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DETERMINATION OF Y_{40} AND Y_{60} COMPONENTS IN THE SHAPES OF RARE EARTH NUCLEI[*]

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ABSTRACT

Differential cross sections for 50-MeV alpha particles of members of the ground state rotational band up to the 6+ or 8+ state were measured in a number of even nuclei of the rare earth region. The data was analyzed under the assumption of a perfect rotor description for the nucleus and a deformed optical interaction between alpha and nucleus by solving the resulting coupled equations. Higher order components Y_4 and Y_6 in the nuclear shapes were determined with precision. A systematic variation of B_4 from positive values in the light rare earths to negative values in the heavy ones is established.

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The rotational spectra of nuclei in the rare earth (A=152 to ~190) and actinide (A=220 to 250) regions, implying large permanent deformations, have been long known. The electric quadrupole moments of these deformed nuclei have been accurately determined by several methods. It is certainly associated largely with a Y_2 component in the nuclear radius. The knowledge about possible higher components Y_4 and Y_6, however, has been very tenuous up to now. Although calculations have predicted a Y_4 component in the uranium region 1,2), the evidence from alpha decay studies is ambiguous 3). Inelastic deuteron scattering near the Coulomb barrier has provided evidence for the existence of such moments in the rare earth nuclei, but the magnitudes and even signs of the moments were not well determined 4). Here we report a systematic study of the shape of deformed nuclei in the rare earth region through excitation of the ground state rotational band by alpha particles 5). We scattered a well collimated and analysed beam of 50-MeV alpha particles on several metallic foils of isotopically pure elements in the rare earth region. Angular distributions were taken with a cooled lithium-drifted multiple counter array. Special care was taken to keep backgrounds low and to maintain resolutions of about 50 keV.

Alpha particles are a good probe of the nuclear surface. Because they are strongly absorbed in the interior we are assured that most of those that are inelastically scattered are involved in a surface interaction. Moreover, even at moderate energy they carry high enough momentum that direct transfer of large units of angular momentum is possible. This is a desirable feature since we want to measure the higher order terms Y_L $(L > 2)$ in the shape, which is possible only when direct excitation of the J=L member of the band competes significantly with the cascade transition. In fact if the direct excitation alone were present, the angular distributions to the J=L state would be determined only by the L'th multipole component of the nuclear field. Since we have measured the differential cross sections to the various rotational levels of the ground state up to the 6^+ and sometimes 8^+ we are therefore in a position to determine, through a careful analysis, the shape up to Y_6 of the nuclear field produced by the ground state.

In our analysis of the data we assume that the alpha-nucleus interaction can be represented by a deformed complex optical potential and that the nucleus is a perfect rotor, at least up to the 8^+ state. We calculate the cross section to members of the ground state band by solving numerically the coupled differential equations that follow from this picture, without further assumptions 6). Coulomb excitation effects were found to be significant and were treated on an equal basis with the nuclear excitation. The multipole expansion of the interaction, the number of partial waves, and the number of coupled channels were all carried to convergence so that we have an exact numerical solution of the scattering model.

Since we treat explicitly the rotations, the optical potential should be essentially the same as that in the neighboring spherical region since in both cases it must carry only the effects of the omitted intrinsic excitations. This is discussed in greater detail elsewhere 6,7). Therefore, the same optical potential was used with only minor adjustments throughout the region from the spherical nucleus Sm^{148} through the deformed region to Hf^{178}. This means that essentially only the shape parameters of the optical potential had to be determined in the analysis. We parameterize the shape according to:

$$R = R_0(1 + \beta_2 Y_{20} + \beta_4 Y_{40} + \beta_6 Y_{60}).$$

Figure 1 shows the data and calculation for Sm^{154}. Notice that the only agreement occurs for $\beta_4 = +0.05$ and that the experimental data is reproduced in detail, including location of diffraction peaks, absolute magnitude of the cross sections, and even depths of the minima of the various states. The dashed and dotted curves show the disagreement with the data that is obtained when β_4 is respectively 0.0 and -0.05. We see in this way that the value of β_4 is quite precisely determined.

The agreement is somewhat improved for almost all the nuclei by including a small negative β_6 term. This term has been included in the best fit curves shown for three of our target nuclei in fig. 2. Here we easily see such changes in the data as shifts in the position of maxima, and smaller amplitudes of oscillation which, we find, imply a change in the sign of β_4 with increasing target mass.

