Title
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Fracture stiffness and aperture as a function of applied stress and contact geometry

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ABSTRACT: An analytical model that assumes variable height asperities and full interaction between them has been developed and implemented numerically to study the behavior of a single fracture under stress. The model has been used to calculate specific stiffness and aperture profiles for specified fracture geometries. The formulation takes account of the deformation of the half-planes defining the fracture which is shown to lead to changes in aperture geometry that are not predicted by other asperity models. A parameter sensitivity study shows that specific stiffness varies significantly depending on the size of the asperities, their height distribution, and their spatial orientation. Curves of stress vs. specific stiffness generated with the model are found to agree in shape and magnitude with those obtained in the laboratory for rock specimens.

1 INTRODUCTION

Understanding the properties of fractures under stress is important in characterizing most geological sites and predicting the behavior of underground engineering structures. The purpose of the present study is to identify the parameters that play an important role in determining the mechanical response of a single fracture to applied loads. An important application of the work is the description of fracture closure as a function of stress. The ability to describe changes in aperture geometry with changes in stress is crucial to predicting fluid flow through fractured rock (see, e.g., Tsang, 1984 and Brown, 1987).

Specific stiffness is a property that defines the relationship between applied stress and fracture deformation. More formally, specific stiffness is defined as the average applied stress divided by the average displacement across the fracture interface in excess of the displacement that would occur if the fracture were not present. Laboratory experiments on single fractures in rock indicate that specific stiffness is initially a sharply rising function with stress that levels off and approaches a constant value (e.g., Goodman, 1976; Bandis et al., 1983; and Pyrak-Nolte et al., 1987).

A common approach to modeling the mechanical deformation of fractures has been to represent the fracture surfaces as parallel planes separated by asperities of varying height. Greenwood and Williamson (1966) modeled the contact between a plane and a nominally flat surface covered by a large number of asperities with heights described by a specified statistical distribution. The asperity tips were taken to be spherical and their deformation calculated from the Hertzian solution for an elastic sphere in contact with a plane. Their model was extended by Greenwood and Tripp (1971) to the case of two rough surfaces in contact. Gangi (1978) used what he termed a *bed of nails* model to describe the permeability of a fractured porous rock as a function of confining pressure. The asperities were modeled as rods with equal spring constants and heights following a power law distribution. Brown and Scholz (1985, 1986), like Greenwood and Williamson, assumed the asperity tips to be spherical and modeled their deformation using the Hertzian solution. However, they also included a term for tangential stresses arising from the oblique contact of spheres so that the stresses at the contacts are not restricted to be normal. Implicit in all the asperity models discussed above is the assumption that the contacts are sufficiently far apart so that they do not interact mechanically. Further, the closure of the fracture is based solely on the deformation of the asperities between the opposing fracture surfaces.

2 THE MODEL

The model that we have developed is also an asperity model but differs from those described above in two important ways. First, the constraint that the deformation at a contact point is independent of the other asperities is removed. Secondly, the deformation of the half-spaces defining the fracture is accounted for in addition to the deformation of the asperities. This formulation can lead to significant changes in void geometry with increasing stress that are not observed when only the deformation of the asperities is considered. An accurate description of the void geometry is important in predicting fluid flow through the fractures because of the highly nonlinear relationship between fluid flow and aperture (Iwai, 1976; Witherspoon, et al., 1980; and Pyrak-Nolte et al., 1987).

For the results presented here, the asperities are modeled as circular disks of varying height. The asperities are not assumed to be mechanically independent. Rather, the force carried by each asperity depends on its height and the heights and proximity of neighboring asperities. The deformation of the asperities is calculated from the elastic compression of the disks. The deformation of the half-spaces defining the fracture is calculated using the Boussinesq solution for displacement beneath a loaded circle assuming a constant stress boundary condition. The deformation at any point on the half-planes is assumed to be a linear combination of the displacements caused by the forces acting on all asperities in the region. The calculation of specific stiffness is based on the average deformation of the fracture surfaces and asperities.

