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Centre vortex removal restores chiral symmetry

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Abstract. The influence of centre vortices on dynamical chiral symmetry breaking is investigated through the light hadron spectrum on the lattice. Recent studies of the quark propagator and other quantities have provided evidence that centre vortices are the fundamental objects underpinning dynamical chiral symmetry breaking in SU(3) gauge theory. For the first time, we use the chiral overlap fermion action to study the low-lying hadron spectrum on lattice ensembles consisting of Monte Carlo, vortex-removed, and vortex-projected gauge fields. We find that gauge field configurations consisting solely of smoothed centre vortices are capable of reproducing all the salient features of the hadron spectrum, including dynamical chiral symmetry breaking. The hadron spectrum on vortex-removed fields shows clear signals of chiral symmetry restoration at light values of the bare quark mass, while at heavy masses the spectrum is consistent with a theory of weakly-interacting constituent quarks.

1. Introduction

Dynamical chiral symmetry breaking is one of the signature features of quantum chromodynamics (QCD), along with the confinement of quarks inside hadrons. These phenomena appear to be emergent properties of QCD, and are generally accepted to originate from some topological feature of the non-trivial QCD vacuum. The centre vortex model [1–8] is a well-known candidate for the origin of confinement, and has been extensively studied on the lattice in both SU(2) and SU(3) gauge theory [9–15]. The role of centre vortices on dynamical chiral symmetry breaking has also been examined on the lattice, where in SU(2) theory [14,16–23] it has been found that they are the fundamental long-range objects responsible. In SU(3), initial studies showed mixed results. While the Landau-gauge AsqTad quark propagator showed no role for centre vortices in SU(3) dynamical chiral symmetry breaking [24], the opposite result was found in the low-lying hadron spectrum [25] using Wilson-type fermions.

Recently, the Landau-gauge quark propagator has been studied using the chirally-sensitive overlap fermion action [26, 27]. There, it was found that removal of centre vortices from gauge field backgrounds results in the loss of dynamical chiral symmetry breaking. Backgrounds consisting solely of centre vortices could reproduce dynamical chiral symmetry breaking after a small amount of gauge field smoothing. This is part of a consistent picture that has emerged from work by the CSSM lattice collaboration demonstrating that smoothed vortex-only gauge fields are able to reproduce a number of salient features of QCD [28]. In addition to macroscopic quantities such as the quark mass function and static quark potential, the microscopic structure of the gluon field has also been examined. It was seen that after smoothing, a gauge field background consisting solely of centre vortices displays a structure of instanton-like objects similar in both size and density to those seen in untouched configurations after cooling, hence providing a mechanism for dynamical chiral symmetry breaking.

In this work we study the role of centre vortices in dynamical chiral symmetry breaking via the low-lying hadron spectrum. While this has been considered previously in Ref. [25], here we offer several significant improvements. Firstly, the work of Refs. [26, 27] has shown that the chiral nature of the overlap fermion action is vital to correctly discern the role of centre vortices in dynamical chiral symmetry breaking, and so it is used here. Additionally, one may be concerned that the procedure of removing centre vortices from gauge field configurations has changed the quark mass renormalization, and so using a Wilson-like action one may have difficulty matching bare quark masses across ensembles. Evidence of this was indeed seen in Ref. [25]. The overlap fermion action, thanks to its lattice-deformed chiral symmetry, does not suffer from additive mass renormalization, and so we may unambiguously compare ensembles with equivalent bare quark masses. In order to study dynamical chiral symmetry breaking, one naturally wishes to minimise the impact of the explicit chiral symmetry breaking induced by the bare quark mass. The overlap action enables us to consider very light masses, with a smallest value considered of $m_q = 13$ MeV.

2. Simulation Details

A centre vortex intersects with a two-dimensional region A of the gauge manifold U if the Wilson loop identified with the boundary has a non-trivial transformation property $U(\partial A) \to z U(\partial A)$, $z \neq 1$, under an element $Z = zI \in \mathbb{Z}_3$ of the centre group of SU(3), where $z \in \{1, e^{\pm 2\pi i/3}\}$ is a cube root of unity. On the lattice we study centre vortices by seeking to decompose gauge links $U_\mu(x)$ in the form

$$U_\mu(x) = Z_\mu(x) \cdot R_\mu(x), \tag{1}$$

in such a way that all vortex information is captured in the field of centre-projected elements $Z_\mu(x)$, with the remaining short-range fluctuations described by the vortex-removed field $R_\mu(x)$. By fixing to Maximal Centre Gauge and identifying $Z_\mu(x)$ as the projection of the gauge-fixed links to the nearest centre element, we produce configurations with vortices removed, as well as configurations consisting solely of vortex matter. Vortex matter is identified by searching for plaquettes with a nontrivial centre flux around the boundary. The procedure used is outlined in detail in Ref. [27]. Throughout this work, we use three ensembles; an original, 'untouched' ensemble (UT) of Monte Carlo gauge fields $U_\mu(x)$, an vortex-only ensemble (VO) consisting solely of centre projected elements $Z_\mu(x)$, and a vortex-removed ensemble (VR) of the remainder fields $R_\mu(x)$.

