Title
STRONG-FOCUSING COCKCROFT-WALTON ACCELERATOR

Author
Wilcox, John M.

Publication Date
1958-02-01

UCRL 3184-Rev.

UNIVERSITY OF CALIFORNIA

Radiation Laboratory

BERKELEY, CALIFORNIA

UCRL-3184-Rev.

UNIVERSITY OF CALIFORNIA

Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

STRONG-FOCUSING COCKCROFT-WALTON ACCELERATOR

John M. Wilcox

February, 1958

Printed for the U.S. Atomic Energy Commission

STRONG-FOCUSING COCKCROFT-WALTON ACCELERATOR

Contents

STRONG-FOCUSING COCKCROFT-WALTON ACCELERATOR

John M. Wilcox

Radiation Laboratory
University of California
Berkeley, California

February, 1955

ABSTRACT

The properties of a strong-focusing Cockcroft-Walton accelerator (with permanent quadrupole magnets installed in each drift tube) have been calculated. It appears that the space-charge repulsion can be overcome so that the machine can accelerate a proton beam of 50 milliamperes or more.

STRONG-FOCUSING COCKCROFT-WALTON ACCELERATOR*

John M. Wilcox

Radiation Laboratory
University of California
Berkeley, California

February, 1956

INTRODUCTION

At the suggestion of James D. Gow, the properties of a Cockcroft-Walton accelerator tube employing permanent strong-focusing magnets have been investigated. Blewett, Christofilos, and Vash,[1] working at Brookhaven National Laboratory, have developed a technique for producing a permanent quadrupole field in small cylindrical magnets by pulsing an extremely large current through suitably shaped electrodes inside the cylinder. In May of 1955 they had magnetized a 1-inch-long cylinder of Indox, 1 inch i.d. and 2.5 inches o.d., to produce a field gradient of 1035 gauss/cm. Such magnets should be ideally suited for insertion in the drift tubes of a Cockcroft-Walton machine. Their focusing is weak compared with that in alternating-gradient synchrotrons, but the resulting small angular divergence of the beam is very desirable.

J. P. Blewett has suggested the use of a succession of electric or magnetic quadrupole lenses to overcome the gap defocusing in a linear accelerator,[2] and a treatment of this problem by means of an impulse-approximation method has been made by Johnston for the Minnesota linac.[3] This impulse method has been applied to the problem described here.

Numerical calculations of the particle trajectories have been performed for an accelerator tube with the following specifications (see Fig. 1):

*Work done under the auspices of the U.S. Atomic Energy Commission.

L = length of sections = repeat length = 2.75 inches
ℓ = length of quadrupole magnets = 1.5 inches
R = radius of electrostatic cylinders = 0.75 inch
number of sections = 20
injection energy = 60 kv
final energy = 460 kv
length of tube = 55 inches
k = gradient of quadrupole magnets = 700 gauss/cm
I = proton current = 50 milliamperes
r_0 = initial radius of injected beam = 0.5 inch
r_f = final radius of beam = approx. 3/16 inch
quadrupole sequence: N S N S N S

BEAM-ORBIT CALCULATIONS

The beam-orbit calculation proceeds in the following manner. There are three forces acting on a beam proton, (a) the space-charge repulsion, (b) the strong-focusing forces, and (c) the electrostatic focusing forces at the gaps. These last become negligible about halfway down the tube. The total radial momentum received by a proton in passing through one drift tube is lumped into an impulse that is assumed to be applied at the center of the tube. The resultant trajectory is a series of straight lines with breaks at the impulse points. This method is of course approximate, but it has the advantage that the calculations can be handled in a finite amount of time and many geometries can be investigated. The greatest error found by the Minnesota group with this method was 15%.

The slope of the trajectory is described as dr/dn, where n is the ordinal number of each drift tube ($Z = nL$), and the change in slope occurring at an impulse point is $\Delta\, dr/dn$. Since the impulse received from each of the above forces is proportional to the radial displacement r, we can define a deflection constant D such that we have

$$D_s = \left(\frac{1}{r} \;\Delta\, dr/dn \right)_{\text{space charge}}, \tag{1}$$

the change of slope due to space-charge repulsion;

$$D_q = \left(\frac{1}{r} \; \Delta\, dr/dn \right)_{\text{quadrupole lens}}, \tag{2}$$

the change of slope due to strong-focusing magnets;

$$D_{es} = \left(\frac{1}{r} \; \Delta\, dr/dn \right)_{\text{electrostatic lens}}, \tag{3}$$

the change of slope due to electrostatic focusing.

Note that a negative value of D implies focusing and a positive value implies defocusing.

In some cases we can calculate D by noting that it is related to the focal length F of a lens element by

$$\frac{L}{F} = D = \frac{1}{r} \; \Delta\, dr/dn, \tag{4}$$

where L is the drift-tube repeat length.

The value of D_s is derived in Appendix A for a beam with cylindrical symmetry as

$$D_s = (L^2/r_n{}^2\; (I/v^3)\; (e/2\pi K_0 m)\; \Delta\, n, \tag{5}$$

where L = drift tube repeat length,

r_n = beam radius,

I = beam current,

v = beam velocity,

e = charge of beam particles,

m = mass of beam particles,

K_0 = permittivity of free space.