3 CHANGES IN FRACTURE APERTURE WITH STRESS

Figure 1a shows a two-dimensional slice through an idealized fracture with the dotted lines representing reference planes. If only the deformation of the asperities is considered, and a normal stress is applied, the reference lines remain parallel and come together by an amount that depends on the deformation of the contacts. For the case of a simple interpenetration model, the surfaces come together in a manner equivalent to allowing the top and bottom fracture surfaces to overlap as illustrated in Figures 1b-1d. If instead, the asperity tips are modeled as spheres and the Hertzian solution is used to calculate the deformation of the tips, the reference planes still remain parallel and come together an amount equal to the deformation of the asperities. For both cases, the dimensions of the asperities themselves are important, but their spatial location on the fracture surface is not.

For comparison, the model described in Section 2 was used to calculate the displacement across the same idealized fracture pictured in Figure 1a. To apply the model, the fracture surface is first discretized as shown in Figure 2a. When a normal force is applied, the reference lines no longer remain parallel because the asperities indent the half-spaces defining the fracture. This is illustrated in Figures 2c and 2d where the dashed lines show the calculated deformation of the reference planes.

Adding in the deformation of the half-plane results in much greater changes in fracture aperture than would occur if just the deformation of the asperities were considered. To see this, compare Figure 2d to Figure 1c. In 1c, the reference lines have come together 22.6% compared to their original position under zero load. In 2d, the average displacement of the two reference planes is 25.7%. Even though the displacement of the reference planes is roughly equal, Figure 2d shows a greater reduction in aperture and appreciable changes in void geometry.

This change in geometry has important implications for fluid flow through the fracture. The asperities have the effect of propping the fracture open while more than average closure occurs in open areas. To illustrate, consider the idealized case of a single circular asperity between two parallel fracture surfaces as pictured in cross-section in Figure 3a. Under zero load, the fracture aperture b is everywhere equal and fluid flow is proportional to b^3 (Snow, 1965). As the load is increased, the fracture surfaces deform as illustrated in Figure 3b. The aperture at the asperity is $b-\delta$ where δ is the deformation of the asperity. The high aperture pockets formed around the asperity are everywhere surrounded by a region of smaller aperture (a). Under stress, the large voids formed around asperities will affect the storativity of the fracture whereas the permeability will be largely controlled by the smaller aperture regions adjacent to the voids.

4 SPECIFIC STIFFNESS

To begin to understand what properties of the fracture surface are important in determining specific stiffness, the model described in Section 2 was used to perform a parameter sensitivity study. In particular, the effects of spatial geometry and the dimension and distribution of heights of the asperities were studied. Finally, model predictions were compared with laboratory measurements.

For a fixed spatial arrangement, the maximum stiffness that can be obtained occurs when all asperities are of equal height. In this case, contact area, and thus specific stiffness, are constant at all stresses. Fixing the heights at 35 microns and holding contact area constant at 25%, the effect of varying the asperity diameter was explored. The results, summarized in Table 1, show that the stiffest configuration (f) is for the smallest disk diameter considered (1 mm). The implication is that the more disperse the contact area, the stiffer the interface.

Next, the effect of varying the height distribution of the asperities was investigated. It is the distribution of heights that determines the rate at which asperities come into contact with increasing stress. In developing realistic fracture models, it is important to consider the spatial correlation of the asperity locations and heights. Nonetheless, as a first step in demonstrating the model, we have considered disks of equal diameter (1 mm) distributed randomly across a planar surface. In reality, fracture surfaces are irregular rather than planar and parallel. However, the deformations calculated for the idealized parallel surfaces can be superposed on the actual irregular surfaces to obtain the true profile.

Curves of specific stiffness were generated using 100 disks distributed in an area of 3.14 cm^2 as shown in Figure 4. The maximum contact area in this case is 25% when all disks are in contact. Using this spatial distribution, specific stiffness as a function of applied stress was calculated for a variety of height distributions. The resulting curves, and histograms of the height densities, are plotted in Figure 5. The results show that for a fixed spatial geometry, a wide range of behavior can be obtained by varying the distribution of heights of the asperities. In all cases, the stiffness curve rises sharply as increasing numbers of asperities come into contact and then begins to level off as the rate at which asperities come into contact diminishes. Eventually, the curve will asymptote when there is no increase in contact area with increasing stress.

