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Signs of Power: A Critical Approach to the Study of Mathematics Cognition and Instruction

By

José Francisco Gutiérrez

A dissertation submitted in partial satisfaction of the

requirements for the degree of

Doctor of Philosophy

in

Education

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Dor Abrahamson, Chair
Professor Alan H. Schoenfeld
Professor Na'ilah Suad Nasir
Professor Martín L. Sánchez-Jankowski

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Abstract

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Doctor of Philosophy in Education

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My dissertation investigates the social power dynamics inherent in a single mathematics classroom and their effects on the quality of students' engagement and therefore learning. I examine the ways in which individual learning and relations of power are mutually constituted through discourse. That is, I view discourse not only as a lens for investigating these phenomena, but as the medium through which mathematical knowledge and power relations simultaneously become objectified, stabilized, and reproduced. To trace this "learning–power imbrication" and its impact on students' identity, agency, and conceptual understanding of mathematics, I conducted a year-long participant ethnography where I immersed myself within a high school mathematics community.

I present a theoretical model of mathematics teaching and learning that captures the fundamentally dialectical relationship between objectification and subjectification, which I refer to as the "obj–subj dialectic." Luis Radford's semiotic–cultural approach to the study of mathematics learning is central to my research on the "obj" side of the dialectic. I focus particularly on his theory of knowledge objectification, which was developed in the context of algebraic-generalization activity. To research the "-subj" side, I combine theoretical perspectives set forth by Anna Sfard and collaborators with those of Rom Harré to analyze the positional identities that emerge during social interaction. I used this theoretical model to conduct a series of qualitative microgenetic analyses of the emergence of the learning–power imbrication during one particular instructional lesson centered on an algebraic-generalization activity.

As students attempted to construct mathematical generalizations, their discursive actions created hierarchical positional identities that they then, in turn, took up, accepted, contested, negotiated, or rejected. Thus some students gain "mathematical ascendency" over others, which is a construct I propose to describe the co-construction of hierarchical subject positions emerging from the obj–subj dialectic during multimodal interactions. Also, whereas the obj–subj dialectic accounted for emergent forms of power, other aspects of the interaction were better explained through the lens of systemic dimensions of power, such as teachers' orientations and the enactment of these orientations. In particular, my data analysis shows that a teacher's "cognitive–conceptual script" resulted in differential learning opportunities.

The research contributes theory refinement and novel methodological techniques for both the learning sciences and critical educational studies. These contributions include: (1) a

qualification of Luis Radford's theory of knowledge objectification, specifically as it relates to students' mathematical generalizations; (2) empirical evidence supporting the general hypothesis that "learning" and "relations of power" are intrinsically reciprocal and mutually constitutive; and (3) analytic techniques for identifying and describing specific mechanisms of power inherent to mathematics instruction and their consequences on learning.

Ultimately, the research presented in this dissertation should inform educational practice. In particular, the signs of power articulated in this work should illuminate and complexify professional discourse and design efforts toward better serving students who have been historically underrepresented in the field of mathematics to navigate both its conceptual and systemic challenges.

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Acknowledgements

I would like to first thank Mr. Lam and his students at César Chávez School for Restorative Justice for welcoming me into their wonderful community. It was an honor to get to know them and "do some math" together.

I would like to thank my dissertation committee, starting with my advisor, Dor Abrahamson. From the very beginning, Dor astonished me with his unflinching support and creative advice. He was a passionate mentor and a person of action—two qualities I strive to emulate. I thank professors Na'ilah Nasir, Alan Schoenfeld, and Martín Sánchez-Jankowski, who inspired me to think differently about my research and its broader impact on teaching. If future readers derive anything of value from my work, they have my committee to thank. All the shortcomings in this dissertation are entirely of my own making.

I wish to thank all my peers and colleagues who trudged with me through the most difficult parts of graduate school. You were allies, editors, and above all, compassionate friends.

Beyond the halls of Tolman, I would like to thank the California rock climbing community—in particular, all my friends at Touchstone. Climbing is the metaphor of my life, and I applied its many lessons to my graduate studies. Thank you, bouldering buddies, for teaching me about inner balance, peace, and perseverance.

Finally, I dedicate this dissertation to my family: to my mother, San Juana; to my father, Miguel; and to my two dearest brothers, Ruben and Angel. My academic path may have taken me farther away physically, but my heart has never been closer to home. I would not be where I am without your support. I would not *be* without your love.

Chapter 1: Introduction

A vignette…

Three high school students, Thalia, Ailani, and Xeni, have been working together on a pattern-finding problem, when their classroom teacher, Mr. Lam, approaches their table and asks questions related to their work. The problem presents a sequence of four geometric figures, and there is a numerical pattern that enables students to make predictions about figures further along the sequence, such as Fig. 10. Taking lead of the group, Thalia has made a prediction about Fig. 10, using an arithmetic strategy that effectively determines figures in the sequence. However, her strategy achieves this goal using a recursive technique, that is, by iteratively progressing from each figure to the next, whereas optimally a strategy would determine figures by algebraic function, without recourse to known items along the sequence. Mr. Lam, in a perfectly sensible pedagogical move, prompts the group to consider much larger numbers, such as Fig. 100, asking, "So we have to know the one before it [Fig. 100]? Is there a way that we don't have to know the one before it?"

Mr. Lam may not be aware of it, but a conflict is about to erupt. Implicit in his question is the assumption that the group's strategy is inefficient. After a 2-sec pause, Thalia stomps her feet and exclaims to Mr. Lam, "Bruh, why you asking all these questions?" Ailani stirs the conflict, interjecting loudly, "YES!" Mr. Lam, keeping his usual calm and professional demeanor, responds to Thalia: "My name is not 'Bruh.' My name is Mr. Lam, and I'm challenging you. That's why I'm asking you these questions, because I want you to get smarter." Thalia buries her face behind her hands and starts laughing sheepishly; her face still hidden behind her hands, she continues: "OK Mr. Lam, why you asking (all these questions)."

Introduction and Objectives

There is more to learning mathematics in a classroom than learning content. I recorded the above vignette while observing Mr. Lam's classroom as part of my ongoing research on the challenges and opportunities for improving mathematics education for students from historically marginalized public school populations, such as African American, Latino/a, and economically disadvantaged students. Thalia's statement of "Why you asking all these questions?" reflects a complex sentiment. She may perceive that she and her peers are being questioned unnecessarily, even interrogated about their participation in the task. Ailani agrees with Thalia's sentiment and, together, their behaviors indicate their sense of Mr. Lam's instruction as coercive at worse, or troublesome at best.

This vignette sparked a variety of questions for me. What does it mean for mathematics students to experience classroom discourse in this way? What aspects of classroom interaction give rise to what I call social-mathematical antagonism? Can this antagonism be leveraged as a resource to support learning? Furthermore, what does it mean for a classroom teacher to associate notions of challenge and struggle with notions of smartness? What is the pedagogical utility of framing mathematics learning as adversarial or as changing intellectual capacity?

Thalia and Ailani's apparent discontent with their mathematics teacher and Mr. Lam's particular views on mathematics pedagogy could be isolated phenomena limited to the envelope of this particular classroom community; it is conceivable that this particular interaction between three high school students, a teacher, and mathematics content is not representative of broader trends. However, by reflecting critically on the results of previous studies that I conducted in similar classroom settings (e.g., J. F. Gutiérrez, 2010, 2013a), I argue that these participants'

experiences are related to broader phenomena endemic to not only mathematics classrooms serving historically marginalized students, but to all classrooms. That is, participating in classroom mathematical discourse necessarily involves engaging in a power dynamic. Through an elaborate case study of the vignette above, I aim to show in this dissertation that mathematical learning is not power neutral, but rather individual learning and power relations are mutually constituted through discourse.

Problem Statement

In this dissertation I investigate the ways in which relations of power, which are inherent to all educational settings, impact students' quality of engagement and therefore learning. Whereas I focus my research efforts on my target population, I theorize that *all* students' participatory behaviors, dispositions toward classroom practice, and mathematical knowledge are mediated by relations of power that shape local instructional contexts and the social interactions therein. To illuminate these issues, I bring to bear complementary learning-sciences and sociopolitical perspectives to expose hidden structures, mechanisms, and processes of power that either enable or hinder classroom mathematics learning.

In this opening chapter I provide background information relevant to my line of research and outline its general objectives. I then introduce the essential conjecture underlying the research presented in this dissertation. Lastly, I provide an overview of the dissertation, chapter by chapter.

Background and General Research Objectives

My scholarly work is motivated by deep concern for equity in mathematics education. It has been well-documented that African American students, Latino/Hispanic students, students with limited English proficiency, and students living in poverty tend to be over-represented in courses considered to be lower-level (e.g. elementary Algebra) and under-represented in higher-level courses (e.g., Algebra II or Calculus) (Oakes, 1990; Oakes, Joseph, & Muir, 2004). My previous studies in northern California schools have corroborated these trends. This dissertation attempted both to understand and disrupt these patterns of inequity by investigating the teaching and learning of mathematics in a single classroom serving students from marginalized sectors of the public school population.

As for a particular curricular content domain that would serve as context for my observations, designs, and analyses, I selected to focus on the disciplinary skill of mathematical generalization. Generalization constitutes a core practice in multiple subject matter domains, such as algebra and geometry, that research has implicated as the *de facto* "gatekeepers" to advanced mathematics courses (Moses & Cobb, 2001; Oakes et al., 2004). In turn, because symbolic representation has been implicated as one of mathematics' fundamental challenges (Kaput, 2007; Schoenfeld, 2007), I draw on semiotics as a means of investigating paths and impediments to students' construction of meaning for mathematical symbols. Yet although I draw heavily from this vibrant and emerging paradigm, it is not without limitations. Mainstream studies of students' meaning-making processes are predominantly framed and analyzed *only* in cognitive terms, and the factors that contribute to the development of conceptual understanding are often assumed to be universal across all student populations and contexts. This orientation does not attend to, and thus fails to understand, the inherently cultural and political nature of mathematics education, and its affordances for an equity-driven agenda are therefore limited.

Inequity in mathematics education is demarcated clearly along lines of race, socioeconomic status, cultural practice, and gender (Martin, 2009; Nasir et al., 2011; Nasir, Hand, & Taylor, 2008; Oakes, 1990). Over the years, scholars have defined and researched inequity mainly along two dimensions. First, the literature has documented and analyzed the systemic barriers that non-dominant students are likely to face, such as tracking, restricting access to advanced-level math courses, and lack of well-prepared and experienced teachers. Second, previous research has also revealed aspects of identity that pose real challenges for mathematics students from marginalized communities (see Chapter 2). Yet still missing from the literature are high-resolution analyses that explore the dynamic interplay of power and mathematical cognition (cf. Hand, 2010). Mainstream cognitive studies must be critically interrogated for their sociopolitical implications (Pais & Valero, 2012), namely, the effects of social power relations on mathematics teaching and learning. To facilitate such an interrogation of cognitive analyses, I rely on sociopolitical perspectives to expose hidden power dynamics that mediate learning opportunities in classrooms. My line of research assumes that power-oriented and cognition-oriented perspectives on learning are not only compatible but also necessarily synergistic. Through designing and implementing instructional units in authentic classroom settings, I aspire to empirically demonstrate the pedagogical utility of this proposed theoretical synthesis.

Accounting for issues of power in learning processes presents theoretical and methodological challenges for educational researchers (see also Valero & Zevenbergen, 2004). I have found three measures useful in moving forward in this line of research: (1) conducting my research with historically under-served youth in empirical contexts that make these issues particularly salient; (2) expanding my theoretical approach to consider sociopolitical perspectives on mathematics education (R. Gutiérrez, 2013); and (3) conducting in-depth analyses of the co-construction of opposition and resistance in mathematics classrooms (Hand, 2010). Utilizing these measures in my previous studies enabled me to articulate a conjecture that underlies this dissertation, which I elaborate on below.

Essential Conjecture – Discourse, Learning, and the Reproduction of Power Relations
The studies presented in this dissertation are part of a line of research that navigates the following conjecture: individual learning and social power relations become imbricated through discourse so as to mutually constitute and express each other. I view discourse not merely as an analytic lens for observing what I refer to as the *learning–power imbrication*,[1] but also as the medium through which mathematical knowledge and power relations simultaneously become objectified, stabilized, and reproduced. In particular, I conceptualize the learning–power imbrication as the reciprocal discursive process whereby (1) students appropriate cultural artifacts (e.g., algebraic symbols and forms) as semiotic means of objectifying personal presymbolic, proto-mathematical knowledge (Abrahamson, 2009; Radford, 2003); yet so doing, (2) students not only adopt a new perspective on the world but they also become cognitively "beholden" and subject to particular discursive practices that temporally and ontologically precede them (cf. "ontological imperialism," Bamberger & diSessa, 2003) and are inherently

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[1] I find "imbrication" the best way to characterize this phenomenon (instead of other terms, such as "overlap" or "intersection"), because imbrication refers to a way things fit into one another, such that their edges are fastened. What strikes me about the transcriptions of math talk that will be featured in this dissertation is that "learning" and "relations of power" are not merely co-present; in some instances, they are co-constitutive and are "cinched" with a single utterance.

hierarchical—that is, power-laden (see Chapter 2 for my full theoretical approach). Thus, acts of (1) objectification are at the same time acts of (2) subjectification.

This dialectical relation between objectification and subjectification—what I hereafter call the "obj–subj dialectic"—is not a novel perspective per se and similar perspectives are found in the literature (e.g., Heyd-Metzuyanim & Sfard, 2012; Sfard, 2007). I have found the construct of a learning–*power* imbrication especially useful, because K-12 mathematics education occurs within a broader nexus of asymmetric power relations that have not been adequately accounted for in the literature.

Thus, the objectives of this dissertation are to conceptualize and empirically explore the role of discourse in the learning–power imbrication and its impact on constructs such as identity, agency, and conceptual understanding (Chapter 2 gives an explanation of these constructs).

Overview of dissertation

Having outlined my research problem and the essential conjecture motivating this dissertation, in the chapters that follow I elaborate on my theoretical and methodological approach, present a series of analyses centered on the same vignette of data, and describe the implications of this dissertation for future design-based research that would have the dual foci of contributing to theory as well as practice of mathematics pedagogy. To help the reader navigate this dissertation, I am providing *Figure 1*, a roadmap to the dissertation, followed by a description of each remaining chapter.
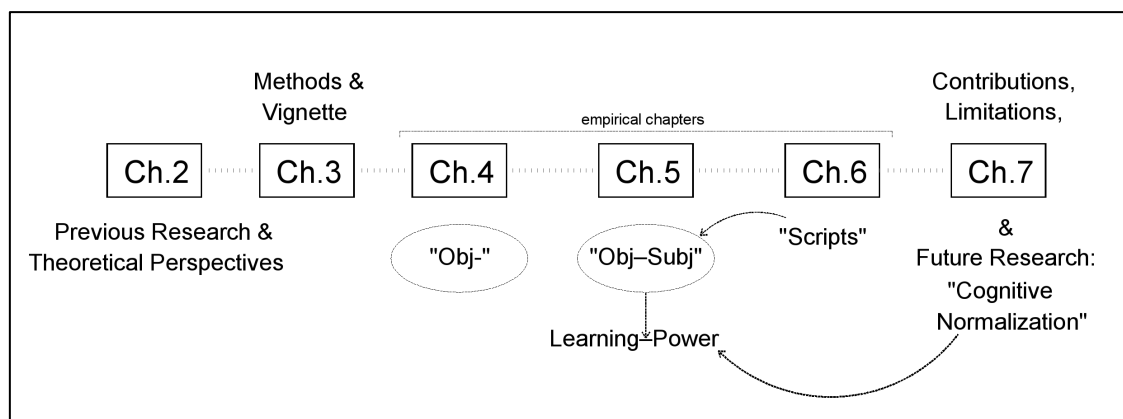
Figure 1. Roadmap to the dissertation. Ch.2 reviews previous research and presents my theoretical perspectives; Ch.3 presents my methodological approach and describes the focal data vignette to be analyzed in the three subsequent chapters; Ch.4 focuses on the "objectification" side of the dialectic; presents an analysis of students' mathematical assertions and milestones; Ch.5 presents an "obj–subj" analysis, exposing the learning–power imbrication; Ch.6 explores the role of teacher "scripts" in the obj–subj dialectic and its impact on the imbrication; Ch.7 summarizes main findings and discusses limitations; offers a Foucauldian interpretation of the learning–power imbrication through the lens of "cognitive normalization," and proposes a new study based on this interpretation.

Chapter 2: I review relevant empirical work pertaining to mathematical thinking and learning as well as studies that focus on power, opposition, and discourse in the practice of pedagogy. Following these literature reviews, I theoretically define an obj–subj dialectic and propose it as a means of investigating my essential conjecture. Luis Radford's (2003) semiotic–

cultural approach to the study of mathematics learning is central to my research on the "obj" side of the dialectic. I focus particularly on his theory of *knowledge objectification*, which was developed in the context of algebraic-generalization activity. To research the "-subj" side, I combine theoretical perspectives set forth by Anna Sfard and collaborators (Heyd-Metzuyanim & Sfard, 2012; Sfard & Prusak, 2005) with those of Rom Harré and colleagues (Harré, 2008; Harré & Moghaddam, 2014) to analyze the *positional identities* that emerge during social interaction vis-à-vis issues related to student individual agency. As students attempt to construct mathematical generalizations, I argue, their discursive actions create *hierarchical* positional identities that must then, in turn, either be taken up, accepted, contested, negotiated, or rejected. It is in this sense that, I argue, "learning" and "relations of power" are intrinsically reciprocal and mutually constitutive.

       Chapter 3: This chapter orients the reader to the data and analyses that will be presented in the remaining chapters. I explain the rationale for my methodology that combines participant ethnography with design-based research principles, data collection techniques, research site, participants, and my researcher positionality in the study. This chapter also presents a detailed narrative of the focal data vignette and begins to analyze some of the main themes that I have discerned from the larger corpus of ethnographic data. This narrative is a "pre-analysis" and is not intended to be comprehensive but rather provide crucial background information that characterizes this particular classroom community and situates the focal vignette within a larger context.

       Chapter 4: Here I present the first analysis of the focal vignette; it focuses on the "obj" side of the dialectic and applies Radford's semiotic–cultural framework to analyze the situated reasoning of three students (Thalia, Ailani, and Xeni) during one particular pattern-finding problem. Putting aside subjectification and looking only at mathematical strategies and tools, I analyze the process of generalizing (i.e., the way that students engage with the problem), and the products that result from this process (e.g., recursive versus closed-explicit generalizations). Firstly, I ask: What semiotic, linguistic, material, and other resources do students bring to bear as they attempt to construct generalizations? Secondly, I ask: How do students navigate the three levels of generalization—Factual, Contextual, and Symbolic—that Radford (2003) has identified as inherent in patterning activity? To pursue these questions, I produced and analyzed a detailed transcription of students' verbal, gestural, and other semiotic actions as they referred to mathematical objects and attempted to construct generalizations during the 35 minute span of data. Findings reveal significant differences between the three students with respect to their (1) participation within each of the semiotic modes, and (2) final generalizations. Furthermore, findings also show that students' reasoning *vacillated* between the F-C-S modes. I conclude that individual differences observed through the lens of objectification *in situ* resulted in a social-mathematical status hierarchy, which is closely examined in the next two chapters.

       Chapters 5 and 6 build on and expand the findings from Chapter 4 and explore a semiotic-linked power dynamic that impacts student agency and identity. The analyses presented in these two chapters launch from an enlarged version of the semiotic–cultural perspective that accounts for processes of subjectification. That is, these chapters take on the explicit focus of the obj–subj dialectic and examine the status hierarchies emerging therefrom.

Chapter 5: This chapter analyzes selected segments of the same 35 minute span of data and explores the tension between "obj," the semiotic resources (e.g., gesture and language, as well as conventional tools such as tables/graphs) to which students have recourse to make mathematical assertions, and "-subj," the hierarchical positional identities that participants co-construct through these multimodal interactions. Findings show that whereas all students made statements that tacitly positioned themselves as having the correct answer (i.e., as mathematical authorities), some students resorted to a broader arsenal of semiotic resources to make their point. These *social-mathematical power encounters* resulted in differentiated status positions, and the students missed opportunities to engage in dialog and collaborate on the shared goals of the task. In this way, I argue that the students' respective semiotic means of objectification also functioned, simultaneously, as semiotic means of subjectification. Thus some students gained *mathematical ascendency* over others, which is a construct that describes the co-construction of hierarchical positional identities emerging from the obj–subj dialectic during multimodal interactions. Mathematical ascendency is a reification of the learning–power imbrication; it is based on the semiotic resources marshaled by individual interlocutors that locate or position them along an emergent hierarchy. Furthermore, a teacher's discursive actions may implicitly mark one type of argument as superior to other forms of argumentation, thus marking both a mathematical hierarchy and a social hierarchy.

By focusing my analysis both on the semiotic and interactional aspects of the obj–subj dialectic, the learning–power imbrication is exposed. The next chapter continues to trace the contours of this imbrication as it unfolds in the focal vignette; as the power dynamic is made visible, potential mechanisms for *manipulating* the imbrication toward pedagogical objectives are also revealed. Furthermore, in moving ahead with the analysis, the limitations of an exclusively content/cognition-based approach to the study of the learning–power imbrication become evident. These limitations are then taken up and addressed in the remaining empirical chapters.

Chapter 6: I present further aspects of the focal data that are not adequately accounted for through the semiotic–cultural perspective alone. The obj–subj dialectic is a necessary but not sufficient theoretical construct for the study of power dynamics in mathematics education. In an attempt to shore up the limitations inherent in a learning-science conceptualization of the obj–subj dialectic, I bring to bear complementary socio-political perspectives that can explicitly account for relations of power not only in local multimodal interactions but also in the systemic organization of mathematics education. Specifically, this chapter presents the construct of *cognitive–conceptual script*, which is based on the work of Kris Gutiérrez and her colleagues (K. Gutiérrez, Rymes, & Larson, 1995), to explore a possible mechanism that links cognition to social structures and vice-versa. Findings show that whereas the obj–subj dialectic accounts for emergent forms of power during moment-to-moment interactions (i.e., mathematical ascendency), there are other aspects of the vignette that are better explained through the lens of systemic dimensions of power such as teachers' orientations and the enactment of these orientations. The main findings for this chapter thus are that the classroom teacher's cognitive–conceptual script were reified and enforced through a pedagogical agenda that: (1) codified and enacted certain "cognitive values" (Goodnow, 1990, as cited in Wertsch, 1998, p. 66) that are socially and historically constructed; (2) was "monologic" (K. Gutiérrez et al., 1995), in that it did not flexibly accommodate students' varied contributions; and (3) was based on an expressly

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stated belief that possibly conflates semiotic mode with mathematical reasoning. Also, students responded to the teacher's cognitive–conceptual script with their own "counter-scripts" that codified and enacted *their* cultural knowledge and forms.

Chapter 7: This final chapter summarizes key findings and contributions to the field of critical mathematics education, and acknowledges limitations of this research. To facilitate a final discussion about future directions, I offer a Foucauldian interpretation of the learning–power imbrication through the lens of "cognitive normalization." Having established the constructs of mathematical ascendency and cognitive–conceptual scripts, I consider Foucault's (1977) theory of power-knowledge to frame these constructs as reifying his critical post-structuralism. I view these phenomena as discursive mechanisms through which status hierarchies emerging from the obj–subj dialectic are co-constructed and thus relations of power are renegotiated or reproduced. I apply Foucault's notion of *normalization* as a meta-lens on mathematics pedagogy (cf. Popkewitz, 2004). I conceptualize mathematics education as indeed part of a broader form of "disciplinary power" (Foucault, 1977) to re-examine both teacher–student and student–student interactions through the lens of *cognitive normalization*. Finally, I conclude this dissertation by proposing future studies. The next steps would be: (1) to continue to build theory pertaining to power relations inherent in mathematics teaching and learning; and (2) to systematically examine the generative hypotheses arising from this dissertation through a conjecture-driven design-based research project (Abrahamson, 2009).

Chapter 2: Relevant Literature and Theoretical Resources

The primary objective of this chapter is to provide background of the theoretical resources that informed this dissertation and orient the reader to the main theoretical claims of the dissertation. I begin by reviewing empirical work pertaining to mathematical thinking and learning as well as studies that focus on power, opposition, and discourse in the practice of pedagogy. Following these literature reviews, I theoretically define an obj–subj dialectic and propose it as a means of investigating my essential conjecture.[2] Luis Radford's (2003) semiotic–cultural framework is central to my analysis of the "obj" side of the dialectic. I focus particularly on his theory of *knowledge objectification*, which was developed in the context of algebraic-generalization activity. To research the "-subj" side, I combine theoretical perspectives set forth by Anna Sfard and collaborators (Heyd-Metzuyanim & Sfard, 2012; Sfard & Prusak, 2005) with those of Rom Harré and colleagues (Harré, 2008; Harré & Moghaddam, 2014) to analyze the positional identities that emerge during social interaction vis-à-vis issues related to agency, status, and authority.

Prior Research

It has been well documented that historically marginalized groups, such as African American, Latino/a, and economically disadvantaged students, are under-represented in higher education and, in particular, in the fields of science, technology, engineering, and mathematics (STEM) (NSB, 2008). As mentioned in the introduction of this proposal, the research further implicates high-school mathematics as the *de facto* "gatekeeper" into academic and technological communities of practice (Ladson-Billings, 1998; RAND, 2003). Namely, access to and completion of rigorous high-school mathematics courses has been shown to be among the strongest predictors of student success in higher education (Oakes et al., 2004). For example, low-income students who take algebra and geometry are almost three times as likely to attend college than those who do not, and students who complete a course beyond Algebra II and then enter college are more than twice as likely to complete college successfully than those who have not taken those advanced courses (Oakes et al., 2004).

In sum, the high-school subject content matter of mathematics appears to serve selectively either as a catalyst that propels students into academic endeavor or an insurmountable obstacle that truncates their academic trajectory and, thus, delimits their capacity to participate in an increasingly technological economy (Moses & Cobb, 2001; Oakes et al., 2004). Therefore, improving access to high-school mathematics education is an important goal toward bridging the academic achievement gap. A central focus of this effort should be on algebra content, because high-stakes exams define "success in school" directly in terms of success in algebra.

In this section I review the literature on student algebraic reasoning that focuses on the interplay between "modeling" and "solving," followed by an overview of the well-documented challenges that (presumably) all algebra learners face. So doing, I introduce the cultural–semiotic perspective as a means of illuminating affective and not just cognitive factors that are critical for mathematics students from marginalized communities specifically. Lastly, I review the literature on social power dynamics, opposition, and discourse in learning situations.

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[2] Whereas it is commonplace to locate operationalization of constructs, etc. under Methods sections, the empirical portions of this dissertation, Chapters 4–7, explain the specific analytic techniques used for their respective analyses.

Empirical Studies on Student Algebraic Reasoning

Modeling Situations and Manipulating Symbols in Algebra Problem Solving
Algebra, viewed as a human practice, can be characterized broadly as involving complex *sense-making* processes (Kaput, 2007; Kooij, 2001; Schoenfeld, 2007). These processes are often further described as demanding two general capacities that both characterize algebraic problem-solving activity and constitute common goals and objectives of curricular design that inform classroom instruction:

1. *Modeling* – actions for making initial sense of a given problem situation, such as creating and expressing generalizations to represent the source situation using increasingly formal semiotic forms (e.g., symbolic expressions, graphs, tables, verbal descriptions, or some combinations thereof).
2. *Solving* – reflecting and operating on those mathematical representations using conventional manipulation procedures to support reasoning about the source situation being modeled.

Distinguishing between these two core capacities has illuminated the challenges faced by my target population. Specifically, my previous studies suggest that discourse plays a more critical role in the development of (1) than (2) for struggling students from historically marginalized groups (see below for further explanation). Before I unpack the details of this assertion, first it is necessary to review the literature on algebra learning challenges that are presumably faced by all learners. So doing will enable me later to leverage a critique of and propose a qualification to the predominant theoretical models pertaining to student algebraic reasoning.

Algebra Learning Challenges: Focus on the Semiotic–Cultural Perspective
Learning algebra has historically been fraught with conceptual challenges (for a review, see Kieran, 2007). For example, the literature has documented cognitive "gaps" that students must traverse as they transition from arithmetic to algebraic forms of reasoning. For example, Luis Radford (2003) applies semiotic analysis to implicate discontinuity in students' spatial–temporal embodied mathematical experience, as they appropriate symbolic notation to express algebraic generalization of non-symbolic situations. This "rupture" designates a conceptually critical shift in the semiotic role of an inscription, such as "*x*," from indexing a specific actual aspect of the problem space, such as the number of "toothpicks" in a geometric construction (see *Figure 2*, below) to meaning any element within the plurality or even infinity of imagined situated extensions of the problem. The *x*, in this case, has to be liberated, so to speak, from the grounding situation from whence it emerged, so that the problem solver can manipulate the algebraic expressions unconstrained by a constant need to evoke the situated meaning of *x*. Consider the "toothpicks" problem (see Figure 1, below), a situation involving an initially unknown general principle governing the relation between a numerical and a geometric sequence.
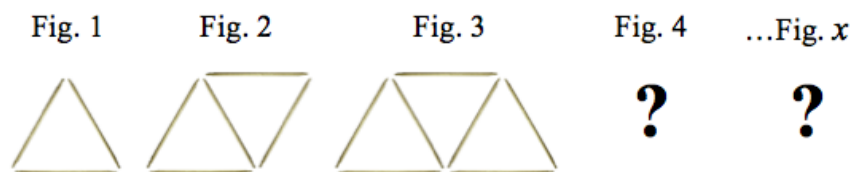
Figure 2. "Toothpicks"— a paradigmatic algebra generalization problem. The task objective is to express the total number of toothpicks in the xth figure. For example, "Fig. 1" consists of 3 toothpicks, "Fig. 2" consists of 5, "Fig. 3" consists of 7, etc., so that the xth figure consists of $2x+1$ toothpicks.

Whereas Radford's rupture lives in the realm of (1) Modeling, researchers have also identified other gaps that live in the realm of (2) Solving. For instance, Filloy and Rojano (1989) identify a stark demarcation, which they call a *didactic cut*, between arithmetic and algebraic forms of reasoning in the context of solving first-degree equations with a single unknown. Equations such as $Ax + B = C$ can be solved using arithmetic means such as counting or inverse operations, whereas equations with unknowns on both sides of the equal sign, such as $Ax + B = Cx + D$, require "operations drawn from outside the domain of arithmetic—that is, operations on the unknown" (p. 19). These scholars conclude that focused instructional intervention is required at such didactic cut points. Note that whereas Filloy and Rojano characterize the arithmetic–algebraic gap in terms of specific mathematical forms ($Ax + B = C$ vs. $Ax + B = Cx + D$) and strategies to deal appropriately with such forms, other researchers would characterize the same gap in more fundamental terms. In particular, Herscovics and Linchevski (1994) maintain that students' difficulty with equations involving double occurrence (e.g., $x + 5 = 2x - 1$) is not so much a didactical issue but rather suggests a deeper, underlying "*cognitive gap*" that can be characterized as a fundamental inability to operate *spontaneously* with or on unknown quantities.

Finally, research findings indicate that the capacities to model and solve algebraic problems do not necessarily develop at the same rate, and the research implicates traditional instruction as determining this developmental differential. Namely, curricular material and teacher practice tend to value symbol manipulation at the expense of creating opportunities for students to practice initially generating these symbols from problem situations (cf. Arcavi, 1994). Thus the two pillars of algebra instruction are to support the development of both types of capacity and to teach students to shift flexibly back and forth between them.

My earlier studies (J. F. Gutiérrez, 2010) support the implication of gaps inherent to algebraic problem solving as foci for productive research. I propose that discourse plays a greater role than has been theorized in explicating these gaps and how they may be forded. In particular, I submit, a critical examination of the role of discourse in algebraic learning reveals that these gaps present affective and not just cognitive challenges. Furthermore, for struggling students from historically marginalized groups, issues of discourse and identity may play a more critical and more nuanced role in the development of the core capacities than has been previously surmised and particularly more so in (1) Modeling as compared to (2) Solving (see elaboration in the "Semiotic–Cultural Approach," below). To set up a discussion regarding the fundamental roles of discourse and identity in learning, I first outline the educational literature that focuses on discourse, social power dynamics, identity, and student oppositional behavior.