After the beam has passed through the first quadrupole lens, it no longer has a circular cross section but has a larger radius in the x direction and a smaller radius in the y direction (see Fig. 2). This effect persists, and increases as the beam goes down the tube. For this calculation the maximum and minimum radii have been averaged to give the radius inserted in Eq. (5). As long as these two radii are not greatly different this should be a good approximation, but near the end of the tube it tends to break down. Of course the space-charge force becomes nonlinear as soon as the beam departs from cylindrical symmetry anyway.

The value of D_q is derived in Appendix B as

$$D_q = L\, k^{1/2}/p^{1/2} \sin k^{1/2}\ell/p^{1/2} \simeq k\ell L/p\,, \tag{6}$$

where L = drift tube repeat length,

ℓ = length of quadrupole magnet,

k = field gradient of quadrupole magnet,

p = momentum of beam particle (gauss-cm).

The value of D_{es} is derived in Appendix C, by use of Eq. (4), which relates D and the focal length of a cylindrical electrostatic lens. Since the lens strengths involved are less than those plotted in graphs such as given by Terman[4] or Zworykin,[5] the focal length had to be calculated with the aid of some numerical theory developed by Zworykin.

Note that as a proton goes down the tube the space-charge repulsion D_s decreases as $1/V^{3/2}$ while the strong focusing D_q decreases as $1/V^{1/2}$, where V is the voltage. However, as the beam is focused, the space-charge repulsion increases as $1/r^2$, so that in fact D_s and D_q may remain roughly proportional. As mentioned above, the electrostatic focusing D_{es} becomes negligible after the first few drift tubes.

We begin the numerical calculations by assuming an initial radius and slope, and calculating the effects of the deflections. At each deflection point, the new slope is obtained from

$$(dr/dn)_{n+} = (dr/dn)_{n-} + D_n\, r_n\,, \tag{7}$$

and the new radius is calculated from the old radius and the slope by

$$r_{n+1} = r_n + (dr/dn)_{n+}\, \Delta n; \tag{8}$$

and since Δn is 1 between drift tubes, we have

$$r_{n+1} = r_n + (dr/dn)_{n+} .$$

The value of D_n used in Eq. (7) is the algebraic sum of D_s, D_q, and D_{es}.

RESULTS AND DISCUSSION

Trajectory changes plotted in this manner are shown in Fig. 2. The calculated curves do not truly represent the actual beam near the end of the path, owing to a failure of the space-charge approximation used, which overestimates the ultimate diverging effect on the initially diverged beam and underestimates the diverging effect on the initially converged beam. The dashed lines in Fig. 2 indicate the general trend of the actual forces on the beam. In practice the beam must be adjusted experimentally by changing the divergence or convergence of the injected beam from the ion source, and by using electromagnets for the final quadrupole magnets so that their strengths can be adjusted.

The quadrupole lenses produce relatively large local changes in the angular deflection of the beam. Therefore, as suggested by Johnston,[3] the first lens is made half-strength in order to decrease the initial enlargement of a beam that is parallel when injected.

Figure 2 represents the trajectory for an initially parallel beam of 50 ma, and Fig. 3 for a beam of 75 ma. The open circles in Fig. 2 indicate the trajectory for a zero-current beam, i.e., no space-charge repulsion. The qualitative behavior of the system can be inferred from these figures. A considerable variation in the injected beam current still does not prevent focusing, but a change in ion-source current from pulse to pulse is reflected in a change of the size and divergence of the output beam. Thus the ion source should be induced to give a uniform output from pulse to pulse, and also each individual pulse should be steady, i.e., have a rectangular wave-form output. Then by adjusting the divergence or convergence of the injected beam and trimming the final electromagnet quadrupoles one can maximize the output beam. It is apparent from the figures that a few degrees of divergence in the injected beam can be tolerated.

The divergence of a 50-ma beam in the absence of the strong-focusing lenses is represented in Fig. 4.

We may investigate the effect of heavy-mass components in the injected beam from the ion source. Equation (5) for the space-charge force includes the factors

$$\frac{1}{mv^3} \sim \frac{1}{1/2\, m\, v^2 \cdot v} \sim \frac{1}{V\frac{\sqrt{2V}}{m}} \sim \frac{\sqrt{m}}{V^{3/2}} ,$$

where V is the voltage. Thus since all particles have the same energy V, the heavy particles contribute to the space-charge repulsion as $\sqrt{m}$, i.e., one milliampere of titanium ions would contribute as much repulsion as 6.9 milliamperes of protons. Thus a "pure" input beam would be desirable.

Breakdowns caused by an electron avalanche traveling in the reverse direction in the tube should be greatly inhibited by the bending action of the quadrupole magnets.

