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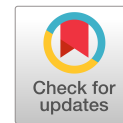
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Estimation of Timing of Liquefaction Using Spectral Energy Ratio

Kil-Wan Ko¹; Robert E. Kayen, M.ASCE²; and Chukwuebuka C. Nweke, A.M.ASCE³

Abstract: The significant reduction in the stiffness of liquefied soil is accompanied by a decrease in the shear wave velocity, which ultimately results in the softening of the liquefied site. Time–frequency response analysis can identify the sudden drop in the frequency of the liquefied site, which has been widely employed to determine the onset of liquefaction. However, using the modal frequency (corresponding to the maximum power at each time step) to identify the timing of liquefaction (t_L) captures the reduction in frequency during earthquakes, but it does not encompass the entire range of frequencies that have changed. Furthermore, previous literature defines t_L as the boundary separating the modal frequency into pre- and postliquefaction time segments, but this estimate does not consider the generation of pore water pressure. Two representative case histories are presented to highlight the limitations of identifying t_L by solely relying on the modal frequency approach that uses a two-step function. As a result, this study introduces an innovative method to identify t_L utilizing the spectral energy ratio (SER), which captures the entire frequency shift. A step-by-step procedure using SER is detailed, and the new estimates of t_L are compared with those derived from previous literature using 30 case histories. To validate the approach, a sensitivity analysis was performed using centrifuge test data from the Liquefaction Experiment and Analysis Projects. Results indicated that incorporating a ramp that accounts for pore water pressure buildup in the trilinear function improved t_L estimation. An optimized SER value of 0.92 was determined for the proposed method. The notable contribution of this study is an enhanced approach of identifying the timing of liquefaction triggering by only utilizing acceleration records without requiring pore water pressure responses. DOI: 10.1061/JGGEFK.GTENG-12442. © 2025 American Society of Civil Engineers.

Author keywords: Soil liquefaction; Spectral energy ratio; Timing of liquefaction; Stockwell transform; Modal frequency; Case history; Centrifuge tests.

Introduction

Soil liquefaction is a phenomenon in which cyclic loading causes saturated granular soil to behave as a liquid-like material due to the generation of excess pore water pressure. Rapid shaking/loading, such as those induced by earthquakes, causes pore water pressure to build up faster than it can dissipate, thereby reducing the effective stresses, which in turn diminishes the shear strength and stiffness of the soil. This abrupt reduction of stiffness and strength in liquefied soil leads to catastrophic consequences in urban areas. The 1995 M_w 6.9 Kobe (Hyogoken–Nanbu) Earthquake induced soil liquefaction that affected the port, resulting in significant deformation of the quay wall and ground settlement (Soga 1998). Similarly, the 2011 M_w 6.2 Christchurch Earthquake caused liquefaction-induced differential settlements, which tilted buildings and led to lateral movement of foundations and bearing capacity failures (Bray and Dashti 2014). Most recently, liquefaction-induced settlement caused flooding, bridge foundation failure, and

transportation corridor damage during the February 2023 Türkiye Earthquakes (M_w 7.8 mainshock and M_w 7.7 aftershock) (Cetin et al. 2023).

The drastic decrease in the stiffness of liquefied soil is accompanied by a reduction of the shear wave velocity, which softens the ground at the liquefied site. During the occurrence of liquefaction, this ground softening causes elongated acceleration responses, a phenomenon that has been recorded at numerous strong motion stations (Hu et al. 2005; Ishihara and Cubrinovski 2005; Unjoh et al. 2012; Yuan et al. 2010; Zhan and Chen 2021). The Stockwell transform, which illustrates time–frequency responses of earthquake records, visualizes sudden frequency changes attributed to liquefaction-induced ground softening. Millen et al. (2021) utilized Stockwell transforms to develop a method of dynamic site response analysis, which incorporated the effect of excess pore water pressure generation and shear strain development of liquefied soil. The frequency of a site is significantly influenced by the subsurface shear wave velocity, the evolution of excess pore water pressure, and shear strain. As such, the frequency change of the site implies synthesized information related to the liquefied soil during earthquakes (Manandhar et al. 2021).

The ground softening that occurs during liquefaction also reduces the shear wave velocity (increases the travel time of shear waves passing through the liquefied soil layer), which subsequently makes shear waves undergo more cycles over the same distance such that more energy dissipation occurs compared to a site that did not liquefy (Millen et al. 2021). This feature of liquefied soil layers is akin to *natural seismic isolation* or the *seismic base-isolation* effect, resulting in a reduction of high-frequency content and an increase of the low-frequency contents of recorded motion (Kokusho 2014). Bouckovalas et al. (2016) found that the *seismic isolation* effect amplified low-frequency energy and attenuated

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high-frequency energy of ground motion as long as a minimum liquefied layer thickness was secured to cause the isolation effect. Note that the entire frequency content of seismic waves is biased to the lower end of the frequency spectrum as the wave travels through the liquefied layer (Kramer et al. 2016). This bias appears as a reduction in the modal frequency over time in the recorded signals. This modal frequency change over time has been identified as a good indicator for identifying the triggering time of soil liquefaction (Glaser 1996; Kostadinov and Yamazaki 2001; Kramer et al. 2016; Özener et al. 2020).

Case histories, as well as laboratory and centrifuge tests, have demonstrated that a significant amount of shear strain accumulates after liquefaction is triggered (Malvick et al. 2008; Kishida and Tsai 2021; Saathoff and Achmus 2023). Therefore, determining the accurate timing of liquefaction triggering (t_L) is essential for better estimating the permanent deformation of liquefied soil. Kramer et al. (2016) introduced a method for identifying t_L using the modal frequency change, its impact on established liquefaction triggering curve and estimates of lateral spreading. Özener et al. (2020) expanded upon findings by Kramer et al. (2016) by delineating a way to identify t_L through a two-step function isolating pre- and post-liquefaction time segments using the modal frequency change over time. Rios et al. (2022) described two methods, strain energy-based and equivalent cyclic stress-based methods, for predicting pore pressure buildup and t_L . Zhang et al. (2023) trained a deep learning model that could distinguish nonliquefied and liquefied motions as well as t_L using the Stockwell transform images. These models and approaches provide robust estimates of t_L , but there are parts that need improvement: (1) the method established by Kramer et al. (2016) utilized the modal frequency change, which cannot represent the entire frequency change during liquefaction; (2) the two-step function prescribed by Özener et al. (2020) divides the modal frequency change into two parts, pre- and postliquefaction, but does not consider a portion for the generation of excess pore water pressure, which can potentially lead to earlier estimates of t_L ; and (3) the model by Zhang et al. (2023) estimates t_L solely using the Stockwell transform.

This study aims to develop a method for estimating the timing of liquefaction using the spectral energy ratio (SER). This method addresses the limitations of the previous models by highlighting the importance of the SER, incorporating a ramp in the two-step function that corresponds to the generation of pore water pressure; this study named the two-step function with the ramp as a trilinear function. To begin, the study reviews the previous methods developed by Özener et al. (2020) and Kramer et al. (2016) with two case histories. In order to identify t_L , the methodology employing the SER is introduced and applied to 30 strong motion records. The estimated t_L for the strong motion records from the newly developed method is then compared with that from the previous method. Finally, the method is validated using centrifuge tests from the Liquefaction Experiment and Analysis Projects [LEAP; alternately referred to by its model number, LEAP-UCD-2017 (Kutter et al. 2020)] by comparing the estimated t_L obtained from accelerometers with the measured t_L from pore water pressure transducers.

Frequency Characteristics of Liquefied Soil

Utilizing the basic concept of fundamental site frequency and its alteration caused by the accumulation of excess pore water pressure, it is possible to account for the frequency shift of liquefied soil. This section elucidates the conceptual impact of liquefaction on the alteration of site frequency. Two case histories are provided

to offer a detailed explanation of the changes in modal frequency, their azimuthal variations, and the implementation of a two-step function. This approach aims to identify t_L as proposed by Kramer et al. (2016) and Özener et al. (2020).

Fundamental Frequency of Liquefied Site

The fundamental site frequency ($f_{so} = V_s/4H$) depends on the shear wave velocity (V_s) and thickness (H) of the soil deposit. Since V_s is a function of mean effective stress (σ'_m) of the soil, f_{so} in the low-strain level is expressed, as below

$$f_{so} = \frac{V_s}{4H} = \frac{\sqrt{G_{\max}/\rho}}{4H} = \frac{\sqrt{G_0(\sigma'_m/P_a)^{0.5}/\rho}}{4H} \quad (1)$$

where $G_{\max}$ = maximum shear modulus at a low strain level; ρ = density of the soil; P_a = atmospheric pressure; and G_0 = shear modulus at 1 atm. Considering the effect of the excess pore water pressure (Δu) on σ'_m , the site frequency (f_s) during liquefaction can be expressed in response to Δu , as below

$$\begin{aligned} f_s &= \frac{\sqrt{G_0(\frac{\sigma'_m - \Delta u}{P_a})^{0.5}/\rho}}{4H} = \frac{\sqrt{G_0(\frac{\sigma'_m(1 - \Delta u/\sigma'_m)}{P_a})^{0.5}/\rho}}{4H} \\ &= \frac{\sqrt{G_0(\frac{\sigma'_m(1 - r_u)}{P_a})^{0.5}/\rho}}{4H} \end{aligned} \quad (2)$$

where r_u = excess pore water pressure ratio ($\Delta u/\sigma'_m$). The conceptual estimation of the fundamental site frequency forms the basis for understanding the frequency change of the soil during liquefaction. Using Eqs. (1) and (2), the ratio of fundamental site frequency change (f_s/f_{so}) can be expressed as a function of r_u , as below

$$f_s/f_{so} = (1 - r_u)^{1/4} \quad (3)$$

Fundamental site frequency decreases in accordance with the r_u buildup (Fig. 1). Notably, the change in frequency becomes

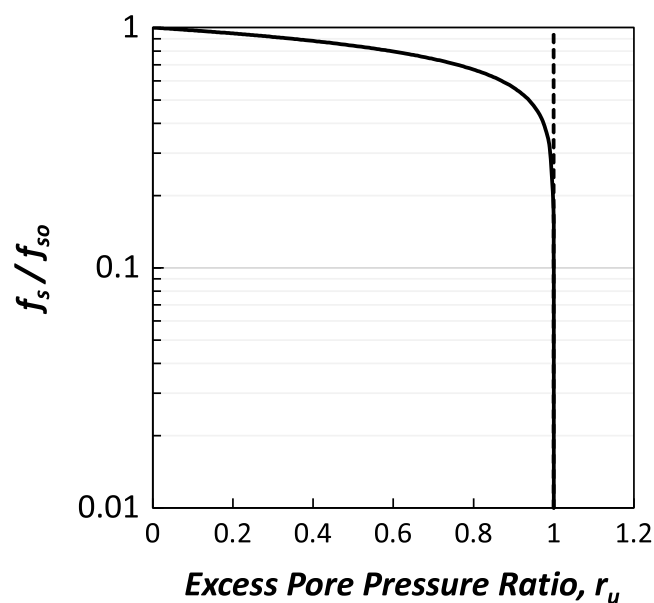


Fig. 1. Ratio of fundamental site frequency change according to excess pore water pressure ratio.

significantly pronounced at approximately $r_u = 0.8$. As an illustrative instance, the fundamental site frequency experiences a halving when $r_u = 0.94$, a value closely associated with liquefaction triggering (Fig. 1). Consequently, strong motion records from liquefied soil exhibit an abrupt decrease in frequency during earthquakes. The subsequent section will analyze time–frequency responses and t_L through two case histories, following the approach introduced by Kramer et al. (2016) and Özener et al. (2020).

1968 Tokachi-Oki Earthquake at Aomori Harbor Station

The 1968 Tokachi-oki earthquake (M_w 8.3) occurred on May 16. The northern portion of Aomori Harbor station facing the sea was constructed on reclaimed land and was fully saturated. This earthquake induced liquefaction that showed surficial manifestation in the form of sand volcanoes and settlements of buildings within close proximity to the station (Yasuda and Tohno 1988). The soil composition beneath the station is comprised of fine sand, silty sand, and gravel, rendering it susceptible to liquefaction. Yasuda and Tohno (1988) performed a stress-based evaluation of liquefaction triggering that showed a factor of safety at the station of less than unity for depths ranging from 0 to 10 m.

