim \rho. \quad (2.47)$$

An e-beam with a finite emittance, ϵ , is composed of electrons that have a finite and intrinsic angular spread. This e-beam, if left to propagate in free space, would expand indefinitely. The rms emittance quantifies the area in the transverse phase-space an e-beam occupies and is given as

$$\epsilon_x = \sqrt{\sigma_x^2 \sigma_{x'}^2 - \sigma_{xx'}^2}, \quad (2.48)$$

where

$$\sigma_x^2 = \langle x^2 \rangle = \int x^2 f_x dx dx', \quad (2.49)$$

$$\sigma_{x'}^2 = \langle x'^2 \rangle = \int x'^2 f_x dx dx', \quad (2.50)$$

$$\sigma_{xx'} = \langle xx' \rangle = \int xx' f_x dx dx'. \quad (2.51)$$

Here, $f_x = f_x(x, x')$ is the normalized e-beam distribution function in the (x, x') phase-space, and $\sigma_x^2, \sigma_{x'}^2$, and $\sigma_{xx'}$ define the mean-squared e-beam size, divergence and correlation respectively. Electrons that are injected into an undulator with an initial angle or displacement from the axis will undergo betatron oscillations. These oscillations reduce the longitudinal velocity on axis. This, in turn, introduces a spread in the resonant wavelength and associated gain reduction,

which has been discussed previously. The e-beam divergence can be written in terms of the Twiss parameters as $\sigma_{x'} = \sqrt{\epsilon_x/\beta_x}$ [44, 22]. Consulting equation 2.45, a constraint on the e-beam emittance needed for effective FEL operation can be defined as:

$$\epsilon_x < \frac{\lambda}{4\pi} \frac{\beta_x}{L_g}. \quad (2.52)$$

One of the more remarkable features of a SASE FEL is the large degree of transverse coherence of the FEL light pulse obtained at saturation. The initial transverse phase-space of the FEL light consists of a large number of spatial modes. Higher order spatial modes, however, naturally have a larger diffraction[45]. Since the beam-radiation interaction is localized within the e-beam, the fundamental mode couples more effectively and has a larger growth rate in a process known as *gain guiding*. This mode eventually becomes the preferred spatial distribution and dominates the transverse profile[35].

Space-charge effects on FEL operation has been studied extensively[46]. The space charge forces (electrostatic e-beam self forces) are repulsive. Thus, they act in direct opposition to the induced microbunching arising from the e-beam and radiation interaction within an undulator. This has a detrimental effect on the FEL gain process. To help quantify this effect (and others), a power-fit formula is presented in Appendix A for the gain length of a high-gain free-electron laser's fundamental mode in the presence of diffraction, uncorrelated energy spread, and longitudinal space-charge effects. It is obtained from a variational analysis using three-dimensional FEL theory. Space-charge effects are certainly present in optical FELs, where the e-beam energy is relatively low. Thus, the results obtained have specific applications to the topic of this thesis.

The approach in Appendix A is inspired by the work of Xie [36], and provides a useful shortcut for calculating the gain length of the fundamental Gaussian

Parameter	Symbol	Value
Period	λ_u	9 mm
Undulator Parameter	K	1.7
Gap	g	1.6 mm

Table 2.1: Cryogenic undulator parameters for the examples considered here.

mode of a free-electron laser having strong space-charge effects in the 3-D regime. The results derived from analytic theory are in good agreement with detailed numerical particle simulations that also include higher-order space-charge effects, supporting the assumptions made in the theoretical treatment and the variational solutions obtained in the single mode limit.

2.3 SASE FEL Examples

It is useful at this point to consider examples of SASE FELs to better understand and illuminate the concepts mentioned above. Here, the characteristic behavior for some specific SASE FEL systems are evaluated. To this end the time-dependent three-dimensional FEL simulation code GENESIS is used to explore the performance of high-brightness e-beams used in tandem with a short-period, high-field cryogenic undulator[47], the parameters of which are found in table 2.1.

The brightness of an e-beam is defined as:

$$B = \frac{2I}{\epsilon_n^2} \propto \frac{Q}{\sigma_t \epsilon_n^2}. \quad (2.53)$$

Here, I is the e-beam current, $\epsilon_n = \beta\gamma\epsilon$ is the normalized emittance and σ_t is the rms length in time. The brightness may be dramatically increased by using lower charge ($Q \sim 1$ pC) e-beams in the electron injector, rather than the nominal design charge of $Q \sim 1$ nC. This is enabled by the scaling laws which define the emittance and beam sizes that can be obtained for a given charge at the photocathode source[6]. Smaller emittance beams are capable of smaller transverse sizes, $\sigma_x^2 = \epsilon_n\beta_x/\gamma$, where β_x is the Twiss coefficient, in the undulator. Simultaneously, smaller emittance e-beams mitigate some of the collective effects that often degrade beam quality during longitudinal compression, allowing for extremely short final bunch lengths and large peak currents. Therefore, these high density e-beams are capable of reaching unprecedented brightness.

High-brightness beams have several implications for FEL performance. The FEL gain parameter, ρ , scales with the beam density, n_0 , like $\rho \sim n_0^{2/3}$ (see equation 2.29). An increase in the e-beam density produces a larger gain parameter. This, in turn, yields an FEL that has a shorter gain length and shorter saturation length. The saturation power also increases. More efficient driving of the gain process, therefore, partially counterbalances the loss of photons due to the decreased number of electrons participating in the FEL interaction. In addition, if the ultrashort e-beams are compressed such that the final bunch length is on the order of the cooperation length (the slippage distance in a gain length), it becomes possible for the beam to emit one longitudinally coherent radiation spike[5], which pertains directly to this thesis.

The use of high-brightness e-beams in a short-period undulator is motivated by the fact that shorter period undulators nominally produce shorter wavelength light. Thus, it becomes possible to use moderate energy e-beams to produce relatively high energy photons. Using a shorter undulator period, however, reduces

the gain parameter. This can be counterbalanced by keeping the undulator parameter, K , relatively large. Modern permanent magnet undulators are limited in the (λ_u, K) parameter space by available materials. A decrease in λ_u is usually accompanied by a decrease in K . The use of cryogenically cooled exotic materials minimizes this limitation.

The first example considered here utilizes a typical e-beam produced from a laser wakefield accelerator (LWFA)[48] and was the original motivation for investigating the cryogenic undulator. In this case, the short-period undulator allows for the production of soft x-ray photons from the e-beam created by the LWFA. Low charge e-beams from more standard photoinjectors are discussed below. The present case illustrates the standard temporal structure of SASE FEL light where the pulse is composed of coherent spikes somewhat randomly distributed along the longitudinal profile. The main parameters for this scenario can be found in table 2.2. This example serves to illustrate the possibilities in realizing an ultracompact FEL driven by an e-beam from a LWFA. It employs an extremely high current ($I = 160$ kA), moderate emittance ($\epsilon_n = 1 \times 10^{-6}$ m-rad), moderate energy spread ($\sigma_\gamma = 0.1\%$) e-beam to reach saturation in the undulator in 3 m as shown in figure 2.4. The output power for this case is an impressive 1.09 TW. It is important to note that the LWFA FEL reaches saturation even though the e-beam does not satisfy the emittance condition stated in equation 2.52. This is because the e-beam possesses an abnormally large current. The FEL saturates in a short distance because a tremendous amount of electrons are packed into a very short space, allowing them to contribute radiation to the fundamental laser mode before diffraction or emittance effects can play a significant role. This is a rather brute force, but effective, method to reach saturation.

The longitudinal profile of the light at saturation, shown in figure 2.5, con-

Parameter	Symbol	LWFA	LCLS	SPARX
Energy (GeV)	E	1.74	4.5	2.1
Charge (pC)	Q	1600	0.25	1.0
Bunch length (μm rms)	σ_ξ	1.2	0.06	0.21
Current (kA)	I	160	0.35	0.7
Normalized emittance (10^{-8} m-rad)	$\epsilon_{n,x(y)}$	100(100)	3.3(3.3)	7.5(3.3)
Energy spread (10^{-3} rms)	σ_γ	1	0.2	0.25
Radiation wavelength ($\text{\AA}$)	λ_r	9.5	1.4	6.56
Beta function (m)	β	4.3	4.8	2
Gain parameter (10^{-3})	ρ	3.6	0.75	1.8
Saturation length (m)	L_s	3	15	10
Cooperation length (nm)	L_c	12.1	8.6	10.6
Output power (GW)	P	1090	4.5	2.4

Table 2.2: Parameters for the three FEL scenarios considered in this section.

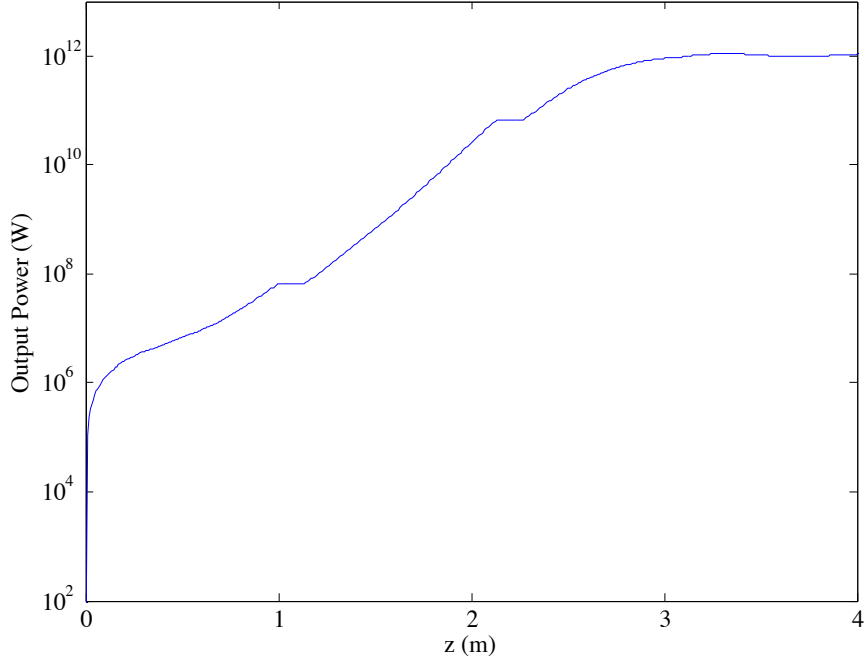


Figure 2.4: Power growth along the undulator for the LWFA FEL.

sists of many spikes. This is because the cooperation length is significantly smaller than the e-beam bunch length. The same spikey signature is found, as expected, in the spectrum in figure 2.6. Figure 2.7 is a two-dimensional plot of the normalized radiation power vs the longitudinal coordinate along the pulse (t) and along the undulator (z). This plot is useful in visualizing the evolution of the individual radiation spikes in the e-beam reference frame as they propagate forward. The three primary regimes of SASE FEL operation are represented in this plot. The start-up regime, where the e-beam first begins to microbunch as it interacts with the initial radiation shot noise, is shown in the region between $z = 0$ m and $z = 1$ m. This is also known as the region of *lethargy*. The region between $z = 1$ m and $z = 3$ m is the exponential gain regime where the power stored in each individual radiation spike grows exponentially. Finally, the region between $z = 3$ m and $z = 4$ m indicates where the FEL has reached saturation.

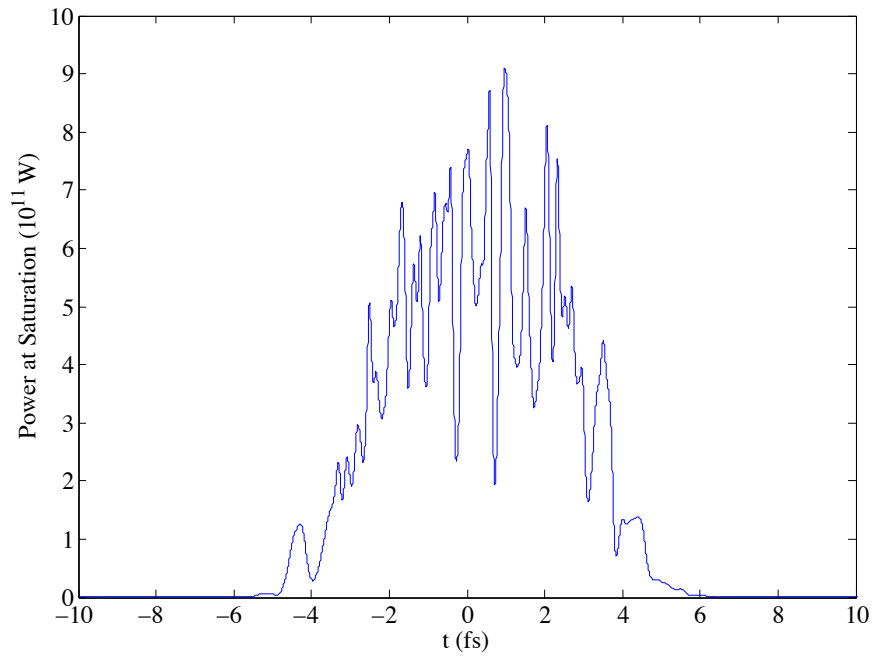


Figure 2.5: Longitudinal profile of the light at saturation for the LWFA FEL.

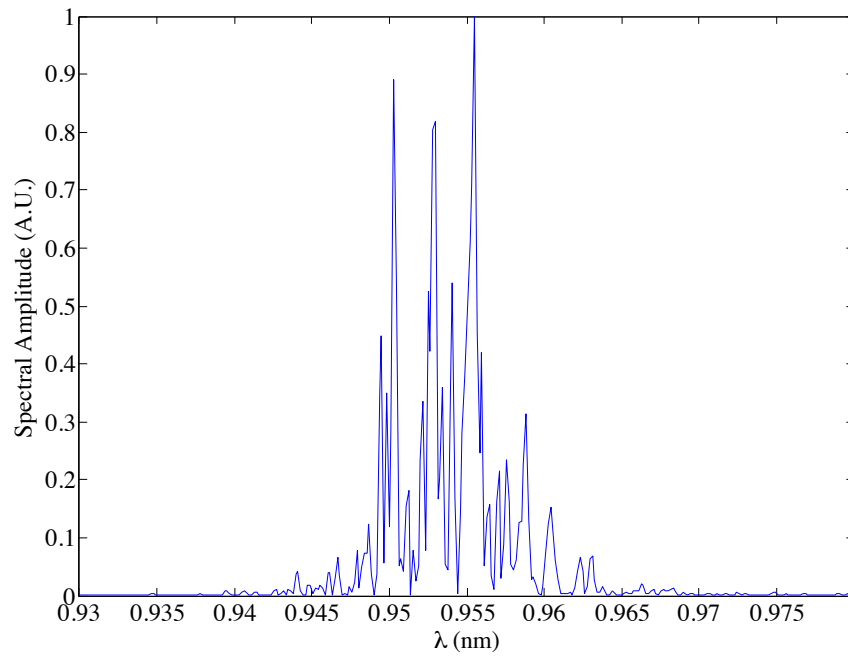


Figure 2.6: Spectrum of the light at saturation for the LWFA FEL.

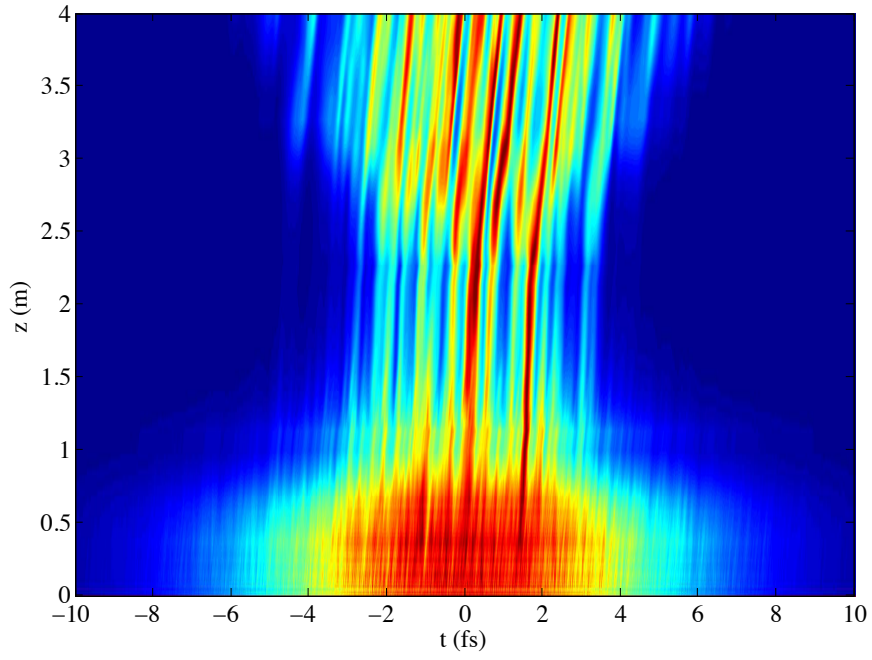


Figure 2.7: Power vs longitudinal coordinate along the pulse (t) and along the undulator (z) for the LWFA FEL. The power is normalized at each z to permit the observation of the spike evolution along the whole undulator.

In the lethargy regime, the light propagates forward at the speed of light. In the exponential gain regime, however, the interaction of the light with the electrons during the gain process slows the propagating wavepackets so that they move forward at the group velocity (see equation 2.43), which is slightly faster than the average e-beam velocity, but not at the speed of light. This is indicated by a change in the slope of a propagating spike (they slip forward less in the e-beam frame). When the FEL reaches saturation the light again propagates forward at a speed near the speed of light and the slope of the propagating spike increases. This phenomenon will be more evident in the next few examples.

The next example uses an ultralow charge e-beam to study the possibility of producing single-spike radiation at the Linac Coherent Light Source (LCLS). The LCLS is the world's first hard x-ray FEL[2]. It nominally runs with an e-beam charge of 250 pC which is capable of producing $\lambda = 1.5 \text{ \AA}$, x-ray photons. The pulse length of the FEL light in this regime normally ranges from 70 to a few hundred femtoseconds. In addition, a low charge scheme of operation ($Q = 20 \text{ pC}$) has also been studied and established[49, 50]. The main goal of this operating mode was to mitigate collective effects during compression so that ultrashort e-beams could be created to produce x-ray pulses that are less than 10 fs. The drive to further shorten the x-ray pulse so that single-spike operation can be achieved is explored here (with the short-period, high-field undulator) for an e-beam with a charge of 0.25 pC.

Start-to-end simulations were performed for this low charge case in the context of the LCLS photoinjector and accelerating sections, which produced a final e-beam bunch length of around $\sigma_\xi \simeq 60 \text{ nm}$ with a peak current of 350 A for a beam with an energy of 4.5 GeV (much lower than the 13.65 GeV e-beam used in the nominal LCLS case). The main parameters for this scenario can be found

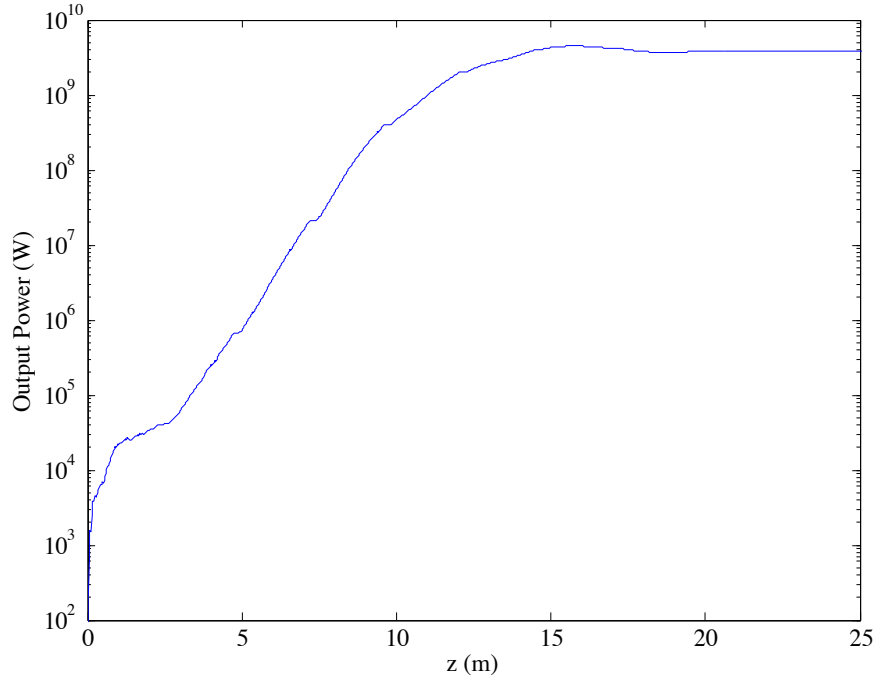


Figure 2.8: Power growth along the undulator for the LCLS FEL case with a 4.5 GeV e-beam.

in table 2.2. Both the longitudinal and transverse phase-space are compact and show no signs of filamentation during compression, which was accomplished by velocity bunching (see section 4.2.2.1) at low energy and chicane compression[51] at high energy. The final emittance in both the x and y planes is $\epsilon_n = 3.3 \times 10^{-8}$ m-rad while the final energy spread is $\sigma_\gamma = 2 \times 10^{-4}$. All of these quantities easily satisfy the conditions mentioned in section 2.2.4. This is vital for pushing the FEL to shorter wavelength operation.

The power growth along the undulator is shown in figure 2.8. Saturation is reached within $z = 15$ m. This is just over a factor of 6 times shorter than what is nominally achieved in the standard LCLS high charge case. The output power at a wavelength of $\lambda = 1.4 \text{ \AA}$ is an impressive 4.5 GW considering the charge is 3 order of magnitude smaller than the standard LCLS case. The longitudinal

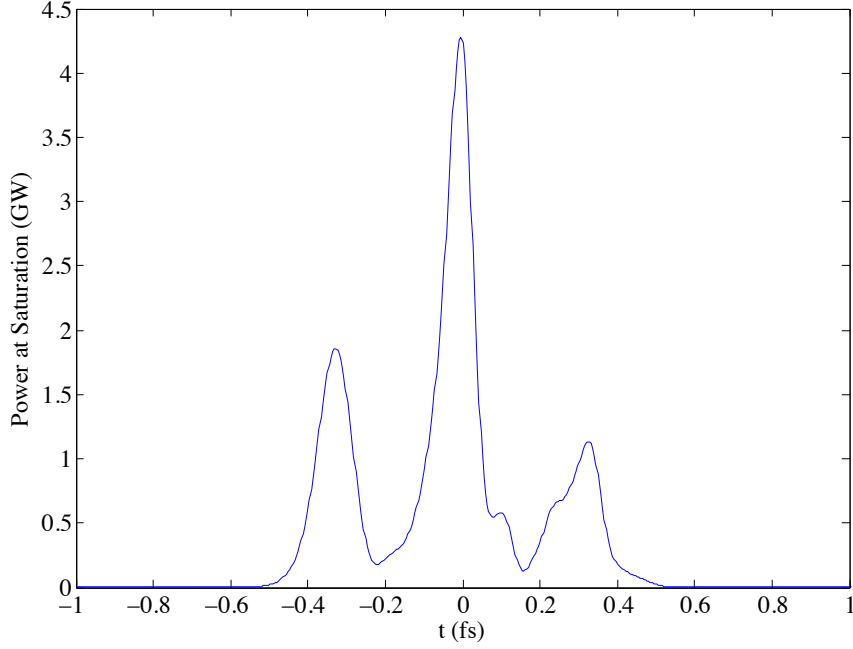


Figure 2.9: Longitudinal profile of the light at saturation for the LCLS FEL case with a 4.5 GeV e-beam.

profile in both the temporal and spectral domains can be found in figures 2.9 and 2.10 respectively. Notice that the typical SASE spiking found in the LWFA scenario is absent. While single-spike operation was not achieved here, due to an extremely short cooperation length, the longitudinal profile of the pulse is cleaned up significantly, with the main spike in the temporal domain possessing a FWHM of less than 0.5 fs.

