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Effects of Temporal and Causal Schemas on Probability Problem Solving

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Abstract

Causal beliefs have been shown to affect performance in a wide variety of reasoning and problem solving. One type of judgment bias that can result from implicit causal models is causal asymmetry -- the tendency to judge predictive inferences as more plausible than comparable diagnostic inferences. In the present study we investigate if the directionality of implicit causal models can also affect application of formal methods, specifically the solution of conditional probability word problems. The study examined temporal and causal schemas, in which the convention is that events are considered in forward direction. Pairs of conditional probability (CP) problems were written depicting events E1 and E2, such that E1 either occurs before E2 or causes E2. Problems were defined with respect to the order of events expressed in CPs, so that $P(E2|E1)$ represents the CP in schema-consistent, intact order by considering the occurrence of E1 before E2; while $P(E1|E2)$ represents CP in schema-inconsistent, inverted order. Participants had greater difficulty encoding CP for events in schema-inconsistent order than CP of events in the conventional deterministic order.

Keywords: Conditional probability; social schemata; causal schemata.

Introduction

Previous research on statistical reasoning has illustrated that individuals often disregard statistically prescribed processes, employing heuristics and biases which may produce invalid inferences (see Barbey & Sloman, 2007; Cohen, 1981; Gigerenzer & Hoffrage, 2007; Kahneman & Tversky, 1979; Krynski & Tenenbaum, 2007; Nisbett, Krantz, Jepson, & Kunda, 1983; Stanovich, Toplak, & West, 2008; and Tversky & Kahneman, 1974, 1980 for extensive treatments of the debate). Improvement in inferential accuracy has been affected by training, expression of likelihood in frequency formats, partitive formulation of the information space, or by expressing real-world application to contexts in which individuals are more likely to reason probabilistically with respect to causal or social schemas (Fox & Levav, 2004; Gigerenzer & Hoffrage, 1995, 2007; Giroto & Gonzalez, 2007; Krynski & Tenenbaum, 2007; Macchi, 2000).

A mathematical word problem's cover story provides real-world context and semantic content, sometimes considered "surface features", because they are usually assumed not to directly affect a problem's technical difficulty or formal solution processes. When a cover story relates to social interactions, however, pragmatic reasoning schemas formed

in response to real-world experiences may be invoked in the mind of the problem solver (e.g., Cheng & Holyoak, 1985; Fong, Krantz, & Nisbett, 1986; Bassok, Chase, & Martin, 1998). The claim made here is that problem solving in probability is particularly affected by domain-specific knowledge in probabilistic reasoning.

Previous research has found discrepancies between formal quantitative probabilistic assessments ("System 2" processing) and on-the-fly, qualitative probabilistic judgments (tapping "System 1"). This prior research has led to a theory of dual-systems representations for probability judgments (Evans & Frankish, 2008; Fox & Levav, 2004; Sloman, 1996; Sloman & Rips, 1998; Smith & Collins, 2009; Stanovich, Toplak, & West, 2008; Tversky & Kahneman, 1983; Windschitl & Wells, 1998). While the two processes have been well-differentiated, it remains to be determined how they may interact in formal problem solving contexts. Individuals have been shown to have a strong bias toward System 1 processes for evaluating social behavior or characteristics qualitatively, while System 2 processes are engaged in symbolic contexts. It may be that the specific interaction of the two processes depends on how problem elements are represented in the mind of the problem solver.

The study reported below examines reasoning effects stemming from the direction of temporality and causation of events in an applied problem's content, examining whether inverting the temporal direction of a schema affects the difficulty of a conditional probability problem. It was hypothesized that it would be easier for individuals to reason forwards regarding temporal or causal events given the deterministic nature of causal and, by extension, temporal schemas (Cheng & Nisbett, 1993; Tversky & Kahneman, 1980). In other words, it should be easier for subjects to calculate the conditional probability of an event given the probability of events preceding it versus calculating the probability of an event given the probability of events occurring later. Similarly, it was expected that problems asking for the conditional probability of an effect given the probability of its cause(s) would be easier than problems asking for the probability of a cause given the probability of its effect(s). It was supposed that inverting the direction of determination should introduce an additional level of difficulty to the problem. It was also expected that the perceived causal strength between events would mediate the effect. The results, described below, yield insights into how probability problems are categorized and solved, what effects

these pragmatic schemas have on problem-solving success, and how they may inform theories of probabilistic reasoning in general.

