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Flory, C.A.

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1981-03-01

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LBL-12385 c.2
Preprint

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Physics, Computer Science & Mathematics Division

Submitted to Physical Review Letters

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Curt A. Flory

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Prepared for the U.S. Department of Energy under Contract W-7405-ENG-48

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NON-PERTURBATIVE CALCULATION OF THE HEAVY QUARK POTENTIAL

Curt A. Flory*

Lawrence Berkeley Laboratory

University of California

Berkeley, California 94720

Abstract

Non-perturbative contributions are incorporated into the heavy quark potential via non-zero vacuum expectation values of gauge invariant operators à la Shifman, Vainshtein, and Zakharov. The derived potential exhibits the appropriate short distance 1/r behavior and the asymptotic linear confining potential. The calculated coefficient of the linear term is in striking agreement with phenomenological potentials that are constructed to reproduce the heavy quarkonia spectra, and gives the proper Regge slope.

*Address after Sept. 1, 1981: Stanford Linear Accelerator Center

Stanford, CA 94305

Within the framework of Quantum Chromodynamics, non-perturbative contributions to certain physical processes can be estimated using the techniques of Shifman, Vainshtein, and Zakharov [1]. In their approach, ignorance of the long wavelength non-perturbative structure of QCD is parameterized by non-zero vacuum expectation values for certain gauge invariant field operators. Once incorporated into the theory, these vacuum expectation values can be experimentally determined. Using some remarkably successful charmonium sum rules [2], they find for the lowest dimensional purely gluonic operator

$$\langle 0 \left| \frac{g^2}{4\pi^2} G^a_{\mu\nu}(0)\, G^{\mu\nu a}(0) \right| 0 \rangle \equiv M_0^4 = (330\ \text{MeV})^4 \quad , \tag{1}$$

where $G^a_{\mu\nu}$ is the gluon field strength tensor.

Intuitively speaking, the problem of including the effects of this non-perturbative vacuum gluon condensate is equivalent to solving the process under consideration in constant uniform color electric and magnetic fields. Of course, the contributions of higher dimensional and possibly non-uniform vacuum fields must also be considered, and either included or shown to be negligible for the relevant process. The effect of this gluon condensate on the quark-antiquark potential can be evaluated quite simply using multipole techniques. The justification for this procedure is that as long as the size of the vacuum gluon fluctuations is larger than the size of the $Q\bar{Q}$ system, the expansion parameter $(\underset{\sim}{k}\cdot\underset{\sim}{r})$ is a small number. Since the vacuum condensate is effectively spatially homogeneous, a lowest order multipole expansion can be used out to relatively large distances. Of course, higher multipoles which couple to gradients of the vacuum fields could contribute when deviations from the homogeneous approximation are

incorporated. However, as will be pointed out, these are unimportant out to a distance of roughly a fermi.

The derivation begins by allowing a static $Q\bar{Q}$ pair which is in a color singlet state and interacting via a 1-gluon exchange potential to couple an arbitrary number of times to the vacuum field, as in Fig. 1. As is known from the 1-gluon exchange potential, the intermediate color octet $Q\bar{Q}$ states are in a repulsive channel, and thus highly virtual with respect to the color singlet incident state. Thus, the vacuum couplings clump into short periods of octet propagation, separated by longer periods of color singlet propagation [3]. Furthermore, the vacuum only contains the color singlet combination $G^a_{\mu\nu} G^{\mu\nu a}$ and pairs of gluon indices must be contracted. Figure 2 results from this reduction of Fig. 1.

The two particle irreducible interaction kernel which describes the vacuum contribution to the color singlet propagation is chosen to be of the form given by Fig. 3a. It is important to note that the color octet Hamiltonian, H_8', is not a complete physical Hamiltonian. It contains the one gluon exchange diagrams and coupling to the vacuum fields with the restriction that the $Q\bar{Q}$ pair remains in a color octet state at all times, with no intermediate color singlet states. This is an obvious requirement as can be seen by reexamining Fig. 2 and recalling that the color singlet states tend to propagate for long time periods. This singlet propagation would, of course, destroy the localization in time necessary to couple to the vacuum field, which is bilinear at a space-time point. Similarly, Fig. 3b describes the vacuum contribution to the color octet state propagation, with the same restriction to color octet intermediate states.

