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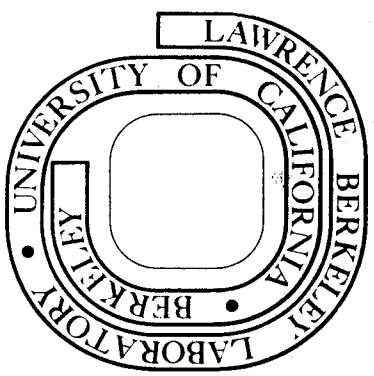

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THE NUCLEAR SURFACE DIFFUSENESS*

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ABSTRACT

A Thomas-Fermi calculation has been performed to determine the dependence of the nuclear surface energy on the diffuseness of the neutron and proton density distributions. The resulting expression is employed for the purpose of predicting the variation in diffuseness to be expected as one moves from light to heavy nuclei, and to extract from measured proton density distributions an estimate of the value of b_0, the diffuseness of semi-infinite nuclear matter. A possible explanation is also given for the fact that some experiments, which are only sensitive to the relative numbers of neutrons and protons in the tail of the nuclear density distribution, give smaller results for the neutron skin thickness than that predicted theory.

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The nuclear radius constant r_0 is a fundamental constant of nuclear physics which is defined in terms of the equilibrium density of infinite nuclear matter by the expression $\rho_0 = \left(\frac{4}{3}\pi r_0^3\right)^{-1}$. The "effective sharp radius" (the terminology and notation of ref. [1] is used throughout this paper) of a nucleus is not simply $r_0 A^{1/3}$, but is smaller for light nuclei which are squeezed by surface tension, and larger for heavy nuclei where dilitation (due to Coulomb and neutron excess effects) begins to dominate. In addition the neutron excess in heavier nuclei is expected to produce a neutron skin of thickness $t = R_n - R_Z$, where R_n and R_Z are the effective sharp radii of the neutron and proton density distributions respectively [2]. A model that includes these features can be used to infer the fundamental quantity r_0 from the experimental measurements of the radii of actual nuclei.

In a similar way the diffuseness of semi-infinite nuclear matter b_0 can be inferred from experimental measurements of the diffusenesses of actual nuclei by using a model that describes the deviations to be expected because of the Coulomb repulsion.

Both the surface energy and Coulomb energy of a nucleus depend on the surface diffuseness. Consequently, variations in diffuseness effect the total energy through the two terms in the expression,

$$E_{\text{diffuseness}} = E_S^0\left[1 + \tfrac{1}{2}(\phi_1\eta_n^2 - 2\phi_2\eta_n\eta_Z + \phi_1\eta_Z^2) + \ldots.\right] + E_C^0\left[1 - \gamma_2\beta_Z^2 + \gamma_3\beta_Z^3 + \gamma_4\beta_Z^4 + \ldots.\right] \qquad (1)$$

In this expression $E_S^0 = a_2 A^{2/3}$, the quantity a_2 being the equilibrium surface energy coefficient of semi-infinite nuclear matter. The second term in the brackets describes the increase in surface energy associated

with deviations of the neutron and proton diffusenesses from their semi-infinite nuclear matter values. The quantities η_n and η_z are defined by the expression

$$\eta_{\substack{n \\ z}} = (b_{\substack{n \\ z}} - b_0)/b_0 \quad , \tag{2}$$

where b_n and b_z are the diffusenesses of the neutron and proton density distributions and b_0 (the fundamental quantity whose value we seek) is their value in the absence of outside influences. A Thomas-Fermi calculation (similar to the ones described in ref. [3]) was performed [4], which yielded the following estimate for the coefficients appearing in eq. (1):

$$\phi_1 = 0.45 \quad ,$$

$$\phi_2 = 0.23 \quad . \tag{3}$$

The second term in eq. (1) represents the Coulomb energy and its dependence on the diffuseness of the proton distribution. The coefficient $E_C^0 = c_1 Z^2/A^{1/3}$, where c_1 is the droplet model Coulomb energy coefficient. The expansion parameter β_z is defined by the expression,

$$\beta_z = b_z/R_z \quad ,$$

where R_z and b_z are "the equivalent sharp radius" and "surface width," respectively, of the proton density distribution.* For a Fermi-function density distribution the coefficients have the values [5],†

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*The reader is reminded that the notation of ref. [1] is used throughout this paper and that an awareness of the definitions given there is essential to the understanding of the work presented here.