Causality and Conditional Probability

With respect to temporal and causal schemas, it is the convention to reason forwards, considering earlier events before those that happen later, and causes before effects, a preference which may affect the perceived strength of a causal relationship (Fernbach, Darlow, & Sloman, 2011; Tversky & Kahneman, 1980). This convention may, in part, explain why, when given a choice, individuals prefer to wager on the outcomes of events which have yet to happen rather than those of past events which may have already occurred (Brun & Teigen, 1990; Fischhoff, 1975, 1976; Rothbart & Snyder, 1970; Wright, 1982).

These temporal asymmetries in reasoning facility or preference might extend to certain conditional probability problems. For example, if event *A* is a cause of *B*, or if *A* merely temporally precedes *B*, then the conditional probability of *B* given *A* may be easier to reason about than the reverse conditional. These reasoning effects may not extend to simple computational problems, since $P(A|B)$ and $P(B|A)$ are equally easy to compute.

One probabilistic reasoning error that we might expect to be related to the hypothesized asymmetries in causal and temporal reasoning is the so-called *fallacy of the transposed conditional* or the *inverse fallacy*, in which $P(A|B)$ is confused with $P(B|A)$ (Bar-Hillel & Falk, 1982; Díaz & de la Fuente, 2007; Krynski & Tenenbaum, 2007; Mackie, 1981; Neath, 2010; Tversky & Kahneman, 1980; Villejoubert & Mandel, 2002). Among these discussions there has been speculation of conditions in which this fallacy is more or less likely to occur, but little evidence demonstrating the phenomenon systematically within judgment under uncertainty. If reasoning about forward-direction causal and temporal CPs is easier and more natural than reasoning about “inverted” CPs, then it might be that this error would arise more often by misinterpreting an inverse-order CP as a forward-direction CP than vice-versa.

Empirical Study

If the ease of reasoning about conditional probabilities shows temporal and causal asymmetries (but formula-based calculations do not), then we would expect to see effects of this directionality on the difficulty of probabilistic reasoning and problem-solving for word problems in which the conditional probabilities are described or given in the problem text, not derived via formula. This study tests if such effects are observed.

Method

Participants. Participants were students enrolled in one of four sections of an introductory course in probability and statistical inference during the Fall 2011 term at a graduate school of education. Two instructors each taught two of the

sections. Number of subjects per condition for each item is detailed in Table 1. A total of 123 students completed the quiz. Data from one student in the causal-intact condition was lost, so total *N* for that item is 122.

Materials. Two test items were presented to each student, embedded in an in-class quiz. The test items are listed in the Appendix. There were two versions of the temporal-schema problem (one invoking the temporally-consistent conditional probability, the other invoking the inverted-order probability), and two versions of the causal schema (consistent and inverted orders).

Procedure. Both instructors gave the quizzes during the fourth week of the course, after covering the topic of conditional probability. Each student saw one temporal schema problem (either order intact or order inverted) and the other version of the causal schema problem. Quizzes were administered during the last 30 minutes of class. Students were permitted to use notes and a calculator.

Results

The dependent variables of interest were 1) correctness of numeric solution, and 2) whether the student encoded conditional probability (CP) correctly. Percentage correct for the two measures is summarized in Table 1, by condition.

Table 1: DVs: Percent accuracy for overall problem solution and encoding of CP (with freq), by condition.

Temporal Items	Intact	Inverted	Total
Solved the problem	58.2 (32)	52.5 (35)	54.5 (67)
Coded CP correctly	74.5 (41)	57.4 (39)	65.0 (80)
<i>N</i>	55	68	123

Causal Items	Intact	Inverted	Total
Solved the problem	25.4 (17)	20.0 (11)	23.0 (28)
Coded CP correctly	65.7 (44)	36.4 (20)	52.5 (64)
<i>N</i>	67	55	122

Separate generalized linear models (GZLM) were fit for the temporal and causal problems on each outcome according to a binary distribution with logit (i.e., log-odds) link function. Each model had two predictors: Order (=intact, inverted) and Instructor.

For temporal problems, the proportion of participants correctly solving the problem was higher for order-intact than for order-inverted items, as hypothesized, but this difference was not significant: $X^2_{LR}(1, N = 123) = 0.52, p = .472$. For the second DV, correct encoding of the given conditional probability information, the hypothesis was confirmed: order (=intact, inverted) significantly predicted correct encoding, $X^2_{LR}(1, N = 123) = 6.02, p = .014$. The odds of correctly encoding CP for the order-intact version relative to order-inverted was estimated as 2.67. The effect of instructor in temporal problems was significant for both DVs: $X^2_{LR}(1, N$

= 123) = 3.93, $p = .048$ for correctness of solution, $X^2_{LR}(1, N = 123) = 4.47, p = .034$ for correctness of CP encoding.