Empirical Studies on Power, Opposition, Identity, and Discourse in Learning Situations
The study of power relations has a long-standing tradition within the social and political sciences. I draw selectively from this vast sociopolitical literature particular seminal conceptual

frameworks that support my focus on discourse in examining and transforming classroom power dynamics. In this dissertation (Ch.7) I therefore utilize Foucault's (1977) theory of "normalization," which conceptualizes power and agency along multiple dimensions. Looking at mathematics education literature, I also draw on critical-sociopolitical scholars who question epistemic paradigms at the basis of dominant curriculum (R. Gutiérrez, 2013; Valero, 2008; Zevenbergen, 1996). Indeed, several of these scholars apply discourse analyses to explore issues of power, participation, and student oppositional behavior (Cornelius & Herrenkohl, 2004; Enyedy et al., 2008; K. Gutiérrez et al., 1995; Hand, 2010; Moschkovich, 1999; Nasir, 2004).

Empirical work on power in mathematics education tends to focus on issues of structural inequity and identity. For example, in a recent review conducted by the Diversity in Mathematics Education project (DiME, 2007), power in mathematics education was analyzed and understood primarily in terms of systemic barriers that non-dominant students are likely to face, such as tracking, restricting access to advanced-level math courses, and lack of well-prepared and experienced teachers. Furthermore, in this review power was also viewed through the lens of "opportunities to learn mathematics" for students from historically marginalized groups. The review implicated the promise of classroom participation structures congruent with students' social–cultural identities.

In sum, the study of power relations in mathematics education is by and large a recent enterprise. That said, there is a growing understanding of structural impediments as well as aspects of identity that pose real challenges for mathematics students from marginalized communities. To this literature, my dissertation contributes high-resolution analyses of the dynamic interplay of power and mathematics learning (cf. Hand, 2010). Namely, I explore how the structural and identity issues that the literature has identified emerge and interact to impact the actual mathematical reasoning of students.

Theoretical Perspectives

Relations of power are endemic to all social interaction (Foucault, 2000). In school classrooms, power relations critically impact learning outcomes by mediating students' *opportunities* to engage the content. I argue that these power relations also affect the *quality* of this engagement, by implicitly reproducing non-agentive dispositions toward mathematics content. Furthermore, I maintain that relations of power are *not* absolute, but can be negotiated, realigned, or created anew through discourse. Indeed, discourse has been implicated within both the mathematics education literature and the sociopolitical literature as an agent of reproduction for better or worse. For example, discourse plays a primary role in conceptual learning (Radford, 2003; Sfard, 2007; Vygotsky, 1978) while at the same time serving as a mechanism by which power is expressed, exercised, and thus perpetuated (Bourdieu, 1991; Davies & Harré, 1991; Foucault, 2000).

Availing of this functional overlap of discourse, I employ theoretical frameworks from both content- and power-oriented fields of inquiry to inform the studies reported in this dissertation. From the learning sciences I present Luis Radford's (2003) *semiotic–cultural approach* to the study of students' algebraic reasoning. After detailing Radford's model of algebraic reasoning, I then "graft" onto it Heyd-Metzuyanim & Sfard's (2012) construct of "subjectifying" and Rom Harré's (1991) "positioning" theory, as a means of investigating issues of identity, agency, and status in mathematical discourse. I thus propose a synthesized theoretical model and its application to expose and analyze the learning–power imbrication in mathematics education. Namely, this novel approach has enabled me to untangle the compound challenges

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faced by students who are attempting to develop a mathematical register even as they are learning new content and, moreover, as they are negotiating, constructing anew, appropriating, or rejecting social identities marked by classroom mathematical discourse. Specifically, the version of the semiotic–cultural approach that I propose—a version that accounts for processes of subjectification—illuminates mathematics learning challenges vis-à-vis issues of identity. I aim to show through my data analysis, that this model exposes apparent tension between students' overt mathematical speech acts and underlying issues of identity and hierarchy that, in turn, intersect with socio-historical narratives of power.

The Semiotic–Cultural Approach to Algebra Learning

Building on Vygotsky's (1962) cultural-historical psychology and Edmund Husserl's (1958) phenomenological philosophy, Luis Radford's (2003) semiotic–cultural approach views learning as an evolving process reflexively co-constrained by cognitive and socio-cultural factors. Specifically, mathematics learning is conceptualized as the process of constructing personal meaning for canonical semiotic artifacts (e.g., algebraic symbols such as the variable "*x*"). Through consolidation and iteration of these constructions, students appropriate the mathematical semiotic artifacts and, reciprocally, build personal meaning for mathematical content as well as fluency with the disciplinary procedures.

Radford's approach takes into account a vast arsenal of personal and interpersonal resources that students bring to bear in solving mathematical situations, including linguistic devices and mathematical tools. A key construct in the framework is *knowledge objectification*, which is defined as the process of making the objects of knowledge apparent (Radford, 2003). For example, a mathematics learner, in an attempt to convey a certain aspect of a concrete object, such as its shape or size, will make recourse to a variety of semiotic artifacts such as mathematical symbols and inscriptions, words, gestures, calculators, and so forth. In algebraic generalization activities involving geometric patterns, however, some of the objects of knowledge are general and therefore "cannot be fully exhibited in the concrete world" (Radford, 2008, p. 87). More broadly, knowledge objects in mathematics are not too cognitively accessible, because they do not exist in the world for empirical investigation (Duval, 2006), that is, these objects are never apparent to perception. Therefore, in order to instantiate (objectify) these ephemeral objects, students must resort instead to personally and culturally available forms such as linguistic, diagrammatic, symbolic, and substantive artifacts as well as the body, which Radford (2003, 2008) collectively terms *semiotic means of objectification* (see also Abrahamson, 2009a).

The power of the semiotic–cultural approach is that critical steps within individual learning trajectories can be explained by noting subtle shifts in the subjective function and status of the semiotic artifacts (Duval, 2006; Sfard, 2007). In particular, mathematics learning in the context of algebraic generalizations can be monitored as subjective transitions along a desired chain of signification, from *Factual*, to *Contextual*, to *Symbolic* modes of reasoning (J. F. Gutiérrez, 2010; Radford, 2010) (see *Figure 3*, below).[3]

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[3] This discussion of Radford's (2003) "F-C-S" framework is meant to be general; Chapter 4 presents an elaborate description of this framework that attends to specific strategies and tools that students bring to bear when dealing with algebraic-generalization tasks.

Factual	Contextual	Symbolic
• Objects and operations are bound to concrete level • E.g., Student utterance: "1 plus 2, 2 plus 3, 3 plus 4…"	• Objects and operations are abstracted and generalized • E.g., "The *figure* plus the *next figure*."	• Objects and operations are expressed through formal symbolism • E.g., "$x + (x+1)$"

Figure 3. "F-C-S" framework (applied to the "Toothpicks" problem; see Figure 1, above): mathematics learning is conceptualized as shifts in semiotic modes.

From this perspective, conceptual understanding is viewed as the capacity to flexibly shift across the three semiotic modes, which consequently requires that students assume agency in making these shifts so as to carefully and incrementally construct personal meaning for conventional semiotic artifacts (e.g., the variable "*x*"). Students' personal acts of generalization from one semiotic mode to the next mark *both* their conceptual understanding *and* their mathematical agency—that is, *agency is tantamount to conceptual understanding* (cf. Jo Boaler & Greeno, 2000). The analyses presented in this dissertation adopt this view of agency-as-conceptual-understanding[4] and explore its implications for students' emerging mathematical and social identities.

To operate in the symbolic mode is predicated on a tacit (if not explicit) alignment with the mainstream classroom discourse (Sfard, 2007). Many students may not experience tension due to shifts in discursive alignment, perhaps because their social identities remain intact and unthreatened by these public acts. However, for students whose mathematical understandings are not couched in the mainstream classroom discourse, these discursive shifts could threaten their social identities and loyalty to their communities, because they perceive the more "mathy" (symbolic) language as indexing the dominant cultural values and ideologies.[5]

Furthermore, returning to Radford's construct of a rupture, note that he describes it as largely a sensuous–cognitive phenomenon. What I identify here is a different kind of rupture that is under-researched, a rupture that is still sensuous yet affective in nature and, through discourse,

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[4] See Gutiérrez (2013a, 2013b) for detailed empirical examples of "agency-as-inference" in classroom practice.

[5] From a sociological perspective, there are mainly two competing interpretations of students' apparent "oppositional" behavior in schools serving historically under-served communities. One perspective is that students are unequivocally *rejecting* schooling practices *because* these practices represent dominant, oppressive cultural norms and values (e.g., see Giroux, 1983; Willis, 1977). Contrary to this perspective, Sánchez-Jankowski (2008) concludes that students' actions primarily *affirm* their own local culture, values, and knowledge and are not their effort to resist the conventional cultural norms of broader society. In this way, students could experience tension in adopting a formal mathematical register not because they seek to flatly reject all things representing broader society, but because it is not immediately clear whether and how the new register is relevant to or affirms their local culture and norms. Indeed, parallel educational research suggests that, given an opportunity, students would seek to affirm their cultural identity within the mathematics classroom (see "Cultural Modeling Framework," Lee, 2006).

becomes imbricated with sociopolitical narratives of power. In what follows, I extend Radford's model to account for issues of positional identity in the "suturing" of the rupture. In particular, I introduce the notion of *discursive subjectification* so as to complement Radford's theory of knowledge objectification.

Dialectical Relation Between Processes of Objectification and Subjectification

Radford's framework is undergirded by the thesis that students appropriate cultural forms as semiotic means of objectification. Similarly, Heyd-Metzuyanim and Sfard (2012) maintain that mathematics learning is the interplay between *mathematizing* (speaking about mathematical objects) and *subjectifying* (speaking about the participants of mathematical discourse). Most relevant to this dissertation is their classification of subjectifying utterances according to their level of "reification." Specifically, they define reification as follows:

> Reifying is the discursive activity of rendering status of an object to something
> that was not necessarily treated this way so far. In our case, this would mean
> reifying a person's actions (such as ''she never tidies up after herself'') into
> ''mental properties'' or ''mental entities'' (such as ''she is very disorganized'' or
> ''she has an attention problem'') thus attributing the person with certain
> permanent qualities. Such reification may be done by the person with regard to
> herself (1st person identifying) or with regard to others (2nd or 3rd person
> identifying, depending on the question of to whom the identifying stories are
> told). (p. 131)

From this perspective, a student's developing mathematical identity is conceptualized as a process of progressive reification, whereby a certain feature of a specific mathematical performance is reified as a permanent feature of that student's routine performance, and reified again as a permanent feature of the student (*Figure 4*).
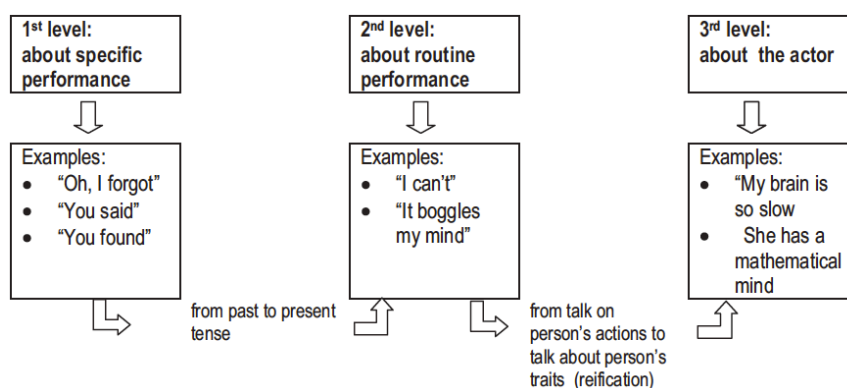
Figure 4. Levels of reifying utterances (adapted from Heyd-Metzuyanim & Sfard, 2012)

Furthermore, progressive reification is a particular sub-type of a more general discursive subjectification that occurs reciprocally with objectification. Specifically, I conceptualize knowledge objectification and discursive subjectification as "two sides of the same coin"—or, rather, as two interleaved cognitive–affective chains that give rise to the learning–power imbrication (*Figure 5*). I applied this synthesized model to observe (1) whether and how students

conditionally appropriate cultural artifacts (algebraic symbols and forms) as semiotic means of objectifying presymbolic knowledge, in relation to (2) reification acts that shape their identity.
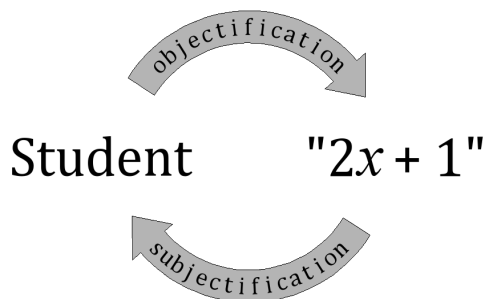
Figure 5. The dialectical relation between objectification and subjectification results in conditional appropriation of cultural artifacts.

Referring to Figure 4 (see on previous page), I have found that 3rd level reifying utterances are rare in data, that is, their spontaneous occurrence in practice is quite rare. The analyses presented in the forthcoming empirical chapters primarily captured subjectification at the 1st and 2nd levels during moments of social interaction. Whereas 3rd level reification can reveal students' "macro" identities (see Wood, 2013, for a recent discussion of "macro" versus "micro" identity in mathematics instruction), my analyses tend to focus on the development of emergent, socially constructed *positional identities*, which I explain next.

Positional Identities and Situated Agency in Mathematical Activity

As students make mathematical assertions during collaborative problem solving, I argue, their discursive action—that is, *situated agency*—creates hierarchical *positional identities* that must then, in turn, either be taken up, accepted, contested, negotiated, or rejected. The term "situated agency" refers to one's ability to make sense of, act upon, or rearrange their environment during goal-oriented activity (cf. "ingenuity," McDermott & Raley, 2011). I use the construct of "positional identity," which is based on the work of Marcy Wood (2013) on "micro-identity" that describes subject positions or "identities enacted in a moment in time" (p. 778). However, I find the construct of *positional* identity especially useful in describing a power dynamic, because the term already points to the mechanism by which hierarchizing occurs—positioning (Harré, 2008; Harré & Moghaddam, 2014). That is, I conceptualize positional identities as hierarchical, because co-constructed subject positions are differentially imbued with social status and authority. Furthermore, these hierarchical positional identities are associated with students' "locations" along the F-C-S trajectory.

I am now in a position to restate and elaborate my essential conjecture. *Figure 6* summarizes the crux of my theoretical argument, which began with Radford's theory of knowledge objectification.
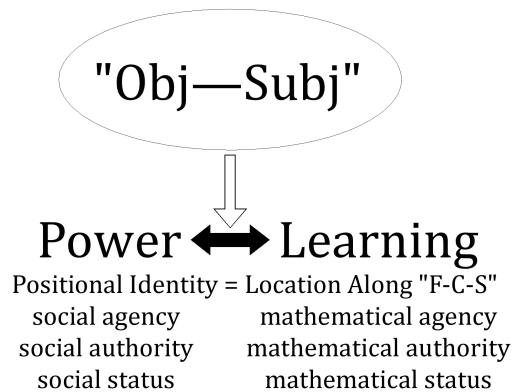
Figure 6. Theoretical conjecture: Students appropriate cultural artifacts and forms as semiotic means of objectification, and thus advance along the "F-C-S" chain of signification; so doing, they simultaneously appropriate positional identities that are differentially imbued with status and authority.

In the sections above, I explain the "obj" side of the dialectic, arguing that a student's personal acts of generalization from one semiotic mode to the next implicitly mark to other students both her conceptual understanding and her mathematical agency. That is, I conceptualize agency as tantamount to conceptual understanding. I further argue, adding "-subj" to the discussion, that as students move along the "F-C-S" chain of signification, they simultaneously appropriate hierarchical positional identities. Namely, operating in the Symbolic mode has higher status, perceived as capable of garnering more authority, than the Contextual and Factual modes.

I used this theoretical model to conduct a series of qualitative microgenetic analyses (Schoenfeld, Smith, & Arcavi, 1991) of the emergence of the learning–power imbrication during one particular instructional lesson centered on an algebraic-generalization activity.

Indeed, the enterprise of mathematics education and the phenomena of mathematics cognition and instruction are multivalent and immensely complex and not a single model can capture them entirely. That said, a critical conceptual analysis of predominant models of students' mathematical reasoning has revealed a close coupling of conceptual understanding and positional identity. Given the centrality of discourse in this process, and given that mathematical discourse, in particular, is inherently power-laden, I conclude that encounters with mathematical signs are necessarily encounters with signs of power. Thus I articulate the construct of a "power encounter," which refers to students' discursive interactions with semiotic systems that temporally and ontologically precede them and, moreover, are embedded in a nexus of asymmetric power relations that are socially and historically constructed. Specifically, *power encounters* with mathematics are salient episodes of classroom interaction where disciplinary discourse serves both explicitly to facilitate a particular learning process and tacitly to instantiate and perpetuate power relations—thus reifying the learning–power imbrication. This dissertation exposes the imbrication in a particular instructional context and identifies possible mechanisms for perturbing, manipulating, and thus favorably reconfiguring power relations inherent to classroom instruction.

Chapter 3: Orientations

The empirical studies presented in this dissertation are part of a larger project investigating both the challenges and opportunities to learn mathematics in classrooms serving students from historically marginalized public school populations, such as African American, Latino/a, and economically disadvantaged students. My broader goal in this larger project is to formulate an understanding of whether/how relations of power impact the quality of students' mathematical engagement and therefore learning. Along those lines, this dissertation utilizes theory and methods from various paradigms, including the learning sciences, critical educational studies, and post-structuralism.

My main strategy is to apply concepts and analytic techniques developed from within each paradigm in steps. To coordinate this work, I centered the analyses on a single vignette of data involving a group of three high school students engaged in a collaborative activity. First, I pursue a line of inquiry centered on the semiotic–cultural approach to the study of mathematical thinking and learning (Chapter 4). Next, I juxtapose aspects of that analysis and re-interpret some of the findings through the lens of the obj–subj dialectic (Chapters 5 & 6). I then apply a critical post-structuralist framework as a meta-lens on mathematics pedagogy and discuss future avenues for design-based research (Chapter 7) that systematically explores the hypotheses generated in this dissertation.

Each of these chapters constitutes a step toward explaining, supporting, and elaborating on my thesis that relations of power and mathematics learning become imbricated through discourse and are co-constitutive. The primary objective, then, of this Orientations chapter is to provide background information to prepare the reader for the data analyses reported in the empirical chapters that follow. This chapter consists of three sections: "Methodology" gives an overview of the general methodological approach and describes the research site, study participants, and data sources[6]; "Instructional Context" provides details of the research project wherein the focal vignette was collected, as well as a detailed narrative of the vignette. This narrative is a "pre-analysis" and is not intended to be comprehensive but rather provide crucial background information that characterizes this particular classroom community and situates the focal vignette within a larger context. Lastly, "Ethnographic Themes" very briefly describes some broader ideas that thread through the remaining chapters of this dissertation.

Methodology

This dissertation research project used a mixed-methods approach, where I combined participant observation with principles from design-based research. In particular, I conducted a three-phase data collection project, which went as follows. In Phase 1, I observed Mr. Lam's classroom over the course of 10 weeks and conducted ongoing in-depth, open-ended interviews with him and focal students. The purpose of this Phase 1 ethnography was to develop a profile of the teacher and students' norms and routine discourse practices. Based on findings from these observations and interviews, in Phase 2 I then worked closely with Mr. Lam to co-design an experimental instructional unit and implemented this unit in his class in one day, over two 85-min. class periods. The first component was a set of challenging tasks that combined geometric objects (called "Spiralaterals") with algebraic reasoning. Aside from these specially crafted

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[6] Detailed explanations of the specific analytic techniques used for Chapters 4–7 are explained separately in those chapters.

tasks, the second component was a novel meta-cognitive prompting strategy. Specifically, before launching students into mathematical activity, the teacher presented a "power" prompt intended to disrupt and favorably reconfigure the existing power dynamics. The prompt was presented in two formats, first as a journal assignment, and second as a whole-class discussion. Both formats were designed to elicit students' perceptions of who can and who cannot be successful with mathematics and to mitigate these status issues at the classroom level. This particular prompting strategy, the teacher and I hypothesized, would foster student empowerment to assume the epistemic agency necessary for developing deep conceptual understanding.

Following the experimental instructional unit, Phase 3 consisted of participant observational fieldwork, including task-based mathematical interviews with focal students. These follow-up interviews were framed as "tutoring" sessions covering the same math content and concepts from the intervention. Furthermore, students were given an opportunity to reflect on those classroom conversations that were spurred by the "power" prompt vis-à-vis the mathematical tasks. Ultimately, these interviews were aimed to help us understand better the challenges and opportunities that some students might have experienced during the design experiment in Phase 2.

In summary, the data generated via participant observation, classroom intervention, and interviews have provided a rich context in which to investigate the role of discourse in the students' concept learning in relation to social power dynamics. For this dissertation, I elected to focus on a 35-min. vignette involving Thalia, Xeni, and Ailani for several reasons. First, that particular class period resembled a typical "day in the life" of Mr. Lam's classroom community. That is, the norms and routines of that period were aligned with the overall profile of this classroom community, as documented in Phase 1 of the project. Other aspects of the instructional intervention were completely unfamiliar to the students, such as the power-prompts, and thus students' behaviors may not be representative of typical whole-class conversations; that said, those novel aspects of the intervention yielded interesting data on students' perceptions of race, math, and school, which later publications will take up. Secondly, the 35-min. vignette contains compelling interactions (by all participants) that clearly bring into relief the tension in the obj–subj dialectic, and thus serves as an exemplary case of the learning–power imbrication.

Research Site and Participants

The specific project wherein the data for this dissertation were collected was implemented to create an empirical context for examining issues of power and mathematics learning. The project, entitled *Critical Mathematics*, was implemented in a single mathematics class at César Chávez School for Restorative Justice (hereafter, "César Chávez"),[7] a small public high school located in a large, diverse urban district in northern California. At the time of the data collection, during the 2013-2014 school year, over 90% of the student body was Latino/a, African American, or recent immigrant students—all from the working-class and low-income neighborhoods surrounding the school.

My research project at César Chávez focused on Mr. Lam's classroom community, which was comprised of a mix of 9th- and 10th-Grade students. During his recruitment interview, Mr. Lam explained that all César Chávez teachers share a vision of educational equity and collectively try to foster a process of "restorative justice" through their daily practices. He reported that César Chávez is becoming increasingly well-known for its approach to social justice pedagogy. For instance, teachers from every department at César Chávez are asked

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[7] All names are pseudonyms.

regularly to speak at conferences and universities about their particular framework for restorative justice-teaching, providing both theoretical and practical resources for teachers to explore how to create "humanizing spaces" that may in turn lead to improved educational outcomes for historically marginalized students. Furthermore, César Chávez has also been featured in local newspapers as an exemplar of the emerging "small schools"[8] movement in northern California and across the US.[9]

Teachers

Aside from Mr. Lam, there were two other adult members of the classroom community that were directly involved in the project: Ms. Urrutia, and myself (hereafter identified in the main text as "Mr. Gutiérrez"; in the transcript students referred to me by my first name, "José"). I here describe each of the three teachers and their role in the project, including the implications of my researcher positionality in the project.

Mr. Lam. Mr. Lam was the regular classroom teacher. He was well-credentialed and experienced with (then) over 6 years teaching in the classroom. He held a National Board Certified Teacher credential and had led numerous workshops locally and nationally on his particular teaching methods. Mr. Lam was involved in all aspects of the project; at every step, there was opportunity for him to voice concern and share his enthusiasm and ideas about the research process.

Ms. Urrutia. Ms. Urrutia was a Master's student and graduate Fellow in a federally funded initiative that sought to (a) prepare beginning mathematicians to enter PhD programs, while at the same time (b) strengthen ties between communities, university researchers, and classroom teachers. As part of her fellowship, Ms. Urrutia worked part-time at César Chávez and her primary charge was to integrate herself into the classroom community as a permanent member; she was the "mathematician in the classroom." Her primary role was not to present new mathematical material, rather it was to work with individual or small groups of students by sitting with them during regular class sessions. Ms. Urrutia was in attendance the vast majority of days that I collected data for this project, including all the days of the instructional intervention that gave rise to the focal vignette.

Mr. Gutiérrez. I was the third adult/teacher member of the classroom community during the 2013-2014 school year. I was a doctoral candidate in my program (educational studies with an emphasis on math, science, and technology) and my role evolved over the course of the project. See "Researcher Positionality" section, below, for full explanation of how my participation shifted from Phase 1 to Phase 2, and again to Phase 3.

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[8] Due to space constraints, I do not provide a full treatise of the ongoing movement toward establishing smaller public schools in the US. But I can briefly mention that one of the main tenants underlying the movement is that public schools are overpopulated and smaller schools, "at their best," can be more effective along various dimensions, such as graduation and matriculation rates, attendance, improved instructional quality, improved teacher working conditions, and ties with community (see Adelman & Taylor, 2009).

[9] I cannot make any comment on the effectiveness of the school's vision and practices, and I am unaware of any formal evaluation that does so (see Limitations section, Chapter 7).

Outside of class, the three of us constantly discussed our students and ways to improve instruction, class culture, and overall morale.

Student Participants

The class consisted of 18 students with diverse academic and mathematical backgrounds. Their group demographics closely matched the demographics of the school. All 18 students agreed to be observed over the school year and to participate in the intervention; 7 students volunteered for interviews during Phase 1, but only 4 of them volunteered to be interviewed during Phase 3.

Students in Focal Vignette. The studies presented in this dissertation are all centered on the same 35-min. vignette involving three students: Thalia (Ta-lee-a), a 9th Grader; Ailani (Ay-lah-nee) was also in 9th Grade; and Xeni (Sheh-nee), a 10th Grader. All three got along well and generally respected and cared for one another; in particular, Thalia and Ailani considered themselves close friends and spent time together outside of school. Thalia and Ailani were involved in focal interviews during Phase 1 but not 3; Xeni was not interviewed.

Researcher Positionality

By spending over 215 hours (over 43 separate days: 33 in the Fall of 2013 plus 10 days in the Spring of 2014) as a participant of this classroom community, I strove to understand, on a practical level, how Mr. Lam enacted his pedagogical practice in a way that embodied his social justice ideals; from a theoretical perspective, I strove to understand the challenges and opportunities he and his students faced when dealing with centralized as well as decentralized forms of power.

The details of Phase 1, the ethnography, required an incremental integration into the classroom community. I began with observations only (from the back of the room, using notepad and pen), and occasionally asked students questions, as they worked in small groups. Over time, I began to join students in their small groups for brief periods of time, so that by the 11th week (the week of the intervention/focal vignette), the students had grown accustomed to me sitting with a particular group for extended periods of time, so that I could gradually increase the time spent with individual students or small-groups. During the focal vignette, for example, I had elected to sit with Team 1, the focal group (of this dissertation), based on Mr. Lam's input and my observations from the previous period. In the video footage, I can be seen working with the group and somewhat "blending in" with the students at their desk. This was all by design, in that it was my intent to shift my role gradually from a "fly on the wall" to a full, participating member of this classroom community.

From the fly on the wall perspective, I was able to document routine norms and practices that characterize this particular class (Lofland, Snow, Anderson, & Lofland, 2006). Gradually, as I expanded my role, I became able to participate during the intervention as a contributing teacher, asking and answering questions of students as they worked on the instructional unit that Mr. Lam and I had designed.

Briefly, the shifts include moving from observation and fieldnotes only and asking clarifying questions, to probing students mathematically during class time, offering one-to-one tutoring during lunch and after school for students who wanted to work with me (a few of them became my main informants through Phase 3), etc. Over time, I found myself experiencing tension between my goal of naturalistic observation (i.e., observing students without intervening) and acting in ways that go against my moral compass. From my spot in the corner of the room,

there were always two or three groups of students whose conversations I could make out most clearly. Sometimes their behaviors were disruptive to the learning process or their actions were creating an emotionally unsafe environment; when I witnessed such behaviors and stayed quiet, I was deeply concerned that I was inadvertently condoning them. After discussing these issues extensively with Mr. Lam and Ms. Urrutia over the course of several weeks, in time I felt comfortable with students and "a part of," so much so that I took up the typical role of an "authority" figure in the room and firmly, but warmly, talked with students when I felt that their behaviors were out of line with keeping a safe learning environment.

By the end of the school year, several students readily identified me as someone they could talk to, not only about math but about other issues as well. That said, I was sure to maintain appropriate boundaries with students, thus I tended to limit topics of conversation to issues related to family background and education, and I would discourage topics related to my personal life (e.g., they constantly asked if I had a girlfriend or if I were married; I would talk about my "job" as a graduate student researcher but not my income). At the end of the school year, a few students had extended personal invitations to me to be at their portfolio reviews.

I conducted interviews with focal students throughout the year. Some of these interviews involved math-related activities, others involved getting more background information about their personal experiences in school, with mathematics, or other outside issues such as details about their family history or their neighborhood.

One major theme that emerged as a result of my involvement in this project was that I felt a tension with regards to competing "agendas." Whereas there was always an overarching "instructional agenda" (to cover designated instructional terrain during all class periods), I was interested in how students interpreted the tasks and their perceptions of the mathematical situations presented to them. I was constantly negotiating the tension between on the one hand, enabling students to satisfy the requirements of a given task, versus providing opportunities for students to engage in more open-ended mathematical inquiry that simply took more time, on the other hand. This tension is not taken up directly in the studies presented in this dissertation, but will be examined in later publications that examine my involvement in the project more closely.[10]

Instructional Context

During the 2013-2014 school year, Mr. Lam's class was formally announced as a Geometry class, but it followed a non-traditional curriculum. Mr. Lam regularly introduced activities that live beyond the traditional content domain of school geometry, and his lesson plans often used tasks that combine algebraic strategies and tools (e.g., pattern-finding and generalization) with geometrical concepts; Mr. Lam viewed Spiralaterals as an exemplar of that kind of task. In the sections that follow, I provide a timeline of the different lessons involving Spiralaterals, followed by an overview of the focal vignette, including a description of the materials and facilitation guidelines.

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[10] In particular, future studies will examine my ideological orientations, the ways in which they were enacted in this educational setting (i.e., my "cognitive–conceptual scripts"; see Chapter 6), and their impact on the quality of student engagement/learning.

Overview of Spiralaterals and Lesson Plans

A Spiralateral is a geometrical object that is drawn on graph paper and is derived from a set of rules, see *Figure 1*. Frank Odds (1973) describes how to construct a basic {1, 2, 3} Spiralateral:

"First draw a segment of unit length to coincide with the edge of a graph square. Turn right through 90° and draw a segment two units long. Again turn right through 90° and draw a segment three units long. A basic pattern of 1-2-3 has now been established. Repeat the same steps again, continuing from the outer end of the three-unit segment. After [three] repetitions of the basic pattern, the segment will join the point at which the diagram started" (p. 121).
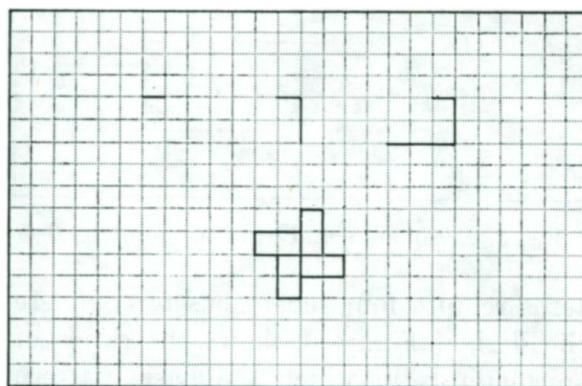

Figure 7. Steps in the construction of a "3-legged" Spiralateral (image taken from Odds, 1973).

