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OPTIMAL TOLLS ON THE BAY BRIDGE

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INTRODUCTION

Corresponding to the two periods within which economic resource-allocation decisions are made, the operator of vehicle guideways has two sets of decisions to make. In the long run (that period within which all inputs are variable, and when investment decisions are made), he must decide on the optimal size (level of construction) of the guideway. In the case of highways, for example, the long-run decision is the number of lane-miles of road to be provided. In the short run, when the capital structure is already in place, the operator must control the use of the facility by a pricing mechanism. In the case of highways, an appropriate toll per vehicle-mile must be decided on.

Of course, the two decisions are not entirely separate. In their fundamental work solving both the long-run and the associated short-run problems for an autos-only highway, Keeler and Small (1977) show that the optimal level of investment is found by equating the marginal consumer benefits to the marginal cost of road capacity, and that the optimal toll at such an optimal investment level is the difference between average variable costs (as perceived by the user) and the short-run marginal cost (evaluated at the optimal investment)

imposed on the system by an additional user.¹ They are able to derive, for a representative superhighway,² both the optimal level of investment and the optimal tolls.³ These last turn out to be considerably higher (in the most congested, or peak, period) than tolls currently charged; for a central city expressway the tolls can reach \$.34 per vehicle-mile.⁴

In this paper I focus totally on the short-run (pricing) decision. The motivation for this is that in most cases it is not true that an investment decision remains to be made. Most of the U.S. guideway stock is already in place, and there are no signs of increased investment activity comparable to that when the Interstate Highway system was built. Rather, the problems now facing the operators of roadways are to keep the existing highway stock from deteriorating (maintenance), and to maximize net social welfare by controlling access to the roads. This is precisely the pricing decision with which I shall be concerned. In essence I shall discuss the question: given that we are faced with an existing system of roads, how ought it to be operated, since we cannot build further?

In the course of answering this question, I extend the results of the Keeler-Small model in the following way. Lacking demand functions for highway travel, they were unable to discuss how the consumers of roadway services might react to the higher tolls indicated by their model.⁵ Since they wrote, however, the state of demand estimation has changed significantly, to the point where we are now able to discuss the dependence of mode-choice decisions in a given time

period on the level of service offered by the modes of travel.

We cannot as yet discuss the joint dependence of mode-choice and time-of-day selection on the characteristics of the travel modes as presented during various time periods. Therefore, in what follows I discuss the pricing decision in the peak period only.⁶ I also assume that all travel taking place in the peak is work-bound; inclusion of peak-period shopping trips is not theoretically difficult, but research on the demand for shopping-trip travel has not reached the requisite level of sophistication, as it has for the work trip.

The structure of the paper is as follows: After a brief description of the Bay Bridge, the operation of which is the subject of the paper, I turn, in Section 2, to the derivation of the optimal short-run tolls corresponding to a given traffic volume. In Section 3 I discuss the demand for roadway services, and show how an equilibrating model may be constructed from the demand models, with the property that at each stage in the equilibrating process the relevant short-run tolls are just those derived in Section 2. Finally, in Section 4, I calculate the best equilibrium tolls for the Bay Bridge under various demand assumptions, and compare them with the actual tolls.

The San Francisco-Oakland Bay Bridge is the main connector between the East and West sides of San Francisco Bay. The span is approximately nine miles long and crosses over Yerba Buena Island in the middle of the Bay. The bridge opened in 1936; until 1958 it had provision for light-rail transit vehicles, operated by the Key System

in Alameda and Contra Costa Counties. In 1958 the bridge was modified: the rails were removed and another deck made available to auto traffic. At present the upper deck handles the west-bound traffic and the lower deck the eastbound, each in five lanes. The toll plaza is located at the eastern end of the bridge: 17 lanes are provided for westbound traffic, and eastbound traffic is free. The construction costs of the bridge were \$70 million up to its opening, and a further \$40 million in 1957 at the time of the renovation. It was cited in 1955 by the American Society of Civil Engineers as one of the seven "Modern Civil Engineering Wonders."⁷

2. SHORT-RUN TOLLS

In this section I derive the social-welfare-maximizing short-run tolls for the given investment in the Bay Bridge. The tolls derived in this section will be optimal under the assumption of an acceptable distribution of income over the users of the bridge. This is a conventional assumption in applied economics--it is made, for example, by Keeler and Small in their investment-and-pricing model⁸--but it needs to be stated. To the extent that the actual income distribution is perceived as sub-optimal, then the tolls to be given here will not be social-welfare-maximizing. In this case the correct procedure would be to consider a model disaggregated along the dimension of income, and to assign "welfare weights" to each income class.⁹

The model to be discussed here involves the use of road capacity by three modes: passenger cars, commuter buses, and trucks. Each type of vehicle is represented by a single average exemplar.¹⁰ For example, for the auto mode we consider as representative a standard car containing 1.46 riders. The question to be answered here concerns the optimal short-run toll for each of the three modes.

In such a model, the optimal toll will have three components. First, since an additional vehicle of whatever type will impose

external costs in the form of increased travel time on the other users of the bridge, there will be a congestion toll reflecting this externality. Second, an additional vehicle will impose other costs on the system: for instance, it will cause the bridge roadway to deteriorate, and it will add to the pollution in the air. Section 2.2 is concerned with estimating the net non-congestion costs. Finally, the method of toll collection (at toll booths) will impose another source of congestion cost, and Section 2.3 describes a simplified model derived from queuing theory to estimate these costs as well.

