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### Publication Date

2024-05-14

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# Radiative Corrections to Light Thermal Pseudo-Dirac Dark Matter

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(Dated: May 16, 2024)

Light thermal dark matter has emerged as an attractive theoretical possibility and a promising target for discovery at experiments in the near future. Such scenarios generically invoke mediators with very small couplings to the Standard Model, but moderately strong couplings within the dark sector, calling into question theoretical estimates based on the lowest order of perturbation theory. As an example, we focus on a scenario in which (pseudo)-Dirac fermion dark matter is connected to the standard model via a dark photon charged under a new  $U(1)'$  extension of the standard model, and we investigate the impact of the next-to-leading order corrections to annihilation and scattering. We find that radiative corrections can significantly impact model predictions for the relic density and scattering cross-section, depending on the strength of the dark sector coupling and ratio of the dark matter to mediator mass. We also show why factorization into the yield parameter  $Y$  typically presented in literature leads to imprecision. Our results are necessary to accurately map experimental searches into the model parameter space and assess their ability to reach thermal production targets.

## I. INTRODUCTION

Of all of the viable scenarios of Dark Matter (DM), one of the most compelling is that of particles that are in thermal equilibrium with the Standard Model (SM) bath in the early Universe. DM with a thermal history is very well motivated because it not only provides a feasible prediction for large non-gravitational interactions between dark and ordinary matter, but is also highly predictive in nature, leading to clear targets for experimental searches. Generically, after the DM has frozen out and its relic density set, it maintains the same interaction strength with the SM today, predicting that it is likely to be observable at a variety of experiments on Earth [1]. A canonical example is that of Weakly interacting massive particles (WIMPs), a class of heavy ( $\sim$  GeV - TeV scale) particles that interact with the SM through a roughly electroweak strength force [2, 3].

However, despite decades of search, heavy ( $\gtrsim$  GeV) WIMPs are yet to be discovered and their parameter space has become tightly constrained [3]. This lack of a clear signal has motivated searches for other visions of DM, including models with masses in the MeV to GeV regime interacting with the SM via new undiscovered forces. While this class of DM is no longer a WIMP in the traditional sense, it can still be in equilibrium at early times and thus represents a thermal target. It is a very attractive prospect as it may provide a road map to a wide and rich dark sector. The possibilities of simplified DM models are vast. However, one of the most appealing scenarios is that of DM interacting with the SM through a new vector boson, often referred to as

a dark photon, corresponding to a  $U(1)'$  extension of the SM [4–6].

Thermal sub-GeV DM, through a dark photon portal is very well motivated and has been the subject of much exploration in the literature. In this scenario, MeV to GeV mass DM can be produced and detected at a variety of current and near-future low energy accelerator experiments [7–28]. Moreover, ambient DM in the Solar System can scatter with electrons and nucleons in small scale direct detection experiments with low energy recoil thresholds [29–31]. In addition, accelerator and underground detector probes are well complemented by astrophysical and cosmological constraints [32–36]. All together, these probes make for an exciting experimental program that shows promise for discovering light thermal DM in the near future.

As an illustrative model, we focus on pseudo-Dirac dark matter interacting with the SM via a dark photon mediator. Current experimental constraints and projections on this model that target the dark matter thermal relic abundance focus on a dark sector coupling  $\alpha_D$  that is stronger than the QCD coupling at the electroweak scale. As a result, the dark sector is strongly coupled and relatively large corrections from higher orders of perturbation theory are expected. In this work, we compute the next-to-leading (NLO) order corrections on both the thermal annihilation and late time scattering processes with electrons. We focus on the NLO corrections at  $\mathcal{O}(\alpha_D)$  and neglect the presumably negligible higher order corrections from the tiny kinetic mixing.

We find that the NLO corrections can be as large as  $\mathcal{O}(10\%)$  for parameters typically discussed in the literature, and are thus necessary to take into account when precisely mapping experimental searches into the parameter space, and when comparing them with

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thermal production milestones which serve as a prime target for a currently growing intensity frontier program. It is worth pointing out that various theoretical constraints on this model resulting from its strong coupling have been considered in the literature, e.g. from the running of the dark sector coupling and the breakdown of perturbation theory [37, 38].

The rest of this article is organized as follows: In section II we describe the reference model, section III is devoted to a full description of the thermal annihilation cross-section, including both leading and next-to-leading order (NLO) processes. We compute the scattering cross-sections with electrons relevant for direct detection searches in section IV. Finally we conclude in section V. The appendix provides some technical details related to the counter-terms in the on-shell renormalization scheme.

## II. PSEUDO-DIRAC DARK MATTER

The basic module we consider consists of two Weyl fermions which play the role of dark matter and are paired by a Dirac mass,  $m_\chi$ . These fermions are neutral under the SM gauge groups, but have equal and opposite charge  $\pm 1$  under a gauged dark  $U(1)'$ , with corresponding gauge boson  $A'_\mu$ . The  $U(1)'$  symmetry is spontaneously broken by the vacuum expectation value of a dark Higgs scalar  $\phi$ , generating a mass for  $A'_\mu$  and (assuming the  $\phi$  charge under  $U(1)'$  is chosen appropriately) Majorana masses for the two Weyl fermions. We further assume that there is kinetic mixing between the dark photon and the SM hypercharge interaction, induced by unspecified UV physics.

In the mass basis, linear combinations of the original Weyl fermions appear as Majorana fermions, with their mixing determined by the Majorana masses and  $m_\chi$ . We follow the standard assumptions in the literature that the Majorana masses are much smaller than  $m_\chi$ . In this limit, the mass splitting between the two Majorana states goes to zero, and the pair can be approximately described as a single “pseudo-Dirac” state  $\chi$ . Strictly speaking, this limit is experimentally ruled out by bounds from the Cosmic Microwave Background [22, 39]. However, relatively small splittings can ameliorate these bounds [22], and the pseudo-Dirac limit is a reasonable approximation to viable models over much of the parameter space.