When iterated, these 2PI kernels can be used to solve for the

full color-singlet Hamiltonian which includes 1-gluon exchange and the vacuum condensate contributions. The simple iteration of Fig. 3a gives the corrected singlet potential

$$H_1 = H_1^0 - H_I \frac{1}{H_8' - H_1^0} H_I \tag{2}$$

where H_1^0 is the 1-gluon ladder exchange potential, and H_I is the gluon-quarkonium interaction Hamiltonian. Similarly, the iteration of Fig. 3b gives for H_8'

$$H_8' = H_8^0 - H_I \frac{1}{H_8' - H_8^0} H_I \ . \tag{3}$$

Note that an important feature of these two equations can be seen by looking at their diagrammatic representation as given by Fig. 4. An iterative solution reveals that only "rainbow diagrams" of vacuum field-$Q\bar{Q}$ coupling contribute in this approximation.

To solve Equations (2) and (3) for H_1 one must know the form of H_I, the gluon-quarkonium coupling. As stated before, the coupling of the uniform vacuum field to the $Q\bar{Q}$ pair suggests a lowest order multipole interaction. The form of this interaction has been derived by several authors [4], and is given by

$$\mathcal{L}_I = \sum_a [- Q_a A_a^0(0,t) + \underset{\sim}{d}_a \cdot \underset{\sim}{E}_a(0,t) + \underset{\sim}{m}_a \cdot \underset{\sim}{B}_a(0,t) + \cdots] \tag{4}$$

where

$$Q_a = g \int d^3r \; \bar{\Psi}\gamma_0 T_a \Psi \tag{5}$$

$$\underset{\sim}{d}_a = g \int d^3r \; \underset{\sim}{r} \; \bar{\Psi}\gamma_0 T_a \Psi$$

$$\underset{\sim}{m}_a = g \int d^3r \; \frac{1}{2} \; (\underset{\sim}{r} \times \bar{\Psi}\underset{\sim}{\gamma} T_a \Psi)$$

with Ψ the $Q\bar{Q}$ wavefunction, $\underset{\sim}{r}$ the $Q\bar{Q}$ separation, and T_a the generators of $SU_c(3)$. Using this interaction and the limit $M_Q \to \infty$, equations (2) and (3) can be evaluated giving

$$H_1 = -\frac{4\alpha_s}{3r} - \frac{1}{18} r^2 \; \frac{1}{\left(H_8' + \frac{4\alpha_s}{3r}\right)} \quad \langle 0 \mid g^2 \underset{\sim}{E}_a^2 \mid 0 \rangle \tag{6}$$

$$H_8' = \frac{\alpha_s}{6r} - \frac{5}{288} r^2 \; \frac{1}{\left(H_8' - \frac{\alpha_s}{6r}\right)} \quad \langle 0 \mid g^2 \underset{\sim}{E}_a^2 \mid 0 \rangle \tag{7}$$

from which the solution for H_1 follows immediately. One finds, using the expression from equation (1)

$$H_1 = -\frac{4\alpha_s}{3r} + \frac{\pi^2 M_0^4 r^2}{18\left(\frac{3\alpha_s}{2r} + \sqrt{\frac{5}{288}}\,\pi M_0^2 r\right)} \; . \tag{8}$$

The interesting features of this potential are that it has the expected coulomb-like behavior $\sim -\frac{4\alpha_s}{3r}$ at short distances, and a linearly rising potential for large r

$$H_1 \xrightarrow[r \text{ large}]{} \frac{2\pi M_0^2}{3}\sqrt{\frac{2}{5}}\; r \; . \tag{9}$$