We have made a preliminary exploration of deeper potentials from which it is already clear that our deformation parameters scale as $\beta\, r_o$ as has been suggested[8]. Accordingly in the presentation of our results we quote values of $\beta\, r_o$, or more precisely $\beta(r_o/1.2)$. The latter choice is made, somewhat arbitrarily, so as to achieve an approximate correspondence with our calculated values reported later. In any case the actual value of β used in our calculation can be obtained from those we list by reference to the value of r_o used for the optical potential. Our results are summarized in Table I and fig. 3.

The deformation β_2 is consistently smaller throughout the region than has usually been believed on the basis of electric quadrupole moments determined by Coulomb excitation experiments. Two points should be stressed in this connection. There is no reason to believe that the charge and nuclear fields have exactly the same shape. Secondly, the quadrupole moment does not define a unique value of β_2 as is sometimes supposed. This is because the Y_4 and Y_6 components in the shape also contribute to the quadrupole moment (and to every other even moment, as does Y_2).

To see whether the values of β_4 determined by our analysis can be understood in terms of the single-particle structure of nuclei, we have calculated this component in the shape by a simple method due to Harada$^{2)}$. The change in energy of the single-particle states due to a Y_4 component in the shape of the Nilsson potential was computed to first order. The equilibrium value of β_4 was determined as the one for which the total energy was a minimum. These calculated values are compared in fig. 3 with the values determined from the analysis of the data. It is gratifying that this simple model reproduces very well the observed trend characterized by larger positive β_4 deformations near the beginning of the deformed region, decreasing to large negative values at the end. A more realistic calculation by Nilsson and his coworkers$^{9)}$ agrees well with our result. This calculation takes into account pairing, core polarization, and Coulomb effects.

We wish to thank Claude Ellsworth for the fabrication of the targets and Noel Brown for his efforts on the computer program.

References

1. K. Kjällquist, Nucl. Phys. 9 (1958) 163
2. K. Harada, Phys. Letters 10 (1964) 80
3. P. O. Fröman, Mat. Fys. Skr. Dan. Vid. Selsk. 1 No. 3 (1957); M. G. Huber, Phys. Letters 13 (1964) 242; H. J. Mang and J. O. Rasmussen, Mat. Fys. Skr. Dan. Vid. Selsk. 2 No. 3 (1962)
4. B. Elbek, M. Kregar, and P. Vedelsby, Nucl. Phys. 86 (1966) 385
5. Some of the experimental results were reported by B. G. Harvey, D. L. Hendrie, O. N. Jarvis, J. Mahoney, and J. Valentin, Phys. Letters 24B (1967) 43; the possibility of establishing a Y_4 component by proton scattering was suggested by R. C. Barrett, Phys. Rev. Letters 14 (1965) 535
6. Norman K. Glendenning, University of California Lawrence Radiation Laboratory Report UCRL-17503 (1967), to be published in the Proceedings of the International School of Physics 'Enrico Fermi' course XL (1967) by Academic Press
7. Norman K. Glendenning, D. L. Hendrie, and O. N. Jarvis, University of California Lawrence Radiation Laboratory Report UCRL-17935
8. N. Austern and J. S. Blair, Ann. Phys. 33 (1965) 32; it should be noted, however, that quadrupole excitations at energies below the Coulomb barrier must scale as $\beta_2 R_c^2$
9. P. Möller, B. Nilsson, and S. G. Nilsson, (private communication)

Table 1

	Sm^{152}	Sm^{154}	Gd^{158}	Er^{166}	Yb^{174}	Yb^{176}	Hf^{178}
$\bar{\beta}_2$	0.246	0.270	0.282	0.276	0.276	0.276	0.246
$\bar{\beta}_4$	0.048	0.054	0.036	0.0	-0.048	-0.054	-0.072
$\bar{\beta}_6$	-0.012	-0.018	-0.018	-0.018	0.0	-0.006	0.0

Deformation parameters, $\bar{\beta} = (r_o/1.2)\beta$ obtained by our analysis of 50 MeV alpha scattering. The optical potential parameters used were about $V = 65.9$ $W = 27.3$ $r_o = 1.44$ $a = .637$ with only very slight adjustments to individual nuclei. The Coulomb potential was generated by a uniform charge distribution with a correct quadrupole moment.