Putting these ingredients together, the resulting theory is described by the SM Lagrangian supplemented by terms describing the dark matter, mediator and dark

Higgs:

$$\begin{aligned} \mathcal{L}_{\text{DM}} = & i\bar{\chi}\gamma^\mu D_\mu\chi - m_\chi\bar{\chi}\chi - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{m_{A'}^2}{2}A'_\mu A'^\mu \\ & + \frac{\varepsilon}{2}X^{\mu\nu}B_{\mu\nu} + |D_\mu\phi|^2 - V(\phi) \end{aligned} \quad (1)$$

where  $\chi$  is a Dirac fermion packaging both of the original Weyl fermions in the limit of zero mass splitting (and thus including both the dark matter and its heavier partner when the mass splitting is taken into account).  $D_\mu \equiv \partial_\mu - ig_D Q_{A'} A'_\mu$  is the covariant derivative for field of  $U(1)'$  charge  $Q_{A'}$ ,  $X_{\mu\nu}$  and  $B_{\mu\nu}$  are the field strengths for the  $U(1)'$  and SM hypercharge bosons, respectively,  $\varepsilon$  characterizes the strength of the kinetic mixing and  $\phi$  is the dark Higgs field. We normalize  $Q_{A'} = 1$  from here on for  $\chi$ . After symmetry breaking, the scalar field acquires a vacuum expectation value (VEV)  $v_\phi$  and can be parameterized in the unitary gauge as  $\phi \rightarrow (v_\phi + H_D)/\sqrt{2}$ , where  $H_D$  is the dark Higgs boson.

Diagonalizing the interactions and assuming  $m_{A'} \ll M_Z$ , the resulting theory contains a dark photon  $A'$  interacting with coupling strength  $g_D$  with the DM,  $\chi$  and strength  $\varepsilon Q_f e$  with SM fields of electric charge  $Q_f$ . The dark photon mass is given by  $m_{A'} = Q_\phi g_D v_\phi$ , with  $Q_\phi$  the charge of the dark scalar. We will assume  $Q_\phi = 2$  such that the dark Higgs can have interactions with the DM Weyl fermions, allowing its VEV to contribute to their Majorana masses. For simplicity, we assume that the mixing between  $\phi$  and the SM Higgs is negligible. The mixing parameter is important when considering the phenomenology of  $H_D$ , but does not play an important role in dark matter annihilation at NLO.

## III. THERMAL FREEZE OUT

For  $m_\chi \leq m_{A'}$ , annihilation of  $\bar{\chi}\chi$  is predominantly into pairs of SM fermions,

$$\chi(p_a) + \bar{\chi}(p_b) \rightarrow f(p_1) + \bar{f}(p_2)$$

where  $p_{a,b}$  label the incoming DM and  $p_{1,2}$  the outgoing final state fermion momenta. This cross section controls both the cosmological relic abundance via freeze-out and the prospects for indirect detection today. Both processes take place for dark matter with typically non-relativistic velocity  $v$ , and can be approximated by the leading ( $s$ -wave) term in the expansion in  $v^2$ . The cross section is related to the matrix element via,

$$\langle\sigma v\rangle = \frac{v}{64\pi s |\vec{p}_a|^2} \int_{t_0}^{t_1} dt \times \overline{|\mathcal{M}|^2}, \quad (2)$$

where  $\overline{|\mathcal{M}|^2}$  is the matrix element, summed/averaged over final/initial polarization states, and  $s \equiv (p_a + p_b)^2$  and  $t \equiv (p_a - p_1)^2$  are the usual Mandelstam variables.

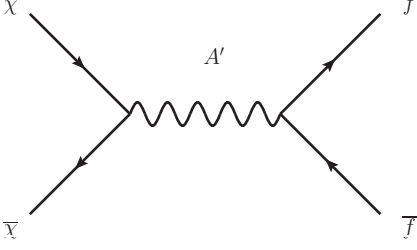


Figure 1. Tree-Level Feynman diagram for DM annihilation into SM fermions

In the non-relativistic ( $v \rightarrow 0$ ) limit  $s \simeq 4m_\chi^2 + m_\chi^2 v^2$ ,  $|\vec{p}_a| \simeq m_\chi v/2$ , and  $t_{0,1} \simeq m_f^2 - m_\chi^2 \mp m_\chi^2 \sqrt{1 - m_f^2/m_\chi^2}$ .

For dark matter masses below  $\lesssim$  GeV, the annihilation rate into hadrons may be inferred by making use of the fact that  $A'$  couples to the same electromagnetic current as the photon. Thus, the effective cross section for annihilation into hadrons at a given center-of-mass energy can be related to the ratio  $R$  measured in  $e^+e^-$  reactions [40]:

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}, \quad (3)$$

at a given center-of-mass energy  $\sqrt{s}$ . The cross section for  $\chi\bar{\chi}$  to annihilate into hadrons is thus:

$$\sigma(\chi\bar{\chi} \rightarrow \text{hadrons}) = \sigma(\chi\bar{\chi} \rightarrow \mu^+\mu^-) \times R(4m_\chi^2).$$

### A. Tree level annihilation

At tree level, the annihilation into SM fermions proceeds via  $s$ -channel exchange of the  $A'$  (see Figure 1). The leading order matrix element  $|\mathcal{M}|_{\text{LO}}^2$  is given by,

$$16\pi^2 \alpha \alpha_D \varepsilon^2 Q_f^2 \times \frac{(D-2)s^2 + 4(m_f^2 + m_\chi^2)^2 + (4s - 8m_f^2 - 8m_\chi^2)t + 4t^2}{(s - m_{A'}^2)^2 + \Gamma_{A'}^2 m_{A'}^2}, \quad (4)$$

where  $D \equiv 4 - \epsilon$  is the dimension of space-time,  $\alpha_D = g_D^2/4\pi$ ,  $\alpha = e^2/4\pi$  and  $\Gamma_{A'}$  is the width of the dark photon, given by

$$\begin{aligned} \Gamma_{A'} = & \Gamma(A' \rightarrow \chi\bar{\chi}) \Theta(m_{A'} - 2m_\chi) \\ & + \Gamma(A' \rightarrow e^+e^-) \Theta(m_{A'} - 2m_e) \\ & + \Gamma(A' \rightarrow \mu^+\mu^-) \Theta(m_{A'} - 2m_\mu) \\ & + \Gamma(A' \rightarrow \mu^+\mu^-) R(m_{A'}^2) \Theta(m_{A'} - 2m_\pi). \end{aligned} \quad (5)$$