2.1 Bridge Congestion Tolls

These tolls are simply the short-run tolls derived by Keeler and Small¹¹ for a given level of capacity. The intuition behind their work is as follows. Denote by Q_t^A , Q_t^B , and Q_t^T the volume of auto, bus, and truck traffic (respectively) in period t . Then for the roadway in question, the maximum attainable speed in period t , S_t , will depend in some way on the traffic level, say $S_t = S_t(Q_t^A, Q_t^B, Q_t^T)$. Consider now the effect on the attainable speed of an additional vehicle, say a car.¹² The time required to traverse a mile of road is $1/S_t$. The additional car increases this time by $\frac{\partial}{\partial Q_t^A} \left(\frac{1}{S_t} \right)$ hours. It increases bus operating time by a similar amount. If the hourly value of time for users of the three modes is, respectively, $\bar{V}_V^A$, $\bar{V}_V^B$, $\bar{V}_V^T$ per vehicle, and if bus operating costs are C_H per hour, then the external cost per mile imposed on the existing traffic stream by the presence of an additional automobile is

$$\tau_t^A = \frac{\partial}{\partial Q_t^A} \left(\frac{1}{S_t} \right) \cdot \{ \bar{V}_v^A Q_t^A + \bar{V}_v^B Q_t^B + \bar{V}_v^T Q_t^T + C_H \} \quad (2.1)$$

and this is precisely the required congestion toll per mile of
 bridgeway. By the same reasoning, the congestion tolls per mile for
 the marginal bus and truck are

$$\tau_t^B = \frac{\partial}{\partial Q_t^B} \left(\frac{1}{S_t} \right) \cdot \{ \bar{V}_v^A Q_t^A + \bar{V}_v^B Q_t^B + \bar{V}_v^T Q_t^T + C_H \} \quad (2.2)$$

and

$$\tau_t^T = \frac{\partial}{\partial Q_t^T} \left(\frac{1}{S_t} \right) \cdot \{ \bar{V}_v^A Q_t^A + \bar{V}_v^B Q_t^B + \bar{V}_v^T Q_t^T + C_H \} \quad (2.3)$$

To make this operational we need, first, estimates of the
 values of time for the different modes, and second, an estimate of
 the attainable speed S_t as a function of traffic volume. The former
 will be discussed in Section 3; as regards the latter, Keeler and
 Small estimated the following speed-flow curve for the Bay Bridge:¹³

$$S_t = 56.8 - .00182Q_t^A \quad (2.4)$$

This must now be modified to reflect a dependence on other types
 of vehicles. To do this, we note, following the Handbook of Transpor-
 tation Engineering,¹⁴ that one bus is equivalent to 1.6 cars, and one
 truck to 2.2 cars. Thus a traffic stream consisting of Q_t^A cars,
 Q_t^B buses, and Q_t^T trucks is equivalent to one consisting of
 $Q_t^A + 1.6Q_t^B + 2.2Q_t^T$ autos only. Hence the required relation between
 the attainable speed on the Bridge and a traffic stream of our three
 types is

$$S_t = 56.8 - .00182 \cdot \{Q_t^A + 1.6Q_t^B + 2.2Q_t^T\} \quad (2.5)$$

and the short-run bridge congestion tolls for a given level of traffic can be obtained by inserting the partial derivatives of equation (2.5) into equations (2.1), (2.2), and (2.3). For the problem under discussion, these tolls take on a particularly simple form: if τ_t^A is the auto congestion toll per mile (as given by equation (2.1)), then

$$\tau_t^B = 1.6 \tau_t^A \quad (2.6)$$

and

$$\tau_t^T = 2.2 \tau_t^A \quad (2.7)$$

2.2 Net Other Costs

Besides congestion on the Bridge (and at the toll-booths, which I discuss in the next section), the marginal vehicle will also force the system to incur other costs. Two of these costs are directly incurred by government agencies: the additional vehicle causes the roadway to deteriorate, resulting in increased maintenance costs; and forces government agencies to provide additional services in the form of police protection, coroner's services, and the like. The other two relevant costs are external, in that they are imposed on other persons. The additional vehicle emits pollutants which do damage to persons and to property; and it gives off noise, perceived by many to be a negative externality. Thus, in order to calculate the full social marginal cost, it is necessary to have estimates of these other costs.

Fortunately, much of the estimation work has already been done, and so we need only draw on the work of other researchers. First, noise costs are assumed to be zero, because of the location of the bridge. Most of the external noise effects will be dissipated over the water; and Keeler and Small estimate¹⁵ that even on a freeway where there is no over-water dissipation, marginal noise costs will be on the order of \$.001-.002 per vehicle mile. It is therefore apparent that discounting completely the marginal noise costs on the Bay Bridge is a reasonable procedure.

As regards pollution costs, these have been definitively studied by Small.¹⁶ He considers pollution damage arising in three ways: through increased human mortality, increased human morbidity, and damage to materials. He is then able to attribute the costs of these three sources to emissions of various types of pollutants, and, given engineering estimates of the emissions of various types of vehicle, to estimate the costs of environmental damage done by each kind of vehicle. The results are shown in Table 1.

Keeler and Small have also studied the marginal maintenance costs to be attributed to an additional vehicle on the road.¹⁷ They find that this cost is \$.00045 per vehicle mile. Interestingly enough, this cost does not vary with the type of vehicle in question: as they state, it is "impossible to discern different marginal maintenance cost for heavy commercial vehicles relative to autos."¹⁸ Note that the costs of painting the span (amounting to approximately \$1 million) do not vary with the bridge traffic, and hence do not

appear in the estimate of maintenance costs given here.

Drawing on the work of D. B. Lee,¹⁹ Keeler, Cluff and Small have estimated the costs of providing public government services to road users.²⁰ These costs, though admittedly rough estimates, come to \$.0045 per auto mile. In Keeler, Small, Viton et al. (1975), it is assumed, in discussing the comparative costs of auto and bus usage, that this figure remains the same per bus-mile.²¹ This assumption is here retained, and extended to cover truck traffic as well.

Table 1 summarizes the discussion to this point. As can be seen from that table, non-congestion costs for autos were about 1.2¢ per vehicle mile, and for trucks and buses, approximately 2.7¢ per vehicle mile. This, however, does not complete the story. For the various types of vehicle already pay for some of these costs through various user-related taxes levied by different agencies. For example, the Federal government taxes gasoline and diesel fuel at the rate of 4¢/gallon; there is a Federal excise tax on tires; and the State of California taxes motor vehicle fuel (with a special reduced rate for transit buses) and imposes license and registration taxes as well. Therefore, in order to discover the net non-congestion costs of vehicle usage, it is necessary to tabulate the schedules of existing taxes. This is done in Table 2.