Mr. Lam came across Spiralaterals at a teacher conference and decided to work on them in class. Ms. Urrutia and I also agreed that they "provide a medium for conjecture, hypothesis testing, and proof" (Schwandt, 1979) and all three of us mused on the fact that the three of us had decades of combined experience working in mathematics yet not one of us had come across Spiralaterals before.

Mr. Lam and his colleagues in the math department dedicated time to the topic and even made Spiralaterals an option for students' end-of-year portfolio assignment. Mr. Lam spent approximately 15 class periods over 7 days on Spiralaterals. The vignette was recorded on Day 5, during the second period of the day, see *Figure 8*.

Class Days with Spiralaterals	Class Info/Date	Description
Prior to the Intervention		
Day 1	Week 9, Weds, Period 1 (2013-11-20)	ML gives PowerPoint presentation to class introducing Spiralaterals, providing basic definition and examples; ML explains that Spiralaterals will be part of an ongoing portfolio assignment due at the end of the school year; students try them.
	Week 9, Weds, Period 2 (2013-11-20)	Students explore Spiralaterals; work in groups and create poster presentations.

Day 2	Week 9, Fri, Period 1 (2013-11-22)	Teacher presents activity intended to highlight more properties of certain 3-legged Spiralaterals, e.g., constructing Spiralaterals with repeated entries such as {1, 1, 1} and {2, 2, 2}.
	Week 9, Fri, Period 2 (2013-11-22)	Students continue their work on exploring patterns/properties of certain 3-legged Spiralaterals, such as permuting code entries, e.g., constructing {3, 5, 7}, {7, 3, 5}, {5, 7, 3}.
Day 3	Week 10, Mon, Period 1 (2013-11-25)	Students explore 3-legged Spiralaterals with "0" as an entry, e.g., {6, 0, 2}; continue group work/posters.
	Week 10, Mon, Period 2 (2013-11-25)	Students explore patterns of *n*-legged Spiralaterals, specifically whether they "close"; continue group work/posters.
	Week 10, Mon, (2013-11-25)	**First Design Meeting**: ML and JFG discuss problems and facilitation guidelines for the intervention (which is to take place the following week).
Day 4	Week 11, Mon, Period 1 (2013-12-02)	Students explore 3-legged Spiralaterals with different construction procedures, such as reversing the headings, e.g., constructing a {1, 2, 3} using the set of headings {up, right, down} and {down, left, up}.
	Week 11, Mon, Period 2 (2013-12-02)	Students practice "decoding" Spiralaterals, i.e., for a given Spiralateral diagram, what's the code to "build" it?

Instructional Intervention

Day 5	Week 11, Weds, Period 1, (2013-12-04)	Students continue practicing decoding 3-legged Spiralaterals; JFG presents "Around the Room" activity involving M1-M8 challenge questions.
	Week 11, Weds, Period 2, (2013-12-04)	Students practice decoding "families" of Spiralaterals and visually identifying patterns in the diagrams, e.g., "What generalizations could you make?" regarding the family: {2, 2, 5}, {4, 4, 7}, {6, 6, 9}; group work on single M-problem.

After the Intervention

Day 6	Week 12, Mon, Period 1 (2013-11-09)	Students given "write up" instructions for Spiralateral projects, but no in-class time allotted today; lunch-time opportunities to work with ML and JFG.
	Week 12, Mon, Period 2 (2013-11-09)	Based on consult with teacher, JFG works individually with Sione, a Grade 9 student, during regular class period, "practicing" Spiralateral constructions.
Day 7	Week 12, Weds, Period 1 (2013-12-11)	No in-class time allotted for Spiralateral work, but students were encouraged to come in during lunch and work with ML or JFG if they had questions about the write up assignment; Sione came in and worked with JFG.

| Week 12, Weds, Period 2, (2013-12-11) | Three Spiralateral questions featured in pre-test review— a game of math "Jeopardy." |

Figure 8. Overview of Spiralaterals lessons during the Fall 2013 semester.

For the instructional intervention (see "Day Five" of Figure 2, above), Mr. Lam and I co-designed a family of Spiralateral pattern-finding problems that required knowledge of their basic construction as well as algebraic strategies and tools. We created eight (14x18 inch) posters for the intervention. Each poster presented the first four figures of a Spiralateral sequence; the task objective was to express a "Code" governing the growth of each sequence, as a set of algebraic formulas in the form of $\{f_1(n), f_2(n), f_3(n)\}$ and whose inputs are the figures' ordinal positions. Furthermore, after we had settled on the different Spiralateral sequences, a careful analysis of each problem led to a ranking system, whereby each poster was assigned a unique challenge level, between "M1" and "M8." See *Figure 9* for examples; see Appendix A for all eight problems.
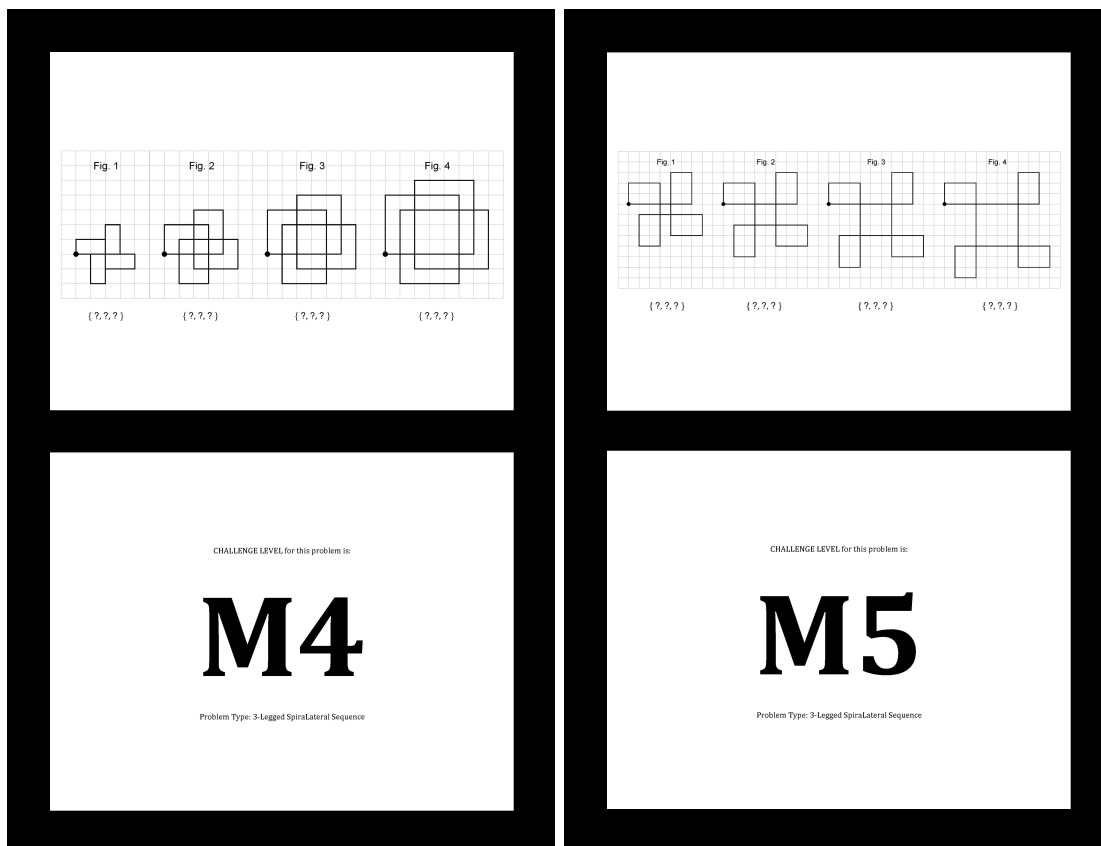

Figure 9. Posters presenting 3-legged Spiralateral sequence problems. The M-number indicates the relative challenge level of a problem, ranging from M1 to M8 (see Appendix A). M5 can be modeled as $\{2, 3, n+5\}$. See next chapter for full description of solution procedure for M5, which is central to the analyses presented in Chapters 4–7.

Overview of Focal Data Vignette

        Classroom Level. The lesson objectives were for each team to collaborate on an M-problem and to address two Team Task questions (see *Figure 10*). A key design feature for implementing the Spiralateral sequences was to substitute increasingly larger numbers (e.g., Fig. 10, Fig. 100[11]) as a way for students to realize that ultimately an arithmetic-recursive strategy is inefficient, thus motivating the need for more powerful tools and strategies such as algebraic generalizing and the use of direct formulas (see Chapter 4, Theoretical Perspectives section, for definitions of "arithmetic-recursive" versus "algebraic" generalizations).
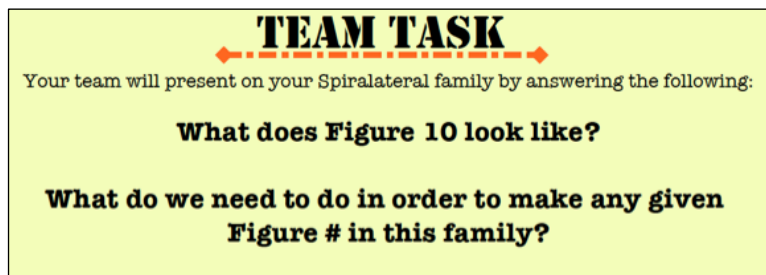
Figure 10. PowerPoint slide displayed on SmartBoard (located at the front of the classroom) with two activity prompts.

        The focal vignette is approximately 35 minutes and begins as Mr. Lam draws the class's attention to the PowerPoint slide, reminds students that the "M-" number on each poster represents its challenge level, and then distributes posters to the teams. Students loudly call out for certain M-problems as Mr. Lam walks around the classroom handing out posters, sometimes shouting across the room, "M1!" and "M6!" Team members negotiate which problem to work on, but the final decision goes to the student designated as the "team captain" for that day.

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[11] To avoid confusion, I will be using "*Figure*" to cite images in the text itself and "Fig." to cite particular enumerated diagrammatic elements in the pattern-generalization tasks.

Figure 11. An image of the classroom community at the start of the focal vignette. Mr. Lam is distributing posters.

Once all the teams received a poster, Mr. Lam announces that each team is expected to present on their problem to one of three teachers in the room (Mr. Lam, Mr. Gutiérrez, or Ms. Urrutia). The teachers walk around, visiting each group and discussing the activity. About 19 minutes into the activity, Mr. Lam addresses the class, asking if they want more time to work on the Team Task. Mr. Lam modifies one of the Team Task prompts, announcing to the class, "What if I gave you figure number X? Y? Z? N? Like, any figure number?" Responding to students' requests, Mr. Lam gives the class "a couple more minutes on this." This format continues for over 16 more minutes; the teachers continue to walk around and visit with different teams. During this final stretch, some students, including one of the focal students, Thalia, indicated that they were growing frustrated or bored with the activity because they had concluded they were "done with everything."

In the 35 minutes in which the vignette took place, Mr. Gutiérrez and Ms. Urrutia visited each group at least twice, and Mr. Lam visited only once with some groups as he was attending to other classroom management issues (a student refused to put her mobile phone away and then refused to "give it up" to Mr. Lam). Once the phone issue was resolved, Mr. Lam got everyone's attention, concluded the Team Task activity, and asked students to form a talking circle per the lesson plan.
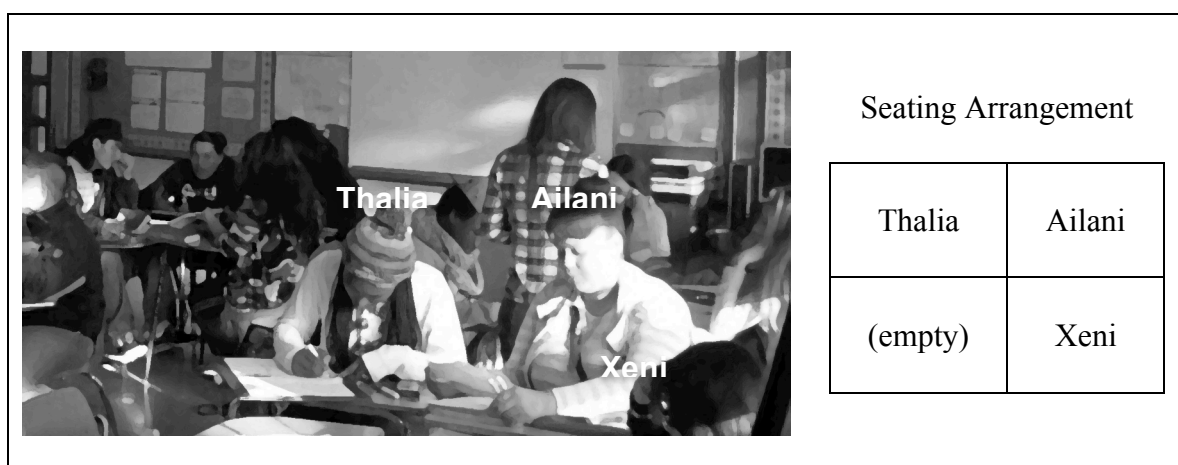
Overview of Team 1.

Figure 12. Focal group. Transcriptions, Tables, and *Figures* will refer to them as "Team 1."

    Table 1 presents a breakdown of Team 1's activity into thirteen sub-episodes. The episodes were determined at points when a participant exited or entered conversation with Team 1. Shaded sub-episodes are analyzed in Chapters 4–7; analyses of entire vignette to appear in future publications.

Table 1.
Breakdown of Vignette.

Episode	Time Stamp	Transcript Lines	Participants	Title (essential quotation)
1.	[00:00–03:35]	1–65	Team 1 w/ JFG & ML	Thalia: "I want a hard one."
2.	[03:35–07:05]	66–87	Team 1 w/ AU	Thalia: "OK we're done."
3.	[07:05–08:13]	88–119	Team 1 w/ ML	Thalia: "Fig. 10?!"
4.	[08:13–09:34]	120–130	Team 1	Thalia: "Fig. 10 will be nineteen!"
5.	[09:34–10:56]	131–162	Team 1 w/ ML	Ailani: "It just added a number!"
6.	[10:56–15:40]	163–182	Team 1 w/ AU	Ms. Urrutia: "Sooo which part is added?"
7.	[15:40–17:35]	183–236	Team 1 w/ JFG	Mr. Gutiérrez: "Convince a friend."
8.	[17:35–19:00]	237–250	Team 1	Thalia: "Did you just get what I just said?"
9.	[19:00–23:14]	251–292	Whole class	Sts.: "We found out the problem. We get 30 points."
10.	[23:14–25:18]	293–341	Team 1 w/ JFG	Mr. Gutiérrez: "He might call on you..."
11.	[25:18–29:54]	342–448	Team 1 w/ ML	Mr. Lam: "My name is not 'Bruh.'"
12.	[29:54–33:09]	449–457	Team 1	Thalia: "That's not how I got it."
13.	[33:09–35:09]	458–507	Team 1 w/ JFG	Mr. Gutiérrez: "It makes me so happy…"

Episode 1. Mr. Lam arrives at Team 1 to hand them a poster. Thalia expresses that she would like a challenging problem, and Ailani exclaims that she would like M8. Mr. Lam tells Ailani that M8 is not available, so she calls for M1 instead. Xeni does not provide any input on which problem to work on.

Thalia and Ailani argue over which M-problem to get. Mr. Lam tells Ailani that "It's your call" and she picks M5. Xeni makes a move to become involved in the activity, by leaning forward and visually tracking the materials and what was happening, but her behaviors also indicate that she perhaps feels slighted or offended that she is not considered a part of the decision-making process and thus begins to read her book.

Episode 2. Thalia "decodes" the first four Spiralaterals and begins to construct a table of values to track her work (see *Figure 9* in Chapter 4, below, for full explanation of "decoding" procedure, and see *Figure 10* for description of Thalia's table of values). Ms. Urrutia walks by Team 1 and asks Thalia which poster they received; Thalia remarks, "I already figured it out." Moments later Ailani turns and asks Thalia for the instructions, giving indication that she had not yet oriented herself to the Team Task. Thalia remarks that they need to find the codes for Figs. 5 and 6. Xeni continues to read from her book.

Episode 3. Mr. Lam's first visit to Team 1; he asks Xeni to put away her book and asks Thalia to put away her mobile device (she has her earbuds in, so presumably she is listening to music). Turning their attention to the task, Thalia is surprised when Mr. Lam asks about Fig. 10, claiming that she needs first to figure out the codes for Fig.'s 6–9 to be able to say something about Fig. 10. As Thalia and Mr. Lam are talking, Ailani leans in and visually tracks their action, indicating that she is following along with what they are discussing. Ailani reaches across the table to grab the poster from Thalia's desk just as Mr. Lam reaches for it too. Ailani clutches the poster with both hands and wrestles it away from Mr. Lam; he asks if they can both see it together but Ailani answers "no." Mr. Lam asks once again about Fig. 10 as he turns to leave the group.

During this episode, Xeni attempts to become involved in the activity, by putting away her book, making eye contact with Mr. Lam, and leaning in as others talk; yet as soon as Mr. Lam leaves, she turns her attention to other things and does not give any further indication that she is invested in the task until past Minute 15 of the vignette (Episode 7).

Episode 4. As Mr. Lam walks away, Ailani holds the poster in front of her face and begins counting aloud, using her fingers to help with the calculation. She announces her solution for Fig. 10 to Mr. Lam, who is standing nearby, but he does not respond initially. There is a long pause and, getting no feedback or input from her group, she writes something down and says aloud to no one in particular, "nineteen." Thalia indicates something on the table of values with her finger, as she asserts that Fig. 10 will be "Fifteen." Ailani adamantly claims that the solution for this given figure is nineteen. Thalia, too, is adamant about the accuracy of her solution. Ailani finally admits that she may have made a counting error. (See Chapter 5, below, for richer description and deeper analysis of Episode 4.)

Episode 5. Mr. Lam returns to Team 1's table, and Ailani and Thalia once again assert their respective solutions regarding Fig. 10. As they argue, Mr. Lam turns his attention to Xeni and asks her questions that would steer her toward acting in relation to the assigned task. Ailani

and Thalia reach an impasse, yet their individual learning trajectories nevertheless indicate forward progress. Ailani articulates a general arithmetic strategy, while Thalia shows that her solution holds for several cases. Mr. Lam does not necessarily act as an arbiter in their argument; rather he challenges Team 1 to come up with a strategy for finding the solution "for any number?" As Mr. Lam walks away, Thalia indicates that she disagrees with his assessment that her solution is insufficient, scoffing "You just keep adding the number, duhhh!" (See Chapter 5, below, for richer description and deeper analysis of Episode 4.)

Episode 6. Thalia sits idly for about a minute then begins to shift around at her desk. She gathers her worksheets, stacks them neatly, and holds them in front of her; she gazes right at the first page, and after a moment's pause she places them down and reaches into her bag and takes out her earphones but does not put them in. She turns to Ailani, and they briefly talk about something not related to the task (musical preferences). Ms. Urrutia approaches Team 1 and sits at the empty desk. Thalia immediately announces that she figured out Fig. 10, explaining that she "adds one to the end of the numbers" and shows that her method holds true for several cases. Ms. Urrutia asks for clarification but does not probe too deeply into Thalia's strategy. After Ms. Urrutia leaves Team 1, Thalia looks around and calls for Mr. Gutiérrez. Xeni continues to read from her book, and Ailani draws something not related to the task.

Episode 7. Mr. Gutiérrez visits Team 1 and inquires about their activity with M5. Thalia immediately remarks that they have already figured out the code for Fig. 10, but Mr. Gutiérrez responds that he is not yet interested in Fig. 10 and asks about Fig. 5. While Thalia explains the sequence, Mr. Gutiérrez turns his attention to Ailani and asks her a question regarding the table of values that Thalia is referring to. Thalia interjects, insisting that, "She [Ailani] didn't do that. It's because I was thinking on my own." Mr. Gutiérrez evokes the frame of "convince a friend" as a means of encouraging Thalia and Ailani to work together, directly instructing Thalia: "I want you to convince Ailani that this [*indicates table of values*] is what's happening here [*indicates poster*]." Both Thalia and Ailani give indication that they "buy in" to the new frame that he proposes. Mr. Gutiérrez makes one final move to encourage Xeni to participate in the conversation, but it does not appear she is listening.

Episode 8. Thalia begins explaining her solution to Ailani, as a response to Mr. Gutiérrez's goading to "convince a friend." Thalia gathers the M5 poster, her table of values, other sheets of paper, and a pencil, and explains the decoding procedure to Ailani. Thalia gestures with her fingers, uses her pencil to indicate certain features of the Spiralateral sequence, and verbalizes her solution procedure aloud. Ailani initially follows along with what Thalia is explaining, but her gaze soon shifts to other things, suggesting that she is no longer following along or listening. Ailani admits that she is "still not convinced!" and they both share a laugh. (See Chapter 5, below, for richer description and deeper analysis of Episode 8.)

Episode 9. Thalia and Ailani chit-chat for a brief moment, while Xeni continues to draw. Thalia sits quietly for a minute, occasionally looking around the room; Ailani sits quietly as well, sometimes singing to herself.

Mr. Lam addresses the class, asking if each team has answered the Team Task questions. Multiple students call out "Yes!" Mr. Lam challenges his students to consider their Spiralateral sequence more generally, stating: "The question around what—about any figure number in your

family, that's still an open question. Because I don't think any group has really been able to explain it—except for Team 2. Team 2 had a pretty good uhm way of thinking about—the question that I asked them was like, what if I gave you figure number X, Y, Z, N? Like any figure number?" Students from Team 6 shout out: "We just did number n!" "We did that!" "We found out the problem! We get thirty points!" Mr. Lam does not respond to Team 6 directly; instead he instructs the whole class to take time to make sense of how diagrammatic elements of a Spiralateral sequence map onto its Code: "So on top of figuring out the code, the other question was like how would the picture look like, right? So yes you could figure out the code based off of the pattern in the code, but can you make sense of the figure number and the picture?"

In the meanwhile at Team 1, Thalia and Ailani look at Xeni's drawing and comment on it. Xeni offers to make a drawing of each of their names using color markers. Ailani grabs color markers and starts drawing too. Thalia looks around the room; she spots Mr. Gutiérrez and asks him to come over to her table.

Episode 10. As Mr. Gutiérrez approaches Team 1, Thalia announces that they have completed the Team Task: "Because I'm—we're done with this [*indicates table of values*]." Mr. Gutiérrez asks Thalia if she has convinced both Ailani and Xeni of the solution for Fig. 10. Thalia confirms she has convinced them. Mr. Gutiérrez asks Ailani a question but she keeps her gaze down and does not give any indication that she acknowledges him. Mr. Gutiérrez turns to ask Xeni a question about Fig. 10, and she too keeps her gaze down at her drawing. Mr. Gutiérrez tells Xeni that "[Mr. Lam] might call on you," to which she responds, "I don't give a damn because I know it." Thalia speaks up for Xeni, adding: "She does know it! She knows.. you have to add one to the end of the numbers." Mr. Gutiérrez asks Xeni to hold up the M5 poster so that everybody can see it. Xeni complies by grabbing the poster with her left hand, but she keeps her gaze down and continues drawing with her other hand. Looking at the poster, Mr. Gutiérrez uses a metaphor to frame an inquiry about M5; he describes the sequence of Spiralateral figures as an actual "family," and he asks: "How are these members of this family related to one another?" Thalia and Ailani immediately try to answer his question, but Mr. Gutiérrez does not respond; instead, he walks away from the group but not before prompting them one last time to look for "fundamental" properties that link all the members of the M5 Spiralateral family to one another. As Mr. Gutiérrez walks away, Thalia, Xeni, and Ailani very briefly talk about Xeni's drawing. Thalia sits idly for a very brief moment, then announces to no one in particular: "I'm done. We figured out Fig. 10." Xeni finishes her drawing and hands it to Thalia. Thalia thanks her then glances over her shoulder and calls out for Mr. Lam.

Episode 11. Mr. Lam arrives at Team 1 and is met by Thalia with a question: "Are we done?" Mr. Lam confirms they are done and that they should report their results to Mr. Gutiérrez. Mr. Lam turns to Xeni and kneels down next to her to talk. He reminds both Ailani and Xeni that the task at hand involves working on M5, not drawing pictures. Thalia defends her team, exclaiming, "We just did it, look it! We just did that." Mr. Lam does not respond to Thalia and instead asks Xeni what she noticed about the Spiralateral pattern. Working together, Mr. Lam, Thalia, and Xeni co-construct a Code for M5. Xeni articulates her version of the final solution using formal symbolism, stating that the last part of the Code is "n plus one." While Mr. Lam tries to unpack what Xeni means by "n," Thalia interjects and gives indication that she is growing frustrated with the conversation and is focused on task completion…which brings us to the opening scenario at the very top of this dissertation.

The solution that Team 1 has articulated, for items beyond Fig. 4, achieves the goal using a recursive technique that relies on known items along the sequence. Mr. Lam prompts the group to consider much larger numbers, such as Fig. 100, asking, "So we have to know the one before it [Fig. 100]? Is there a way that we don't have to know the one before it?" After a 2-second pause, Thalia stomps her feet and exclaims to Mr. Lam, "Bruh, why you asking all these questions?" Ailani stirs the conflict, interjecting loudly, "YES!" Mr. Lam calmly responds to Thalia: "My name is not 'Bruh.' My name is Mr. Lam, and I'm challenging you. That's why I'm asking you these questions, because I want you to get smarter." Thalia buries her face behind her hands and starts laughing sheepishly; her face still hidden behind her hands, she continues: "OK Mr. Lam, why you asking (all these questions)."

Mr. Lam continues the mathematical conversation and challenge the group with "So could you even figure out Fig. 100 right now?" adding the parameter of "I don't tell you what Fig. 99 looks like, can you tell me Fig. 100?" Thalia explains her method for Fig.'s 1–10, but Mr. Lam insists that her method is insufficient for items much further along the sequence. Ailani leans back in her chair, throws her arms behind her, looks to Mr. Lam and leans forward as she exclaims: "Well maybe—OK so (like) two three and you don't have to know all of them, you know the first—you can know the original one and then! And then figure out Fig. 99." As she speaks, Ailani pounds the table with the marker in her hand and uses a tone of voice suggesting to Mr. Lam that she aims to "settle the score" once and for all in this confrontation. Thalia interrupts Ailani, softly clapping her hands together as she speaks, accentuating each word as she talks: "Why you talking so ratchet?" Mr. Lam weighs in with: "Don't get hyped up against me." As Mr. Lam is speaking, Thalia points to him and says: "Ratchet. Say it, don't get ratchet." Mr. Lam does not respond to Thalia's comment and instead steers the conversation back to the mathematics at hand, asking about Fig. *n*. The conversation continues for a few more minutes, until finally Xeni arrives at a complete solution of "n plus five." (See Chapter 6, below, for richer description and deeper analysis of Episode 11.)

Episode 12.[12] Xeni explains to Thalia her final solution, and Thalia summarizes their work and announces that they are done with the task: "Oh so like if you said Fig. 10 equals fifteen because it'll be plus five? Ohhhhh OK that's why you said fifteen. Oh OK I get you. OK you've convinced me. We're done." The three of them talk about something not related to the task; while they talk, Thalia constantly looks over her shoulder in the direction of Mr. Gutiérrez and eventually catches his attention and calls him over.

Episode 13. When Mr. Gutiérrez arrives at Team 1, Thalia announces, "We're done with everything." He picks up the poster and holds it out in front of him so that it is visible to everybody (he made a similar action in Episode 10, above), and asks: "What is the most important or most interesting thing about the family of M5?" Ailani looks up from her notebook, glances at Mr. Gutiérrez, then at the poster, then returns her gaze to her notebook. Xeni and Thalia talk over each other, trying to answer his question. Thalia starts but then gives Xeni the turn. Xeni explains that there is a general code for each figure and articulates a complete solution

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[12] The footage of this episode contains inaudible portions that last several seconds or more, due to increased noise in the class and the fact that Xeni's talk is directed away from the camera. But occasionally students' utterances can be heard clearly and the transcription of Episode 11 reflects these missing turns of talk. I analyze only those utterances that are clearly audible.

of the form {2, 3, n+5}. Mr. Gutiérrez asks questions to determine if their strategy holds for known cases, such as Fig. 4, Fig. 5, and Fig. 6. As Mr. Gutiérrez, Thalia, and Xeni are talking, Mr. Lam addresses the class and wraps up the Team Task. Mr. Lam calls for a Talking Circle and the students begin to reconfigure their desks into a circle. The focal data vignette ends.

Ethnographic Themes

In this final part of this Orientations chapter, I very briefly describe emerging themes that I have discerned from the larger corpus of ethnographic data. So doing, I aim to characterize this particular classroom community and situate the forthcoming analyses of the focal vignette within a larger context. Furthermore, by providing this information, the reader may get a sense of Mr. Lam's agenda, frames, and ideological orientation, which are relevant to the analyses presented in this dissertation (particularly Chapter 6, where I discuss the notion of "scripts").

Social–Mathematical Antagonism

The interactions in the vignette represent relations of power that manifest and express themselves in the form of what I call *socio-mathematical antagonism* among teachers and students. This antagonism is not necessarily a negative or repressive phenomenon, but rather takes on friendly and productive qualities at times. It is observable throughout all 13 episodes and, according to the ethnographic data, is a regular feature of the culture of this classroom community. That said, I submit, social-mathematical antagonism is not unique to this classroom. I have observed both productive and unproductive versions of this antagonism in other classroom settings (e.g., J. F. Gutiérrez, 2013a) but I have not conceptualized these phenomena until now. The following chapters present deeper analyses of certain antagonistic moments and their impact on Thalia, Ailani, and Xeni's mathematical reasoning, agency, and identity.

Restorative Justice through Mathematics Pedagogy

My larger project at César Chávez documents the ways in which Mr. Lam attempts to foster a process of restorative justice through his daily teaching practice. Mr. Lam's pedagogical approach consists of tools and tactics that attempt to make explicit connections between mathematical practices and issues related to social justice and liberation from oppression. His agenda, frames, and ideological orientation are best characterized with a single metaphorical "equation" that he repeatedly presented to his class and which was repeated to me during Phase 1 interviews: "Math=Thinking=Liberation." This equation was taken up in journaling assignments, small group activities, and whole-class discussions; students were encouraged to critically reflect on the meaning of this equation, its implications, and its relevance to their lives. This dissertation does not directly take up an analysis of these activities. Rather, my intent is to show (in Chapter 6) that Mr. Lam's "equation" played a role in his moment-to-moment decision-making, and that his actions had an impact on the quality of the students' reasoning and agency.

The High Status of Mathematical Generalizing in Symbolic Form

Mr. Lam believes that "generalizing" is an exemplar of mathematical reasoning or thought-process. Although his math course is formally announced as a Geometry class, Mr. Lam provides opportunities for students to engage in algebraic generalization activities. He expressed during his recruitment interview that he considered "generalizing" as one of the hallmarks of algebra readiness. Additionally, during Week 8 of Phase 1, he commented that if a new student proclaims that she/he is good at algebra or that she/he does not need to retake an algebra course, Mr. Lam would assess them with pattern-generalizing problems. "Generalizing is more

important than presenting advanced topics [in school algebra]," he stated, and that it was perhaps more important than other content domains (such as Geometry). It appears that Mr. Lam places a premium on generalizing as a practice, and he strives to support and foster this capacity. In particular, Mr. Lam associates mathematical generalization with the ability to "see," "critique," and "understanding systems," which is a core capacity that he seeks to foster in his students as part of his vision for social justice pedagogy. That said, Mr. Lam's view of what he considers authentic generalizing emphasizes "abstractions" that involve "letters." That is, in his instruction Mr. Lam at times conflates the cognitive action of generalizing with using symbols. This makes it possible to overlook other authentic forms of generalizing that do not rely on reasoning with or manipulating alpha-numeric symbols. His particular view on generalizing was enacted during the focal vignette.