Here,

$$\Gamma(A' \rightarrow \chi\bar{\chi}) = \frac{1}{3} \alpha_D m_{A'} \left(1 + \frac{2m_\chi^2}{m_{A'}^2}\right) \sqrt{1 - \frac{4m_\chi^2}{m_{A'}^2}}, \quad (6)$$

$$\Gamma(A' \rightarrow f\bar{f}) = \frac{1}{3} \varepsilon^2 \alpha m_{A'} \left(1 + \frac{2m_f^2}{m_{A'}^2}\right) \sqrt{1 - \frac{4m_f^2}{m_{A'}^2}}, \quad (7)$$

are the partial decay widths into DM and SM fermions, respectively. For much of the parameter space of interest,  $m_{A'} \geq 2m_\chi$  and  $\varepsilon \ll 1$ , leading to  $\Gamma_{A'} \simeq \Gamma(A' \rightarrow \chi\bar{\chi})$ .

In the non-relativistic limit, the thermally-averaged tree-level cross section is

$$\langle \sigma v \rangle_{\text{LO}} = \frac{8\pi \alpha \alpha_D Q_f^2 \varepsilon^2 (2m_\chi^2 + m_f^2)}{(4m_\chi^2 - m_{A'}^2)^2 + \Gamma_{A'}^2 m_{A'}^2} \sqrt{1 - \frac{m_f^2}{m_\chi^2}}, \quad (8)$$

which agrees with Refs. [10, 41] in the  $m_f \rightarrow 0$  limit.

In the literature, it is common to factor out the combination  $Y \equiv \varepsilon^2 \alpha_D (m_\chi/m_{A'})^4$  which controls tree-level annihilation in the  $m_{A'} \gg m_\chi$  limit. This leads to imprecision. First, the relevant parameter space is typically  $m_{A'} \simeq m_\chi$ , and errors of order  $m_\chi^2/m_{A'}^2$  are typically substantial. Second, the higher order corrections to the annihilation rate considered below are  $\mathcal{O}(\alpha_D^2)$ , and thus do not factorize in the same way. And finally, various constraints and experimental prospects do not themselves factorize in the same way, which can make comparison using it as a parameter rather misleading. For this reason, we focus on the direct model parameters  $\{\varepsilon, \alpha_D, m_\chi, m_{A'}\}$  in our analyses.

### B. NLO Corrections

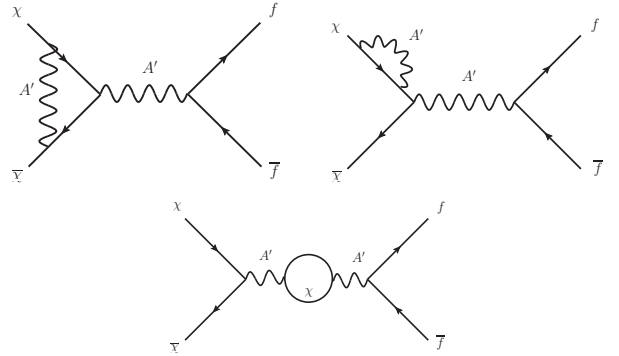


Figure 2. Representative Feynman diagrams for the one-loop corrections to  $\chi\bar{\chi} \rightarrow f\bar{f}$ .

At  $\mathcal{O}(\alpha_D^2)$ , the annihilation cross section receives virtual corrections in the form of self-energy corrections to the dark photon propagator and incoming dark matter wave functions and a correction to the  $A'-\chi-\bar{\chi}$  vertex (see Figure 2). In the regime of  $m_{A'} \geq 2m_\chi$ , an additional emitted  $A'$  would always be off-shell, leading the

real emission corrections to be effectively higher order in  $\varepsilon$  such that they can be safely neglected. The  $\mathcal{O}(\alpha_D^2)$  correction to the annihilation process is given by the interference between the leading order and next-to-leading order matrix elements in the non-relativistic limit, summed and averaged over the final and initial polarizations,

$$\overline{\delta\mathcal{M}^2} = 2 \operatorname{Re} \left\{ \overline{\mathcal{M}_{\text{LO}}^* \times \mathcal{M}_{\text{NLO}}} \right\}. \quad (9)$$

We compute the one loop diagrams in the non-relativistic limit with the aid of `FeynCalc` [42], interfere them with the LO matrix elements, and reduce the resulting expressions via the Passarino-Veltman procedure [43] to scalar

$$\begin{aligned} \langle\delta\sigma v\rangle_{\text{NLO}} = & -\frac{\alpha_D \langle\sigma v\rangle_{\text{LO}}}{3\pi} - \frac{\alpha_D \langle\sigma v\rangle_{\text{LO}}}{6\pi m_\chi^2 (4m_\chi^2 - m_{A'}^2)} \left\{ -2(m_{A'}^2 - 4m_\chi^2) \right. \\ & \left( m_\chi^2 (m_{A'}^2 + 2m_\chi^2) (2B'_0(m_{A'}^2; m_\chi^2, m_\chi^2) + 3B'_0(m_\chi^2; m_{A'}^2, m_\chi^2)) + A_0(m_{A'}^2) - A_0(m_\chi^2) \right) \\ & + 24m_\chi^4 B_0(m_{A'}^2; m_\chi^2, m_\chi^2) + (m_{A'}^4 - 14m_{A'}^2 m_\chi^2 + 40m_\chi^4) B_0(m_\chi^2; m_{A'}^2, m_\chi^2) \\ & + (m_{A'}^4 + 4m_{A'}^2 m_\chi^2 - 32m_\chi^4) B_0(4m_\chi^2; m_\chi^2, m_\chi^2) - 24m_\chi^4 B_0(4m_\chi^2; m_\chi^2, m_\chi^2) \\ & \left. + (m_{A'}^6 + 4m_{A'}^4 m_\chi^2 - 20m_{A'}^2 m_\chi^4 - 48m_\chi^6) C_0(m_\chi^2, m_\chi^2, 4m_\chi^2; m_\chi^2, m_{A'}^2, m_\chi^2) \right\} \end{aligned} \quad (10)$$