Allocation of these taxes to each vehicle mile traveled is straightforward once we have determined the characteristic of the three vehicles in question. These characteristics are listed in the first three notes of Table 2, and it is they that largely determine

Table 1
Non-Congestion Costs--1972

<u>\$ / Vehicle--Mile</u>					
<u>Mode</u>	<u>Pollution</u> ¹	<u>Noise</u> ²	<u>Government</u> ³	<u>Maintenance</u> ⁴	<u>Total</u>
Auto	\$.0069	\$0.0	\$.0045	\$.00045	\$.01186
Bus	.0220	0.0	.0045	.00045	.02695
Truck	.0218	0.0	.0045	.00045	.02675

Notes

1. Source: Small (1975, p. 28). San Francisco weighting factor 2.3.
2. See text.
3. Source: Keeler, Small, Viton, et al. (1976, p. 111)
4. Source: Keeler and Small (1977, p. 10)

Table 2

Existing Taxes

Mode	\$ / Vehicle Mile						
	State ^d Fuel	Fed ^e Fuel	Fed ^f Oil	Fed ^g Tires	Fed ^h Sales	Fed ⁱ Parts	License & Registration ^j Total ^k
Auto ^a	\$.00805	\$.00293	\$.00024	\$.00035	\$.00245	\$ 0.0	\$.00143 \$.01545
Bus ^b	.00862	.00690	0.0	.00149	.01166	0.0	.00151 .03018
Truck ^c	.01281	.00466	.00118	.00149	.01003	.00234	.00366 .03617

Notes

- a. Standard Car, 1972, price \$2,696 (Keeler et al., 1975, p. 94). Annual mileage 10,000. Gets 13.67 mpg (Statistical Abstracts of U.S., 1976, Table 960). Ratio of oil to gas consumption 1:186 (Keeler et al., 1975, p. 84).
- b. Diesel transit bus, 1972 price \$41,000 (Viton, 1974, p. 17). Annual mileage 36,000 (AC Transit data). Gets 5.80 mpg (Statistical Abstract of U.S., 1976, Table 960).
- c. 2-axle, 6-tire diesel truck (tare weight 10,000 lbs). 1972 price \$12,000. Annual mileage 12,000 (Statistical Abstracts of U.S., 1976, Table 932). Gets 8.59 mpg (Statistical Abstracts of U.S., 1976, Table 960). Uses .75 quarts oil/1,000 miles (Claffey, 1971).
- d. Tax is \$.07/gallon for auto and truck; \$.01/gallon for transit bus (Highway Statistics, 1972, p. 11).
- e. Tax is \$.04/gallon on both gasoline and diesel fuel.

Table 2 (continued)

Notes (continued)

- f. Tax is \$.06/gallon. Transit buses are exempt (Highway Statistics, 1972, p. 64, note 7).
- g. Tax is \$.10/lb. Tax on auto tire: \$2.30; on bus and truck tire, \$6.20. Tire life in all cases is assumed to be 25,000 miles.
- h. Rates are given in Highway Statistics, 1972, p. 64. The taxes per mile are computed on the age distribution of the California vehicle fleet (autos), the AC Transit fleet (buses), or the U.S. truck fleet (trucks). The annual cost assumes lifetimes of 10 years (auto) or 15 years (truck and bus) and an interest rate of 6%.
- i. Tax is 8% of sales price; autos and transit buses are exempt (Highway Statistics, 1972, p. 64, note 7). Estimate of maintenance expenses from Transport Statistics of the United States, 1972, based on experience of the Western District. See also Table 4 infra. It was assumed that parts accounted for 50% of the \$.0585/vm maintenance repairs expenses.
- j. Licenses: \$3.25 per year (Highway Statistics, 1972, p. 40). Auto registration: \$11.00/year; truck registration and weight fee \$121 (California Income Tax form 540A, 1973); bus registration \$51.00/year, based on average California receipts for buses (Highway Statistics, 1972, pp. 36, 42).
- k. Gross weight per truck assumed to be 22,000 lbs. (Claffey, 1971). Thus there is no use tax, payable on gross weight of 26,000 lbs. or more. Autos and transit buses pay no use tax (Highway Statistics, 1972, p. 64).

the costs in the body of the table. In selecting the vehicle characteristics care was taken to use those of California vehicles wherever possible; thus, for example, the transit bus considered is one used by the Alameda-Contra Costa Transit (AC Transit) District, and these buses constitute the vast majority of transit vehicles crossing the bridge.²² Where it was not possible to find the characteristics of specifically Californian vehicle usage, recourse was had to the Statistical Abstract of the United States (1976) in order to get characteristics approximating the average vehicle of the type in question. This procedure is likely to be accurate in the case of autos and buses, where model differences are relatively small. It will be less accurate in the case of trucks, subject to wide model variations. Thus, it is important to stress that the type of truck considered here (a 2-axle diesel truck of tare weight 10,000 lbs.) is only one of a number of vehicles which might have been selected.

Comparing Tables 1 and 2, we can see that, for the three representative vehicle types studied here, each vehicle more than pays, through the existing system of taxes related to vehicle usage, for the four non-congestion costs of vehicle usage. This conclusion may be of interest in its own right. It does not, however, imply that vehicles pay in taxes more than their full social costs: there is the external congestion cost considered in the last section, and, for the Bay Bridge in particular, there is the induced congestion at the toll booths. This last is the final component in the determination of short-run tolls, to which I now turn.

2.3 Congestion at the Toll Booth

By entering the traffic stream on the bridge, the additional vehicle causes increased congestion at the toll booths as well as on the span itself. As with bridge congestion, this congestion also imposes costs on other drivers; and in this section a simple model is proposed which enables us to conclude that these costs are negligible.

To discuss the extent to which an additional vehicle imposes costs on the existing toll-booth queues, it is first of all necessary to know just how long it takes a vehicle to clear a booth. Leslie Edie (1954) has studied this problem in connection with the operations of the Port Authority of New York. He concludes that the minimum times to move forward to the booth after the previous vehicle moves off, to pay the toll, and then to get underway again are 8.0 seconds, 9.5 seconds, and 11.0 seconds for passenger cars, buses, and trucks respectively.²³ However, it is easily seen that these values cannot apply to the Bay Bridge. On April 19, 1972, between 7:00 and 7:30 am, 4,871 vehicles moved through the toll plaza.²⁴ This traffic was made up of 4466 autos, 157 buses, and 319 trucks.²⁵ If they had cleared the toll plaza at Edie's rates, it would have taken 11.3 hours of toll-booth operations, but with 17 lanes open only 8.5 lane-hours of operation took place. Thus we may conclude that Edie's minimum vehicle times are overstated in the aggregate by at least 33%. I assume that the times are overstated in the same proportion (as they apply to the Bay Bridge), and in what follows the

applicable times are 5.4 seconds, 6.4 seconds, and 7.4 seconds for autos, buses, and trucks respectively.