†For a Fermi-function the surface width b is related to the diffuseness parameter z by the expression $b = \pi/\sqrt{3}\ z$.

$$\gamma_2 = 5/2$$

$$\gamma_3 = 3.02168$$

$$\gamma_4 = 1 \qquad (5)$$

For the liquid drop model coefficients a_2 and c_1 we will use the values of a_2 and r_0 from the latest droplet model fit to nuclear masses, fission barriers and radii [6], which are

$$a_2 = 20.69 \text{ MeV} \quad , \text{ and}$$

$$r_0 = 1.18 \text{ fm} \quad , \text{ hence}$$

$$c_1 = \frac{3}{5}\frac{e^2}{r_0} = 0.7322 \text{ MeV} \; . \qquad (6)$$

For purposes of illustrating the consequences of eq. (1) it is sufficient to set $R_Z = R = r_0 A^{1/3}$ in (4), to retain only the terms in γ_2 and γ_3 in the Coulomb energy, and to expand the Coulomb energy to second order in η_Z. The resulting energy expression is

$$E = E_S^0 \phi_1 \left[\text{constant} + \tfrac{1}{2}(\eta_n^2 - 2\varepsilon\eta_n\eta_Z + \eta_Z^2) \right.$$
$$\left. - C(1-\delta)\eta_Z - C(1-2\delta)\eta_Z^2 + \ldots . \right] \quad , \qquad (7)$$

where

$$\varepsilon = \phi_2/\phi_1 \quad ,$$

$$\delta = \frac{3}{2}(\gamma_3/\gamma_2)(b_0/r_0)A^{-1/3} \quad ,$$

$$C = (5c_1/a_2\phi_1)(b_0/r_0)^2 Z^2 A^{-5/3} \; . \qquad (8)$$

Minimization with respect to η_n and η_Z results in the following expressions for the equilibrium values:

$$\eta_z = C(1-\delta)/[(1-\epsilon^2) - C(1-2\delta)] \ ,$$

$$\eta_n = \epsilon\eta_z \ . \qquad (9)$$

Figure 1 is a schematic contour drawing which illustrates how the surface energy increases when the neutron and proton diffusenesses differ from b_0. The upper part of the figure corresponds to the values of ϕ_1 and ϕ_2 we calculated using a Thomas-Fermi approximation [4]. The Coulomb repulsion creates a driving force which causes the proton diffuseness to increase. As a consequence of the orientation of the surface energy contours the neutron diffuseness also increases and the point describing the system moves along the dashed line until a new equilibrium is obtained. In some sense this system might be described as "symmetry dominated" since the neutron and proton diffuseness change in such a way as to try to maintain the N/Z ratio throughout the surface. If ϕ_2 had had the opposite sign then the contours would have appeared like those in the lower part of the figure. Adding the Coulomb repulsion would have increased the proton diffuseness and caused the neutron diffuseness to *decrease*. Such a situation could be described as "compressibility dominated" since the neutron diffuseness changes in such a way as to try to maintain the total matter diffuseness as nearly constant as possible.

In order to estimate the value of b_0 we have calculated the values of b_z for nuclei along beta-stability [using Green's approximation [7] that $N-Z = 0.4\ A^2/(200+A)$] assuming various values of b_0. These predictions are compared with the experimental results in Fig. 2a, from which we estimate that

$$b_0 = 0.82 \pm 0.05 \text{ fm.} \qquad (10)$$

Error bars are shown on the experimental points in Fig. 2 when they are available in the literature [8]. Most of the smaller error bars are associated with a single measurement. These values are probably unrealistic since the scatter of the results is usually larger when more than one measurement is made for a particular nucleus. Most of the larger error bars are associated with multiple measurements. In addition to the fact that the quality of the data limits the accuracy to which b_0 can be determined, it should be recognized that some scatter is expected because of shell effects. The diffuseness of the neutron or proton density distribution surely depends, to some extent, on the particular single particle configuration [9]. There is also some evidence that single particle effects show a preference, in heavy nuclei, for nuclear potential wells which are more diffuse for protons than for neutrons [10].