Figure Captions

Fig. 1. Differential cross-sections for 50 MeV alpha particles scattered from Sm^{154}. The coupled channel calculations corresponding to values of β_4 equal to +0.05, 0.0, and -0.05 are compared. In the latter two cases the optical potential radius and β_2 were readjusted so as to achieve the best possible agreement with the 0^+ and 2^+ state. These results illustrate the extreme sensitivity to β_4 especially since differences in the 4^+ and 6^+ cross sections of an order of magnitude appear at certain angles.

Fig. 2. Coupled channel calculations of the differential cross sections for 50 MeV alpha particles of three nuclei exhibiting respectively positive, zero, and negative values of β_4 are compared with the data. The shape parameters in each case are exhibited in the figures.

Fig. 3. Solid lines indicate the values of β_4 calculated in first order perturbation theory based on the Nilsson scheme. The values of β_4, multiplied by $(r_o/1.2)$, that were obtained from an analysis of the scattering data are shown by solid dots. The error bars indicate our feeling of the precision with which the parameters can be extracted.

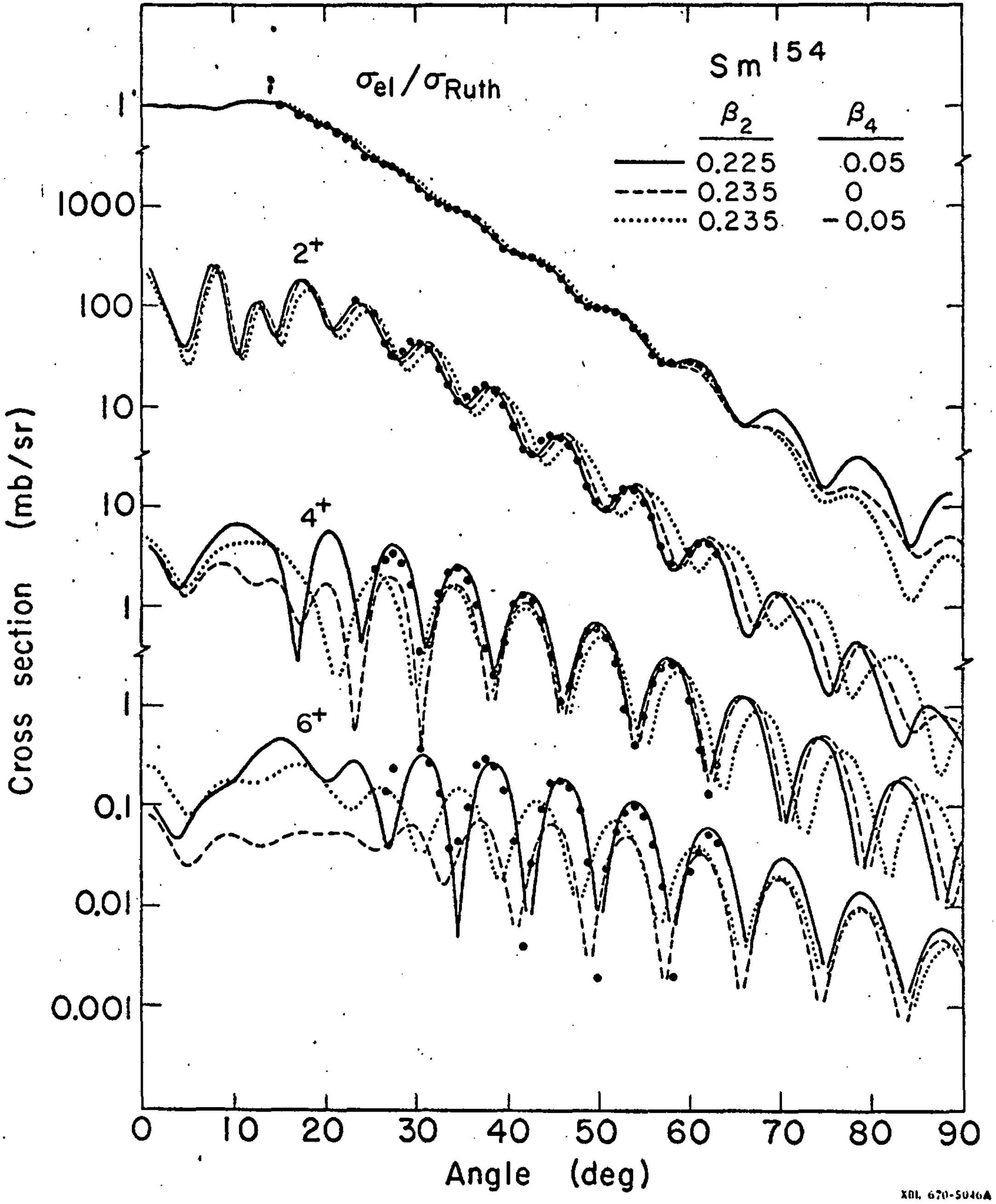
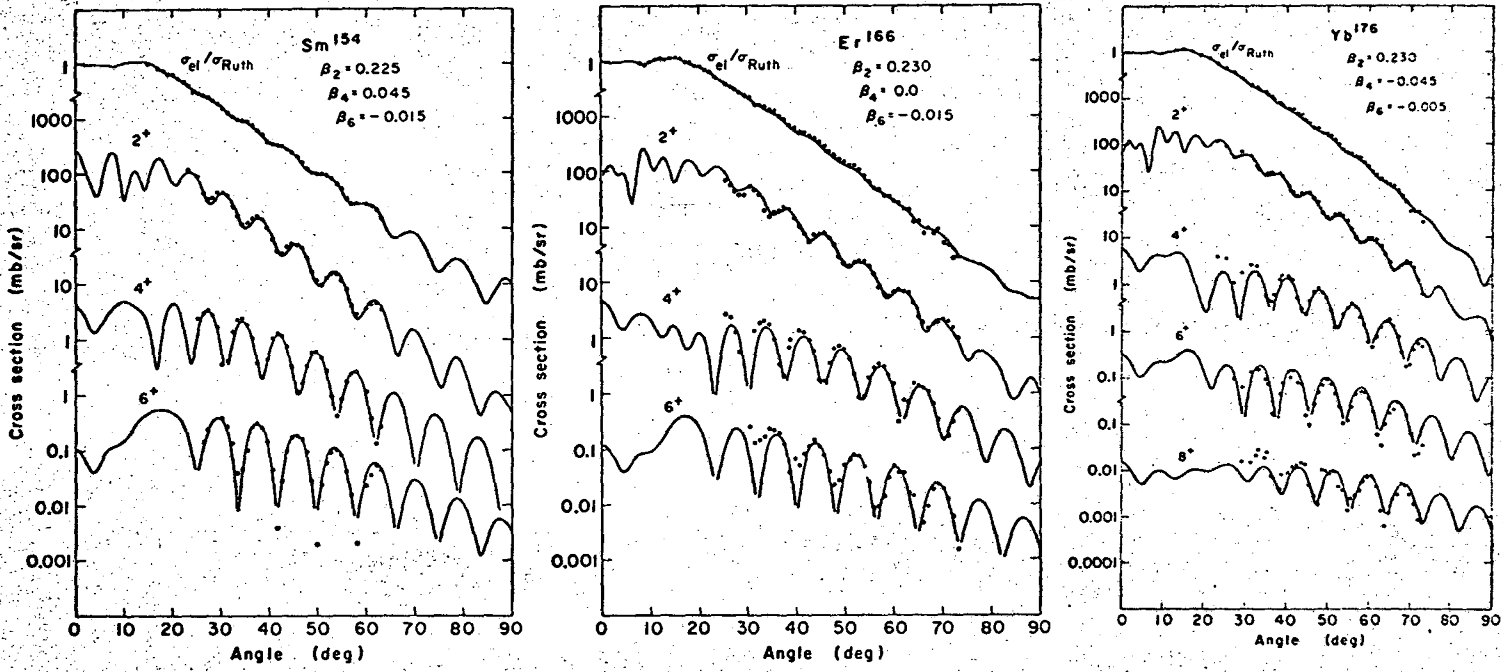