For the causal-schema problems, order (=intact, inverted) significantly predicted correct coding of CP, $X^2_{LR}(1, N = 122) = 11.71, p < .001$, confirming the hypothesis. The odds of expressing CP correctly for order-intact items was estimated as 3.70, relative to order-inverted items. However, order did not significantly predict overall problem solution success, $X^2_{LR}(1, N = 122) = 1.21, p = .272$. The effect of instructor was significant for both DVs: $X^2_{LR}(1, N = 122) = 3.90, p = .048$ for correctness of solution, $X^2_{LR}(1, N = 122) = 3.98, p = .046$ for correctness of CP encoding.

One reason why consistency with temporal or causal schema is related to correct encoding of CP, but not with overall solution success, is that encoding CP is just one of many steps that must be carried out to solve a problem correctly. Table 2 presents the conditional probability of problem solution success given correct encoding of CP.

Table 2: Conditional probability of correct answer given correct CP-encoding, by condition.

Item Type	Intact	Inverted	Total
Temporal	.78	.76	.77
Causal	.39	.55	.44

Error analyses. A set of post-hoc analyses examined the frequency of several known errors in probability problem solving. Written protocols were analyzed with respect to deviations from the correct solution for each problem type (see Appendix). Table 3 summarizes the observed frequencies of the two most common probabilistic-reasoning errors exhibited in these data, the transposed conditional (substituting $P(A|B)$ for $P(B|A)$) and “compound substitution”, in which the conditional probability $P(A|B)$ is confused with the joint-event probability $P(A \wedge B)$. Other errors included miscalculations, otherwise incorrectly formalizing the problem, and some idiosyncratic errors. Another commonly documented type of error, *simple substitution* (substituting $P(A)$ or $P(B)$ for $P(A|B)$), did not occur in these data.

The hypothesized asymmetries in reasoning about intact and inverted-order temporal/causal schemas are reflected in an asymmetry in the rate of transposition errors: misinterpreting $P(A|B)$ is much more common than misinterpreting $P(B|A)$. For temporal items, no participants in the intact-order condition made transposed conditional errors. GZLM could not be used to analyze these data because zero cells yield complete separation in the data, thus unreliable estimates. A chi-square test of independence between condition (intact, inverted) and this error type was therefore evaluated using Fisher's exact test, and was significant, $X^2(1, N = 43) = 11.12, p < .001$. The number of students making compound substitution errors in temporal items was too low for statistical analysis.

For causal items, more errors were of the transposed conditional type for the order-inverted condition, and the

difference was marginally significant, $X^2_{LR}(1, N = 58) = 4.81, p < .057$. Students in the order-inverted condition were significantly more likely to commit compound substitution errors, $X^2_{LR}(1, N = 58) = 6.66, p < .010$.

Table 3: Observed percentages (with frequencies) of selected encoding errors, by condition.

Temporal Items	Intact	Inverted	Total
Transposed Conditional	0 (0)	51.7 (15)	34.9 (15)
Compound Substitution	7.1 (1)	6.9 (2)	7.0 (3)
Other Errors	92.9 (13)	41.4 (12)	58.8 (25)
<i>N</i>	14	29	43

Causal Items	Intact	Inverted	Total
Transposed Conditional	8.7 (2)	31.4 (11)	22.4 (13)
Compound Substitution	60.9 (14)	25.7 (9)	39.7 (23)
Other Errors	30.4 (7)	42.9 (15)	37.9 (22)
<i>N</i>	23	35	58

Discussion

The rationale behind this study was to demonstrate the effect of temporal or causal order on coding conditional probability and overall solution success. When a problem text presents conditional probabilities in verbal form, they are more likely to be encoded correctly as conditional probability $P(A|B)$ when the natural temporal or causal order of events is $A \rightarrow B$ rather than $B \rightarrow A$.

Temporal order significantly affected encoding conditional probability (CP). The odds ratio for encoding CP correctly in temporal items was 2.66, indicating that a student receiving an order-intact item was 2.66 times more likely to encode conditional probability correctly than a student receiving an order-inverted item, controlling for instructor. In an error analysis of students who did not encode CP correctly for temporal items, only students receiving order-inverted items transposed the order of events. Order effects were also demonstrated in the causal-schema problems. Participants were more than three times more likely to encode CP correctly for order-intact items than for order-inverted items, controlling for instructor. Error analyses using GZLM indicated that a marginally greater proportion of students made the transposed conditional error in the order-inverted condition, controlling for instructor.