Chapter 4: Semiotic–Cultural Analysis of Mathematics Participation and Learning

This chapter reports on a semiotic–cultural analysis of the focal data vignette. Radford's theory of knowledge objectification constitutes one side of the obj–subj dialectic, which is the emphasis of this chapter. In particular, I apply his Factual-Contextual-Symbolic ("F-C-S") framework (Radford, 2003, 2010) to analyze the quality of learning underlying Thalia, Ailani, and Xeni's discursive productions, which will become most relevant in later chapters when discussing the impact of relations of power on individual learning. Therefore, the aims of this chapter are to conduct a basic F-C-S analysis and provide a synoptic view of the mathematical milestones reached by the students in the focal vignette.

The findings from this first empirical study reveal significant differences between the three students with respect to their (1) participation within each of the F-C-S semiotic modes, and (2) final generalizations. Furthermore, students' reasoning *vacillated* between the F-C-S modes. I conclude that individual differences observed through the lens of objectification *in situ* resulted in a social-mathematical status hierarchy, which is closely examined in the remaining chapters of this dissertation.

A Semiotic–Cultural Perspective on Spiralateral Geometry

The work of mathematics-education scholar Luis Radford draws from Lev Vygotsky's (1962/1962) cultural-historical psychology as well as Edmund Husserl's (1958) phenomenology to explain the emergence of students' algebraic reasoning. For example, Vygotsky highlights the indexical nature of children's pre-linguistic activity, including the use of pointing gestures to draw others' attention to objects in the their environment (i.e., the use of a "protolanguage"), whereas Husserl, in like vein, suggests that conceptual content is intrinsically rooted in former sensory experience (Husserl, 1958). Integrating these two perspectives, Radford focuses on the roles that the human body, discourse, and signs play in students' spontaneous communication acts within collaborative problem solving; broadly, he explicates mathematical reasoning as a *praxis cogitans* of meaning making—a praxis that is inextricably dependent upon and generating of semiotic artifacts (Radford, 2006, p. 5). Mathematics learning is thus a mediated, distributed, and dynamically reciprocal process in which students' emerging presymbolic knowledge is iteratively objectified in progressively sophisticated forms of historically and culturally endowed semiotic systems. One such semiotic system—which is prevalent in the mainstream of school mathematics programs—is algebra. Viewed from the semiotic-cultural perspective, "algebraic thinking is a particular form of reflecting mathematically" (Radford, 2003) [p. ?], that is, algebra is about "using signs to think in a distinctive way" (Radford, 2008, p. 87).

To illuminate the content and process of this distinctive way of thinking characteristic of algebra, Radford elaborates a theoretical framework for describing and analyzing students' reasoning during collaborative engagement with pattern-generalization algebraic activities. Mr. Lam and I adapted Radford's framework to create the family of Spiralateral problems used for this dissertation project (see Chapter 3 – Orientations, for detailed description of the instructional intervention). I now use the M5 Spiralateral sequence (*Figure 7*, below) as a context to elaborate on Radford's framework, which is central to my data analysis.

A key construct in Radford's framework is *knowledge objectification*, which is defined as the process of making the objects of knowledge apparent (Radford, 2003). For example, a

mathematics learner, in an attempt to convey a certain aspect of a concrete object, such as its shape or size, will make recourse to a variety of semiotic artifacts such as mathematical symbols and inscriptions, words, gestures, calculators, and so forth. In patterning activities, however, some of the objects of knowledge are general and therefore "cannot be fully exhibited in the concrete world" (Radford, 2008, p. 87). More broadly, knowledge objects in mathematics are not too cognitively accessible, because they do not exist in the world for empirical investigation (Duval, 2006), that is, these objects are never apparent to perception. Therefore, in order to instantiate (objectify) these ephemeral objects, students must resort instead to personally and culturally available forms such as linguistic, diagrammatic, symbolic, and substantive artifacts as well as the body, which Radford (2003, 2008) collectively terms *semiotic means of objectification* (see also Abrahamson, 2009).
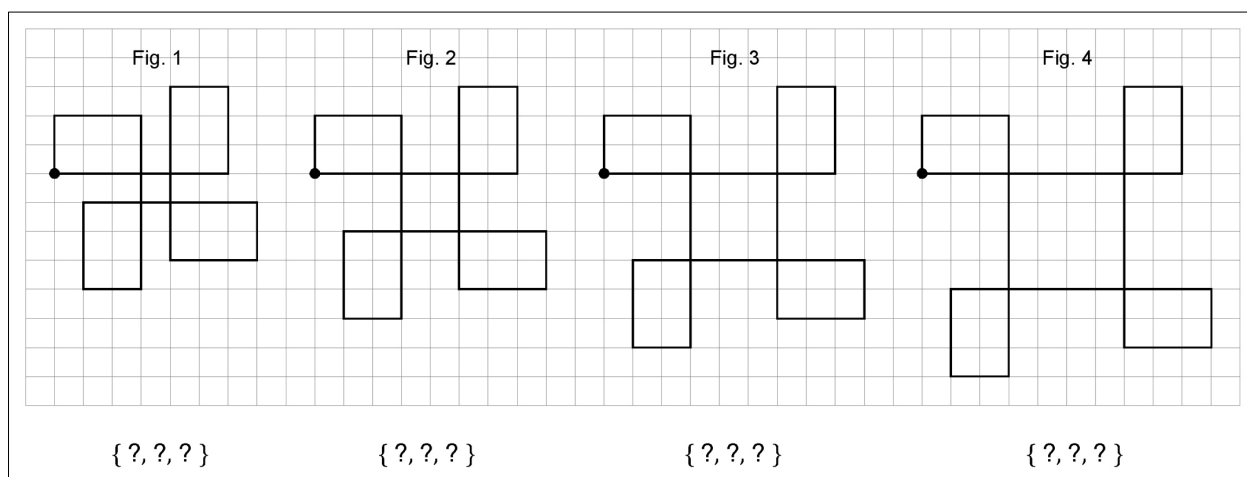

Figure 13. M5, a 3-legged Spiralateral problem. For each figure in the family, students determine its "code" (lowercase "c"), the algorithm consisting of three numerical entries that determine the respective lengths of the legs, which in turn determine the overall size and shape of each figure.
   Ultimately, the task objective is to express a "Code" (capital "C"), the set of generalizations $\{f_1(n), f_2(n), f_3(n)\}$ that determines the code for any Spiralateral figure along the sequence given its ordinal position n. For example, the code for "Fig. 1" is {2, 3, 6}, "Fig. 2" is {2, 3, 7}, "Fig. 3" is {2, 3, 8}, "Fig. 4" is {2, 3, 9}, etc., so that "Fig. n" is {2, 3, $n+5$}. (See also *Figure 9*, below, for a detailed description of the Spiralateral "decoding" procedure applied to Fig. 1.)

        As they attempt to construct generalizations from an initial set of numeric and/or figural cues such as those in the M5 Spiralateral problem, students often resort to using one of two strategies (Radford, 2008). The following elaborates on these strategies (using examples from the empirical data discussed later in this chapter), and *Figure 14* will offer a summary of this somewhat complex structure.
   1. The first, *naïve induction*, is characterized by a process of "guess and check." For example, a student might propose a simple rule for $f_3(n)$, such as "the figure-number plus one" and check its validity on a few cases. Finding that the proposed rule does not satisfy one or more cases, the student might then propose an entirely different rule or modify her previous rule, for example, "the figure-number plus five" and check its validity as well, and so on.

2. The second and more sophisticated strategy, *generalizing*, is expressed as an active search for recurrent relationship structures and/or patterned commonalities within or between constituent elements in the problem space (Radford, 2008). This latter strategy, in turn, typically produces two types of generalization, *arithmetic* and/or *algebraic*.

   2.1. Arithmetic generalizations typically take the form of a recursive solution, which only *in*directly expresses any term in the sequence. In the M5 Spiralateral sequence, for example, students readily notice that the size of the middle square of each consecutive figure increases by one unit, while the size of the "ears" remain the same across all figures, as 2x3 rectangles. In turn, a student might map these aspects onto the codes, noticing that the first and second code entries are constant, while the third code entry always increases by a factor of 1 with respect to the previous figure, an observation expressible as the arithmetic generalization $\{f_1(n), f_2(n), f_3(n)\} = \{2, 3, f_3(n-1)+1\}$. The commensurate *explicit solution* $\{f_1(n), f_2(n), f_3(n)\} = \{2, 3, n+5\}$, however, is more powerful (see below), because it determines directly any item along the infinite sequence (Radford, 2008).

   2.2. Making the leap from recursive to explicit solutions involves generalizing a pattern *algebraically*, which requires: grasping a commonality C of some particulars in a sequence (say p_1, p_2,...p_k); generalizing C to subsequent terms of the sequence (p_{k+1}, p_{k+2}, p_{k+3}...); transforming C into a hypothesis; and using this hypothesis to *directly* express any term in the projected sequence (Radford, 2006, 2008). Now, the preferred strategy for accomplishing this cognitive work is not naïve induction (i.e., to formulate an explicit solution through a process of guess and check). Rather, the preferred strategy of generalization would be based on an observation regarding the size of each figure and using this observation as a *warrant* to explicitly *deduce* the length of leg-3 for the nth figure (see also the "architecture of algebraic pattern generalization" Radford, 2008). Specifically, a student needs to observe that the figure-number is the side length of the square in a given figure (e.g., Fig. 4 contains a 4x4 square); one must add this segment to the total length of leg-3 in that figure—the "n" of "$n+5$". Furthermore, in addition to the side length of the middle square, note that leg-3 includes two other segments of length 2 and length 3, respectively, which thus requires the addendum of a total of 5 units—the "+5" of "$n+5$." If and when this cognitive work is accomplished, the student would be able to directly express any figure along the sequence—thus generalizing the pattern *algebraically* and formulating a closed-explicit solution.

      2.2.1. Finally, Radford's framework also distinguishes among three types of algebraic generalizations in accordance with their level of generality. A *Factual generalization* applies to objects, including signs viewed as concrete entities (e.g., lines and numerals in a diagram), as well as to arithmetic operations bound to these objects (e.g., utterances such as "2 plus 1 plus 3, 2 plus 2 plus 3, 2 plus 3 plus 3").

      2.2.2. A *Contextual generalization* generalizes objects as well as the actions upon the objects—it is a scheme that acts on abstract yet contextually situated objects. Thus, particular figures that are farther along the M5 Spiralateral sequence and are not necessarily perceptually available

concrete entities (e.g., "Fig. 25" and "Fig. 26") are replaced with generic expressions (such as "the figure" or "the next figure").

2.2.3. Lastly, *Symbolic generalizations* are expressed using formal algebraic notation, for example, the expression "the figure plus five" is reformulated as "$n+5$." (See *Figure 8*, below, for a summary as well as more examples that anticipate the empirical data discussed later in this paper).

Solution Strategies	Naïve Induction Characterized by a process of "guess and check" E.g., "It's '$n+1$'… No. Try '$n+2$'… No. … Try '$n+5$'… YES!"	Generalizing Active search for recurrent relationships/commonalities among constituent elements in the problem space; typically produces two types of generalizations, *arithmetic-recursive* and/or *closed-explicit*.		
Generalization Types		Arithmetic Generalization Recursive ($f_i(n)$ is expressed in terms of the previous figure) E.g., $f_i(n) = f_i(n-1)+b$.	Algebraic Generalization Explicit formula ($f_i(n)$ is expressed in terms of the figure's ordinal position) E.g., $f_i(n) = n+5$	
Levels of Generality		Factual Objects and operations are bound to concrete level E.g., "2 plus 4 plus 3, 2 plus 5 plus 3, 2 plus 6 plus 3…"	Contextual Objects and operations are abstracted and generalized E.g., "2 plus the figure plus 3" E.g., "the figure plus 5"	Symbolic Objects and operations are expressed through formal symbolism E.g., "2+n+3" E.g., "$n+5$"

Figure 14. Semiotic-cultural taxonomy of algebraic pattern-finding solution processes (adapted from Radford, 2006, to the M5 Spiralateral sequence problem).

A Qualification of Radford's Framework

The new study presented in this chapter stems from my previous research studies that apply the semiotic-cultural framework to classroom data. I realized that inherent to Radford's framework were what might be called three *semiotic modes* of action, in which students might participate during collaborative engagement with patterning tasks. These modes are linguistic but not psychological, that is, by 'mode' I am referring to the surface features of student utterances, temporarily putting aside the question of semiotic grounding. A student can be operating in any of the three F-C-S modes during authentic generalizing acts or while engaged in naïve induction (see examples in *Figure 14*, above).

Moreover, the results from one particular study (J. F. Gutiérrez, 2010) suggest that students' respective patterns of participation *within* each of these semiotic modes can be ostensibly similar, thus giving teachers the impression that all students shared similar learning experiences during collaborative activity. Thus, I proposed the distinction that differentiates two meanings of "generalization": the *process* of generalizing (i.e., the way that students engage with the problem), and the *product* that results from this process. This distinction is important, because researchers and/or teachers are liable to attribute understanding to students even when such attribution is not necessarily warranted. In particular, a student's utterance may be interpreted as marking the *production* of a semiotic generalization, when in fact the student is

only appropriating and elaborating on the product of another student's generalization process, still within the same semiotic mode.

Furthermore, the proposed process-vs.-product distinction led me to articulate a certain pedagogical principle, that generalizing activity is not merely to create opportunities for students to operate *in* each of the symbolic modes, but rather to support them as they shift *along* the desired chain of signification from the Factual through to the Symbolic mode—making sure that each and all students perform the cognitive work necessary for grounding the target signs. Lastly, the F-C-S framework, in its current formulation, might suggest that students monotonically progress from one mode to the next, yet previous findings (Gutiérrez, 2010, 2013) indicate this is not necessarily the case.

To help clarify these issues, I examined the resources that students use in *navigating* the various semiotic spaces inherent to situated problem solving in an ecologically valid instructional context. In particular, this study asks:

- What mathematical strategies and tools do the students bring to bear during the pattern-finding activity?
- How do students navigate the F-C-S levels of generality from beginning to end of a collaborative pattern-finding activity?

Next I describe the specific methods used to address these questions.

F-C-S Methods

Empirical Context. This chapter presents the analysis of three students' situated reasoning during one particular pattern-finding problem involving the M5 Spiralateral problem. Data consist of a 35 minute span of classroom video. See Chapter 3, for more details about the participants in this study.

Analysis Techniques. This chapter focuses on the level of generality of students' multimodal discursive utterances. The interactions were analyzed in accord with techniques developed previously (J. F. Gutiérrez, 2010, 2013a); operationalization of the F-C-S modes are carefully detailed in these previous publications. Here I summarize the procedures briefly, as following.

I produced and analyzed a transcription of students' verbal, gestural, and other semiotic actions as they referred to mathematical objects and attempted to construct generalizations. All student utterances were coded for their semiotic mode (Factual, Contextual, Symbolic) and, in particular, I evaluated whether or not each utterance reflected a generalization *to* a particular mode or merely a reiteration *in* the mode.

Before I present the findings, it should be helpful to first present an overview of Thalia, Ailani, and Xeni's mathematical activity during the vignette. With this brief overview, it is my intention to prepare the reader for quantitative analysis of the data that will be reported upon in the Results section that follows as well as the qualitative analysis presented in the remaining Chapters of this dissertation.

Overview of Student Mathematical Activity

Thalia's Participation and Milestones. At the onset of the activity, Thalia did not have any trouble decoding the M5 poster. She immediately noticed that the code for a given Spiralateral could be determined using a procedure previously discussed in class, that of tracing

and measuring the constituent line segments. For example, Thalia used a pencil to trace the three orthogonal "legs" of "turn-1" of Fig. 1 (see *Figure 15*), verbalizing as she went: "this one is two up, three over, one two three… [indicates six units of third leg]." Using this method she worked out the Spiralateral code for Fig. 1 as {2, 3, 6}.

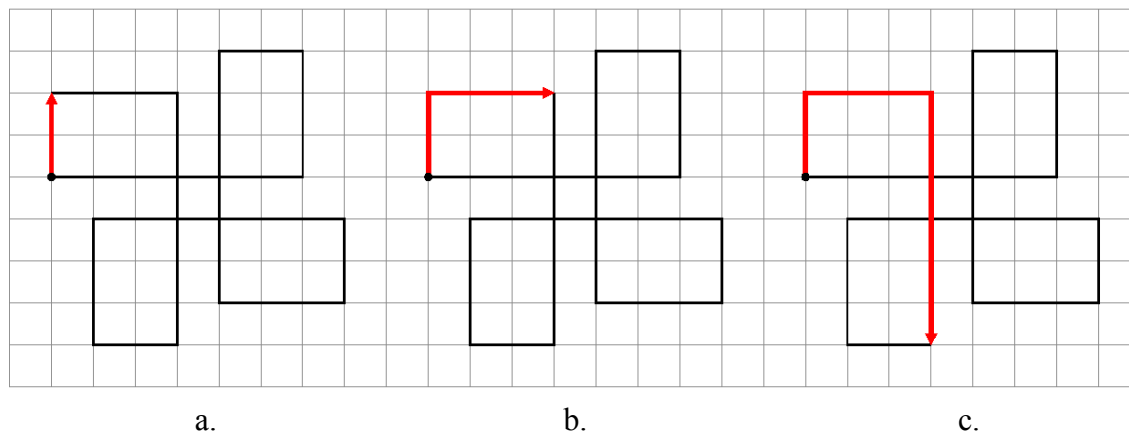a.   b.   c.

Figure 15. Decoding procedure for 3-legged Spiralaterals, which Thalia applied to Fig. 1 of M5: (a) the first "leg" of "turn-1" extends upward vertically from the starting dot (two units); (b) the second leg of turn-1 extends to the right (three units), making a 90° angle with the first leg; and (c) the third leg extends downward vertically (six units), making a 90° angle with the second leg, thus completing turn-1. Continuing from there, the process of confirming that turn-2, turn-3, and turn-4 all follow the same numerical pattern of {2, 3, 6}, ultimately showing that turn-4 terminates at the starting point.

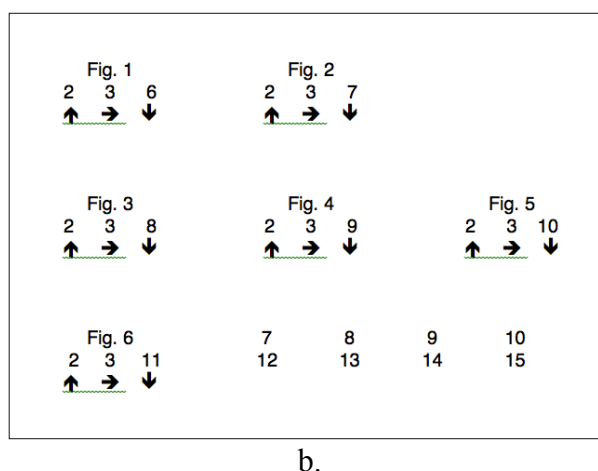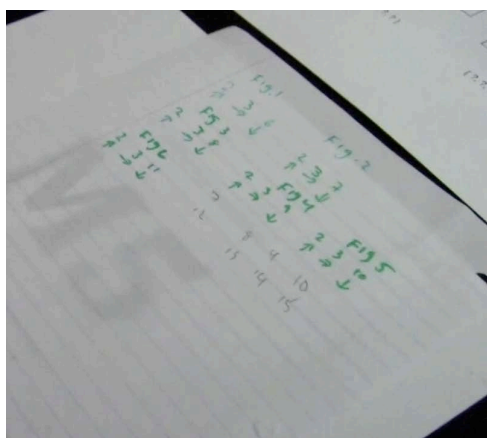a.   b.

Figure 16. (a) an image of Thalia's work and (b) its typed reproduction showing codes for Figs. 1–10, which she used to articulate an arithmetic-recursive Code for M5.

After some prompting from Mr. Lam, Thalia decoded other figures further along the sequence (Figs. 2–10) and wrote her work on a sheet of paper (*Figure 16,* above). Working with a table of values, and verbalizing her patterns to Ailani, Xeni, and the teachers, Thalia expressed a Code for M5, in the form: $\{f_1(n), f_2(n), f_3(n)\} = \{2, 3, f_3(n-1)+1\}$, which states that the code of a given figure (Fig. *n*) is dependent on the previous (Fig. *n*–1), such that the first and second entries remain constant while the third entry increases by a summand of 1 with respect to the

previous figure (an arithmetic-recursive formula). Thalia championed her Code and argued its accuracy adamantly to Ailani as well as the three teachers (Mr. Lam, Ms. Urrutia, and Mr. Gutiérrez), that is, until Xeni became involved about two-thirds of the way into the vignette, at which point she took the lead on the mathematical activity and guided Thalia to a complete solution in the form $\{f_1(n), f_2(n), f_3(n)\} = \{2, 3, n+5\}$.

Aliani's Participation and Milestones. At the start of the activity, Ailani's first efforts were as much about getting herself oriented to the general objectives of the Team Task as they were about positioning herself socially and elevating her status among her peers and the teachers (more on this issue later, see Chapters 6 and 7). Soon after Thalia got going on her work, Ailani became involved in the task by asking Thalia for the instructions, but it was not until Mr. Lam prompted Team 1 (on his first visit) that Ailani looked at the M5 poster and began taking action in relation to completing the tasks in the domain of scrutiny.

Ailani's first mathematical contribution was in the form of a prediction about the third entry of the code for Fig. 10, in the form $f_3(10) = 19$. She arrived at this value by employing a method that used her fingers to count and make a calculation; Ailani did not give any indication that she was using the Spiralateral decoding procedure that Thalia had been using. Later on, Ailani articulated a Code for M5, in the form $\{f_1(n), f_2(n), f_3(n)\} = \{$ __, __, $f_3(n-1)+1\}$ (i.e., she made no reference to the first or second entries of the Code, only the third). At the very end of the vignette, Ailani made one final mathematical contribution, in the form $\{f_1(n), f_2(n), f_3(n)\} = \{$ __, __, $f_3(n)+5\}$; however, further qualitative analysis in light of the teacher–student interaction (Chapter 7) reveals that she appropriated the speech form of others so as to articulate what on the surface appears to be a correct solution.

Xeni's Participation and Milestones. Xeni does not overtly participate in the activity up until minute 25 of the 35-min. vignette. At the very beginning of the activity, as Mr. Lam was distributing the M-posters to the class, Xeni gave some indication that she wanted to become involved in the activity but perhaps felt shunned or thwarted by the others, and so she elected not to invest her time/energy until much later in the activity; it took much prompting and encouragement by the teachers for Xeni to participate. For the first 25 minutes, Xeni reads a book, sits idly, draws pictures, writes notes, and engages other students off-task.

At Episode 11, Xeni finally orients herself toward participating in the mathematical discourse. So doing, she reiterated what Thalia and Ailani had already established, a Code for M5 in the form $\{f_1(n), f_2(n), f_3(n)\} = \{$ __, __, $f_3(n-1)+1\}$. Xeni, too, made no reference initially to the first or second entries, only the third. Xeni attempted to rearticulate her Code using formal symbolism, and thus made a contribution that states $\{f_1(n), f_2(n), f_3(n)\} = \{2, 3, n+1\}$, which was her attempt at using the variable "n" but perhaps in error. That is, Xeni's "n" did not index ordinal position, rather it referred to the third entry of the previous figure, $f_3(n-1)$. In any event, Xeni perceived the third entry, which she called "n," as an indeterminate quantity. Ultimately, with some prompting from Mr. Lam and Mr. Gutiérrez, Xeni articulated a new Code for M5 as $\{f_1(n), f_2(n), f_3(n)\} = \{2, 3, n+5\}$, which states that the code of a given figure is dependent only on its ordinal position, such that the first and second entries remain constant while the third entry is equal to "the figure number plus five." Xeni explained the Code to Thalia who was then able to reiterate the solution to Mr. Gutiérrez in the final episode of the vignette.

Study Results & Discussion

Analysis of students' utterances in terms of semiotic mode revealed that students' reasoning *vacillated* between the three modes. This analysis also revealed that not all students' operated in each of the three modes, and that only Thalia and Xeni were able to co-construct a closed-explicit[13] generalization.

I present quantitative findings of: (1) the number of times each student was operating within each semiotic mode, including visual representations of the students ostensible semiotic trajectories (OST's) that show the vacillating nature of students' actions from the beginning to end of the activity; (2) the number of times each student resorted to using a particular strategy (generalizing vs. naïve induction); and (3) the number of times each student produced semiotically grounded generalizations (i.e., grounded in concrete elements of the initial problem space).

Table 2, below, presents the number of times a student produced an utterance from within each mode.

Table 2.
Students' Total Utterances in Each Ostensible Semiotic Mode

| Student | Semiotic Mode | | | |
	Factual	Contextual	Symbolic	Total
Thalia	50	19	0	69
Ailani	6	5	0	11
Xeni	10	10	3	23
Total	66	34	3	103

$p < 0.01$; $X^2 (4, N = 103) = 14.78$

Chi-Square analysis of *Table 2* revealed a difference between the students with respect to their participation within each of the three semiotic modes during the 35 minute span of data ($X^2 (4, N = 103) = 14.78$, $p < 0.01$). That is, putting aside contextual factors and looking only at mode, Thalia's, Ailani's, and Xeni's participation were all different. Moreover, in coding for ostensible semiotic mode, Radford's theory predicts a particular outcome, regarding the individual shifts from one mode to another. Namely, the systemic unavoidable cognitive-sensuous "rupture" (see Chapter 2) from the Contextual/Factual modes to the Symbolic mode implies that there would be fewer instances of students operating in the Symbolic mode as compared to the Factual and Contextual. The Chi-Square analysis of aggregated F-C-S modes indeed bore this out, revealing a statistically significant difference, with the fewest instances in the Symbolic mode.

The Factual and Contextual modes made up the bulk of sense-making activity and were crucial for all three students' articulating generalizations. Only one student, Xeni, operated in the Symbolic mode, which in situ resulted in a social-mathematical status hierarchy; later chapters closely examine this semiotic-linked power dynamic and its impact on student agency and identity.

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[13] For the remainder of this dissertation, the terms "closed-explicit" and "algebraic" will be used interchangeably to indicate the same type of generalization.

Students Ostensible Semiotic Trajectories.

      The graphs that follow represent students' semiotic-mode participation sequences. *Figure 17* provides a synoptic left-to-right view of each student's participation through the lens of the semiotic mode they are in. Every node in the graph represents a student utterance coded as articulated in one of the three semiotic modes. In the sections below, I examine Thalia, Ailani, and Xeni's ostensible semiotic trajectories (OST's). In this section I draw attention to just a few features of each graph and save a deeper analysis for later chapters.
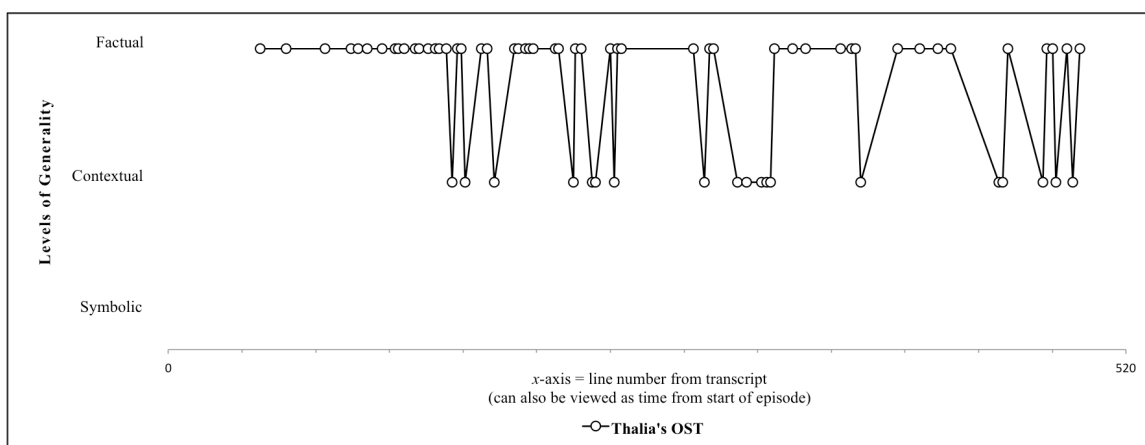

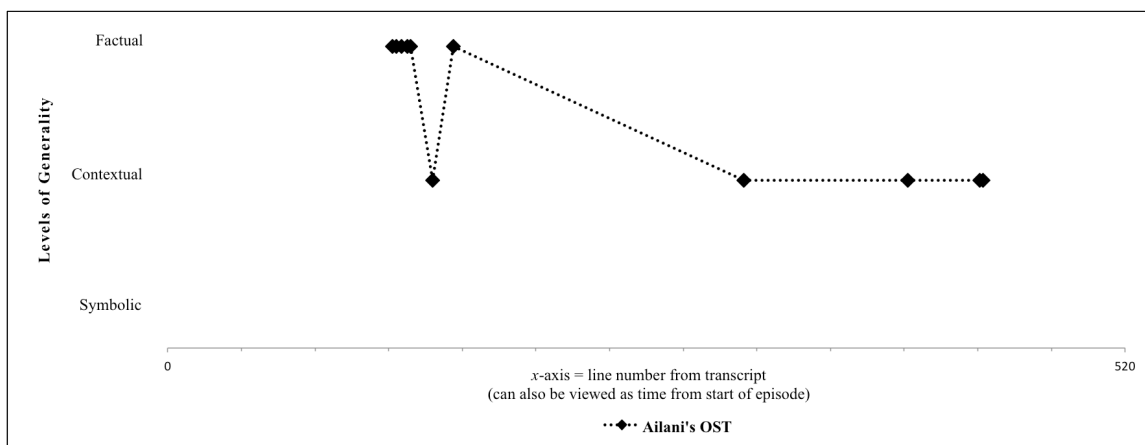*Figure 17a*. Thalia's participation sequence (ostensible semiotic trajectory).

Figure 17b. Ailani's participation sequence (ostensible semiotic trajectory).
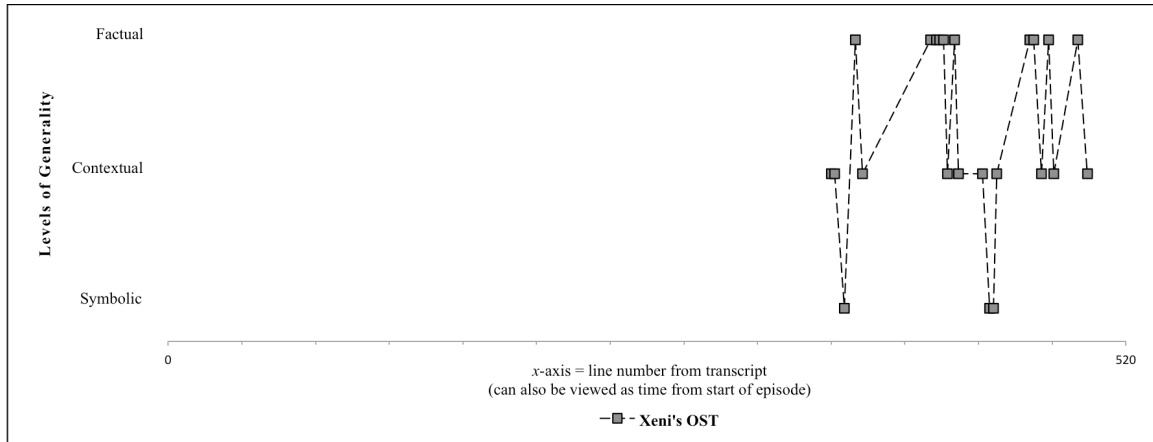
Figure 17c. Xeni's participation sequence (ostensible semiotic trajectory).

Figure 17. Representations of Thalia, Ailani, and Xeni's ostensible semiotic trajectories.

Thalia and Ailani enter the mathematical discourse at the Factual level, and they both vacillate between the Factual and Contextual modes. Furthermore, neither Thalia nor Ailani operate in the Symbolic mode. Xeni, on the other hand, enters the mathematical discourse in the Contextual mode and endeavors to operate in the Symbolic mode right away; and yet, as we will see below, her generalizations—i.e., the final *products* of her generalizing process—are ultimately articulated in the Contextual mode only, not in the other two modes.

