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FURTHER RESULTS ON THE FLOW OF A MAXWELL
FLUID THROUGH AN ABRUPT CONTRACTION

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Summary

We report new finite element simulations of creeping flow of an upper-convected Maxwell fluid through a sudden planar contraction whose re-entrant corner is defined by a circular arc of small radius. A series of finite element meshes is used, with increasing resolution near the rounded corner. The latter is approximated by either C^0 or C^1 finite element mappings. We find that a limit point in the numerical solution family emanating from the Newtonian result is responsible for the loss of convergence of the iterative scheme beyond some critical value of the Weissenberg number We . The location of the limit point in We -space is not very sensitive to rather extensive mesh refinement, and is virtually invariant to the choice of either C^0 or C^1 discretizations of the rounded corner. These results suggest that the limit point is an intrinsic property of the Maxwell fluid. No definitive conclusion can be drawn, however, for the numerical solutions obtained at values of We close to the limit point show spurious wiggles near the corner.

1. Introduction

Since the work of Beris, Armstrong, and Brown [1], there has been increasing evidence that the failure of mixed finite element methods to provide convergent iterates beyond some critical value of the Weissenberg number We is due to the occurrence of a *limit point* in the family of numerical solutions emanating from the Newtonian result. Whether the limit point is an intrinsic property of the exact solution family or only an artifact of the discretization procedure is an important question which has attracted much attention over the last few years (for a detailed discussion, see the review by Keunings [2]).

In a recent paper, Keunings [3] carried out an extensive mesh refinement experiment for the flow of Maxwell and Giesekus fluids through an abrupt planar contraction. The author used a sequence of meshes which are increasingly refined near the re-entrant corner. A limit point in the solution family emanating from the Newtonian result was found to be responsible for the loss of convergence of the iterative scheme at some critical value of the Weissenberg number. The location of the limit point in We -space is given in Table 1 as a function of mesh refinement. With the Giesekus model, the limit point does not stabilize to a mesh-independent location, and thus appears to be spurious. As seen from Table 1, intermediate mesh refinement largely delays the occurrence of the limit point, but further refinement is not very helpful in that regard. It is worth noting that the numerical solutions obtained in [3] with the Giesekus model are well-behaved (i.e. oscillation free), even for values of We close to the limit point.

Inspection of Table 1 shows that the picture is very different with the upper-convected Maxwell fluid. The limit point seems to settle in a mesh-independent location, as seen by Yeh et al. [4] in their simulations of circular contraction flows. It would thus appear that a true limit point (i.e. a limit point of the exact solution family) has been identified in the present flow problem. With the most refined mesh, however, the limit point occurs at a much reduced value of We . This disturbing fact has also been observed by Brown et al.[5] with a different numerical method. In contrast to the results obtained with the Giesekus model, the numerical solutions computed with the Maxwell fluid near the limit point are polluted by spurious oscillations which emanate from the re-entrant corner.

The above results clearly show that intensive mesh refinement with conventional mixed finite element techniques is not very helpful in the presence of viscoelastic stress singularities. Unfortunately, the nature of stress singularities for commonly used viscoelastic models remains largely unknown. In a recent paper, Lipscomb et al.[6] have shown that the stress singularity can be computed explicitly for the second-order fluid model in flows satisfying the Giesekus-Tanner-

Huilgol theorems. In the case of creeping flow through a planar contraction, the Newtonian extra-stress has a singularity of the type $r^{-0.455}$, where r is the distance from the singular point (Moffatt [7]). It is shown in [6] that the corresponding stress singularity for the second-order fluid goes like $r^{-0.455} + We r^{-0.911}$. This result establishes the singular nature of the Newtonian limit: there always exists some region $r \ll We^{1/0.911}$ of the corner where viscoelastic and Newtonian stress fields differ by an arbitrarily large amount. We also see that the dominant term of the stress singularity is the *square* of the Newtonian singularity. The numerical difficulties are thus expected to be much more significant than with a Newtonian fluid. Even though the above analytical results only apply to the second-order fluid, it has been observed numerically [6] that the $r^{-0.911}$ dependence holds for the upper-convected Maxwell fluid, at least for very low We and in a region close to (but excluding) the re-entrant corner. The results of Table 1 can thus be interpreted as follows: meshes that are sufficiently refined in the vicinity of the re-entrant corner render computations with the Maxwell fluid extremely difficult, much more difficult indeed than with a Newtonian fluid; above some degree of mesh refinement, approximation errors will grow to the extent that an artificial limit point is produced, whose location in We -space goes to zero with increased resolution.