The following simple model will be used to analyze congestion at the toll-booths. Three traffic streams converge on the 17 booths; the distributions of arrivals at the plaza are independent Poisson variates. I assume that the distribution of vehicle types going through a particular booth is just the distribution of vehicle types in the traffic stream generally. Thus, the problem can be analyzed in terms of the congestion at a single toll-booth.²⁶ If the vehicle times derived in the last paragraph are exponential random variables, then the traffic at a toll-booth is a simple birth-and-death queuing process, known in the literature as an M/M/1 queue.²⁷ I discuss the equilibrium queue.

Denote by Q_t^A , Q_t^B , and Q_t^T the number of vehicles per hour of the three types, and by $q_t = (q_t^A, q_t^B, q_t^T)$ the proportions of each vehicle type. Then our assumptions imply that in the t -th hour (on average) $Q = (Q_t^A + Q_t^B + Q_t^T)/17$ vehicles pass through the single booth analyzed. If the vehicle times are denoted μ_t^A , μ_t^B , and μ_t^T , then write $\mu = \mu_t^A q_t^A + \mu_t^B q_t^B + \mu_t^T q_t^T$ as the weighted average vehicle time.²⁸ One can then show that (provided the system is ergodic, i.e., satisfies $Q/\mu < 1$), the expected number of vehicles in the queue is given by $N = \rho/(1-\rho)$, where $\rho = Q/\mu$, and that total expected system time (i.e., time spent in the queue plus time paying the toll) is $T = N/Q$.²⁹

An additional vehicle alters the proportions q_t and the

weighted average μ , changing them to q_t' and μ' . In the same way as before we can define N' and T' , the new expected queue size and system time. We now define the expected increase in system time to be $\Delta T = T' - T$. The number of vehicles so affected is N . From the assumed distribution of traffic, the given occupancy rates of the vehicles and the values of time of the different occupants, we can calculate the average value of time in the traffic stream. We then take as an estimate of the external cost imposed on the traffic stream by the appearance of another vehicle the quantity

$$\Delta T \cdot N \cdot V_v^* \quad (2.8)$$

where V_v^* is the average value of time.³⁰

What is the magnitude of this cost? It can be calculated that it is extremely small. Suppose that the arrival rates referred to at the beginning of this section hold for the subsequent half-hours as well, as do the occupancy rates.³¹ We assume the values of time shown in Table 3 (these will be justified in the following section), which summarizes the data. Then one may calculate that for an additional auto, $N = 8.81335$ vehicles, $\Delta T = .000013092$ hours, and $V_v^* = \$8.21/\text{hour}$. Thus the required congestion toll for the toll plaza operation is \$.00097 per car. This is clearly negligible, as might perhaps have been expected. The average queue is small, and will only be augmented with probability $1/17$; and the weighted average vehicle time is 5.56 seconds. With numbers of these magnitudes, an additional vehicle will not cause much time loss.

This concludes the derivation of the short-run bridge tolls.

Table 3Data for Calculations of Auto Toll-Booth Congestion Toll

<u>Mode</u>	<u>Vehicles</u> ¹	<u>Vehicle Time</u> ²	<u>Value of Time</u> ³	<u>Occupancy</u> ⁴
Auto	8,932	5.4	\$3.00	1.56
Bus	314	6.4	3.00	37.0
Truck	638	7.4	9.00	1.1

Number of Booths: 17

Notes

1. Per hour. See text.
2. Seconds per vehicle.
3. Per occupant. See text.
4. Source: Homburger (1972).

These consist of the quantities (in the case of passenger cars) given by equations (2.1) plus (2.8); and the difference between the last columns of Tables 1 and 2. Recall that all these were derived for a given traffic volume. The next section justifies this approach, and derives values of time for the three modes in question.

3. DEMAND AND EQUILIBRATION

This section completes the model. In Section 3.1 I discuss the mode-choice decisions of commuters faced with two modes (auto and bus), each offering different service characteristics. Section 3.2 discusses the price elasticity of demand of trucking bridge services. In both these sections values of time for users of the modes are derived. Finally, Section 3.3 shows how an equilibrium model may be formulated from the demand studies of Sections 3.1 and 3.2, and the tolls derived (for a given traffic level) in Section 2.

3.1 Commuter Work-Trip Mode Choice

Following Hausman and Wise (1978), consider a representative customer i with characteristics a_i , facing two alternatives in work-trip mode choice: a bus, characterized by an observable vector x_i^B and a car, characterized by x_i^A . For example, a_i might include the income and age of the representative consumer, x_i^B the time taken to complete the trip by bus and the fare, x_i^A gasoline prices, etc. Suppose the consumer has a utility function U_i , linear in parameters and defined over the alternatives, such that

$$U_i^A = U_i^A(x_i^A, a_i) = Z_i^A / \beta \quad (3.1)$$

(and similarly for V_i^B)

where Z_i^A is a vector of arithmetic combinations of the elements of X_i^A and a_i , and β is a vector of parameters.

We now suppose that an individual is distinguished from the average by a stochastic term reflecting unobservable differences in the modes or among individuals.³² Thus we may write

$$U_i^A = U_i^A(X_i^A, a_i) + \varepsilon_i(X_i^A, a_i) = Z_i^A/\beta + \varepsilon_i^A \quad (3.2)$$

$$U_i^B = U_i^B(X_i^B, a_i) + \varepsilon_i(X_i^B, a_i) = Z_i^B/\beta + \varepsilon_i^B \quad (3.3)$$

where the ε s are random variables. The individual is assumed to make that choice which maximizes his utility. Due to the presence of the stochastic term, we can analyze only the probability that a given choice will be made. The probability that individual i chooses a car is

$$\begin{aligned} P_i^A &= \text{Prob}[U_i^A + \varepsilon_i^A > U_i^B + \varepsilon_i^B] \\ &= \text{Prob}[\varepsilon_i^B - \varepsilon_i^A < U_i^A - U_i^B] \\ &= \text{Prob}[\varepsilon_i^B - \varepsilon_i^A < (Z_i^A - Z_i^B)/\beta] \end{aligned} \quad (3.4)$$