We also see that Thalia's participation sequence is consistent across the 35-min. vignette, as indicated by her many "nodes" and relatively shorter gaps between them, as compared to Ailani's sequence. Ailani's sequence consists of less nodes and larger gaps, thus indicating less involvement than Thalia. In the in-depth analysis that follows in this chapter (and later chapters as well), I will show that despite Ailani's OST reflecting the longest gaps, she nevertheless positions herself constantly as an authority in the conversation, as someone who can evaluate and challenge the validity of knowledge of the other participants.

Xeni's graph clearly shows that she became involved in the mathematical discourse much later, but once in the conversation her participation sequence indicates consistent involvement, showing no major gaps (as compared to, say, Ailani's sequence). Moreover, notice that Xeni's initial move to the Symbolic was followed immediately after by a move to the Factual, after which point her activity vacillated between the Contextual and Factual modes; as the in-depth analysis will show, below, these transitions sometimes occurred on a turn-by-turn basis, each turn reflecting one of Xeni's complete propositions, and at other times Xeni shifts back-and-forth between the Contextual and Factual modes *within the same utterance* (see below).

The Process vs. Products of Student Reasoning

To further illustrate the differences between the students' mathematical reasoning, I scrutinized each of their utterances in terms of their semiotic functioning. Namely, of the 103 utterances coded for their semiotic mode, I identified, analyzed, and coded all instances of student attempts to ground a generalization *to* a particular mode; there were a total of 40 utterances of this kind. *Table 3* shows the number of times a student employed naïve induction or generalizing as a solution strategy.

Table 3.
Total Instances of Student Reasoning-Strategy Use

Student	Strategy		Total
	Naïve Induction	Generalizing	
Thalia	0	20	20
Ailani	3	3	6
Xeni	3	11	14
Total	6	34	40

$p<0.01$; X^2 (2, $N = 40$) = 9.75

Chi-Square analysis of *Table 3* revealed a difference between the students with respect to their inclination to use the two types of reasoning (X^2 (2, $N = 40$) = 9.75, $p<0.01$). Xeni primarily used generalizing as a strategy; Ailani used naïve induction and generalizing equally; and Thalia used only generalizing as her main reasoning strategy. It is interesting to compare/contrast the *process* (students' OST's) with their *products* (semiotically grounded generalizations), below.

Table 4.
Total Instances of Students' Semiotically Grounded Generalizations

Student	Generalizations		Total
	Arithmetic	Algebraic	
Thalia	15	2	17
Ailani	2	0	2
Xeni	2	6	8
Total	19	8	27

$p<0.01$; X^2 (2, $N = 27$) = 11.34

Table 4 revealed a difference between the students with respect to their productions of two types of generalizations (X^2 (2, $N = 27$) = 11.34, $p<0.01$). Thalia's generalizations were mostly arithmetic; Ailani's contributions resulted in two arithmetic but no algebraic generalizations; and Xeni articulated more algebraic generalizations than arithmetic ones.

Figure 18, below, locates these generalizations in the students' participation sequences. Looking at Thalia's generalizations (both types, *arithmetic* and *algebraic*, *Figure 18a*), her participation sequence indicates that she is consistent in both the process of generalizing and articulating its products. That said, Thalia's discursive productions that are more mathematically powerful (*Figure 18b*) came toward the end of the problem-solving activity and, moreover, were co-constructed with contributions by Xeni and the teachers. Ailani's two arithmetic generalizations happened quite apart from each other (*Figure 18c*). The two occurred as Ailani was in conflict or tension with others, and they were both "forged" in the Contextual mode. Xeni's semiotic constructions resulted in the more sophisticated generalization form, that of an algebraic generalization which she explained to Thalia and to the teachers. Interestingly, all of Xeni's generalizations were articulated in the Contextual mode (*Figure 18d*), though her reasoning process shifted between all three modes. Xeni's two initial generalizations were arithmetic, and the remaining six were accomplished algebraically (*Figure 18e*).
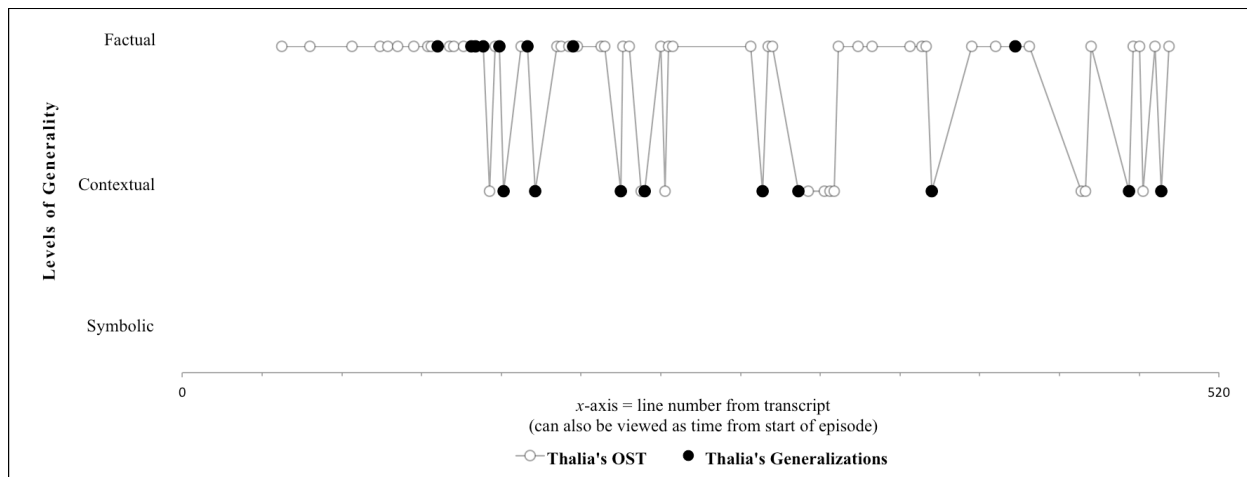
Figure 18a. Thalia's arithmetic-recursive and closed-explicit generalizations.
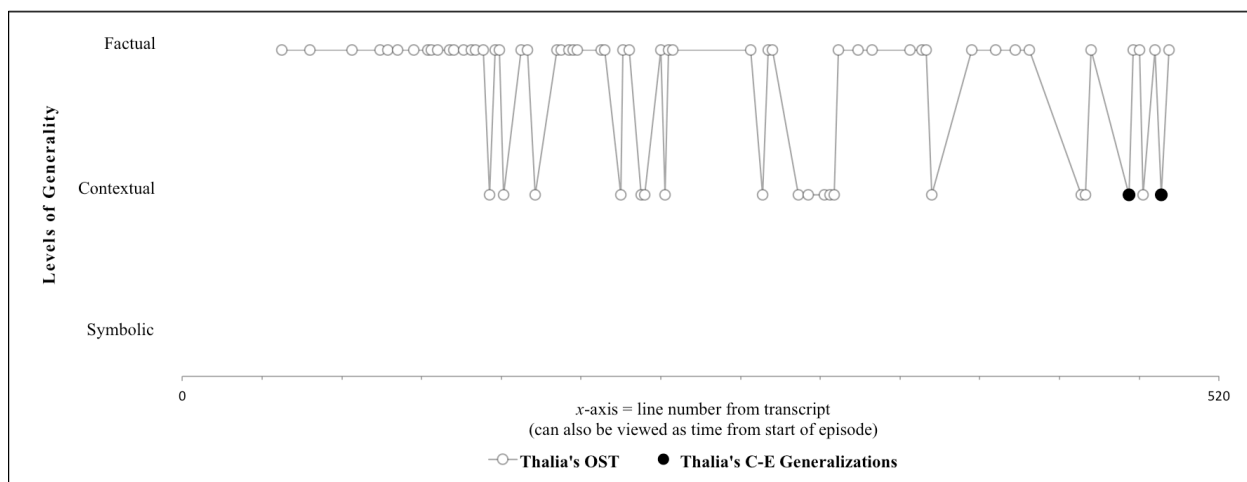

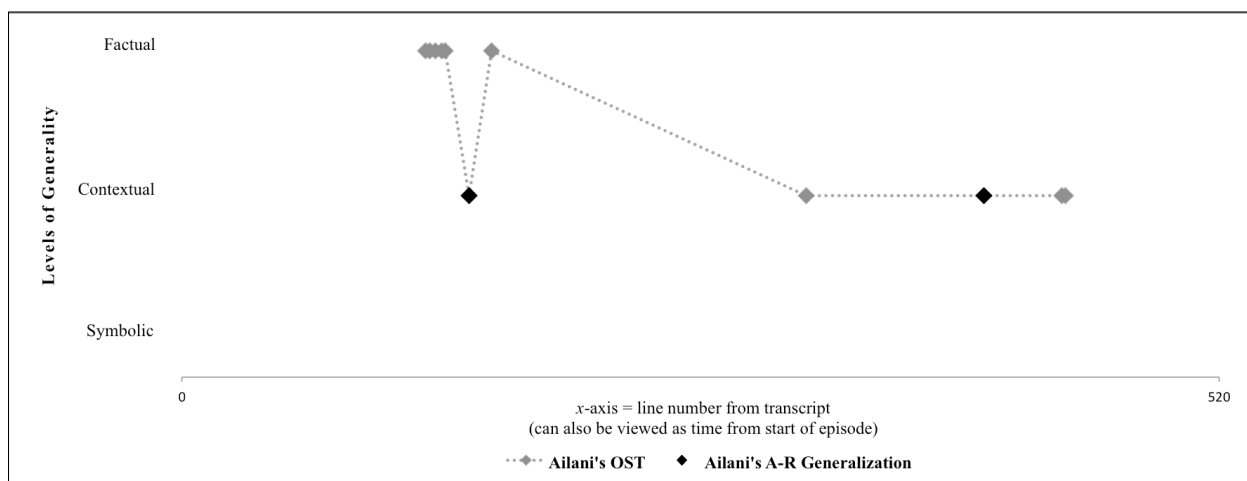*Figure 18b*. Thalia's closed-explicit generalizations only.

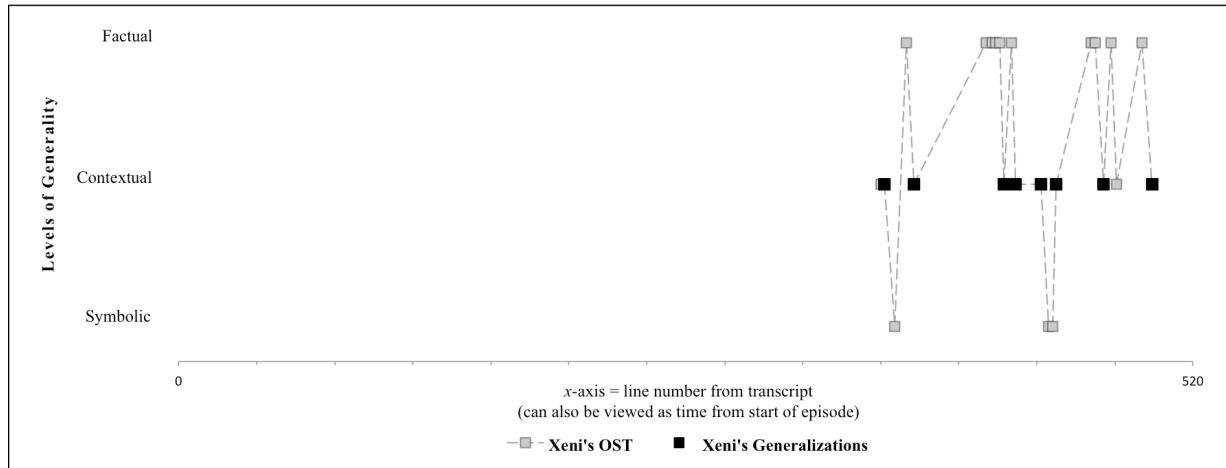*Figure 18c*. Ailani's arithmetic-recursive generalization.

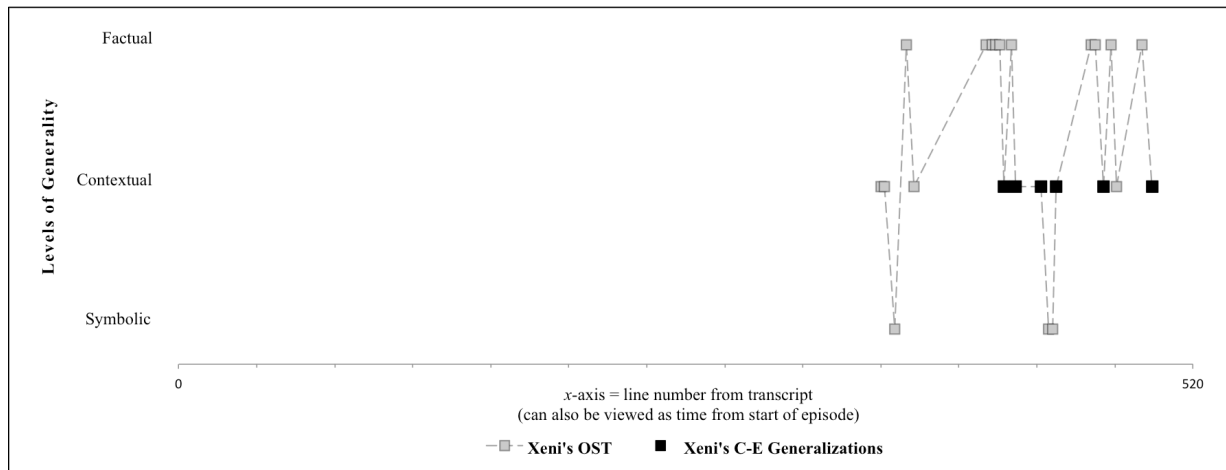*Figure 18d*. Xeni's arithmetic-recursive and closed-explicit generalizations.

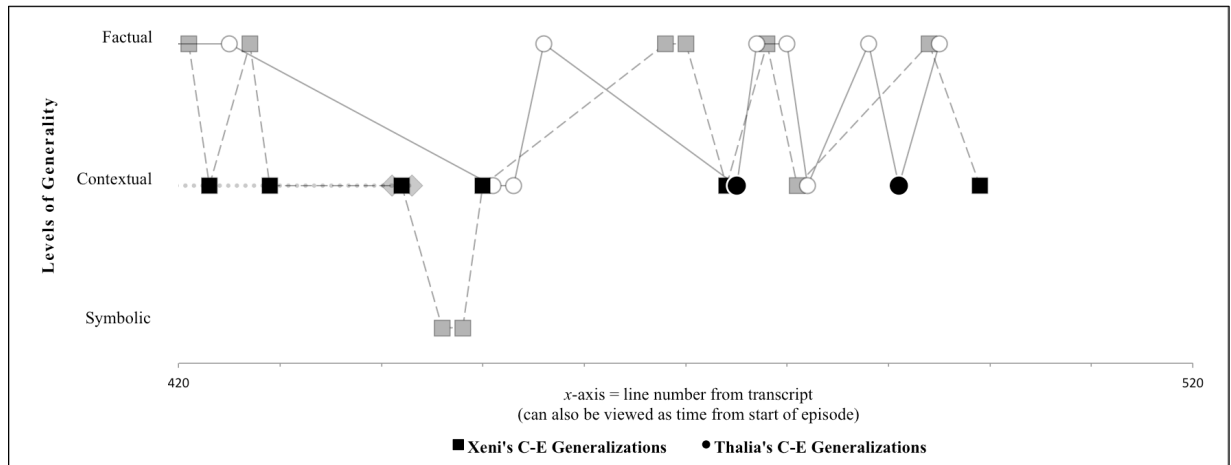*Figure 18e*. Xeni's closed-explicit generalizations only.

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Figure 18f. Lines 420–520, reflecting final 6 1/2 minutes of Team Task activity.

Figure 18. Representations of Thalia, Ailani, and Xeni's OST's and semiotically grounded generalizations.

Thalia was equally likely to express an arithmetic-recursive generalization in either the Factual or Contextual modes, suggesting a vacillatory nature to developing conceptual fluency, as students navigate certain semiotic spaces. Looking at just her closed-explicit articulations (*Figure 18b*), these were co-constructed with Xeni. Xeni's explanation could have been cast in the Contextual mode, which led to Thalia appropriating the solution procedure and thus appropriating the semiotic mode as well. Similarly, Ailani was equally likely to operate in either the Contextual or Factual mode, yet both instances of an arithmetic-recursive solution were articulated in the Contextual mode (*Figure 18c*). Xeni's semiotic trajectory, as mentioned just above, reflects that: (a) she became involved late; yet when she did, she (b) operated in all three modes. Her actual mathematical generalizations, most interestingly, were all cast in the Contextual mode (*Figure 18d* and *12e*). Her symbolic contributions bore no actual mathematical power—at least not any more power than her other contributions. Furthermore, as we shall see in Chapter 5, these utterances cast in the Symbolic mode gave her interpersonal/relational power and reproduced her higher status among her other two team members.

Study Conclusions

The analyses presented here foreground mathematical content and strategies and focus on individual student cognition. Framing the students' actions in this way, with the emphasis on mathematics and cognitive performance, is proposed as an important approach in mathematics educational research, because it highlights possible leverage points to intervene and increase mathematical knowledge, equalize opportunities for students' voices to be heard, and thus strive for productive engagement and relational equity (Jo Boaler, 2008) in the classroom.

Focusing on the strategies and tools involved in students' objectification processes, we observed that the Factual and Contextual modes made up the bulk of sense-making activity and were crucial for all three students' articulating generalizations. However, only Xeni operated in the Symbolic mode and articulated a closed-explicit solution, which she later explained to Thalia.

In the following chapters, I show that individual differences *in situ* result in a social-mathematical status hierarchy. Chapters 5 and 6, for example, closely examine a semiotic-linked power dynamic and its impact on student agency and identity. OST's are, at least in part, motivated by a desire to hold the more privileged semiotic space. Looking ahead to Chapter 7, I ultimately argue that the three semiotic modes are actually proxies for broader semiotic spaces inherent to mathematical activity beyond patterning tasks. I shall discuss mathematical discourse as occurring in, or rather *creating*, "semiotic spaces" that are socially hierarchical. That is, the Symbolic mode may be perceived by teachers and students as having higher status than the Contextual and Factual modes. This perception, at least in part, stems from a broader historical narrative about mathematics education that prioritizes the use of formal symbolism/registers over informal language and other communicative means (such as gesture, rhythm, cadence, repetition, etc.).

Chapter 5: The Objectification–Subjectification Dialectic in Mathematical Discourse

The previous chapter provided an overview of the students' mathematical activity and the vacillating nature of their reasoning across the 35 minute span of data. This chapter adds much needed detail to the story of how Thalia, Xeni, and Ailani navigated the various mathematical–semiotic spaces in relation to their social standing in the group. Specifically, I present a series of analyzed transcriptions that explore a tension between the semiotic resources (e.g., gesture and language, as well as conventional tools such as tables and graphs) to which the students took recourse to make mathematical assertions, and the hierarchical positional identities that they co-constructed through these multimodal interactions. I focus on nuanced aspects of the dual processes of objectification (Radford, 2003) and subjectification (Davies & Harré, 1991; Harré, 2008; Heyd-Metzuyanim & Sfard, 2012; Sfard & Prusak, 2005) as a method of exploring this dialectic (i.e., "obj–subj").

An examination of the social-mathematical antagonism, through the lens of the obj–subj dialectic, reveals that power dynamics and new mathematical understandings are co-constitutive through public discourse. With the data presented in this chapter, I hoped to illustrate that co-construction of power relations and the co-construction of knowledge are not necessarily distinct processes. To be sure, each of these processes can be independently instantiated in the practice of pedagogy, but I maintain that they also become co-constitutive at certain points in the discourse—thus the learning–power imbrication emerges.

Operationalizing Subjectification

When certain verbal utterances do not directly indicate a human actor—for example, when they only mention specific mathematical objects—under Sfard's scheme those utterances would not be coded as subjectifying. Under their general approach, Sfard and colleagues (2012) open up new analytic avenues dealing with what might thus be called *direct* subjectification; however, what I show in my data and analysis, below, are instances where the referents of verbal utterances do not directly involve human actors but nevertheless involve *indirect* subjectification. That is, some verbal utterances/actions mark subjectification through tacit positioning (Harré, 2008) (see Ch. 2 for more theoretical background, and see data Episode 4, below).

Obj–Subj in a Numerical Patterning Task

I here present qualitative, microgenetic analyses (Schoenfeld et al., 1991) of a series of selected transcript segments. These segments represent the bulk of students' mathematical activity during the first 19 minutes of the 35-min. vignette (see Chapters 6 and 7 for analyses of those other episodes). By conducting separate, detailed, and sequential analyses of Thalia, Ailani, and Xeni's behaviors during group work, I aim to elaborate on and extend findings from Chapter 4 by illustrating the tension in the obj–subj dialectic and how Ailani and Thalia co-construct a mathematical hierarchy.

This section presents three sub-episodes from the larger vignette: Episodes 4, 5, and 8. The following transcripts include notes about movement, gaze, and other actions in the form of descriptions inserted in the text. For an explanation of transcript conventions, see *Table 5*.

Table 5.
Summary of Transcript Conventions

Symbol	Description
/	Self-interruption
=	No gap between utterances
//	Marks beginning and end of overlapping utterances
two dots ".." at end of text	Very slight pause, less than a second
(2 sec)	2-sec pause
repeated letters	E.g., " generrraliiize," marks lengthened syllable, each repeated letter equals one "beat"
italics	Marks stress
CAPITAL LETTERS	Increased volume
(??), ((this))	Unclear or inaudible reading, tentative reading
Bracketed notes	e.g., "[*shifts gaze from Ailani to Mr. Lam; laughs*]," marks other voice qualities or actions
(a), (b)	Links talk to image of that turn

Episode 4 – Time Stamp [08:13–09:34] – Team 1

Episode 4 begins just as Mr. Lam walks away from the focal group's table.

Just before leaving the group, Mr. Lam was in the middle of clarifying the instructions for Team 1, when he and Ailani fought over the M5 poster. They reached for the poster at the same time, but Ailani gained possession of it, clutching the poster with both hands, exclaiming "Can I seeee it?!" Mr. Lam released his grip on it, responding, "Yesss you can see it. You don't have to like—can I see it while you see it?" Mr. Lam reached one more time for the poster, but Ailani pulled away and kept it in her hands, just as Mr. Lam gets called to another group. Ailani held the poster out in front of her using both hands, as if to say, 'I got this.'

As Mr. Lam walks away, Ailani holds the poster in front of her face and begins counting aloud. As she counts, she releases her grip to free both of her hands to use as counting aids (see detailed transcription, below). Then she slightly torques her body, reaches behind her, and places her hand on the shoulder of Mr. Lam, who is crouching at another group's table behind her. She announces her solution for Fig. 10, but Mr. Lam does not respond initially. She moves her body out of torque to face her group again. There is a long pause. Getting no feedback or input from her group, she begins to write something down and says aloud to no one in particular, "nineteen."

While Ailani is counting and working directly with the poster, Thalia's gaze is down at her desk and she is adding entries to her table of values. Then, without lifting her gaze at all, Thalia indicates something on the table of values with her finger, as she asserts that Fig. 10 will be "Fifteen." Ailani adamantly claims that the solution for this given figure is nineteen. Thalia, too, is adamant about her solution, and she resorts to a complex array of communicative means to make her point. As Thalia talks, she enters a rhythmic cadence that accentuates her utterance of the last entry of each code in the sequence, using both of her index fingers in concert to articulate a numerical pattern that she is verbalizing for the first time (Line 128, below). Ailani's gaze is on Thalia's hands as she talks. When Thalia is finished (Line 129), Ailani nods her head slightly, momentarily shifts her gaze to Xeni, shifts it back to Thalia's desk/table of values, and

exclaims, "I say nineteen." Her gaze shifts around again for a few seconds, and the excerpt ends as Ailani acknowledges a possible counting error.

Below I present the Episode 4 transcription in its entirety, followed by a line-by-line analysis of the role of semiotic resources in the students' obj–subj processes.

Line# Speaker   Utterance/Action/Image

Seating Arrangement

Thalia	Ailani
(open)	Xeni

122   Ailani:

a.
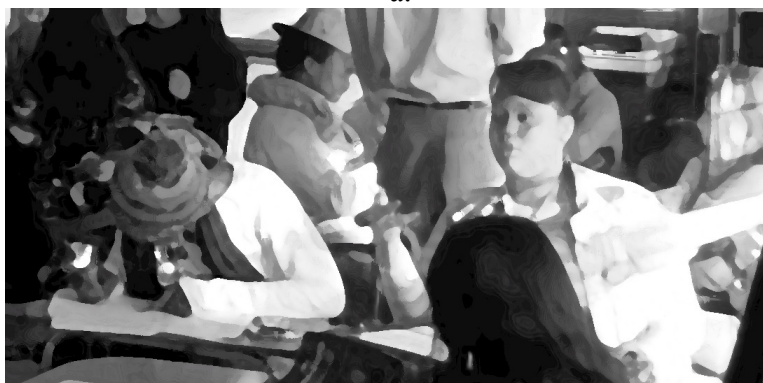
b.

c.

d.

[*holds poster in front of her face with both hands and counts aloud; as she counts, her right hand comes free and she begins counting with her fingers* (a)*; she then lowers the poster and continues counting fingers on both hands* (b)*; momentarily gazes in Xeni's direction as she counts aloud* (c)] "Eleven twelve.. thirteen.. fourteen, fifteen, sixteen, seventeen, eighteen, nineteen." [*getting no response from Xeni or Thalia, Ailani reaches back* (d) *and taps Mr. Lam on the shoulder and leaves her hand there for a second*] "Fig. 10 will be (1 sec) nineteen." (10 sec) [*to Xeni or to no one in particular*] "((Fig. 10)).. nineteen."

123	Thalia:	[*gazes down at her sheet; indicates entries in her table of values*] "Fig. 10 will be fifteen."
124	Ailani:	[*responds to Thalia*] "Nineteen."
125	Thalia:	[*gazes down; shakes head no*] "Fig. 10 will be fifteen."
126	Xeni:	[*leans her head on her hand; unclear where her gaze is*]
127	Ailani:	[*loudly, almost shouting*] "Fig. 10 will be nineteen!"

| 128 | Thalia: | |

e.

f.

[*still without lifting her gaze from her sheet, indicates entries with her finger as she talks in a rhythmic cadence* (e)] "It goes two-three-six, then it goes two-three-seven, then it goes two-three-eight, then it goes two-three-nine, then it goes two-three-ten, then it goes two-three-eleven, twelve, thirteen, fourteen, fifteen." [*keeps gaze down, continues working* (f)]

| 129 | Xeni: | [*adjusts in her seat; there is no audio for her at this point, it is possible that she might have responded to something Thalia or Ailani said, but one cannot tell from the footage*] |

| 130 | Ailani: |

g.

[*briefly gazes in Xeni's direction, but it is unclear if she actually addresses Xeni; likely addressing no one in particular*] "I say nineteen. (4 sec) I'm not even sure I counted that right, but who cares?" [*moves poster to the empty desk across her* (g)]

Analysis & Discussion. An analysis of the above transcription reveals at least two important components. First, this excerpt reflects Thalia's and Ailani's first mathematical objectifications. I should note upfront that in examining their interaction, I want to table the issue of whether Thalia or Ailani had the correct answer. Rather, I wish to focus on the semiotic means they each used to objectify their emergent understandings.

Ailani's semiotic means of objectification consisted of a counting strategy and verbal speech (with a certain illocutionary force) to assertively express a partial solution, in the form of $f_3(10) = 19$. Ailani counted on from a known quantity (Fig. 4) but she had not yet indicated that she was attempting to generalize a numerical pattern. In contrast, Thalia too resorted to verbal speech, yet she also used rhythm, gesture, and repetition, and a mathematical table as semiotic resources. Entering a rhythmic cadence (Line 128) enabled Thalia to objectify an arithmetic-recursive generalization,[14] in the form of $\{f_1(n), f_2(n), f_3(n)\} = \{2, 3, f_3(n-1)+1\}$ which, in this context, is more informative than Ailani's solution.

The second component this transcription reflects is a dynamic process of subjectification. To examine the dynamics of subjectification during this interaction, I focus on whether/how Thalia and Ailani refer to themselves and each other. In the transcript above, Thalia did not refer to herself or Ailani directly. Looking at Thalia's pronominal usage, what we see is that the referents of her speech did not directly involve human actors. And yet, Thalia made statements that implied *she* has the correct answer and *Ailani* does not. Ailani, too, made statements that tacitly positioned herself as having the correct answer. Thalia, however, resorted to a broader arsenal of semiotic resources to make her point. Additionally, Thalia did not make eye contact with Ailani, instead keeping her gaze on her work. This social-mathematical power encounter resulted in differentiated status positions. Ailani's effort to appropriate mathematical authority was challenged, and an opportunity was missed to engage in dialog and collaborate on the shared goal, to determine Fig. 10.

In sum, we observe in Episode 4 that both Thalia and Ailani attempted to gain *mathematical ascendency* over the other, which is a construct I claim that describes the hierarchical status positions that were co-constructed through this multimodal interaction. Furthermore, the hierarchy was based on the semiotic resources marshaled by and pitted against individual interlocutors. In this way, Thalia's and Ailani's respective semiotic means of objectification simultaneously functioned as semiotic means of subjectification.

The notion of mathematical ascendency bears out in the rest of the vignette, and Thalia, Ailani, and Xeni all make moves to gain and assume it at different times in the activity.

Sfard suggests that mathematics learning involves subjectifying and objectifying—two related yet separate processes. I argue that subjectification, as a broader process that includes "subjectifying" as Sfard defines it (i.e., as directly referring to participants of a discourse), also happens when participants make objectifications that, in so doing, only indirectly or tacitly refer to other participants. If we had been looking at subjectifying utterances only as defined by Sfard

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[14] See also Radford (2003, 2008) for more on rhythm and cadence as semiotic means of objectification.

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and colleagues, we would highlight only one single incident, when Ailani admits her counting strategy's inaccuracy (Line 130). But analysis of this single turn of talk, although a crucial one, does not capture the dynamics of implicit positioning in Line 122–129. As Thalia and Ailani made mathematical assertions, they co-constructed positional identities that must then, in turn, either be taken up, accepted, contested, negotiated, or rejected.

Episode 5 – Time Stamp [09:34–10:56] – Team 1 with Mr. Lam

This episode begins immediately where Episode 4 leaves off, as Mr. Lam returns to Team 1 in response to Ailani's tap on his shoulder (see Episode 4, above). Ailani and Thalia once again assert their solutions regarding Fig. 10 and attempt to establish their mathematical ascendency by appropriating authority. As they argue, Mr. Lam turns to Xeni and asks her questions that would steer her toward acting in relation to the assigned task. What I aim to show in the detailed analysis, below, is that although Ailani and Thalia argue their respective solutions and appear to be reaching an impasse, their individual learning trajectories nevertheless show movement from the Factual to the Contextual modes of reasoning and, moreover, *that these advancements were forged in and through the social mathematical power dynamic.*

Below I present the Episode 5 transcription in its entirety, followed by a line-by-line analysis of the role of semiotic resources in the teacher and students' obj–subj processes, which in turn led to the construction of new mathematical understandings as well as situated, semiotic-based relations of power among all the participants.

Line#	Speaker	Utterance/Action/Image
131	Mr. Lam:	[*to Ailani*] "Alright what's the question? What was the question? You came up with how many/the code?"
132	Ailani:	"Nineteen."
133:	Mr. Lam:	"Nineteen?"
134	Thalia:	[*loudly*] "No! No. [*softly, counts to herself; keeps her gaze down*] Twelve.. fourteen.. [*to Mr. Lam*] It'll be fifteen."
135	Mr. Lam:	

a.

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		[*walks around to Xeni's side of the table; slides poster closer to her* (a)] "Fifteen? //Xeni, what do you think?"
136	Thalia:	//"It'll be fifteen.// I think I did this right but I might be wrong."
137	Xeni:	"(??)"
138	Mr. Lam:	"Hmm."
139	Xeni:	[*turns her gaze away from Mr. Lam, down to her desk*] "I don't know."