The two-dimensional plot of the normalized radiation power vs the longitudinal coordinate along the pulse (t) and along the undulator (z) is shown in figure 2.11. The three regimes of SASE FEL operation are once again evident. However, only three spikes reach full saturation in this case. The distinction in the propagation velocities between the exponential gain regime and saturation is more clearly evident in this figure where the transition is seen to occur around

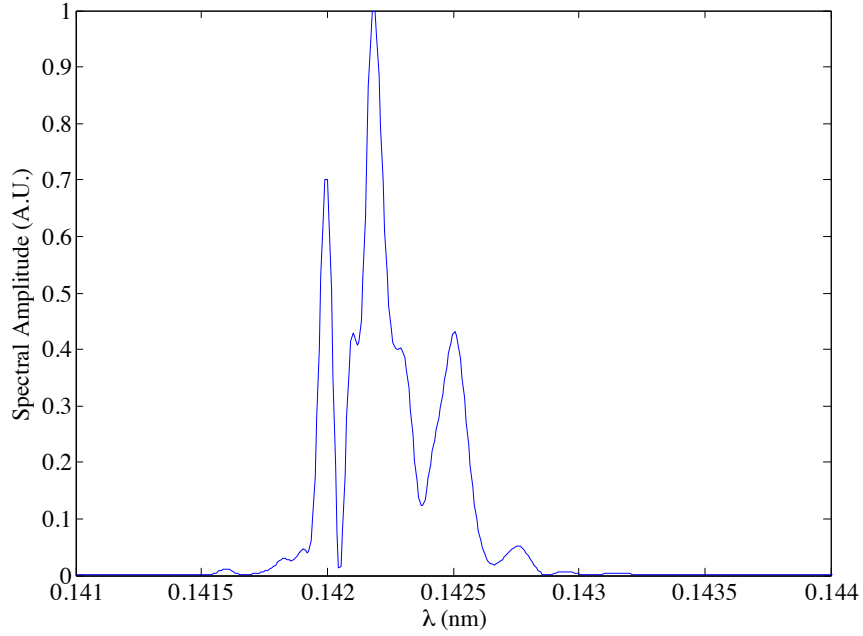


Figure 2.10: Spectrum of the light at saturation for the LCLS FEL case with a 4.5 GeV e-beam.

the $z = 12$ m mark.

The final example discussed in this section thoroughly exploits the advantages of coupling low charge e-beams to the short period, high field undulator. This case examines the usage of a low charge e-beam ($Q = 1$ pC) from the SPARX FEL[52], which is currently under design in Frascati, Italy. The e-beam in this case is compressed by velocity bunching at low energy and further compressed at high energy by a magnetic chicane. This produces a final bunch length of $\sigma_\xi = 0.21 \mu\text{m}$ and a peak current just over $I = 700$ A. The transverse emittances in this case are $\epsilon_{n,x(y)} = 7.5(3.3) \times 10^{-8}$ m-rad. The larger horizontal emittance is a consequence of coherent synchrotron radiation effects in the chicane. Other parameters of interest for this scenario are given in table 2.2.

The power growth along the undulator is shown in figure 2.12. Saturation is

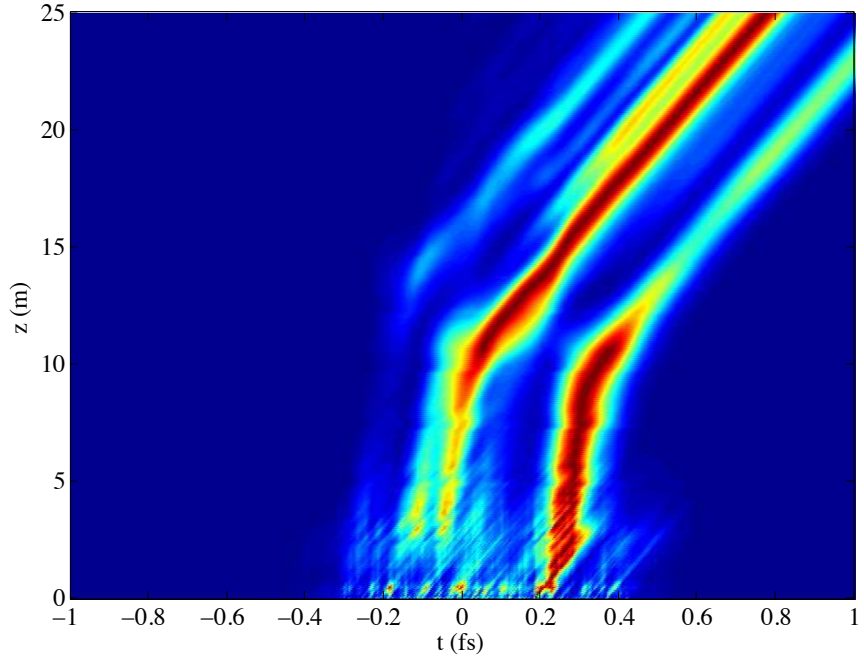


Figure 2.11: Power vs longitudinal coordinate along the pulse (t) and along the undulator (z) for the LCLS FEL case with a 4.5 GeV e-beam. The power is normalized at each z to permit the observation of the spikes evolution along the whole undulator.

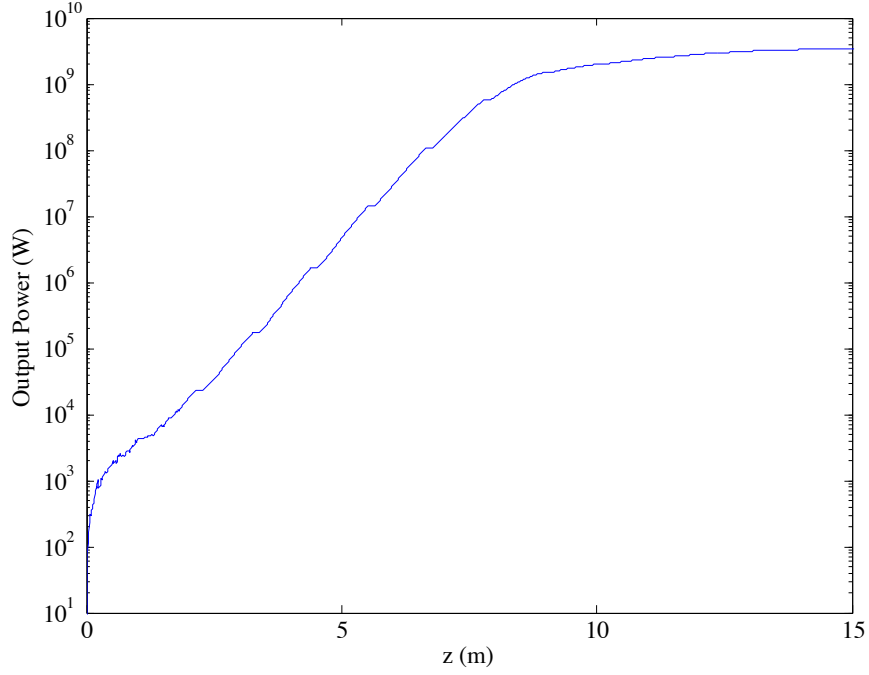


Figure 2.12: Power growth along the undulator for the SPARX FEL.

reached within $z = 10$ m and at a wavelength of $\lambda = 6.56$ Å. This is only about ~ 4 times the normal LCLS wavelength, and was achieved with a relatively modest e-beam energy of 2.1 GeV. The longitudinal profiles in both the temporal and spectral domains are shown in figures 2.13 and 2.14, respectively. Despite such a short cooperation length, the ultrashort e-beam in this scenario was capable of producing quasi-single-spike radiation in the soft x-ray wavelegnth regime with an rms pulse length of 0.35 fs.

The two-dimensional plot of the normalized radiation power vs the longitudinal coordinate along the pulse (t) and along the undulator (z) is shown in figure 2.15. The three regimes of operation are clearly evident in this plot. The discontinuities in radiation propagation velocity between the lethargy and exponential gain regimes occurs around $z = 2.5$ m while the discontinuity in propagation ve-

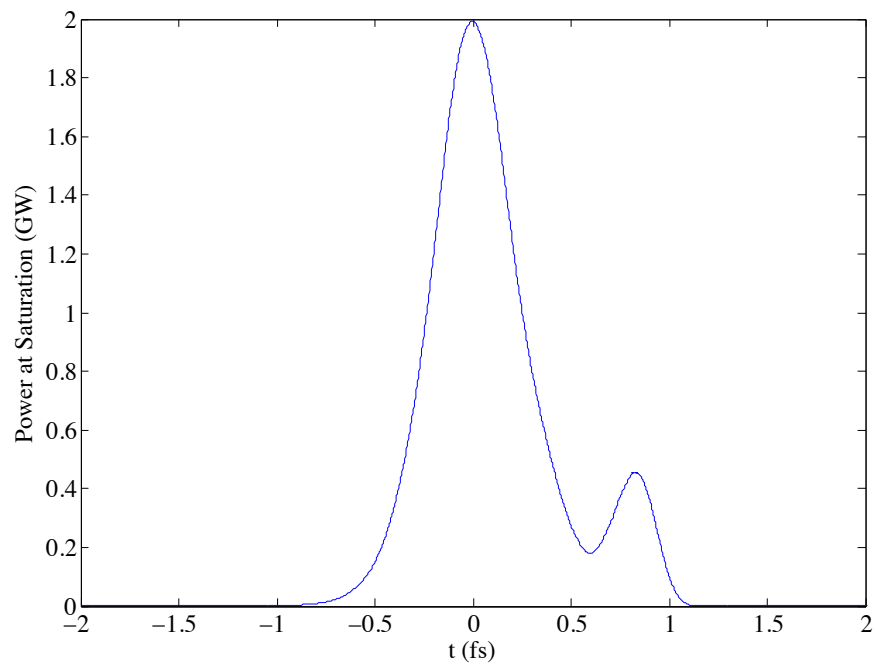


Figure 2.13: Longitudinal profile of the light at saturation for the SPARX FEL.

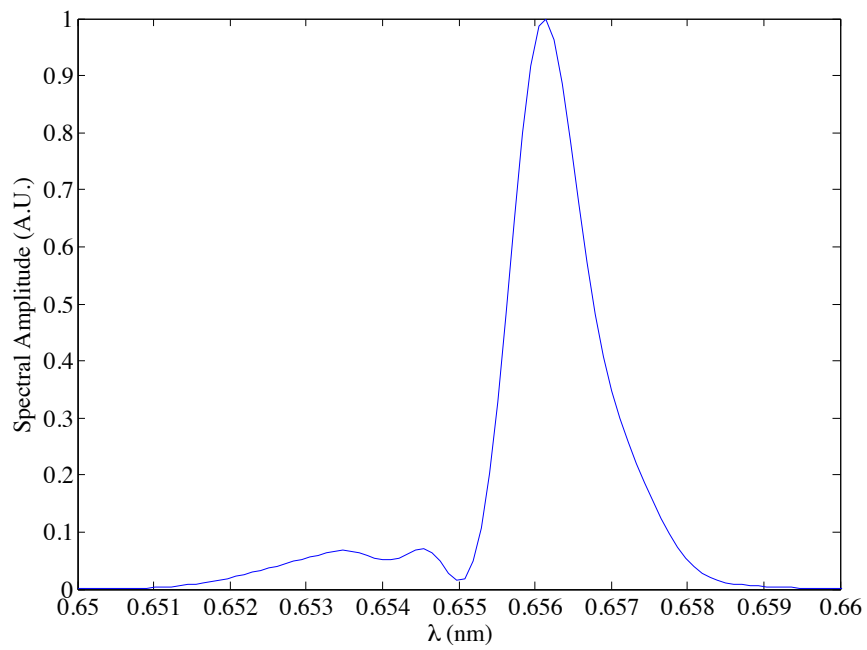


Figure 2.14: Spectrum of the light at saturation for the SPARX FEL.

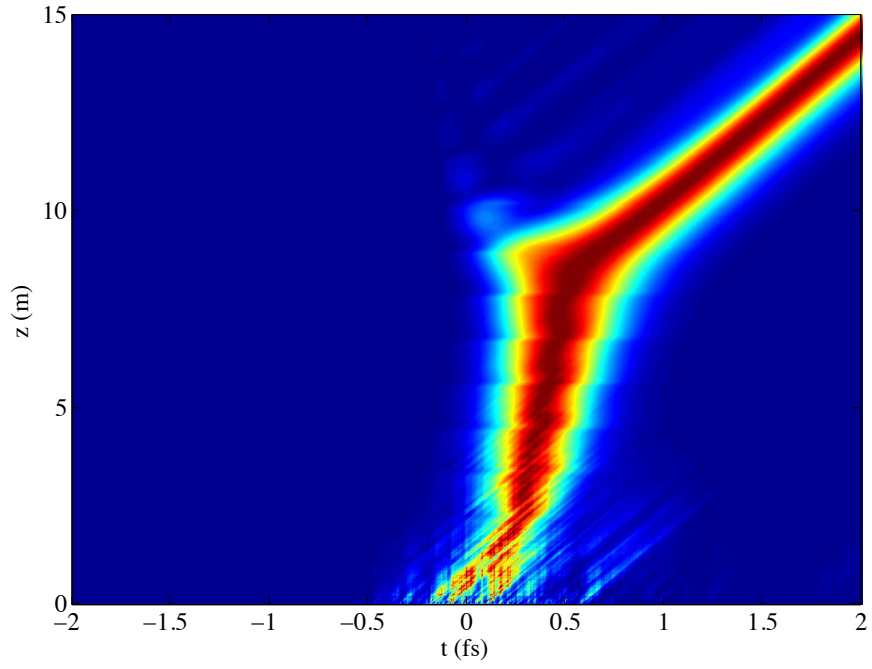


Figure 2.15: Power vs longitudinal coordinate along the pulse (t) and along the undulator (z) for the SPARX FEL. The power is normalized at each z to permit the observation of the spikes evolution along the whole undulator.

locity between the exponential gain regime and saturation occurs around $z = 10$ m. This plot also clearly shows the amplification of one dominant, coherent spike of radiation.

CHAPTER 3

Transient-Grating Frequency-Resolved Optical Gating

3.1 Introduction

The basic operating principles behind FROG were explored in Chapter 1, where the mechanisms used to generate the FROG trace and extract the pulse amplitude and phase information were first introduced. This chapter describes in detail the specific TG diagnostic geometry that was implemented at the SPARC FEL test facility in Frascati while emphasizing the difficulties in using such an advanced diagnostic on a FEL. In addition, the physics on nonlinear optics, with an emphasis on third-order processes, is briefly explored in order to more clearly identify how the FROG signal field is created. This process is approached from two complementary perspectives. The first perspective aims at describing the production of the signal field from a diffraction grating picture. Here, two intense pulses of light interfere in a third-order nonlinear optical crystal and spatially modify the medium properties. A third pulse of light interacts with this grating and can diffract into a unique direction if the *Bragg* condition is satisfied. This diffracted signal can therefore be isolated and studied. The second perspective evaluates the establishment of a third-order nonlinear polarization from the interaction of three intense pulses of light within the nonlinear optical crystal. This polarization acts

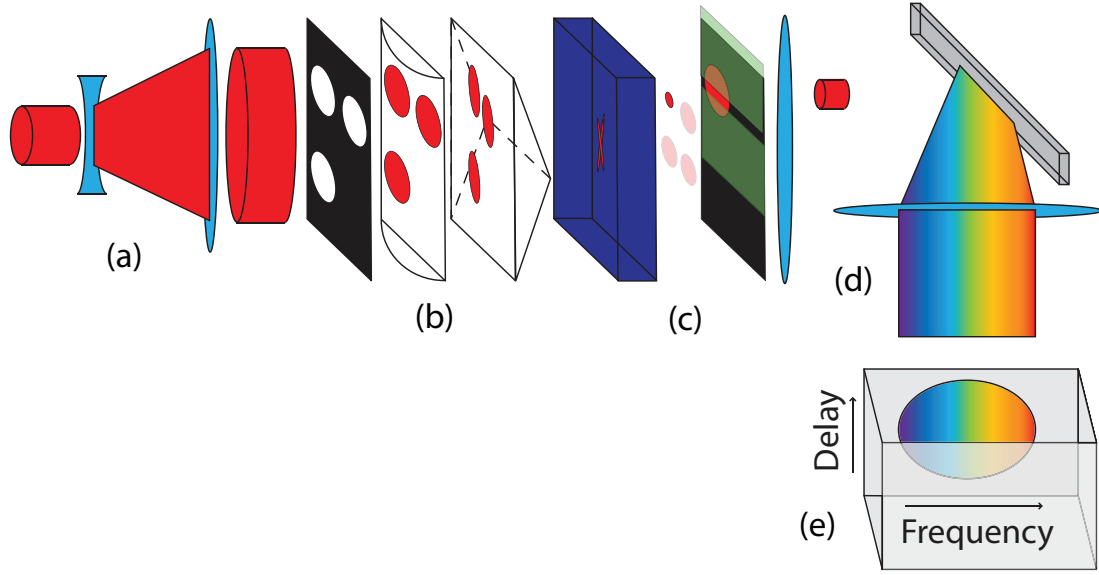


Figure 3.1: Schematic of transient-grating FROG diagnostic: (a) beam expander; (b) Input mask, cylindrical lens, Fresnel bi-prism; (c) nonlinear optical medium, output mask, knife edge slit; (d) Focusing optics, diffraction grating; (e) CCD camera.

as a source term for a fourth light pulse only if the conditions for phase-matching are satisfied.

3.2 Diagnostic Geometry

The TG FROG diagnostic used to extract the longitudinal profile was constructed with the unique capabilities and challenges of measuring FEL output in mind, particularly in its extendibility to short wavelength operation[53, 54]. It was based on a geometry first proposed and demonstrated by Lee[55]. The diagnostic geometry as constructed is shown in figure 3.1. A very early iteration of the diagnostic is also shown in figure 3.2, where many of the optics in the setup are spaced far apart in order to clearly label the individual components. Later versions of the diagnostic have been made more compact. The light entering

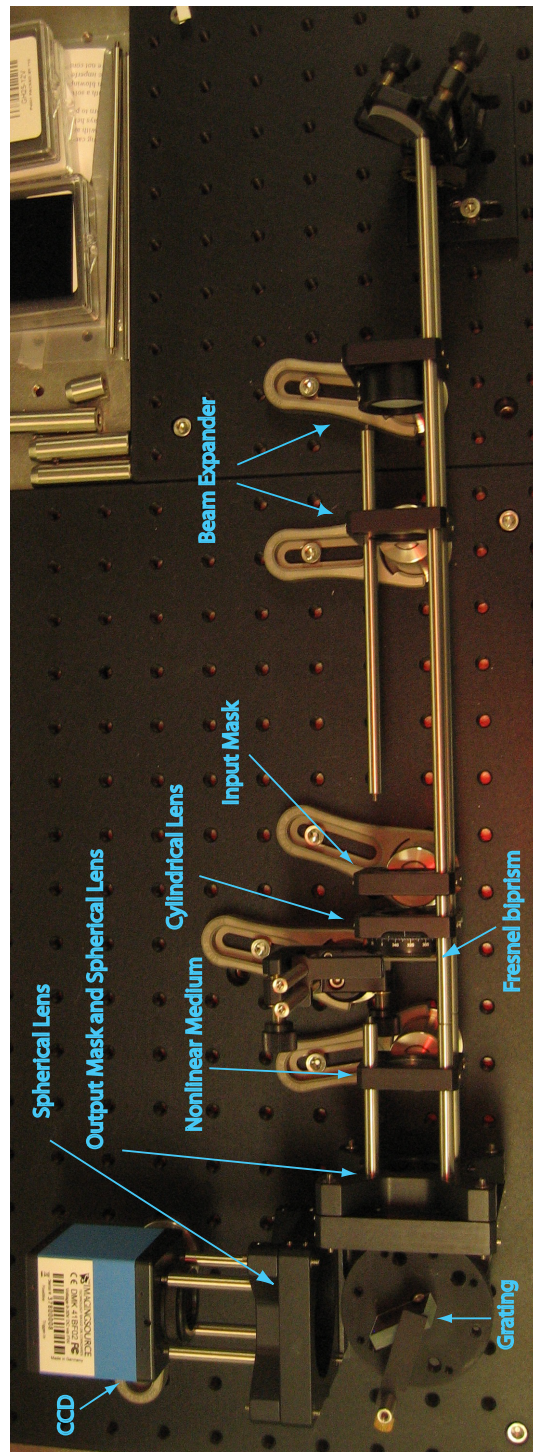


Figure 3.2: Image of the diagnostic during the design phase at the UCLA PE-GASUS laboratory.

the FROG device is first expanded to a relatively large diameter using a basic telescope, allowing the generation of three identical copies of the input beam after passage through a mask. The mask was machined from aluminum and was black anodized with a matte finish to reduce reflections and stray light. It contains three holes each with a diameter of 5 mm. These holes were placed in such a way that the diagnostic operated in the familiar BOXCARS configuration[56]. The application of the mask is vital as it avoids the use of multiple delay stages and beam splitters, and associated alignment difficulties, to produce the three copies of the beam. More importantly, however, the beam expansion serves to transversely filter the pulse. This mitigates possible sources of corruption in the FROG reconstruction algorithms that may result from nonuniform transverse intensity profiles as well as spatial (transverse) frequency chirp. Spatial chirp, as mentioned in section 1.4.1, is prevalent in FEL light pulses and is a result of the angular dependence of the FEL resonance condition on the emission angle[57]. This quadratic dependence (see equation 1.45) can clearly be seen in figure 3.3. These spectral images were taken with the in-vacuum spectrometer at the SPARC FEL facility for an e-beam experiencing exponential gain in the SASE mode. Here, the horizontal axis serves as the dispersive direction and is wavelength calibrated while the vertical axis indicates the position on the vertical entrance slit of the spectrometer. The details of this spectrometer are discussed in chapter 4. Spatial chirp is clearly evident in the spectrum taken after the first undulator module (UM in figure) and persists through the gain process as can be seen in subsequent undulator sections. This is indicated by the quadratic dependence of the wavelength on the vertical transverse dimension.