Results are calculated on 50 pure gauge-field configurations using the Lüscher-Weisz $\mathcal{O}(a^2)$ mean-field improved action [29], with a $20^3 \times 40$ volume at a lattice spacing of 0.125 fm. We use the FLIC operator [30–33] as the overlap kernel, with negative Wilson mass $m_w = 1$. As per Refs. [26, 27], 10 sweeps of cooling are performed on the vortex-only ensemble in order to ensure the smoothness condition required for locality of the overlap operator.

The hadron interpolating fields we use here are listed in Table 1. Note that we consider only isovector mesons in order to avoid disconnected contributions.

Table 1. A list of the meson and baryon interpolators considered herein.

Meson	I, $\mathrm{J^{PC}}$	Operator
π	$1, 0^{-+}$	$\bar{q}\, \gamma_5\, \frac{\tau^a}{2}\, q$
ρ	$1, 1^{--}$	$\bar{q}\, \gamma_i\, \frac{\tau^a}{2}\, q$
a_0	$1, 0^{++}$	$\bar{q}\, \frac{\tau^a}{2}\, q$
a_1	$1, 1^{++}$	$\bar{q}\, \gamma_i\, \gamma_5\, \frac{\tau^a}{2}\, q$

Baryon	I, $\mathrm{J^P}$	Operator
Nucleon	$\frac{1}{2}, \frac{1}{2}^+$	$[\, u^{\mathrm{T}}\, \mathrm{C}\, \gamma_5\, d\,]\, u$
Δ	$\frac{3}{2}, \frac{3}{2}^+$	$[\, u^{\mathrm{T}}\, \mathrm{C}\, \gamma_i\, u\,]\, u$

Table 2. Values of the overlap mass parameter, μ, considered, with corresponding bare quark masses in physical units, using $a = 0.125$ fm.

μ	m_q (MeV)
0.004	13
0.008	25
0.012	38
0.016	50
0.032	101
0.040	126
0.048	151
0.056	177

The hadron spectrum is calculated with bare quark masses varying over a large range. The values of the overlap mass parameter μ and the corresponding bare quark mass are given in Table 2. 100 iterations of Gauge-invariant Gaussian smearing [34, 35] are performed at the fermion source and sink. Fixed boundary conditions are applied in the temporal direction, with our quark sources placed at $n_t = 10$ relative to the lattice length of 40. We have investigated the use of the variational method [36, 37], and found no significant improvement in our ability to discern the ground state signals salient to our investigation. Hadron effective masses are extracted in the standard way, with uncertainties obtained via a second-order single-elimination jackknife analysis.

3. Vortex-Only Spectrum

The light hadron spectrum on the untouched and vortex-only ensembles is presented for the four light quark masses in Fig. 1. Turning first to the lightest quark mass at $m_q = 13$ MeV, results for the untouched spectrum are as expected. The pion, rho, nucleon, and Delta all have clear signals, and sit slightly heavier than their physical values. The vortex-only ensemble is able to reproduce all qualitative features of the spectrum, although masses are slightly lower. This is most likely an artifact of cooling [26, 27, 38]. Notably, in both cases the pion is much lighter than the rho, a clear signal that it retains its nature as a pseudo-Goldstone boson, and thus that dynamical chiral symmetry breaking is present on the vortex-only ensemble. A clear separation between the nucleon and Delta baryons is also maintained.

The behaviour seen at the lightest quark mass is replicated across the remaining seven quark masses considered. There are clear signals for all four hadrons in both the untouched and vortex-only cases. Again, the masses on the vortex-only ensemble are slightly smaller than those in the untouched ensemble, but the qualitative features with regard to the ordering of the different hadrons are reproduced.