140   Mr. Lam

b.

[*to Xeni*] "OK what might help? You think maybe writing in the code might help? [*slides poster closer to her and indicates the "{?, ?, ?}" along the bottom of each figure* (b)] If you write in the code and look for a pattern?"

141   Thalia:

c.

"It'll be fifteen. [*waves a sheet of paper in front of Mr. Lam but he ignores it and keeps his gaze on Xeni; Thalia gazes up at Mr. Lam for the first time, reaches for second sheet of paper and hands it to Mr. Lam* (c)] I already wrote all the codes."

142   Ailani: [*gaze down at her desk; she drawing*]

143   Mr. Lam

d.

[*grabs second sheet of paper from Thalia and places it in front of Xeni on her desk*] "Oh good she's got the codes. So maybe you can find a pattern based off of her codes. [*talks in a singsong voice as he points with his fingers to values in Thalia's table* (d)] Two three six, two three seven//"

144   Ailani:

e.

//[*interjects loudly* (e); *to Mr. Lam*] "It just added a number!"

145	Thalia:

f.

//[*interjects loudly* (f)*; to Mr. Lam*] "It just goes six seven //eight nine ten eleven twelve thirteen fourteen fifteen."

| 146 | Mr. Lam: | [*nods head, either agreeing or counting along or both*] "Two three eight, two three nine, two three ten.." |

147	Thalia:

g.

[*with a tone suggesting that the pattern should be obvious to Mr. Lam* (g)] "Eleven twelve thirteen fourteen fifteen."

148	Mr. Lam:	"So what do you notice about the first two [*referring to the first two entries of the code*]?"
149	Thalia:	"What do you mean?"
150	Mr. Lam:	"What do you ah—"

h.

151	Thalia:
		[*makes repeating arching gesture with her finger, across the figures in the poster* (h)] "It goes six seven eight nine ten eleven twelve thirteen fourteen fifteen."
152	Ailani:	[*adjusts in her seat; without lifting her gaze, loudly*] "No, it don't!"
153	Xeni:	[*no indication she is contributing to the converstaion; yet she could be observing/listening*]
154	Thalia:	"It does. It always adds//"

155 Ailani:

j.

//[*to Thalia*] "Bruh, it's like ten [*lifts gaze from her desk to Mr. Lam* (j)] but you added one or two."
156 Mr. Lam: [*to Thalia, indicating entries in her table of values*] "Two three—

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no but—no but look, it says two then three then //six."

157	Thalia:	//"Six."//
158	Mr. Lam:	"Then this one says two then three then seven. // Two three eight.
159	Thalia:	//[*with a tone suggesting that the pattern should be obvious to Mr. Lam; talks fast, indicating entries in the table with her finger*] "It goes two three and then eight,// and then it goes two three and then nine, and then it's going to be two three and then ten, and then eleven twelve thirteen fourteen fifteen."

160   Mr. Lam:

k.

m.

n.

[*to Thalia*] "OK so two three fifteen, you're saying?"
[*to Ailani; makes repeated arching gesture with his right hand, marking each "entry" of the code with each motion* (k)] "OK that's what she meant when she said fifteen. So that code, it's staying two [*gestures up-down* (m)], three [*gestures up-down*], and then something [*gestures up-arching over to the right-down, palm up; starts to walk away* (n)].
[*to the group*] OK so how would you find for any/for any number?"

p.

161   Thalia:

[*throws hands in the air; posture and facial expression suggest frustration* (p)*; to Xeni*] "You just keep adding the number, duhhh!" [looks to Ailani, then looks to Xeni]

162   A. & X.:   [*no response*]

Analysis & Discussion. Earlier in Episode 4, we observed that Thalia's and Ailani's semiotic means of objectification also served as semiotic means of subjectification, and as such these semiotic resources formed the basis of a social-mathematical hierarchy. Here in Episode 5 the power dynamic continues and the students marshaled their semiotic resources to resolve an overt mathematical conflict, but we also observe that the teacher marshaled his semiotic resources, as well, to orient Xeni toward acting in ways that are relevant to the problem-solving domain. When Mr. Lam walked away from Ailani and Thalia's side of the table to Xeni's side and asked her, "What do you think?" he positioned Xeni as the center of attention and her (non)participation as the critical aspect of the situation, even as Ailani was vying for his attention regarding Fig. 10.

Thalia too fought for Mr. Lam's attention, for him to recognize that she had done all the work, when she stated "I already wrote all the codes" as she waved the sheet of paper in Mr. Lam's view. Her tone of voice suggested that he need not bother Xeni with that task because it

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was "already" completed. Mr. Lam maintained his "critical cool"[15] at this moment as well, and finally accepted Thalia's gesture to look at her codes. In that moment, Ailani interjected with, "It just added a number!" (Line 144), which is an utterance iterated in the Contextual mode as it referred to a general procedure and was not tied to concrete elements in the problem space. What is most interesting to note at this point, is that this is the first student utterance coded in the Contextual mode, whereas all previous utterances were articulated at the Factual level of generality. Thalia, too, interjected at the exact moment that Ailani articulated her Contextual statement (Line 145), with a re-articulation of her Factual recursive generalization: "It just goes six seven eight nine ten eleven twelve thirteen fourteen fifteen." Mr. Lam responded to Thalia's and not Ailani's contribution, asking a question about the pattern that Thalia was verbalizing. Ailani's contribution went unacknowledged and, as a consequence, Ailani went unrecognized as having achieved a greater level of generality than all the other student participants.

We see here that Ailani's semiotic means of objectification have increased in generality, suggesting increase in her mathematical sophistication and understanding. However, when we look closely at Ailani's remark to Thalia, "Bruh, it's like ten but you added one or two" (Line 155), we see that the mathematical basis of her assertion is uncertain. Ailani is not sure if one needs to add 1 or add 2 from the last known quantity (Fig. 4), so she adjusted the repeated summand to account for the difference between Thalia's answer of "fifteen" and her "nineteen." Despite the uncertainty Ailani is indicating here in Episode 5, and despite Ailani having admitted the possibility of a faulty counting strategy at the end of Episode 4, she nevertheless continued to hold on to this solution. She is personally invested in the task and will carry this solution through to the end. (The next chapter and analysis of Episode 11 show that her statement of "added one or two" comes back into play.)

Thalia responded to Ailani's assertions with a statement that reflects her first teeter into the Contextual mode of reasoning, at Line 154: "It does. It always adds—" but it was cut off by Ailani.

Both Ailani and Thalia made statements that express a mathematical procedure, but the illocutionary force of their utterances also positioned them as capable of asserting knowledge and thus established them as authority figures in the discussion. I claim that through these complex interactions, the threshold to operate in the Contextual mode was lowered because their status and identity were at stake. That is, the inherent power struggle of the conversation spurred them on to position themselves as authority figures within that dynamic, and what resulted were Ailani and Thalia's first Contextual statements.

With regard to Mr. Lam's role in the evolving dynamic. At Line 143, there was a notable shift in Mr. Lam's discourse from general discusive tactics (to encourage Xeni to participate) to

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[15] This is another emerging theme that I have discerned in the ethnographic data. "Critical cool" refers to a personal characteristic or mode of instruction of Mr. Lam. It is aligned with and informed by the César Chávez's ethos of teaching for restorative justice, specifically their pedagogical principles of "exuding a warm demeanor" and "controlling the mode" (explicitly stated in school materials, such as posters and the school's website). Mr. Lam reported that this approach requires an exorbitant amount of patience and care on his part; that the key to maintaining his critical cool is responding affirmatively to student behavior and not just reacting or pillorying. Related to this, Mr. Lam constantly practices "pausing" mid-sentence, to buy time for both him and his students to consider what is happening/the circumstances at hand.

operating in the Factual mode, when he highlighted certain aspects of the problem situation, marking them as imporant with his tone and cadence.

Mr. Lam revoiced Thalia's recursive generalization (Line 159), but he shifted the semiotic space from the Factual to the Contextual level of generality. So doing, Mr. Lam did not reiterate the additive/recursive component of Thalia's code, only that the first two entries remained constant while the variable was the third entry (Line 160: "It's staying two, three, and then something"). As Episode 5 came to an end, Mr. Lam walked away and tossed a final question to the team, asking them to consider cases beyond just the first ten figures (Line 160: "for any number?"). Thalia scoffs at Mr. Lam's final question, with one final statement in the Contextual mode that, for her, captures the complete solution (Line 161: You just keep adding the number—duhhh!"). Thalia may have interpreted the subtext of Mr. Lam's question as evaluating her solution as insufficient, and she disagreed with his assessment. Mr. Lam's question is a common tactic used in these kinds of instructional contexts involving figural patterns, intended for students to realize that recursive strategies are limited when dealing with cases much further down the line. Thalia's comments point to a mismatch between her perception of the requirements of the task and Mr. Lam's expectations. As we will see in Chapter 6, Thalia's frustration will ensue, all the way until Episode 11, where an overt conflict erupts between her, Mr. Lam, and Ailani. This conflict momentarily leaves the discursive frame of "mathematical practice" to a broader ideological frame. The next episode, Episode 8, the last one presented in this chapter, highlights other aspects of the role of semiotic resources in the social mathematical power dynamics.

Episode 8 – Time Stamp [17:35–19:00] – Team 1

At the end of Episode 5, above, Ms. Urrutia approaches Team 1 and inquires about their poster; Thalia is the only one to talk with her; Ailani draws and Xeni reads from her book. Prompted by Ms. Urrutia, Thalia explains that she had "figured out Fig. 10," that "it stays two-three" (referring to the first and second entries) and "it just adds one to the end of the numbers" (referring to the third entries).

Mr. Gutiérrez too visited Team 1 to inquire about their activity with M5 (Episode 7). In their interaction, Mr. Gutiérrez employed several tactics for guiding all three students to take action in relation to the task. So doing, he evoked the frame of "convince a friend" as a means of encouraging Thalia and Ailani to work together, directly instructing Thalia (Line 203): "I want you to convince Ailani that this [*indicates table of values*] is what's happening here [*indicates poster*]." Mr. Gutiérrez thus positions Thalia as the center of the discursive activity, as though she is the one producing "the truth" of which the others have to become "convinced." This framing is "one way" and not too collaborative in nature. Mr. Gutiérrez's later utterances attempt to perhaps lessen the effect of this positioning, by drawing on his knowledge of an idiom that he had heard from professional mathematicians and mathematics educators. He instructed Thalia to "first convince yourself of something," "and the next step is to convince a friend," and then "once you guys are convinced, convince the enemy" (see Mason, Burton, & Stacey, 1982; Tall, 2009).

The idiom of "convince a friend" frames the interaction between Thalia and Ailani in Episode 8, which we will see next. Encouraging the interaction in this way, Thalia is positioned as the "more competent peer," and Ailani is positioned as a "friend" who needs convincing of the accuracy of both the process and the product. This positioning is problematic, however, because Ailani is positioned as a "friend" who is not party to generating those products or experiencing the process herself. As the transcription below will demonstrate, Thalia is clearly driving the

problem-solving process, while Ailani is not recognized as a valid contributor but only as a "sounding board" for Thalia's individual developing fluency.

      Episode 8 begins just as Thalia begins explaining her solution to Ailani, as a response to Mr. Gutiérrez's goading to "convince a friend."

Line# Speaker   Utterance/Action/Image

238   Thalia:

a.

b.

[*taps Ailani on the shoulder, returns gaze to poster* (a)] "OK look look OK look look look. You see this dot? Start right here, right? [*very briefly gazes at Ailani; indicates starting dot and line segments of Fig. 1 using her finger and the tip of her pencil* (b)] You go up two then you go over three and then you go down six, one two three four five six. OK."

239   Xeni: [*opens her math notebook and puts her markers away, which suggests she is preparing to enter the conversation*]

c.

240   Thalia:

[*continues explaining to Ailani* (c)] "And you keep doing that same thing ((it lead to)) back to the point right there.

241   Xeni: [*briefly leans in toward Thalia and Ailani, which suggests she is observing their interaction; but then takes out another marker and begins drawing again*]

d.

242   Thalia:

e.

[*spontaneously lays pencil flat to indicate figures using a sweeping gesture* (d)] You do the same sequence ((over and over and over again)). And then on Fig. 2 (??).. this is [*traces segments of Fig. 2*

with the tip of her pencil and then looks at Ailani but she has turn her gaze away/down (e)] this is up two over three down seven. You see that? You see that?"
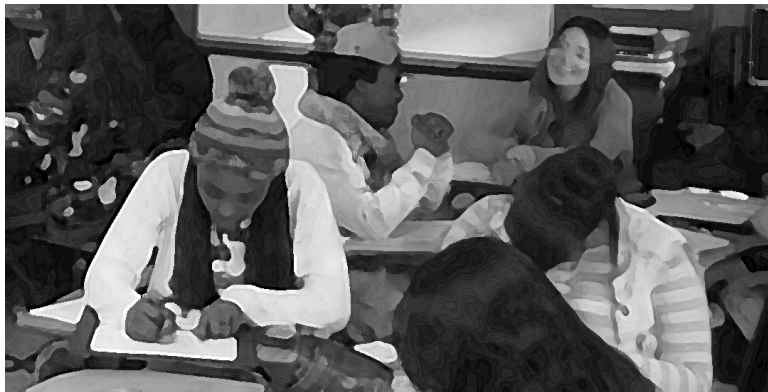

f.

243   Ailani:

[*gaze is on poster and Thalia* (f)] "Yeah."

g.

244   Thalia:

[*using tip of her pencil, Thalia traces over each line segment of each leg for Fig.'s 2–4* (g)] "You go up two, over three, down seven, then up two, over three, down seven, up two over three down seven, up two over three down seven til you get to that point right there. Then this one is up two over three down EIGHT, up two over three down eight, up two over three down eight, up two over three down eight.. this one is up two over three down NINE.

245   Ailani:

h.

[*sighs loudly as she readjusts in her seat* (h)]

246   Thalia:

j.

[*in a tone of voice that suggests she is frustrated or upset* (j)] "Did you just get what I just said?"

247   Alani:

k.

(1 sec) [*looks at Thalia with what appears a quizzical expression; shakes head "no"* (k)]

248   Thalia:

m.

[*shakes her head as she remarks one last thing to Ailani*] "Bruh, José didn't get that. That was mighty easy." [*looks over her shoulder, across the room in Mr. Gutiérrez's direction; announces loudly* (m)] "OK José, my dood!"

249   Ailani:

n.

p.

[*also loudly, in Mr. Gutiérrez's direction* (n)] "I'm still not convinced!" [*turns away from Thalia, smiling* (p)]

250   Thalia:

q.

[*laughing and smiling* (q)] "Stop playyying."

Analysis & Discussion. In the above transcription, Thalia used a variety of semiotic resources, similar to Episodes 4 and 5 (above), including a gesture that involves a "sweeping" motion with the pencil across the figural cues (see Line 242, image-d). These semiotic actions enabled her to express a type of proto-generalization, where the *actions* are generalized but not the objects (i.e., the numerical inputs). The individual inputs and their functional relation to the ordinal position of the figure are not yet revealed. Thalia displays great confidence and enthusiasm when explaining her generalized recursive action.

This episode began as Thalia made a bid for Ailani's attention by saying "look look." Ailani drew her gaze to Thalia's fingers/pencil and followed her actions, but then Ailani broke her gaze just at the moment when Thalia shifted her actions/utterances from the Factual to the Contextual mode. That is, Ailani followed along with Thalia when she went over individual movements that traced the contours of the first Spiralateral, but once each turn of the figure was accounted for, Thalia generalized the action, "and you keep doing that same thing"… and precisely at that moment Ailani looked away.

From the analyst's perspective, this could be merely a coincidence and has no bearing, theoretically, on the relation between interactive participation and semiotic mode, but it nevertheless suggests the possibility that students could subjectively experience such moments as marking a shift in discourse, that the discursive "game" has changed in some way. Some students, if they have limited experience operating in the contextual mode, might "check out" (stop paying attention) at this point.

Another possible explanation is that Thalia's explanation could have just been "too wordy" and therefore "burdensome" to Ailani because it did not allow any time for Ailani to practice or reflect on those actions that Thalia was verbalizing and gesturing. After slow and careful analysis of these interactions, my best theoretical interpretation of this moment is that Ailani was following along at the level of Factual action but then, as Thalia's instruction "got away from her" (i.e., became too wordy), Ailani may have recognized the F-to-C shift as a natural "break" in the talk—an opportunity for her to take an attentional break as well.

This analysis is very difficult, because it is further complicated by the fact that Ailani is distracted by the color markers that are in her hand. It could be that Ailani was not listening—or not in a position to listen well—from the get-go. That said, although Ailani is "playing" with the markers in her hand, her gaze is on the poster and on Thalia's actions throughout the exchange.

Toward the end of their interaction there is a mismatch between Thalia's semiotic actions and her statement, "Did you get what I just said?" Thalia did not merely "say" what was happening—she took action with her hands and interacted with the gridded space presented on the M5 poster in very sophisticated ways that are very challenging to capture completely, succinctly, with mere verbal description. For example, she used the tip of her pencil to precisely trace the contours of Figs. 2, 3, and 4, and she called out the length and heading of each leg (e.g., "up two, over three, down seven"). For every utterance that Thalia expressed verbally, there was a co-occurring deictic gesture—that is, a *physical* action. Yet Thalia's question suggested that Ailani could or should have been able to follow along with just the *verbal speech* to make sense of the problem.

Ailani may have been trying to save face earlier in the interaction, when she answered "yeah" when Thalia first checked for understanding (Line 242: "You see that?"). And although Ailani may have found it challenging to follow along with Thalia's propositions, Ailani nevertheless continued to participate under the "convince a friend" frame of interaction. She might have perceived a higher social status in her position as the one who needs convincing; she took up this frame as an affordance for interaction, but it ultimately did not seem to help her mathematically. Ailani refused to give up her mathematical authority, but she lost her footing in the interaction. Ultimately, Ailani reluctantly admitted that she did not "get" what Thalia said, and thus Thalia assumed the mathematical ascendency over her. Ailani did not give any indication that she could or would fight back mathematically, instead she just maintained her position of "not convinced" and found status in that position.

I do not believe that Thalia meant to "lose" Ailani on purpose, to purposefully confuse or overwhelm Ailani with her explanation. In fact, as mentioned earlier, Thalia and Ailani are good friends and spend time together outside of school (note their amicable exchange in Line 250, image-q, above). But all things considered, the interactions between Thalia and Ailani across these three episodes implicate them both—as well as the teacher—as unwitting agents in the reproduction of status hierarchies through mathematical discourse.

Chapter 6: "My name is not 'Bruh'": Manipulating the Learning–Power Imbrication

One of the overarching aims of this dissertation is to take a critical look at mathematics cognition and instruction. The perspectives I present thus far, in Ch.4 and Ch.5, are articulated primarily in the parlance of the learning sciences, which has given my analyses "bottom-up" traction from student cognition toward target semiotic structures. Ch.4 is oriented toward content and concept learning and provides a comprehensive investigation of students' generalizations across the 35 minute span of data. Ch.5, through the lens of an obj–subj dialectic, critically analyzes the hierarchical positional identities that emerged vis-à-vis issues related to individual student agency and authority. In this present chapter, I continue the obj–subj analysis of the vignette, focusing on one extended episode involving the focal group and Mr. Lam. Specifically, this chapter explores the role of teacher "scripts" in the obj–subj dialectic and its impact on the emerging imbrication.

Operationalizing Cognitive–Conceptual Scripts in Mathematics Pedagogy

The obj–subj dialectic is a necessary but not sufficient theoretical construct for the study of power dynamics in mathematics education. In an attempt to shore up the limitations inherent in a learning-science conceptualization of the obj–subj dialectic, I bring to bear complementary socio-political perspectives that can explicitly account for relations of power not only in local multimodal interactions but also in the systemic organization of mathematics education.

I present the construct of *cognitive–conceptual script*, which is based on the work of Kris Gutiérrez and colleagues (K. Gutiérrez et al., 1995), to explore a possible mechanism that links cognition to structures and vice-versa. Whereas the obj–subj dialectic accounts for emergent forms of power during moment-to-moment interactions (i.e., mathematical ascendency; see Ch.5), there are other aspects of the vignette that are better explained through the lens of systemic dimensions of power such as teachers' orientations and the enactment of these orientations in practice.

Gutiérrez, Larson, & Ryhmes (1995) connect day-to-day classroom interactions to a larger theoretical framework based on a poststructuralist critique of education. Building on the work of Bakhtin (Bakhtin, 1981) and his notion of dialogic meaning and social heteroglossia, Kris Gutiérrez and colleagues use scripts as a means of analyzing a teacher's power in the classroom. Applying Bakhtin's literary theory to the microanalysis of classroom settings, they argue that a teacher's power is maintained through a "monologic script" that is characterized by a highly rigid and narrow epistemological stance. They define scripts along two crucial, inter-connected dimensions. First, scripts are orientations that develop after repeated interactions in local contexts; and second, these orientations are influenced by cultural values and norms (K. Gutiérrez et al., 1995). A teacher and students' actions in a particular moment could reify and enforce these scripts, or resist them via *counter-scripts*. My analysis presented in this chapter builds on this work by operationalizing the notion of scripts within a specific content domain, algebraic generalization.

The two dimensions of scripts, just mentioned above, have practical implications for the teaching and learning of mathematics. During instruction, a teacher's enacted scripts take the form of pedagogical moves that either implicitly or explicitly evaluate what counts as valid mathematical knowledge and participation. This, in turn, creates both mathematical and social

status hierarchies that prioritizes certain forms of argumentation over others (e.g., formal symbolism has a higher status than explanations involving gesture and other communicative measures).

Using the construct of scripts, I analyzed an extended episode from the focal vignette to determine the teacher's pedagogical orientations, their underlying causes, the enactment of these orientations, and their effect on student engagement. I operationalized Mr. Lam's *cognitive–conceptual script* in terms of the epistemic orientations he used to interpret the activity of students and to guide his own participation in the interaction. Similarly, students enacted a cognitive–conceptual *counter-script* when they disagreed or did not comply with the teacher's view of appropriate mathematical participation.

The larger corpus of ethnographic data suggests that Mr. Lam's cognitive–conceptual script consisted of a "monologic" view of generalizing. I mentioned in Ch.3, that Mr. Lam viewed the disciplinary skill of mathematical generalizing as an essential component of his pedagogical practice. The ethnographic data revealed that he considered authentic generalizing as "abstractions" that involve "letters." Mr. Lam at times conflated the cognitive action of generalizing with using formal symbolism; in other words, his statements blurred important distinctions between *reasoning* and *semiotic mode*. This caused him in some instances to overlook other authentic forms of generalizing that did not rely on alpha-numeric symbols. These elements of a cognitive–conceptual script, pertaining to generalizing specifically, were reified during the vignette, as we will see next.

Manipulating the Imbrication

The following video transcripts include notes about movement, gaze, and other actions in the form of descriptions inserted in the text. For an explanation of other transcript conventions, see Table 5, above, in Chapter 5.

Episode 11 – Time Stamp [25:18–29:54] – Team 1 with Mr. Lam

As Episode 11 begins, the attitudes and sentiments of the students toward Mr. Lam are best characterized with the expression, "a chill in the air." Thalia's responses are sometimes curt, in a way that suggests she is frustrated with Mr. Lam, or her teammates, or the general activity. Ailani's responses to the teacher reflect similar sentiments. Xeni's decision to read her book and draw for the majority of activity certainly contributed to the initial dynamic between the students and Mr. Lam, even after Mr. Lam's repeated invitations and encouragements to Xeni to become involved. Yet throughout this episode, Mr. Lam's tone during his interactions with the students can be aptly described as professional and even diplomatic. He challenges the students personally and mathematically, and he does so in a respectful manner.

The social mathematical antagonism that we saw in earlier episodes features prominently in this episode as well. This antagonism takes the form of adversarial interactions among the participants, namely between Mr. Lam, Thalia, and Ailani; but it also takes the form of productive engagement, where one particular encounter between Ailani and Mr. Lam, for example, may have spurred Ailani once again to ascend to a higher level of generality (i.e., operating in the Contextual mode).

This episode sees Xeni finally become involved in the conversation. Prior to Episode 11, she gave little indication that she was invested in the mathematical activity. With the sections that follow, I aim to demonstrate tha Xeni's contributions were privileged over those of the other

students. In particular, Ailani's contributions are ignored and Thalia's are not fully sanctioned by the teacher.

As the episode moves along, the tone of the conversation changes from "a chill in the air" (i.e., adversarial and apparently unproductive) to happy, somewhat joyous, and excited. Also, I argue, throughout not only this particular episode but the entire vignette, Ailani, Thalia, and Xeni's antagonism toward Mr. Lam, one another, and the activity, paradoxically, reflects and even propels their increasing interest and involvement. There is a lot at stake for everyone, not only to "save face" and play "the school game," but to express themselves authentically in this environment and display themselves and their teammates as competent. The antagonistic dynamic sometimes worked against the group's productivity, but I maintain that it fostered students' personal, deep engagement with not only content but with each other, supported personal expressivity, and created numerous opportunities for the three students to struggle with and defend their emerging identities as competent doers of mathematics. Furthermore, it is interesting to note that amid the authority figure, Mr. Lam, the three students align. In particular, Thalia and Ailani have their "semiotic battles" over signs of power (see Episodes 4, 5, and 8), but in this episode they come together as a "united front" against the teacher. Through it all, the students' displays of mathematical reasoning were at times not ideal, and the interactions took confusing turns, but my main argument is that: (a) Mr. Lam's and the students' actions reflect their collective attempts to manage the power dynamic, which (b) is imbricated with the actual math talk.

I now present the transcript for Episode 11, breaking it down into smaller sections and interleaving the analysis plus discussion for each section. I highlight how the power dynamic shifts and how these shifts impact students' co-construction of the solution.

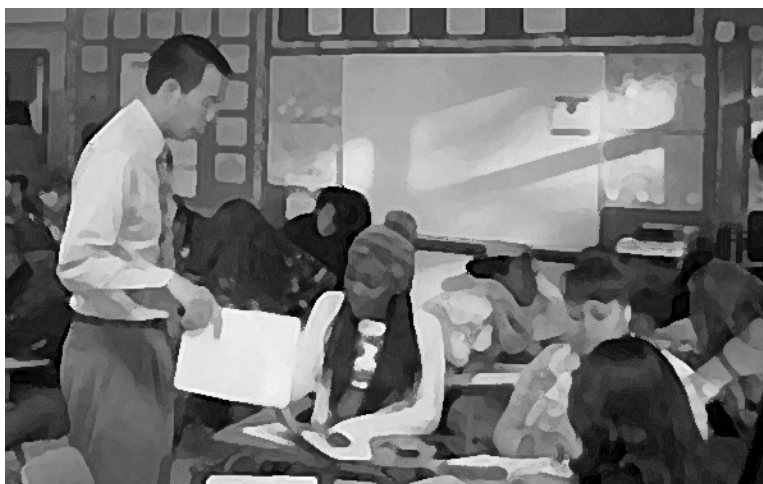| Line#	Speaker	Utterance/Action/Image

a.
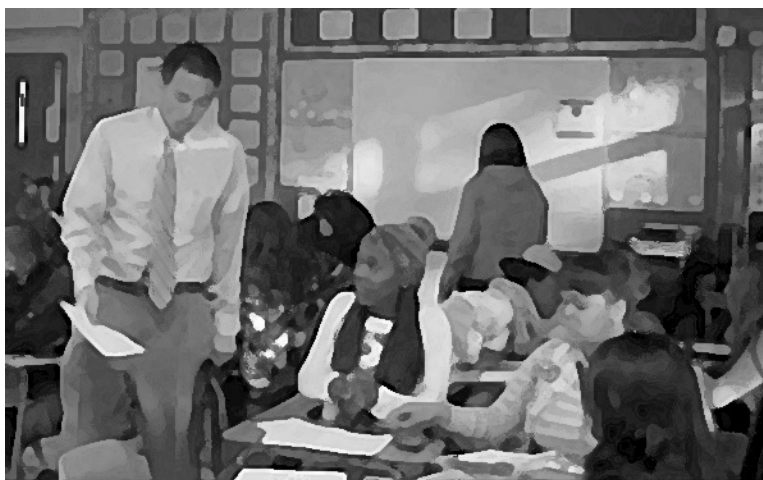
342	Thalia:	[*to Mr. Lam as he approaches the group* (a)] Are we done?"
343	Mr. Lam	[*stands next to Thalia*] "Yeah did you explain it to him [*Mr. Gutiérrez*]?"
344	Thalia:	[*in a brusque tone that suggests she is frustrated*] "Explain what?"

| 345 | Mr. Lam: | |

b.

[*indicates poster and other items on the table* (b)] "Uh what you guys saw in your.."

346	Thalia:	"How I got Fig. 10?"
347	Mr. Lam:	"Yeah."
348	Thalia:	"Yes I explained it. Why you got her bag?"
349	Mr. Lam:	"Cause [*Petra, another student*] went to the bathroom."

| 350 | Mr. Lam: | |

c.

[*kneels next to Xeni* (c)*; addressing Xeni and Ailani*] "So/ay ay hey hey/I think if I were asking to ah get like a nice thing to put on my board like I did for Thalia then maybe you guys would be successful in the task//"

351	Xeni:	//[*hands Ailani her drawing*]//
352	Mr. Lam:	"=but the task right now is looking at this [*picks up M5 poster and orients it to face Ailani and Xeni*] and coming up with a pattern."
353	Thalia:	"I already explained the pattern."
354	Mr. Lam:	[*to Xeni*] "You see how I gave you a task? This is what I was wanting you—"
355	Ailani:	[*quickly glances at the board, presumably at the two guiding questions, then gazes down at a sheet of paper and starts drawing*]

356   Thalia:

d.

[*holds up table of values/codes* (d)] "We just did it, look it! We just did that."

357   Mr. Lam:

e.

"OK so, if weee just did it, I want to hear from Xeni [*places his hand on Xeni's shoulder* (e)]."

358   Thalia: "OK."

As Mr. Lam approached the group, Thalia immediately asked, "Are we done?" with a tone that suggests that she wants to reach a conclusion with the activity straightaway. In interacting with Mr. Gutiérrez and Ms. Urrutia in previous episodes, Thalia had given much indication that she believed she had satisfied all the requirements of the task when she articulated her recursive strategy in both the Factual and Contextual modes. She showed signs of frustration with the activity structure and with the fact that her work had not yet been fully sanctioned by one of the teachers.

Using subjective narrative, Thalia commented that "I already explained the pattern [to the other teachers]," and thus positioned herself as having satisfied the requirements of the task, as she understood it. As Xeni was getting challenged by Mr. Lam to participate, Thalia exclaimed that "we just did that" to express solidarity with Xeni and to signal to Mr. Lam that the team had collaborated (they had not). Mr. Lam was not fooled. He challenged their mascarade by singling out Xeni even more. Interestingly, this was not the first instance where Thalia spoke up for Xeni.

In Episode 10, Mr. Gutiérrez was attempting to orient Xeni toward taking action related to the task, when Thalia jumped in (Line 309): "She [Xeni] does know it! She knows you have to add one number—you have to add one to the end of the numbers."

Thalia once again, as we have seen throughout the data presented in this dissertation, positioned herself as a matheamtical athority. But in this opening section, we see that she did not express her solution with the same variety and complexity of semiotic resources that she had used in previous episodes. Instead, she resorted to verbal utterance to make her point to Mr. Lam. She attempted to draw his attention to her work (table of values) but Mr. Lam instead chose to address the situation with Xeni (to encourage her participatition).

The transcript below picks up with Mr. Lam questioning Xeni about the task. In particular, we'll see his tactics shift across various semiotic modes. Just as the students must navigate the semiotic spaces associated with generalizing activity (see Chapter 4), so too must Mr. Lam navigate these spaces with them.

| Line#	Speaker	Utterance/Action/Image
359	Mr. Lam:	[*to Xeni*] "What did you notice about any figure number? So alright let's start//"
360	Xeni:	//"Any figure number in.."//
361	Mr. Lam:	"=what was the code for Fig. 10?"
362	Xeni:	

f.