For comparison, the specific stiffnesses calculated using the model are compared to a curve obtained by L. Pyrak-Nolte et al. (1987) in the laboratory for a granite core sample with a fracture oriented perpendicular to the axis of the core. In making the model calculations, a Young's Modulus of 50.0 GPa and a Poisson's ratio of 0.2 was used to coincide with the values of the granite used in the laboratory experiment. For the laboratory data, specific stiffness was calculated by taking the inverse of the tangent slope to the stress vs. displacement curve. The curve obtained from the laboratory data is plotted in Figure 5 as the bold line. As can be seen from the figure, the curves obtained using the model, assuming a maximum contact area of 25% and asperity heights distributed between zero and 100 microns, bracket the results obtained in the laboratory for this particular specimen. The best matches are with curves 3 and 4 which have height distributions that are more tightly clustered than those used to generate curves 1 and 2.

The horizontal lines in Figure 5 show the maximum stiffness that can be achieved using the particular spatial configuration shown in Figure 4. The curves represent the limiting case that occurs when all asperities are of equal height. Curves 5, 6, and 7 correspond to asperity heights of 30, 50, and 80 microns, respectively. The difference between the lines is due to the difference in the compressibility of the asperities; the shorter asperities are less compressible and thus create a stiffer interface. The small difference between the lines indicates that the absolute asperity height is not as important as the distribution of heights.

The spatial orientation of the disks is also an important parameter. When all disk heights are made equal, the only difference between the arrangement in Figure 4 and arrangement f in Table 1 is the spatial distribution of the disks. The symmetric distribution f yields a specific stiffness of 54.3 E12 Pa whereas the more clustered arrangement of Figure 4 gives a value of approximately 9.0 E12 Pa. In other words, for this particular case, holding contact area, disk diameter, and height constant, specific stiffness decreases roughly 83% in going from a symmetric spatial distribution of contact to a more clustered distribution. As previously discussed, the predicted curves in Figure 5 were derived for a specified spatial geometry. The sensitivity of specific stiffness to the assumed spatial distribution may explain in part the discrepency between observed and predicted results.

5 CONCLUSIONS

In modeling the behavior of a single fracture with an asperity model, it has been shown that accounting for the deformation of the half-spaces defining the fracture leads to greater reductions in aperture than would be calculated if only the deformation of the asperities were considered. In addition, the spatial geometry of the asperities becomes important leading to differential deformation of the fracture surfaces that can result in significant changes in void geometry. This change in aperture geometry with stress is important in fluid flow calculations.

The spatial geometry of the asperities, their dimension and height distribution have all been shown to be important parameters in determining fracture stiffness. For a constant contact area, small, dispersed contact points form a stiffer interface than large or clustered contact areas. The distribution of heights affects both the shape and magnitude of the stress-stiffness curve. The more nearly equal the heights, the stiffer the fracture is. The distribution of heights is found to be more important than the absolute height. Finally, the proposed model has been shown to yield values for specific stiffness that are consistent with those observed in the laboratory for single fractures in natural rock.

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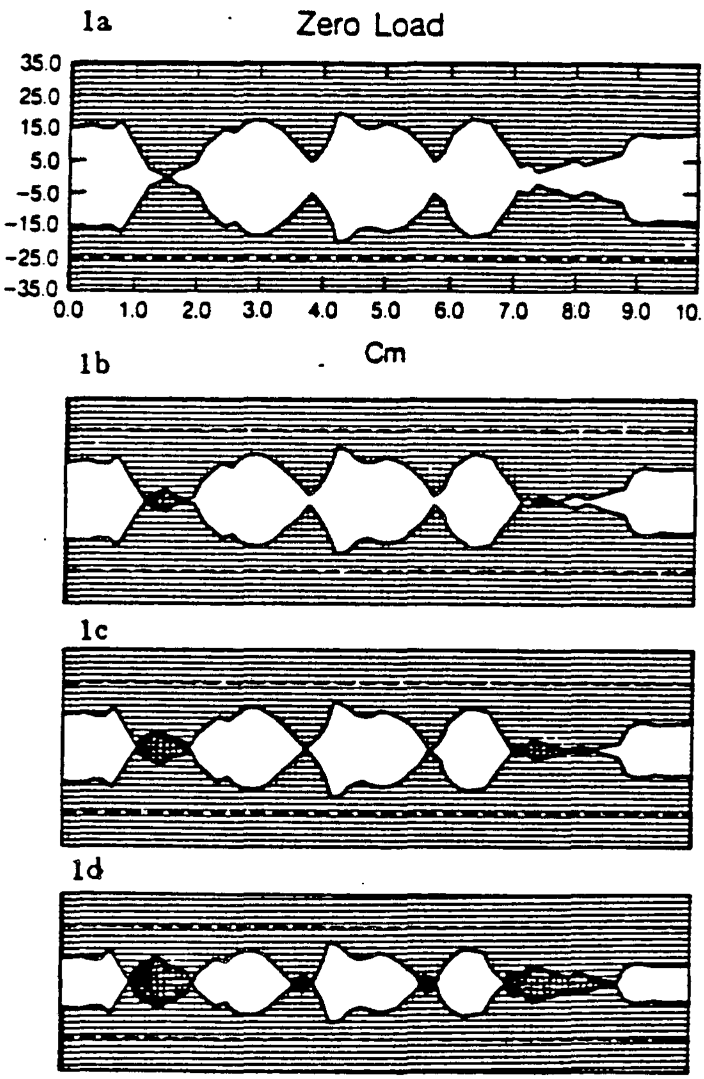