After their generation, the probe (from the lower hole in the mask) and two gate pulses (from the upper holes in the mask) are passed through a cylindrical lens, which brings each pulse to a line focus within the third-order nonlinear

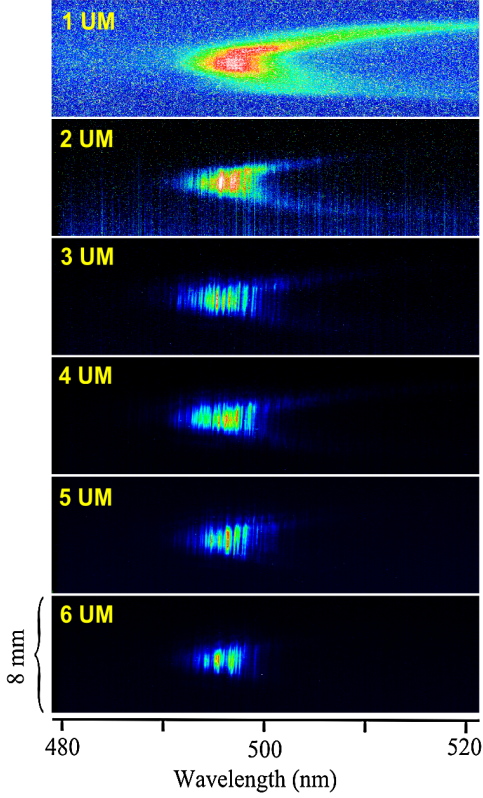


Figure 3.3: Evolution of the SPARC SASE FEL spectrum during the exponential gain process[57].

optical medium. A 2 mm thick UV grade fused silica crystal was used as the third-order nonlinear material because it supports broadband (UV - IR) operation and is less dispersive than other materials. Before reaching the nonlinear medium, the pulses are passed through a Fresnel bi-prism with an apex angle of 160 degrees. This crosses the probe and gate pulses at a large angle within the nonlinear optical medium. A variable delay is created between the interfering beams that is mapped along the vertical transverse dimension, allowing the diagnostic to make single-shot measurements. This concept is more clearly illustrated in figure 3.4. The ability to make single shot measurements is critically important for SASE FEL light pulses, where there is a large shot-to-shot variability due to

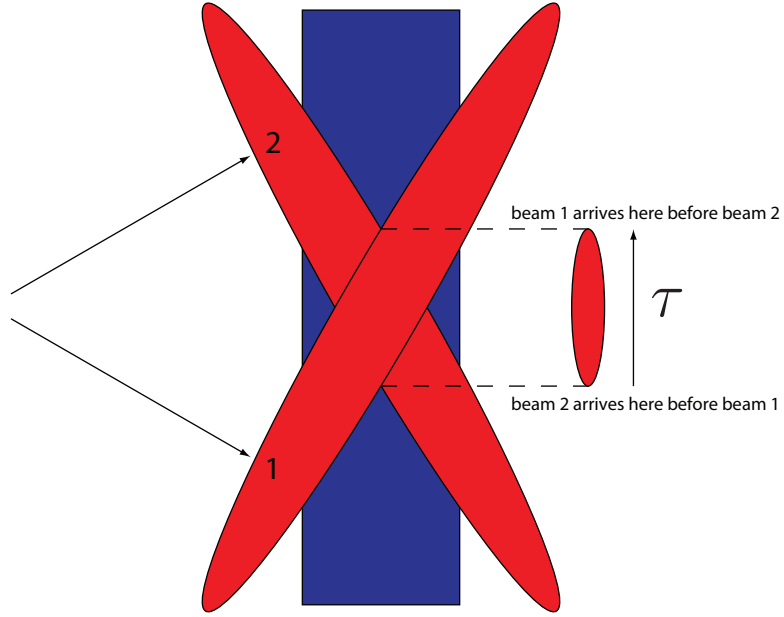


Figure 3.4: Two beams that are crossed at an angle within a nonlinear optical medium map a relative delay of the signal field to a transverse dimension.

the stochastic startup nature. A major advantage of the Fresnel bi-prism is that it is automatically aligned in space as well as in time. The interference of the beams within the nonlinear medium generates an autocorrelation signal field $E_{sig}(t, \tau) \propto E(t)|E(t - \tau)|^2$, by a transient-grating nonlinear interaction termed degenerate four-wave mixing (DFWM)[58, 59]. This process is described in detail below. Here, τ is the relative delay between the gate and probe pulses. This signal is not symmetric with respect to delay, and, as a result, does not have ambiguity in the direction of time. Also, the third-order nonlinear optical process does not suffer from the phase-matching constraints between the signal and pump fields that limit second-order nonlinear processes. This permits phase matching across a broad spectral range, which is ideal for a tunable laser source such as the FEL; it is limited only by the transparency of the nonlinear optic. The signal field is created at the same frequency as the pump and probe pulses,

as opposed to the second-harmonic generated in second-order nonlinear optical processes. This makes it challenging to filter scattered or stray light, arising from spontaneous undulator emission, or radiation that diffracts (overcoming the gain guiding) out of the FEL upstream. Once the signal field is optimally isolated, it is passed through a custom fabricated spectrometer consisting of focusing optics and a holographic reflective diffraction grating where it is spectrally resolved along the transverse dimension orthogonal to the delay direction, and imaged into a CCD camera to yield a FROG trace. The line focus in the nonlinear optic serves as the entrance slit to the spectrometer. The diffraction grating is mounted on a remotely controlled motorized rotation stage. Changing the angle at which the light impinges on the grating changes the wavelength of light that ultimately reaches the CCD camera. This is the only diagnostic realignment that is necessary when tuning the FEL (or any other laser source) to a different resonant wavelength. The expression for the trace in this particular geometry is

$$I_{FROG}^{TG}(\omega, \tau) \propto \left| \int_{-\infty}^{\infty} E(t) |E(t - \tau)|^2 \exp(-i\omega t) dt \right|^2. \quad (3.1)$$

Iterative Fourier transform algorithms are then used to retrieve the complex electric field, $E(t)$. While the TG FROG concept is not new[60, 61], it should be noted that the geometry implemented here has minimal alignment degrees of freedom and operates on a single-shot basis; key features that help simplify a complex measurement at an FEL.

3.3 Nonlinear Optics

The workhorse in any FROG diagnostic is the nonlinear optical process that is used to generate the signal field from the interference of the probe and gate pulses

within a nonlinear optical medium. The time information of the unknown pulse is encoded in this signal field. Here, the operating principles behind the DFWM process used for this particular FROG diagnostic are outlined and follows closely the analysis in [58, 62, 20].

3.3.1 Basic Concepts

When an electric field is applied to a dielectric material it will establish a macroscopic quantity known as the polarization, $\vec{P}(\omega)$. If the electric field is not too strong the polarization will exhibit a linear dependence on the electric field, $\vec{E}(\omega)$:

$$\vec{P}(\omega) = \epsilon_0 \boldsymbol{\chi}(\omega) \cdot \vec{E}(\omega). \quad (3.2)$$

Here, $\boldsymbol{\chi}(\omega)$ is the optical susceptibility of the material and is, in general, a tensor (indicated by the bold symbol). The response of the medium to an applied electric field is usually described by a complex index of refraction, $\tilde{\mathbf{n}}(\omega)$:

$$\tilde{\mathbf{n}}(\omega) = \sqrt{1 + \boldsymbol{\chi}(\omega)} = \mathbf{n}(\omega) + i\mathbf{K}(\omega). \quad (3.3)$$

Here, $\mathbf{n}(\omega)$ is the index of refraction and $\mathbf{K}(\omega)$ is the attenuation constant within the medium. For an isotropic and linear medium, $K(\omega)$ and $n(\omega)$ are related by the familiar *Kramers-Kronig* relations[21]:

$$n(\omega) = 1 + \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{K(\omega')}{\omega' - \omega} d\omega', \quad (3.4)$$

$$K(\omega) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{n(\omega') - 1}{\omega' - \omega} d\omega'. \quad (3.5)$$

It is understood that the integrals should include only the principle part in the complex ω' plane.

The polarization can act as a source term for new radiation at the frequency ω through the relevant Maxwell equations. When the applied electric field of the light is intense enough, however, the material response, and thus $\chi(\omega)$ itself, can exhibit a complicated dependence on the field $\vec{E}$. In this case, the polarization can be expanded into a power series in $\vec{E}$ [17, 63] to obtain:

$$\vec{P} = \vec{P}^L + \vec{P}^{NL} = \epsilon_0 \left[\chi^{(1)} \cdot \vec{E} + \chi^{(2)} \cdot \vec{E} \cdot \vec{E} + \chi^{(3)} \cdot \vec{E} \cdot \vec{E} \cdot \vec{E} \right]. \quad (3.6)$$

This expansion is generally referred to as the *constitutive relationship*. There are many different behaviors that can result from the various terms in the expansion. For the purposes of this thesis, however, only a nonlinear response arising from the third-order nonlinear optical susceptibility, $\chi^{(3)}$, will be considered.

The third-order polarization which results from the intense interaction of light pulses is described by

$$P_i^{(3)}(\omega) = \epsilon_0 \chi_{ijkl}^{(3)}(\omega) E_j(\omega_1) E_k(\omega_2) E_l(\omega_3) \quad (3.7)$$

where the indices i , j , k , and l refer to the Cartesian coordinate axes, $\chi_{ijkl}^{(3)}$ is the relevant tensor element from one of the $3^4 = 81$ elements of the third-order nonlinear susceptibility, $\chi^{(3)}$, and the Einstein summation convention for repeated indices is assumed. The symmetry properties, which reduce the number of relevant terms, and specific calculation of the elements of the third-order susceptibility is beyond the scope of this work. From equation 3.7, it is immediately apparent that the nonlinear polarization can oscillate, and thus act as a source of new radiation, at the frequencies $\omega = \pm\omega_1 \pm \omega_2 \pm \omega_3$. In general, the third order nonlinear polarization, $\vec{P} = \vec{P}(\vec{k}, \omega)$, also has a spatial/directional dependence on $\vec{k}$, which arises from the propagation direction of the interfering intense pulses of

light, $\vec{E} = \vec{E}(\vec{k}, \omega)$. Rather than continuing down the path of the mathematical description of the generation of new light from a driving nonlinear polarization term, it is useful at this point to consider the simple grating picture of nonlinear optics specific to the case of the TG FROG device.

3.3.2 Transient-Grating Interactions

The optical properties of matter can become spatially modulated in the presence of two interfering intense light waves, i.e. interference between two laser sources. This interference produces a spatially periodic intensity and/or polarization material excitation which couples to the refractive index and/or absorption coefficient detailed in equation 3.3 above. The periodic spatial modification of the material properties acts as a diffraction grating. Diffraction gratings created in this way only exist in the presence of the interfering light pulses and turn off when the pulses are not present. Thus, these gratings are often referred to as dynamic, or transient, gratings. Of interest to this thesis is a DFWM process by which a probe beam is diffracted from a material grating caused by the interference of two pump beams within a third-order nonlinear optical material.

The production of this type of grating can be illustrated by considering a simple case of plane-wave interference as seen in figure 3.5. Here, two pump laser pulses of wavelength λ_p and wave vectors $\vec{k}_1$ and $\vec{k}_2$, electric field amplitudes of $\vec{E}_1$ and $\vec{E}_2$ and intensities of I_1 and I_2 are crossed at an angle θ within a nonlinear optical material. These quantities are all measured within a material with an index of refraction, n . The wave vectors $\vec{k}_1$ and $\vec{k}_2$ are confined to the Cartesian x-y plane and are defined as

$$\vec{k}_{1,2} = k_z \hat{z} \pm k_x \hat{x}. \quad (3.8)$$

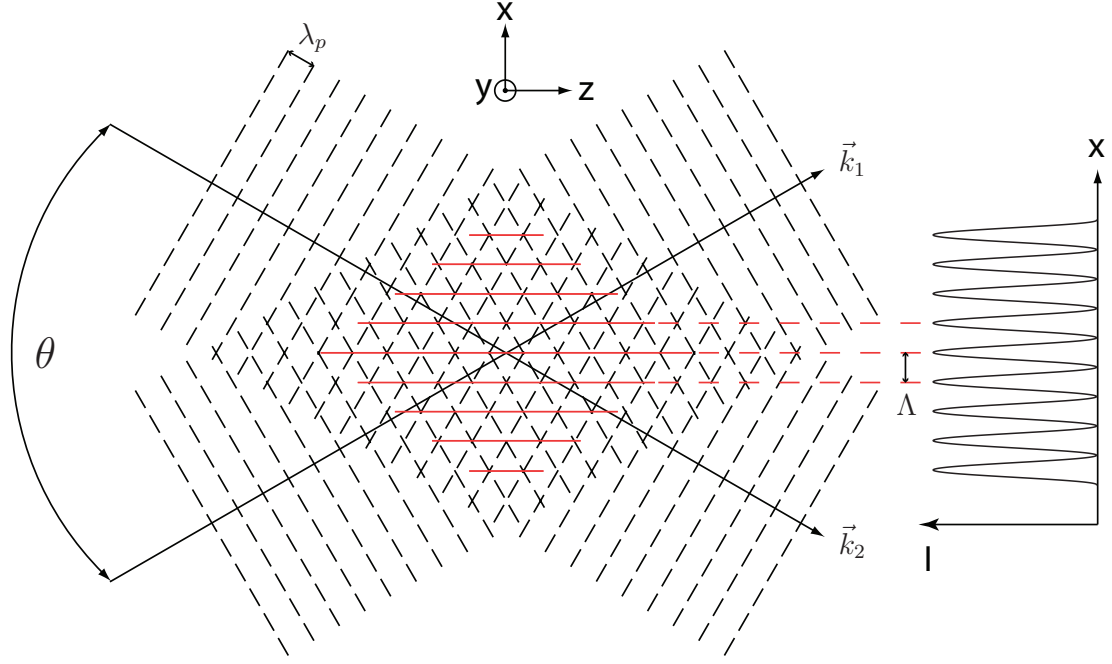


Figure 3.5: Production of a material excitation grating by interference of two plane waves.

The interaction of these pulses creates an interference pattern, defined by the grating vector $\vec{q}$, which is

$$\vec{q} = \pm(\vec{k}_1 - \vec{k}_2) = \pm 2k_x \hat{x}. \quad (3.9)$$

The spatial period of the induced grating is

$$\Lambda = \frac{2\pi}{|\vec{q}|} = \frac{\lambda_p}{2\sin\left(\frac{\theta}{2}\right)} \quad (3.10)$$

and is a function of the pump pulse wavelength and interaction angle. For these

two interacting plane waves the total time dependent field is given by

$$\vec{E} = \left(\vec{E}_1 e^{ik_x x} + \vec{E}_2 e^{-ik_x x} \right) e^{(ik_z z - \omega_p t)} + c.c. \quad (3.11)$$

The intensity distribution is therefore

$$\begin{aligned} I &= \frac{1}{2} n \epsilon_0 c \left(\vec{E} \cdot \vec{E}^* \right) \\ &= I_1 + I_2 + 2\Delta I \cos(2k_x x) \end{aligned} \quad (3.12)$$

where

$$\Delta I = \frac{1}{2} n \epsilon_0 c \vec{E}_1 \cdot \vec{E}_2^* \quad (3.13)$$

is the intensity modulation and the asterisk indicates a complex conjugation (*c.c.*). Equation 3.13 is the quantity of interest for transient grating creation. This quantity is, in general, a tensor. It may contain elements, depending on the material symmetry properties, that allows crossed polarized interfering beams to create gratings in *anisotropic* materials.

The intensity modulation of the interfering pump pulses in a medium couples to an excitation of the material and forms a spatial modulation of the material properties. The type of excitation can vary from scalar (temperature) to vector (electric field) to tensor (stress, strain, distribution of excited molecules). Therefore, it is convenient to write the material excitation in tensor form:

$$\Delta X_{ij} = g_{ijkl} \Delta I_{kl}, \quad (3.14)$$

where g is a coupling constant and ΔX describes the excitation. Regardless of the type of excitation, the material grating couples to either the index of refraction, n (amplitude grating), the absorption, K (phase grating), or both. The modulation

of the material properties with amplitude ΔX is accompanied by the creation of a grating with amplitudes

$$\Delta n = \frac{\partial n}{\partial X} \Delta X, \quad (3.15)$$

$$\Delta K = \frac{\partial K}{\partial X} \Delta X. \quad (3.16)$$

This can also be combined for the complex refractive index:

$$\Delta \tilde{n} = \frac{\partial \tilde{n}}{\partial X} \Delta X, \quad (3.17)$$

where the tensor character of ΔX has been ignored.

Gratings induced by the processes mentioned above can be probed by a third pulse of light with field $\vec{E}_3$, wave vector $\vec{k}_3$ and intensity I_3 . The induced transient grating can diffract the probe beam into various directions which depends strongly on the grating orientation $\vec{q}$, the incident direction of the probe pulse $\vec{k}_3$, and the material sample, and thus grating, thickness. The discussion here is confined to the analysis of *thick* gratings. Thick gratings, as a consequence of constructive interference, can only be efficiently probed if the diffracted wave satisfies the Bragg diffraction condition:

$$\vec{k}_{s,m} = \vec{k}_3 + m\vec{q}. \quad (3.18)$$

Here, $\vec{k}_{s,m}$ is the diffracted wave and m can take the values $m = \pm 1, \pm 2$, etc. A complete and detailed discussion of the grating creation and detection is beyond the scope of this work but can be found in reference [58].

Figure 3.6 shows the four wave interaction geometry used in the TG FROG diagnostic while figure 3.7 shows a detailed k-vector diagram. The top two beams

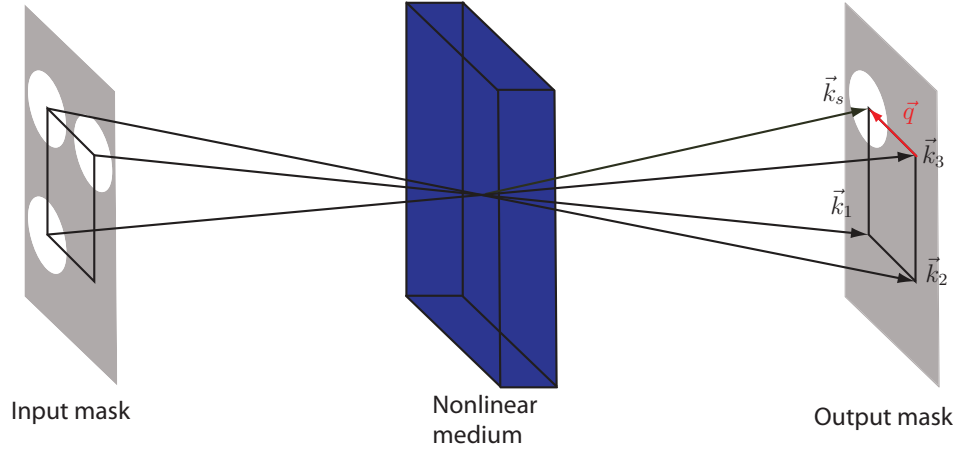


Figure 3.6: TG interaction geometry.

that are generated by the input mask with wave vectors $\vec{k}_1$ and $\vec{k}_2$ interfere in the nonlinear medium and form a material excitation grating. This grating is defined by the grating vector $\vec{q}$ in both figures. The third beam generated by the input mask partially diffracts from this grating into a unique direction as defined by equation 3.18 and is indicated in figures 3.6 and 3.7 by the wave vector $\vec{k}_s$. This diffracted signal propagates in a unique direction and can be segregated from the three input beams by the use of an output mask. In this way, the signal field generated by the nonlinear interaction, which contains the pulse time information, can be isolated and spectrally resolved.

3.3.3 Nonlinear Polarization

It was shown (equations 3.6 and 3.7) that ultrashort light pulses with sufficiently high energy could drive a nonlinear response within an optical medium. The nonlinear response takes the form of a nonlinear polarization term in Maxwell's equations and can act as a source of new radiation (the FROG signal field) at a frequency, and in a direction, that is dependent on the interacting pulses. A

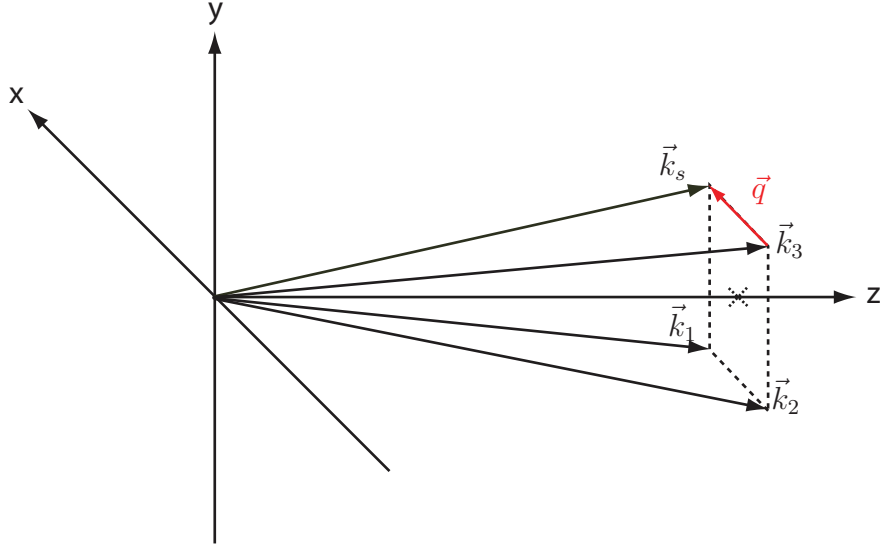


Figure 3.7: Four wave mixing for the TG FROG diagnostic.

third-order nonlinear response is considered here in the context of the TG FROG diagnostic geometry shown in figures 3.6 and 3.7.

Some assumptions must initially be made in order to make the analysis more tractable. First, it is assumed that the focusing conditions of the interacting beams are such that the confocal parameter is much longer than the longitudinal interaction length within the nonlinear optical medium. Under this condition, the wavefronts of the interacting beams are nearly flat and the generation of the signal field can be considered in the longitudinal direction only while ignoring transverse effects. Second, it is assumed that neither the three interacting beams nor the signal field that they generate are absorbed in the nonlinear medium. Third, it is assumed that the nonlinear interaction is a low efficiency process and is purely third order. Under this assumption none of the interacting beams is depleted and the coupled set of equations describing this four-wave interaction is reduced to one for the signal field.