where  $A_0$ ,  $B_0$ , and  $C_0$  are scalar integral functions, and  $B'_0$  is the derivative of  $B_0$  with respect to its first argument. We find that the contribution of the dark scalar to the dark photon self energy is quantitatively quite small and thus does not change the results by much, whereas the analytic expression is rather complicated. Hence, for brevity we omit it from the expression above. The full annihilation cross section to  $\mathcal{O}(\alpha_D^2)$  is:

$$\langle\sigma v\rangle = \langle\sigma v\rangle_{\text{LO}} + \langle\delta\sigma v\rangle_{\text{NLO}} + \mathcal{O}(\alpha_D^3). \quad (11)$$

The impact of the one-loop correction can be summarized by the quantity  $\langle\delta\sigma v\rangle_{\text{NLO}}/(\alpha_D \langle\sigma v\rangle_{\text{LO}})$ , which characterizes the relative change compared to the leading order cross section with the  $\alpha_D$  dependence scaled out. In Figure 3, we plot this ratio as a function of the ratio of  $m_{A'}/m_\chi$ , for  $m_\chi = 1$  MeV for different values of  $m_H/m_{A'}$ . For  $m_H \geq m_{A'}$ , the inclusion of the dark Higgs negligibly impacts the annihilation cross-section, whereas in the light dark Higgs regime,  $m_H \ll m_{A'}$ , the contribution is positive, though subdominant compared to the contributions from virtual dark photons. These results indicate that for  $m_{A'}/m_\chi \lesssim 4$ , the corrections are positive, and can be as large as tens of percent near resonance where  $m_{A'} \sim 2 m_\chi$ . For larger mass ratios, the impact of the higher order corrections is to decrease the net annihilation rate by an amount that flattens out for very large mass ratios. We find that, provided  $m_{A'}$

integrals which we evaluate numerically using `LoopTools` [44]. We cross-checked our results by hand as well as by using `Package-X` [45]. We find that the ultraviolet divergences cancel between the DM wave function corrections (see appendix A) and the vertex correction, as is expected based on the analogue of the Ward identity for  $U(1)'$ . UV divergences in the one-loop correction to the dark photon propagator cancel against the  $\delta Z_{A'}$  and  $\delta m_{A'}$  counter-terms, renormalized in the on-shell scheme (see appendix B for details), resulting in a final expression that is finite.

The resulting correction to the annihilation cross section is expressed as

and  $m_H$  are specified as a ratio to  $m_\chi$ , the quantity  $\langle\delta\sigma v\rangle_{\text{NLO}}/(\alpha_D \langle\sigma v\rangle_{\text{LO}})$  is very insensitive to the mass of the dark matter itself over the entire range of interest.

### C. Relic Density

To compute the relic density we follow the description of refs. [46, 47] where it was pointed out that care is required in the treatment of the number of relativistic degrees of freedom as a function of temperature for sub-GeV dark matter annihilations. Hence, we solve the Boltzmann equation for the comoving number density of dark matter,

$$\frac{dY}{dx} = \frac{s\langle\sigma v\rangle}{Hx} \left[ 1 + \frac{1}{3} \frac{d(\ln g_s)}{d(\ln T)} \right] (Y_{eq}^2 - Y^2). \quad (12)$$

We refer the reader to ref. [46] for discussion of the relevant quantities. Ref. [46] presented results for the target cross section which reproduces the observed DM density only down to  $m_\chi \sim 100$  MeV. To cover the relevant parameter space, we implement their formalism, reproducing their results in the regime that they covered, and extending them down to  $\sim 1$  MeV DM masses.

In Fig. 4 we show plots of kinetic mixing as a function of DM mass for a few representative values of the

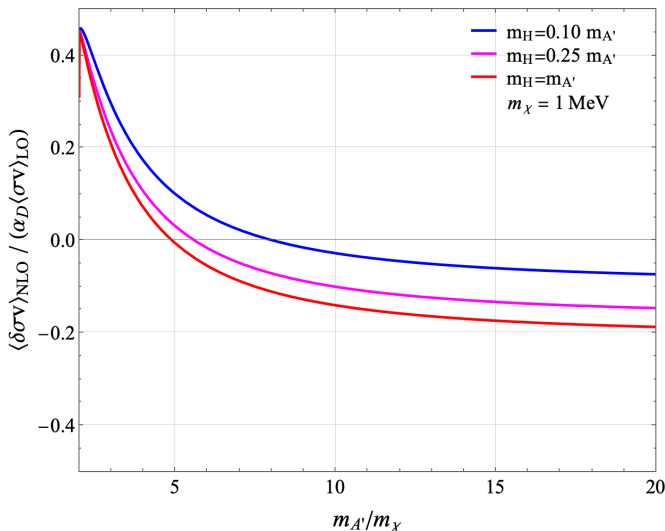


Figure 3. The ratio  $\langle \delta\sigma v \rangle_{\text{NLO}} / (\alpha_D \langle \sigma v \rangle_{\text{LO}})$  as a function of  $m_{A'}/m_\chi$ , for  $m_\chi = 1$  MeV and for different ratios of  $m_H/m_{A'} = 0.1, 0.25$  and  $1$ , represented by the blue, magenta and red lines, respectively.

ratio of  $m_{A'}/m_\chi$ , obtained by scanning through the kinetic mixing for each DM mass, finding the combination resulting in the observed relic abundance  $\Omega h^2 = 0.12$  [48]. Each plot shows the required kinetic mixing parameter for three different values of the dark sector coupling  $\alpha_D$ , corresponding to the LO only (solid lines) and NLO + LO (dashed lines) computations respectively. For  $m_{A'}/m_\chi = 3$ , which is typically chosen as a benchmark parameter point in the literature, we also indicate the current experimental constraints (a combination of both accelerator and astrophysical probes [24]) as the gray shaded region. In that panel, green, purple and orange dotted lines are projections from the upcoming LDMX,  $M_3$  and Belle II experiments, respectively.