Strong motion recordings captured at the station also revealed evidence of liquefaction during the 1968 Tokachi-oki earthquake (Fig. 2). Undrained cyclic loadings liquefy the soil which loses its shear stiffness and results in longer periodic motions. Continued cyclic loading leads to the dilational responses of the liquefied soil beyond the phase transformation line, thereby increasing the effective stress and causing dilation spikes (Kutter and Wilson 1999). This can be observed in the acceleration records for N–S and E–W directions, which display longer periodic motions and dilation spikes occurring after approximately 30 s [Fig. 2(a)]. Peak ground accelerations (PGAs) are 0.22 g at 39 s and 0.19 g at 36 s for N–S and E–W directions, respectively.

Liquefaction triggering typically transpires when greater stresses are applied to the soil during earthquakes. Thus, this study establishes a low threshold acceleration of 0.04 g, and only portions of acceleration records exceeding 0.04 g were subjected to the

Stockwell transform. To account for the frequency changes in both horizontal motions, the Stockwell transform combines N–S and E–W components by taking the norm of their individual power spectrograms. The Stockwell transform was normalized based on the maximum amplitude of each time step to clearly visualize frequency changes over time. This method of using the Stockwell transform was uniformly applied to all acceleration records utilized in this study. The Stockwell transform of the recorded motions distinctly reveals the frequency changes in the time–frequency domain [Fig. 2(b)]. Initially, the motions have a significant energy in the 5–10 Hz range, but at around 20 s, a pronounced frequency drop occurs, followed by a continuous decline. Beyond 60 s, a substantial amount of energy is centered around 0.5 Hz.

As introduced by Özener et al. (2020), the combined Stockwell transform in Fig. 2(b) produces the modal frequency, representing a single frequency corresponding to the maximum power at each time bin in the Stockwell transform [Fig. 3(a)]. The modal frequency change follows the same trend as the Stockwell transform. Nonetheless, the frequency change in recorded motions can also be affected by the generation of secondary waves due to earthquake motions interacting under boundary effects. For instance, surface waves can also induce an abrupt decrease in frequency irrespective of liquefaction triggering. However, while liquefaction results in a decrease in frequency regardless of the direction of acceleration records, surface waves induce a frequency reduction in specific directions due to polarization. In this context, azimuthal modal frequency facilitates the differentiation of these two effects [Fig. 3(b)]. By employing two perpendicular ground motions, rotated ground motions were computed to determine the modal frequency change over time for each direction across a range of 0 to 180 degrees. For the 1968 Tokachi-oki earthquake, the azimuthal modal frequency plot shown in Fig. 3(b) exhibits a reduction in frequency of less than 1 Hz at approximately 15 s for modal frequencies in the 45-degree rotated directions (N–W and S–E) that is not observed in the N–E and S–W directions. Therefore, 15 s cannot be identified as t_L . However, after approximately 20 s, an appreciable reduction of frequency becomes evident in all directions that is associated with liquefaction.

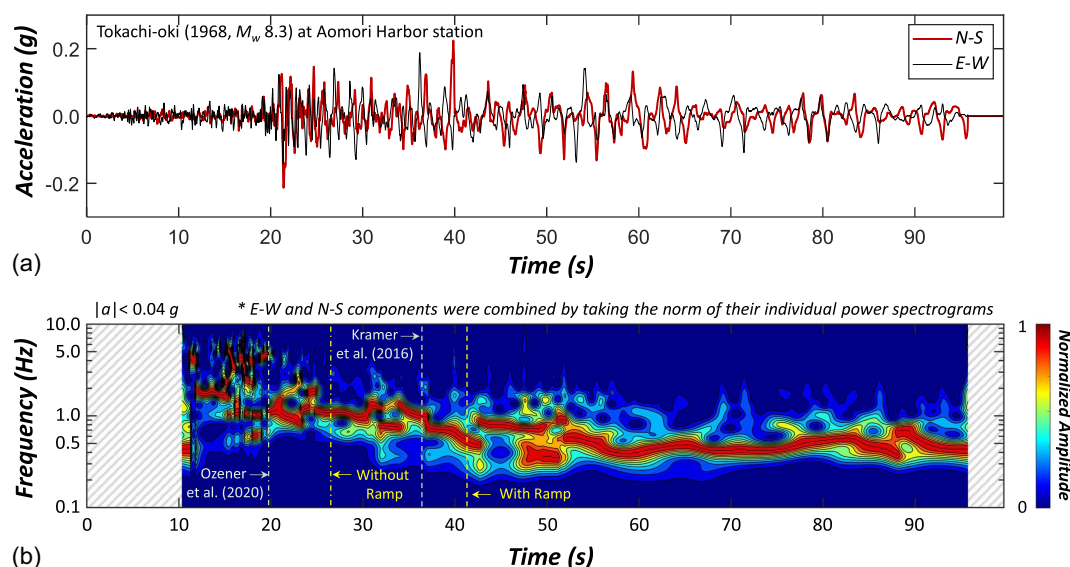


Fig. 2. Acceleration and time–frequency responses during the 1968 Tokachi-oki earthquake recorded at Aomori Harbor station: (a) acceleration time histories for N–S and E–W directions; and (b) Stockwell transform of horizontal motions. Dotted lines in the Stockwell transform indicate the timing of liquefaction from this study, Özener et al. (2020), and Kramer et al. (2016).

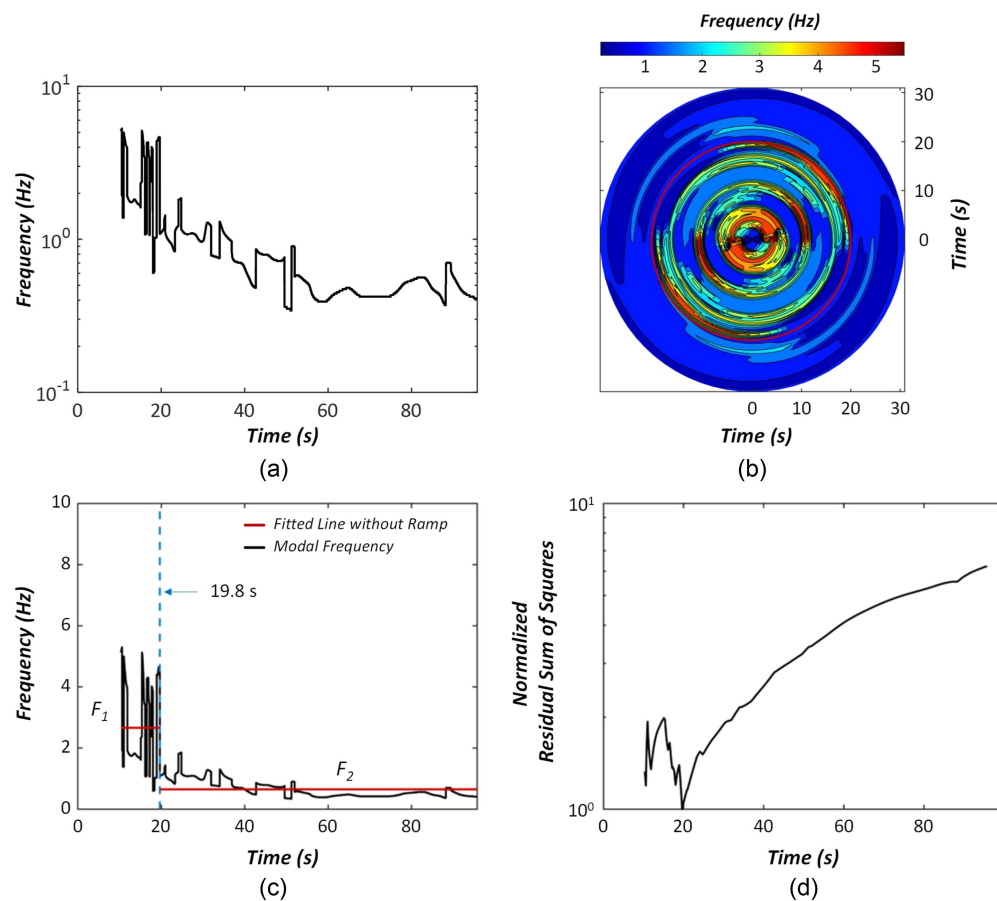


Fig. 3. Modal frequency change and timing of liquefaction during the 1968 Tokachi-oki earthquake recorded at Aomori Harbor station: (a) modal frequency change from combined Stockwell transform; (b) azimuthal modal frequency; (c) a two-step function for modal frequency change; and (d) normalized residual sum of squares.

Greenfield (2017), Kramer et al. (2016), and Özener et al. (2020) devised a method to determine t_L using a two-step function. The process to find the optimized two-step function unfolds as follows: (1) the two-step function segregates the modal frequency change into two segments corresponding to nonliquefied and liquefied zone at an arbitrary time (τ) when the step occurs; (2) the function computes two modal frequencies, F_1 and F_2 , as the mean modal frequencies from 0 to τ and τ to the end of the shaking, respectively [Fig. 3(c)]; (3) it calculates the residual sum of squares between the two-step function and modal frequency change over time; (4) the computed residual sum of squares signifies the residual of each τ and the procedure repeats with τ sweeping from 0 to the end of shaking [Fig. 3(d)]; (5) the optimal two-step function is identified when a minimum value of the normalized residual sum of squares is achieved, and the corresponding τ demarcating the two steps in the optimized function indicates t_L . Detailed steps of this procedure are described in Özener et al. (2020).

Using the two-step function, t_L is determined as 19.8 s during the 1968 Tokachi-oki earthquake, as recorded at Aomori Harbor station [Fig. 3(d)]. Moreover, the estimated t_L is consistent with the value derived from the azimuthal modal frequency plots. Employing and comparing these two methods for identifying the timing of liquefaction is compelling and some observations can be made with regards to their effectiveness. First, liquefaction does not manifest abruptly; thus, a function could be introduced to track the generation/buildup of excess pore water pressure. Second, this case history demonstrates the noisiness of modal frequency versus

time, which complicates the identification of the distinct reduction in frequency that represents a precise timing.

1995 Kobe (Hyogoken–Nanbu) Earthquake at Amagasaki Station

The 1995 Kobe (Hyogoken–Nanbu) earthquake (M_w 6.9) struck the Kobe–Osaka region in Japan on January 17. The Amagasaki station in Osaka is situated near estuaries, leading to the presence of loose and fine alluvial sand deposits overlying a thick alluvial clay layer (Shibata et al. 1996). The sand layers beneath Amagasaki consist of relatively uniform particle sizes, increasing its susceptibility to liquefaction. The Amagasaki station is approximately 10 km from the eastern edge of the fault (Kagawa et al. 1996), and numerous sand boils were observed in its vicinity.

Ground motion acceleration recorded at the station showed PGAs of 0.28 g at 10 s and 0.33 g at 12 s for the N–S and E–W directions, respectively [Fig. 4(a)]. Given the station's proximity to the fault, recorded earthquake waves for both components already contained low-frequency energy. At around 8 s, when the earthquake wave arrived at the station, the Stockwell transform reveals high amplitude energy at a frequency of approximately 1.5 Hz [Fig. 4(b)]. Due to the near-fault effect, the frequency drop attributed to the liquefaction is not clearly discernible. Modal frequency with indistinct reductions in frequency at 13 s and 16 s and azimuthal modal frequency changes with indistinct frequency changes demonstrate the ambiguity and challenges in identifying t_L (Fig. 5).

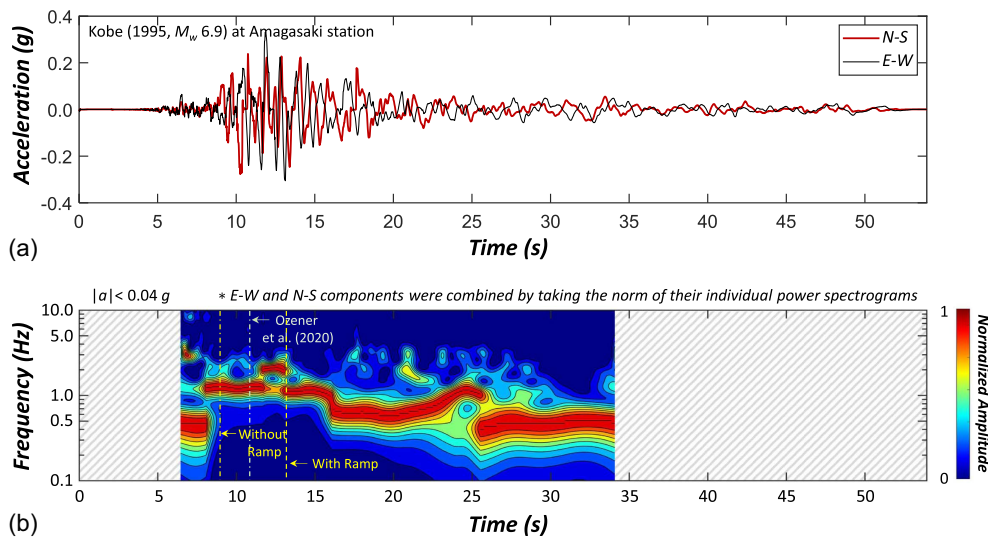


Fig. 4. Acceleration and time–frequency responses during the 1995 Kobe (Hygoken Nanbu) earthquake recorded at Amagasaki station: (a) acceleration time histories for N–S and E–W directions; and (b) Stockwell transform of horizontal motions. Dotted lines in Stockwell transform indicate the timing of liquefaction from this study and Özener et al. (2020).