For reasons given by Domencich and McFadden (1975), the random variables ε are often assumed to have the extreme value distribution.³³

Under this assumption, the mode-choice probabilities can be shown to satisfy

$$\frac{P_i^A}{P_i^B} = \frac{P_i^A}{1-P_i^A} = \frac{e^{Z_i^A/\beta}}{e^{Z_i^B/\beta}} \quad (3.5)$$

(where the first equality follows from the assumption that only two modes are available). Then

$$\ln \frac{P_i^A}{1-P_i^A} = (Z_i^A - Z_i^B) / \beta \quad (3.6)$$

and Domencich and McFadden show how, given observations on Z_i^A and Z_i^B and knowledge of the choices actually made by a sample of individuals, the unknown parameter vector β can be estimated by maximum likelihood methods.³⁴

Kenneth Train reports the following two-mode logit model estimated on a sample of 771 Bay Area commuters (t-statistics in parentheses):³⁵

$$y = \ln \frac{P^A}{1-P^A} = - \frac{3.80}{(5.07)} - \frac{.0343}{(5.60)} \left(\frac{AC-F}{W} \right) - \frac{.0233}{(2.80)} (AIV-BIV) + D \quad (3.7)$$

where

AC = Auto cost per round trip (\$)

F = Bus round-trip fare (\$)

W = Post tax wage (¢/minute)

AIV = Auto round-trip in-vehicle time (minutes)

BIV = Bus round-trip in-vehicle time (minutes)

D = Other terms which, as will appear in section 3.3, need not concern us.

From this we may derive the marginal value of in-vehicle time for commuters (note that it will be the same irrespective of the mode chosen) as a per cent of the wage as follows:³⁷

$$V_V^A = V_V^B = \frac{\delta AIV}{\delta AC} = \frac{\delta BIV}{\delta F} = \frac{\delta y}{\delta AC} / \frac{\delta y}{\delta AIV} = W \frac{-0.0233}{-0.0343} = .68W \quad (3.8)$$

That is to say, the marginal value of in-vehicle time is 68% of the post-tax wage. At the observed average post-tax wage of \$4.85 per hour, the value of time is \$3.30 per hour.

3.2 Trucking Demand

As in the previous section, two distinct problems face us. We must first determine the correct social marginal valuation of time, and then discuss the relationship of the demand for trucking use of the bridge to the price charged. It turns out that we cannot, in our present state of knowledge, provide complete answers to these two questions, as we were able to do in the last section for commute travel, and we will eventually have to solve the second by means of reasonable parametrizations.

The social marginal value of time will have two components. First, longer travel time will be directly costly to the truck operator in the form of increased driver wages.³⁸ In addition, there will be costs imposed on the shipper of the goods carried by the trucker if his shipment arrives late. Using a model similar to that of the last section, Levin has studied the mode choices of shippers of manufactured goods.³⁹ The shipper value of time is expressed as:⁴⁰

$$C = r^* V t \quad (3.9)$$

where t = time (years)

V = value of shipment (¢/hundredweight)

r^* = subjective discount rate

Levin reports several attempts to estimate r^* . Using the arithmetic average of the calculated values of r^* in his models 1, 2, and 3, we take $r^* = 1.132$.⁴¹ Then C may be calculated to be $\$.00065 V^*t$ per hour, where V^* is the value of the shipment in dollars per ton. Using Levin's estimate that the average value of goods shipped is

\$1,000/ton,⁴² we see that an estimate of the shipper-value of delay is \$.65/ton/hour. For the payload assumed on our standard truck in Table 2, 6 tons, the marginal social value of delay is, then, the product of truckers' wages and the observed occupancy of the truck, plus \$3.90, per hour.

Turning now to the second task, it would be desirable to use Levin's truck mode-share equation to estimate directly the dependence of demand for trucking road-services on the price charged. However, without restrictive assumptions, this is not now possible. For example, while for peak commuter demand it is not unreasonable to assume inelasticity with respect to toll variations over the time of day (at least until "flex time" becomes more generally accepted), the possibilities of scheduling truck trips to avoid peak tolls cannot be dismissed.⁴³ Levin's approach gives us no way to incorporate this aspect of truck demand. In this paper I adopt a parametric approach, and examine (in Section 4) the dependence of the results on various assumed values of truck price-elasticity of demand. This elasticity may be taken to reflect either the fact that increased costs may be passed on to shippers, causing them to change modes, or any scheduling considerations (whether regarding routes or times of travel) relevant to demand.

In order to apply such an assumed elasticity, it is necessary to know the base costs to which changed costs in the form of the tolls of Section 2 can be compared. Table 4 shows the operating costs of truckers in the ICC's Western District.⁴⁴ We also require

Table 4
1972 Western District Truck Operating Costs¹
(per vehicle mile)²

Maintenance	\$.0880
Transportation ³	.0784
Terminal	.2256
Traffic Management	.0305
Insurance and Safety	.0365
Administration	.0504
Depreciation ⁴	<u>.0460</u>
TOTAL	\$.5554

Notes

1. Source: ICC (1972).
2. Vehicle miles (excluding those attributed to rented vehicles):
 3.011×10^9 .
3. Excluding purchased transportation; wages separately accounted for, and not included here.
4. Source: American Trucking Association (1974), for U.S. generally.

a description of a plausible trip. For the purposes of this paper, I assume that this trip is 50 miles long: approximately from Palo Alto to Berkeley. Table 5 shows the composition of the trip. Naturally, to the extent that actual trips are longer than the one assumed here, the less will be the effect of toll increases on bridge demand by truckers, toll charges being a progressively smaller fraction of total cost. However, I may at this point anticipate the results of Section 4 and note that the optimal tolls are such that their magnitude will not alter significantly the number of trucks on the road.⁴⁵ We have evidence that the average Pacific Region haul is considerably longer than that assumed here--359 miles in 1972⁴⁶--but this cannot be used as a guide to the average haul over the Bay Bridge, since traffic bound for the Pacific Northwest or Southern California would presumably not use the bridge at all.