[*takes poster from the table, holds it up with her left hand and indicates certain elements with her right* (f)] "OK so for Fig. 10 we just have to add uhm from here on it has like one/we just add one to this part to the last.."

363	Mr. Lam:	"Yeah, to the last one. So it's/what's the first two [*code entries for Fig. 10*]?"
364	Xeni:	"Uhm.."
365	Thalia:	[*interjects*] "Two three."

366   Mr. Lam:
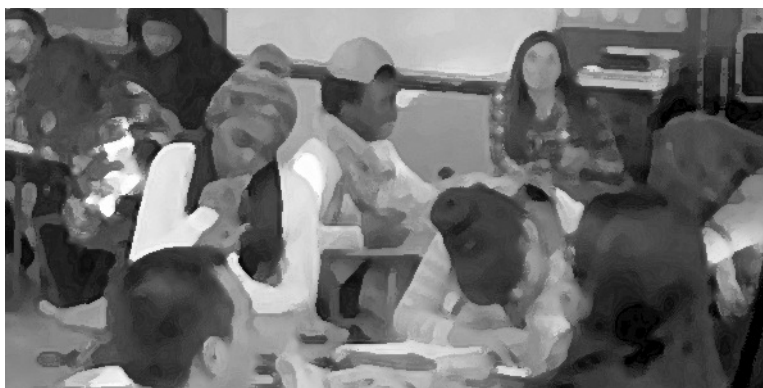
g.

[*picks up what Thalia said* (g)] "Two three, no matter what."

367   Xeni:

h.

"Yeah and then the last number which is n.. [*makes a sweeping gesture with her hand in the air* (h)] n plus one."

Mr. Lam began his inquiry by operating in the contextual level of generality, asking Xeni if she noticed anything about "any figure number," but then decided to instead start off with a Factual case (Fig. 10). Xeni's response shadowed Mr. Lam's—she too began to formulate a response in the Contextual mode but then shifted her attention back to concrete elements (Lines 359–362), observing that "for Fig. 10 […] we just add one to this part to the last." Xeni's statement represents an arithmetic-recursive generalization regarding the third entry of the Code (this generalization is reiterated in the transcript that follows, just below, and she elaborates on it to include the first and second entries). Considering all the contextual factors surrounding this utterance, however, Xeni's "we" indicated her compliance with Mr. Lam's request to become involved in the discussion, as well as a contribution to the mathematical discussion. Xeni positioned herself as: (a) willing to enter the discourse; but at the same time, (b) to enter it powerfully—already attempting to articulate statements in the Contextual level of generality.

As Xeni entered the mathematical discourse, her pronominal usage started as grounded in concrete elements and used subjective narrative, but there was already indication that she tended to operate in—or jump to—an objective, impersonal pronominal mode.

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Xeni continued to position herself as a mathematical authority by jumping to the Symbolic mode, referring to the third entry as "the last one which is n—n plus one." Xeni took the discourse to the Symbolic mode and rearticulated her generalization of $\{2, 3, f_3(n-1)+1\}$ as $\{2, 3, n+1\}$—which was her attempt at expressing a closed-explicit generalization *to* that mode (Gutiérrez, 2010) but perhaps in error. That is, Xeni's "*n*" did not index ordinal position, rather it referred to the third entry of the previous figure, $f_3(n-1)$. Xeni perceived the third entry, which she called "n," as an indeterminate quantity, that is, an element in flux.

Next, Mr. Lam acknowledges that he is following along with Xeni's use of the term "n" and is attempting to unpack its semiotic content. It is clear that Xeni has moved the discourse into the Symbolic mode; what is not clear however, is whether or not her statement of "n+1" was intended to be direct algebraic formula or an arithmetic-recursive solution. Mr. Lam astutely recognizes this ambiguity and asks for clarification, below.

Line#	Speaker	Utterance/Action/Image
368	Mr. Lam:	"Yeah so n plus one, so the n for Fig. 1 here was sssix?"
369	Xeni:	"Yeah."
370	Mr. Lam:	"OK so two-three-six, two-three-seven, two-three-eight, two-three-ninnne"//
371	Thalia:	//"Then ten, and then eleven."//
372	Mr. Lam:	"=so then Fig. 10 was?"
373	T. & X.:	"Fifteen."

374   Mr. Lam:

j.

<center>k.</center>

[*makes a similar sweeping gesture with hand in a "cup" position; to Thalia and Xeni* (j & k)] "Two three fifteen. OK so how would you figure out for any number?"

375	Xeni:	"Uhm//
376	Thalia:	//[*to Mr. Lam, again in a brusque tone that suggests she is hurrying and irritated by Mr. Lam*] "Just add it."//
377	Xeni:	"=just add one more//"
378	Mr. Lam:	//"But.."//

<center>m.</center>

379 Xeni:

"=[*makes a "backward sweeping" gesture with her left hand* (m)] from the first one."

Working together, Xeni, Thalia, and Mr. Lam implicitly articulated an arithmetic-recursive generalization of the form:

$$\{f_1(n), f_2(n), f_3(n)\} = \begin{cases} \{2, 3, 6\} \text{ if } n = 1. \\ \{2, 3, f_3(n-1)+1\} \text{ if } n \geq 2. \end{cases}$$

Mr. Lam used similar linguistic devices as Thalia, such as repetition and rhythm ("two-three-six, two-three-seven, two-three-eight, two-three-ninnne") as a way of cueing a generalization that would produce the code for figures further down the sequence, such as Fig. 10.

Xeni's semiotic resources largely involved indexing with her finger, verbal summation, and a backward sweeping gesture indicating the iterative nature of her method (to "add one more from the first one").

Thalia interjected at multiple points during this exchange, even though it was clear that Mr. Lam was addressing Xeni. Thalia's posture and tone suggest she is frustrated and becoming increasingly irritated with each turn of talk. Whereas on the surface, it appears that the group is collectively achieving a milestone (i.e., co-constructing the recursive generalization above, using formal symbolism, etc.), two confrontations are on the horizon.

<center>78</center>

In order to prepare the reader for the first confrontational interaction to be described below, it is important to rewind for a moment and review Thalia's mathematical behaviors thus far.

It appears there was a status or satisfaction for Thalia in "being done" and getting recognition and credit for her work. Throughout the vignette, there was much evidence that suggests Thalia wanted to be recognized both as an independent thinker and as part of her group. For example, at one point during Episode 7, Mr. Gutiérrez asked Ailani about whether her mathematical contributions were reflected in the table of values. Thalia responded emphatically (Line 202): "Oh no she didn't do that—it's because I was thinking on my own [*places tip of index fingers on sides of her head*]." Curiously though, as much as Thalia claimed ownership over the artifacts and ideas she had produced, she also readily defended Xeni, like we observed in the opening section above, and was willing to work closely with Ailani in earlier episodes.

Up to this point, as we enter the next section below, Thalia seems satisfied with their solution procedure, to "add one more" to the third entry of the code, and her actions do not indicate that she ascribes or acknowledges any utility to Mr. Lam's inquiries. The data strongly suggest that as far as Thalia is concerned, she and her team members have satisfied all the requirements of the Team Task yet their procedure has not been fully sanctioned. Below, just as Mr. Lam poses a question intended to expose the limitations of their recursive method, Thalia's patience reaches a tipping point and she speaks up forcefully, insisting that Mr. Lam clarify the goals and objectives of the task at hand.

Line#	Speaker	Utterance/Action/Image
380	Mr. Lam:	

n.

		[*makes the same sweeping/cupped gesture with hand* (n)] "The first one. But how/so we have to know the one before it?"
381	Xeni:	"Yeah."
382	Mr. Lam:	"Is there a way that we don't have to know the one before it?" (2 sec)

383   Thalia:

p.

[*stomping her feet; to Mr. Lam, talking with her hand in front of her mouth* (p)] "Bruh, why you asking all these questions?"

384   Ailani:

q.

[*quickly raises her gaze from her paper to Mr. Lam* (q)] "Yes!"

385   Mr. Lam:

r.

[*accentuating his words with a "pinching" gesture* (r)] "My name is not 'Bruh.'//

386   Thalia: //[*buries her face in her hands and starts laughing sheepishly*]//

| 387 | Mr. Lam: | |

s.

"=[*still making the pinching" gesture* (s)] "My name is Mr. Lam, and I'm challenging you. That's why I'm asking you//

| 388 | Thalia: | //[*talks with her face in her hands*] "OK Mr. Lam why you asking (all these questions)."// |

| 389 | Mr. Lam: | "=these questions, because I want you to get smarter. So.." |

At first glance, one might interpret this interaction as a moment where Thalia 'lashed out' inappropriately 'against teacher,' expressing her frustration with the mathematical activity and with Mr. Lam's constant questioning. In the moment, a teacher might have read Thalia's apparent recalcitrance as her unwillingness to continue with the discussion because she wants to be "done" with math. From the data however, it is clear that Thalia was frustrated with the situation and her sentiment was powerfully expressed. I argue that her outburst was not so much a case of her 'lashing out' inappropriately or immaturely 'against teacher,' but rather her outburst constituted a valid concern, to understand "why you asking all these questions?" The volume of her voice, accompanied by the stomping of her feet, shows that her outburst was emotively fueled, and yet I claim that her actions, to their core, emanated from a sense of ownership and authenticity that resulted from her engagement with the mathematics and her participation as a member of the group. In other words, Thalia's question—what I would deem her primary action in this instance—was relevant to the mathematical discussion: she inquired about Mr. Lam's expectations for an appropriate solution procedure, and insisted on clarification about what exactly she and her team needed to do in order to satisfy all the requirements of the task. Her secondary or accompanying actions, which consisted of stomping her feet, raising the volume and using a sharp tone of her voice, and referring to Mr. Lam as "Bruh," may have been interpreted as an act of disrespect toward the teacher. Calling Mr. Lam "Bruh" did not acknowledge his role as the teacher nor his authority.

The data indicate that Mr. Lam responded to Thalia's secondary actions and not her primary one—initially.

Perhaps Mr. Lam viewed Thalia's secondary behaviors (raised voice, stomped feet, etc.) as an indictment on his teaching or his motives for teaching, thus he responded not to the actual content of her question, but responded instead to these secondary behaviors. So doing, Mr. Lam's response shifted the discourse outside of the realm of the mathematical activity that was occurring in the here-and-now to a discursive space beyond mathematical practice or math class. He expressed a general sentiment: "I want *you* to get smarter" (my italics), which I argue does not refer solely to Thalia but to *all* his students. That is, this is an ideological statement whose

core assumptions govern Mr. Lam's pedagogical practice (see Chapter 3 for further descriptions of Mr. Lam's expressed ideological beliefs and their relation to his teaching practice)—a microcosm of which is reflected in his dealings with his students during Episode 11.

Mr. Lam's tone of voice remained respectful, diplomatic, and professional. He did not lose his "critical cool" by any means; he maintained focus on the task at hand and provided a clear response to Thalia's inquiry. His voice rose slightly as he spoke, but I would not deem this as an over-reaction; it was a measured response that may have been intended to be motivating. That said, his reply did connote problematic assumptions. His reply of "I want you to get smarter," might assume that Thalia (and all other students) were not smart already or smart enough. Furthermore, from a broader critical perspective, his reply couched mathematical practice within a "smartness" frame that scholars have argued is detrimental or has little benefit to (mathematics) education (e.g., Horn, 2007; Leonardo & Broderick, 2011). This, I claim, constitutes a power-driven misalignment between Thalia's question (the primary behavior/content) and Mr. Lam's response that attended to Thalia's secondary behavior. With his statement, "I'm challenging you […] because I want you to get smarter," Mr. Lam enacted his ideological beliefs about the nature and role of mathematics learning in the life of a child.

One plausible interpretation of this particular exchange, given the wealth of ethnographic data on Mr. Lam's orientations toward teaching and working with youth, is that his response was meant to be *empowering* for Thalia. In the moment, however, his comment of "get smarter" was counterproductive. In the footage, Thalia raised her hands to her face, lowered her head slightly and sunk lower into her seat, indicating that she had become self-conscious or even ashamed, and thus was compelled to retreat or hide behind her hands (see Line 387, image (s), above). Whereas Mr. Lam generally intends to empower his students by offering certain ideational resources (Nasir & Cooks, 2009) about mathematical practice (see Chapter 3), and in this particular moment perhaps meant to empower Thalia and her teammates by this meta-script, the net effect was that he trampled her agency: Mr. Lam momentarily impeded her ability to participate in the conversation with confidence, and impeded her *mathematical* agency in particular. Having re-established the boundaries and 'rules for conduct' (i.e., you cannot refer to teacher using slang, such as "Bruh"—which maintains teacher's and students' respective classroom roles and the relations of power inherent in those roles), he then enacted his cognitive–conceptual script, immediately returning to using tactics that aim to guide the students toward a normative understanding of the solution procedure.

In the segment that follows, Ailani continues the confrontation with Mr. Lam while Thalia stays "hidden away" behind her hands. Mr. Lam shifts the discourse back to a mathematical sphere and poses questions to the group. Thalia comes "out of hiding" to make a contribution but it is immediately challenged by Mr. Lam (he asserts standards of productivity and conceptual understanding), thus stoking Thalia's frustration even more.

Line# Speaker   Utterance/Action/Image

390   Ailani:

t.

[*scowls at Mr. Lam; under her breath* (t)] "(??)"

391   Thalia:

u.

[*torques her body toward Ailani, peeks at her from behind her hands and starts laughing* (u)]

392   Mr. Lam:

v.

[*to Ailani* (v)] "Thank you for holding it."
[*to group*] "Two three and then? (1 sec) That's the real question! Can you figure it out //without knowing"

w.

393	Ailani:	
		//[*does not look at Mr. Lam, keeps gaze down at her drawing; loudly* (w)] "YES, YOU COULD! YES, you could!"//
394	Thalia:	//[*rubs hands over her face, burying herself more; laughs again from behind her hands* (w)]//
395	Mr. Lam:	[*leans in toward Ailani*] "WITHOUT knowing the ones before it?"

x.

396	Ailani:	
		[*gaze still down* (x)] "YES!"
397	Mr. Lam:	[*to group*] "So could you even figure out Fig. 100// right now?"

| 398 | Thalia: | |

y.

		//"Like I just looked at// this picture [*reaches across her desk, indicates poster figures* (y)] and they didn't even give me the sequences. I just looked at it// and counted the thingy."
399	Mr. Lam:	//"Yes but if I don't give// you the picture and I don't tell you what Fig. 99 looks like, can you tell me Fig. 100?"
400	Ailani:	[*gazes down at her paper*] "Yeah."
401	Thalia:	"Yeah."
402	Mr. Lam:	"What would it be? You know the first two, right? Two three and then what's that last one? That's my question. How can you generrraliiize so that you don't relyyy on the ones before it?"
403	Thalia:	

z.

[*sighs loudly; rubs her face* (z)]

Ailani first entered this confrontational encounter at Line 384, in the previous section above, by shouting "Yes" in support of Thalia's interrogation of Mr. Lam. She added to the confrontation by making a comment under her breath (Line 390, inaudible) which, judging by Thalia's and Mr. Lam's responses, was likely a snide remark. Mr. Lam attempted to repair the conversation and to get it back on track by asking a question at the group level (see Line 392). Ailani continues to assertively position herself as an authority in the conversation, exclaiming "Yes," that it is possible to figure out a pattern "without knowing the ones before it." Mr. Lam continues to push the issue, challenging the group with "So could you even figure out Fig. 100 right now?" adding the parameter of "I don't tell you what Fig. 99 looks like, can you tell me Fig. 100?"

Throughout this particular section, the social-mathematical antagonism took on the form of overlapping speech and interruptions at various points in the interaction. Thalia's response (Line 398) took on the form of a subjective narrative and contained unspecified objects ("thingy"). Mr. Lam did not ask Thalia to unpack the ambiguous referents within her narrative. Instead, he 'agreed then disagreed' with Thalia's method; he acknowledged her method of "looking at this picture" and counting-on, but then he quickly moved to demonstrate how her method might fall short for figures much further along the sequence, such as determining Fig. 100 without any knowledge of the Fig. 99. Governed by his cognitive–conceptual script, Mr. Lam did not decide to look at the strategy up close, rather he highlighted its limitations and thus tacitly positioned it as insufficient. Mr. Lam continued in critiquing the insufficiency of Thalia's

strategy even after both Ailani and Thalia had acknowledged that "yeah" it was possible. He asks in Line 402: "How can you gennneraliiiize so that you don't relyyy on the ones before it?"

It is important to state that I do not wish to pillory Mr. Lam's pedagogical approach here. In fact, the particular series of prompts and tactics that he employed (i.e., asking about Fig. 100 without relying on the previous figure) are commonly used and intended for students to realize the limitations of an arithmetic-recursive strategy and thus the need for more powerful tools such as algebraic generalization. At the same time, these prompts establish a *minimum standard* that had not yet been reached by the group (in the next chapter, I conceptualize this phenomenon through the Foucauldian lens of "normalization"). Furthermore, Mr. Lam's question (Line 402) assumes that the students have not yet been engaged in authentic generalizing. The data from this vignette, combined with evidence from the Phase 1 ethnography (see Part 3 of Chapter 3, above), suggest that Mr. Lam was oriented toward overly valorizing one particular form of generalizing (Symbolic closed-explicit solutions), which made him liable to overlook other forms of authentic generalizing. I argue that Thalia's work *did* reflect authentic generalizing, although in the form of recursive solution procedures. Thalia's utterances reflect the work that she put into developing personal meaning of the source situation and into progressing in her modes of semiotic action from the Factual to the Contextual (see transcription above, particularly those turns leading up to Line 383). It is not surprising then, that Thalia became so frustrated, because her mathematical work and competence had not been acknowledged.[16]

I argued in previous chapters that the threshold to operate in the Contextual and Symbolic modes is perhaps lowered during high-stakes interactions, where status and identity are negotiated and challenged. In the section that follows, I aim to further illustrate this point with another example of how the power struggle inherent in the conversation may have triggered Ailani to position herself as a mathematical authority and, notably, doing so in the Contextual mode without any clear evidence up to this point that she had established herself successfully in the Factual mode. Furthermore, the following section also shows how the students shift the discourse away from the realm of mathematics to other cultural frames, which in turn serves to shift the power dynamic.

Line# Speaker   Utterance/Action/Image

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[16] This, at a time when Thalia was determined to participate in math class. During a whole-class conversation that took place immediately after this focal lesson, Thalia mentioned that she was feeling "bipolar," meaning that she did not want any part of the activity during Period 1, but in Period 2 she just "wanted to do work." There is evidence from the onset of this focal vignette, that Thalia sought a challenge and expected to meet that challenge, for example, when she stated that she "wants a hard one [M-problem]." As the activity went along, Thalia maintained a high level of enthusiasm and engagement (as compared to her team mates, Ailani and Xeni); Thalia constantly took time to care for her work materials, including all her worksheets, making sure her work area was organized and tidy.

404   Ailani:

aa.

ab.

[*leans back in her chair, throws her arms behind her* (aa), *looks to Mr. Lam then leans forward as she begins to talk to Mr. Lam, pounding the table with the marker as she talks* (ab)] "Well maybe/OK so (like) two three and you don't have to know all of them, you know the first/you can know the original one and then!// And then figure out ///Fig. 99."

405   Mr. Lam: //"OK, so.."//

406   Thalia:

ac.

		///[*interrupts Aliani, softly clapping her hands together as she speaks, accentuating each word as she talks* (ac)] "Why are you talking so ///ratchet?"
407	Mr. Lam:	"OK yeah don't get, don't get hyped up//against me."
408	Thalia:	

ad.

		//[*quickly points to Mr. Lam* (ad)] "*Ratchet.*// Say it, don't get *ratchet.*"
409	Mr. Lam:	[*unclear where Mr. Lam gaze is here, but it appears he turns his gaze away from Thalia and addresses Xeni and Ailani*] "So alright let's look at Fig. 1. What was n for Fig. 1?"
410	Thalia:	"Six." [*starts laughing*]
411	Ailani:	[*giggles*] "(??)"
412	Mr. Lam:	[*to Thalia and/or Ailani*] "One was six. Two was seven. Three was eight. What do you notice about the pattern between four and nine?
413	Ailani:	[*in a high-pitched voice that suggests she is not taking Mr. Lam's inquiry seriously*] "I don't know, BUDDY BUDDY!
414	Mr. Lam:	"One and six, two and seven, three and eight, four and nine, what do you notice?"
415	Thalia:	"Ohhh myy god." [*gazes at the poster*]
416	Xeni:	

ae.

[*counts on her fingers* (ae)]

According to Radford's theory of knowledge objectification (the "obj" side of the dialectic), Ailani's utterance (Line 404) marks a crucial shift along the F-C-S chain of signification. Specifically, the last part of her utterance reflects a generalized action or operation, which was articulated in the Contextual mode, when she claimed: "you can know the original one and then add on" in order to "figure out Fig. 99." Her recursive-arithmetic procedure, from a semiotic perspective, was expressed in a higher level of generality than her previous contributions, which indicates the development of her understanding of the problem. Flagging this shift-to-Contextual as a milestone for Ailani, at this juncture it is important to now examine carefully the mathematical content of Ailani's contribution vis-à-vis the dynamics of subject positioning that occurred, that is, to look at the "-subj" side of the dialectic.

One possible interpretation of what we can attribute to Ailani's mathematical understanding, a generous one, is that Ailani understood the arithmetic procedure and how to execute it, thus this is a case of expressing declarative knowledge. Or, it could be that Ailani picked up the word forms from Thalia and Xeni, who had been verbalizing the same arithmetic solution procedure just a few moments earlier, and thus Ailani appropriated those word forms without actually having understood the mathematics inherent in those forms. This latter explanation is indeed quite plausible, given that her articulation, on the surface, is more inchoate than Thalia's and Xeni's productions. Tabling this issue for the moment (unpacking the mathematical content/referents of Ailani's utterance), I would also focus on the fact *that* she spoke. By acting at this particular moment, Ailani positioned herself as someone capable of making a contribution to the discursive space, and as someone who can "stand up to teacher." My analysis of not only this section of transcript, but as a general analytic approach, hinges on the notion that *the co-construction of both power relations and mathematical knowledge can be reflected within a single interaction or even within a single utterance*. At Line 404, Ailani made a single move that both contributed to the mathematical discourse and resisted Mr. Lam's positioning of herself and the group in relation to his authority.

Thalia and Mr. Lam responded not to the mathematical content of Ailani's comment, but to the way she expressed it. Thalia directly reproached her, using the derogatory term "ratchet," suggesting Ailani was speaking in an unpleasant or distasteful manner. Mr. Lam, perhaps unaware of this word's derogatory meaning, did not in turn admonish Thalia for inappropriate use of language and disrespecting her peer; instead he agreed with Thalia, adding "don't get hyped up against me," which tacitly, if inadvertently, condoned Thalia's behavior. Thalia capitalized on this moment, sensing she had the upper hand, and pushed the boundaries of safe and appropriate classroom conduct a little further. By turning her attention to Mr. Lam and insisting that he use the inappropriate term, "say it, don't get ratchet," Thalia attempted to perturb the power dynamic, thus appropriating social status and authority. In particular, she momentarily reversed the roles so that she was "playing teacher" and Mr. Lam was now "playing student."

Through it all, Ailani's contribution went unacknowledged. Based on Thalia's and Mr. Lam's responses, it appears only Ailani's tone of voice and posture were recognized (possibly as "hostile") but the content of her proposition was overlooked.

From this point on, Ailani gave less and less indication that she was following along mathematically, even though she continued to participate in the discourse. Most of her contributions from here on were not related to the task, but all her actions nevertheless serve to position herself in particular ways within this shifting dynamic (see Lines 411 and 413, above).

Mr. Lam continued to prompt and question what Xeni had stated earlier and thus shifted the discourse back to the Symbolic mode and grounded the notion of an indeterminate quantity in known cases (Line 409: "What was n for Fig. 1?"). But not all participants were party to this particular type of discourse, and so Mr. Lam jumped in to answer his own question (Lin 412). He ultimately privileges one student's contributions (Xeni's), which had been cast in the Symbolic mode, and thus directly promotes the dominant cognitive–conceptual script. This gives status to Xeni (an already high-status student), and sanctions Thalia's marginalization of not only Ailani's discursive contribution, but her personhood when she refers to her as talking "ratchet."

In the next segment of transcript, we will observe that only Xeni was able to respond to Mr. Lam's line of questioning that was cued in the Symbolic mode, and only she was able to take very crucial first steps toward shifting conceptually, from recursive relationships to closed-explicit ones. Lines 417–450, below, constitute the longest, final section of Episode 11. Before moving forward though, I would remind that just above we left the last section with Xeni counting on her fingers, while Mr. Lam, Thalia, and Ailani were all talking. Xeni's counting strategy will soon lead to an insight of a numerical pattern between the figure number and the code, which she is excited to share with Mr. Lam. However, as will be shown, Xeni is not the only one who makes a mathematical insight; both she and Thalia speak up at the same moment, but again Mr. Lam focuses on Xeni's statement.

Line#	Speaker	Utterance/Action/Image

417   Mr. Lam: [*to Xeni*] "You want me to write those numbers down?"

af.

418   Thalia:

[*reaches across table, indicates something on poster* (af)] "Ohh oh oh yo //I get it!"

419   Xeni: //[*points to the poster; looks at Mr. Lam*] "Plus five!"//

420   Thalia: [*briefly looks at Mr. Lam then shifts her gaze back to poster*] "You// add one to six and then you add two and then [*makes swirling gesture*] dah dadah dadah dadah. I don't know."

421   Xeni: //"Plus five."//

422   Mr. Lam: "What did you say, Xeni?"

423   Xeni: "Plus five."

424   Mr. Lam: [*referring to the poster*] "So Fig. 1, 2, 3, 4, 5, and then the n was

90

six seven eight nine ten. So you're saying what plus five?"

425	Xeni:	"The figure.. figure number plus five."
426	Mr. Lam:	"Plus five. So you take this [*figure number*].. add five. So if I asked for Fig. 10, [*to Thalia*] you told me it was?"
427	Thalia:	"Fifteen."
428	Mr. Lam:	"Fifteen. If I told you Fig. 100, you're telling me.."
429	Xeni:	"A hundred and.. five."
430	Mr. Lam:	"A hundred and five. So the code would be what?"
431	Xeni:	"Plus five—figure number plus five."
432	Mr. Lam:	"Figure number plus five//"

ag.

| 433 | Thalia: | //[*in a tone that suggests she is frustrated* (ag)] "OK there are we done now?"// |
| 434 | Mr. Lam: | "=so it'll be two three and then figure number plus five." |

ah.

435	Thalia:	[*same frustrated tone* (ah)] "OK are we done now?"
436	Mr. Lam:	"I don't know. *Are* you done?"
437	Xeni:	"Yeah."
438	Thalia:	[*glances at Xeni's and makes a gesture suggesting they are in agreement*] "Yeah, we're done."
439	Mr. Lam:	"So if I asked you figure number n, could you tell me what it

would be?" [*stands up*]
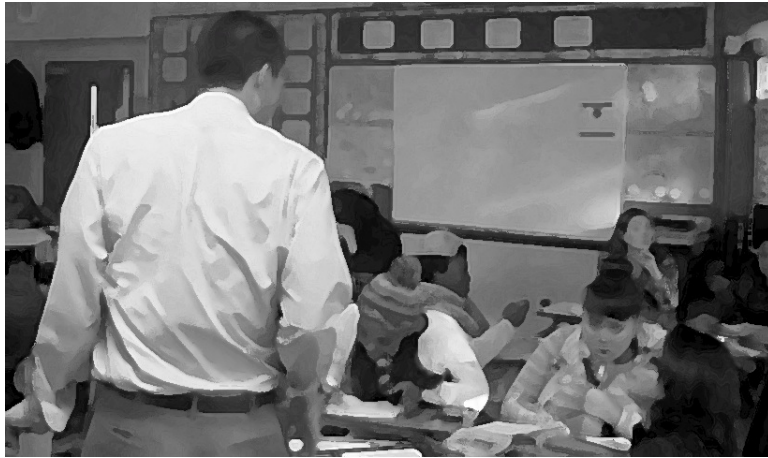
440   Thalia:

aj.

[*looks down at her paper* (aj)] "Dude stop asking/I mean Mr. Lam!
Stop //asking all these questions!"

441   Xeni: //"Plus five."//

442   Mr. Lam: [*his tone and posture shifts to more playful; smiles to Xeni*]
                    "What/what plus five?"

443   Ailani:

ak.

[*to Mr. Lam, as he stands to walk away* (ak)] "The NUMBERS
plus five! The FIGURE number."

444   Xeni: "The figure number plus five."

445   Ailani: [*quickly shifts her gaze from Thalia, to Mr. Lam, to Xeni*] //"ONE
                    TWO PLUS FIVE!"

446	Mr. Lam:	
		am.

		//[*smiling* (am)] "So figure number n, what would that be?"
447	Thalia:	[*aloud, to no one in particular*] "OK, do we have all this stuff down now?"
448	Xeni:	[*loudly, to Mr. Lam who has walked across the room*] "N FIVE!"
449	Mr. Lam:	[*from across the room, still smiling*] "N five?"
450	Xeni:	"I mean FIVE N!"

Xeni arrived at her closed-explicit formula by counting on her fingers and noticing a pattern, when she announced "Plus five!" Recall that this was in response to Mr. Lam's direct question about the numbers he was articulating in a turn of talk just prior to this last section (Line 414: "One and six, two and seven, three and eight, four and nine, what do you notice?"). This utterance found Xeni in the Facutal mode, as Mr. Lam cued it, and then she followed by referring to specific codes in the table of values.

As Xeni made the caluculation with the aid of her fingers, Thalia was noticing another recursive numerical pattern in the table. Thalia inferred that the code for Fig. 1 contained the "seed" for each subsequent code; starting with the code for Fig. 1, {2, 3, 6}, "You add one to six" to get the code for Fig. 2, {2, 3, 7}, "and then you add two.." to get the code for Fig. 3, {2, 3, 8}; and with the utterance, "and then dah dadah dadah dadah," she made a swirling gesture with her finger to objectify that this process continues.

It is interesting to note that both Thalia and Xeni attempted to get Mr. Lam's attention, by speaking to him and looking directly at him. Thalia acted first and Xeni interjected, yet Mr. Lam decided to go with Xeni, thus privileging her over other students once again. Perhaps Mr. Lam interpreted the last part of Thalia's comment of "I don't know" as an indication that she did not want to go further. Whatever triggered his decision-making, what resulted was that Thalia's semiotic actions (what she was indicating on the poster and what she was verbalizing) were overlooked, similar to what had happened to Ailani just moments before. In fact, Thalia's behaviors from this point indicate that she was still engaged and invested in the task objective.

From an analyst perspective, Mr. Lam made interesting moves as he navigated the various semiotic spaces on his end. In one instance, we saw him shift between all three semiotic modes, from the Factual, to the Symbolic, to the Contextual within a single turn (Line 424), when he stated: "(Factual) So Fig. 1, 2, 3, 4, 5, (Symbolic) and then the n was six seven eight nine ten. So you're saying (Contextual) what plus five?" But in situ these shifts did not mark anything significant for all of the students, only Xeni.

Thalia again expressed that she was annoyed with Mr. Lam and his belabored approach to the problem. It is interesting to note that her statements changed from a question—"Why you asking all these questions?"—to an emphatic request or behest—"Stop asking all these questions!"

The data strongly imply that Thalia's frustration with Mr. Lam and with the activity stems from her confusion surrounding (what she perceived as) a belabored demand for a task that, she felt strongly, they had successfully completed already. It is not the case, I submit, that Thalia focused on task completion so she and her friends could be "done with math." Rather, I claim, her tacit criteria for what qualified as a satisfactory solution had long been met. Mr. Lam however, had different criteria for what qualified as a satisfactory solution: the generalization was complete if and when it was applied successfully to the nth figure. In response to Thalia's assertion that they were "done" with the task, Mr. Lam posed a determining question (Line 439): "So if I asked you figure number n, could you tell me what it would be?" While Xeni and Ailani pick up the remainder of the turns in this particular episode with Mr. Lam, Thalia sorted all the sheets of paper (table of values) and organized the materials on her desk, signaling that she was ready to move on from this activity.