Numerically, the long distance potential is

$$H_1 \xrightarrow[r \text{ large}]{} (.144 \text{ GeV}^2)\; r. \tag{10}$$

It is interesting to compare the coefficient of the linearly rising term with phenomenological potentials that have been fit to the heavy quarkonia data. Fitting the upsilon spectrum to a coulomb plus logarithmic plus linear potential [5] gives

$$H_1^{exp} \xrightarrow[r \text{ large}]{} (.155 \text{ GeV}^2)\; r \; . \tag{11}$$

If one demands that the phenomenological potential reproduces a Regge slope of .9 GeV^{-2}, one finds

$$H_1^{exp} \xrightarrow[r \text{ large}]{} \frac{r}{2\pi\alpha'} \longrightarrow (.143 \text{ GeV}^2)\; r \; . \tag{12}$$

There is spectacular agreement between our derived potential and these phenomenological potentials inferred from the data. Our potential however, has an analytic interpolating form between the previously conjectured short distance coulomb-like potential and the long distance linear confining potential. Detailed calculations of the quarkonium spectra using the potential of equation (8) are forthcoming.

It is also of interest to see the effects of the vacuum magnetic field on the $Q\bar{Q}$ system. Using the same techniques as above, one finds an effective "hyperfine Hamiltonian"

$$\Delta H_1^{hyperfine} = -\frac{\pi^2 M_0^4 \left(1 - \frac{1}{3} S(S+1)\right)}{6M_Q^2\left(\frac{3\alpha_s}{2r} + \sqrt{\frac{5}{288}}\,\pi M_0^2 r\right)} \tag{13}$$

where S is the spin of the meson and M_Q is the quark mass. Note that the mass of the $^1S_0$ state is suppressed with respect to the $^3S_1$, but a quantitative confrontation with experiment awaits a numerical analysis.

Finally, the validity of our approximation scheme must be addressed--principally the neglect of higher order multipoles. It is a simple exercise to estimate the size of the next order contributions to our process using one very reasonable assumption. It is conjectured that since the scale of the vacuum fluctuations of $G^a_{\mu\nu}G^{\mu\nu a}$ is roughly M_0, higher dimensional operators that occur in the expansion by increasing the number of derivatives or fields differ only in their power of M_0. With this model it is easy to show that the next order term in the interaction Lagrangian of equation (4) that contributes is smaller than the lowest order term by a factor of roughly $\frac{1}{3!}\left(rM_0/2\right)^2$. This is less than a 10% correction until r gets out to distances greater than a fermi. Thus, our potential should probably not be used beyond this distance, which puts very little constraint upon low lying energy level calculations.

Acknowledgments

I thank J. Sheiman and M. Suzuki for useful discussions. This research was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U. S. Department of Energy under Contract No. W-7405-ENG-48.

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Figure Captions

Figure 1: Quark-antiquark pair interacting via 1-gluon exchange, and coupling to the vacuum gluon condensate.

Figure 2: Clustering of the quarkonium-vacuum field interactions about regions of color octet propagation.

Figure 3: (a) 2PI kernel describing the vacuum contribution to the color singlet $Q\bar{Q}$ propagator.
(b) 2PI kernel describing the vacuum contribution to the color octet $Q\bar{Q}$ propagation.

Figure 4: Diagrammatic representation of equation 3 exhibiting the rainbow structure of the vacuum summation.