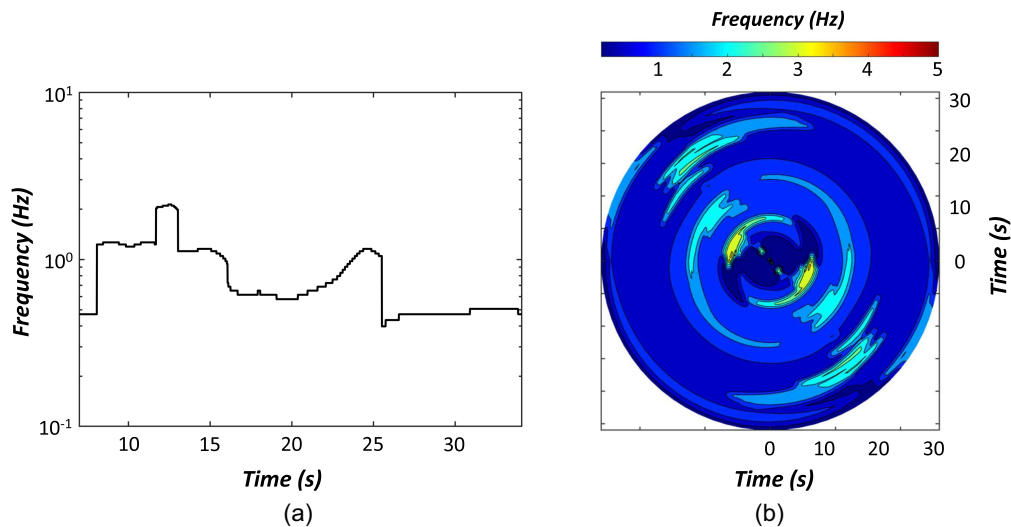


Fig. 5. Modal frequency change during the 1995 Kobe (Hygoken Nanbu) earthquake recorded at Amagasaki station: (a) modal frequency change from combined Stockwell transform; and (b) azimuthal modal frequency.

Estimation of Timing of Liquefaction Triggering Using Spectral Energy Ratio

Spectral Energy Ratio

Tracking changes in modal frequency and its azimuthal variations represent an important advancement in estimating the timing of liquefaction. However, the case histories presented highlight the complexity of identifying the timing of liquefaction as a reflection of distinct changes in the modal frequency. For example, Özener et al. (2020) and Kramer et al. (2016) estimated t_L as 19.8 s and 36.5 s, respectively, during the 1968 Tokachi-oki earthquake recorded at Aomori Harbor station [Fig. 6(a)]. In the frequency domain, the modal frequencies at 19.8 s and 36.5 s are nearly identical, measuring 1.09 and 0.98 Hz, respectively [Figs. 6(b and c)]. However, in terms of the overall frequency distribution, the energy at 19.8 s comprises relatively higher frequencies, whereas at 36.5 s,

the majority of energy is dominated by lower frequency components. To quantitatively capture the complete frequency change over time, this study introduces the concept of the SER, as follows:

$$\text{SER} = \frac{\int_{f_L}^{f_H} |X(f)| df}{\int_{0\text{Hz}}^{10\text{Hz}} |X(f)| df} \quad (4)$$

where $X(f)$ = Fourier amplitude as a function of frequency (f); and f_L = arbitrarily chosen frequency affecting the area labeled as B in Fig. 6(b). Put simply, the SER calculates the ratio between the area of B and area of A in Fig. 6(b). SER depends on f_L and is computed for each time step of the Stockwell transform, so that the SER becomes a function of f_L and time (t).

SER three-dimensional plots vividly illustrate the comprehensive frequency shift over time during earthquakes (Fig. 7). These figures present the variation of SER over time by altering the value of f_L and the vertical projection of the SER during 1968

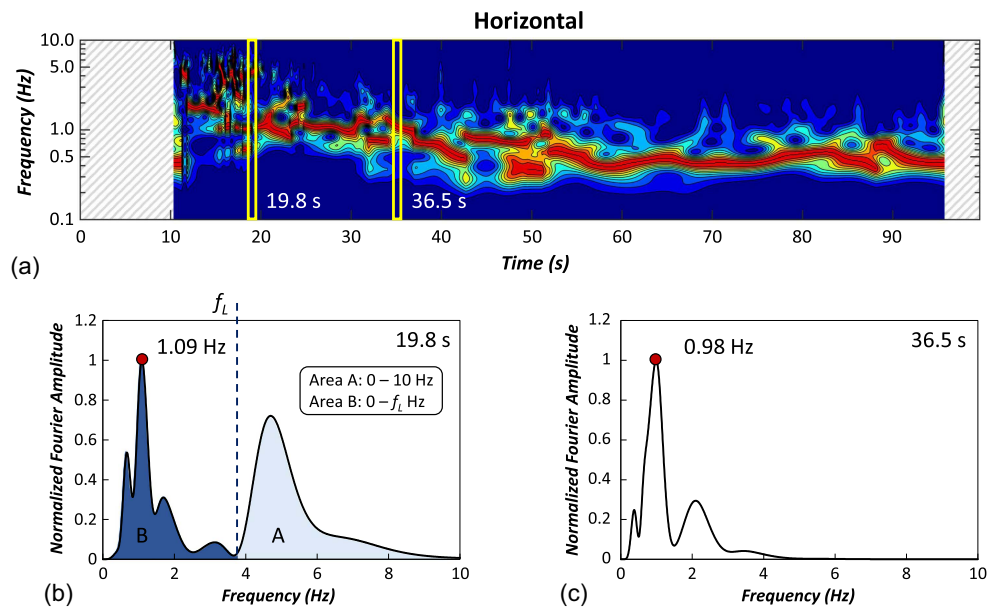


Fig. 6. Stockwell transform and frequency domain at each time step during the 1968 Tokachi-oki earthquake recorded at Aomori Harbor station: (a) combined Stockwell transform; (b) frequency domain at 19.8 s; and (c) frequency domain at 36.5 s.

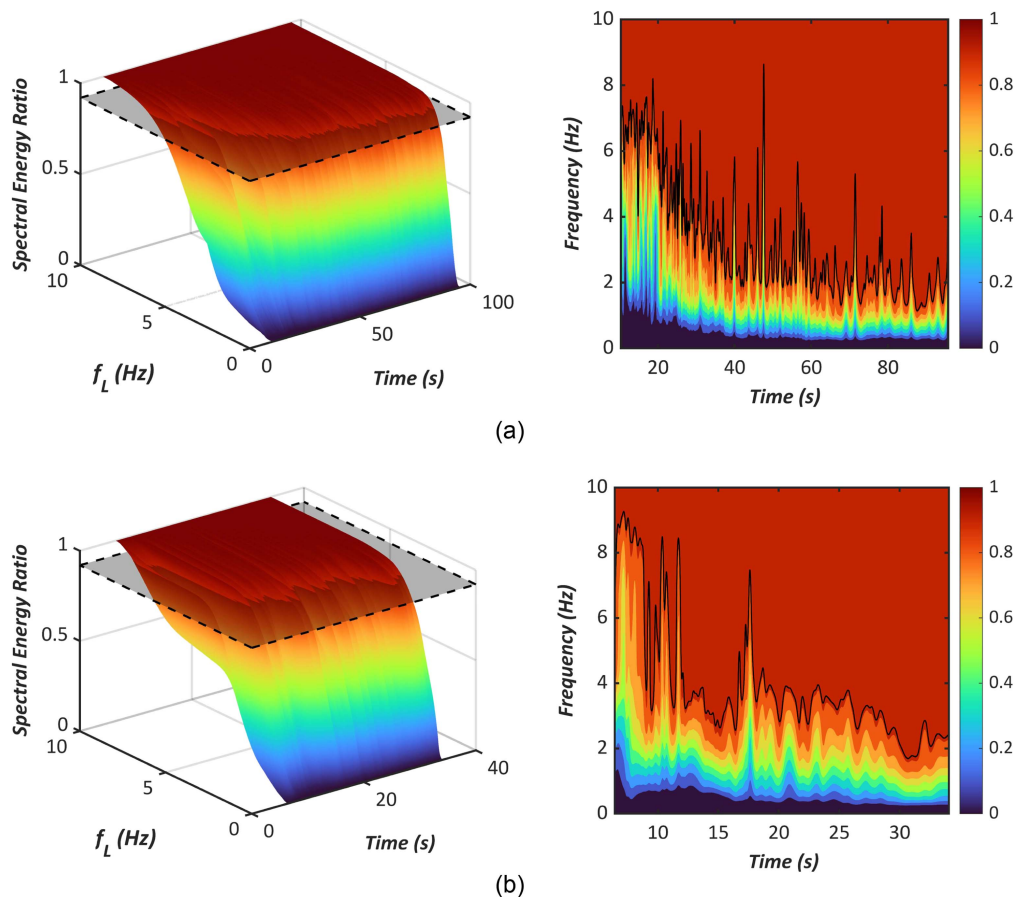


Fig. 7. SER over time for each f_L and vertical projection of SER: (a) 1968 Tokachi-oki earthquake recorded at Aomori Harbor station; and (b) 1995 Kobe (Hyogoken Nanbu) earthquake recorded at Amagasaki station. Horizon and black lines in SER plots correspond to spectral energy ratio of 0.92.

the Tokachi-oki earthquake recorded at Aomori Harbor station [Fig. 7(a)] and 1995 Kobe (Hyogoken Nanbu) earthquake recorded at Amagasaki station [Fig. 7(b)]. A plane corresponding to $f_L = 0$ Hz exhibits $SER = 0$ over time because there is no

frequency energy at 0 Hz. Conversely, SER remains constant at unity over time when $f_L = 10$ Hz. From a top-down view of the three-dimensional plots, the frequency change over time can be observed at a particular value of SER . For both case histories,

the frequency change corresponding to a SER of 0.92 distinctly elucidates how the frequency boundary (f_L) for 92% of the frequency energy decreases over time. In the case of 1968 Tokachi-oki earthquake [Fig. 7(a)], the frequency initiates at around 7 Hz and gradually drops to 2 Hz at 40 s. Interestingly, the 1995 Kobe earthquake record notably indicates the onset of frequency drop from 5 s to 13 s in contrast to the modal frequency change [Fig. 5(a)].

Step-by-Step Procedure to Estimate Timing of Liquefaction Using Spectral Energy Ratio

Using the SER, a methodology for estimating t_L is formulated. Note that the developed methodology incorporates a ramp corresponding to pore water pressure buildup within the two-step function, which is the trilinear function. The procedure for estimating t_L can be outlined as follows:

1. Apply the Stockwell transform to two horizontal components of strong motion records. To consolidate the two horizontal motions into a single Stockwell transform, combine the N-S

- and E-W components by taking the norm of their respective power spectrograms. Only portions with acceleration records greater than 0.04 g are considered for the Stockwell transform.
2. Compute the SER for each time step using the Stockwell transform via Eq. (4).
3. Obtain frequency change over time corresponding to a SER of 0.92, denoted as $f_{L,SER=0.92}(t)$. The selection of SER value of 0.92 is based on sensitivity analysis, which will be elaborated in the section "Sensitivity Analysis of Spectral Energy Ratio."
4. Fit the trilinear function to $f_{L,SER=0.92}(t)$. The start time for excess pore pressure buildup is marked at τ and the duration for the buildup is Δt . This implies that the pressure buildup ends at $\tau + \Delta t$. Both τ and Δt are arbitrary times, making the trilinear function, denoted as $s(t, \tau, \Delta t)$, a function of both τ and Δt . Specifically, the ranges for τ and Δt are defined as follows: $t_0 < \tau < t_f$ and $\Delta t: 0 \leq \Delta t < t_f - t_0 - \tau$. Here, t_0 and t_f indicate the times corresponding to the start and end of time domain for $f_{L,SER=0.92}(t)$. The trilinear function, $s(t, \tau, \Delta t)$, is computed, as follows:

$$s(t, \tau, \Delta t) = \begin{cases} F_1 = \frac{1}{\tau - t_0} \int_{t_0}^{\tau} f_{L,SER=0.92}(t) dt, & t_0 \leq t \leq \tau \\ F_u = \frac{F_2 - F_1}{\Delta t} (t - \tau) + F_1, & \tau < t < \tau + \Delta t \\ F_2 = \frac{1}{t_f - (\tau + \Delta t)} \int_{\tau + \Delta t}^{t_f} f_{L,SER=0.92}(t) dt, & \tau + \Delta t \leq t \leq t_f \end{cases} \quad (5)$$

where F_1 and F_2 = mean values of the $f_{L,SER=0.92}$ from t_0 to τ and $\tau + \Delta t$ to t_f , respectively; and F_u = ramp for excess pore water pressure buildup and the linear function connecting two points, (τ, F_1) and $(\tau + \Delta t, F_2)$. If Δt is equal to zero, the trilinear function becomes the same as the two-step function described in Özener et al. (2020).