The equation that describes the propagation and growth of the signal field in the $+z$ direction inside the crystal can be obtained from Maxwell's equations and is given by[17]:

$$\frac{\partial^2}{\partial z^2} E_s(z, t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} D_s(z, t) = \mu_0 \frac{\partial^2}{\partial t^2} P^{(3)}(z, t), \quad (3.19)$$

where

$$D_s(z, t) = \int \epsilon(t - t') E_s(z, t') dt' \quad (3.20)$$

is the linear electric displacement $D_s(z, t) = \epsilon_0 E_s(z, t) + P^{(1)}(z, t)$ [21]. Much like the analysis presented in chapter 2, it is useful at this point to move into the frequency domain for multiple reasons. For one, the spectrum of the signal field is easily measured by a standard spectrometer while the temporal intensity is not. The second reason will be illuminated shortly. Moving to the frequency domain by writing the signal field and polarization as a Fourier superposition of monochromatic waves transforms equation 3.19 into

$$\frac{\partial^2}{\partial z^2} \tilde{E}_s(z, \Omega) + k_s^2(\Omega) \tilde{E}_s(z, \Omega) = -\mu_0 \Omega^2 \tilde{P}^{(3)}(z, \Omega). \quad (3.21)$$

Here, the transformation is made into the Ω frequency space to easily distinguish the frequency of the signal field from the input waves,

$$k_s^2(\Omega) = \frac{\Omega^2 \tilde{\epsilon}(\Omega)}{c^2} = \frac{\Omega^2 n(\Omega)^2}{c^2} \quad (3.22)$$

is the frequency dependent wave vector of the signal field, $\tilde{\epsilon}(\Omega)$ is the Fourier transform of the relative permittivity, $\epsilon(t)$, and $n(\Omega) = \sqrt{\epsilon(\Omega)}$ is the index of

refraction. The plane wave approximation

$$\tilde{E}_s(z, \Omega) = \tilde{\mathbb{E}}_s(z, \Omega)e^{ik_s(\Omega)z} \quad (3.23)$$

is now made to further simplify equation 3.21 using the slowly varying amplitude approximation (see equation 2.3). Substituting equation 3.23 into 3.21 produces:

$$2ik_s(\Omega)\frac{\partial}{\partial z}\tilde{\mathbb{E}}_s(z, \Omega) = -\mu_0\Omega^2\tilde{P}^{(3)}(z, \Omega)e^{-ik_s(\Omega)z}. \quad (3.24)$$

A similar analysis could have been performed in the time domain rather than the frequency domain. However, the slowly varying amplitude approximation would have to have been made in the time domain to neglect the second order temporal derivative. This approximation is not valid for pulses that are only a few cycles long. Thus, the analysis presented here is valid for ultrashort pulses of any temporal duration and illustrates the power of the frequency domain representation used throughout this thesis. Equation 3.24 can be solved by integrating over the length of the interaction, L :

$$\tilde{\mathbb{E}}_s(L, \Omega) = \frac{ic\mu_0\Omega}{2n(\Omega)}\int_0^L \tilde{P}^{(3)}(z, \Omega)e^{-ik_s(\Omega)z}dz. \quad (3.25)$$

Following equation 3.7 the third order nonlinear polarization can be written in the following form:

$$\begin{aligned} \tilde{P}^{(3)}(z, \Omega) = & \int \int \chi^{(3)}(\omega_1, \omega_2, \Omega + \omega_1 - \omega_2) \tilde{\mathbb{E}}_1(z, \omega_1) \tilde{\mathbb{E}}_2^*(z, \omega_2) \\ & \times \tilde{\mathbb{E}}_3(z, \Omega + \omega_1 - \omega_2) e^{-i(\Omega + \omega_1 - \omega_2)\tau} \\ & \times e^{i[k_1(\omega_1) - k_2(\omega_2) + k_3(\Omega + \omega_1 - \omega_2)]z} d\omega_1 d\omega_2. \end{aligned} \quad (3.26)$$

Here, $\tilde{\mathbb{E}}_i$ is the frequency representation of the three interacting fields in the

plane wave approximation (equation 3.23), τ is the relative delay between pulse three and pulses one and two, the exponential factor $e^{-i(\Omega+\omega_1-\omega_2)\tau}$ arises from the Fourier shift theorem, and χ^3 includes the (potentially dispersive) frequency-dependence of the nonlinear susceptibility. The integrals in equation 3.26 indicate that the nonlinear polarization should contain the contributions from all possible permutations of the three interacting fields weighted by the nonlinear susceptibility. Inserting equation 3.26 into equation 3.25 and integrating over the interaction length for a low-efficiency process ($\tilde{\mathbb{E}}_i = \text{constant}$) produces a signal field of the form:

$$\begin{aligned} \tilde{\mathbb{E}}_s(\Omega, \tau) = & \frac{ic\mu_0\Omega L}{2n(\Omega)} \int \int \chi^{(3)}(\omega_1, \omega_2, \Omega + \omega_1 - \omega_2) \tilde{\mathbb{E}}_1(\omega_1) \tilde{\mathbb{E}}_2^*(\omega_2) \tilde{\mathbb{E}}_3(\Omega + \omega_1 - \omega_2) \\ & \times \text{sinc}\left(\frac{\Delta k(\Omega, \omega_1, \omega_2)L}{2}\right) e^{i\frac{\Delta k(\Omega, \omega_1, \omega_2)L}{2}} \\ & \times e^{-i(\Omega+\omega_1-\omega_2)\tau} d\omega_1 d\omega_2, \end{aligned} \quad (3.27)$$

where

$$\Delta k(\Omega, \omega_1, \omega_2) = k_1(\omega_1) - k_2(\omega_2) + k_3(\Omega + \omega_1 - \omega_2) - k_s(\Omega) \quad (3.28)$$

is the phase mismatch. The signal field will be optimally produced if the three interacting fields are phase matched ($\Delta k = 0$). For the special case of the TG FROG diagnostic, which utilizes a DFWM process, $\omega_1 = \omega_2 = \omega_3 = \Omega$ and phase matching is automatically fulfilled because the signal field satisfies the Bragg condition (equation 3.18). Additionally, if the approximation $\chi^{(3)} = \text{constant}$ is made, equation 3.27 reduces to equation 3.1 for the signal field intensity.

CHAPTER 4

Description of the Experiment

4.1 Introduction

As mentioned previously, one promising method to increase the longitudinal coherence of FEL radiation relies on electron beam phase space manipulations that preferentially concentrate the gain along a small longitudinal portion of the e-beam. The method utilizes an energy-chirped e-beam and an appropriately chosen undulator taper (secular variation of the undulator field amplitude or period along the nominal beam propagation direction, z). This method is described in detail elsewhere[64, 65, 66], but can be summarized as follows: the resonant frequency of light amplified in a FEL is determined by, among other things, the e-beam's mean energy (see equation 1.45). The amplified light propagates forward with respect to the electrons because it's group velocity (given by equation 2.43) is larger than the e-beam velocity (given on average by equation 1.64). In an energy-chirped e-beam this velocity mismatch brings an amplifying radiation spike out of resonance as it slips forward in the e-beam frame, inhibiting the gain, unless an appropriate undulator taper is applied to compensate for the local change in resonance. In the case of a linear energy chirp, the 'mean' local change in resonance can be described as

$$\gamma(s) = \gamma(s_0) + \alpha(s - s_0). \quad (4.1)$$

Here, $s = z - \bar{\beta}_z ct$ is the longitudinal position in the e-beam frame, s_0 is the position where a radiation spike begins amplification and α quantifies the energy chirp. Because the amplifying radiation within an undulator slips forward one radiation wavelength in one undulator period during exponential gain, equation 4.1 can be recast in the following form:

$$\gamma(z) = \gamma_0 + \alpha z \frac{\lambda_r}{\lambda_u} \eta. \quad (4.2)$$

This equation describes the change in energy that a radiation spike experiences in the laboratory frame. It is now possible to express the undulator taper that compensates for the local change in resonance due to slippage as a function of the distance along the undulator. It is found on axis to be

$$K(z) = 2\sqrt{(\gamma_s + \alpha z \eta \omega_u / \omega_r)^2 \omega_u / \omega_r - 1/2}, \quad (4.3)$$

where γ_s is the normalized e-beam energy at the longitudinal position where the radiation spike begins amplification and η is a factor that accounts for the different light propagation velocities in different stages of the amplification process, e.g. exponential gain or saturation[65]. For a short electron beam[5] it is possible for only one coherent radiation spike undergoing exponential gain and propagating at the appropriate velocity to match the e-beam energy chirp and associated undulator taper; this spike eventually reaches saturation.

The e-beam energy chirp with compensating undulator taper experiment was conducted at the SPARC FEL Test Facility in Frascati[57]. The facility as it currently exists is shown in figure 4.1. High brightness electron beams are created at SPARC using an injector that consists of a 1.6 cell RF (radio frequency) photocathode gun, followed by three traveling wave (TW) linear accelerating

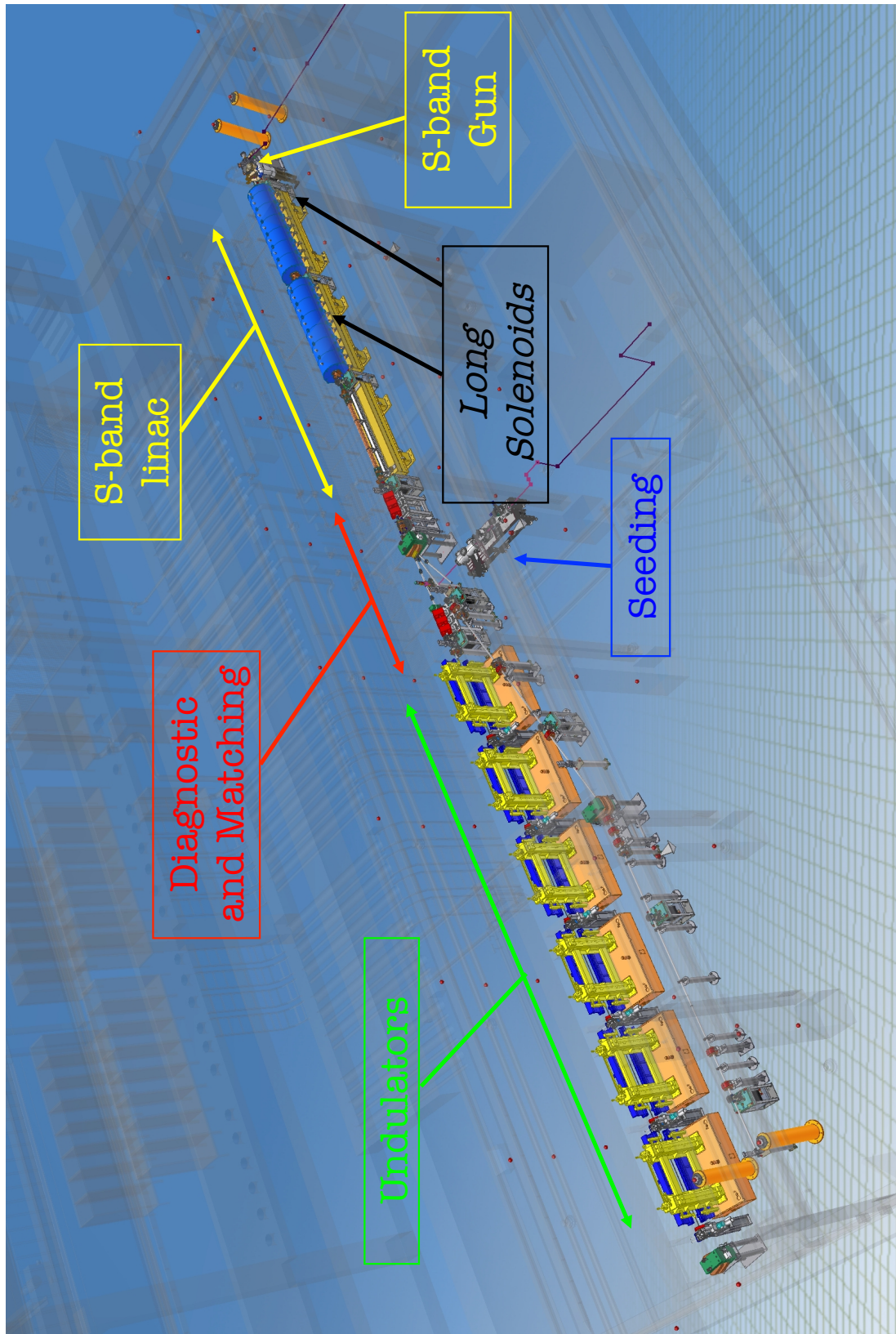


Figure 4.1: General SPARC layout.

(linac) sections, two of which are embedded in focusing solenoids. Longitudinal e-beam compression is obtained by running the first linac section off crest near the zero-crossing phase, where the first solenoid is used to optimize the emittance compensation process[67]. This method, termed *velocity bunching*[68], gives an increase in the peak current while leaving a residual energy chirp in the electron beam longitudinal phase space[69, 70]. Two quadrupole triplets along with a dipole based magnetic spectrometer and RF deflecting cavity allow the measurement of time-projected and time-resolved (slice) longitudinal and transverse e-beam parameters. These same transport optics match the beam into the undulator, which is comprised of 6 independent, variable gap sections. The undulator sections were discretely tapered through gap adjustment in accord with the above prescription for $K(z)$, and for a radiation spike propagating with group velocity near the speed of light. After undergoing exponential gain and entering saturation, the FEL light exiting the last undulator section was directed to a diagnostic station where it was input to the FROG device.

At this point it is useful to discuss relevant previous experimental results in chirped SASE FELs and the applications of FROG diagnostics to provide context for the SPARC scenario. In [71] Andonian, et al., studied the effects a chirped e-beam could have on the spectral characteristics of the output radiation and found the resulting light exhibited an anomalously large bandwidth[72]. In [73] Li, et al., used a negatively chirped e-beam, where the electrons in the head of the bunch had a higher energy than the electrons in the tail, to compensate for the light's intrinsic positive chirp that naturally arises from the SASE process. Velocity bunching, however, results in a positively chirped electron beam, which serves to reinforce this intrinsic chirp. In the present experiment, the undulator tapering serves to compensate for both the intrinsic SASE chirp as well as the e-beam chirp, while simultaneously compensating for the electron energy loss to

the FEL radiation. This scheme effectively preserves the narrowband radiation nominally produced in a standard non-chirped single-spike SASE FEL as seen in [65]. However, it should be noted that no direct time-domain measurement had been achieved in [65] and the pulse length at the Fourier limit was only estimated from the spectral width. An advanced diagnostic, therefore, was implemented that could simultaneously perform the measurement, while pushing the diagnostic paradigm to shorter wavelength operation. Undulator tapering has also been studied previously in the context of seeded FELs by Wang, et al., in [74] where significant efficiency enhancement and spectral narrowing were first observed. The present experiment, however, uses an undulator taper in a SASE FEL to suppress radiation growth where the change in resonance due to slippage is not compensated by an e-beam energy chirp. Superradiant FEL pulses generated by a coherent seed, such as those observed with a SHG FROG by Watanabe, et al., in [75] also share many similar characteristics to single-spike SASE pulses. The physical mechanisms that govern their growth and propagation, however, are significantly different. In addition, the TG FROG yields the direction of time unambiguously.

This chapter presents a very brief introduction to the SPARC FEL test facility where the bulk of the thesis work was completed, with emphasis placed on relevant aspects related to the e-beam energy chirp and undulator tapering experiment mentioned above.

4.2 The SPARC Test Facility

The SPARC project, based at the INFN (Istituto Nazionale di Fisica Nucleare) and originally proposed in March of 2002, is a research and development test facility that was designed to explore the challenges and technological issues inherent in

establishing a world class SASE FEL based x-ray source[76]. Moreover, SPARC was meant to be utilized as the injector prototype for the SPARX project[77], a SASE FEL facility for the production of coherent radiation in the wavelength range between 13 and 1 nm. There were originally four phases involved in the construction of this facility:

1. 150 MeV advanced photoinjector test facility
2. SASE FEL test facility in the visible wavelength range
3. X-ray optics/monochromator research and development
4. SASE FEL coherent soft x-ray source

The goals of the first phase of the project were to generate high brightness e-beams able to drive SASE FELs in the visible wavelength region and to investigate the RF e-beam compression technique known as velocity bunching. The project is currently in the second phase of development, where the high brightness e-beams delivered by the advanced photoinjector are used to drive a SASE FEL optimally tuned around $\lambda \sim 530$ nm. It was during this time period that this thesis work was carried out. Relevant components of the facility are discussed below.

4.2.1 The SPARC Laser

The production of high brightness beams for use in FEL experiments invariably begins with a high quality laser system. The SPARC photoinjector drive laser system[78] is capable of producing high quality, uniform flattop transverse profile, ultraviolet (UV) pulses with a temporal duration normally between 5 – 10 picoseconds (ps) and a rise and fall time (the time needed for the pulse intensity to grow from 10% to 90% of it's maximum value and vice versa) of around 1 ps.

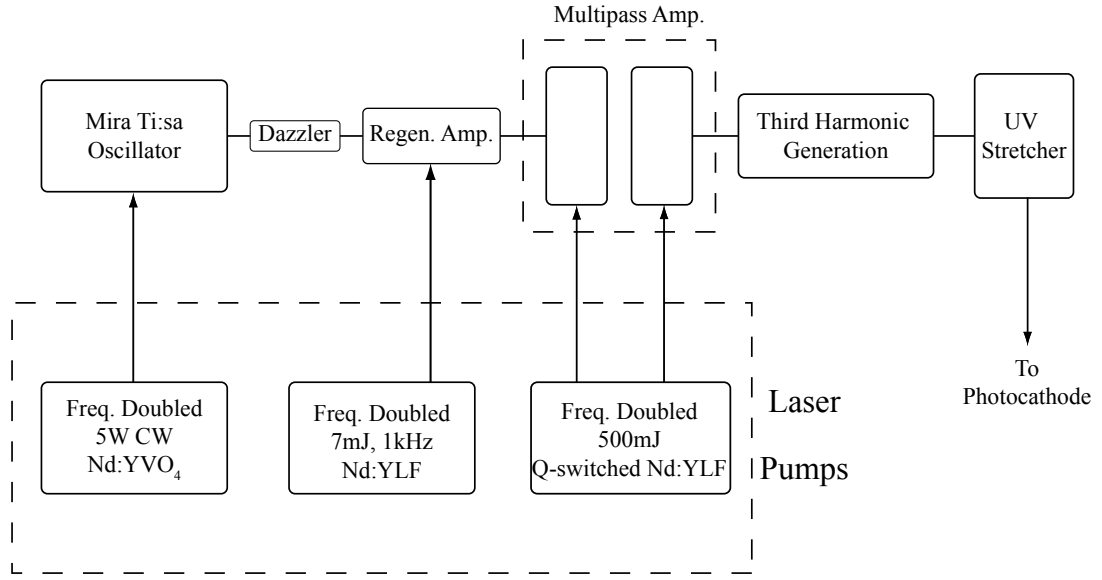


Figure 4.2: The general SPARC laser system layout.

The laser system, as seen in figure 4.2, is based on a Ti:sapphire oscillator manufactured by CoherentTM, which produces ~ 100 femtosecond (fs) long pulses that are synchronized with the 2856 MHz accelerating field of the photocathode gun and linac sections. It is pumped by the second harmonic of a Nd:YVO₄ Verdi laser, also from CoherentTM, which supplies 5 W CW power at $\lambda = 532$ nm. An acousto-optic programmable dispersive filter called the *DAZZLER*[79], used to modify the spectral amplitude and phase in order to control the laser temporal profile in conjunction with a UV stretcher, is placed between the oscillator and regenerative pre-amplifier, which is pumped by a 7 W frequency doubled Nd:YLF laser. Further amplification occurs in two double pass stages that are excited by a separate frequency doubled Nd:YLF laser with an energy around 0.5 J per pulse. This system is capable of producing 50 mJ pulses at a repetition rate of 10 Hz at $\lambda = 800$ nm. The pulse power from the amplifier is usually lowered, however, to around 20 mJ to limit nonlinear propagation effects. After amplification the IR

Parameter	Symbol	Value
Wavelength	λ	266 nm
Energy	E	50 – 150 μ J
Pulse length (FWHM)	l_c	4 – 10 ps
Rise time	τ	1 ps
Spot size (hard edge)	σ_r	~ 1 mm

Table 4.1: Typical SPARC laser parameters for this thesis.

(infrared) pulses undergo a third harmonic generation process through the use of two type I beta barium borate (BBO) crystals to produce a few hundred fs long UV pulses with an energy per pulse ~ 3 mJ at $\lambda = 266$ nm. This frequency conversion is required to generate photons with enough energy to overcome the work function of the cathode in the photoinjector. At this point the pulse is sent through a UV stretcher where the pulse is elongated and shaped in time in accordance with the DAZZLER system[80, 81]. After stretching, the pulse is sent through a circular aperture to obtain a transverse flat-top laser spot. The laser is then imaged on the photocathode by an appropriate optical system. Typical SPARC drive laser parameters for this thesis are shown in table 4.1.

4.2.2 The SPARC Photoinjector and Linac Sections

The SPARC RF photoinjector, often referred to simply as a ‘gun’, is one of the most recent iterations of the 1.6 cell S-band (2856 MHz) BNL/UCLA/SLAC type[82]. This particular generation (as seen in figures 4.3(a) and 4.3(b)) overcomes some of the previously known deficiencies in the gun performance. The

gun is run at a high gradient of $105 - 120$ MV/m, which demands $11 - 15$ MW of RF power. It is fitted with a copper (Cu) cathode that typically has a quantum efficiency of about 10^{-4} and a work function of around 4.59 eV. The cathode is illuminated under quasi-normal incidence by the SPARC photoinjector drive laser mentioned above that has been shaped to have a spatially and temporally flat-top, uniform intensity profile. The photon energy of 4.66 eV (corresponding to $\lambda = 266$ nm) is large enough to cause the photoemission of electrons, which are then accelerated to high energy (~ 5.6 MeV) by the RF field. The e-beam charge is monitored by an insertable Faraday cup located just in front of the gun and allows for the measurement of the correct phase setting between the drive laser and the gun RF accelerating field.

The violent acceleration in the gun helps to reduce space-charge induced emittance growth. Further control of the emittance growth is accomplished through a process known as *emittance compensation*[83]. This process begins with the e-beam being focused by a solenoid magnet approximately 20 cm in length placed immediately after the gun and ends at the exit of the third linac section where the e-beam energy is about 150 MeV (near on crest acceleration) and the emittance is frozen. The first linac section is placed 1.5 m downstream of the cathode coinciding with a laminar e-beam waist as called for in the ‘Ferrario’ working point[84]. Normally, an e-beam in this regime would undergo emittance oscillations produced by transverse space-charge forces. Accelerating the beam in this condition, known as the *invariant envelope*, damps these oscillations producing a normalized emittance that reduces to a steady state minimum when properly matched into the accelerating channel. The SPARC gun regularly produces e-beams with normalized emittances in the range of $\epsilon_n = 1 - 2$ mm-mrad.

The SPARC facility uses three SLAC-type (S-band, 2856 MHz) TW linac

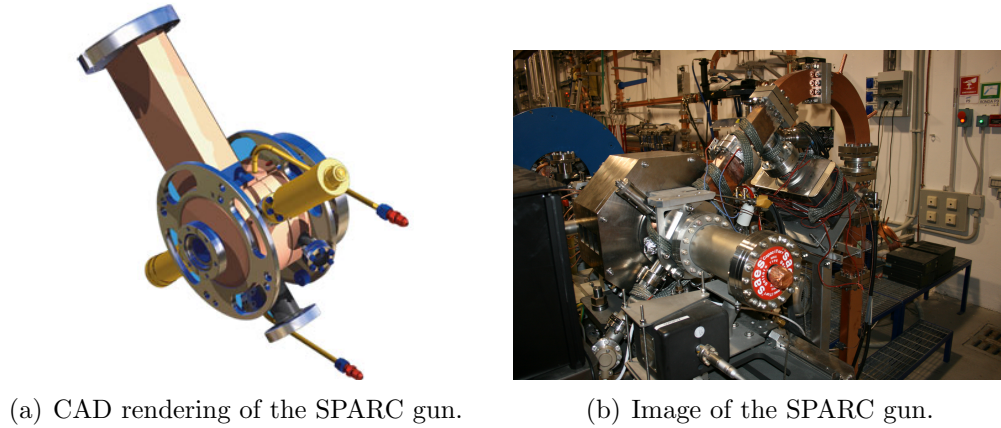


Figure 4.3: Depictions of the SPARC photoinjector.

sections to boost the final energy of the e-beam to 150 – 200 MeV. The first two linac sections are embedded in long solenoids, which provides extra focusing during the emittance compensation process. Each of the solenoids is composed of a long iron yoke containing thirteen individual coils, which produces a solenoidal magnetic field that is capable of reaching a maximum value of 1.8 kG. An image of the linac sections is shown in figure 4.4.