For temporal-schema problems, the conditional probability for solving a problem correctly, given that conditional probability was correctly encoded, did not differ for both order conditions (Table 2). For the causal-schema items, fewer students in the intact condition correctly solved the problem correctly (given having encoding CP correctly) than did students in the inverse condition. This may be explained by fact that the causal items required calculating the probability of the *complement* of the given event in addition to encoding CP, introducing an additional source of noise into the DV of solution correctness. In fact, this complementary event was linguistically signaled by the phrase “DID NOT”

in the order-inverted problem (but not in the order-intact version), perhaps introducing a source of bias.

The results from the error analyses indicate that there may be schema-specific effects related to temporal and causal order. Recall that the likelihood of committing a compound-substitution error in causal items was significantly higher in the order-intact condition (Table 3). Since types of errors coded were mutually exclusive, it makes sense that in absence of making a transposed conditional error, students erring in encoding CP may be more likely to make another systematic error. Compound substitution errors are fairly common when interpreting conditional probability, thus these data do not necessarily indicate that mistaking joint for conditional probability is specifically affected by temporal or causal order. In other words, these error analyses may demonstrate that transposed conditional errors are sensitive to order effect, while compound substitution errors are *not* specific to order effect.

Findings from this study seem related to a phenomenon of judgment bias demonstrated by Tversky and Kahneman (1980). They showed that, in cases in which $P(A) = P(B)$, and $P(A|B) = P(B|A)$, individuals were more likely to judge $P(B|A)$ as greater than $P(A|B)$ if they believed that A was a cause of B . The results of our study demonstrate that, possibly as a result of the strength of causal direction, students are more prone to recognize and correctly encode a conditional probability that preserves the natural “forward” direction of cause to effect.

General Discussion

Results from the present study support the idea that there are differential schematic effects on probability problem solving. Systematically altering the causal or temporal schema depicted in a probability problem's cover story is shown to affect the correct encoding of conditional probability as well as the type of errors, which in turn affects correctly solving a problem. Importantly, effects of pragmatic reasoning schemas are demonstrated for formal probability calculations and encoding usually assumed to be characterized by System 2 reasoning processes, which have typically been shown to be resistant to heuristics and biases.

The study showed that for both temporally- and causally-related events, participants were more likely to encode given conditional probabilities incorrectly when the events are expressed in inverse order. In addition, those errors were more likely to transpose the events from inverse order to intact order than from intact to inverse order. For problems depicting causally-related events, participants were also more likely to incorrectly encode CP when solving for CP expressing events in inverse order.

This finding may be considered in light of the phenomenon of the *fallacy of the time axis*, as illustrated by Falk (1986). In a within-subjects study, participants were asked to consider the events of drawing two marbles from an urn containing two black and two white marbles. Asked first to evaluate the probability of drawing a second white marble after having drawn a first white marble without replacement,

$P(W_2|W_1)$, most participants provide the correct answer with relative ease. Next, asking the same participants to consider $P(W_1|W_2)$, a significant proportion of participants reply that the question is meaningless. Of those who attempt to solve the problem, many indicate that the probability is 1.00 or incorrectly solve the item without considering the probability of the conditioning event.

Although the results from the current study parallel Falk's (1986), the results discussed here generalize her results by demonstrating that the fallacy also occurs in encoding CP in formal probability problem solving, typically considered to be governed by System 2 processes. Furthermore, to date there has been little evidence or explanation put forth concerning processes influencing the occurrence of the fallacy of the transposed conditional. Villejoubert and Mandel (2002) documented that frequency formats reduced the number of transposed conditional errors, however, their offered explanation was only that “people simply confuse $p(H|D)$ with $p(D|H)$ because the latter *sounds* a lot like the former.”

Krynski and Tenenbaum (2007) speculated that transposed conditionals are more likely when $P(A|B)$ is estimated as roughly equivalent to $P(B|A)$, as proposed by Tversky and Kahneman (1980). Our findings suggest that the transposed conditional error also can occur when $P(A|B)$ and $P(B|A)$ are not equal. Rather, our results suggest that that problem solvers tend to exhibit the error only (or mainly) when the temporal or causal order of events in the problem contradict the natural causal or temporal ordering of the depicted real-world events. But note that these order effects in CP may not necessarily affect overall difficulty of a problem over-and-above encoding CP.