Actually, we can go one step further and assume, on the basis of Table 1, that a *true* limit point exists at $We \approx 0.6$ for creeping, symmetric flow of an upper-convected Maxwell fluid through a planar contraction. In view of the above discussion, it is indeed quite plausible that the viscoelastic stress singularity overwhelms the numerical results computed with the most refined mesh to the extent that it induces an artificial limit point before the true limit point could ever be reached. A numerical experiment by Lipscomb et al.[6] gives some ground to this hypothesis. The computations described in [3] were repeated using the most refined mesh with a single change: the Maxwell relaxation time was set to zero in the minute elements containing the re-entrant corner, thus maintaining the strength of the singularity at its Newtonian value. Interestingly enough, the limit point moved from 0.112 (see Table 1) to about 0.6, i.e. close to the hypothesized limit point of the exact solution family. Unfortunately, the numerical solutions for We close to 0.6 were still polluted by spurious node-to-node oscillations generated at the re-entrant corner. We must thus conclude that the mixed finite element technique used in [3] and [6] is unable to prove conclusively the existence of true limit points in viscoelastic flow problems endowed with stress singularities.

The present note complements [3] and [6] with new simulations of the flow of a Maxwell fluid through a planar contraction whose re-entrant corner is defined by a *circular arc* of small radius. Rounding the corner should eliminate the significant numerical difficulties associated with the stress singularity, thereby

enabling us to test the possible existence of a true limit point in this flow geometry. As in [3], we use a series of finite element meshes with increasing resolution in the vicinity of the rounded corner. The numerical technique is identical to that used in [3] and [6], the only difference being related to the discretization of the rounded corner. Two approximation techniques are invoked to that end: the classical isoparametric finite element mapping, which results in a corner representation of class C^0 , and a new finite element mapping, referred to as superparametric transformation, which allows for a corner representation of class C^1 .

As in [3], we find that a limit point in the numerical solution family emanating from the Newtonian result is responsible for the loss of convergence of the iterative scheme beyond some critical value of We . The location of the limit point in We -space is not very sensitive to mesh refinement, and virtually identical critical values of We are found with both C^0 and C^1 approximations of the rounded corner. This suggests that the upper-convected Maxwell fluid has a true limit point in the present flow problem. Unfortunately, the numerical solutions obtained for values of We close to the limit point exhibit spurious oscillations near the corner. It is thus impossible to draw a definitive conclusion on the basis of the present numerical experiments.

2. Governing equations and numerical technique

In the present note, we consider two-dimensional, symmetric creeping flows of an upper-convected Maxwell fluid through a sudden 4:1 contraction. The transition between the contraction wall and the downstream slit is defined by a circular arc of radius $0.2H$, where H is the half-thickness of the downstream slit (Fig.1). In dimensionless form, the governing equations read

$$\mathbf{T} + We \overset{\nabla}{\mathbf{T}} = 2\mathbf{D} ,$$

$$-\nabla p + \nabla \cdot \mathbf{T} = 0 , \tag{1}$$

$$\nabla \cdot \mathbf{v} = 0 ,$$

where $\mathbf{T}$ denotes the extra-stress tensor, $\mathbf{D}$ is the rate of strain tensor, p is the indeterminate pressure, $\mathbf{v}$ is the velocity vector, and the superscript ∇ stands for the upper-convected derivative [8]. We define the Weissenberg number We by

$$We = \frac{\lambda V}{H}, \quad (2)$$

where λ is the relaxation time, and V is the average velocity in the downstream slit of half-thickness H . Conventional boundary conditions are specified along the boundary of the computational domain: fully-developed velocity and extra-stress fields at the upstream section, no-slip at the wall, fully-developed velocity field at the downstream section, and symmetry conditions at the plane of symmetry.