3.3 Equilibration

In this section the supply side of Section 2 and the demand side of Sections 3.1 and 3.2 are integrated into a simple cobweb model of convergence to equilibrium.⁴⁷ The idea behind this is simple. Denote by $Q_t^0 = (Q_t^0 A, Q_t^0 B, Q_t^0 T)$ the presently observed traffic stream--the number of cars, buses, and trucks arriving at the bridge during period t . Corresponding to this traffic stream there will be a set of optimal short-run tolls, as derived in Section 2. But if these tolls were imposed, the mix of traffic would change. There would be two distinct effects. First of all, the mode choices

Table 5Assumed Composition of Average Truck Trip

<u>Component</u>	<u>Length (mi.)</u>	<u>Speed (mph)</u>
Freeway	37	() ¹
Bridge	9	() ¹
Arterials	5	15

Note

1. Freeway speed assumed to be same as bridge speed, determined by equation (2.5).

of commuters would no longer be the same. If τ^* is a vector of currently charged tolls per occupant (indexed by mode) and ${}^0\tau$ a vector of the optimal short-run tolls, then the individual auto rider experiences a cost change $\Delta {}^0\tau^A = {}^0\tau^{*A} - {}^0\tau^A$, and the costs to the bus passenger will change by $\Delta {}^0\tau^B = {}^0\tau^{*B} - {}^0\tau^B$, assuming, as we shall do, that all increases are passed on to the bus rider through increased fares.

We know the current mode split at toll ${}^0\tau^*$. The effect of the new short-run tolls will be to change the log mode-split by the amount

$$\frac{-.0343}{W} (\Delta {}^0\tau^A - \Delta {}^0\tau^B)$$

as can be seen on examination of equation (3.7).⁴⁸ Adding this term to the original log mode-split gives the predicted log mode-split which would result from imposition of the new tolls. From this, the values ${}^0P^A$ and ${}^0P^B$, the new probabilities that a commuter will choose the auto and bus modes, can be calculated. From the assumption of Section 1 that all travel in the peak is work-trip travel, we can calculate the new distribution of commuter traffic.⁴⁹

Similarly, if η is the assumed truck price elasticity, we can compare the old and new costs for the trip described in Table 5 and compute the effect on truck traffic.⁵⁰ The result will be a new vector of peak traffic, denoted by 1Q_t . In precisely the same way as before, a short-run toll vector ${}^1\tau$ corresponding to 1Q_t may be calculated, and "round 2" values of ${}^1P^A$, ${}^1P^B$, and a new vector of traffic 2Q_t obtained. At the n-th stage, we compare ${}^{n-1}Q_t$ with

$n-2_{Q_t}$. If each element in the one is within 1 vehicle of the corresponding element in the other, we say that an equilibrium is attained.⁵¹

It is an equilibrium in the sense that imposition of the tolls

$(n-1_{\tau}^A, n-1_{\tau}^B, n-1_{\tau}^T)$ will cause the distribution of traffic to go

to $(n-1_{Q_t}^A, n-1_{Q_t}^B, n-1_{Q_t}^T)$ and stay there: that is to say, it is a

stable point. Thus we have constructed a sequence of problems, each

characterized by a given distribution of traffic, and with the move-

ment from one problem to another governed by the demand side estimated

in Sections 3.1 and 3.2.⁵² The result is an equilibrium set of tolls:

this justifies the approach of Section 2, which derived tolls as a

function of a given traffic demand.

With these preliminaries done, I turn now to the estimation of the equilibrium Bay Bridge tolls.

4. TOLLS ON THE BAY BRIDGE

Table 6 summarizes the existing work-bound traffic situation on the bridge.⁵³ I assume that the effect of charging differential tolls in the peak and off-peak periods is that eventually all peak traffic will be spread out over a two-hour peak with traffic characteristics found by averaging those in the present peak and near-peak periods. Recall, from the previous discussion, that auto and truck occupancy rates are considered fixed; and that all traffic in the peak is work-bound. Bus operating costs varying with vehicle hours are \$12.09 per hour, as in Viton (1974).

The Bay Bridge now collects tolls in the westbound direction only, and it is traffic in this direction that is the focus of the analysis here. That is to say, we analyze the morning peak. The afternoon trip, with most of the traffic flowing east, is assumed to have the same composition as the morning, westbound traffic. This assumption is less easy to justify than the one assigning all and only the work-trip traffic to the morning peak: it is clear that people have considerably more flexibility in their arrival times at home than they do in their report-for-work times. Nonetheless, I assume that the evening peak is just the return trips of the morning peak

Table 6
Observed and Assumed Traffic Westbound¹

<u>Time</u>	<u>Autos</u>	<u>Persons Per Auto</u>	<u>Buses</u>	<u>Passengers Per Bus</u>	<u>Trucks</u>	<u>Persons² Per Truck</u>
7:00-7:30 am	4,466	1.56	157	37.0	315	1.1
7:30-8:00	3,530	1.53	191	39.5	285	1.1
8:00-8:30	3,589	1.38	102	37.5	269	1.1
8:30-9:00	3,332	1.32	45	35.2	354	1.1
Spread peak --per hour	7,459	1.46	248	37.9	612	1.1

Notes

1. Source: Homburger (1972) and text
2. Assumed. See Homburger (1972)

traffic. Thus, the optimal one-way collection is twice the calculated short-run marginal cost.

Table 7 shows the equilibrium situation. As we can see from that table, the results do not vary significantly with the assumed truck demand elasticity. I shall therefore discuss the results for the case of unit truck demand elasticity. First, all tolls are very much higher than at present. Equilibrium auto tolls increase by a factor of 2.7. For the buses, the toll paid per vehicle goes up by a factor of 4.4. However, because of the increased bus usage--an additional six per cent of the commuters use the buses--the toll per passenger increases from 1.3¢ to 5.2¢. Even though this is large in percentage terms, it is small when compared to the 60¢ increase in per-passenger auto tolls.