Figure 1. Schematic diagram of the change in aperture with increasing stress for a cross-section through an idealized fracture assuming a simple interpenetration model. The reference lines have come together 13%, 23%, and 32% in Figures 1b, 1c, and 1d, respectively.

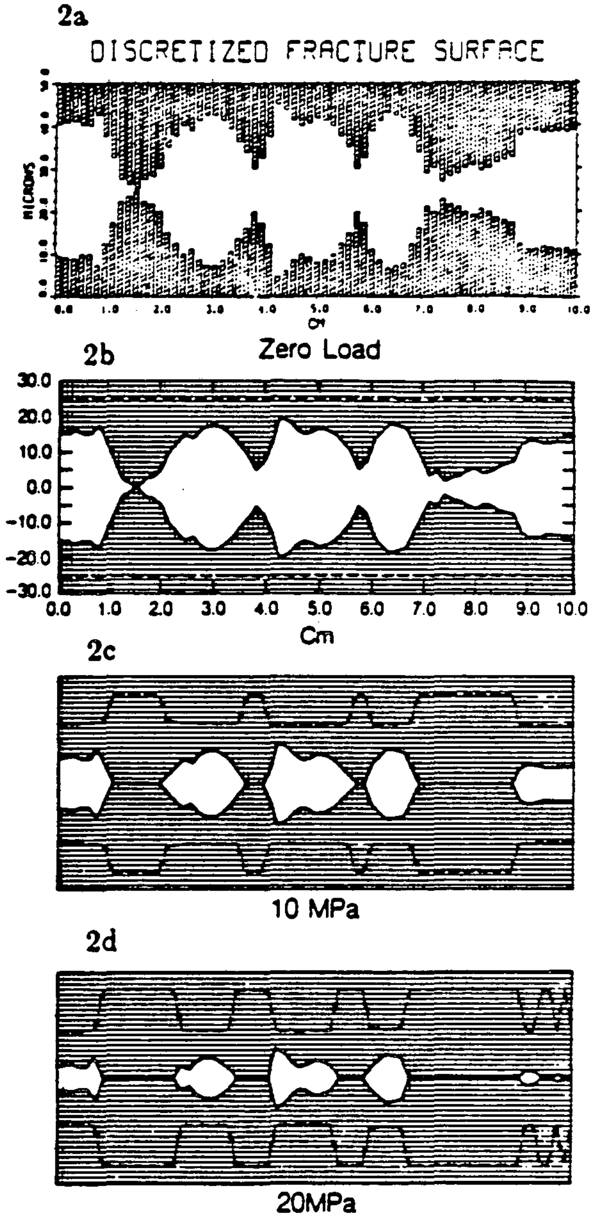


Figure 2. Calculated change in aperture with increasing stress using the model described in Section 2 for the same idealized fracture pictured in Figure 1a. The reference lines have come together 17% and 26% in Figures 2c and 2d, respectively.