The current profile of the emitted e-beam is intimately related to, and influenced by, the temporal profile and total energy of the drive laser. The SPARC photoinjector regularly produces peak currents of around $I = 60$ A. The combination of low normalized emittance coupled with relatively large peak currents results in extremely *high brightness* e-beams. This is usually considered an important figure of merit at advanced accelerator based light sources. A peak current of $I \sim 60$ A, however, while large, is not enough to push the SPARC FEL into saturation. As reported in [57], the SPARC FEL nearly achieved saturation at a wavelength of $\lambda = 500$ nm using an e-beam with a peak current of $I \sim 53$ A. There is a need, therefore, to compress the e-beam to achieve higher currents while simultaneously maintaining other critical e-beam properties, such as the

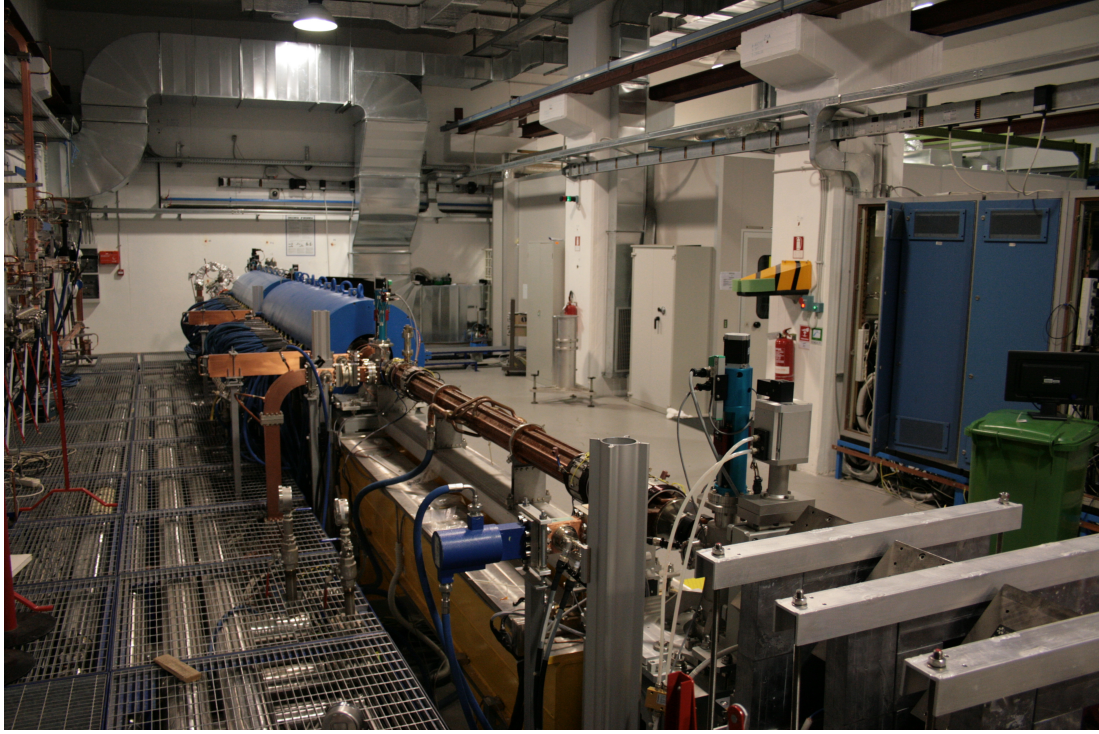


Figure 4.4: Image of the three SPARC linac sections and two focusing solenoids.

emittance and energy spread, at low levels. To this end, the SPARC facility employs the RF based compression technique termed velocity bunching, rather than using magnetic compression with a chicane[51], to reach large peak currents. This process has been studied extensively in the past. A brief qualitative description is given here.

4.2.2.1 Velocity Bunching

In the velocity bunching technique, the e-beam is injected into a long RF structure at the zero crossing field phase. If the beam has a slower velocity than the RF phase velocity it will slip back to a phase where the field is accelerating. While it slips back the tail of the beam encounters a larger accelerating field, and thus

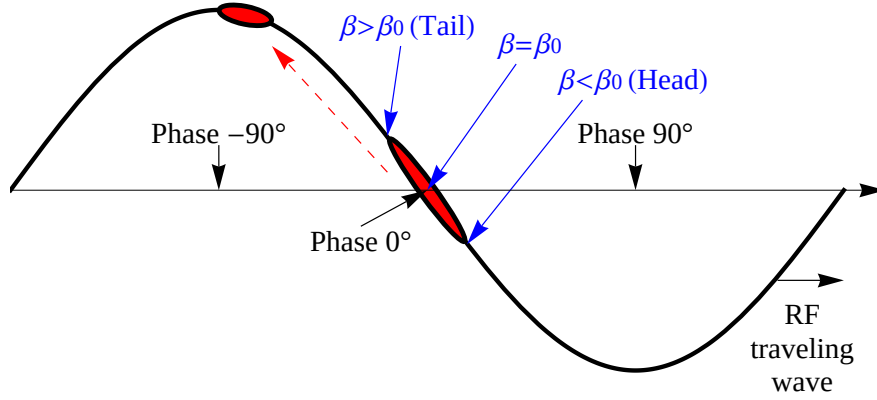


Figure 4.5: Interaction of an e-beam with an accelerating field near the zero crossing phase illustrating the velocity bunching concept.

gains more energy, than the head of the beam. The beam develops an energy chirp and compresses because the energy difference between the head and tail changes the relative positions of the electrons. Compression and acceleration, therefore, occur simultaneously within the same linac section. The maximum energy gain, however, is less than what would be obtained for on crest acceleration. This process is more clearly illustrated in figure 4.5.

Velocity bunching can be further understood by considering the interaction of an electron with a sinusoidal longitudinal accelerating electric field (averaged over fast oscillations) of the form:

$$E_z = E_0 \sin(kz - \omega t + \phi_0). \quad (4.4)$$

The electron motion obeys a time-independent hamiltonian for this system given

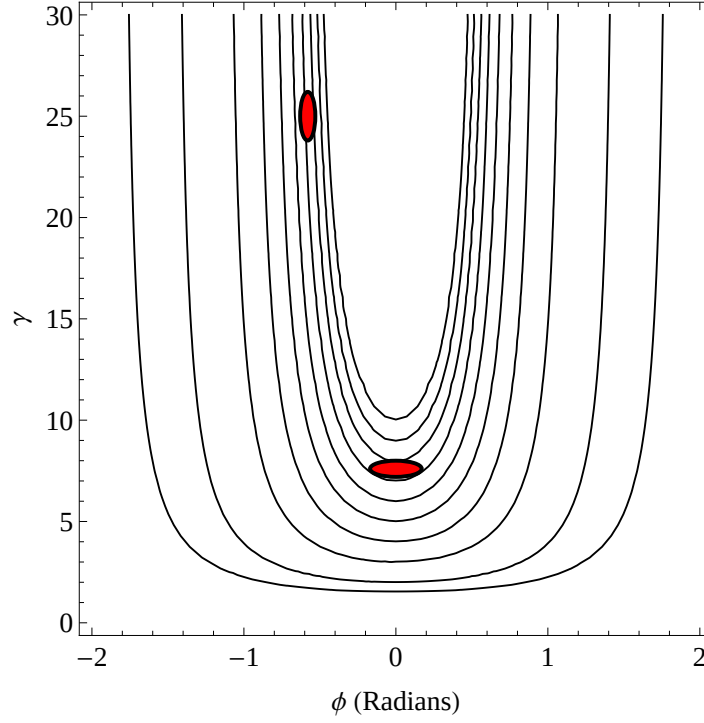


Figure 4.6: Phase space rotation of an e-beam in the velocity bunching picture.

by:

$$H = \gamma - \sqrt{\gamma^2 - 1} - \kappa \cos(\phi), \quad (4.5)$$

which is thus a constant of the motion. Here, $\kappa = eE_0/mc^2k$ is a dimensionless vector potential amplitude for the wave. Electron trajectories can be tracked and plotted in phase space using the equations of motion derivable from the hamiltonian, and follow the contours shown in figure 4.6. An e-beam injected near $\phi = 0$ will begin to slip in phase. As the particle is accelerated the phase slippage slows until it stops, at which point the phase contours in figure 4.6 are vertical. An obvious phase space rotation has occurred and the particle bunch has been compressed.

The total compression is limited by various factors, including nonlinear RF

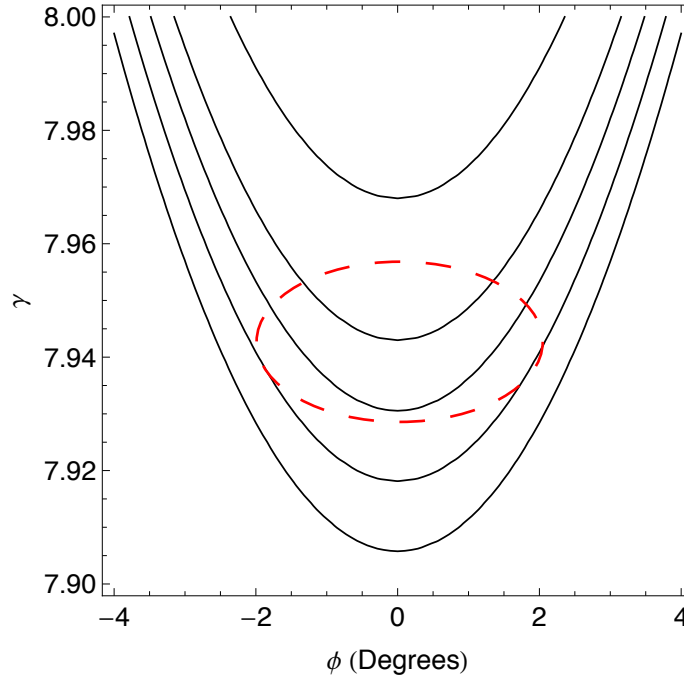


Figure 4.7: Phase space contours at injection. The e-beam phase space is shown as a dotted red ellipse.

forces, which increase the longitudinal emittance. One can see this qualitatively by analyzing the contour lines at injection as seen in figure 4.7. Here, the contour lines do not follow the outline of the longitudinal phase space ellipse. As the contours straighten out it distorts the ellipse imparting a nonlinear correlation thereby increasing the emittance as seen in figure 4.8. An excellent discussion of the limiting factors involved in the velocity bunching scheme is found in [70], where compression factors in excess of 10 have been reported.

4.2.3 Electron Beam Characterization

The diagnostic system at the SPARC facility was designed to optimize the transport of the e-beam from the creation at the photoinjector, through acceleration

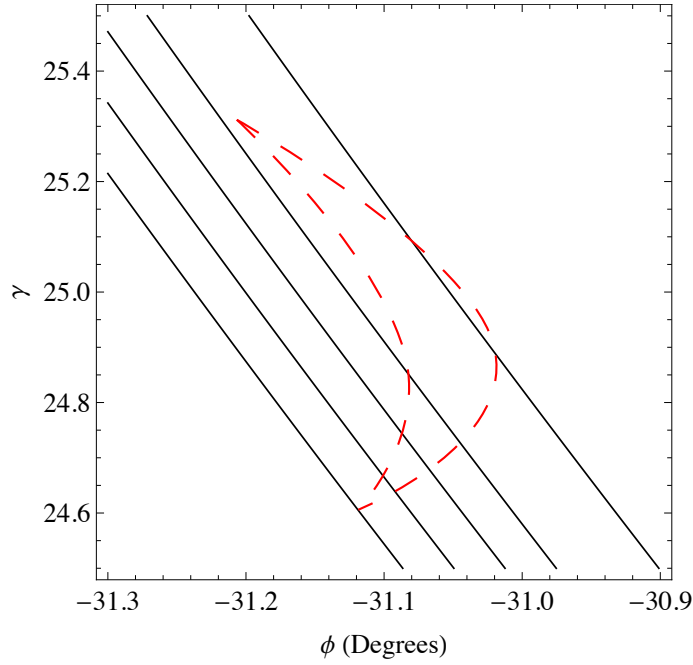


Figure 4.8: The e-beam phase space illustrating longitudinal emittance growth.

in the linac sections, and ultimately through the passage in the undulator sections. In addition, the transfer line from the linac sections to the undulator was meant to measure the main e-beam parameters and to optimally match the e-beam into the FEL. The e-beam envelope and Twiss parameter evolution is reconstructed by measuring the rms beam size on four screens along the linac: at the entrance to each RF structure and at the linac exit. Further beam measurements are made along a seven meter transfer line that also serves to match the e-beam to the undulator sections using a host of six quadrupoles arranged in two triplets. The diagnostic elements along this transfer line are shown in figure 4.9. The first quadrupole triplet is used to measure the projected emittance on a 100 μm thick Ce:Yag screen, which is mounted on a linear actuator for quick insertion and extraction via remote control, by the quadrupole scan technique[22, 23]. The read-out is through a CCD camera and frame grabber. A

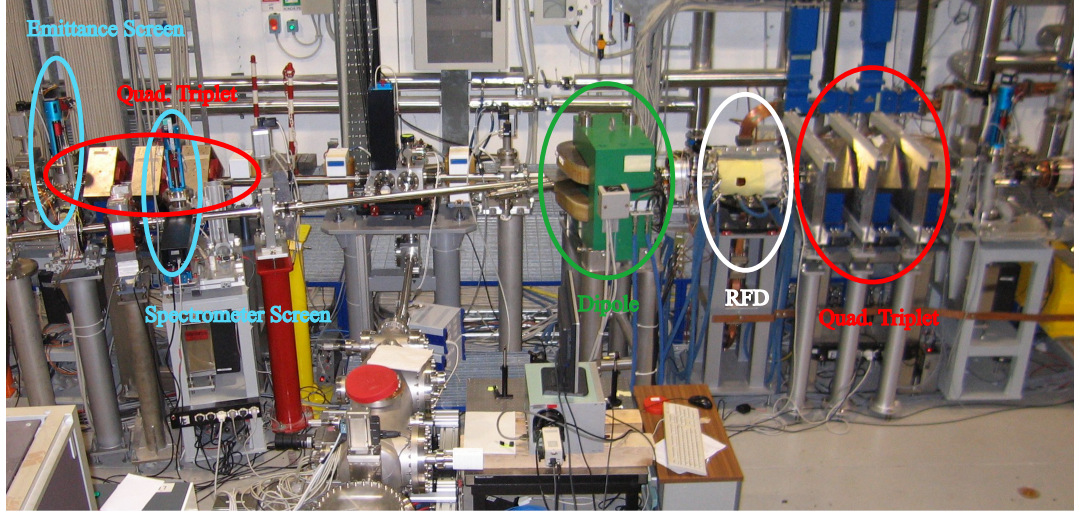


Figure 4.9: Image of the transfer line and the diagnostic/matching optics.

dipole-based magnetic spectrometer is used to deflect the e-beam onto a separate $100\ \mu\text{m}$ thick Ce:Yag screen along a dogleg off the main line allowing for the measurement of the beam energy and energy spread. In addition, an S-band, five cell, standing-wave RF deflecting cavity (RFD) can be used in combination with the first quadrupole triplet and and magnetic dipole spectrometer to measure slice longitudinal (bunch length, energy spread, trace space) and transverse (emittance) e-beam parameters. For typical SPARC e-beam parameters the time resolution is limited to around ~ 90 fs while the energy resolution is limited to about ~ 5 keV for an e-beam with an energy of 150 MeV[85].

4.2.4 The SPARC Undulators

The SPARC undulator[86] (as seen in figure 4.10) is composed of six independent, adjustable gap, permanent magnet sections arranged in the Halbach configuration. The parameters for the undulator are listed in table 4.2. The individual un-

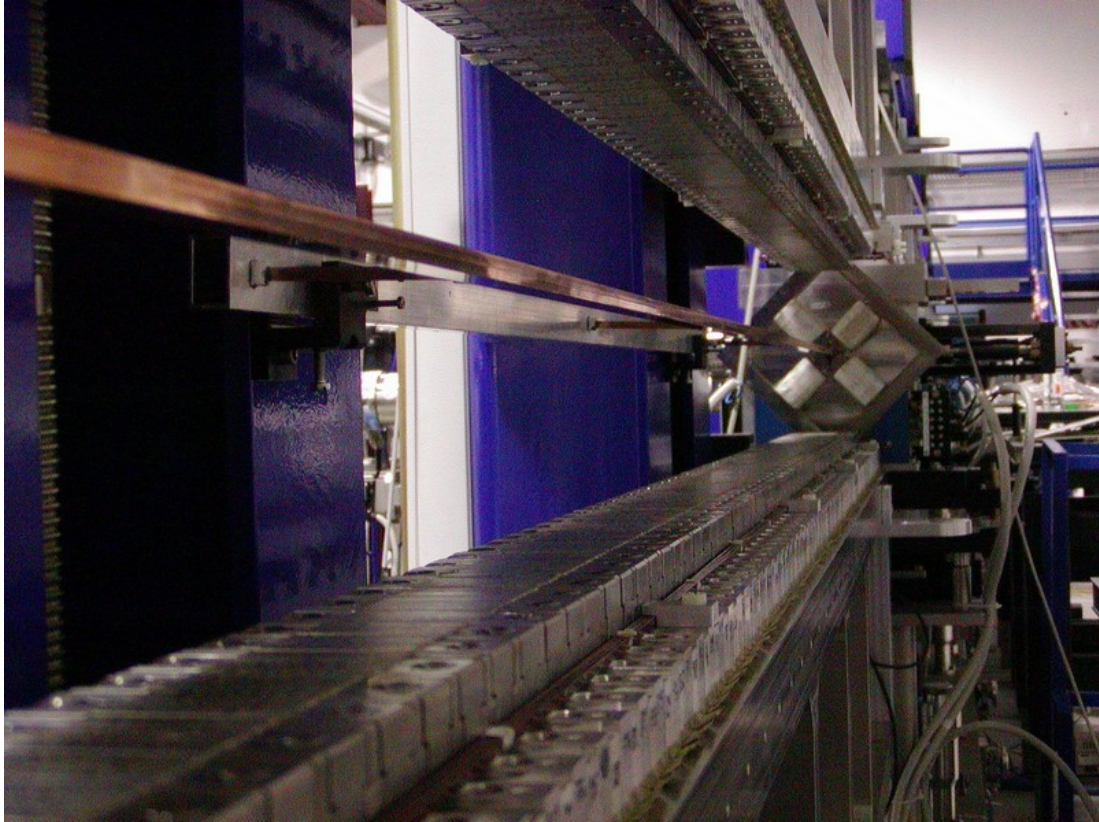


Figure 4.10: Image along the axis of one SPARC undulator section. A focusing quadrupole magnet can be seen at the end of the section.

Parameter	Value
Period	2.8 cm
Undulator Length	2.156 m
No. of Periods	77
Gap (nom/min/max)	0.958/0.82/2.5 cm
K (nom/min/max)	2.145/3.2/0.38
Remanent Field	1.31 T

Table 4.2: SPARC undulator parameters.

undulator sections are separated by breaks that host horizontal focusing quadrupole magnets for e-beam transport as well as radiation and e-beam diagnostic stations. Each diagnostic station is equipped with linear actuators that provide for the insertion of alumina screens to monitor the e-beam size and position as well as separate aluminum mirrors to extract the FEL radiation. The transport of the e-beam through the undulators is realized with a FODO lattice where the vertical focusing is provided by the undulator sections[26] and the horizontal focusing is provided by the quadrupoles. Steering magnets are included in the design of the quadrupole magnets as additional coils. Matching the e-beam into the undulator sections is determined by imposing periodic conditions on the Twiss parameters of the FODO period consisting of the undulator and drift section hosting the quadrupole and by equalizing the transverse average beta functions ($\bar{\beta}_{x,y}$).

4.2.5 The SPARC Control System

The SPARC control system was designed and built with the evolution of the machine in mind[87]. It is versatile enough to accommodate new beamline elements and diagnostic systems as the facility has matured, from the beginning, as an advanced photoinjector to a SASE FEL in the optical regime. Ultimately, it will be able to integrate new technologies as the facility further advances to a fully developed x-ray FEL facility. It is connected to all the major accelerator elements as well as data acquisition devices including, but not limited to: the laser, RF and vacuum systems, magnet power supplies, electron beam diagnostics, and FEL radiation diagnostics. The main operations in an accelerator control system consists of:

- Acquiring data/information
- Displaying data/information
- Data/information analysis
- Storing data/information
- Executing commands

To simplify all of these aspects, the SPARC control system was designed on a three level architecture. The first level encompasses the control room where the human interface exists. This allows the operator to control different aspects of the accelerator, access the logbook to share information, and store the information on an internal database. The second level consists of the front-end CPU's that execute commands given by the operator, process the requested data (either by the operator or by automatic commands), and save the information to the internal database. The third level consists of the acquisition boards that actually obtain

the data from the various accelerator elements. All three levels are connected by a Gigabit Ethernet LAN.

The SPARC control system utilizes Labview, a well known and robust rapid application development software, for the control of the various accelerator beam-line elements and for data acquisition. In addition, various Matlab (which can be easily integrated with Labview), Mathematica, and MathCad routines have been introduced and incorporated into the control system to facilitate online and offline analysis of acquired data.

CHAPTER 5

Results and Analysis of the Chirp-Taper Experiment

5.1 Introduction

The measurement of FEL light from an energy-chirped electron beam and a tapered undulator was performed at the SPARC FEL test facility in Frascati, Italy from 2010 – 2011. The first results were obtained in the spectral domain[65] using an in-vacuum spectrometer[88, 89] located after the last undulator section. This spectrometer covers the spectral range from 35 – 560 nm and can operate in both the single shot and integrated modes. The CCD camera and upstream optics were energy calibrated and allowed for the simultaneous measurements of the FEL pulse energy. Most of the FEL light statistics from the chirp/taper experiment were measured with this spectrometer and are presented in [65].

The excellent results obtained in the spectral domain prompted the recalibration of the TG FROG diagnostic from 400 nm (where it was initially calibrated for seeded FEL experiments) to 530 nm where the chirp/taper experiment was optimized. This chapter describes in detail the first time-domain measurements, obtained in the summer of 2011, using the TG FROG diagnostic.

Parameter	Symbol	Value
charge	Q	250 pC
peak current	I	264 A
mean e-beam energy	E	113.1 MeV
e-beam energy chirp	α	$-2.5 \text{ keV}/\mu\text{m}$
rms slice energy spread	σ_γ	5×10^{-3}
norm. x(y) emittance	$\epsilon_{n,x(y)}$	2.3(1.6) mm-mrad
undulator period	λ_u	2.8 cm
undulator parameter	K	1.31
radiation wavelength	λ	530 nm

Table 5.1: Experimental e-beam and undulator parameters

5.2 Experimental Electron Beam and Undulator Parameters

The e-beam used in the energy-chirp and undulator taper experiments was created, accelerated, and compressed according to the prescription in section 4.1. The projected and slice e-beam parameters were measured along the diagnostic and matching section before entrance to the undulator. The experiment was done at a mean energy of $E = 113.1 \text{ MeV}$, a charge of $Q = 250 \text{ pC}$, a peak current of $I = 264 \text{ A}$, and with a projected normalized x(y) emittance of $\epsilon_{n,x(y)} = 2.3(1.6) \text{ mm-mrad}$ (measured at 85% of the bunch charge). These parameters are summarized in table 5.1. The measured current profile along with the slice energy

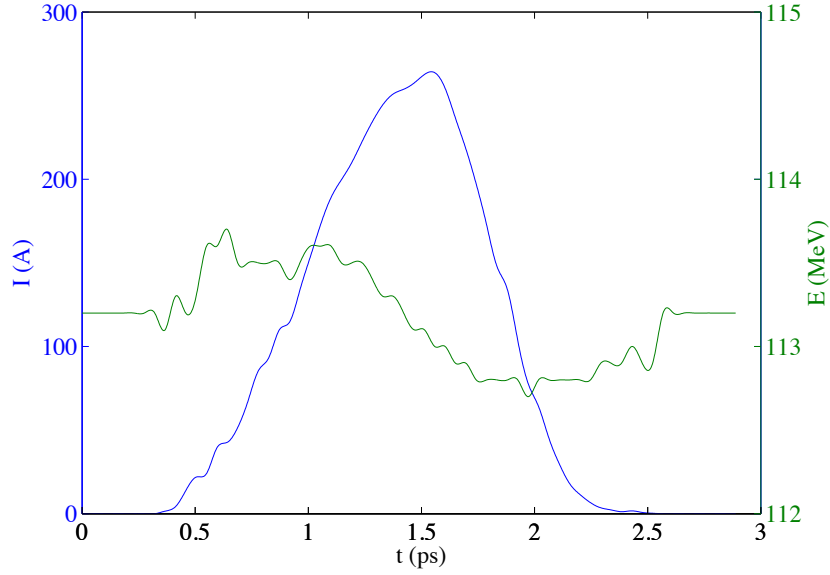


Figure 5.1: E-beam longitudinal current profile (blue) and slice energy (green).

and slice energy spread can be found in figures 5.1 and 5.2, respectively. The measured longitudinal phase space at the linac exit after velocity bunching shows a residual energy chirp of $\alpha \simeq -2.5 \text{ keV}/\mu\text{m}$. The rms slice energy spread, while not constant along the longitudinal profile of the beam, averages around $\sigma_\gamma \sim 5 \times 10^{-3}$.