APPENDIX

A. SPACE-CHARGE REPULSION CONSTANT

The radial impulse received by a beam particle from space-charge repulsion while traveling a length L in the Z direction is

$$\Delta p_r = e\, E_r\, \Delta t, \tag{a}$$

where

$$\Delta t = \frac{\Delta Z}{v} = \frac{L \Delta n}{v}, \tag{b}$$

and from Poisson's equation for cylindrical symmetry we have

$$E_r = -\frac{\partial V}{\partial r} = \frac{\rho r}{2K_0}, \tag{c}$$

where ρ is the charge density and K_0 is the permittivity of free space. We also have

$$\Delta\left(\frac{dr}{dn}\right) = \Delta\left(L\,\frac{dr}{dZ}\right)$$

$$= \Delta\left(L\,\frac{dr}{dt}\,\frac{dt}{dZ}\right)$$

$$= \frac{L}{v}\,\Delta\left(\frac{dr}{dt}\right)$$

$$= \frac{L}{v}\,\frac{\Delta p_r}{m} \tag{d}$$

Now, D_s is defined as $\frac{1}{r}\,\Delta\left(\frac{dr}{dn}\right)_{\text{space charge}}$, and, substituting in Eqs. (a), (b), (c), and (d), we obtain

$$D_s = \frac{1}{r}\ \Delta\left(\frac{dr}{dn}\right) = \frac{L^2}{v^2}\ \frac{e\rho}{2K_0 m}\ \Delta n\ . \qquad \text{(e)}$$

Assuming a uniform charge distribution, we have

$$\rho = \frac{I}{\pi r_n^2 v} \qquad \text{(f)}$$

and thus Eq. (5) is

$$D_s = \frac{L^2}{r_n^2}\ \frac{I}{v^3}\ \frac{e}{2\,\pi\, K_0\ m}\ \Delta n\ . \qquad (5)$$

As a general check on the calculations and on the impulse approximation, the spreading of an initially parallel proton beam of 10 ma at 100 kv was calculated, and found to agree within a few percent with the analytic results by E. R. Harrison[16] (see Fig. 5).

B. ALTERNATING-GRADIENT DEFLECTION CONSTANT

From the definition of the deflection constant D we have D = L/f, where f is the focal length of a lens, and L is the repeat length. Courant et al.[17] give, for the focal length of a quadrupole lens,

$$f = \frac{1}{K^{1/2}\ \sin\,(K^{1/2}\ \ell)}\ , \qquad (15)$$

where

$$K = \frac{1}{B\,R}\ \ \frac{d\,B_x}{d\,y}\ , \qquad (16)$$

ℓ = length of quadrupole magnet,

p = BR; R is radius of curvature in field B,

B_x = ky for the quadrupole magnet.

Therefore we have

$$K = \frac{k}{p} \quad \text{and} \quad f = \frac{p^{1/2}}{k^{1/2} \sin\left(\frac{k^{1/2}\ell}{p^{1/2}}\right)} ,$$

and thus

$$D_q = \frac{L}{f} = \frac{L\,k^{1/2}}{p^{1/2}} \sin \frac{k^{1/2}\,\ell}{p^{1/2}} \simeq \frac{k\,\ell\,L}{p} .$$

Alternatively, D_q can be derived in the manner of Appendix A. The radial impulse received by a beam particle in passing through a quadrupole magnet of length ℓ is

$$\Delta p_y = F_y \Delta t = \frac{B_x\,e\,v}{c}\,\Delta t, \qquad \text{(a)}$$

with

$$B_x = k\,y \qquad \text{and} \quad \Delta t = \frac{\ell}{v} ,$$

where z is measured down the tube, and x and y are orthogonal.

We have $\Delta p_y = \dfrac{k\,y\,e\,\ell}{c}$, (b)

and, as developed in Appendix A,

$$\Delta\left(\frac{dy}{dn}\right) = \frac{L}{v}\,\frac{\Delta p_y}{m} . \qquad \text{(c)}$$

Substituting, we obtain

$$D_q = \frac{1}{y}\,\Delta\left(\frac{dy}{dn}\right) = \frac{k\,e\,\ell\,L}{m\,v\,c} . \qquad \text{(d)}$$

Now, we have

$$\text{Force} = \frac{B\,e\,v}{c} = \frac{m\,v^2}{r}$$

and

$$p = B\,r = m\,v\,\frac{c}{e} .$$

Then we have

$$D_q = \frac{k\,\ell\,L}{m\,v\,\frac{c}{e}} = \frac{k\,\ell\,L}{p} ,$$

which agrees with the previous results.

C. ELECTROSTATIC FOCUSING CONSTANT

The deflection constant D is again calculated from the relationship $D = L/f$, where f is the focal length of an equal-diameter cylindrical lens. This focal length is plotted by Terman[4] and Zworykin,[5] but not for the weak lenses encountered here. Therefore it has been computed with the aid of some theory developed by Zworykin.[5] We deal with a small value of V_{object}/V_{image}, so that the lens is weak and can be considered a thin lens. For this case, Formula 13.35 on page 437 of Zworykin gives

$$\frac{1}{f} = \frac{3}{16} \left(\frac{V_{ob}}{V_{im}}\right)^{1/4} \int_a^b \left(\frac{V'}{V}\right)^2 dZ , \qquad \text{(a)}$$

where the integral is taken over the gap. Also on page 379, Fig. 11.10 (b), Zworykin has plotted the voltage V and derivative V' over the gap between two equal-diameter cylinders. Thus a simple numerical integration can be done to yield the required focal lengths, which are shown in Fig. 6. Some focal lengths were computed by this method in the range included in the graph in Terman,[4] and were found to agree.