5. Compute the total residual, $R(\tau, \Delta t)$, as the difference between the frequency change for SER = 0.92, $f_{L,SER=0.92}(t)$ and the trilinear function, $s(t, \tau, \Delta t)$, as below

$$R(\tau, \Delta t) = \frac{2}{F_1 + F_2} \int_{t_0}^{t_f} (f_{L,SER=0.92}(t) - s(t, \tau, \Delta t))^2 dt \quad (6)$$

$R(\tau, \Delta t)$ represents the difference between the trilinear function and $f_{L,SER=0.92}(t)$ at each τ and Δt .

6. Normalize the residual function, $R_n(\tau, \Delta t)$, by the minimum residual value, as below

$$R_n(\tau, \Delta t) = \frac{R(\tau, \Delta t)}{\min(R(\tau, \Delta t))} \quad (7)$$

By using τ and Δt when $R_n(\tau, \Delta t) = 1$, the timing of liquefaction triggering is estimated as $\tau + \Delta t$.

Timing of Liquefaction Estimated Using SER with the Trilinear Function and Two-Step Function for Case Histories at Aomori Harbor Station and Amagasaki Station

In order to assess the effect of the ramp function on the estimation of t_L , the two-step function (without a ramp) and trilinear function (with a ramp) were applied to $f_{L,SER=0.92}(t)$.

Fig. 8 shows plots of the two-step function [Fig. 8(a)] and the trilinear function [Fig. 8(c)] applied to the frequency change at SER = 0.92 from recorded ground motions during the 1968 Tokachi-oki earthquake at Aomori Harbor station. Also shown are the normalized residuals corresponding to both functions [Figs. 8(b and d)]. The normalized residual sum of squares for the two-step function [Fig. 8(b)] corresponds to the horizontal projection at $\Delta t = 0$ of Fig. 8(d), which implies that the function does not take into account the pore water pressure buildup during the shaking. With the absence of the ramp in the two-step function, the residual function demonstrates the minimum residual at 26.5 s, which signifies t_L . This estimate of $t_L = 26.5$ s, $s(t, 26.5$ s), divides the $f_{L,SER=0.92}(t)$ into nonliquefied and liquefied zones [Fig. 8(a)]. Upon introducing the ramp via the trilinear function, the residual function evolves into a three-dimensional plot with τ and Δt [Fig. 8(d)]. The three-dimensional residual function reveals the minimum residual at τ of 13.83 s and Δt of 27.51 s, resulting in the determination of t_L as 41.34 s. The trilinear function, $s(t, 13.83$ s, 27.51 s), further delineates the $f_{L,SER=0.92}(t)$ into nonliquefied, liquefied, and pore water pressure buildup zones [Fig. 8(c)].

The same procedure was applied to records from the 1995 Kobe (Hyogoken Nanbu) earthquake records at Amagasaki station (Fig. 9). When the ramp is omitted from the two-step function, the residual analysis indicates 9 s as the estimate of t_L [Fig. 9(b)]. The optimized two-step function corresponds to $s(t, 9$ s) [Fig. 9(a)]. Interestingly, a series of comparable residuals with minimum values are noted within 9 to 13 s, which suggests an uncertainty of t_L within this interval. In contrast, when considering the ramp function via the trilinear function, the three-dimensional residual plot distinctly exhibited a minimum residual at τ of 7.45 s and Δt of 5.76 s, so that t_L is determined as 13.21 s ($\tau + \Delta t$). The optimized trilinear

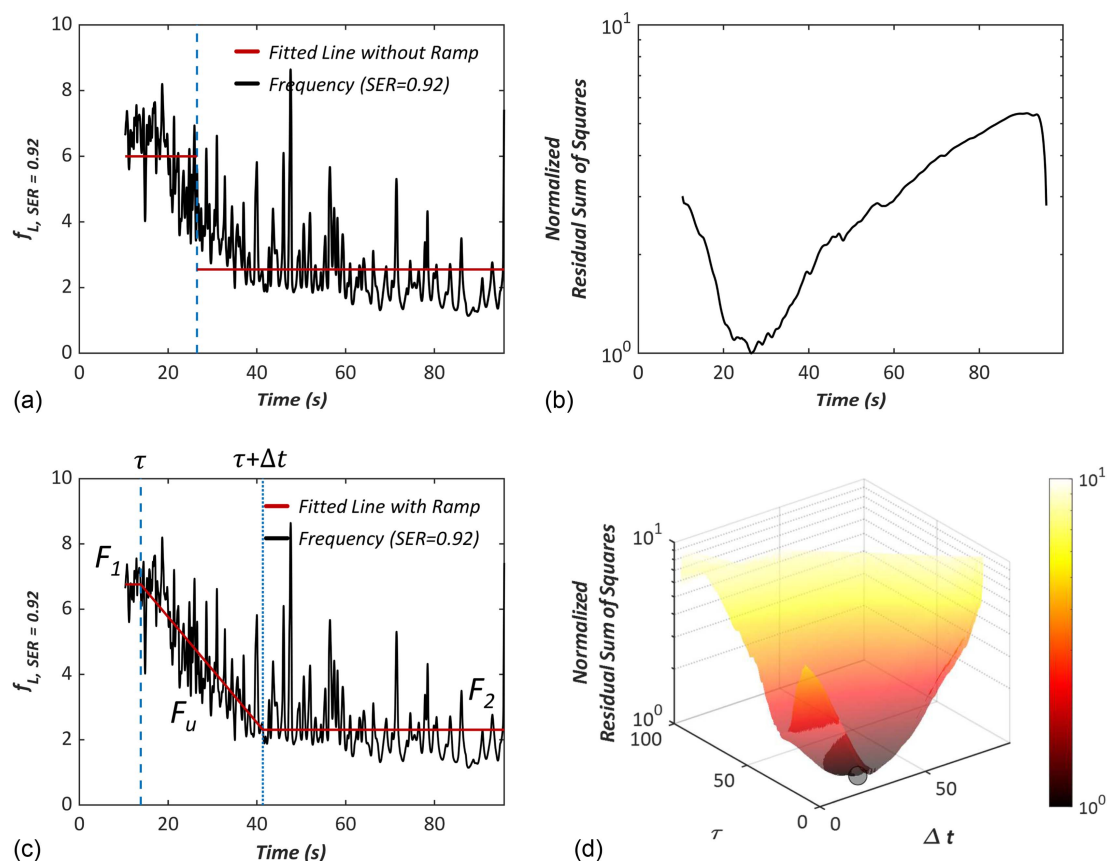


Fig. 8. Frequency change, the optimized two-step function and trilinear function, and residual function using recorded strong motions during 1968 Tokachi-oki earthquake recorded at Aomori Harbor station: (a) frequency change corresponding to SER of 0.92 and the optimized two-step function; (b) residual function of the two step function; (c) frequency change corresponding to SER of 0.92 and the optimized trilinear function; and (d) residual function of the trilinear function.

function corresponds to $s(t, 7.45 \text{ s}, 5.76 \text{ s})$ [Fig. 9(c)]. Comparing the proposed approach to the modal frequency change with the two-step function [Fig. 9(c) versus Fig. 5(a)] highlights the robustness of applicability of the SER and trilinear function combination approach due to a consistent description and identification of distinct frequency reduction trends.

Comparison of Timing of Liquefaction for Strong Motion Records from 30 Case Histories

Timing of liquefaction from this study and previous literature is compared in Fig. 10. For the records during the 1968 Tokachi-oki earthquake, t_L obtained using SER with the trilinear function, 41.3 s, is similar to that from Kramer et al. (2016), which is 36.5 s [Figs. 2(b) and 10(a)], whereas the timing from Özener et al. (2020) at 19.8 s precedes shear wave arrival. Timing, t_L , estimated using the SER with the two-step function is 26.5 s. A similar trend is observed in the case of 1995 Kobe earthquake records [Figs. 4(b) and 10(b)]. Timing of liquefaction using SER with the ramp, 13.2 s, is later than that from Özener et al. (2020), 10.9 s, and that without the ramp, 9 s. The timing without ramp, 9 s, is earlier than the shear wave arrival.

The proposed methodology was applied to 30 case histories with ground motion recordings available at locations where liquefaction occurred (Table 1). Fig. 10(c) shows a comparison of the t_L obtained using SER with the trilinear function proposed in this study to the timing of liquefaction from Kramer et al. (2016),

Özener et al. (2020), and Zhang et al. (2023). Zhang et al. (2023) obtained t_L using a time–frequency image through a deep learning model resulting in separate t_L values for N–S and E–S motions. To facilitate the comparison in Fig. 10(c), the points representing the estimated t_L from Zhang et al. (2023) are the average of the N–S and E–S component t_L estimates. As shown in Fig. 10(c), t_L from this study is typically longer than that of the three other methods due to the inclusion of the ramp. The t_L from this study is closest to that from Zhang et al. (2023) as indicated by the proximity of the fitted line of their estimates to the 1:1 line in Fig. 10(d). To determine t_L , Zhang et al. (2023) trained their model using 1,323,400 cases from numerical models, allowing for the accurate estimation of timing through r_u . This implies that the method developed in this study is comparable to the deep learning model whose t_L has already been validated using excess pore water pressure records.

Validation and Sensitivity Analysis Using Centrifuge Tests

Geotechnical centrifuge tests are effective physical models for simulating field stress conditions and prototype behavior of soil, foundation, and structure system. This study employed LEAP which was international collaborative research aimed at producing high-quality centrifuge test data for validating and calibrating numerical constitutive models (Kutter et al. 2020). LEAP–UCD–2017