The SPARC undulators are optimized to produce SASE FEL radiation around $\lambda \sim 530 \text{ nm}$. The nominal undulator parameter, tuned to produce radiation at this wavelength given the average e-beam energy, was $K = 1.31$. The e-beam was matched into the undulator sections after the measurement of the relevant Twiss parameters along the transfer line between the linacs and undulators. Matching the e-beam into the undulators consists of applying periodic boundary conditions on, and equalizing, the transverse beta functions $\bar{\beta}_{x,y}$ averaged over a FODO period. The FODO period consists of an undulator section and the drift between

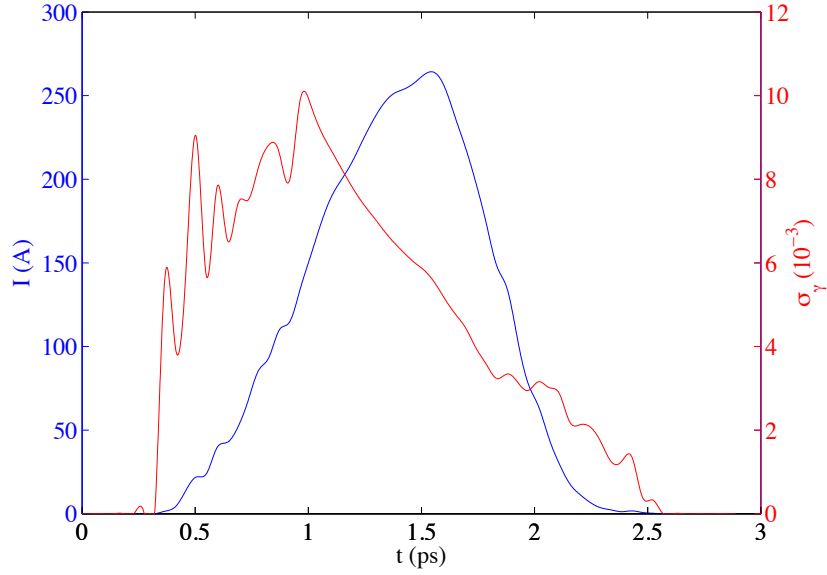


Figure 5.2: E-beam longitudinal current profile (blue) and slice energy spread (red).

two adjacent undulators hosting the horizontal focusing quadrupole magnets. Each of the undulator sections were then individually and discretely tapered in accord with the prescription found in equation 4.3 for a radiation spike beginning amplification at the back of the e-beam and propagating forward near the speed of light. The undulator matching and tapering conditions are modeled and calculated in a MATHCAD workbook written by Dr. Luca Giannessi, which has been integrated into the SPARC control system. After each undulator section was tuned to the calculated value, small adjustments to the gap (and thus undulator K parameter) were made while minimizing the spectral width on the in-vacuum spectrometer. This process had to proceed quickly due to linac phase drifts and made obtaining ancillary data extremely difficult.

Linac phase jitters of $\pm 1^\circ$, which affects the injection phase of the e-beam into the first accelerating section, cause current fluctuations of up to 100 A. This

relatively large fluctuation is due to the extreme sensitivity of velocity bunching on the injection phase. In addition to the phase jitter, there existed a slow phase drift associated with temperature fluctuations that made accelerating and compressing the e-beam in the velocity bunching mode extremely challenging. This slow phase drift dramatically affected the measured e-beam parameters and made matching and focusing the e-beam into and through the undulator sections impossible after roughly 30 minutes of operation. Thus, only a finite amount of data could be collected for any single set of measured running conditions. Nevertheless, conditions were stable just long enough for the TG FROG to obtain clear enough images to accurately reconstruct the longitudinal profile of the SASE light at saturation by running the FEL in the energy-chirp and undulator tapering mode.

5.3 TG FROG Image and Reconstruction

The FEL in the e-beam energy-chirp and undulator tapering scenario reached saturation in the sixth undulator section. The energy of a typical pulse averaged around $E_{SASE} \sim 30 \mu\text{J}$, which was measured by both the in-vacuum spectrometer and by a joule meter housed on an external (to vacuum) diagnostic station. In addition to the joule meter, the diagnostic station hosts a photodiode (used for synchronizing an external laser pulse and the e-beam for seeded FEL experiments) and the TG FROG diagnostic. The FEL light was directed to the TG FROG diagnostic after optimizing the undulator tapering by maximizing the FEL energy and minimizing the spectral width. The average energy of the FEL pulse at saturation was just large enough to initiate the third-order nonlinear optical process necessary for obtaining the time information from the FROG diagnostic. A typical experimental FROG trace (false color) is shown in Fig. 5.3 for the

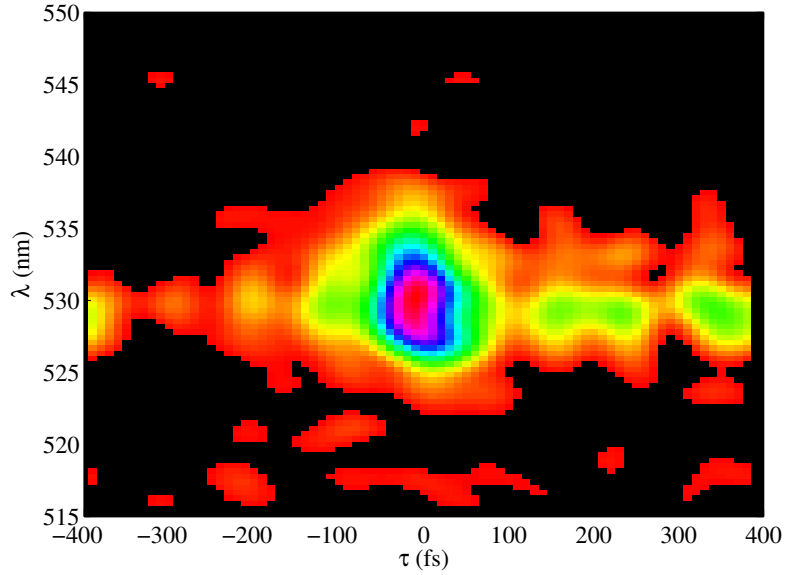


Figure 5.3: Experimental FROG trace.

experimental parameters listed in table 5.1. The raw image was post-processed by centering the trace and using only a low-pass filter to smooth over high-frequency noise. It should be noted, however, that noise has still managed to survive and slightly corrupt the image. This is likely the result of light that has diffracted out of the FEL before the onset of saturation and that has reflected in multiple directions off of the undulator vacuum pipe. This scattered light enters the diagnostic at various off-axis angles and is difficult to suppress because it is at the same frequency as the FROG signal field. One strength of the reconstruction process, however, is the ability of the algorithm to sift through the noise because of the robust over-sampling involved[90].

There are some features of the experimental FROG trace that are worth mentioning. The main body of the pulse is very slightly tilted. This is indicative of a pulse that has a longitudinal chirp. The trace is also not symmetric with

respect to time. There is a very weak, trailing tail that occurs just after the main body of the trace. Thus, by inspection of the trace alone, and without a full numerical reconstruction, one can surmise many of the longitudinal pulse characteristics. This is part of the power of the FROG technique.

The experimental FROG trace was fed into a reconstruction code that was written in MATLAB to extract the longitudinal profile of the pulse through the use of two-dimensional phase retrieval algorithms. The algorithms converged and yielded the reconstructed trace found in figure 5.4. The FROG error[20] for this reconstruction was $\sim 1\%$. Any error $\leq 1\%$ is considered a good value. Therefore, the FROG retrieval is excellent for this case considering the amount of noise that survived the filtering process. The FROG error, however, is not the only measure by which the reconstruction can be evaluated (as mentioned in section 1.3.2). The reconstructed FROG trace also shows excellent overall agreement with the experimental trace, thereby lending more credence to the reconstruction process. Many of the key features of the experimental trace are well represented. These features include the shape of the leading edge, the asymmetry in the main body, and the length and shape of the trailing tail.

The normalized temporal profile for the light from the reconstruction is found in figure 5.5 while the normalized spectrum is found in figure 5.6. The reconstructed longitudinal profile is composed of a dominant coherent spike with a FWHM of $\delta\tau_{FWHM}^R = 98$ fs followed by a weaker trailing tail. The reconstructed spectrum is composed of a dominant spike centered at $\lambda = 530$ nm with a FWHM of $\delta\lambda_{FWHM}^R = 1.4$ nm.

One advantage of using the FROG technique is that the temporal and spectral content, along with the pulse's phase information can be extracted simultaneously from the associated trace. This allows a rigorous determination of the time-

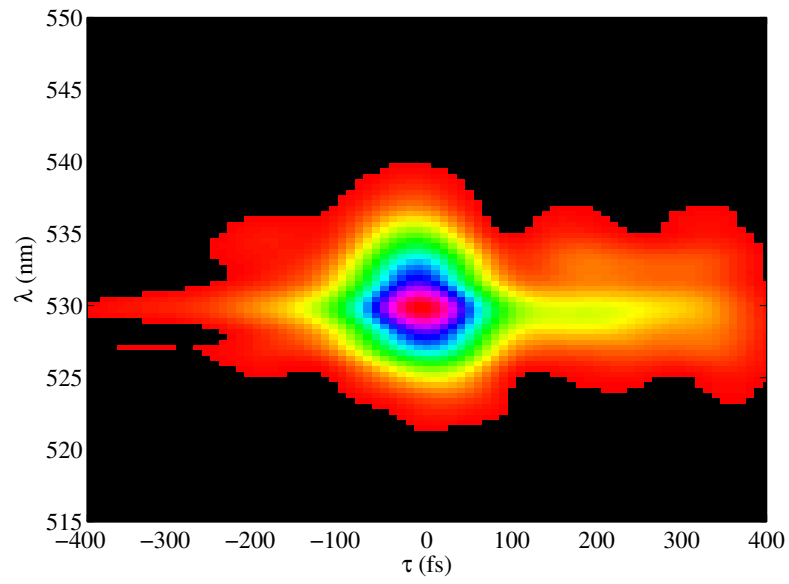


Figure 5.4: Reconstructed FROG trace.

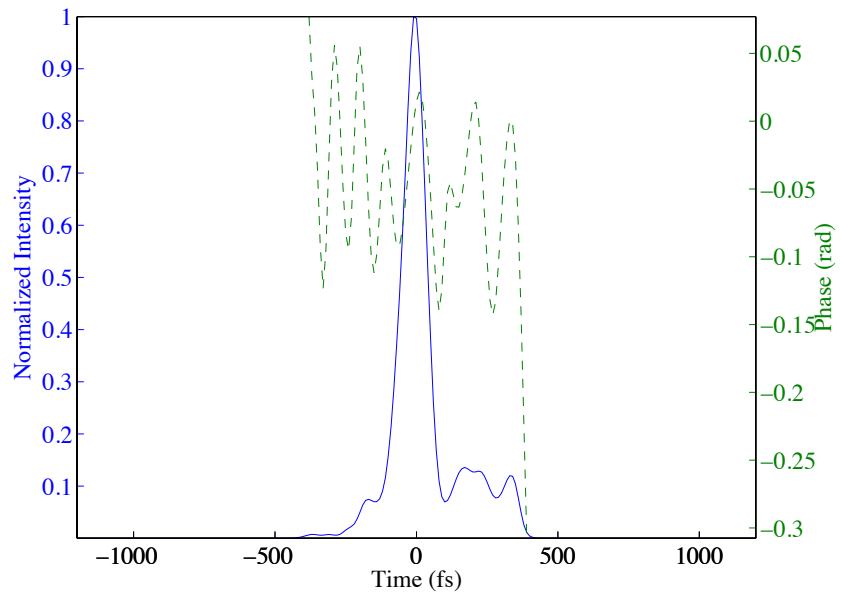


Figure 5.5: Reconstructed normalized temporal profile.

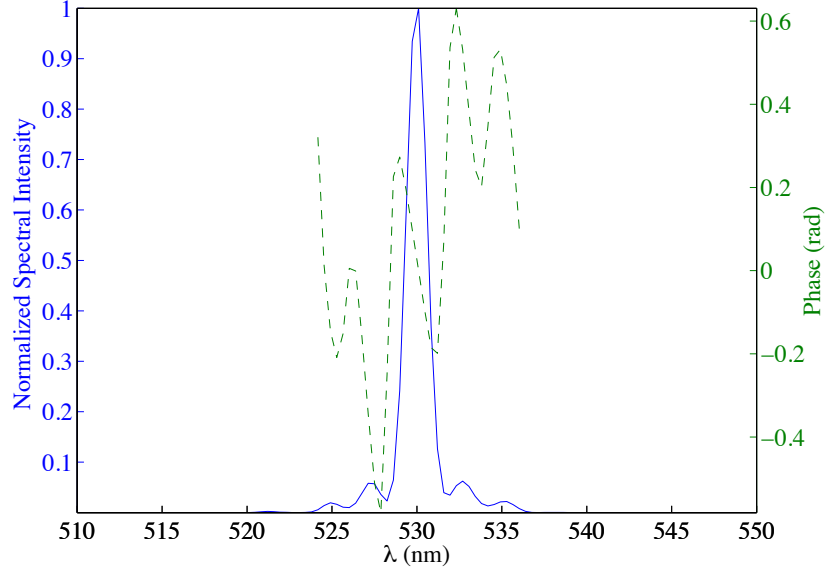


Figure 5.6: Reconstructed normalized spectrum.

bandwidth product[14], $TBP = \tau_{rms}\omega_{rms}$, where

$$\omega_{rms}^2 = \int_{-\infty}^{\infty} E'(t)^2 dt + \int_{-\infty}^{\infty} E(t)^2 \phi'_t(t)^2 dt. \quad (5.1)$$

Here, $E(t) = \sqrt{I(t)}$ is the electric field's real amplitude, $\phi_t(t)$ is the temporal phase minus the mean frequency, the prime indicates differentiation with respect to time, and the intensity, $I(t) = |E(t)|^2$, is normalized to have unity time integral. This expression for the rms bandwidth contains contributions from variations in pulse amplitude as well as those due to phase changes. A similar relationship holds for τ_{rms}^2 :

$$\tau_{rms}^2 = \int_{-\infty}^{\infty} \tilde{E}(\omega)^2 d\omega + \int_{-\infty}^{\infty} \tilde{E}(\omega)^2 \phi'_\omega(\omega)^2 d\omega. \quad (5.2)$$

Here, $\tilde{E}(\omega) = \sqrt{S(\omega)}$ is the spectral amplitude, $\phi_\omega(\omega)$ is the spectral phase and

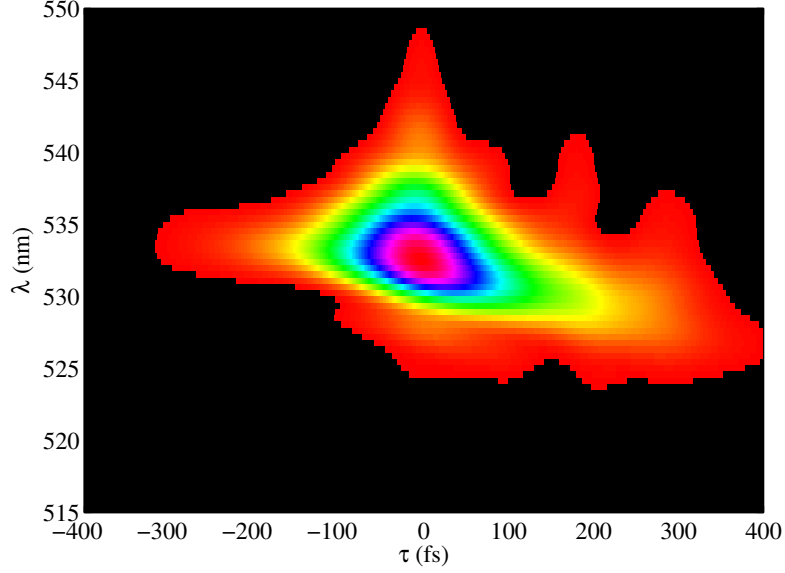


Figure 5.7: Simulated FROG trace.

where the prime now indicates differentiation with respect to ω . The pulse in figure 5.5 has a time-bandwidth product $TBP \sim 1.2$. This indicates that single-spike radiation production was nearly achieved while operating in the chirp and taper scenario.

5.4 Comparison With Simulation

The results from the experimental reconstruction are compared to fully time-dependent, 3-D particle-based simulations using the FEL code GENESIS. Here, the experimentally measured e-beam parameters from table 5.1 were used to specify the input electron particle distribution. Post-processing the simulation data with an algorithm to extract the light's longitudinal profile at the transverse location indicated by the input mask in figure 3.1 results in the simulated FROG trace found in Fig. 5.7. The agreement in the details of the reconstructed

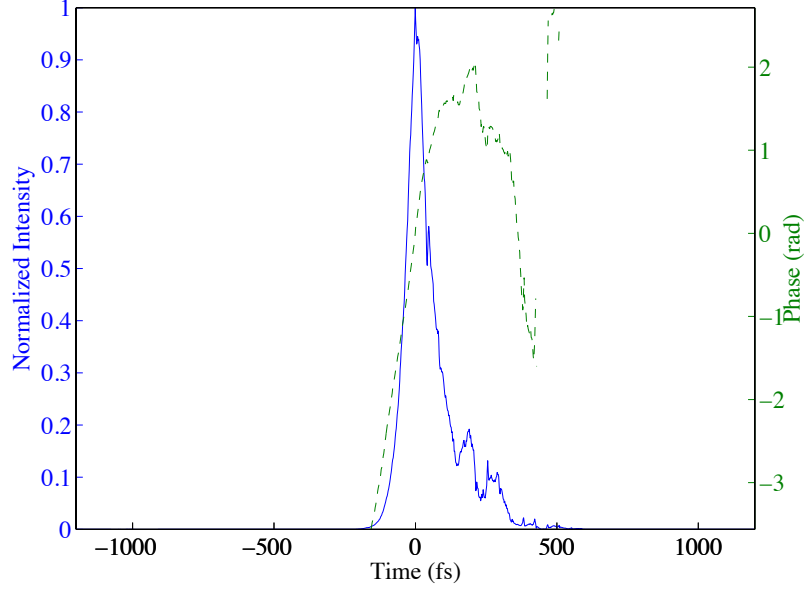


Figure 5.8: Simulated normalized longitudinal profile.

and simulated FROG traces is striking. There are several distinct features which are well reproduced by the simulated FROG trace, such as the asymmetry in the main body, indicating a slight chirp, as well as the length and shape of the leading and trailing tails. The normalized longitudinal profile of the light from the simulations of the e-beam energy-chirp and undulator tapering can be found in figure 5.8. The simulated temporal profile is in excellent agreement with the reconstructed profile. It is composed of a single dominant coherent spike with a FWHM of $\delta\tau_{FWHM}^G = 91$ fs and a weaker trailing tail. It is interesting to note that there remains a small positive chirp in the phase in both the reconstructed and simulated temporal profiles that is likely a result of the discrete nature of the undulator tapering. Other features, such as a nonlinear curvature in the longitudinal e-beam energy-chirp, could also have an affect on the light's temporal phase. The two-dimensional plot of the normalized radiation power vs the

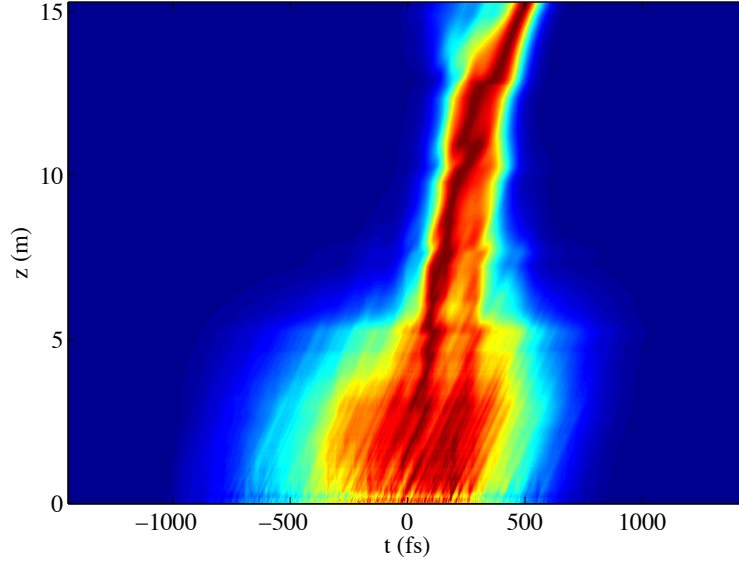


Figure 5.9: Power vs longitudinal coordinate along the pulse (t) and along the undulator (z).

longitudinal coordinate along the pulse (t) and along the undulator (z) is shown in figure 5.9. It is clearly evident that the combination of the e-beam energy chirp and undulator tapering produced one single dominant coherent spike for the SASE FEL. The total energy of the pulse from simulation was $E_{gen}^t \sim 28 \mu\text{J}$, which is comparable to the experimentally measured energy.

One concern inherent with use of the TG FROG on a FEL is that the on axis light is discarded by the input mask. Fig. 5.10 compares the off-axis and total power from the reconstructed and simulated data. While the full power profile shows a more pronounced tail, the main features of the pulse are still captured by the off axis data gathered by the FROG system. The subtlety of interpreting the results from the TG FROG is an inherent challenge in FELs (as opposed to standard lasers), due to their non-negligible spatial chirp. Nevertheless, the current measurement is seen to be robust.

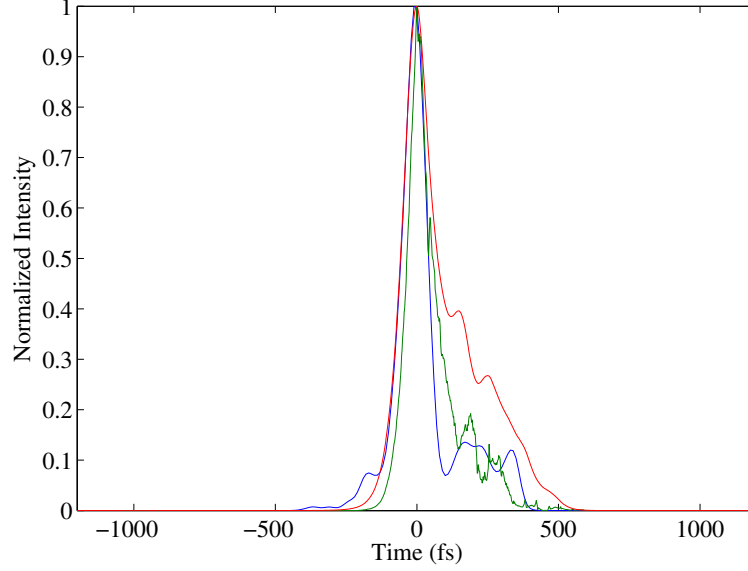


Figure 5.10: Comparison between the reconstructed intensity (blue), simulated power at the mask (green), and total pulse simulated power (red).

Results from simulations for an energy-chirped e-beam in the absence of a compensating undulator taper were examined to underscore the significance of the present experiment. The simulation was identical to the one presented above except that the undulator sections were not tapered. The temporal profile of the resulting light, which had a total energy of $E_{gen}^{nt} \sim 16 \mu\text{J}$, is shown in figure 5.11. The profile is no longer dominated by a single coherent spike. Rather, there are multiple distinct lasing modes that are present. In addition, since the total energy of the pulse, which is less in this case, is distributed over multiple radiation spikes, the energy per spike is significantly less. The two-dimensional plot of the normalized radiation power vs the longitudinal coordinate along the pulse (t) and along the undulator (z) is shown in figure 5.12 for this case. It is clear that more than one coherent spike of radiation propagates and is amplified in the absence of an undulator taper.