The governing equations (1) are discretized by means of the mixed Galerkin/Finite Element technique known as method MIX1 (see e.g. [3]). Velocity and extra-stress fields are approximated using P^2-C^0 shape functions, while the pressure field is discretized by means of P^1-C^0 polynomials. Fig. 1 shows a partial view of the three finite element meshes used in the present work. The total number of nodal unknowns is 3257, 11550, and 33239, respectively. We discretize the flow domain by means of 9-node quadrilateral and 6-node triangular isoparametric elements. The resulting approximation of the rounded corner is thus of class C^0 . Alternatively, we use a C^1 representation of the rounded corner by applying a special geometrical mapping to the corner elements only. This superparametric transformation maintains C^0 continuity of the mapping *within* the flow domain (see [9] for details).

3. Results and conclusions

For each mesh of Fig. 1, we have computed a numerical solution family parameterized by We using Newton's iterative scheme and a first-order continuation procedure. The starting point is the Newtonian solution ($We = 0$). Newton's method provided quadratically convergent iterates up to a critical value $\overline{We}$. Every attempt to obtain convergent iterates for $We > \overline{We}$ failed, even for increments of We of the order of 10^{-4} . Furthermore, the sign of the determinant of the Jacobian matrix would oscillate during these unsuccessful iterations, which implies the singularity of the Jacobian matrix in the vicinity of $\overline{We}$. These facts indicate the presence of a limit point of the numerical solution family emanating from the Newtonian result.

Listed in Table 2 are our best available estimates of the location We_{lim} of the limit point, as a function of mesh refinement and the type of approximation (C^0 or C^1) of the rounded corner. As in [3], we define We_{lim} as $\overline{We} + \Delta \overline{We}/2$, where $\Delta \overline{We}$ is the smallest increment of We used in the unsuccessful attempts to pursue the calculations beyond $\overline{We}$. Inspection of Table 2 reveals that the location of the limit point in We -space is not very sensitive to intensive mesh refinement. (A similar observation was made recently by Brown et al.[5] in simulations of

axisymmetric flows through a rounded contraction carried out with a different mixed finite element technique.) Furthermore, virtually identical critical values are obtained with both C^0 and C^1 discretizations of the rounded corner.

These results suggest that the computed limit points reflect the presence of a true limit point of the continuous problem located at $We \approx 0.8$. Assuming the validity of this conclusion *and* the existence of a true limit point in the case of the abrupt re-entrant corner (cfr. Introduction), the effect of rounding the corner would thus be to displace the location of the true limit point from $We \approx 0.6$ to $We \approx 0.8$. Examination of the quality of the numerical solutions for We close to We_{lim} does not, however, allow us to draw a definitive conclusion about the existence of a true limit point. As in [3] and [6], we find that the computed velocity and stress fields are quite smooth for sufficiently small values of We , but develop spurious wiggles near the corner when We approaches We_{lim} . These wiggles are quite similar to those described in [3] and need not be documented here. It may thus well be that the computed limit points are the result of excessive approximation errors. Improved discretization procedures are clearly needed to settle this issue.

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Figure 1.

Schematic of the flow through a sudden contraction with a rounded corner, and partial view of the three finite element meshes.

Captions for Tables

Table 1. Location of the limit point as a function of mesh refinement (abrupt re-entrant corner [3]); corner element sizes are made dimensionless with the half-thickness of the downstream slit

Table 2. Location of the limit point as a function of mesh refinement, for both C^0 and C^1 approximations of the rounded corner (Maxwell fluid)

Degrees of freedom	Size of corner element	Location We_{lim} of the limit point	
		Giesekus model	Maxwell model
3139	0.25	0.805	0.873
5059	0.20	1.05	0.565
3046	0.05	4.545	0.556
11172	0.02	0.408	0.588
40974	0.005	0.610	0.112

Table 1. Location of the limit point as a function of mesh refinement (abrupt re-entrant corner [3]); corner element sizes are made dimensionless with the half-thickness of the downstream slit

Degrees of freedom	Size of corner elements	Location of limit point	
		C^0 approx.	C^1 approx.
3257	0.08	0.978	0.997
11550	0.04	0.716	0.722
33239	0.02	0.834	0.816

Table 2. Location of the limit point as a function of mesh refinement, for both C^0 and C^1 approximations of the rounded corner (Maxwell fluid)

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