Sm^{154}
β_2
β_4
0.225 0.05
0.235 0
0.235 -0.05
$\sigma_{el}/\sigma_{Ruth}$
2^+
4^+
6^+
1
1000
100
10
10
1
1
0.1
0.01
0.001
Cross section (mb/sr)
0
10
20
30
40
50
60
70
80
90
Angle (deg)
XBL 670-5046A

Figure 1



XBL 670-5054

Figure 2

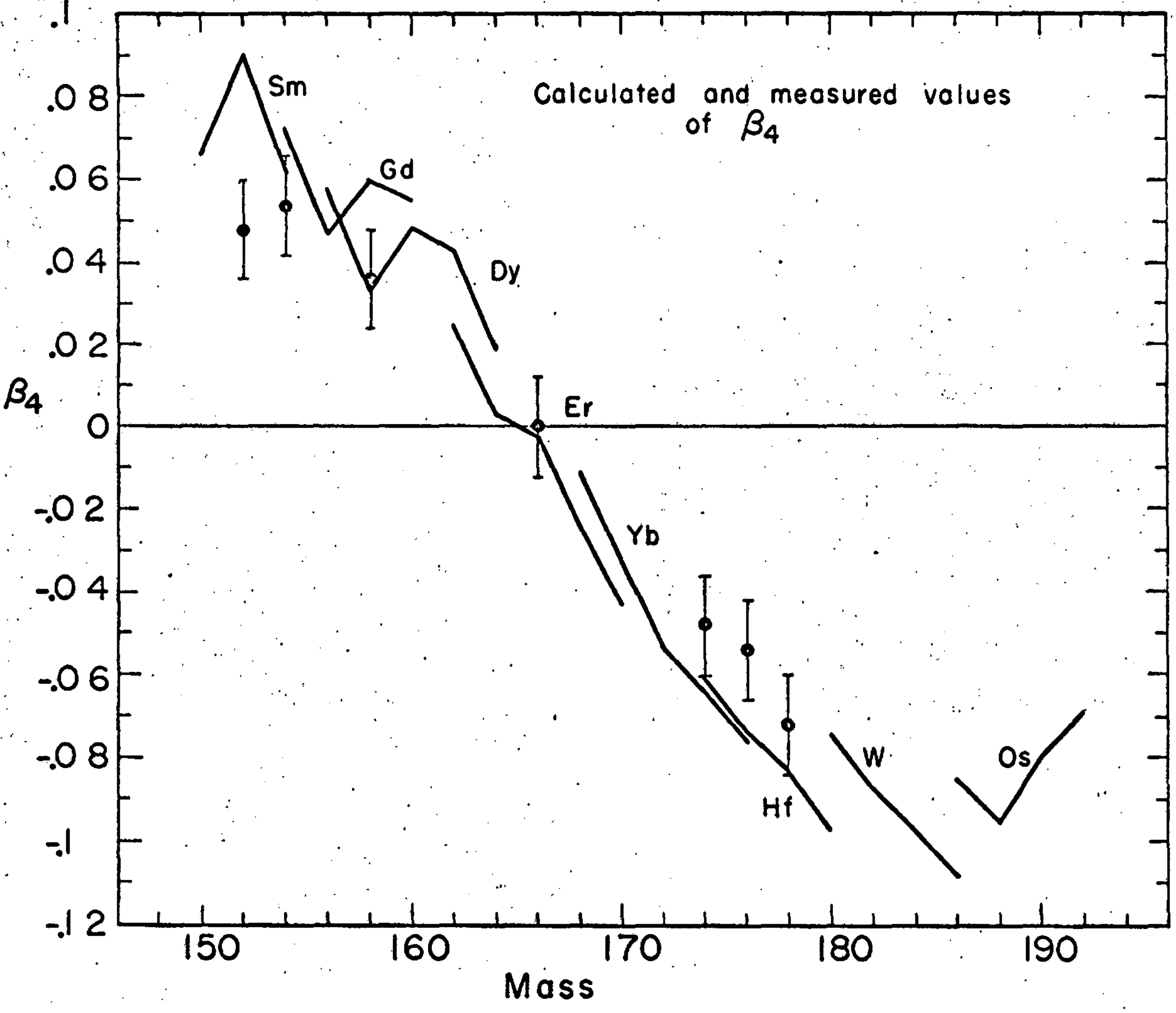


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Figure 3