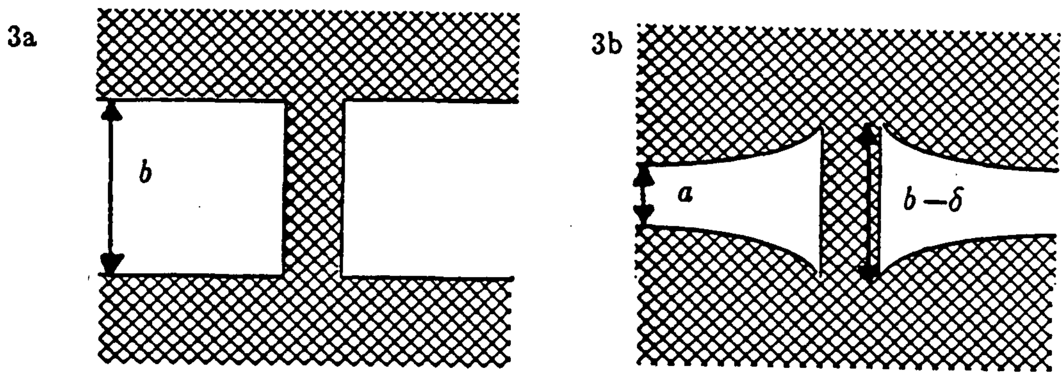
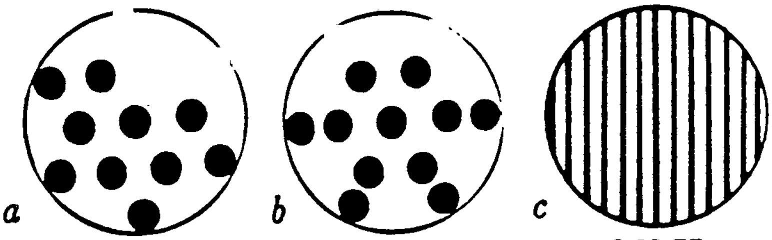
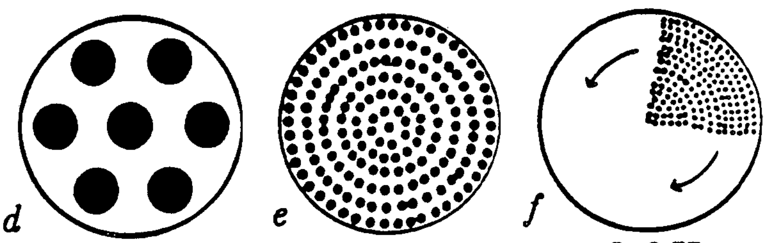


Figure 3. Cross-section of aperture for a single asperity between parallel plates under zero load (a) and after a load is applied (b).



	a	b	c
Estimated stiffness (MPa/m):	1.31 E7	1.30 E7	3.23 E7
Contact area:	25%	25%	25%
Disk diameter (cm):	0.7	0.7	Strip width = 0.1



	d	e	f
Estimated stiffness (MPa/m):	1.18 E7	1.65 E7	5.43 E7
Contact area:	25%	25%	25%
Disk diameter (cm):	1.0	0.2	0.1

Table 1. Specific stiffnesses calculated for spatial arrangements with equal contact areas of 25% but varying disk sizes.

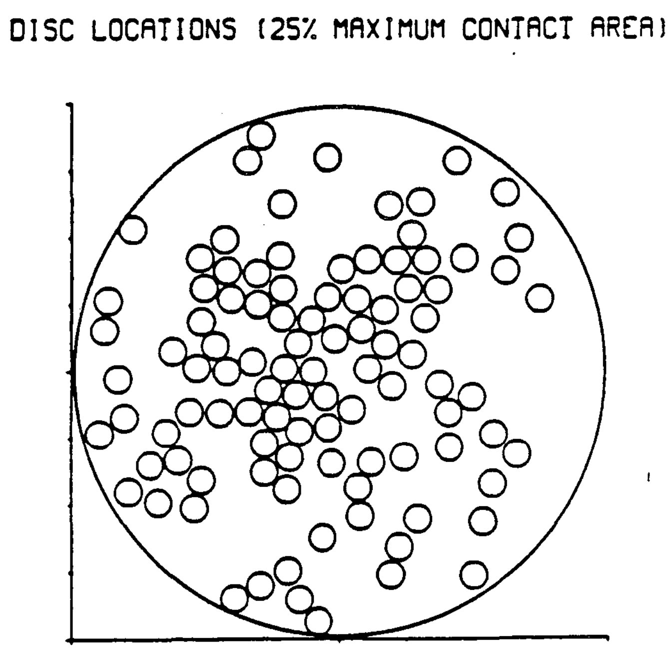


Figure 4. Spatial orientation of discs used to generate the curves of stress vs. stiffness plotted in Figure 5. Disc diameters are 1mm.