In the lower part of Fig. 2 the predicted diffusenesses (b_n for neutrons and b_z for protons) are plotted against the mass number A for nuclei along beta-stability. At the bottom of the figure the neutron skin thickness predicted by the droplet model is plotted [6]. If it were not for the neutron skin the diffuseness of the total matter distribution b_ρ (neutrons plus protons) would simply be the weighted mean of b_n and b_z that is shown by the dashed line. The actual value of b_ρ is always larger since the separation between the neutron and proton surfaces contributes to the apparent diffuseness. The relationship between b_ρ, b_n, b_z and the skin thickness t is

$$b_\rho^2 = \left(\frac{N}{A}\right) b_n^2 + \left(\frac{Z}{A}\right) b_z^2 + \left(\frac{N \cdot Z}{A^2}\right) t^2 \qquad (11)$$

Figure 3 shows the consequences of employing the value $b_0 = 0.82$ fm and eqs. (9) to predict the diffusenesses of the heavy nucleus ^{208}Pb. The upper part of the figure shows the proton and neutron density distributions with

$$b_n = 0.91 \text{ fm, and}$$
$$b_z = 1.00 \text{ fm,} \qquad (12)$$

as calculated here, and with the effective sharp radii R_n and R_z separated by a skin thickness ($t = 0.40$ fm) as predicted by the droplet model [6]. The lower part of the figure shows the local nuclear asymmetry,

$$\delta(r) = [\rho_n(r) - \rho_z(r)]/\rho(r), \qquad (13)$$

as a function of position in the nucleus. The solid curves correspond to fixing both the neutron and proton diffuseness at $b_n = b_z = 1$ fm and varying the surface stiffness coefficient Q in the droplet model so as to produce different values of the neutron skin thickness t, as shown in Table 1. If the diffusenesses are allowed to assume the values predicted here (12), the dot-dashed curves result. The important consequence of this set of calculations is that they illustrate how an experiment that measures the N/Z ratio in the tail of the nuclear density distribution can give misleading results regarding the neutron skin thickness, if the neutron and proton diffusenesses are artificially constrained to have the same value. Note that at 9 fm from the center of the nucleus in Fig. 3, the dot-dashed curve corresponding to case (b) ($t = 0.20$ fm) has bent downward so as to give the same value of δ as would be obtained for $t = 0$ and $b_n = b_z$. Since the diffuseness of the proton distribution is certainly greater than that of

the neutron distribution, even if the actual values differ somewhat from those predicted here, it is easy to understand why no conclusive experimental evidence has been obtained for the existence of a neutron skin. Droplet model considerations [3] make it clear that a neutron skin must exist for heavier nuclei but the difference in diffuseness makes it difficult to observe when the experiments are only sensitive to the relative numbers of neutrons and protons in the tail of the nuclear density distribution.

Recognition of the nuclear diffuseness degree of freedom also has important implications in the field of collective nuclear excitations. In principle, both $T = 0$ and $T = 1$ nuclear diffuseness vibrations are possible and these may be of monopole, quadrupole, or higher multipole order. The excitation energies are probably quite high because the restoring force is large and the inertial parameters are small. In addition these vibrations are probably strongly mixed with the corresponding bulk vibrations because of the inertial coupling between them.

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TABLE 1.[a] Labels for curves in Fig. 3.

Curve Fig. 3	Surface stiffness (Q in MeV)	Skin thickness for ^{208}Pb
(a)	17	0.40
(b)	51	0.20
(c)	∞	0.00

[a] See ref. [6] for the droplet model expressions governing the relationship between Q and t, and the values of the other coefficients employed in the calculation.

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FIGURE CAPTIONS

Fig. 1. Schematic drawing of surface energy contours to show how the energy increases when the neutron and proton diffusenesses vary from their semi-infinite nuclear matter value b_0. The upper figure corresponds to our Thomas-Fermi calculations [4]. The lower figure corresponds to a hypothetical set of contours that would result if the coefficient ϕ_2 had the opposite sign.