Since, as we have seen, each vehicle type currently pays for more than its share of the non-congestion externalities, and since it has been shown that congestion at the toll booth is negligible, it follows that it is the congestion externality that is responsible for the high tolls. It is interesting to note that even with the new tolls, the diversion of traffic to the mass transit mode is small. As planners, concerned with maximizing social welfare, this should not concern us. We can have no reason to quarrel with the preferences of consumers, provided only that they do not treat scarce resources as free, as they would if external effects were not accounted for. That 48% of the commuting public would choose to pay the high full cost of their actions and still stay with the private

Table 7

Equilibrium Bay Bridge Tolls and Volumes¹

Truck Demand Elasticity	Autos Per Peak Hour	Toll		Buses Per		Toll		Bus		Trucks Per		Toll	Bridge Speed
		Per Car	Per Bus	Peak Hour	Per Hour	Per Car	Per Bus	Share	Peak Hour	Peak Hour	Per Truck		
Current ²	7,459	\$0.50	\$0.50	248		\$0.50	\$0.50	46%	612		\$1.72	40.1 mph	
.50	6,640	1.37	2.24	248		2.24	2.24	52%	606		2.99	41.6	
.75	6,641	1.37	2.23	248		2.23	2.23	52%	603		2.98	41.6	
1.00	6,643	1.37	2.23	248		2.23	2.23	52%	600		2.98	41.6	
1.25	6,644	1.37	2.23	248		2.23	2.23	52%	597		2.98	41.6	
1.50	6,645	1.36	2.23	248		2.23	2.23	52%	594		2.98	41.6	
2.00	6,647	1.36	2.22	248		2.22	2.22	52%	588		2.97	41.6	

Notes

1. Tolls rounded to two places of decimals; assumes one-way collection.
2. For derivation of current traffic volumes (spread-out peak), see text. Traffic volumes are for direction of greatest flow.

car tells us only that people must value the convenience of the car highly. In these circumstances efforts further to increase the attractiveness of mass transit will actually lead to decreases in social welfare.

Indeed, this has been the tendency on the Bay Bridge. In the latter part of 1972 the Toll Bridge Authority reduced the bus toll from 50¢ to 10¢. The first row of Table 8 shows the optimal tolls conditioned on the distribution of traffic after the new toll schedules went into effect.⁵⁴ In both cases the optimal price ratio of auto to bus tolls is .61. However, holding auto tolls constant and reducing bus tolls changes the actual price ratio from 1.00 to 5.00, which increases rather than reduces the distortion.⁵⁵

These calculations assume that the trucking sector views the tolls as 2-way tolls, that is to say, that the trucks crossing the bridge in the morning peak make a return trip in the evening peak. It has already been suggested that this is an implausible assumption, and that scheduling to avoid high tolls is a very real possibility. To investigate the effects of relaxing this assumption, suppose that all truckers make only one peak trip. Thus, in distinction to the perception of commuters, truckers see the single toll collected as a one-way toll. The second and third rows of Table 8 show the equilibrium tolls calculated under this assumption for the two different toll schedules. The effects are small, particularly on the commuting population.

We may therefore conclude, first, that the efficient allocation

Table 8

Equilibrium Bay Bridge Tolls¹ and Volumes² Under Alternative Assumptions

Assumption ³	Autos Per Peak Hour		Buses Per Peak Hour		Toll Per Bus		Bus Share		Trucks Per Peak Hour		Toll Per Truck		Bridge Speed	
New toll sche- dule (bus toll 10¢)	6,296	\$1.39	252		\$2.26		54%		823		\$3.02		41.3 mph	
Truckers per- ceive toll as 1-way toll; old toll sche- dule (bus toll 50¢)	6,646	1.36	248		2.23		52%		592		2.97		41.6	
Truckers per- ceive toll as 1-way toll; new toll sche- dule (bus toll 10¢)	6,301	1.38	252		2.25		54%		811		3.01		41.4	

Notes

1. One-way collection.
2. For spread-out peak.
3. Truck elasticity of demand 1.0.

of transportation resources in the Bay Area, given that we cannot alter the amount of capacity in place, requires considerably higher tolls than currently observed. And, secondly, that an optimal toll structure would have only small effects on the transit usage.

A final question concerns the feasibility of implementing the higher tolls: could they be implemented at all? It would appear that they could. The Bay Bridge falls under the California Toll Bridge Authority Act.⁵⁶ In 1950, the Attorney General of California held that the Authority has ample power to fix and make changes in tolls,⁵⁷ and in another opinion given in 1959 he held that the toll schedule (for commercial vehicles) should be such that the vehicles using the bridge will bear their equitable portion of fees collected.⁵⁸ If the "equitable portion" is taken to mean "full social marginal cost," as economists would argue that it should, then there is in this opinion scope for implementing the tolls derived in this paper. As a matter of procedure, however, the Authority may be required to hold public hearings before raising the tolls. In 1977 the Golden Gate Bridge Highway and Transportation District raised the one-way toll on the Golden Gate Bridge from 75¢ to \$1.00. A complaint was filed in the California Superior Court in San Mateo County alleging that the District violated the California Environmental Quality Act by not holding hearings on the proposed toll increase. The court held for the plaintiffs.⁵⁹ Thus, the effect of this decision is that unilateral action by the Authority is open to successful challenge in the courts. But note that an evaluation of a proposed toll

increase along the lines of the California Environmental Quality Act would require precisely the information discussed in this paper, that is to say, a discussion of externalities and the effects of toll changes on the flow of traffic. So, although hearings are required, the data developed in this paper contain the needed information to alter the Bay Bridge tolls to their optimal levels.⁶⁰

FOOTNOTES

- ¹T. E. Keeler and K. A. Small, 1977, p. 4.
- ²Ibid., p. 11. See particularly the last paragraph on p. 11.
- ³Ibid., p. 18.
- ⁴Ibid. Interest rate 12%; value of time \$2.25 per vehicle-hour.
- ⁵Keeler and Small are not unaware of this limitation: ibid., p. 4.
- ⁶As will appear later, the "peak" considered in this paper is a "spread-out" peak derived from actual conditions.
- ⁷Cited on a flyer, "Toll Bridges of California," used by the California Department of Transportation.
- ⁸Keeler and Small, 1977, p. 3.
- ⁹This is done for a general pricing and investment model in a forthcoming paper by George S. Cluff.
- ¹⁰For details of the representative vehicles, see notes 1, 2, and 3 to Table 2, infra.
- ¹¹Keeler and Small, 1977, p. 17.
- ¹²The derivation of the congestion tolls for the other modes is precisely analogous.
- ¹³Keeler and Small, 1975, p. 34. Their reported equation has been altered to reflect the fact that exactly five lanes in each direction are available, and that the optimal capacity of the road is 2,000 cars per hour.
- ¹⁴Handbook of Transportation Engineers, 1976, p. 322.
- ¹⁵Keeler and Small, 1975, p. 52.