REFERENCES

1. Blewett, Christofilos, and Vash, Phys. Rev. 99, 652 (1955).
2. J.P. Blewett, Phys. Rev. 88, 1197 (1952).
3. Lawrence Johnston, Mimeographed Report, University of Minnesota, July 1, 1953.
4. F.E. Terman, Radio Engineers' Handbook (McGraw-Hill, New York, 1943), p. 326.
5. Zworykin, Morton, and Ramberg, Electron Optics and the Electron Microscope (Wiley, New York 1945).
6. ~~L. Ruby and J.D. Gow, Phys. Rev. 99, 663 (1955). Also UCRL-3103, 1955 (unpublished).~~
7. E.R. Harrison, Some Notes on the Acceleration of Relatively Large Current Proton Beams, AERE - G/R-991, Sept. 1952.
8. Courant, Livingston, and Snyder Phys. Rev. 88, 1190 (1952).

FIGURE CAPTIONS

Fig. 1. Drift-tube geometry.

L = 2.75 in. = length of sections;

r = 0.5 in. = initial radius of beam;

R = 0.75 in. = radius of drift tubes;

ℓ = 1.5 in. = length of quadrupole magnets.

Fig. 2. Beam radius as a function of drift-tube number for 50-ma current.

Fig. 3. Beam radius as a function of drift-tube number for 75-ma current.

Fig. 4. Beam radius as a function of drift-tube number without strong focusing.

Fig. 5. Beam divergence caused by space charge. (Note that Fig. 8 of Harrison would give points ~10% farther to right.)

Fig. 6. Focal lengths of equal-diameter cylindrical electric lenses.