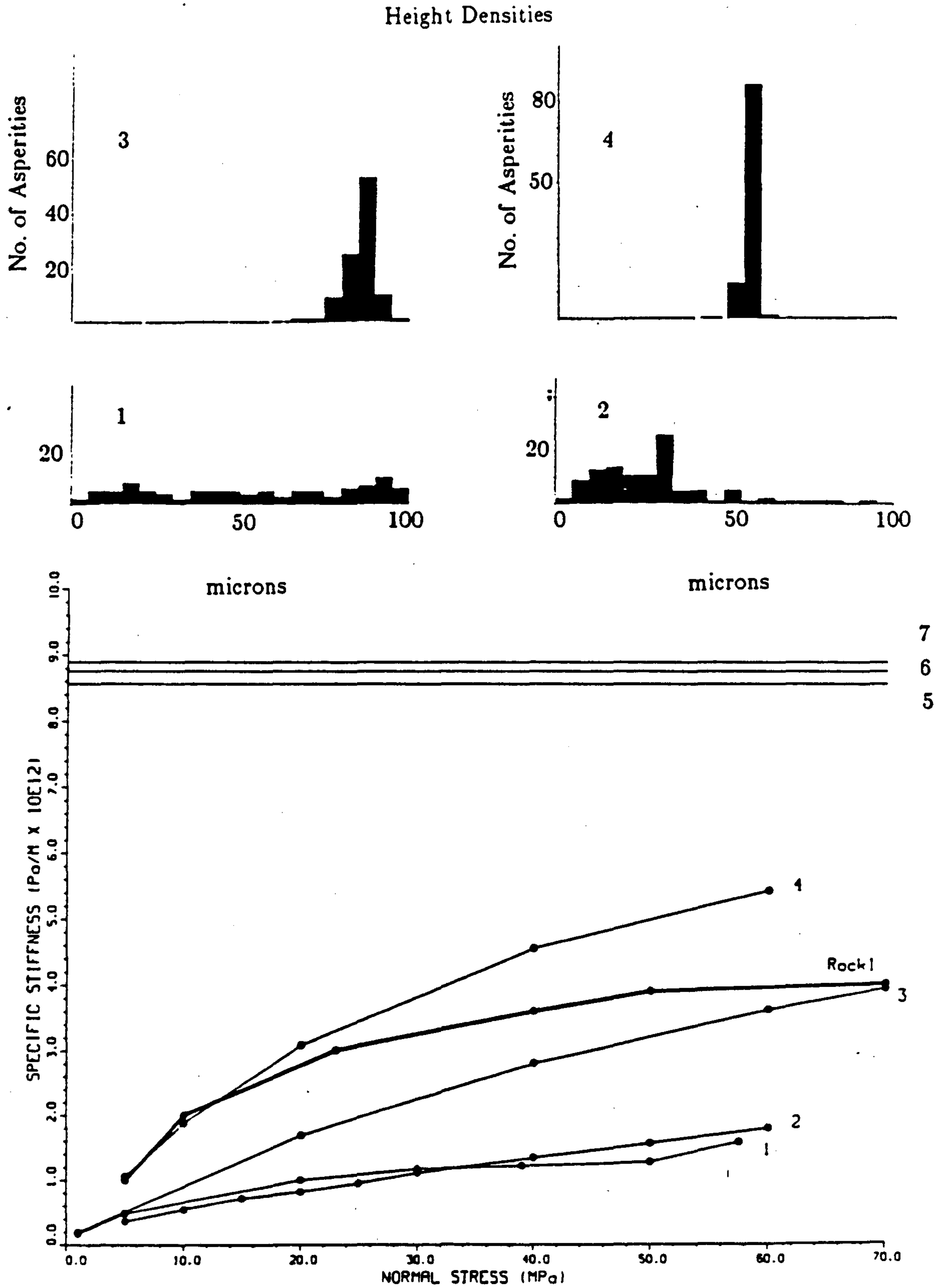


Figure 5. Curves (1-4) of stress vs. specific stiffness for the spatial geometry shown in Figure 4 and different height distributions (plotted as histograms). The bold line is a curve obtained from laboratory data for a granite core specimen containing a single fracture. Lines 5-7 are the values of stiffness calculated assuming the asperities to be of equal height (heights of 30, 50 and 80 microns, respectively).