#### IV. DARK MATTER DIRECT DETECTION

In this section we investigate NLO corrections to light pseudo-Dirac DM scattering with the SM in direct detection experiments. Due to crossing symmetry, the corrections we consider here are described by similar diagrams as in Fig. 2. Since the dark photon kinetically mixes with the SM photon, DM can scatter universally with both leptons and nucleons. Given the tiny momentum transfers and our parameter space of interest for which  $m_{A'} \gtrsim m_\chi$ , the interaction can be approximated as an

effective four-point fermion interaction, integrating out the mediator.

Following ref. [49], we factorize the matrix element as:

$$|\overline{\mathcal{M}}_{EFT}|^2 = |c_{EFT}|^2 \frac{1}{4} \sum_{spin} (J_\chi^\mu J_\mu^f)^\dagger (J_\chi^\mu J_\mu^f), \quad (13)$$

where  $J_\chi^\mu = \bar{u}_\chi \gamma^\mu u_\chi$  and  $J_\mu^f = \bar{u}_f \gamma_\mu u_f$  represent the dark sector and SM currents respectively. Here  $f$  can represent either electrons or nucleons. The Wilson coefficient  $c_{EFT}$  includes contributions from the tree level dark photon exchange, plus NLO corrections from the vertex correction, DM self-energy and the dark photon vacuum polarization, analogous to those represented in Fig. 2. Given our focus on sub-GeV DM and the prospects for its detection at upcoming experiments, we restrict our attention to DM scattering with electrons. However our general computations can be extended to nucleon scattering with the incorporation of the appropriate form factors for either spin-dependent or spin-independent scattering.

The differential DM-electron scattering cross-section as a function of the momentum transfer is usually written

$$\frac{d\sigma}{dq^2} = \frac{\bar{\sigma}_e}{4\mu v^2} |F_{DM}(q)|^2. \quad (14)$$

Here  $\bar{\sigma}_e$  is defined as the free scattering cross-section at a reference value  $q = \alpha m_e$  [29, 50], which typifies the momentum of an electron bound in an atom in the detector. For the DM mass range, and ratios with the mediator mass ( $m_{A'}/m_\chi$ ) we consider in this work,  $q \sim \mu v \ll m_{A'}$ , and hence  $F_{DM}(q) \rightarrow F_{DM}(q=0) \sim 1$ .

Similarly to the annihilation case, the cross-section for scattering with electrons up to  $\mathcal{O}(\alpha_D^2)$ , can be parameterized as:

$$\bar{\sigma}_e = \bar{\sigma}_e^{\text{LO}} + \bar{\sigma}_e^{\text{NLO}} + \mathcal{O}(\alpha_D^3). \quad (15)$$

At zero momentum transfer, the LO scattering cross-section is given by [29, 50]

$$\bar{\sigma}_e^{\text{LO}} = \frac{16\pi\alpha\alpha_D\varepsilon^2\mu^2}{m_{A'}^4}, \quad (16)$$

where  $\mu \equiv m_\chi m_e / (m_\chi + m_e)$  is the reduced mass between the DM and the electron. The  $\mathcal{O}(\alpha_D^2)$  correction can be written in terms of scalar integrals as:

