Title
FARADAY CELL SOLENOIDS FOR AUTOMATIC ELLIPSOMETERS

Authors
Mathieu, H.J.
Muller, R.H.

Publication Date
1973-08-01

Submitted to
Surface Science (Letter to the Editor)

LBL-1836
Preprint

c. 1

FARADAY CELL SOLENOIDS FOR AUTOMATIC ELLIPSOMETERS

H. J. Mathieu and R. H. Muller

August 1973

Prepared for the U. S. Atomic Energy Commission
under Contract W-7405-ENG-48

LBL-1836

DISCLAIMER

FARADAY CELL SOLENOIDS FOR AUTOMATIC ELLIPSOMETERS

H. J. Mathieu and R. H. Muller

Inorganic Materials Research Division, Lawrence Berkeley Laboratory and
Department of Chemical Engineering; University of California
Berkeley, California 94720

In the course of our construction of a self-compensating ellipsometer[1] based on principles reported in this Journal by H. P. Layer,[2] we have found certain limitations in the published design that may have been responsible for some of the problems encountered by the previous author and could be of interest to others who want to build a similar system.

The operation of this ellipsometer is based on the use of Faraday cells for magnetically rotating the plane of polarization of polarizer and analyzer. A control-loop is driven by a signal that is derived from an AC modulation of the Faraday rotation. This signal indicates the direction of rotation needed to reach extinction. Since Faraday rotation over even a modest dynamic range of polarizer and analyzer azimuth (10° for Layer's design) requires the manipulation of sizeable magnetic fields, the inductance of Faraday coils needs careful consideration. Both an adequate AC modulation amplitude and a fast response of azimuth rotation (to be referred to as DC response) are, in practice, limited by coil inductances.

a. DC Response

The response time of an automatic ellipsometer is inversely proportional to the slew rate S of the Faraday rotation, which is defined as (symbols listed at the end)

$$S = \frac{d\theta}{dt} \quad . \tag{1}$$

The Faraday rotation θ is given by Eq. (2)

$$\theta = VH\ell \tag{2}$$

and the magnetic field strength by Eq. (3)[3]

$$H = \frac{0.2\pi NI}{\sqrt{\ell^2 + r^2}} \; . \tag{3}$$

Equations (1) through (3) show that the slew rate is proportional to the rate of change of the solenoid current

$$S = aN \frac{dI}{dt} \; , \tag{4}$$

with

$$a = \frac{0.2\pi V\ell}{\sqrt{\ell^2 + r^2}} \; . \tag{5}$$

Fast changes of solenoid current are, however, hindered by self-inductance,

$$-\frac{dI}{dt} = \frac{E}{L} \; . \tag{6}$$

Thus, Eq. (4) becomes

$$S = -\frac{aNE}{L} \; . \tag{7}$$

For a single layer solenoid the inductance is

$$L = \frac{4\pi^2 r^2 \mu N^2 \cdot 10^{-9}}{\ell} \; . \tag{8}$$

Equation (7) can therefore be written in the form of

$$S = -\frac{a}{b} \; \frac{E}{N} \; , \tag{9}$$

with

$$b = \frac{4\pi^2 r^2 \mu \cdot 10^{-9}}{\ell} \quad . \tag{10}$$

Equations (9) and (10) show that the voltage E applied to the solenoid and the number of turns N of the solenoid determine the slew rate of the Faraday cell.

Based on the properties of Layer's system,[4] one can derive a theoretical slew rate[5] for his solenoid of S = 469°/s. This value assumes the availability of a satisfactory control signal and is mainly limited by the high number of turns of the coil. In order to achieve the desired response time in the range of milliseconds,[1] a slew rate of about 1500°/s appears necessary.[6]

b. AC Response

The voltage for an AC modulation current of amplitude I_o is determined by Eq. (11).

$$E = -\omega L I_o \cos\omega t \tag{11}$$

Application of Eqs. (2) and (3) gives for the AC modulation amplitude of the Faraday rotation (in degrees)

$$\theta_o = -a \frac{NE}{\omega L} \quad , \tag{12}$$

or

$$\theta_o = -\frac{a}{b} \frac{E}{\omega N} = \frac{S}{\omega} \quad . \tag{13}$$

With Eq. (13) applied to Layer's solenoid,[4] one derives an azimuth oscillation amplitude of $\theta_o = 0.15°$

In order to derive a satisfactory control signal from the AC modulation to achieve a resolution in azimuth of 0.01° or better, an azimuth oscillation of θ_o = 1-2° amplitude is necessary.[1,7] Also, the control signal must be derived from an average of at least 5-10 cycles of the AC modulation. With the modulation frequency of 500 Hz employed by Layer, a response time of one millisecond can therefore not be achieved. Increasing the modulation frequency to overcome this problem would result in an even smaller modulation amplitude (due to coil inductance).

c. Power Dissipation and Dynamic Range

The Faraday cell solenoid proposed by Layer, dissipates about 340 W at 10 A DC,[8] not counting the power due to AC modulation.