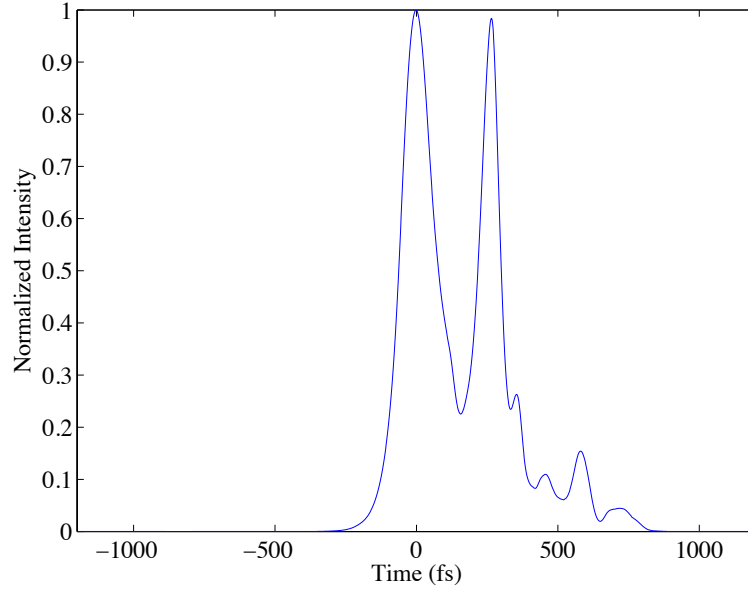


Figure 5.11: Simulated normalized longitudinal profile in the absence of undulator tapering.

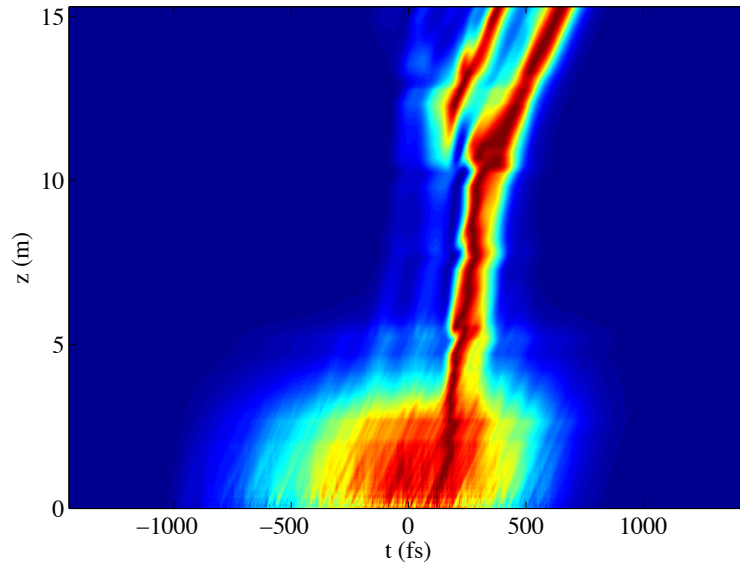


Figure 5.12: Power vs longitudinal coordinate along the pulse (t) and along the undulator (z) in the absence of undulator tapering.

5.5 Conclusion

Self-amplified spontaneous emission free-electron lasers are capable of producing coherent and tunable radiation on the time and length scales of atomic and electronic processes. This fourth generation light source allows the greater scientific community to probe deeper into regions of higher spatial, temporal and energy resolutions. SASE FELs, however, nominally produce longitudinally incoherent radiation that is a collection of randomly distributed coherent spikes. Much effort has been placed on producing a single longitudinally coherent radiation spike. To this end, longitudinal e-beam phase space manipulation techniques that can concentrate the FEL gain along only a small portion of the e-beam longitudinal profile have been investigated for their promising characteristics.

This dissertation reports the first direct time-domain measurement of a single coherent radiation spike from a SASE FEL amplifier employing an energy-chirped e-beam coupled with a tapered undulator. This energy-chirp/undulator taper scheme effectively preserves the FEL gain only where an appropriate undulator taper compensates for the detuning experienced by a propagating radiation spike as it slips forward in the e-beam frame. The experiment was done in the optical wavelength regime at the SPARC FEL test facility in Frascati, Italy where the unique combination of velocity bunching, which leaves a residual longitudinal e-beam energy-chirp during compression, and undulator tapering allowed for the observation of this phenomenon. This measurement provides further insight into methods that can be used to shape the SASE FEL longitudinal profile to enhance coherence properties.

In addition, the measurement was taken with an advanced TG FROG diagnostic that has the potential to significantly extend FEL short pulse measurement systems to short wavelength. The FROG pulse measurement and reconstruction

technique has become the standard for ultrafast lasers in the optical regime. The TG FROG diagnostic used in this dissertation employs a third-order nonlinear optical process to generate the signal field, which is both the system's primary strength and weakness. On one hand, third-order processes are notably weaker than second-order processes. Furthermore, the radiation field directly on-axis, where the intensity is the highest, is discarded. These aspects, taken in combination, demand greater input field strength, which is normally obtained only at saturation in a FEL. On the other hand, phase-matching limitations and the need for frequency doubling are removed by use of TG FROG, allowing its application into the deep UV. In this spectral region, however, dispersion in the device's focusing optics becomes a limiting factor in the reconstruction process. To mitigate this effect in the future, it would be useful to use an all reflective geometry, as has been demonstrated by Nagy, et al. in [91].

The measured SASE FEL pulse from the energy-chirp/undulator taper experiment had a temporal FWHM of $\delta\tau_{FWHM}^R = 98$ fs and a measured time-bandwidth product of $TBP \sim 1.2$, indicating that the Fourier limit was nearly achieved. The TBP can be measured on a single-shot basis because the FROG diagnostic technique simultaneously measures the temporal and spectral amplitudes and phases. This is a major advantage that FROG has over its competitors. The measurement was compared to in-depth particle simulations using the FEL code GENESIS. There exists excellent overall agreement in both the measured and simulated FROG traces as well as the measured and simulated temporal profiles. The e-beam energy-chirp and undulator tapering modality could potentially be used to produce coherent, extremely bright x-ray FEL pulses. In addition, TG FROG could potentially be used, with appropriate optical changes, at short wavelength FEL facilities as a means of longitudinally measuring high power, ultrafast light pulses.

APPENDIX A

Gain length fitting formula for free-electron lasers with strong space-charge effects

A.1 Introduction

The work presented here is ancillary to the main theme of the thesis; that being the e-beam energy-chirp and undulator tapering experiment at the SPARC facility. It illustrates, however, a different and complimentary analytic approach to FEL theory other than what was presented in chapter 2. The main goal of this work was to illustrate the detrimental effects that longitudinal space-charge forces have on the FEL gain process and relies heavily on the three-dimensional theory developed by Hemsing, *et al.*, in [92, 93, 94].

The distance along the undulator it takes for the power of the emitted light to increase by a factor of e during the exponential growth regime is known as the power gain length (as seen in chapter 2). It is given in the one-dimensional, cold-beam limit as

$$L_{1D} = \frac{1}{2\sqrt{3}k_u\rho}, \quad (\text{A.1})$$

where ρ is the well-known Pierce parameter[39], given by

$$\rho = \left[\frac{K[JJ]}{4\sqrt{2}} \frac{\theta_p \gamma_z}{k_u \gamma} \right]^{\frac{2}{3}}. \quad (\text{A.2})$$

Here, the Lorentz factor relating the average longitudinal beam motion to the laboratory frame is $\gamma_z = \gamma / \sqrt{1 + \frac{K^2}{2}}$, the relativistic plasma wavenumber is

$$\theta_p = \sqrt{\frac{2I_e}{I_A \gamma_z^2 \sigma_x^2}}. \quad (\text{A.3})$$

and σ_x is the rms electron beam size, I_e is the electron beam peak current and $I_A \simeq 17\text{kA}$ is the Alfvén current.

The power gain length is one of the most important parameters in the design of a high-gain FEL, as it determines the overall size (and thus cost) of the undulator system needed for the FEL to reach saturation. One-dimensional theory, however, does not entirely capture the complexity of high-gain FELs operating in the infrared or visible wavelength region where various strong three-dimensional effects can be detrimental to performance and ultimately degrade the gain. For these 3D FELs the gain length L_g exhibits a sensitive and complicated dependence on diffraction (the tendency of the light to spread while propagating), the detuning from resonance, and the uncorrelated energy spread and longitudinal space-charge in the e-beam. Given the large number of physical parameters that influence the dynamics, it is useful to describe these effects using scaled parameters that individually represent the essential features of the FEL system[36]. Here, focus is placed on the three of highest relevance to optical regime FELs: the diffraction parameter, $\eta_d = L_{1D}/2k\sigma_x^2$, which quantifies the extent to which transverse effects contribute to the gain for a FEL with wavelength $\lambda = 2\pi/k$; the scaled energy spread parameter $\eta_\gamma = 2k_u\sigma_\gamma L_{1D}$, which captures the contribution of the rms uncorrelated e-beam energy spread σ_γ ; and $\bar{\theta}_p = 2\theta_p L_{1D}$, the space-charge parameter, which is scaled to be twice the plasma phase advance over a one dimensional gain length. Even in terms of this reduced set of param-

ters, determination of L_g for design and optimization of the FEL system requires either numeric solutions to the governing equations or fully 3D numerical particle simulations, which are often time consuming. Therefore, it would be useful to quickly evaluate the performance of the FEL system without having to resort to these more time intensive and computationally costly methods.

For short-wavelength FELs in which emittance effects can play an important role and where space-charge can be neglected ($\bar{\theta}_p \rightarrow 0$), Xie[36] provided a power fit formula that has proven to be extremely useful for quickly predicting the growth rate of the dominant Gaussian optical mode. The condition of neglecting space-charge is usually easily satisfied for these high energy ($\gamma \sim 10^4$), short wavelength FELs such as the Linac Coherent Light Source (LCLS)[2] where $\rho \sim 10^{-4}$ and $\bar{\theta}_p L_{1D} \ll 1$. However, many FELs of current interest do not operate in this regime. The longitudinal space-charge field begins to counteract the microbunching process as $\bar{\theta}_p$ nears unity (θ_p approaches L_{1D}^{-1}), which occurs as $\rho \rightarrow \theta_p \lambda_u$ as evidenced in Eq (A.2). Space-charge effects are of interest for low energy Raman FELs where the beam charge density is sufficiently high[95, 96, 97]. The performance of the FEL in this limit is fundamentally changed by collective space-charge effects. Of relevance recently are IR-optical self amplified spontaneous emission (SASE) high-gain FELs that are based on very high brightness electron beams at relatively low energy ($\gamma \sim 10^2$), and thus potentially are susceptible to both space-charge and diffraction effects. For instance, the ~ 800 nm VISA FEL[98] was characterized by $\rho \sim 5 \times 10^{-3}$, giving $L_{1D} \simeq 10$ cm, but with diffraction and space-charge effects the actual measured gain length was 18 cm. While diffraction is always a notable degrading effect to the gain for IR-optical FELs, space-charge may often play a larger role than the electron beam energy spread or emittance.

FELs also have long been considered candidates as high average power light sources[99] since the e-beam that acts as the gain medium does not suffer from thermal loading and excitation bandwidth constraints that limit conventional sources. FEL oscillators first demonstrated the progress made in realizing this application[100]. More recently, there has been increased interest in obtaining high average power using high-gain amplifiers operating at ~ 100 MeV e-beam energies and $I_e \sim 1$ kA currents where longitudinal space-charge will strongly affect the FEL performance[101, 102]. The e-beam used in such FELs will operate in a space-charge dominated, rather than emittance dominated, mode, and thus the dominance of space-charge over emittance effects in the longitudinal FEL dynamics should not be surprising. Similarly, the investigation of longitudinal space-charge waves in high-brightness beams is still currently of great theoretical interest[103, 104], following past emphases on understanding transverse beam-plasma oscillations[83]. Taking this into account, it should be noted that there is currently no handy formulation for quickly predicting important FEL characteristics when space-charge has significant influence, despite the fact that there has been strong historical interest and recent experimental investigations into FELs that operate under these conditions. While some progress has been made to this end in [105], the numerical algorithm produced therein suffers the same time-consuming constraints as mentioned previously.

For the above reasons, the power fit approach of Xie is revisited, including in the fit analysis space-charge as a relevant parameter while ignoring the negligible effect of e-beam emittance, ϵ_x , in the limit that $\eta_e = 2kL_1D\epsilon_x^2/\sigma_x^2 \rightarrow 0$, or $\eta_e \ll 2\eta_\gamma$ in the presence of energy spread, and $\sqrt{\eta_e\eta_d} \ll 1$. The analysis of [106, 107] is followed and the three dimensional integro-differential FEL equations for the FEL field amplitudes are solved in the limit where the FEL signal field is dominated by the fundamental Gaussian mode. Similar to Xie, the gain length of the three

dimensional mode L_g is expressed as

$$\overline{\delta k_i} = \frac{L_{1D}}{L_g} = \frac{1}{1 + \Lambda_{0,0}}, \quad (\text{A.4})$$

where $\overline{\delta k_i}$ is maximized at the optimal detuning to yield the shortest possible gain length and $\Lambda_{0,0}$ is expressed as a power fitting formula that is a function of η_d , η_γ and $\bar{\theta}_p$. The fit is obtained from numerical solutions to the analytic theory presented in the next section, and shows excellent overall agreement with results from numerical particle simulations performed with Genesis 1.3[9] that also include higher-order space-charge distributions. It should be emphasized that the results obtained in the following sections are in general limited to FELs that operate from the optical to the far IR, or otherwise satisfy the stated parametric constraints.

A.2 Analytic model

The three dimensional high-gain FEL equations in the presence of uncorrelated energy spread and non-negligible space-charge effects have been explored in previous work[106]. Under a slowly-varying transverse field approximation, $\mathbf{E}_\perp(\mathbf{x}, t) = \text{Re} \left[\tilde{\mathbf{E}}_s(\mathbf{x}_\perp, z) e^{ik(z-ct)} \right]$, the integro-differential equation that describes the FEL field amplitude evolution along the undulator is given as,

$$D(z)\tilde{E}_s = i \frac{k\mu_0 e c^2}{\gamma m v_0^2} \int_0^z dz' \left[\frac{[JJ] e K^2}{4\gamma^2} \tilde{E}_s + \frac{e}{k^2} D(z') \tilde{E}_s \right] \times \int_{-\infty}^{\infty} d\eta \frac{\partial F_0}{\partial \eta} e^{i\left(\frac{k}{\gamma^2} \eta - \theta_0\right)(z'-z)} \quad (\text{A.5})$$

where $D(z) = \nabla_\perp^2 + 2ik \frac{\partial}{\partial z}$ is the paraxial wave operator, $\eta = (\gamma_1 - \gamma)/\gamma$ is the relative energy deviation of an electron with energy γ_1 from the nominal beam

energy γ , $F_0 = F_0(\mathbf{x}_\perp, \eta)$ is the unmodulated e-beam distribution which stays fixed during transport, $\theta_0 = k/\gamma_z^2 - k_u$ is the detuning, and $\gamma_z^2 = 1/(1 - v_0^2/c^2)$ is the relativistic longitudinal energy factor. Zero initial modulation in the beam has been assumed, as has the condition $k\sigma_x/\gamma_z \gg 1$, which stipulates that the e-beam profile is large compared to the microbunching wavelength in the moving frame. Under this latter constraint the space-charge fields are assumed to be predominantly longitudinal. Further, to make the analysis more tractable, the transverse variation of the charge density distribution is also neglected.

In the single-mode limit, the FEL is assumed to be dominated by a fixed-profile transverse mode that grows exponentially in amplitude along z . The field can then be written simply as

$$\tilde{E}_s = E_{p,l} u_{p,l}(\mathbf{x}_\perp) \exp(i\delta k z) \quad (\text{A.6})$$

where $E_{p,l}$ is the mode amplitude and $\delta k = \delta k_r - i\delta k_i$ is the complex wavenumber associated with the FEL process. By inserting this into (A.5) one can calculate the gain length $L_g = 1/2\delta k_i$ of the (p, l) mode in the linear regime, where the dominant, exponentially growing solutions are characterized by $\delta k_i > 0$. The modal profile distribution $u_{p,l}(\mathbf{x}_\perp)$ is any suitable function with indices p and l to describe the mode of interest.

Following [107], a Laguerre-Gaussian basis provides a convenient description for a FEL with cylindrical symmetry:

$$u_{p,l}(\mathbf{x}_\perp) = \exp\left[il\phi - \frac{r^2}{w^2}\right] \left(\frac{r\sqrt{2}}{w}\right)^{|l|} L_p^{|l|}\left(\frac{2r^2}{w^2}\right), \quad (\text{A.7})$$

where $L_p^{|l|}$ is a Laguerre polynomial. With an uncorrelated Gaussian e-beam distribution, $F_0 = n_0 (\sqrt{2\pi\sigma_\gamma^2})^{-1} \exp[-r^2/2\sigma_x^2 - \eta^2/2\sigma_\gamma^2]$, Equation (A.5) reduces

in the single mode limit to an expression that yields the complex wavenumber δk for the (p, l) mode,

$$\left[\overline{S}_1 - \overline{\theta}_p^2 \mathbb{F}_{p,l} \right] \left[\overline{\delta k} + \frac{\eta_d}{\alpha} (2p + |l| + 1) \right] = - \left(\frac{2}{\sqrt{3}} \right)^3 \mathbb{F}_{p,l}. \quad (\text{A.8})$$

Scaled variables have been introduced: $\overline{\delta k} = 2L_{1D}\delta k$ is the scaled complex wavenumber and $\alpha^2 = w^2/4\sigma_x^2$ is the complex spot size parameter. The energy spread contribution to the gain is given by the term,

$$\overline{S}_1 = -2\sqrt{2\pi}\eta_\gamma^3 \left[\int d\overline{\eta} \frac{\overline{\eta} \exp(-\overline{\eta}^2/2\eta_\gamma^2)}{\overline{\delta k} - \overline{\theta} + 2\overline{\eta}} \right]^{-1}, \quad (\text{A.9})$$

where $\overline{\theta} = 2L_{1D}\theta_0$. The coupling between the optical modes and the transversely Gaussian e-beam profile is given by,

$$\mathbb{F}_{p,l} = \frac{(2p + |l|)!}{p!(p + |l|)!} \frac{\alpha^{2p}}{(\alpha + 1)^{2p + |l| + 1}} {}_2F_1 \left(-p; -p; -2p - |l|; 1 - \frac{1}{\alpha^2} \right), \quad (\text{A.10})$$

where ${}_2F_1(a; b; c; x)$ is the hypergeometric function.

By virtue of the $\overline{\theta}_p^2 \mathbb{F}_{p,l}$ term in (A.8), the effect of longitudinal plasma oscillations for higher order FEL modes are included. This is a quasi-three dimensional extension of a purely one dimensional model that takes into account, as a first-order approximation, the modification to the space-charge field profile due to the structure of the optical modes that map to the transverse microbunching profile. Averaged over the beam, this modifies the effective plasma wavenumber by the coupling factor $\mathbb{F}_{p,l}$.

In general, solutions to (A.8) can be found for a mode (p, l) by application of the variational condition $\delta(\overline{\delta k})/\delta\alpha = 0$ as in[108, 36]. The scaled complex

wavenumber $\overline{\delta k}$ and spot size parameter α can then be determined by solving the two resulting equations.

A.2.1 One dimensional limit

In the 1D limit, the transverse variation of the e-beam and the fields is neglected. The diffraction parameter becomes vanishingly small $\eta_d \rightarrow 0$, and the degeneracy of the modes sends the coupling factor to unity $\mathbb{F}_{p,l} \rightarrow 1$. Equation (A.8) is then given simply as,

$$\left[\overline{S_1} - \overline{\theta_p}^2 \right] \overline{\delta k} = - \left(\frac{2}{\sqrt{3}} \right)^3. \quad (\text{A.11})$$

In the additional limit of vanishing energy spread for a cold-beam, $\eta_\gamma \rightarrow 0$, Eq (A.9) reduces to $\overline{S_1} = (\overline{\delta k} - \overline{\theta})^2$ and the familiar cubic equation for $\overline{\delta k}$ of the 1D high-gain FEL is obtained,

$$\left[(\overline{\delta k} - \overline{\theta})^2 - \overline{\theta_p}^2 \right] \overline{\delta k} = - \left(\frac{2}{\sqrt{3}} \right)^3. \quad (\text{A.12})$$

Figure A.1 depicts how the scaled gain of the cold-beam 1D system varies with the detuning for several values of the space-charge parameter[46]. At resonance ($\overline{\theta} = 0$) and in the absence of space-charge effects ($\overline{\theta_p} = 0$) dominant solutions to (A.12) are simply $\overline{\delta k} = 2L_{1D}\delta k = 1/\sqrt{3} - i$, and the 1D gain length is retrieved from $L_{1D} = 1/2\delta k_i$. In the collective regime where space-charge effects are significant, it is straightforward to show that the shortest gain length (the maximum of $\overline{\delta k}_i$) is obtained at a detuning of $\overline{\theta} = -\overline{\theta_p}$, in the limit $\overline{\delta k}_i \ll \overline{\theta_p}$ [109]. The enhanced oscillatory effects of space-charge on the FEL-induced charge wave serve to move the frequency into optimized resonance at this detuning.

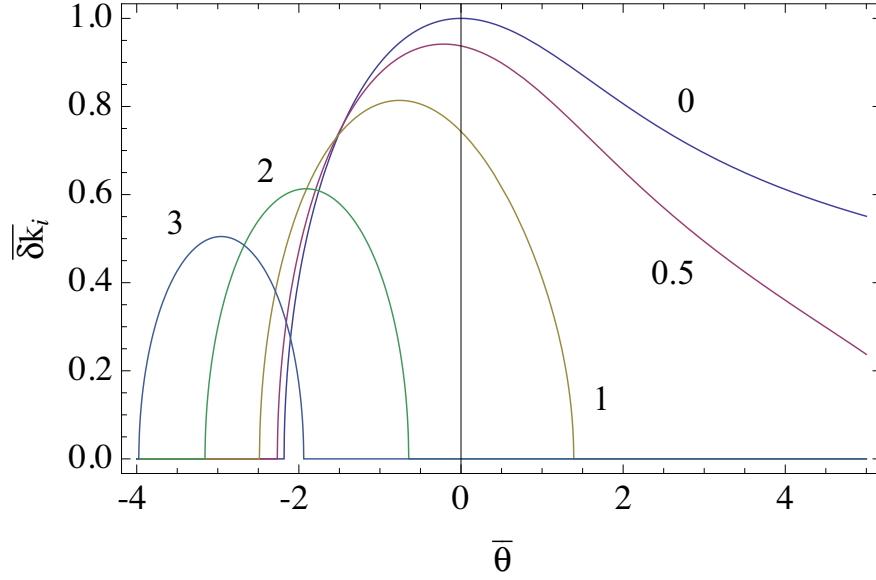


Figure A.1: Detuning gain curves in the 1D limit for $\bar{\theta}_p=0, 0.5, 1, 2$ and 3 .