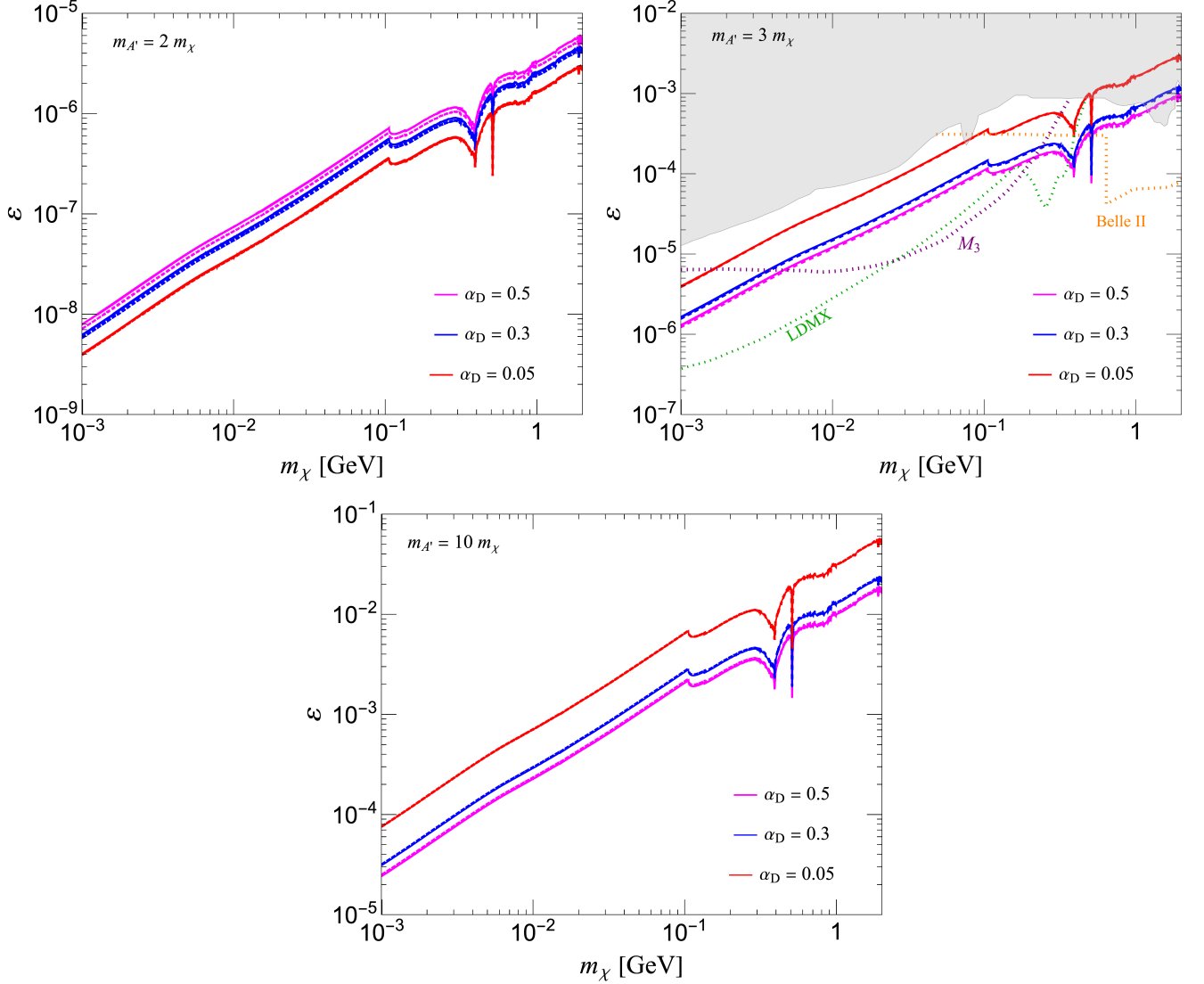


Figure 4. Value of  $\varepsilon$  producing the observed relic abundance of dark matter as a function of  $m_\chi$ , for  $\alpha_D = 0.05, 0.3$  and  $0.5$  (red, blue and magenta lines respectively) based on LO only (solid lines) and NLO + LO (dashed lines) cross sections and for the indicated value of  $m_{A'}$  on each panel. The dark Higgs mass is chosen to be  $\geq m_{A'}$ , such that its contribution is negligible. For the  $m_{A'} = 3m_\chi$  panel, the gray shaded regions are the exclusions from past accelerator experiments, and the green, purple and orange dotted lines are projections from the upcoming LDMX,  $M_3$  and Belle II experiments respectively [24].

$$\begin{aligned}
 \bar{\sigma}_e^{\text{NLO}} = \frac{\alpha_D \bar{\sigma}_e^{\text{LO}}}{6 \pi m_{A'}^2 m_\chi^2} & \left\{ m_{A'}^2 (m_{A'}^2 + 2m_\chi^2) \left( 3B_0(m_\chi^2; m_{A'}^2, m_\chi^2) - 3(m_{A'}^2 - 2m_\chi^2) C_0(m_\chi^2, m_\chi^2, 0; m_\chi^2, m_{A'}^2, m_\chi^2) \right. \right. \\
 & \left. \left. + 2m_\chi^2 (2B'_0(m_{A'}^2; m_\chi^2, m_\chi^2) + 3B'_0(m_\chi^2; m_{A'}^2, m_\chi^2)) \right) - 8m_\chi^4 B_0(m_{A'}^2; m_\chi^2, m_\chi^2) \right. \\
 & \left. + (-3m_{A'}^4 - 6m_{A'}^2 m_\chi^2 + 8m_\chi^4) B_0(0; m_\chi^2, m_\chi^2) \right\} \quad (17)
 \end{aligned}$$

In Fig. 5, we show the direct detection constraints on the model. The left panel characterizes the importance of the one-loop correction, parameterized by the quantity

$\bar{\sigma}_{e\text{NLO}}/(\alpha_D \bar{\sigma}_{e\text{LO}})$  in terms of the ratio  $m_{A'}/m_\chi$  for 1 GeV and 1 MeV DM masses.