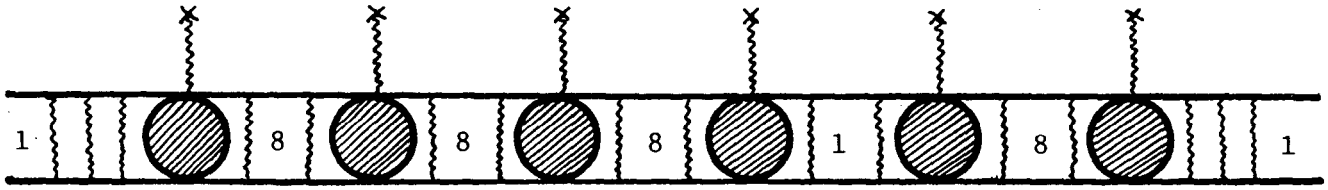


FIGURE 1

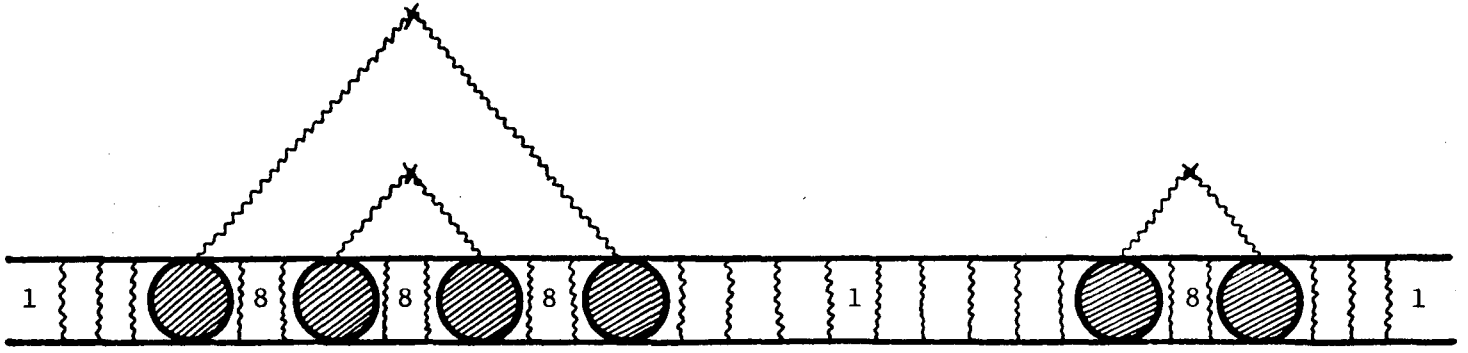
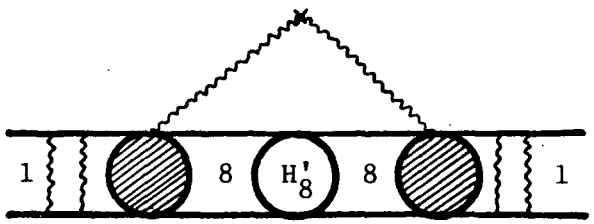
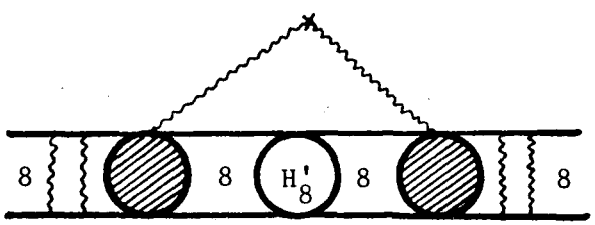


FIGURE 2



(A)



(B)

FIGURE 3

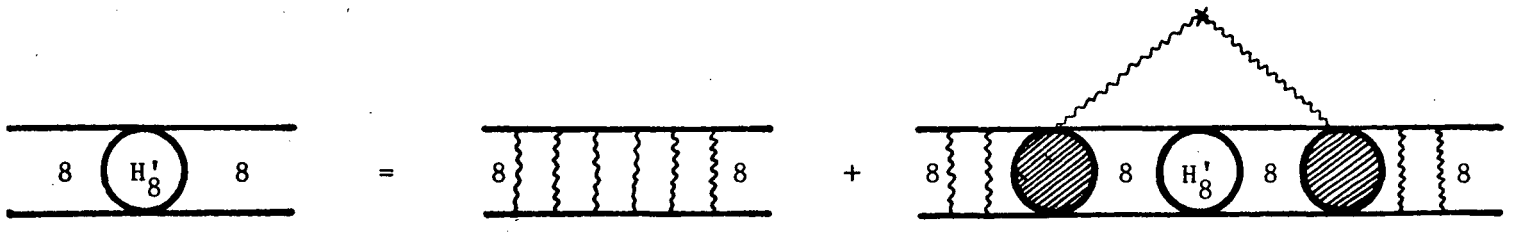


FIGURE 4

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