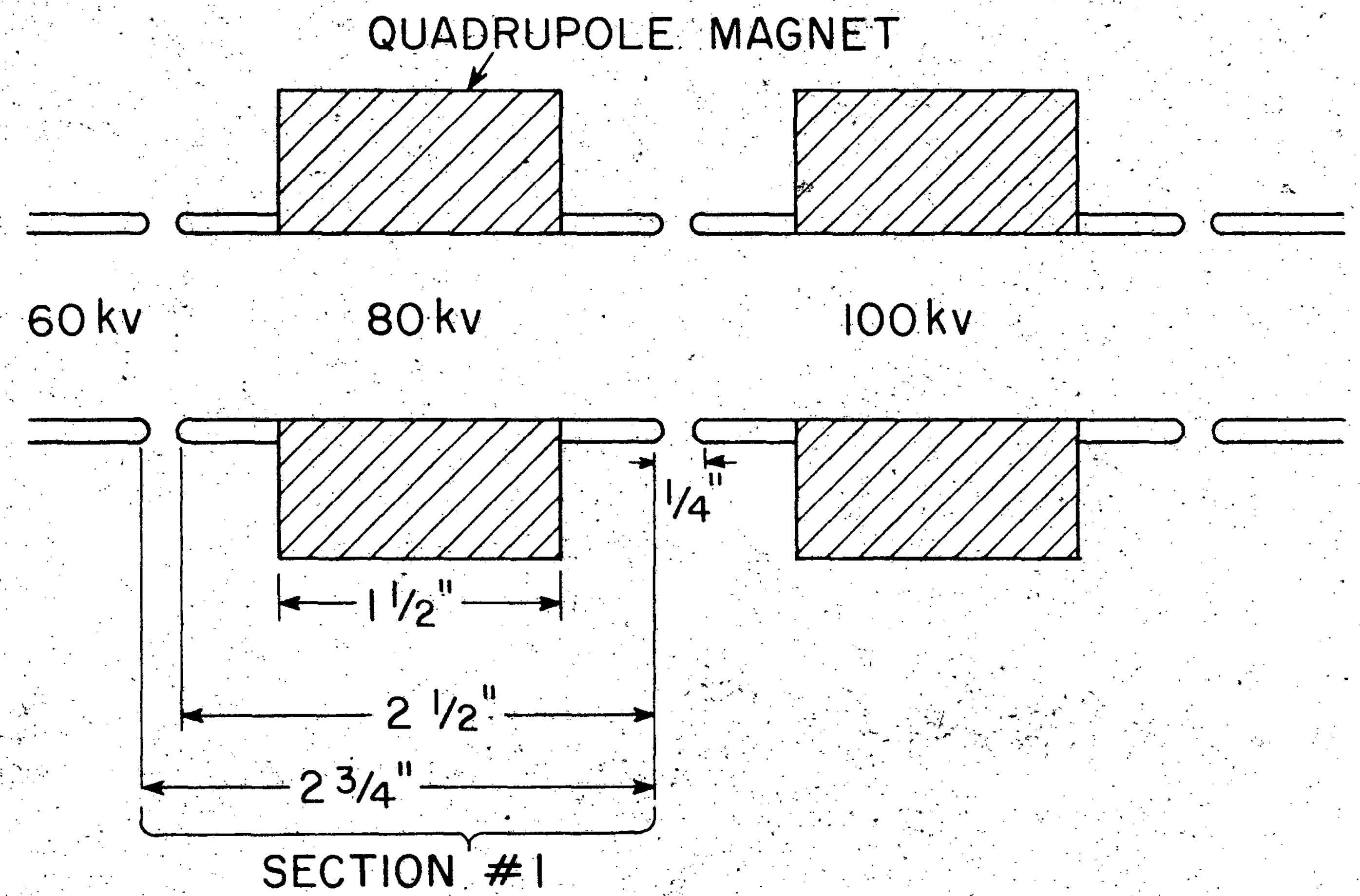
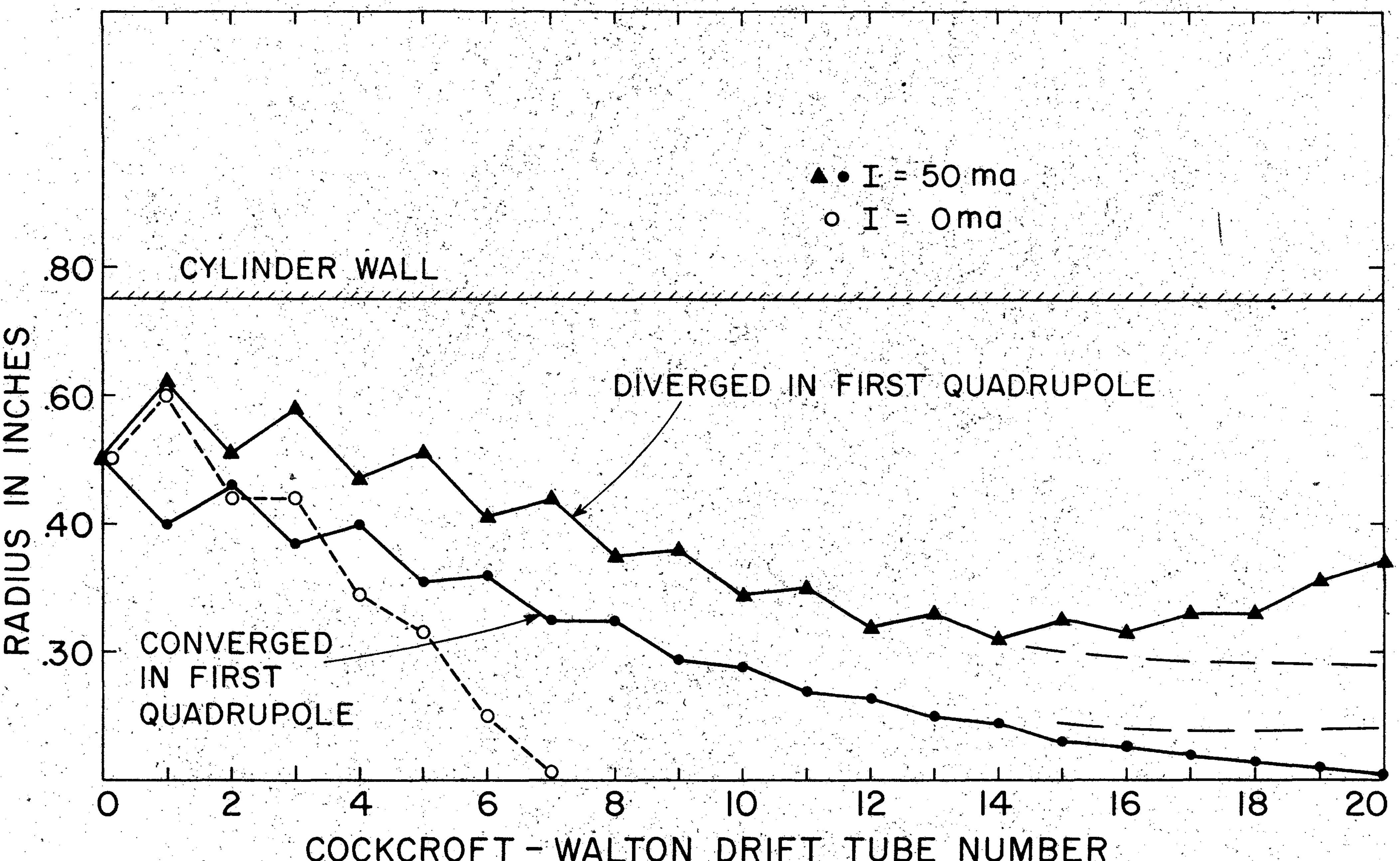


Fig 1

24221-1



▲● I = 50 ma
○ I = 0 ma
CYLINDER WALL
DIVERGED IN FIRST QUADRUPOLE
CONVERGED IN FIRST QUADRUPOLE
RADIUS IN INCHES
.80
.60
.40
.30
0
2
4
6
8
10
12
14
16
18
20
COCKCROFT - WALTON DRIFT TUBE NUMBER