These results demonstrate that investigating the specifics of how conditional probability is interpreted in context may inform a bias produced by the order of events depicted in probability problems and their relation to pragmatic reasoning schemas induced by real-world experiences. If, in the cover story of a problem (and in the underlying real-world pragmatic schema), event A occurs before event B or causes event B , then the conditional probability $P(B|A)$ reflects a schema-consistent, intact order of events, translated as “the probability that B occurs given that A has occurred,” reflecting a deterministic, forward-looking time perspective. In contrast, translating $P(A|B)$ as “the probability that A occurs given that B has occurred” seems nonsensical given the problem script. Representing and reasoning about $P(A|B)$ demands consideration of events in inverted chronological order and may be validly translated as only “the probability that A has occurred given that B has occurred,” requiring retrospective time perspective. These two representations of conditional probability reflect two different types of reasoning about uncertainty (Hacking, 1975; Fox & Ülkümen, 2011). While $P(B|A)$ may be addressed in the predictive, aleatory sense, $P(A|B)$ must be considered with epistemic evaluation postdictively.

The phenomenon we document may also have implications for education. Schematic effects on probability problem

solving may inform instruction on statistical inference via consideration of how the two types of probability judgments are theoretically distinct. Evidence from the present study supports an interpretation which addresses this difference in reasoning as parallel to the process of statistical inference and has implications for statistical education and scientific reasoning generally.

Specifically, in evaluating evidence from a sample, the process of hypothesis testing requires considering the conditional probability of finding a result in light of the null hypothesis H_0 , a preexisting fact in the world, which *in fact* either is, $P(H_0) = 1$, or is not, $P(H_0) = 0$. So while results from the present study indicate that it is easier to conceptualize $P(\text{some observed phenomenon} | H_0)$, hypothesis testing evaluates $P(H_0 | \text{some observed phenomenon})$. In addition, in evaluating the validity of statistical inferences, students are taught to consider the likelihood of *Type I* and *Type II* errors, terms which have become shorthand for particular conditional probabilities (Neath, 2010). Training in statistical inference demands a level of mastery of understanding CP in inverted order, which is inconsistent with causal schemas and may warrant more classroom discussion with respect to the nuances and implications for hypothesis testing.

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Appendix: Test Items & Solutions

Temporal-schema items

Order Intact. You are waiting to meet your friend, who is coming from work. He phones saying he will get on the next bus. From experience you know that if he catches an express bus his chances of being on time are 90%, but if he catches a local bus his chances of being on time are 65%. You also know that 60% of the buses that stop by his work are locals, thus he has a 60% chance of catching a local bus today. What is the probability that he arrives on time?

Order Inverted. You are waiting to meet your friend, who is coming from work. He phones saying he will get on the next bus. From experience you know that when he arrives on time, 90% of the time he has caught an express bus, but when he arrives late, 65% of the time he has caught an express bus. You also know from experience that he is late 60% of the time, thus you figure that he has a 60% chance of being late today. What is the probability that he catches an express bus?

Solutions: *E*: Catch express bus; *OT*: Is on time

<p>Order Intact</p> $P(OT E) = .9$ $P(OT E^c) = .65$ $P(E^c) = .6$ $P(OT) = P(OT E)P(E) + P(OT E^c)P(E^c)$ $= (.9)(1 - .6) + (.65)(.6)$ $= .36 + .39$ $= .75$
<p>Order Inverted</p> $P(E OT) = .9$ $P(E^c OT) = .65$ $P(OT^c) = .6$ $P(E) = P(E OT)P(OT) + P(E^c OT)P(OT)$ $= (.9)(1 - .6) + (.65)(.6)$ $= .36 + .39$ $= .75$

Causal-schema items

Order Intact. At a journalism school, a professional ethics exam is given to all students at the end of their first year. Extensive research has established that the probability that a student studies specifically for this exam is 70%. The overall proportion of students who pass the exam is 92%. Exactly 66% of the students will study for the exam and pass it. If we know that a student has studied specifically for the exam, what is the probability that the student FAILS?

Order Inverted. At a journalism school, a professional ethics exam is given to all students at the end of their first year. Extensive research has established that the probability that a student studies specifically for this exam is 70%. The overall proportion of students who pass the exam is 92%. Exactly 66% of the students will study for the exam and pass it. If we know that a student has passed the exam, what is the probability that the student DID NOT study specifically for it?

Solutions: *P*: Pass exam; *S*: Studies

<p>Order Intact</p> $P(P) = .92$ $P(S) = .7$ $P(S \cap P) = .66$ $P(P^c S) = \frac{P(P^c \cap S)}{P(S)}$ $= \frac{.7 - .66}{.7}$ $= \frac{.04}{.7}$ $= .057$	<p>Order Inverted</p> $P(P) = .92$ $P(S) = .7$ $P(S \cap P) = .66$ $P(S^c P) = \frac{P(S^c \cap P)}{P(P)}$ $= \frac{.92 - .66}{.92}$ $= \frac{.26}{.92}$ $= .283$
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