We saw in previous sections that Ailani entered the conversation at confrontational moments between Thalia and Mr. Lam, and Ailani's actions in those particular moments largely stoked rather than resolved the conflict. When Thalia challenged Mr. Lam at Line 440, just above, Ailani had an altogether different kind of response as before. Previously, Ailani had jumped in, in solidarity with Thalia; but here, Ailani jumped into Xeni's space, answering Mr. Lam's question regarding Fig. n, stating loudly, "The numbers plus five!"

Xeni, too, answers Mr. Lam's question about the nth figure, stating "Plus five." Xeni's statement was missing any reference to the original quantities and only mentioned an addend, thus Mr. Lam immediately asked Xeni for the same clarification (Line 442): "What plus five?" It was at this point that Ailani attempted to enter the conversation, but her contributions went unacknowledged, again, as Mr. Lam responded only to Xeni's comments (Lines 443–446). Perhaps Mr. Lam viewed Ailani as attempting to "shark" Xeni's turn, or perhaps he didn't pick up on what Ailani was saying due to overlapping talk. In any event, Ailani made clear attempts to participate in the conversation, but Mr. Lam talked over her and addressed only Xeni. Ailani was thus excluded and deprived of an opportunity to access the ideas that were being talked about.

Radford's semiotic–cultural framework sheds some light on Ailani's complex problem solving. Ailani verbally articulated a closed-explicit formula, "the numbers plus five," in response to Mr. Lam's question. Analyzing the video footage and transcription however, I diagnosed Ailani's final contributions as ungrounded generalizations. Ailani gave no indication previously that she had been party to the crucial steps that Xeni and Thalia had been making (regarding the functional relationship between the figure numbers and the codes). Ailani attempted to enter the discourse by expressing mathematical statements that were previously articulated by her peers. The data imply that Ailani had not made the necessary connections inherent in the expression (Line 443): "the [*figure*] numbers plus five." I conclude that in the final moments of this episode, Ailani merely appropriated and reiterated what others had previously stated.

Through the lens of knowledge objectification, it may be instructive to assess Ailani's contributions here, as I have done just above, based on their mathematical accuracy and semiotic grounding. However, Radford's theory does have limitations in interpreting this interaction.

Whereas Radford's framework sheds light on whether and how students' discursive contributions are semiotically grounded, it does not explain why students choose to take up, fight for and hold, or give up their "semiotic turf." To illustrate this point, I want to shift the analytic focus to how Ailani's idea remained with her during the entire 35 minute lesson.

Ailani carried her previous strategy of "you added one or two" (Episode 5, see Lines 151–155) all the way through to Episode 11. That is, she fought against Thalia's, Xeni's, and even Mr. Lam's mathematical ascendency, standing her ground and making one final assertion based on her knowledge of the situation. Her statement of "One two plus five" is a combination of the work that Xeni had just done and Ailani's work from Episodes 4 and 5, when she had used a counting-on strategy to determine Fig. 10. Radford's framework would indeed have an analyst mark this as a case of naïve induction, where her generalization is the result of a random association, jumbling of two solutions, or just a random guess altogether. But looking at her situated agency, on the other hand, revealed so much more about Ailani's participation in the leading discourse. That is, we observe throughout the vignette that the environment was arranged in particular ways, and Ailani at times was quite successful at re-arranging that environment (e.g., when she "won" possession of M5 from Mr. Lam), but sometimes not (e.g., here at the very end of Episode 11, where her contributions went unrecognized) (cf. "ingenuity" by McDermott & Raley, 2011; in particular, see their take on "the precise timing of ability in school").

Ailani never abandoned her solution in the face of adversity, though she did admit that she did not follow Thalia's explanation in Episode 8 (previous chapter). Ailani and Thalia challenged one another throughout the vignette and resisted being positioned lower along a hierarchy that (metaphorically) lurked in the background. Ailani and Thalia defended their propositions, stubbornly so. Xeni's contributions, on the other hand, were not met with so much adversity. She was asked for clarification by Mr. Lam and the other teachers, but her social–mathematical status was never directly challenged. Most importantly, her status was implicitly reproduced when Mr. Lam called on her over others at every turn. Xeni also positioned herself as a participant willing and able to enter the Symbolic mode of action, whereas others did not do so unprompted. So doing, perhaps operating in the symbolic mode caught the attention of the teacher, which he took as a sign of "thinking" or authentic generalizing. This type of knowledge or way of knowing was not attributed to Thalia or Ailani during this episode.

In summary, an analysis of Mr. Lam's discursive action and in-the-moment decision-making revealed that his instructional moves were at times governed by a cognitive–conceptual script. This script took the form of a pedagogical agenda that: (1) codified and enacted certain ideological dimensions of his pedagogical orientations[17]; (2) was mathematically monologic, in that it did not flexibly accommodate students' varied contributions; and (3) was based on an expressly stated belief that possibly conflates semiotic mode with mathematical reasoning. This analysis also revealed that students responded to the teacher's cognitive–conceptual script with their own "counter scripts" that codified and enacted their cultural knowledge and forms.

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[17] See Wertsch's (1998) discussion on "power and authority" as inherent properties of mediated action. In particular, the ideological aspects of Mr. Lam's general pedagogical orientation reflect certain "cognitive values" (Goodnow, 1990, as cited in Wertsch, 1998, p. 66) that are socially and historically constructed. Algebraic generalizations, specifically, and symbolic versions of those generalizations, specifically, is the broader narrative of "what counts" in school math.

Chapter 7: Contributions, Limitations, and Future Research

This final chapter summarizes key findings and contributions, and acknowledges some limitations of this research. To facilitate a final discussion about future directions, I offer a Foucauldian interpretation of the learning–power imbrication through the lens of "cognitive normalization."

Contributions and Conclusions

The main theoretical contribution of this dissertation is a model of mathematics learning that accounts for processes of objectification and subjectification in multimodal interaction. I developed this model based on my previous research as well as findings emerging from my three data-analysis chapters. Ultimately, I maintain that students' personal acts of generalization feature the following properties:

- ("Obj") Shifts from one semiotic mode to the next mark both conceptual understanding and mathematical agency. That is, agency is tantamount to conceptual understanding.
- ("-Subj") As students move along the F-C-S chain of signification, they simultaneously appropriate hierarchical positional identities. Namely, operating in the Symbolic mode has higher status and is perceived as capable of garnering more authority as compared to the Contextual and Factual modes.
- Emerging tensions in the obj–subj dialectic give rise to a learning–power imbrication in practice.

Furthermore, the analyses in this dissertation showed that these theoretical mechanisms played out in actual moments of teaching and learning in ways that impacted the quality of student engagement. We observed in Chapters 5 and 6, for example, that during antagonistic interactions, wherein status was at stake and identity struggles were involved, the threshold for students to shift along the F-C-S chain of signification was perhaps lowered. Moreover, we observed in Chapter 6 that the imbrication was amenable to contextual as well as systemic factors, such as teacher and students' scripts. The findings from Chapter 6 suggested that the threshold for teachers to enact "monologic" scripts lowers during interactions where the *teacher's* identity, agency, and authority were in question.

Moving forward, these observations now lead me to articulate a general hypothesis regarding a fundamental relationship between power and learning: *perturbing the power dynamic participants of mathematical discourse to appropriate agency, status, and authority—thus spurring new mathematical cognitions and understandings.* This hypothesis indeed warrants further investigation as part of future research projects. I take up these issues and discuss some limitations of the obj–subj perspective next.

Limitations and Future Research

In this section I present limitations of this research, related to (1) methodology, and (2) theory and practice.

Methodology

      This dissertation presented a series of analyses centered on the same focal data vignette. Whereas I reported on the full results of a content- and cognition-oriented study that analyzed the students' mathematical assertions and milestones across the entire 35-min. vignette (Ch. 4), this dissertation reports partial results of the qualitative microgenetic analyses of that same vignette. I selected the sub-episodes for Ch. 5 and Ch. 6 because those data highlighted the tension in the obj–subj dialectic involving the regular classroom teacher, Mr. Lam, and three of his students, Xeni, Thalia, and Ailani. The data in those sub-episodes demonstrated a "proof-of-concept" of the learning–power imbrication. That said, there were compelling interactions in the remaining sub-episodes that are relevant to this line of research, such as Episode 7, where Mr. Gutiérrez evoked the frame of "convince a friend" as a means of getting Thalia and Ailani to work together (see Episode 8, Ch. 5). Additionally, there were other interactions involving Mr. Lam and other groups of students that perturbed the learning–power imbrication in different ways. These analyses/results will be reported in future publications.

      Another limitation in this research is related to the school site wherein the data for this project were collected. My research at this site focused on a single teacher and his classroom. I cannot make any comment on whether the school's restorative justice vision and practices impacted student achievement outcomes. However, the ethnographic data does suggest that at least some students shared Mr. Lam's values, goals, and motivation. This project was not set up to document students' perceptions of restorative justice per se, but students occasionally remarked on the broader values of the school, and Mr. Lam regularly spoke to students about the school's value-system and its vision. These issues are indeed relevant for an analysis of power dynamics in school and the ways in which these dynamics play out in classrooms. The analyses/results of the specific teacher practices that attempted to enact César's Chávez's vision of restorative justice will be reported in future publications as well.

Theory and Practice: A Foucauldian Interpretation of the Politics of Mathematics Pedagogy

      In this dissertation, I "looked down" at a focal data vignette involving three mathematics students and their teacher. I described emergent interactions and determined whether/how mathematics learning was supported in particular moments in time. I marked opportunities that were created, completely missed, or constrained in some way. Furthermore, I conducted my analysis in a particular mathematical content domain, that of mathematical generalization. So doing, I was able to carry out my analysis in dialogue with theory and methods from the learning sciences. I theorized, operationalized, and demonstrated a fundamentally dialectical relationship between mathematical objectification and subjectification. At the center of the mathematical activity described in this dissertation are three "semiotic modes" that both students and teachers take recourse to when dealing with patterning tasks in a school setting. I now argue that the F-C-S semiotic modes are actually proxies for broader semiotic spaces inherent to mathematical activity beyond algebraic-generalization tasks. I consider mathematical discourse as occurring in, or rather *creating*, "semiotic spaces" that are socially hierarchical. That is, "The Symbolic" is *generally* perceived as having higher status than "The Contextual" or "The Factual." This semiotic hierarchy stems from a broader historical narrative about who can and who cannot be successful with mathematics, where one's mathematical competence is perceived in terms of fluency with formal mathematical symbolism.

      One limitation of this conceptualization of mathematical discourse and power is that its political implications may not travel farther than the realm of mathematics classrooms. It can be

argued that the obj–subj dialectic that I have formulated in this dissertation, as a semiotic-linked status hierarchy expressed in-and-through the use of semiotic resources, accounts for local instantiations of "mathematical subjectification" recognized only in the "closed space of the mathematics classrooms" (Pais & Valero, 2012, p. 16). This is a theoretical limitation that future studies can directly take up. In the final section, below, I take a brief "look up" from my situated data analysis to explore how the general role of the teacher (and the role of schooling) potentially contribute to broader processes of subjectification, such as the kind Pais and Valero (2012) draw attention to:

> From a Foucauldian point of view, it can be argued that this type of subjectification is a technology of the self, set in operation by expert teachers and researchers, with the effect of providing an effective governmentalisation of the learner into a reduced form of identity as mathematics learner that has to converge towards the social norms of a mathematical culture. (p. 17)

Pais and Valero (2012) point out that: (a) the categories of "teaching" and "learning" are determined by the educational research community in a way that reproduces political status-quo that emphasizes "learning" and "content" as the primary locus of investigation in the lives and times of children in schools; and (b) a learning-sciences approach to research on objectification and subjectification is confined to this already restricted political realm, to the realm of "acquisition of mathematics" and "being with others" in mathematics classrooms. Radford's theory of knowledge objectification, in combination with Sfard's theory of subjectifying, have been rigorously defined and elaborated on in the context of mathematics learning—this is both a strength and a weakness of their theory. I maintain however, that the obj–subj perspective that I offer can be extended to address the concerns raised by Pais and Valero (2012). They write, "the emphasis on mathematical objectification and subjectification makes the research gaze to ignore all the 'non-mathematical' complexities that both teachers and learners experience in their everyday practice" (p. 16). This dissertation is my first step toward taking on the challenge that Pais and Valero pose to the research community. Acknowledging the theoretical limitations of a *mathematical* obj–subj dialectic, next I offer the buds of a broader Foucauldian data analysis and thus push toward articulating a critical interpretation of mathematics pedagogy as the general process of *cognitive* normalization.

Cognitive Normalization. Broadly, normalization is the discourse phenomenon whereby people are categorized as either normal or abnormal (Foucault, 1977). Tansy Hardy (2004) argues that normalizing in the classroom is a technique of power. She observes:

> Connecting the notion of normalization with power reveals the process that determines what becomes valid knowledge in the classroom and how that knowledge can be expressed and by whom. It is the normalization process that determines, for example, who "has the difficulties" and who does not. (Hardy, 2004, p. 109)

Previous research shows that normalizing occurs in a variety of educational settings (Gore, 1995; Hardy, 2004), and recent studies demonstrate that normalizing—among other techniques of power—can stymie the implementation of reform-oriented regimes in math and science education (e.g., Donnelly, McGarr, & O'Reilly, 2014). These studies highlight that the

normalizing exercise of power impacts learning outcomes by mediating students' *opportunities* to engage the content. Yet, in line with my general methodological and theoretical approach outlined in this dissertation, I conjecture that normalizing also affects the *quality* of this engagement. Furthermore, I maintain that, through discourse, normalization is unfortunately liable to be recreated, and yet it also stands to be renegotiated, reconfigured, and thus ameliorated.

Thomas Popkewitz (2004), too, observes the normalizing function of certain aspects of mathematical practice. He writes:

> The teacher must observe the interactions of students as related to the norms of forming a logical argument. Yet the classroom focus on students' learning the rules of argument is not only a process of modeling of truth. It is also a process of normalizing the inner characteristics of the students through modeling of the social as a means of constructing knowledge. (Popkewitz, 2004, p. 19)

The data analysis presented in this dissertation revealed aspects of this normalizing pedagogy, enacted through Mr. Lam's cognitive–conceptual scripts. The analysis also revealed students' negotiation and resistance to that normalizing exercise of power. I argue that the vignette and analysis presented in this dissertation are not an isolated case, but are part of a larger trend in mathematics education whereby equity discourse becomes mangled—or rather, imbricated—with actual mathematical practices; this *structural* learning–power imbrication is achieved through reform policy and rhetoric (Schneider, 2015).

Leveraging a critique of the broader discourse of mathematics educational research, Popkewitz (2004) argues that equity-oriented perspectives found in reform documents may in some instances perpetuate a normalizing pedagogy that reifies the practices of social inclusion and exclusion.

> The reference to *all* children in the reports represents both more and less than a simple principle of equity. The phrase "all children" not only reiterates a political and social principle but also functions as a pivoting point to distinguish two human kinds in the standards and research—the child who has all of the capacities to learn, problem solve, and achieve in schooling, and the child who is of a different human kind, the *disadvantaged*. (Popkewitz, 2004, p. 23, original italics)

Therefore, the notion of a script that enacts these ideological orientations toward mathematics cognition and instruction represents the articulation and enforcement of a *minimum cognitive–conceptual standard* that all students are expected to achieve. In my data, for example, students were differentiated and set apart according to a standard of generalizing. Looking beyond just my data however, I conceptualize mathematics education as indeed part of a broader form of "disciplinary power" (Foucault, 1977). As such, through the mechanism of scripts, the practice of mathematics pedagogy *cognitively normalizes* students when it:

- *evaluates* student action in relation to a cognitive–conceptual norm;
- *differentiates* among students according to their apparent progress toward this norm;
- *hierarchizes* students in terms of their "level" of understanding/achievement in relation to this norm;

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- *homogenizes* students when it asserts or demands a group norm; and
- *excludes* students whose behaviors fall below a minimum threshold and are therefore deemed "abnormal."

The five dimensions of this framework were manifest in the 35-min. vignette. My dissertation examined a subset of a much larger ethnographic data corpus consisting of 60+ hours of classroom footage, fieldnotes, and student interviews gathered over the course of a school year. Further studies will systematically analyze two aspects of the entire corpus: (a) teacher-facilitated class discussions; and (b) teacher interventions in group tasks. So doing, I will identify and code emerging categories representing distinct forms of participation and learning surrounding students' encounters with normalizing power across these two settings. These studies will examine how cognitively normalizing in Mr. Lam's classroom might have differentially impacted mathematics learning for women, racial minority students, and students who are non-native English speakers.

Finally, following this data analysis and theory building, the next step would be to systematically examine the generative hypotheses arising from this research through a conjecture-driven design-based research project (Abrahamson, 2009; Abrahamson, Trninic, Gutiérrez, Huth, & Lee, 2011). The specific conjecture to be explored, which was stated above, is as follows: *perturbing the power dynamic enables participants of mathematical discourse to appropriate agency, status, and authority—thus spurring new mathematical cognitions and understandings*. Maintaining the obj–subj dialectic as the analytic focus, results from a proactive, critically oriented design-based approach can give us traction on the specific mechanisms of power inherent to mathematics instruction and their consequences on learning. So doing, we could find ways to better support students who have been historically underrepresented in the field of mathematics to navigate both its conceptual and systemic challenges.

When students appropriate mathematical symbolism, per force they reproduce signs of power. It is our society's moral charge to democratize access to these powerful signs while remaining vigilant that signified power is wielded charitably, equitably. With this dissertation, I have attempted to illuminate routes to agency, signs to empowerment.

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Appendix A: Spiralateral Problems

Figure 19. Images (a) through (h), below, of specially crafted (14x18 inch) posters used in the intervention. Each poster presents the first four figures of a Spiralateral sequence; the task objective is to express a "Code" governing the growth of each sequence, as a set of algebraic formulas in the form of $\{f_1(n), f_2(n), f_3(n)\}$ and whose inputs are the figures' ordinal positions.
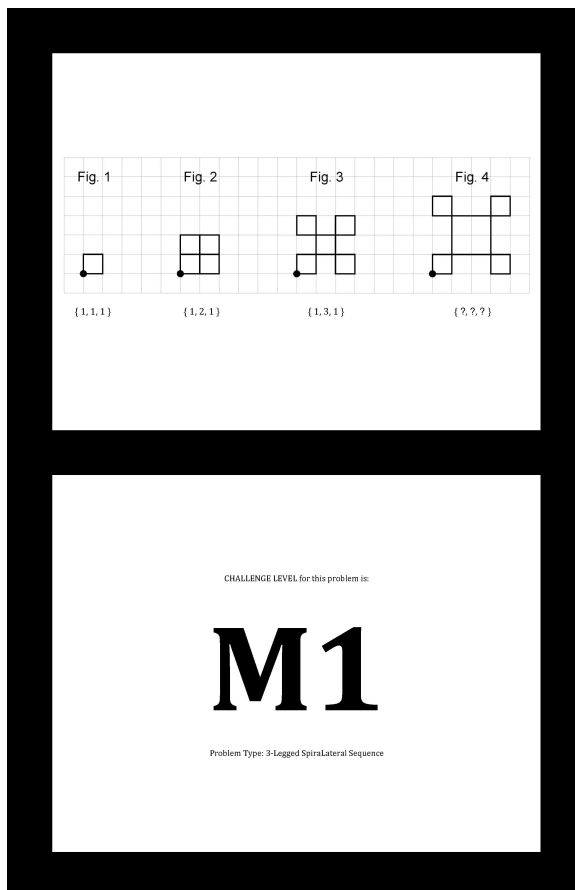
Figure 19a. M1 sequence can be modeled as $\{1, n, 1\}$
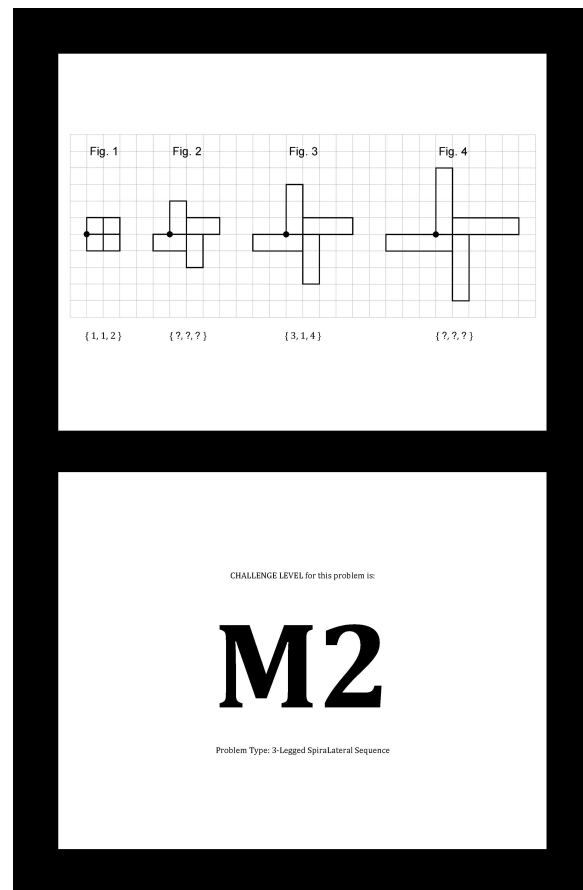
Figure 19b. M2 sequence can be modeled as $\{n, 1, n+1\}$.
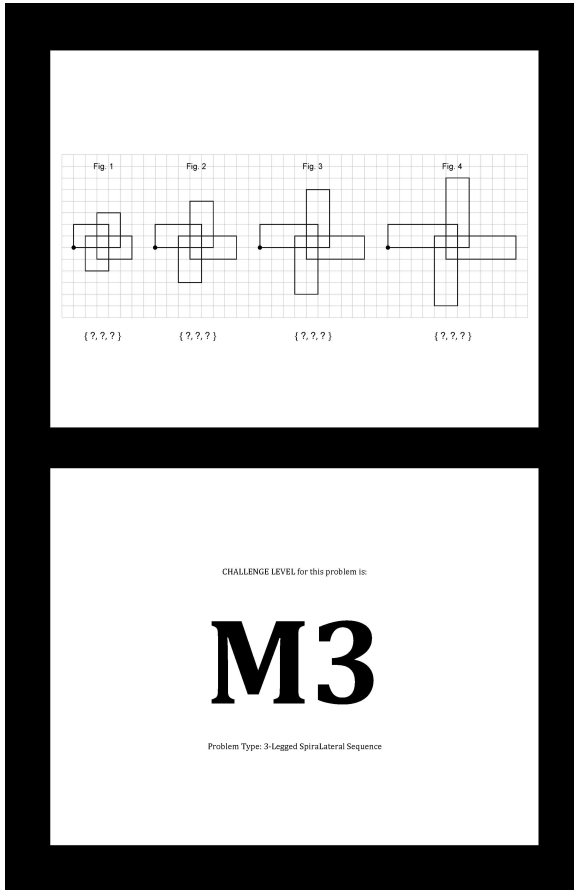
Figure 19c. M3 sequence can be modeled as {2, *n*+2, *n*+3}.
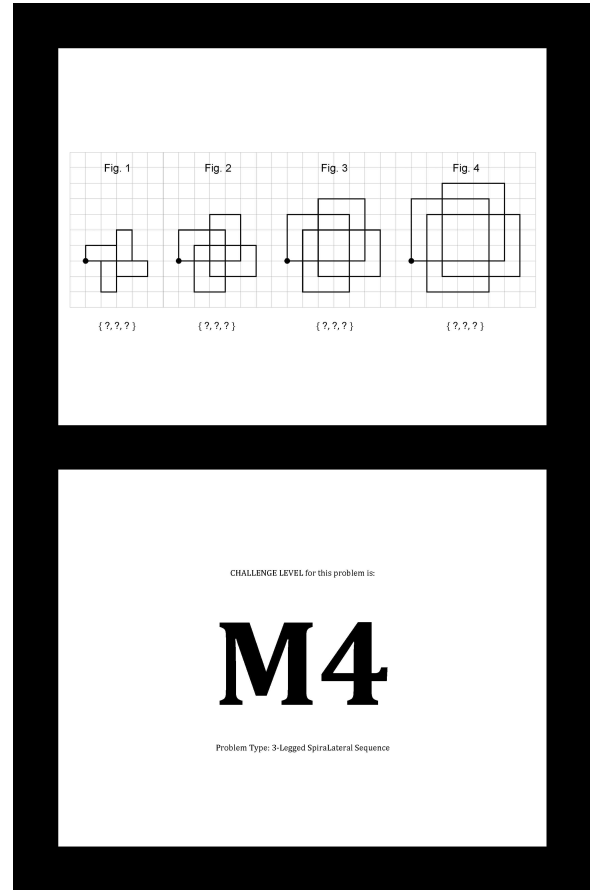
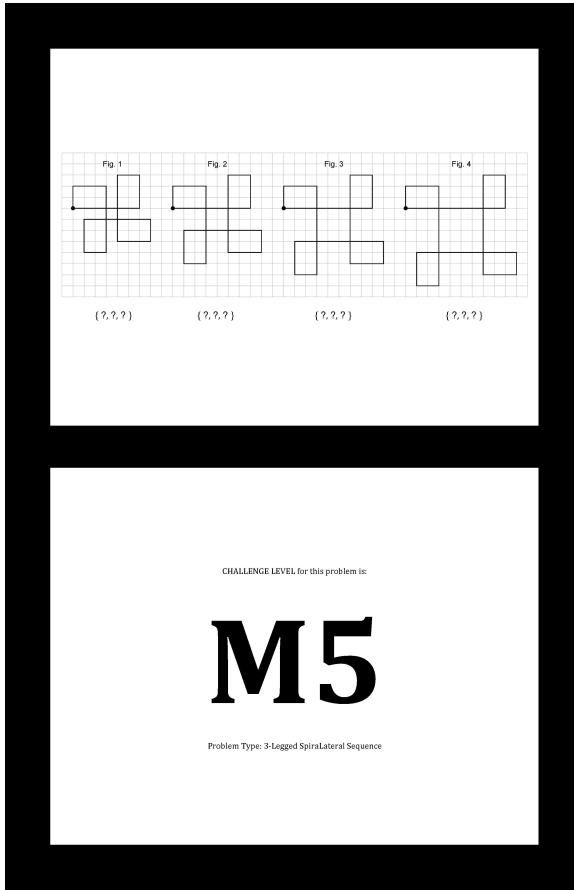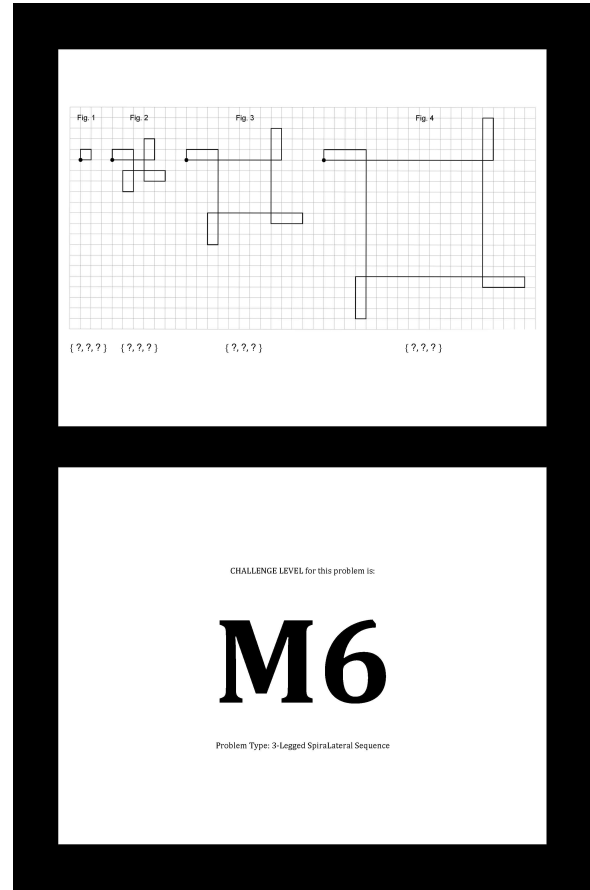*Figure 19d*. M4 sequence can be modeled as {*n*, *n*+1, *n*+2}.

Figure 19e. M5 sequence can be modeled as {2, 3, *n*+5}.

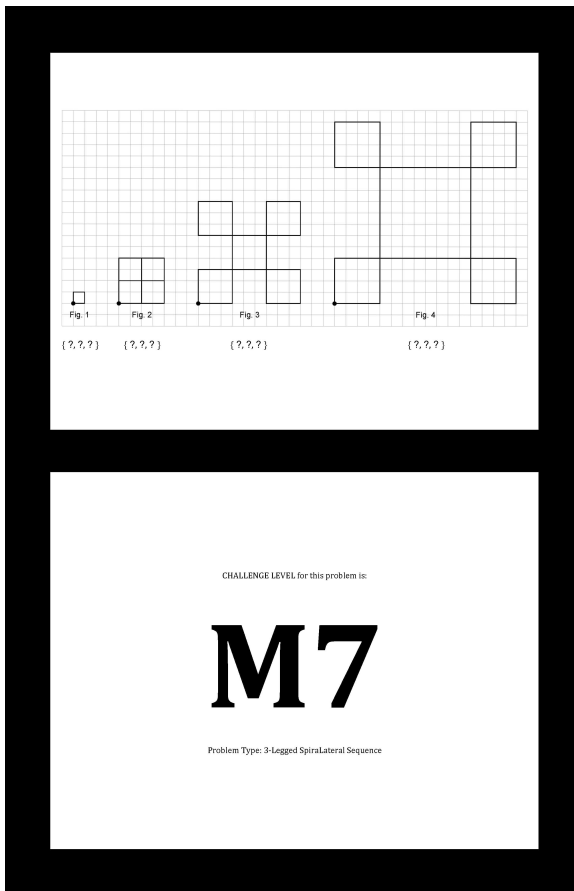*Figure 19f*. M6 sequence can be modeled as {1, *n*, n^2}.
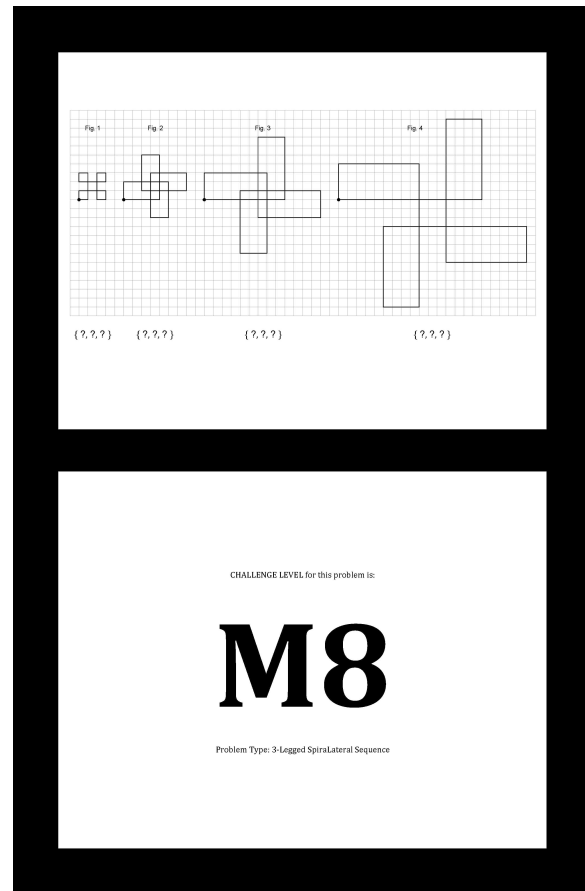
Figure 19g. M7 sequence can be modeled as $\{n, n^2, n\}$.

Figure 19h. M8 sequence can be modeled as $\{n, 2n+1, n^2\}$.