Fig 2

24222-1

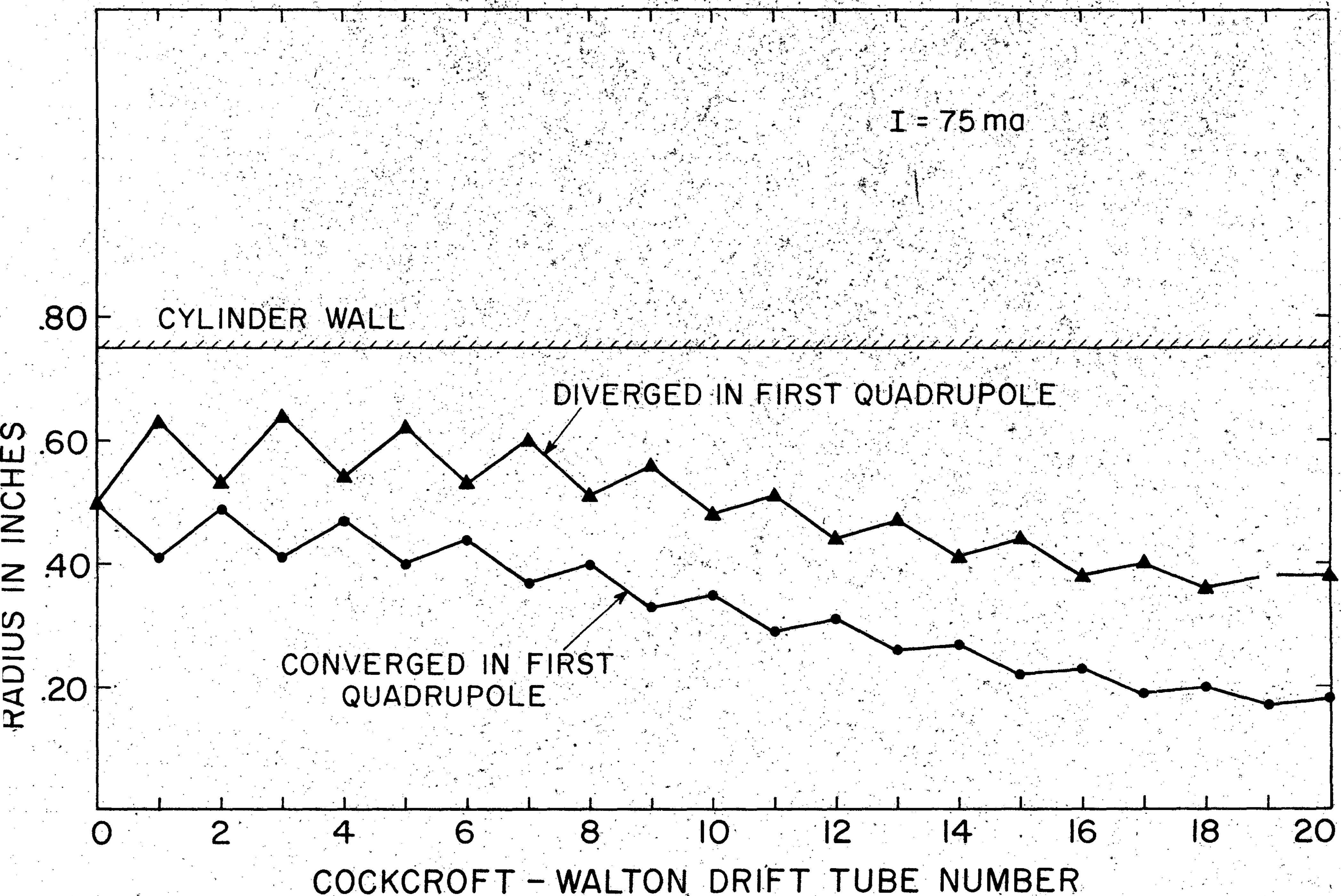
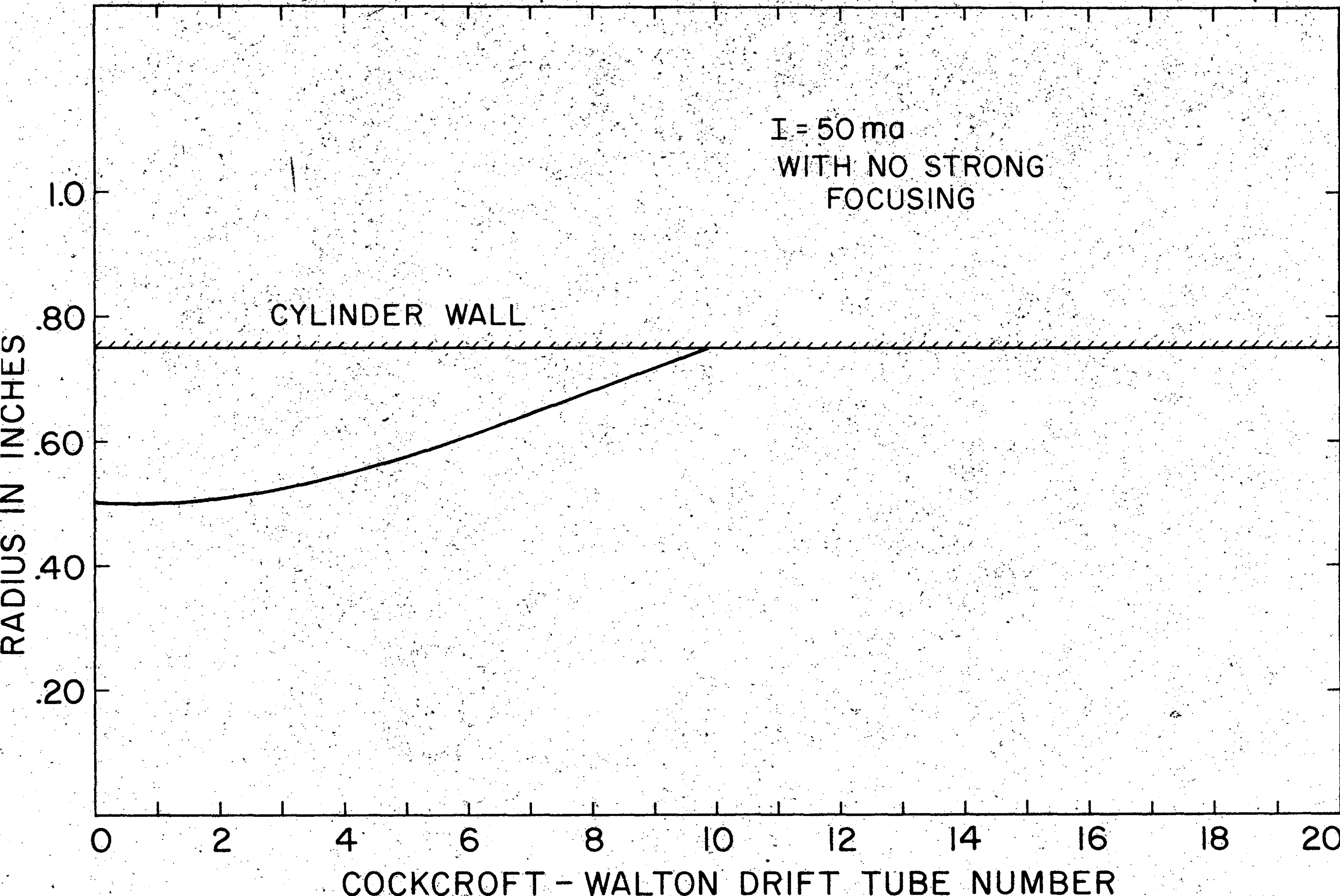