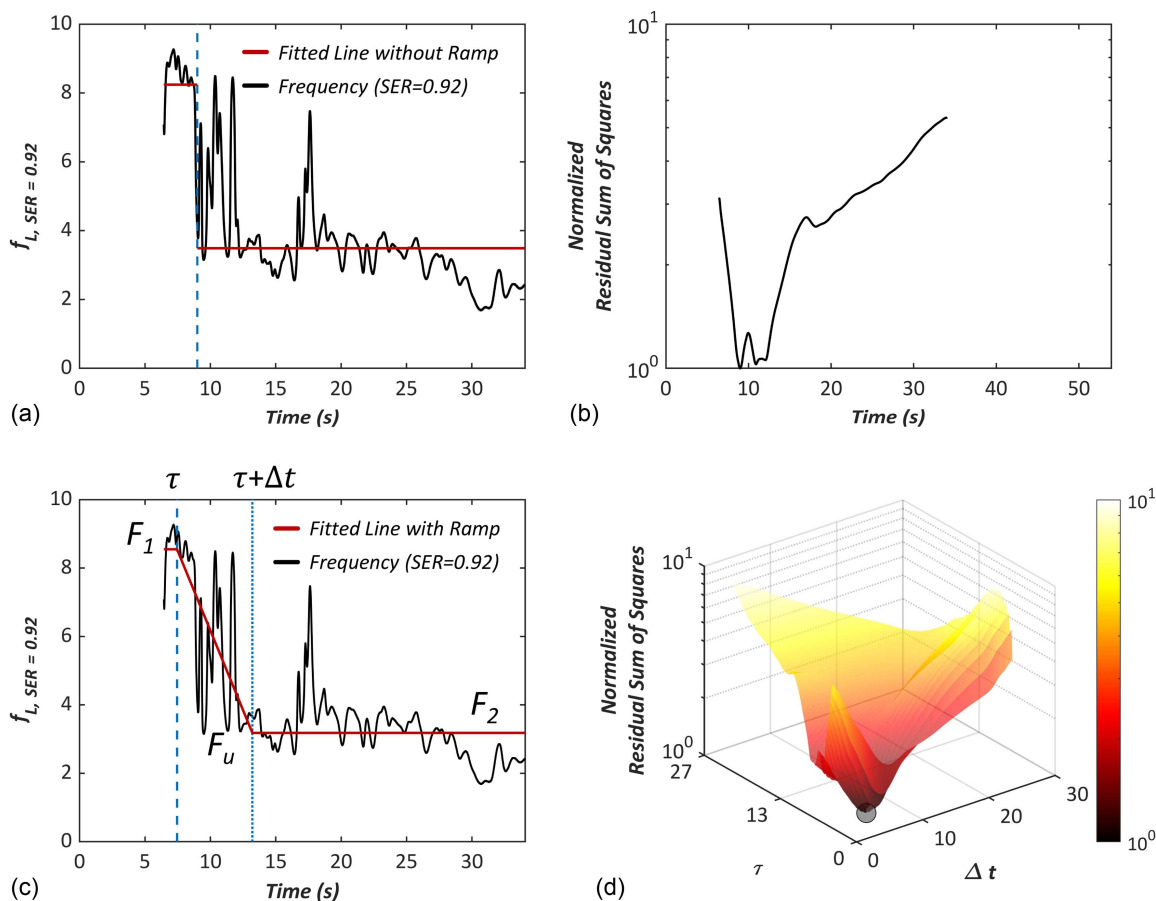


Fig. 9. Frequency change, the optimized two-step function and trilinear function, and residual function using recorded strong motions during 1995 Kobe (Hygoken Nanbu) earthquake recorded at Amagasaki station: (a) frequency change corresponding to SER of 0.92 and the optimized two-step function; (b) residual function of the two step function; (c) frequency change corresponding to SER of 0.92 and the optimized trilinear function; and (d) residual function of the trilinear function.

specifically generated and shared high-quality centrifuge data associated with dynamic responses and liquefaction of saturated granular soils from nine centrifuge centers. Given that the centrifuge tests include acceleration and pore water pressure responses simultaneously, LEAP-UCD-2017 offers a valuable resource for validating the proposed methodology. The validation is achieved by comparing predicted t_L obtained through the developed method with the measured t_L from the centrifuge tests.

Centrifuge Tests

The LEAP-UCD-2017 project employed a mildly sloping ground with an incline of 5 degrees [Fig. 11(a)]. The sloping ground was reconstituted by a pluviation method using Ottawa F-65 sand (ElGhoraiby et al. 2020). To ensure consistency and comparability across different centrifuge centers, a rigid box was utilized for the centrifuge tests. This approach helped avoid complexities associated with boundary conditions in laminar boxes at various centrifuge centers and provided uniform conditions for testing (Zeghal et al. 2018). After dry pluviation of the sand, a viscous fluid (a mixture of methylcellulose and water) was used to fully saturate the ground model abiding by the LEAP guidelines (Kutter et al. 2020). This viscous fluid was used to ensure that the seepage flow of fluid in prototype scale was compatible with field scale behavior. Vertical arrays of accelerometers (AH1–AH4) and pore pressure

transducers (P1–P4) were deployed at the center of the test model to measure horizontal acceleration and pore water pressure responses at different depths of the centrifuge model during the shaking. Vertical spacing of the vertical arrays for accelerometers and pore pressure transducers was 1 m at the prototype scale, respectively, and the distance between each accelerometer and pore pressure transducer was 0.5 m at the prototype scale.

To ensure consistency in the experiments despite the use of varying sizes of the rigid boxes at different centers, the LEAP-UCD-2017 applied different centrifugal accelerations to the model to simulate the exact prototype model at each centrifuge center. A smooth ramped sine wave was used as an input motion. The duration of 20 s and frequency of 1 Hz for the input motion were the same for each test. LEAP-UCD-2017 organized the test matrix by varying the relative density of the ground model and the peak acceleration of the input motion [Fig. 11(b)]. The resulting data set comprised seven centrifuge tests that each produced four acceleration records and pore pressure responses providing a total of 28 acceleration responses that were used to validate the proposed method.

Sensitivity Analysis of Spectral Energy Ratio

Vertical projection of SER reveals different frequency changes depending on the horizon of SER values [Fig. 11(c)], as demonstrated

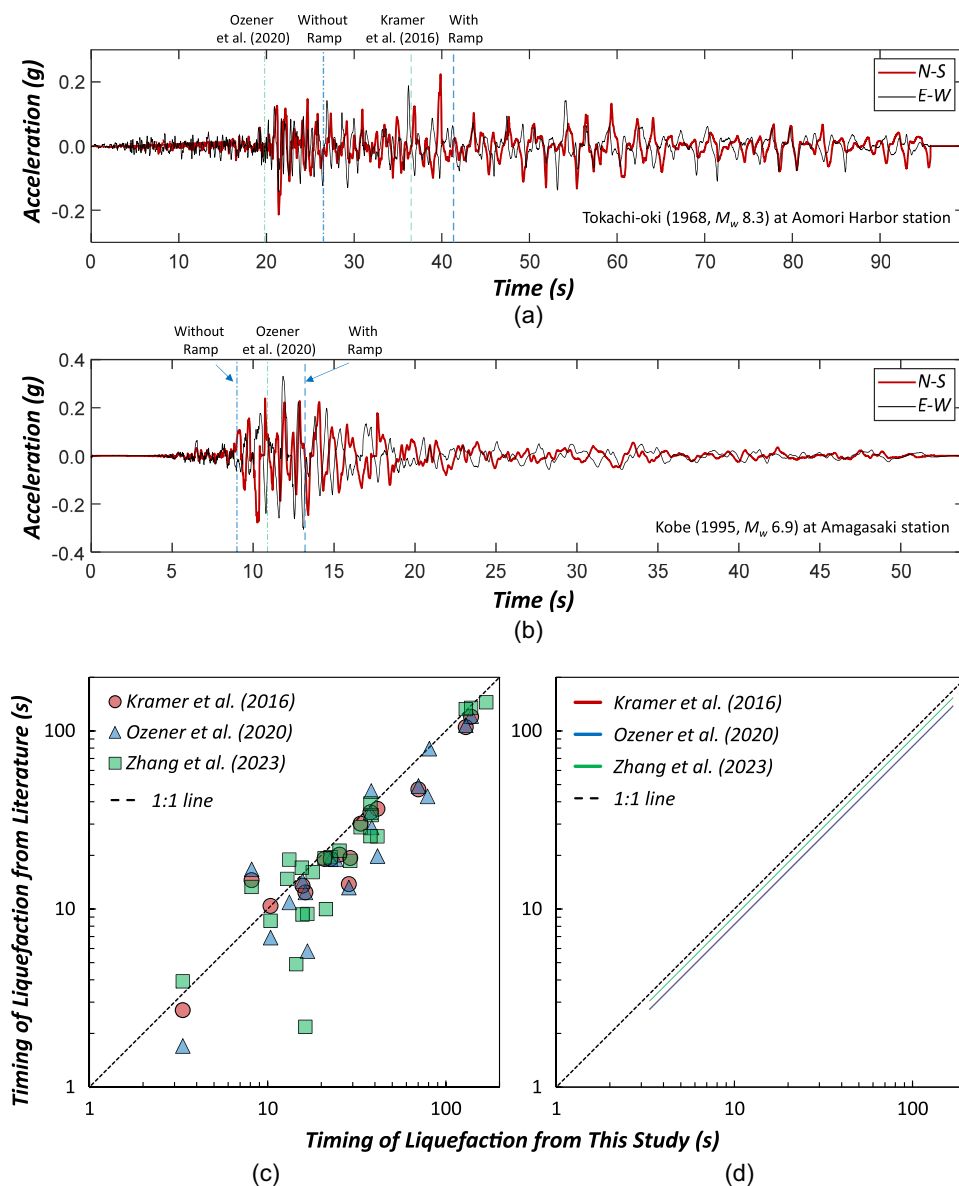


Fig. 10. Comparison timing of liquefaction triggering: (a) 1968 Tokachi-oki earthquake recorded at Aomori Harbor station; (b) 1995 Kobe (Hygoken Nanbu) earthquake recorded at Amagasaki station; (c) comparison of 30 case histories using Kramer et al. (2016), Özener et al. (2020), and Zhang et al. (2023); and (d) fitted lines using data in Fig. 10(c).

by the case histories shown in Fig. 7. To determine an optimized SER value that better predicts t_L , this study conducted sensitivity analysis using centrifuge tests.

The sensitivity analysis involved the use of 15 different SER values ranging from 0.3 to 1.0 [Fig. 12(a)]. The proposed method was used to predict the t_L following the procedure outlined in the section “Step-by-Step Procedure to Estimate Timing of Liquefaction Using Spectral Energy Ratio” for each SER value. To examine the effect of including the ramp in the function, predicted t_L was obtained based on the two-step function and the trilinear function, separately. The measured t_L was obtained from pore water pressure transducers to serve as benchmarks. To assess the accuracy of t_L predictions, linear fitted lines without y-intercepts were applied to the data. A slope approaching 1 indicates better SER suitability for prediction because it means the fitted line is close to a 1:1 line between measured and predicted t_L [Fig. 12(b)].

The slope of the fitted line consistently approaches 1 as SER values increase from 0.3, with a sharp decrease beyond SER of 0.93. SER values closer to 1 or 0 correspond to frequencies that are closer to 10 Hz or 0 Hz over time, respectively. This suggests that the trend of frequency drop cannot be effectively captured by the two-step function under such conditions when using extremely high or low SER. This phenomenon also affects the R-squared value of the fitted line [Fig. 12(c)]. The sensitivity analysis reveals that the trilinear function consistently outperforms the two-step function which lacks the capability of accounting for pore water pressure buildup. Considering the slope of the fitted line and the R-squared values, this study selected an optimized SER value of 0.92. This value provides a clearer representation of the frequency drop trend and yields more accurate t_L predictions. To investigate uncertainty ranges of t_L as a function of SER, 13 different SER values (i.e., 0.3, 0.4, 0.5, 0.7, 0.8, 0.85, 0.88, 0.9, 0.91, 0.92,

Table 1. Timing of liquefaction from strong motion records of 30 case histories

Earthquake	Year	Magnitude	Station	This study using spectral energy ratio		Timing from Kramer et al. (2016) (s)	Timing from Özener et al. (2020) (s)	Timing from Zhang et al. (2023) (s)
				Timing with ramp (s)	Timing without ramp (s)			
Niigata	1964	7.7	Kawagishi-cho	10.38	7.81	10.4	6.9	EW: 8.61 NS: 8.54
Tokachi-Oki	1968	8.3	Aomori Station	41.34	26.50	36.5	19.8	EW: 26.68 NS: 24.43
Imperial Valley	1979	6.6	Bond's Corner	16.7	6.08	—	5.8	EW: 9.97 NS: 8.76
Nihonkai-Chubu	1983	7.7	Aomori Station	78.85	62.3	—	42.9	—
Superstition Hills	1987	6.6	Wildlife	28.52	19.06	13.8	13.2	—
Loma Prieta	1989	6.9	Alameda Naval Air Station	16.30	12.83	12.4	12.4	EW: 1.8 NS: 2.56
Loma Prieta	1989	6.9	Treasure Island	15.71	15.40	13.5	14.3	EW: 9.88 NS: 8.73
Kushiro-Oki	1993	7.8	Kushiro Port	38.4	34.77	N/A	28.7	EW: 36.5 NS: 30.83
Kobe	1995	6.9	Kobe JIS	3.34	2.69	2.7	1.7	EW: 4.69 NS: 3.15
Kobe	1995	6.9	Port Island	8.12	7.24	14.5	16.7	EW: 12.86 NS: 13.72
Kobe	1995	6.9	Amagasaki	13.21	9	—	10.9	EW: 19.75 NS: 18.13
Kobe	1995	6.9	Takatori	14.48	5.28	—	—	EW: 3.97 NS: 5.82
WesternTottori	2000	6.6	Sakaimitato	22.57	19.3	19.1	19.1	EW: 19.06 NS: 20.02
Tokachi-Oki	2003	8.3	HKD086	37.72	31.11	34.9	34.9	EW: 41.17 NS: 36.95
Niigata-Chuetsu	2007	6.6	NIG018	24.06	23.16	N/A	19	—
Darfield	2010	7	HPSC	33.22	24.66	30.1	N/A	EW: 29.62 NS: 27.89
Darfield	2010	7	GDLC	37.78	37.35	—	—	EW: 25.19 NS: 26.01
Christchurch	2011	6.2	CCCC	29.08	24.02	19.3	—	EW: 17.46 NS: 19.81
Christchurch	2011	6.2	CHHC	25.3	21.42	20.2	—	EW: 22.00 NS: 20.46
Christchurch	2011	6.2	CMHS	20.86	20.38	19.1	—	EW: 18.98 NS: 19.54
Christchurch	2011	6.2	REHS	17.91	15.73	—	—	EW: 15.73 NS: 16.43
Christchurch	2011	6.2	PRPC	12.90	12.37	—	—	EW: 15.63 NS: 13.87
Christchurch	2011	6.2	HPSC	15.54	15.29	—	—	EW: 17.54 NS: 16.57
Christchurch	2011	6.2	SHLC	21.22	20.69	—	—	EW: 8.69 NS: 11.2
Tohoku	2011	9	CHB024	138.69	119.42	120	121	EW: 132.9 NS: 136.33
Tohoku	2011	9	IBR014	129.19	115.94	105	107.7	EW: 132.82 NS: 132.34
Tohoku	2011	9	MYG013	70.1	60.91	46.9	49.1	—
Tohoku	2011	9	MYG010	38.13	38.09	—	46	—
Tohoku	2011	9	Onahama Port	80.74	79.85	—	79.5	—
Tohoku	2011	9	TKY017	168.62	143.83	—	—	EW: 144.73 NS: 144.35