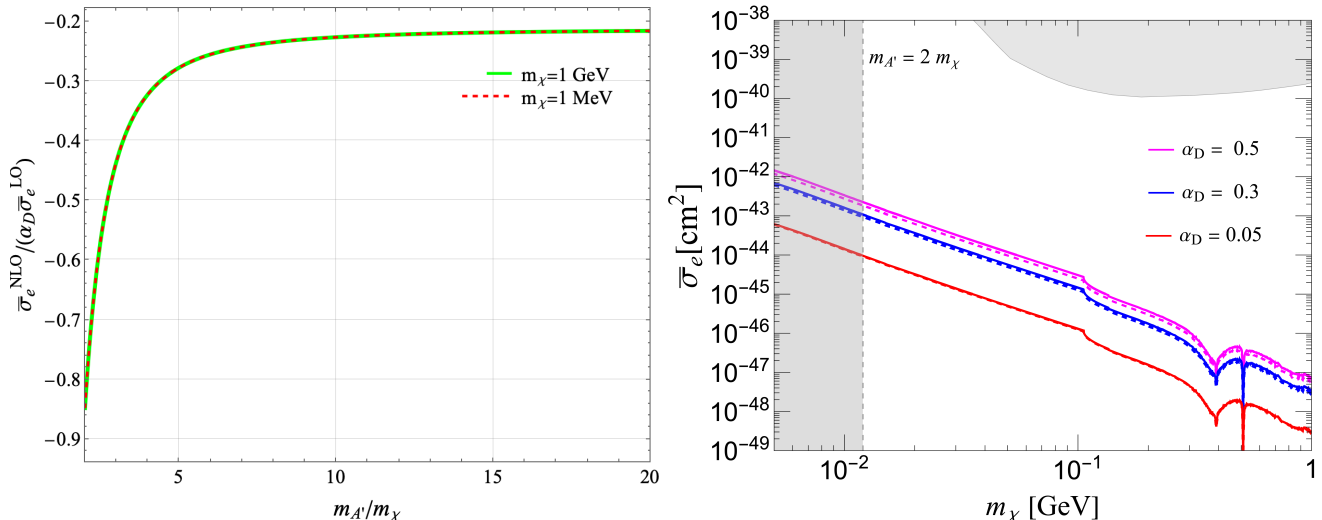


Figure 5. Dark matter direct detection parameter space. The left panel shows the ratio of NLO to LO electron scattering cross-section vs  $m_{A'}/m_{\chi}$  ratio for  $m_{\chi} = 1$  MeV (red, dashed line) and  $m_{\chi} = 1$  GeV (green solid line). The right panel shows dark matter-electron scattering cross-section vs  $m_{\chi}$  for dark matter thermal relic abundance with  $\alpha_D = 0.05, 0.3$  and  $0.5$ , represented by the red, blue and magenta lines respectively. We show both the LO only (solid lines) and NLO + LO (dashed lines) for the  $m_{A'}/m_{\chi} = 2$  case. The upper gray shaded region represents the current model independent direct detection constraints from a combination of the XENON1T, PANDAX and SENSEI experiments and the vertical gray shaded region is the constraint from  $\Delta N_{eff}$ . All constraints were obtained from ref. [31].

In the right panel of the figure, we show the averaged DM-electron scattering cross-section vs DM mass. The red, blue and magenta lines represent DM produced through the thermal freeze-out mechanism for benchmark choices of  $\alpha_D = 0.05, 0.3$  and  $0.5$  respectively. The dashed lines are NLO + LO contributions, while the solid lines are only at LO. For illustration, we only show the case for  $m_{A'}/m_{\chi} \simeq 2$ . We see a clear dependence on the dark sector coupling  $\alpha_D$  and differences between the NLO + LO and LO only contributions.

Also shown as the grey shaded region are the model independent constraints on this parameter space. Below  $\sim 10$  MeV, there is a stringent bound from big bang nucleosynthesis and  $\Delta N_{eff}$ , whereas the upper grey shaded region indicates bounds from a combination of direct detection experiments, including XENON1T, PANDAX and SENSEI (obtained from ref. [31]).

## V. DISCUSSION AND CONCLUSIONS

In this work, we revisit the computations for sub-GeV thermal dark matter processes. We consider a highly motivated and sought after scenario, in which (pseudo)-Dirac dark matter couples to the SM via a massive dark photon, representing a compelling benchmark model of dark matter. We focus on the corrections at order  $\alpha_D$ , which is typically invoked to be large in order to obtain the observed relic density of the dark matter.

We compute the NLO corrections for thermal (pseudo)-Dirac dark matter annihilating in the early

universe, as this represents a very important target for upcoming experiments. We find that the NLO corrections can be as large as 10's of percents, depending on the  $m_{A'}/m_{\chi}$  ratio and the strength of the dark sector coupling constant  $\alpha_D$ . While this is not a dramatic difference compared to the targets inferred from tree level calculations, it is an important result to establish the ability of proposed experiments to discover or constrain MeV scale dark matter, and to precisely quantify what we will learn from such experiments in the future. Furthermore, our results show that factorization into the ‘‘yield parameter’’  $Y$ , as is common practise in the literature, can be misleading, especially in the resonance region, where the NLO corrections are more significant.

For completeness, we also show the direct detection limits based on DM-electron scattering. We find that the NLO corrections are slightly more visible in the resonance region and become less significant for larger mass ratios (similar to the annihilation case). We leave a more dedicated study of pseudo-Dirac dark matter with larger mass splittings as well as other Lorentz structures of interest to future studies.

## ACKNOWLEDGEMENTS

We thank Basudeb Dasgupta for useful discussions. GM acknowledges support from the UC office of the President through the UCI Chancellor’s Advanced Postdoctoral Fellowship. GM is also supported in part by the National Sciences and Engineering Research Coun-



cil of Canada. TMPT and GM are supported in part by the U.S. National Science Foundation under Grant PHY-2210283. This work was performed in part at Aspen Center for Physics, which is supported by National Science Foundation grant PHY-2210452.

### Appendix A: Dark Matter Self-Energy

In terms of the Passarino-Veltmann scalar functions, the dark matter self-energy takes the form

$$\begin{aligned} \Sigma(p) = & \frac{\alpha_D}{8\pi} \left[ \frac{(\epsilon - 2)\not{p}}{p^2} \left( A_0(m_{A'}^2) - A_0(m_\chi^2) \right) \right. \\ & + (-m_{A'}^2 + m_\chi^2 + p^2) B_0(p^2; m_{A'}^2, m_\chi^2) \\ & \left. - 2m_\chi(\epsilon - 4)B_0(p^2; m_{A'}^2, m_\chi^2) \right], \end{aligned} \quad (\text{A1})$$

where  $\not{p} \equiv \gamma^\mu p_\mu$  and  $\epsilon = 4 - D$ . The dark matter wave-function counter-term in the on-shell renormalization scheme,  $\delta Z_2 = d\Sigma/d\not{p}, (\not{p} \rightarrow m_\chi)$ , is given by:

$$\begin{aligned} \delta Z_2 = & -\frac{\alpha_D}{8\pi m_\chi^2} \left[ (\epsilon - 2) \left( m_{A'}^2 (-B_0(m_\chi^2; m_{A'}^2, m_\chi^2)) \right. \right. \\ & \left. \left. + A_0(m_{A'}^2) - A_0(m_\chi^2) \right) \right. \\ & \left. + 2m_\chi^2 (m_{A'}^2(\epsilon - 2) - 4m_\chi^2) B_0'(m_\chi^2; m_{A'}^2, m_\chi^2) \right]. \end{aligned} \quad (\text{A2})$$

The dark matter mass counter-term in the on-shell scheme,  $\delta m_\chi = \Sigma(p), (\not{p} \rightarrow m_\chi)$  can be similarly extracted, but does not enter into the computation of annihilation or scattering with electrons at NLO.