I = 75 ma
.80
CYLINDER WALL
DIVERGED IN FIRST QUADRUPOLE
.60
.40
.20
CONVERGED IN FIRST
QUADRUPOLE
RADIUS IN INCHES
0
2
4
6
8
10
12
14
16
18
20
COCKCROFT - WALTON DRIFT TUBE NUMBER

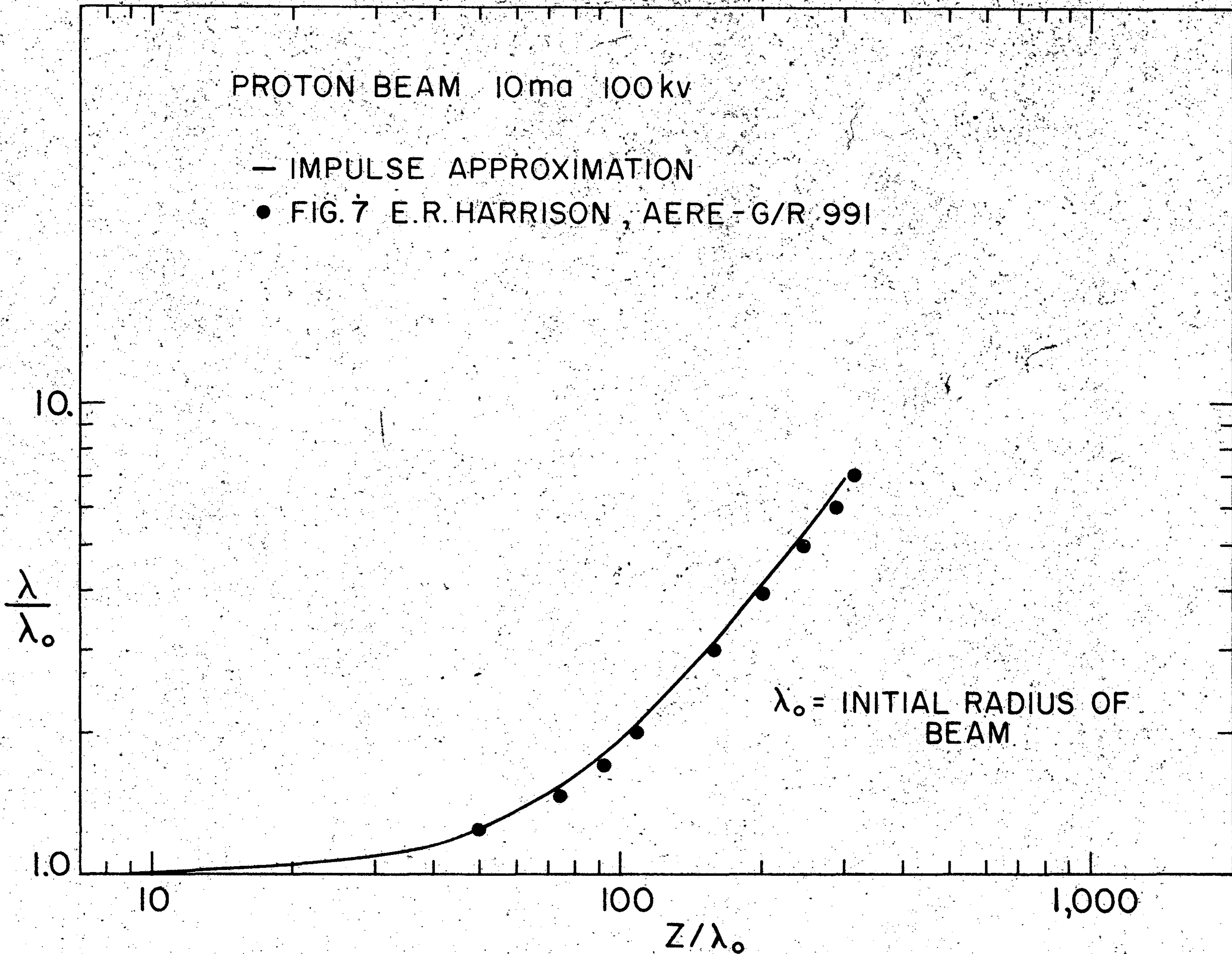
Fig 3



I = 50 ma
WITH NO STRONG
FOCUSING
CYLINDER WALL
RADIUS IN INCHES
1.0
.80
.60
.40
.20
0
2
4
6
8
10
12
14
16
18
20