0.93, 0.94, and 0.95) were used to calculate the mean and standard deviation of t_L at the shallowest depth. A4 and P4 were employed to estimate and measure t_L for each test, respectively [Fig. 12(d)]. Tests showing clear frequency changes in the SER three-dimensional plots (such as test cases CU2, Ehime1, RPI1, and ZJU2) resulted in lower standard deviations. However, the

lower uncertainties were not necessarily associated with the accuracy of the method for predicting t_L .

As a representative example, identification of t_L using records from AH4 and P4 of UCD2 is shown in Fig. 13. The $f_{L,SER=0.92}$ of AH4 exhibits initial frequencies close to 9 Hz, but a significant frequency drop occurs between around 6 s to 9 s [Fig. 13(a)].

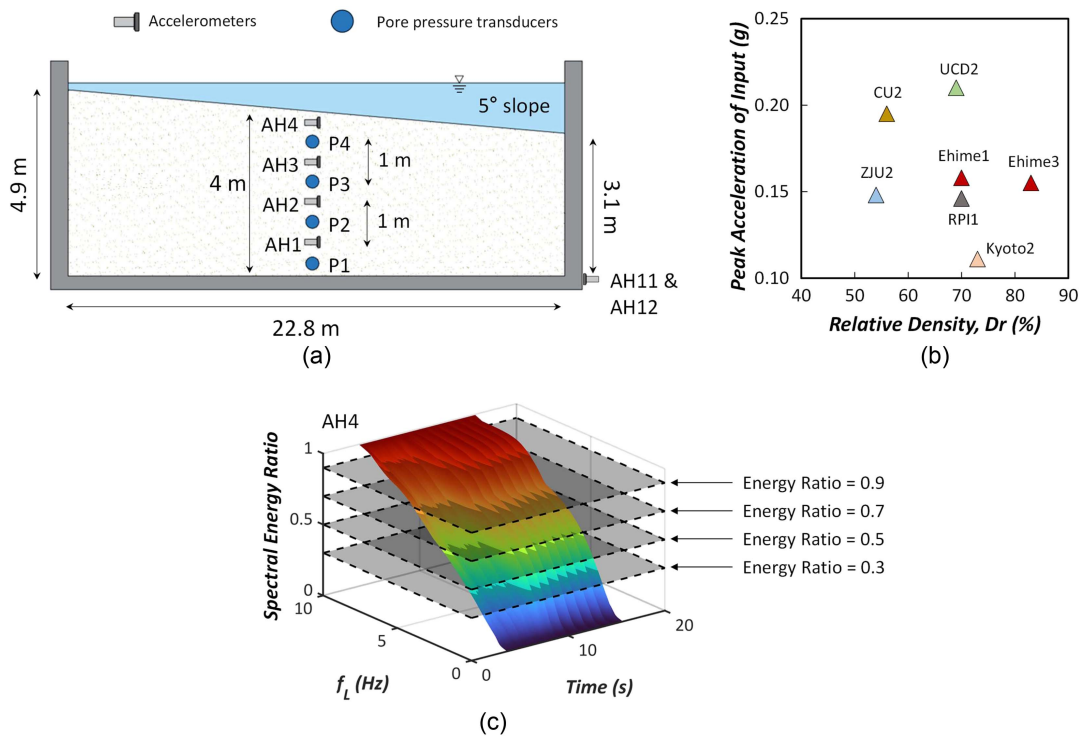


Fig. 11. Centrifuge tests of Liquefaction Experiment and Analysis Projects (LEAP-UCD-2017): (a) section view of LEAP-UCD-2017 test model in prototype scale; (b) peak acceleration of input and relative density of the test used for this study; and (c) concept of sensitivity analysis by varying SER using AH4 accelerometer from UCD2.

After 10 s, there is a fluctuation of frequency from lower to higher values due to the dilative behavior of the soil. The LEAP tests used the 1 Hz sinusoidal input motion and Ottawa F-65, poorly graded uniform sand, which tends to be dilative and generates de-liquefaction shock waves during shaking under undrained condition (Kutter and Wilson 1999). The two-step function indicates t_L of 7.89 s [Fig. 13(b)] whereas the trilinear function results in 8.95 s as t_L [Fig. 13(c)]. The measured t_L from excess pore water pressure records during the shaking is 8.79 s [Fig. 13(d)]. The function with the ramp provides a t_L prediction that closely aligns with the measured timing, whereas the function without the ramp estimates t_L close to the r_u of 0.5, which occurs prior to liquefaction triggering.

Discussion and Limitations

Frequency drops are not solely associated with liquefaction triggering but also with factors such as S-wave arrival, surface waves, and site effects. These potential contributors introduce uncertainties in pinpointing the timing of liquefaction triggering. Therefore, this section delves into the uncertainties and limitations of the model in identifying the timing of liquefaction triggering using other earthquake case history data.

2018 Anchorage, Alaska Earthquake (M_w 7.1) at NSMP 8027 and K211 (Fire Department)

Typically, reduction in frequency due to the triggering of liquefaction occurs independent of the direction of acceleration records while the arrival of S-waves and surface waves can cause a reduction in frequency that varies with specific directions due to polarization. Assessments using azimuthal modal frequency changes can serve as a means to distinguish these frequency reduction effects.

In certain instances, however, the arrival of S-waves and surface waves can also lead to frequency drops irrespective of the direction of acceleration records. In these scenarios, comparing the Stockwell transform of horizontal and vertical motions can serve as a reliable indicator to identify the cause of the frequency reduction and potentially quantify their contributions to the uncertainty in estimating the timing of liquefaction triggering.

This section examines liquefaction triggering using earthquake records from two recording stations (NSMP 8027 and K211—fire department) during the November 30, 2018 Anchorage, Alaska earthquake (M_w 7.1). Fig. 14 shows the acceleration time histories at these two stations. Subsurface investigation data for the NSMP 8027 site were not publicly accessible, making consideration of the Stockwell transform as a viable approach to identify liquefaction triggering. Cabas et al. (2021) suggested that cyclic softening or isolated pockets of liquefied soil were suspected at the NSMP 8027 site, but severe liquefaction likely did not occur based on a comparison of the Stockwell transforms between the horizontal and vertical motions. The changes in azimuthal modal frequency show the prevalence of near absolute low modal frequencies (< 1.5 Hz) at approximately 50–54 s [Fig. 15(a)], while the onset of low frequencies in the Stockwell transform of vertical motion is observed around 50–55 s [Fig. 15(c)]. Cabas et al. (2021) identified the arrival of surface waves at 51.6 s due to the low frequencies in the vertical motion, which is not indicative of severe liquefaction cases. However, using the developed model based on SER with a ramp [Figs. 16(a and b)] leads to an estimated timing of liquefaction at this site of 51.2 s.

For the K211 (fire department) site, Cabas et al. (2021) identified the timing of liquefaction triggering to be approximately 44 s. This determination was made from the Stockwell transform of N–S and vertical motions without using E–W motion. The horizontal time–frequency response in the N–S direction exhibited an abrupt

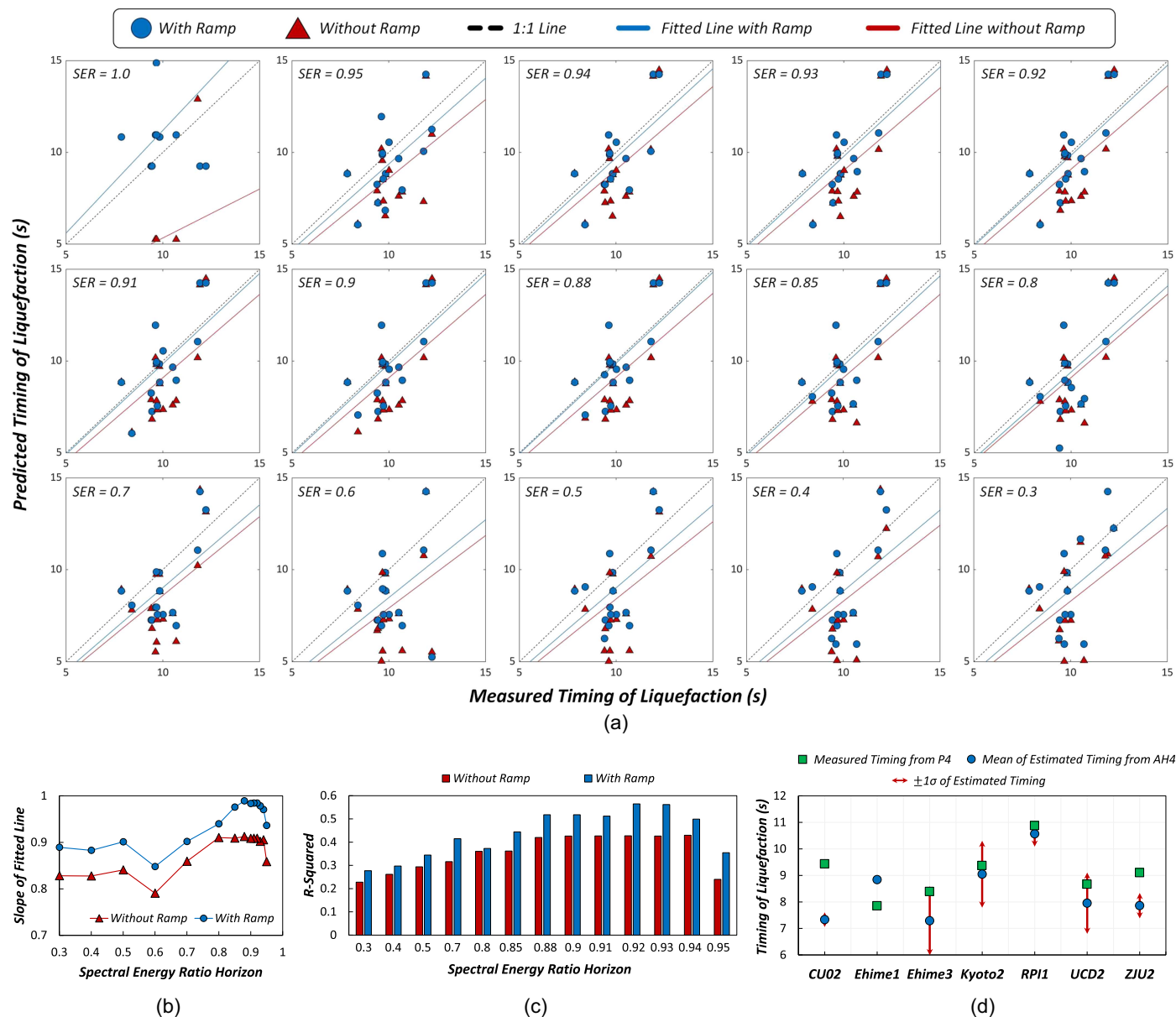


Fig. 12. Sensitivity analysis of SER with and without ramp: (a) comparison between measured and predicted timing of liquefaction; (b) slope of fitting curve with spectral energy ratio horizon; (c) R-squared of fitting curve with spectral energy ratio horizon; and (d) mean and standard deviation of estimated timings of liquefaction from AH4 using 13 different SERs and measured timings from P4 (1 m depth).

reduction in frequency at around 44 s, whereas those in the vertical motion did not. The azimuthal modal frequency of K211 showed frequency energies at around 2 Hz even after 44 s in the N–S direction, and after 50 s, modal frequencies in all directions dropped to less than 1 Hz [see Fig. 15(b)]. The Stockwell transform of the vertical motion revealed low frequencies around 1 Hz after 51 s [see Fig. 15(d)]. When it comes to the SER method, which combines frequency energies in both horizontal directions [see Fig. 16(c)], a fitted line with a ramp displayed a noticeable frequency drop from 49 to 55.8 s [see Fig. 16(d)]. According to the developed model, this drop estimates the timing of liquefaction at this site to be 55.8 s, whereas Cabas et al. (2021) identified a frequency drop at around 51.4 s that was attributed to the arrival of surface waves.