Fig. 2. In the upper part of the figure calculated and experimental values of the diffuseness of the proton distribution b_z are plotted against the atomic mass number A. The different curves correspond to different values of the fundamental constant b_0. In the lower part of the figure the calculated values of the neutron skin thickness t and the diffusenesses b_n, b_z, and b_ρ are given for nuclei along beta-stability.

Fig. 3. The neutron and proton density distributions predicted by the droplet model [6] and eqs. (9) are shown in the upper part of this figure. The local nuclear asymmetry is shown in the lower part of the figure for the three different cases listed in Table 1. The solid curves correspond to $b_n = b_z = 1$ fm, while the dot-dashed ones correspond to the values predicted here (12).

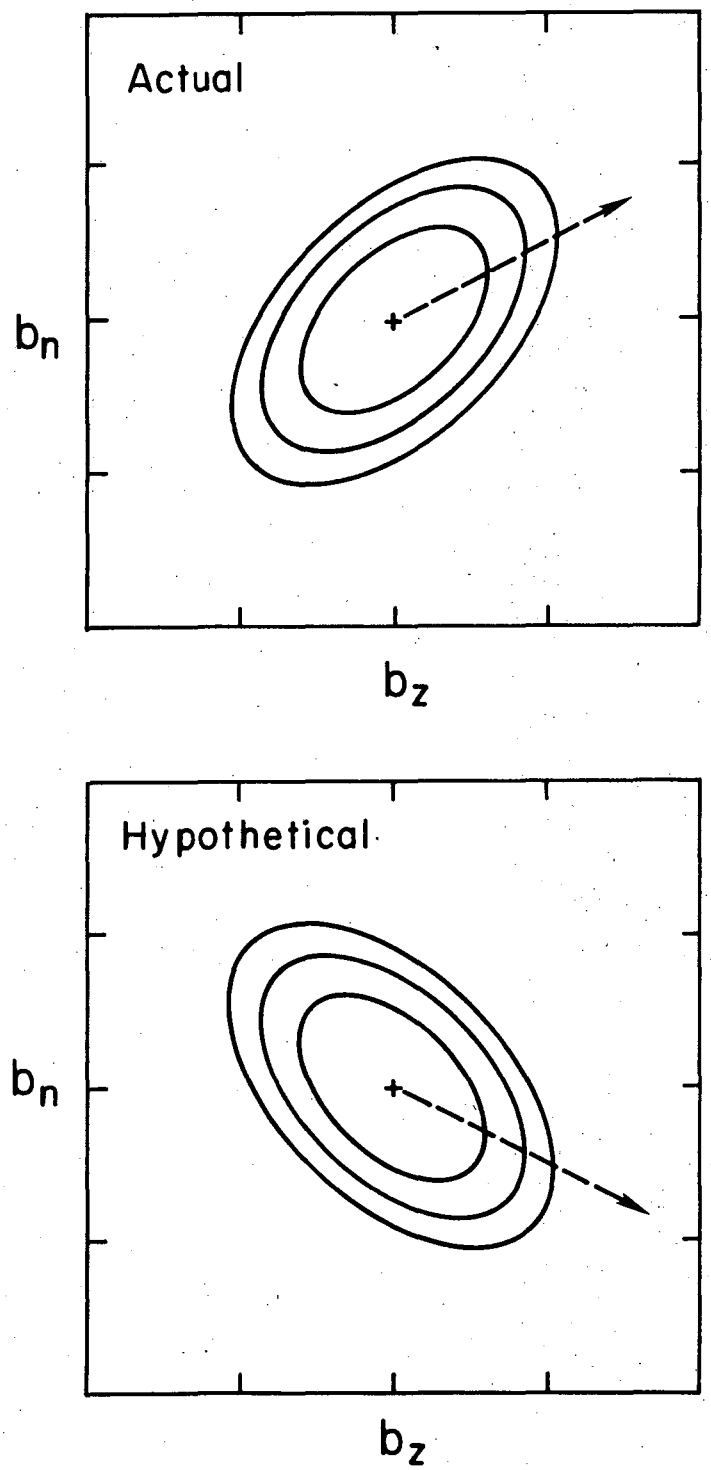

Fig. 1

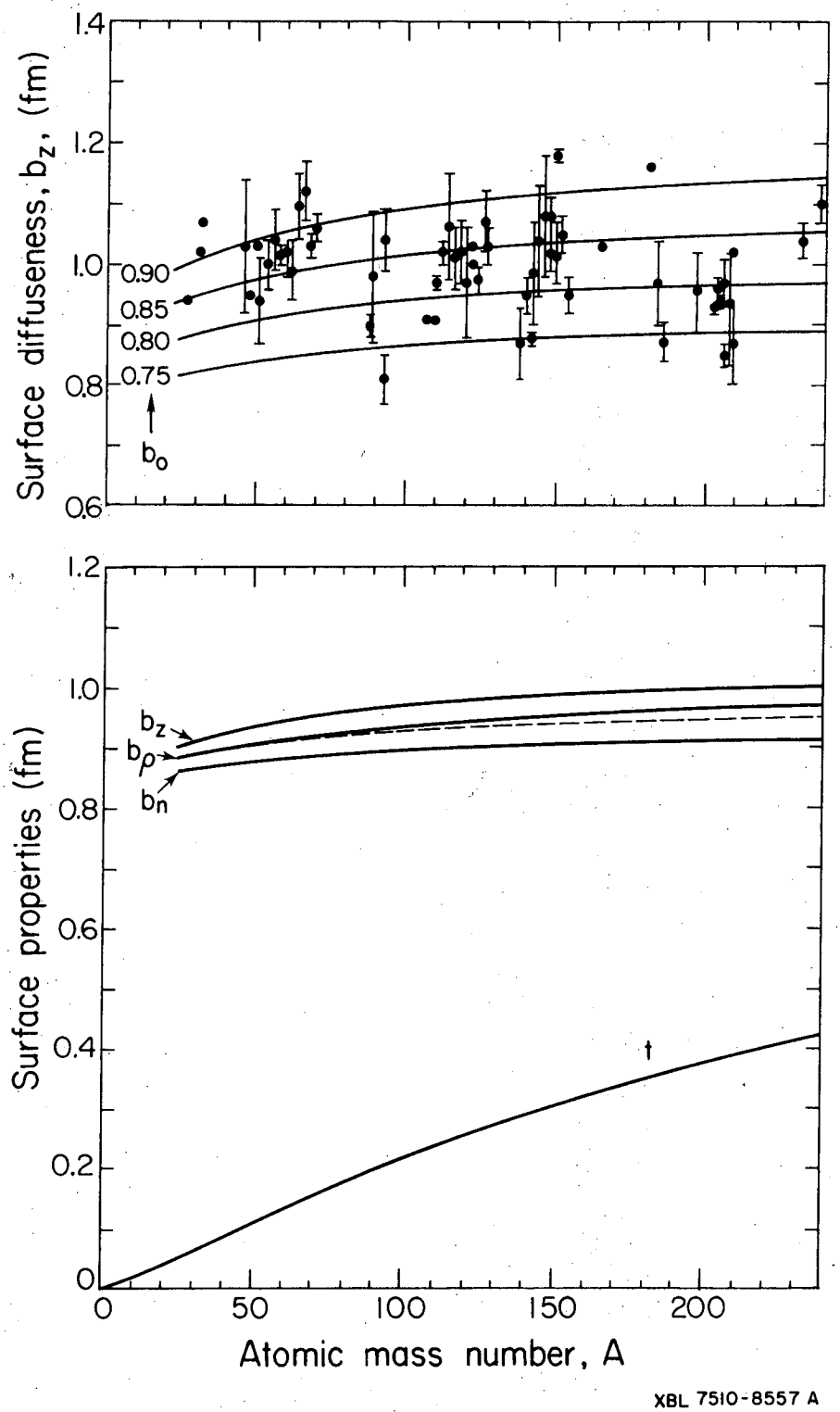
Surface diffuseness, b_z, (fm)
1.4
1.2
1.0
0.8
0.6
0.90
0.85
0.80
0.75
b_o
Surface properties (fm)
1.2
1.0
0.8
0.6
0.4
0.2
0
b_z
b_ρ
b_n
t
0
50
100
150
200
Atomic mass number, A
XBL 7510-8557 A