### Appendix B: Dark Photon Self-Energy

The tree level dark photon propagator in the unitary gauge takes the form:

$$i D_{\mu\nu}^{(0)}(k^2) = \frac{i(-g_{\mu\nu} + k_\mu k_\nu / m_{A'}^2)}{k^2 - m_{A'}^2} \quad (\text{B1})$$

where  $k$  is the momentum of the  $A'$ . The self energy correction can be expressed as

$$\Pi_{\mu\nu}(k^2) \equiv g_{\mu\nu} \Pi_1(k^2) + k_\mu k_\nu \Pi_2(k^2) \quad (\text{B2})$$

where the scalar function  $\Pi_1(k^2)$  characterizes the transverse component and contributes to the  $S$ -matrix, whereas the longitudinal component given by  $\Pi_2(k^2)$  does not contribute when coupled to a conserved current. The mass counter-term for the massive dark photon is

$$\delta m_{A'}^2 = \text{Re}[\Pi_1(m_{A'}^2)] \quad (\text{B3})$$

and the wave function renormalization counter-term is

$$\delta Z_{A'} = Z_{A'} - 1 \simeq -\text{Re} \left[ \frac{d\Pi_1(m_{A'}^2)}{dk^2} \right] \quad (\text{B4})$$

where  $Z_{A'}$  is the wave function renormalization for a massive dark photon. The renormalized correction to the scalar part of the dark photon propagator is thus [51]

$$\Pi_{\text{ren}}(k^2) = \Pi_1(k^2) - \delta m_{A'}^2 + (k^2 - m_{A'}^2) \delta Z_{A'} \quad (\text{B5})$$

We discuss the one-loop contributions from the dark matter and the dark Higgs to the dark photon self-energy separately, below. At NLO, these two classes of contributions are simply summed together.

#### 1. Contribution from Dark Matter

Using dimensional regularization, the one-loop correction to  $\Pi_1(k^2)$  with the DM fermion in the loop takes the form:

$$\begin{aligned} \Pi_1(k^2) = & \frac{\alpha_D}{2\pi(\epsilon - 3)} \left( (4m_\chi^2 - (\epsilon - 2)k^2) B_0(k^2; m_\chi^2, m_\chi^2) \right. \\ & \left. + 2(\epsilon - 2)A_0(m_\chi^2) \right) \end{aligned} \quad (\text{B6})$$

where  $\epsilon = 4 - D$ .

The dark matter contribution to the mass counter-term is

$$\begin{aligned} \delta m_{A'}^2 = & \frac{\alpha_D}{2\pi(\epsilon - 3)} \text{Re} \left( 2(\epsilon - 2)A_0(m_\chi^2) \right. \\ & \left. + (4m_\chi^2 - m_{A'}^2(\epsilon - 2)) B_0(m_{A'}^2; m_\chi^2, m_\chi^2) \right) \end{aligned} \quad (\text{B7})$$

and the DM contribution to the wave function renormalization counter-term  $\delta Z_{A'}$  is

$$\begin{aligned} \delta Z_{A'} = & -\frac{\alpha_D}{2\pi(\epsilon - 3)} \text{Re} \left( (\epsilon - 2)B_0(m_{A'}^2; m_\chi^2, m_\chi^2) \right. \\ & \left. + (m_{A'}^2(\epsilon - 2) - 4m_\chi^2) B_0'(m_{A'}^2; m_\chi^2, m_\chi^2) \right) \end{aligned} \quad (\text{B8})$$

#### 2. Dark Higgs Contribution

The one-loop correction  $\Pi_1(k^2)$  to the dark photon self-energy due to dark Higgs is

$$\begin{aligned} \Pi_1(k^2) = & \frac{\alpha_D}{16\pi(\epsilon - 3)k^2} \left[ \left( -2k^2 (m_{A'}^2(2\epsilon - 5) + m_H^2) \right. \right. \\ & \left. \left. + k^4 + (m_{A'}^2 - m_H^2)^2 \right) B_0(k^2; m_{A'}^2, m_H^2) \right. \\ & \left. + A_0(m_H^2) ((23 - 8\epsilon)k^2 + m_{A'}^2 - m_H^2) \right. \\ & \left. - (A_0(m_{A'}^2) (k^2 + m_{A'}^2 - m_H^2)) \right] \end{aligned} \quad (\text{B9})$$

where  $\epsilon = 4 - D$ . The contribution to the mass counter-term is

$$\begin{aligned} \delta m_{A'}^2 = & \frac{\alpha_D}{16\pi m_{A'}^4(\epsilon - 3)} \text{Re} \left[ \left( -4m_{A'}^4(\epsilon - 3) \right. \right. \\ & \left. \left. - 4m_{A'}^2 m_H^2 + m_H^4 \right) B_0(m_{A'}^2; m_{A'}^2, m_H^2) \right. \\ & \left. - A_0(m_H^2) (8m_{A'}^2(\epsilon - 3) + m_H^2) \right. \\ & \left. + (m_H^2 - 2m_{A'}^2) A_0(m_{A'}^2) \right] \end{aligned} \quad (\text{B10})$$

and the contribution to the wave function renormal-

ization counter-term is

$$\begin{aligned} \delta Z_{A'} = & \frac{\alpha_D}{16\pi m_{A'}^4(\epsilon - 3)} \text{Re} \left[ m_H^2 (m_H^2 - 2m_{A'}^2) \right. \\ & B_0(m_{A'}^2; m_{A'}^2, m_H^2) \\ & + m_{A'}^2 \left( 4m_{A'}^4(\epsilon - 3) + (m_H^2 - m_{A'}^2) A_0(m_{A'}^2) \right. \\ & \left. + 4m_{A'}^2 m_H^2 - m_H^4 \right) B_0'(m_{A'}^2; m_{A'}^2, m_H^2) \\ & \left. + (m_{A'} - m_H)(m_{A'} + m_H) A_0(m_H^2) \right]. \end{aligned} \quad (\text{B11})$$

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