A.2.2 Fundamental mode

For the fundamental Gaussian mode $(p, l) = (0, 0)$, the variational constraint $\delta(\bar{\delta k})/\delta\alpha = 0$ applied to the 3D single mode expression in (A.8) yields the equations

$$\begin{aligned} \left(\bar{S}_1 - \frac{\bar{\theta}_p^2}{1+\alpha} \right) \left(\bar{\delta k} + \frac{\eta_d}{\alpha} \right) + \left(\frac{2}{\sqrt{3}} \right)^3 \frac{1}{1+\alpha} &= 0, \\ - \left(\bar{S}_1 - \frac{\bar{\theta}_p^2}{1+\alpha} \right) \frac{\eta_d(1+\alpha^2)}{\alpha^2} + \bar{\theta}_p^2 \left(\bar{\delta k} + \frac{\eta_d}{\alpha} \right) &= \left(\frac{2}{\sqrt{3}} \right)^3. \end{aligned} \quad (\text{A.13})$$

Figures A.2, A.3, and A.4 illustrate how η_d , $\bar{\theta}_p$ and η_γ affect the gain curves of the fundamental mode according to (A.13). Each contribution pushes the peak of the gain curve into the detuning region $\bar{\theta} < 0$ where the e-beam energy is above the resonant energy. The maximum in the detuning curve varies in a complicated way with the parameters that specify the FEL. While these effects have been studied

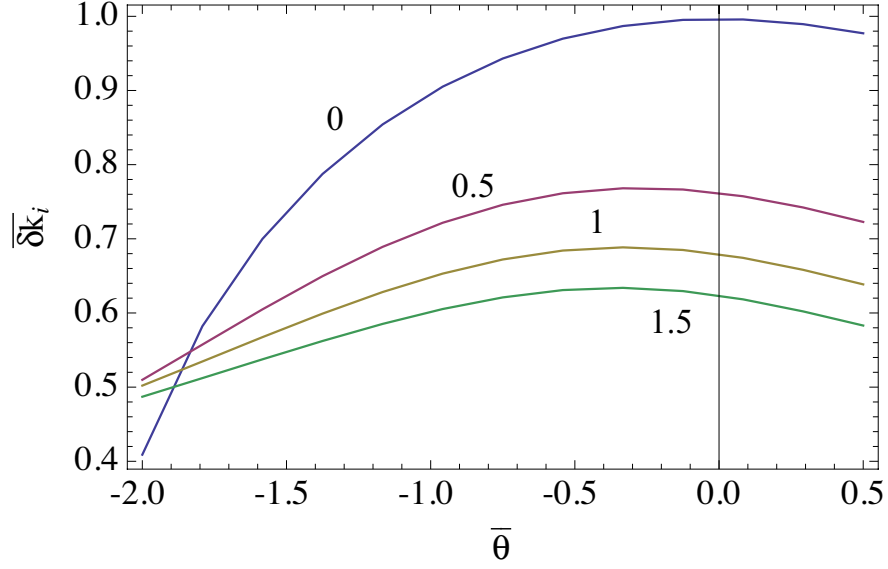


Figure A.2: Detuning curves of the fundamental (0,0) mode for $\eta_d=0, 0.5, 1$, and 1.5 with $\bar{\theta}_p, \eta_\gamma=0$.

previously[35, 46] they serve to demonstrate that the desired power fit formula, calculated over a wide range of accessible parameters that still yield high-gain solutions, would be a useful tool over numerical solutions of the full equations for quick determination of the peak gain.

A.3 Power fit formula

Numerical solutions to the full variational equations taken from Eq (A.8) are fit to a power formula over the constituent parameter space of $\eta_d, \eta_\gamma, \bar{\theta}_p$ and $\bar{\theta}$. At the detuning value that minimizes the gain length for given values of $(\eta_d, \eta_\gamma, \bar{\theta}_p)$,

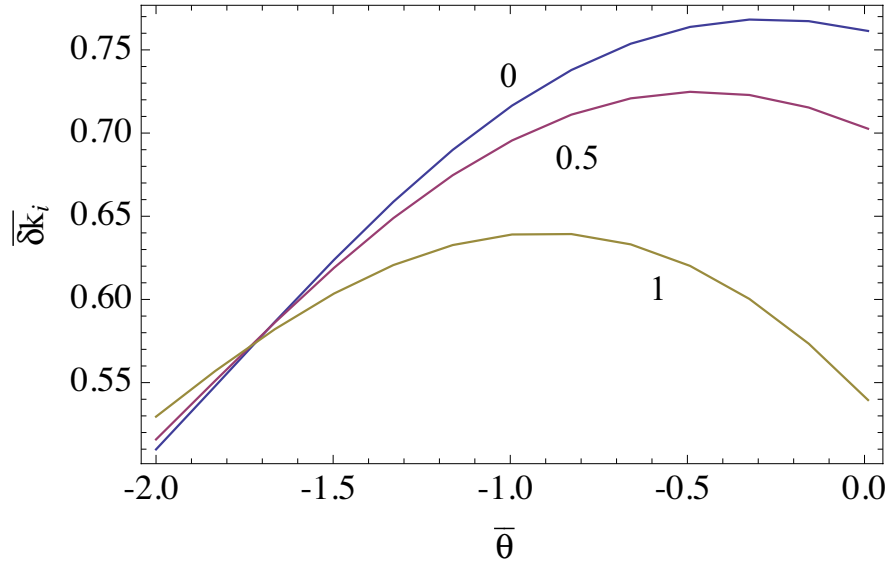


Figure A.3: Contribution of space-charge on 3D system: $\bar{\theta}_p=0, 0.5$, and 1 with $\eta_\gamma=0$, $\eta_d = 0.5$.

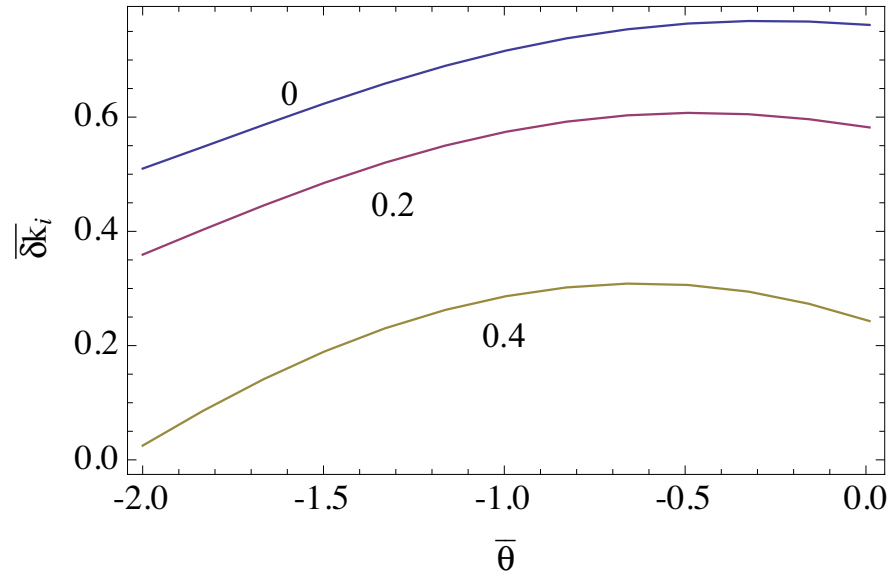


Figure A.4: Contribution of energy spread on 3D system: $\eta_\gamma=0, 0.2$, and 0.4 with $\bar{\theta}_p=0$, $\eta_d = 0.5$.

the fit is found to be,

$$\begin{aligned}\Lambda_{0,0} = & 0.450\eta_d^{0.570} + 3.00\eta_\gamma^2 + 0.196\bar{\theta}_p^{1.91} + 51.0\eta_d^{0.950}\eta_\gamma^3 \\ & + 0.0988\eta_d^{0.230}\bar{\theta}_p^{1.21} + 0.0375\eta_\gamma^{0.875}\bar{\theta}_p^{12.7} \\ & + 2.35\eta_d^{11.9}\eta_\gamma^{14.9}\bar{\theta}_p^{11.4}\end{aligned}\tag{A.14}$$

Note the precise agreement with Xie's fitting formula for the parameters η_d and η_γ when $\bar{\theta}_p = 0$. To illustrate the utility of this formula and in the spirit of Xie the representative parameters for the MW class FEL amplifier from [101] are used, where $E = 65\text{MeV}$, $K = 1.26$, $\lambda_u = 1.8\text{cm}$, $\epsilon_n = 2.0\text{mm} - \text{mrad}$, $I = 600\text{A}$, $\sigma_\gamma = 5 \times 10^{-4}$, $\sigma_x = 100\mu\text{m}$. The scaled parameters for this case are $\eta_d = 0.641$, $\eta_\gamma = 0.028$, and $\bar{\theta}_p = 0.400$. The effects of diffraction, energy spread and longitudinal space-charge conspire to extend the gain length from $L_{1D} = 0.081\text{m} \rightarrow L_g = 0.114\text{m}$.

A.4 Simulation and Results

The FEL simulation code Genesis 1.3[9] was used to evaluate the accuracy of the fitting formula in (A.14). Nearly 14000 time-independent simulations were performed using a large range of rms transverse electron beam sizes, relative rms energy spreads, and beam currents in order to sample a significant portion of the scaled parameter space where the analytical model is applicable (see Table A.1). As with the analytic model, the detuning was adjusted to obtain the shortest gain length for each set of parameters. At each optimized detuning value a scaled value of the gain is obtained, denoted by $\overline{\delta k_i}^G$.

To quantify the agreement between the analytic model fit in (A.14) with

Parameter	Symbol	Values
Diffraction	$\eta_d = L_{1D}/2k\sigma_x^2$	0.04 (10^{-4}) – 0.98 (1)
Energy spread	$\eta_\gamma = 2k_u\sigma_\gamma L_{1D}$	0 (10^{-4}) – 0.34 (0.4)
Space-charge	$\bar{\theta}_p = 2\theta_p L_{1D}$	0.16 (10^{-4}) – 0.42 (1)

Table A.1: Scan parameters. Simulations (Theory)

results from Genesis simulations, a variance is defined of the form

$$\sigma^2 = \frac{1}{N} \sum_j^N \left(\overline{\delta k_i}(j) - \overline{\delta k_{i,j}}^G \right)^2, \quad (\text{A.15})$$

where $\overline{\delta k_i}(j)$ is the value of the scaled gain from the analytic fit at the point $(\eta_{d,j}, \eta_{\gamma,j}, \bar{\theta}_{p,j})$, and $\overline{\delta k_{i,j}}^G$ is the numerical value from Genesis at the same point. Good agreement is found, with a variance of $\sigma^2 = 5.4 \times 10^{-4}$ over the entire parameter space. For comparison, an independent power fit was performed according to the data points from Genesis. This fit is denoted by the scaled gain function $\overline{\delta k_i}^F = \overline{\delta k_i}^F(\eta_d, \eta_\gamma, \bar{\theta}_p)$. Using the fit function $\overline{\delta k_i}^F$ in place of the analytic fit function $\overline{\delta k_i}$, the variance was nearly the same value, $\sigma^2 = 4 \times 10^{-4}$, demonstrating that the model in (A.14) provides a reliable measure of the scaled gain length. Figure A.5 shows the qualitative consistency between the analytic contour from (A.14) and data points obtained through Genesis simulations for the specific case of $\eta_\gamma = 0$.

Figure A.6 shows the relative difference between the analytic and simulated fits, given by $\sigma^F = 1 - \overline{\delta k_i}^F / \overline{\delta k_i}$. The two fits agree across the scaled parameter space for $\eta_\gamma = 0$ to within 4%. Figure A.7 also shows the relative difference, in this case as a function of energy spread and space-charge for $\eta_d = 0.5$. This plot

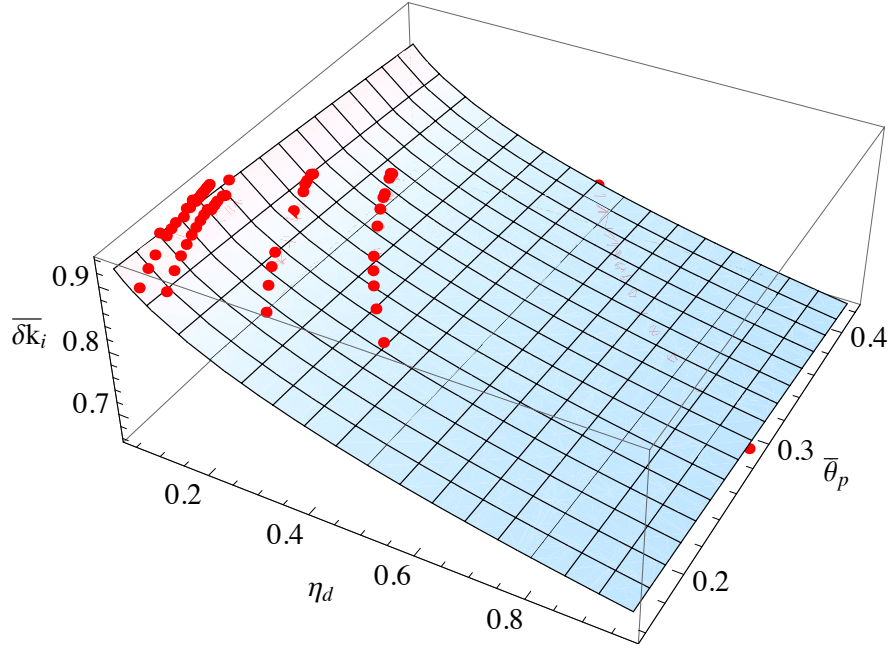


Figure A.5: Analytic fit contour and simulated data points for $\eta_\gamma = 0$.

indicates that the two fits are in better agreement as the energy spread increases. Therefore, the 4% relative difference seen in figure A.6 where $\eta_\gamma = 0$ is the largest error found across the scaled parameter space.

It is worth noting that, as the diffraction parameter increases, the difference between the analytic and simulated fits increases as well. This can be seen in Figure (A.5) where the analytic contour lies slightly above the simulated data values for large values of the diffraction parameter. This illustrates the limitations of the theory as the FEL system becomes more three dimensional in nature. It occurs because, as the diffraction parameter increases and the FEL becomes more sensitive to 3D effects, the transverse mode profile of the FEL light also increases. When the optical mode becomes larger than the e-beam, the shape of the profile begins to differ from that of the simple Gaussian assumed in the single-mode analytic model. This behavior is depicted in Figure A.8 where the theoretical

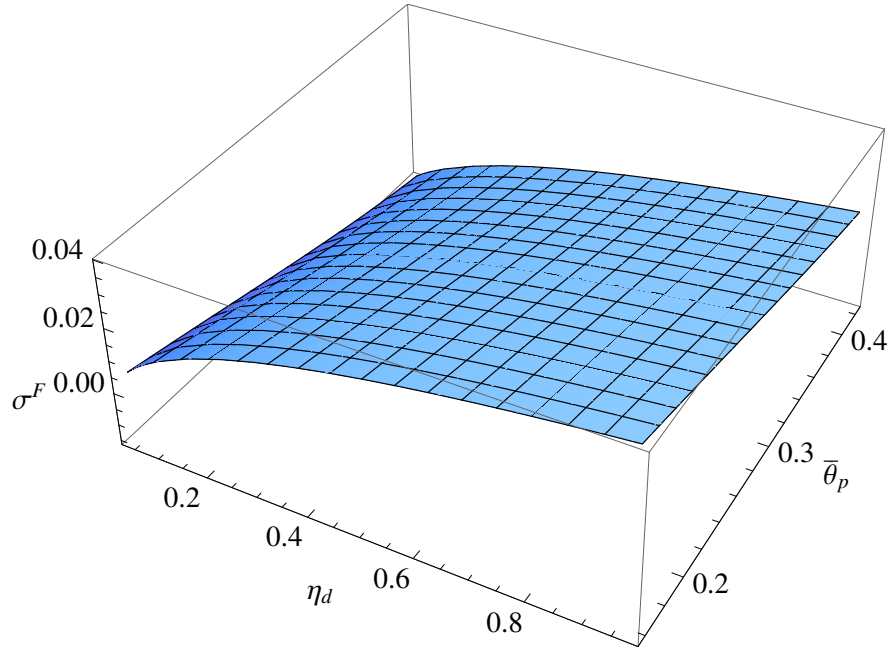


Figure A.6: Relative difference between the analytic and simulated fits for $\eta_\gamma = 0$.

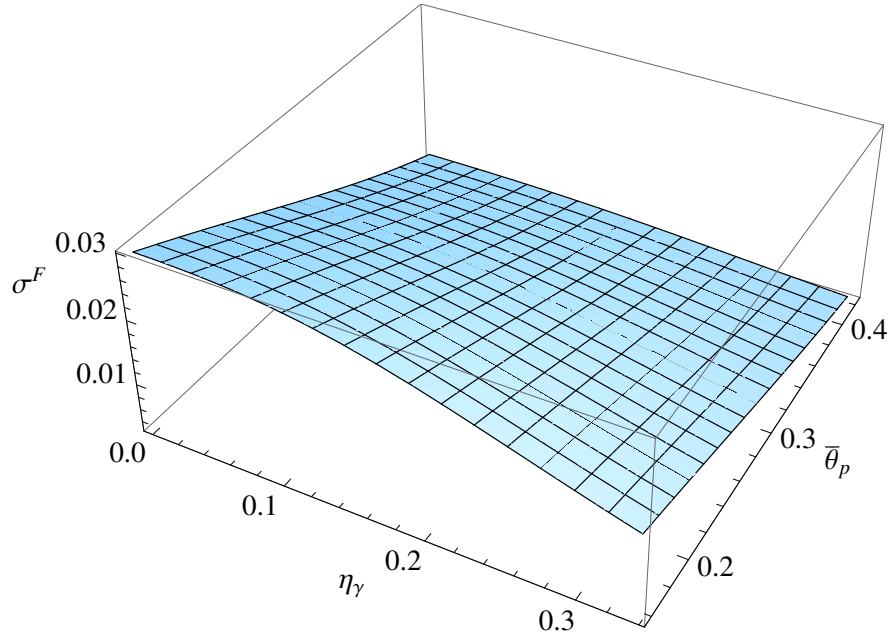


Figure A.7: Relative difference between the analytic and simulated fits for $\eta_d = 0.5$.

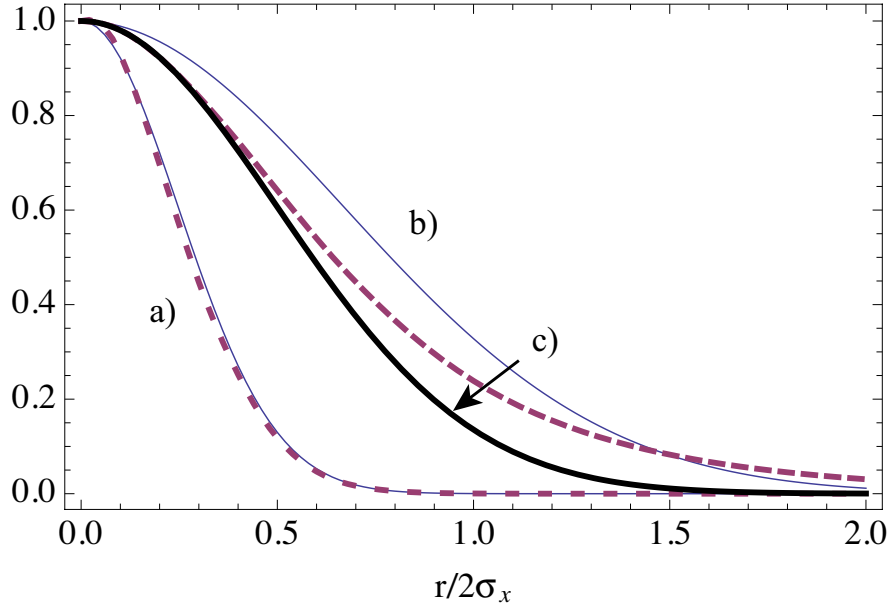


Figure A.8: Comparison of the transverse normalized intensity mode profile between simulation and theory for two cases: (a) a quasi-1D scenario with $\eta_d = 0.04, \eta_\gamma = 0, \bar{\theta}_p = 0.23$ where the mode profile is contained within the e-beam, and (b) a 3D scenario with $\eta_d = 0.97, \eta_\gamma = 0, \bar{\theta}_p = 0.28$ where the edges of the mode profile lie outside the e-beam. Genesis results are represented as dashed lines while the solid, blue lines represent results from theory for both cases. The e-beam is shown in black (c).

and simulated optical profiles for two different regimes are compared. For small η_d the system tends to be more 1D and the optical mode is well inside the e-beam. The agreement is excellent in this case, both in the predicted optical spot size and in the gain length. But as η_d approaches unity, the large optical mode calculated from Genesis clearly has richer structure, and does not precisely match the profile predicted from theory. This occurs because the region of the laser field near the e-beam center becomes narrowed due to gain compared to the region farther from the axis. The result is a sharper profile. The inability of the single Gaussian mode description to precisely match the optical profile for strongly 3D FELs is the primary source of the few percent error between the single mode

solutions and results from simulations for the predicted gain length. Future work could strive to eliminate this small error by instead using a non-Gaussian mode better suited to describe the pinched optical profiles of strongly 3D FELs.

It is also interesting to note that the agreement between simulated and analytic fits actually improves as both the space-charge and the energy spread increase. This can clearly be seen in Figure A.7, where the relative difference between the analytical and simulated fits is shown as a function of η_γ and $\bar{\theta}_p$ for $\eta_d = 0.5$. Figure A.9 shows that the agreement extends to the predicted profile, where despite a near doubling of the space-charge contribution, the profiles closely match. This illustrates that the presence of strong space-charge effects are well modeled by the theory presented.

Note that the numerical fit to the Genesis data points, $\overline{\delta k_i^F}$, was omitted. This result was not included for a variety of reasons. The power fit that was performed does not yield a unique solution, and the coefficients that are recovered from the fit are subject to multiple factors. These factors include the minimization method, the function used to minimize the residuals, and the constraints applied to initial parameter values. Regardless of these variables, every fit that was performed yielded similar results, where the agreement with the analytic fit was accurate to within at most $\sim 4\%$. Also, the probed parameter space from the Genesis simulations did not span the entire space where the analytic model is applicable, as evidenced in table A.1. In addition, Genesis includes only longitudinal space-charge and does so on a Fourier decomposition basis. The inclusion of more Fourier coefficients in the simulation increases the accuracy of the space-charge calculation at the expense of computing time and resources. The power fits obtained through simulation were in better agreement with the analytic model as more Fourier coefficients were included. For these reasons, only the results of

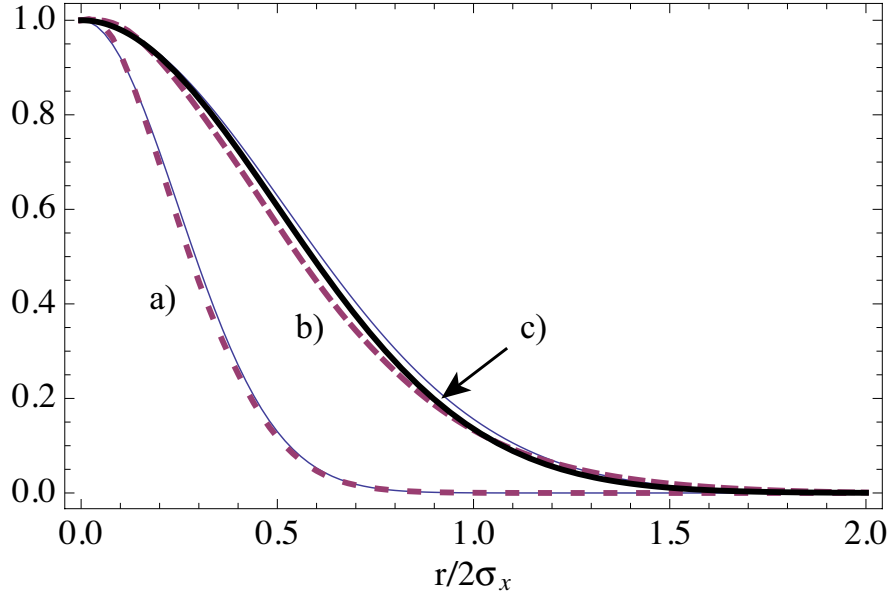


Figure A.9: Comparison of the transverse normalized intensity mode profile between simulation and theory for two cases: (a) $\eta_d = 0.04, \eta_\gamma = 0, \bar{\theta}_p = 0.23$ where the mode profile is contained within the e-beam and (b) $\eta_d = 0.45, \eta_\gamma = 0, \bar{\theta}_p = 0.42$ where the mode profile approximately matches the e-beam. Genesis results are represented as dashed lines while the solid, blue lines represent results from theory for both cases. The e-beam is shown in black (c).

the analytic fit are quoted.

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