Fig. 2

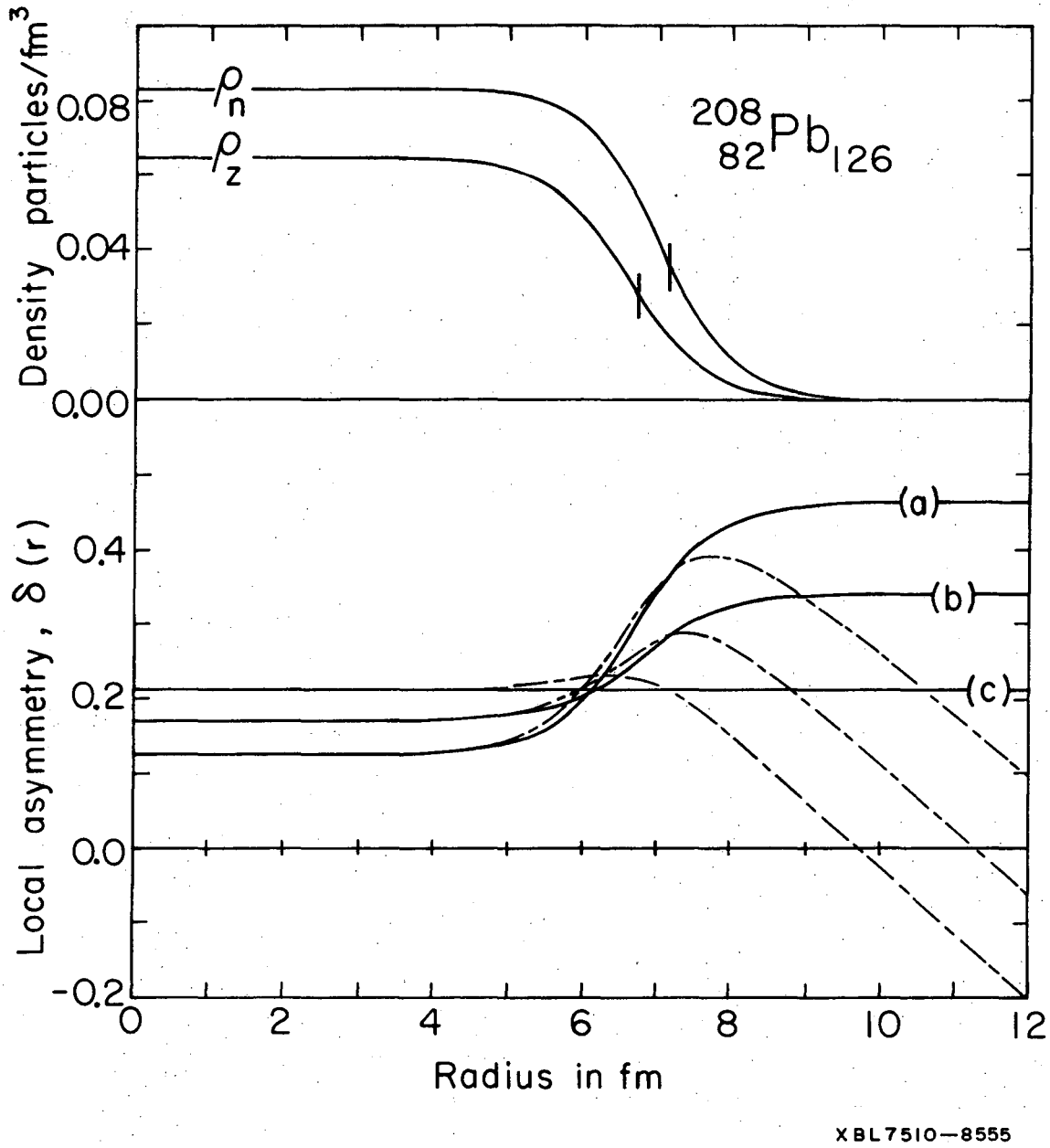
Density particles/fm³
0.08
0.04
0.00
ρ_n
ρ_z
$^{208}_{82}Pb_{126}$
Local asymmetry, δ (r)
0.4
0.2
0.0
-0.2
(a)
(b)
(c)
0
2
4
6
8
10
12
Radius in fm
XBL7510—8555

Fig. 3

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