As demonstrated by the records at NSMP 8027 and K211, a comprehensive assessment of both the vertical and horizontal time–frequency responses is needed to ensure proper identification

of the onset of liquefaction. The additional evaluation of the vertical Stockwell transform allows for the ability to differentiate reductions in frequency due to liquefaction triggering from those due to the arrival of S-waves or surface waves, and also consider various factors that may influence the acceleration responses at strong motion stations.

2011 Christchurch (M_w 6.2) at Kaiapoi North School (KPOC)

To assess the performance of the proposed SER method in a non-liquefiable setting, it was applied to strong motion data recorded at Kaiapoi North School (KPOC) during the 2011 Christchurch earthquake (M_w 6.2). This station is primarily comprised of well-graded gravel (GW) and well-graded sand (SW) with an average shear wave velocity for the upper 30 m ($V_{s,30}$) of 257 m/s and an average

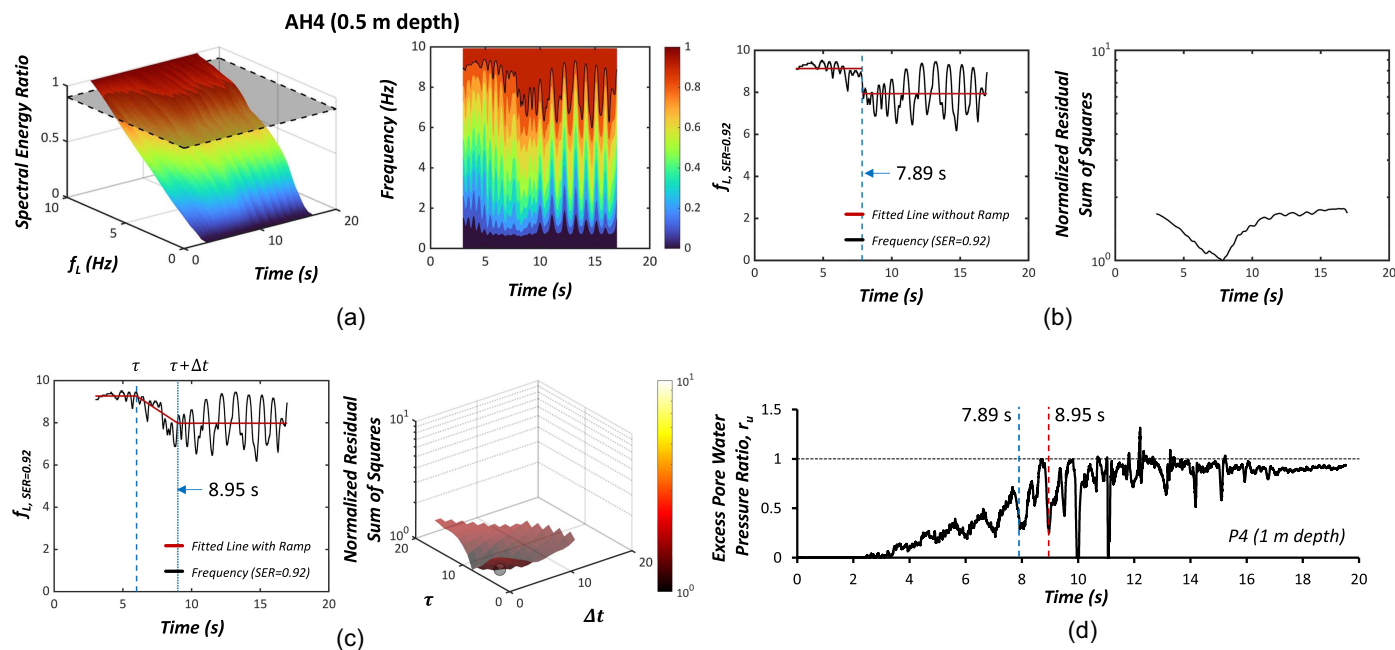


Fig. 13. Representative result of sensitivity analysis from UCD2 centrifuge data using AH4 accelerometer and P4 pore water pressure transducers: (a) spectral energy ratio of AH4 (0.5 m depth); (b) frequency change at SER of 0.92, the optimized two-step function, and residual function of the two-step function; (c) frequency change at SER of 0.92, the optimized trilinear function, and residual function of the trilinear function; and (d) measured pore water pressure response of P4 (1 m depth).

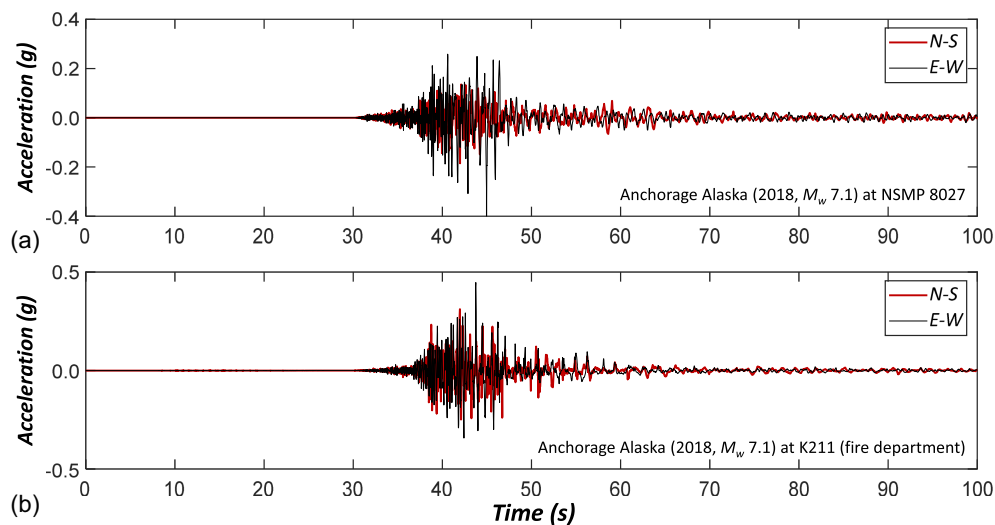


Fig. 14. Acceleration time histories during the 2018 Anchorage, Alaska earthquake: (a) at NSMP 8027; and (b) K211 (fire department).

adjusted standard penetration test (SPT) blow count, $SPT-(N_1)_{60}$, value of 40.8, indicating highly nonliquefiable conditions. Notably, postearthquake reconnaissance did not observe any surficial evidence of liquefaction at this site (Ilgaç 2022).

The recorded acceleration responses at KPOC during the earthquake exhibit no discernible evidence of undrained soil behavior, such as dilation spikes or flat responses [Fig. 17(a)]. Furthermore, the Stockwell transform of horizontal motions reveals no significant frequency drop throughout the earthquake ground motions [Fig. 17(b)]. Consistent with this observation, SER plots [Fig. 17(c)] corroborate the findings of the Stockwell transform, showing no

indication of frequency drop. Despite the absence of clear evidence of frequency drop, the model identifies a liquefaction triggering time of 21.32 s based on a fitted line with a ramp [Fig. 17(d)]. However, the presence of multiple plausible fitted lines with ramps, as indicated by the normalized residual sum of squares, underscores the ambiguity in identifying the precise timing of liquefaction triggering. In addition, the duration of the ramp associated with excess pore water pressure buildup ($\Delta t = 0.26$ s) was significantly insignificant. This observation suggests that the recorded motion lacks a distinct liquefaction-induced frequency drop, thereby reinforcing the designation of the site as nonliquefiable during the earthquake event.

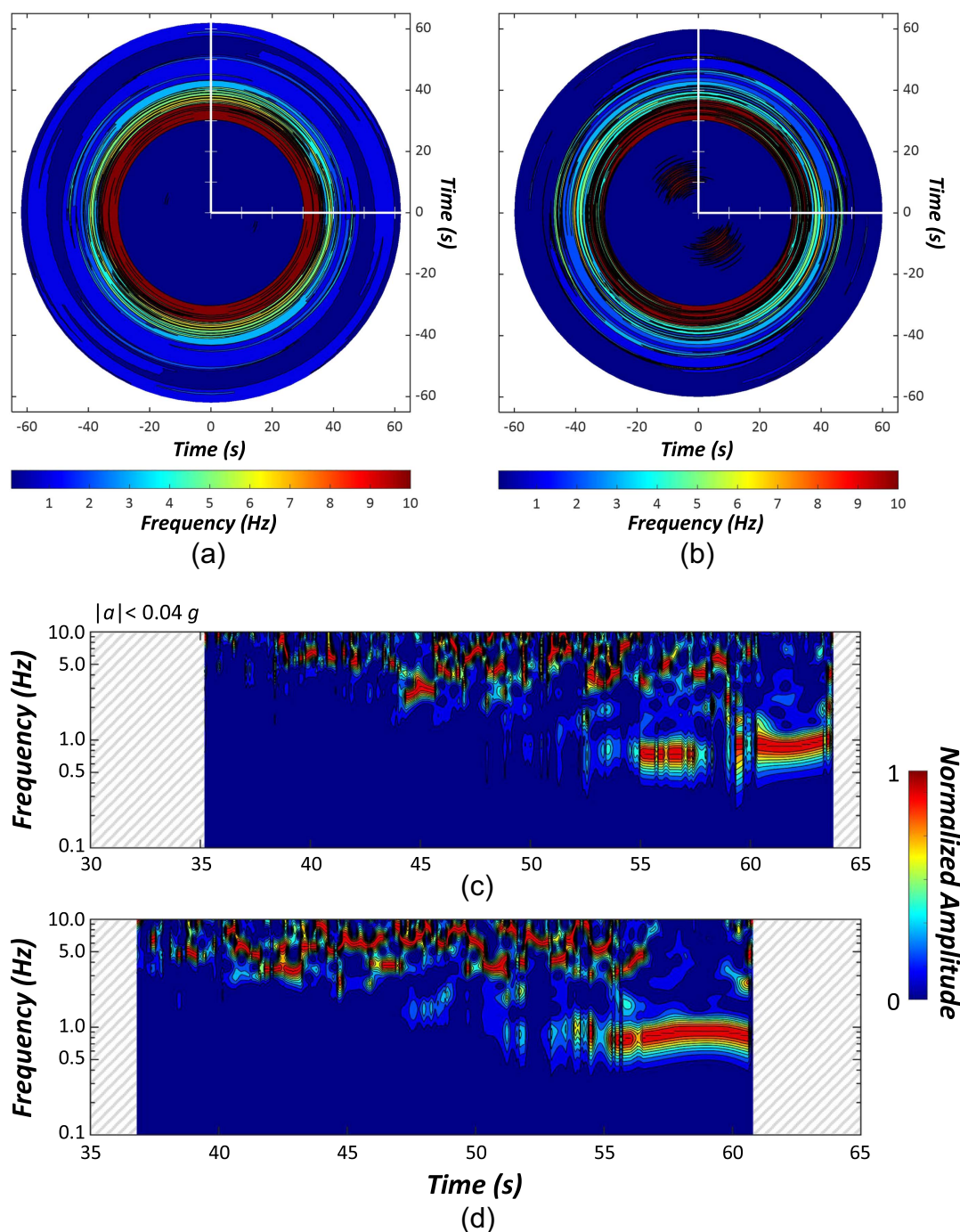


Fig. 15. Modal frequency change during the 2018 Anchorage, Alaska earthquake: (a) azimuthal modal frequency recorded at NSMP 8027; (b) K211 (fire department); (c) Stockwell transform of vertical motions recorded at NSMP 8027; and (d) K211 (fire department).