- ¹⁶K. A. Small, 1975.
- ¹⁷Keeler and Small, 1977, p. 10.
- ¹⁸Ibid.
- ¹⁹D. B. Lee, 1972.
- ²⁰T. E. Keeler, G. S. Cluff, and K. A. Small, 1974.
- ²¹T. E. Keeler, K. A. Small, P. A. Viton, et al., 1975, p. 111.
- ²²See W. Homburger, 1972, p. 6.
- ²³L. Edie, 1954, p. 122, Table IV (left-handed booth).
- ²⁴Homburger, op. cit., p. 1.
- ²⁵Ibid., p. 3. The bus and truck data were in fact collected two days before the data on total traffic. For the purposes of this paper it is assumed that each day's collection is an unbiased estimate of the true values, and that, therefore, it is legitimate to use the data as if they had been collected simultaneously. Note that no major traffic disruptions took place in the period under notice: ibid., p. 2.
- ²⁶Note that this assumption does not allow for special lanes for one or more types of vehicle, as, for example, special bus or 3-or-more-per-car lanes.
- ²⁷For example, Kleinrock, 1975.
- ²⁸These are now expressed in hourly units.
- ²⁹Kleinrock, op. cit., pp. 96-98.
- ³⁰Note that this quantity depends on which type of vehicle is being discussed, since this alters the vector q_t .
- ³¹The data from Homburger, op. cit., show that in fact the previous and subsequent half hours have considerably less traffic than this. Thus the toll derived here is an overestimate.
- ³²Hausman and Wise, 1978, p. 407. This is the second of two explanations of the stochastic term that they give.
- ³³Domencich and McFadden, 1978, p. 61 ff. A random variable X has the extreme value distribution if

$$\text{Prob}[X \leq a] = \exp[e^{-a}].$$

³⁴ Ibid., pp. 120 ff.

³⁵ Train, 1976, Model 9.

³⁶ Essentially, the reason is that we analyze changes from the current mode split, and all that varies is the term AC-F, since all traffic moves across the bridge at the same speed.

³⁷ The relation between the V 's derived here and the terms $\bar{V}$ in Section 2.1 is that $\bar{V}$ is the product of V and the corresponding occupancy factor for the mode in question. That is, the V 's are values of time per person hour, the $\bar{V}$'s values of time per vehicle hour.

³⁸ All other costs are assumed to vary with vehicle-miles.

³⁹ Levin, 1978. By aggregating, Levin is able to estimate probability functions as linear relations, rather than in the logistic form required by equations (3.6) and (3.7) of Section 3.2. The modes available were truck, rail boxcar, and piggyback.

⁴⁰ Ibid., p. 33.

⁴¹ Ibid., p. 33. Model 4 is not used, since the coefficient of b_2 is not well-estimated.

⁴² Ibid., p. 32. Levin notes that this is slightly above his sample average, derived from 42 3-digit commodity groups included in the Census of Manufacturers and the Census of Transportation (ibid., p. 29).

⁴³ Casual empiricism and a 3000-mile trip on I-80 suggest that, on the driver level at least, scheduling to avoid open weigh-stations is common. This issue is discussed further in Section 4.

⁴⁴ ICC, 1972, pp. 24-25.

⁴⁵ See Table 7, infra, $\eta = 2.0$.

⁴⁶ American Trucking Association, 1974, p. 37.

⁴⁷ See Viton, 1977, for more details on this procedure.

⁴⁸ This justifies the term D in the version of equation (3.7) used here, since the change in the log mode split is not changed by the terms summarized in D.

⁴⁹ As mentioned in Section 1, auto occupancy is considered fixed. The same is supposed for truck traffic. Such an assumption is strained in the case of bus commuters. To remedy this, the following

assumption on bus agency behavior is made: if more commuters choose to ride the bus, they will be accommodated on the existing bus fleet up to the point at which there are 50 patrons per bus. When the number of patrons exceeds 50/bus, more buses will be run. This does not seem an unreasonable view of the way transit companies actually operate. To anticipate a result of the next section, more buses never have to be run.

⁵⁰ The elasticity used was the arc elasticity computed with respect to the original full cost (including time values).

⁵¹ The question may be raised whether this will always converge to an equilibrium. No general proof is attempted, but it may be noted that in no case reported here, in Viton (1977), or in Cluff (1978)--which last discusses an exceptionally wide variety of problems--did an equilibrium fail of attainment.

⁵² McFadden, Talvitie et al., 1977, discuss an equilibrium process involving use of the Scarf algorithm (Scarf and Hansen, 1976), but this seems too complex a process for the relatively straightforward problems discussed. It may be noted that, using the approach in the text, convergence took at most five rounds.

⁵³ See note 25 on the collection of the observed traffic situation.

⁵⁴ Traffic details from the September 1972 Bay Bridge Survey (Homburger, 1972).

⁵⁵ This assumes no "second-best" effects. See Lipsey and Lancaster, 1957.

⁵⁶ West's California Codes, Section 30601.

⁵⁷ 15. Opinions of the Attorney General 105 (1950).

⁵⁸ 34. Opinions of the Attorney General 217 (1959).

⁵⁹ John Mallick, County of Marin, v. Golden Gate Bridge et al., in California Superior Court, San Mateo County; Number 217298, 217299.

⁶⁰ On July 5, 1978, a law (SB 1733) became effective which held that a public agency need not comply with the provisions of the California Environmental Quality Act in raising tolls, when the new tolls were needed to "meet operating expenses, purchase or lease supplies and materials . . . to obtain capital funds . . . (or) to maintain service." (Legislative Counsel's Digest, SB 1733.) This has been interpreted as settling the cases against the Golden Gate Bridge (note 59 supra). However, in order to set optimal tolls for the sake of economic efficiency, it appears that hearings under the California Environmental Quality Act are necessary. Note that since the writ of the Toll Bridge Authority runs only on bridges and immediate approaches, the possibility of levying bridge tolls in order to approximate optimal freeway tolls, as per Keeler and Small (1977), does not arise.

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