COCKCROFT - WALTON DRIFT TUBE NUMBER

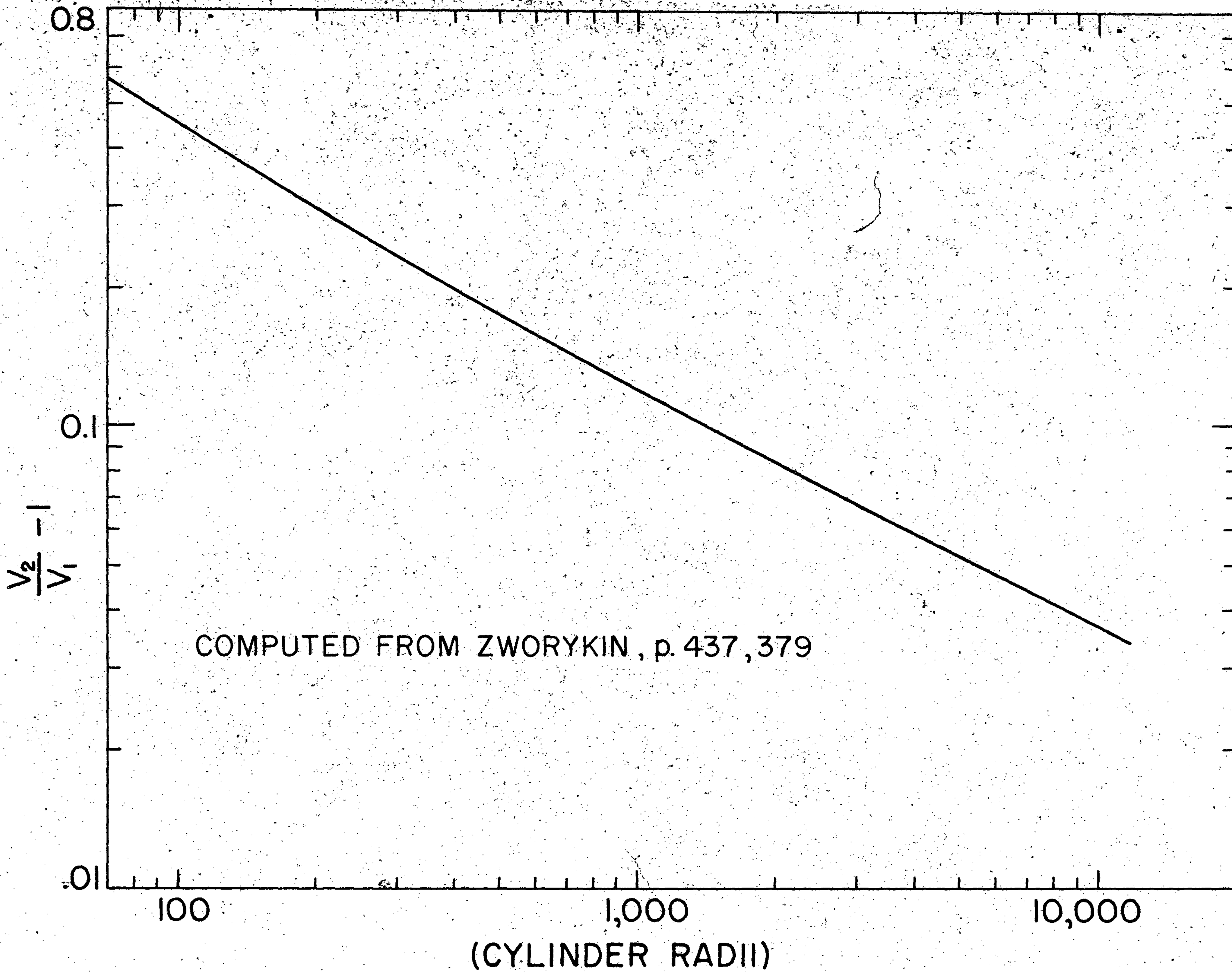
Fig 4

24224-1



PROTON BEAM 10 ma 100 kv
— IMPULSE APPROXIMATION
• FIG. 7 E.R. HARRISON, AERE-G/R 991
λ₀ = INITIAL RADIUS OF BEAM
10.
1.0
10
100
1,000
λ/λ₀
Z/λ₀

Fig 5



0.8
0.1
.01
$\frac{V_2}{V_1} - 1$
COMPUTED FROM ZWORYKIN, p. 437, 379
100
1,000
10,000
(CYLINDER RADII)

Fig 6

24226-1