Uncertainties and Limitations

Based on the sensitivity results obtained from centrifuge tests, employing the SER and integrating ramps as fitted lines in the time–frequency response leads to more accurate estimations of timing of liquefaction triggering. Given that the centrifuge data used for the sensitivity analysis only encompasses responses from a liquefied uniform soil strata (i.e., each increment of depth below the surface is identical in density), the method developed in this study proves most beneficial for sites where clear manifestations such as sand boils, ground fissures, or building settlements unequivocally indicate liquefaction triggering. Additionally, the timing of liquefaction

estimated from the SER method will be more uncertain in field case histories since the centrifuge data calibrating SER are based on harmonic loading with a simple soil profile. The R-squared value of the fitting curve for $SER = 0.92$ falls below 0.6 suggesting inherent uncertainties such as the potential for liquefaction triggering to occur within the timeframe of the ramps.

Furthermore, as demonstrated by the assessment of the 2018 Anchorage, Alaska earthquake at NSMP 8027 and K211, as well as the 2011 Christchurch earthquake at Kaiapoi North School, determining the timing of liquefaction requires a comprehensive evaluation approach. Representative case histories, such as the timing of liquefaction triggering at Port Island during the 1995

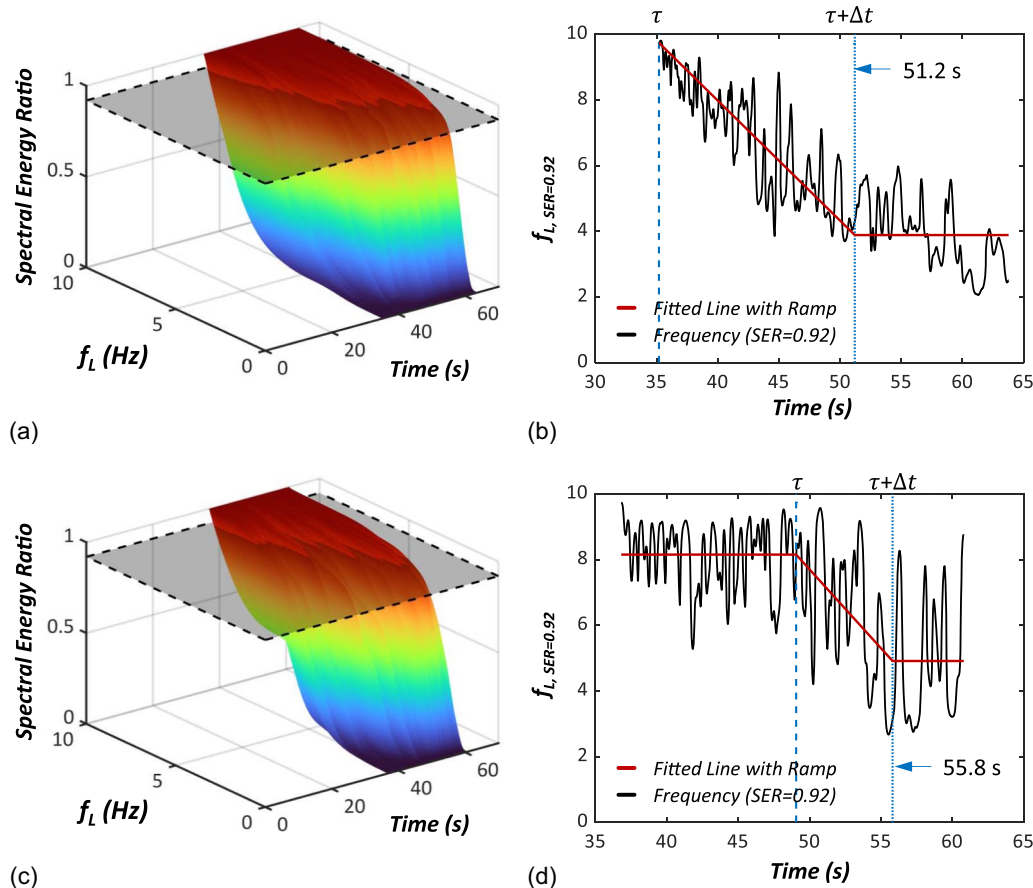


Fig. 16. SER and fitted lines using the 2018 Anchorage, Alaska earthquake records: (a) SER of NSMP 8027; (b) frequency change at SER of 0.92 and the optimized trilinear function of NSMP 8027; (c) SER of K211 (fire department); and (d) frequency change at SER of 0.92 and the optimized trilinear function of K211 (fire department).

Kobe earthquake and at Wildlife during the 1987 Superstition Hills earthquake, vary depending on the study (Elgamal et al. 1996; Zeghal et al. 1996; Bonilla et al. 2005; Yuxian et al. 2005; Özener et al. 2020). This variability underscores the need for additional considerations, such as the Stockwell transform for vertical motions and azimuthal modal frequency changes, which facilitate distinguishing the impacts of S-wave arrival, surface wave arrival, and site effects on frequency drops. Ultimately, interpretations are contingent on the specifics of each earthquake case history. Therefore, a comprehensive evaluation—accounting for the Stockwell transform of vertical motions, surficial manifestation, azimuthal frequency variation, SER, and other factors—should be conducted. Moreover, multiple uncertainties are inherent in such analyses.

Summary and Conclusions

The sudden decrease in frequency during earthquakes has commonly been utilized as an indicator of liquefaction triggering attributed to the softening of saturated soil under undrained condition. The conventional approach to determine liquefaction timing involves analyzing modal frequency change, which represents a singular frequency corresponding to the maximum power at each time bin in time–frequency responses. However, this approach often fails to capture the entirety of the frequency shift during earthquakes. This study introduces a novel method for identifying the onset of liquefaction using the concept of spectral energy ratio (SER), which encompasses the entire frequency change. Using 30 case histories,

this study predicted and compared the timing of liquefaction from previous literature. Moreover, the developed method was validated with the centrifuge test results. This study enhanced the approach of identifying timing of liquefaction triggering by utilizing only acceleration records without requiring pore water pressure responses. The main findings of this study are summarized as follows:

1. The application of modal frequency and azimuthal modal frequency changes proved unsuitable for two specific case histories: the 1968 Tokachi-oki earthquake recorded at Aomori Harbor station and the 1995 Kobe (Hygoken Nanbu) earthquake recorded at Amagasaki station. The modal frequency drop observed in the 1968 Tokachi-oki earthquake case occurred prior to shear wave arrival and attributed to the absence of the ramp in the two-step function accounting for pore water pressure buildup. The 1995 Kobe earthquake records did not exhibit clear frequency drop trends when relying on modal frequency changes.
2. This study introduced the concept of SER to quantify comprehensive frequency change during earthquakes. The use of a SER of 0.92 highlighted distinct frequency reduction trends, even in the case of the 1995 Kobe earthquake records. The ramp in the trilinear function corresponds to pore water pressure buildup during shaking.
3. The proposed method in this study was compared with Kramer et al. (2016), Özener et al. (2020), and Zhang et al. (2023) using 30 case histories. The SER method yielded later timing than t_L from the other methods due to inclusion of ramp via the trilinear function. The predicted timing aligned more closely with that

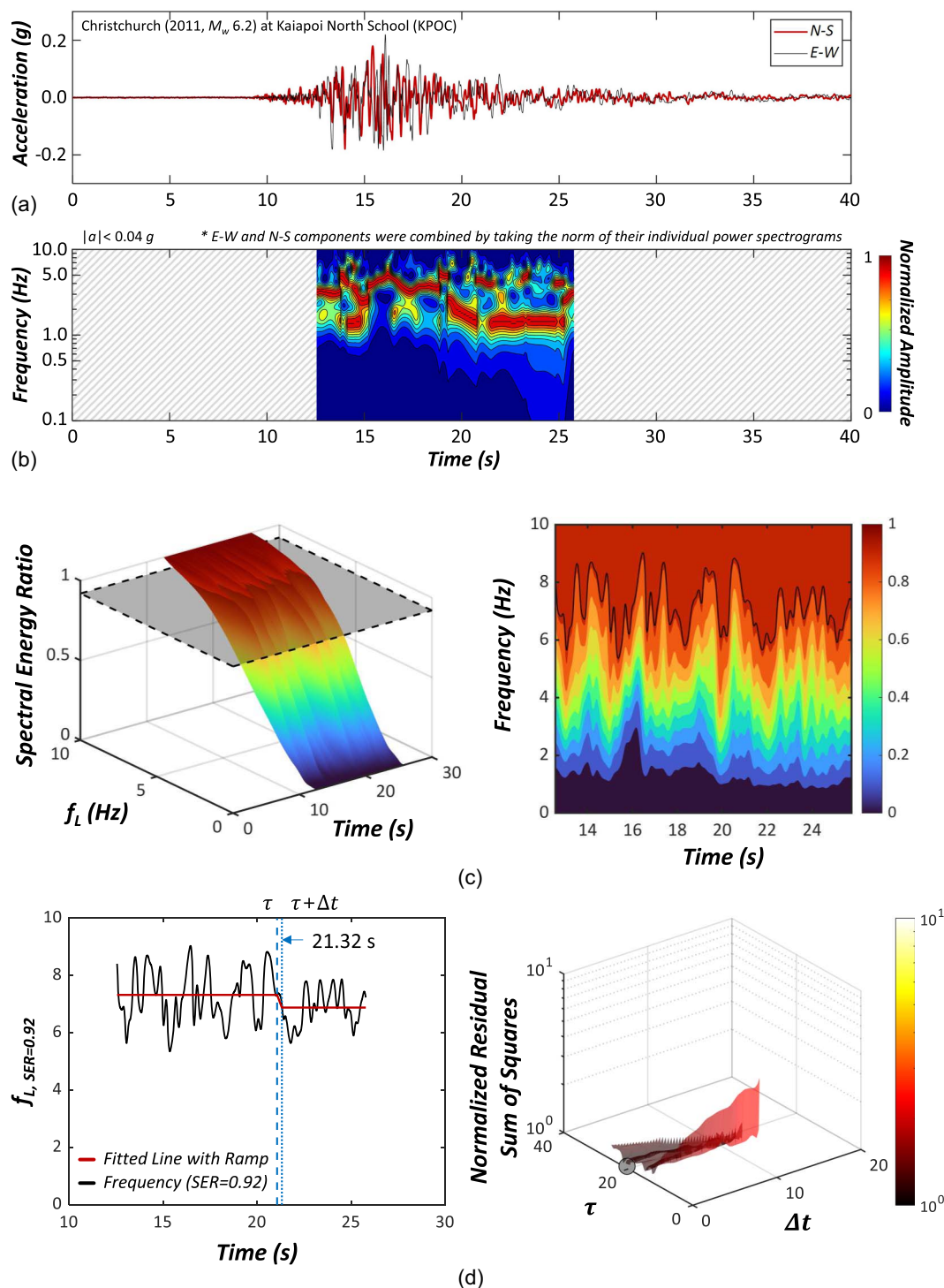


Fig. 17. Applying the model to nonliquefied earthquake records during the 2011 Christchurch earthquake recorded at Kaiapoi North School (KPOC): (a) acceleration time histories; (b) Stockwell transform of horizontal motions; (c) SER of KPOC recorded motions; and (d) frequency change at SER of 0.92, the optimized trilinear function, and residual function of the trilinear function.

from Zhang et al. (2023) that employed a model trained with a substantial number of numerical cases (1,323,400) to accurately estimate timing through r_u . In other words, the developed method better estimated t_L from case histories.

4. Validation and sensitivity analysis were conducted using LEAP tests, involving seven centrifuge tests with varying relative density and peak acceleration of input motion. The results of

the sensitivity analysis suggested an optimized SER value of 0.92, ensuring better identification of t_L . Centrifuge test results demonstrated that the method provided robust and better estimates of t_L when incorporating the ramp via the trilinear function.

5. Given that the most significant deformations during liquefaction typically occur after the triggering, the ability to accurately

identify t_L facilitates more precise assessment of consequence and liquefaction triggering. For instance, the Arias intensity-based liquefaction triggering curve (Kayen and Mitchell 1997) can be improved by utilizing t_L to cut the Arias intensity at the estimated timing, which represents the energy capacity of liquefied soil. This study will utilize the developed method to enhance the Arias intensity-based liquefaction triggering curve in the future.

Data Availability Statement

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

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