Since the solenoid was air-cooled, one can estimate an initial temperature-rise (neglecting heat losses) of about 32°/min[9] for operation at full current (10A). Temperature changes during operation are undersirable, because they result in a variation of the Verdet constant and in time-dependent birefringence of the Faraday cell core, that cannot be tolerated in an ellipsometer. Also, continuous service of this solenoid at full current would probably not be possible.

d. Summary

The design of solenoids for Faraday cell-operated ellipsometers has to resolve the following dilemma: A large dynamic range (optical rotation) of the instrument requires the generation of fairly high magnetic fields, that are normally obtained by the use of a large number of turns in the solenoid, although a fast response requires a reasonably low self-inductance. On the other hand, the AC modulation needed for fast response and high resolution calls for the use of high frequencies of large amplitude, which requires a very low self-inductance or a small number of turns. The single coil employed by Layer is not able to provide this combination of capabilities, but the use of separate coils for AC and DC components, as proposed earlier by Winterbottom[10] appears more suitable. In addition, both coils should contain a minimum number of turns (particularly the AC coil) and be designed to carry high currents (particularly the DC coil).

LIST OF SYMBOLS

a	constant defined in Eq.(5)
b	constant defined in Eq.(10)
E	voltage applied to solenoid (AC or DC)
H	magnetic field strength (DC)
I	current flowing through solenoid (DC)
I_o	current amplitude of AC modulation
ℓ	length of solenoid
L	self-inductance of solenoid
N	number of turns of solenoid
μ	permeability of solenoid core
ω	angular frequency of AC modulation
r	radius of solenoid (mean)
S	slew rate of ellipsometer azimuth rotation
t	time
θ	Faraday azimuth rotation due to DC current
θ_o	Faraday azimuth oscillation amplitude due to AC current
V	Verdet constant of Faraday cell core

REFERENCES AND NOTES

1. H. J. Mathieu and R. H. Muller, LBL-2228, to be submitted to Rev. Sci. Instr.
2. H. P. Layer, Surf. Sci. 16, 177 (1969).
3. It is assumed that the length ℓ of the solenoid is not large compared to the radius r. H gives the magnetic field strength at the end of the solenoid. Our own experience shows,[1] that it is preferable in connection with the design of a Faraday solenoid to use the magnetic field strength at the axial end of the solenoid rather than at its axial center.
4. The following data, some of which had to be inferred from other evidence, have been used in the analysis of Layer's solenoid:

$N = 1000$ turns (Ref. 8)

$V = 1.8\times10^{-3}\ \frac{\text{deg}}{\text{A}} = 0.086\ \frac{\text{min}}{\text{Oe cm}}$

$\ell = 7.5$ cm

$r = 2.1$ cm (Ref. 8)

$E = -10$ V (Ref. 11)

$\omega = 2\pi\times500$ rad.

$a = 1.09\times10^{-3}\ \frac{\text{deg}}{\text{A}}$

$b = 2.32\times10^{-8}$ H

$\frac{a}{b} = 4.69\times10^{4}\ \frac{\text{deg}}{\text{V s}}$

$L = b\,N^{2} = 23$ mH

5. Our own experience shows[1] that the theoretical slew rate is larger than what can be obtained experimentally. Layer has determined a slew rate of 4°/s, which was, however, limited by the slew rate of his recorder.

6. R. H. Muller and H. J. Mathieu, LBL-1857, submitted to Applied Optics.
7. B. D. Cahan and R. F. Spanier, Surf. Sci. 16, 166 (1969).
8. Layer used #19 gauge magnet wire. Actually his solenoid must have been built of 14.5 layers of 69 turns each to give 1000 turns over a length of 7.5 cm. Based on this information, one can calculate the radius of the first layer of the solenoid to be 1.27 cm and that of the outermost layer 2.87 cm, giving a mean radius r = 2.1 cm. The ohmic resistance of this wire of 1000 turns (130 m) is calculated to be 3.4Ω, the inductance 23 mH.
9. This estimate was based on the uniform heating of a solenoid made up of 750 g copper, specific heat 0.39 Ws/g°C and 200 g of insulating material (bakelite) of specific heat 1.68 Ws/g°C.
10. A. B. Winterbottom in Ellipsometry in the Measurement of Surfaces and Thin Films, E. Passaglia, R. R. Stromberg, J. Kruger, eds. (Natl. Bur. Std. Misc. Publ., 1964) Vol. 256, p. 97.
11. Assumed voltage according to Layer's Fig. 2.

TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720