At low quark masses, the signal for the a_0 and a_1 mesons is too poor to allow study. We show these results exclusively at the three heaviest quark masses, 126, 151, and
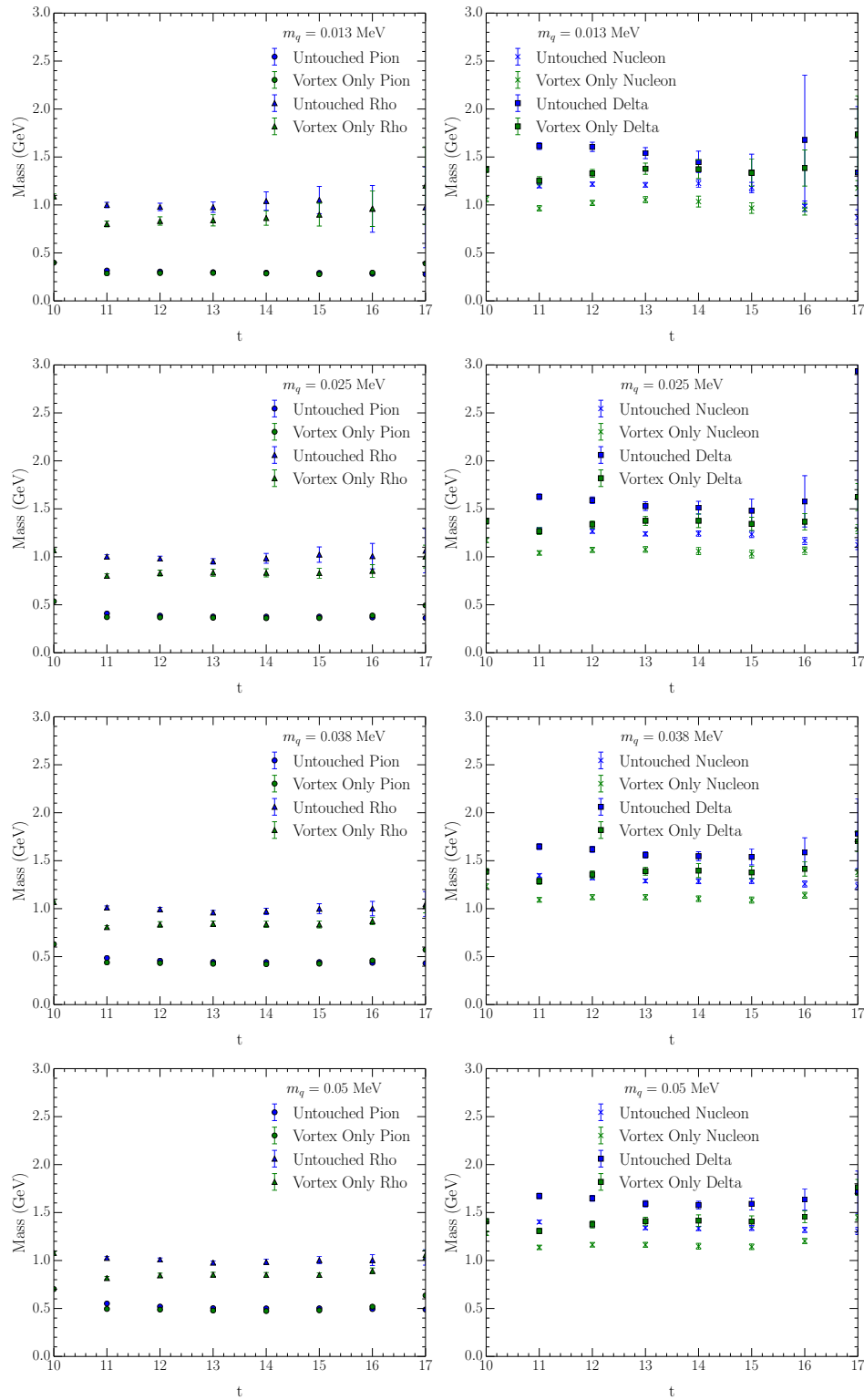
Figure 1. The effective masses for the low-lying mesons (left) and baryons (right) on the untouched (blue) and vortex only (green) ensembles. Results are shown for light bare quark masses with values of $m_q = 13,\ 25,\ 38,\ 50$ MeV from top to bottom respectively.
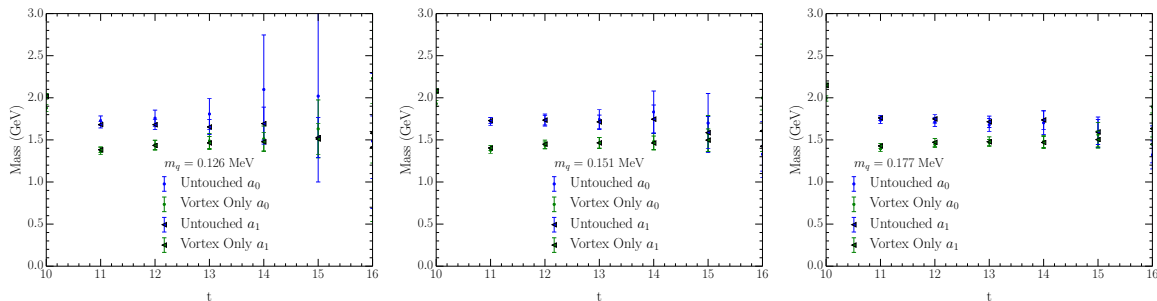
Figure 2. The effective masses for the a_0 and a_1 mesons on the untouched (blue) and vortex-only (green) ensembles, at bare quark masses of 126 (left), 151 (middle), and 177 (right) MeV.

177 MeV, in Fig. 2. At these masses, the a_0 and a_1 are approximately degenerate on the untouched ensemble, and this is reproduced on the vortex-only ensemble. Again, while the masses are slightly lower, the qualitative features of the hadron spectrum are reproduced.

In pure gauge theory on the lattice, the η' meson cannot gain mass from repeated $q\bar{q}$ annihilation due to the lack of disconnected fermion loops. Therefore it does not have the relatively large mass given to it by the axial anomaly in QCD. This leads to an $\eta' - \pi$ 'ghost' state in the scalar meson channel, which gives a negative-metric contribution to the two-point correlator [39]. This effect is most pronounced at low quark masses; at higher quark masses this two-particle ghost state is much higher in energy than the single-particle state, and thus does not contribute at large Euclidean times. This phenomenon is responsible for the difficulties in measuring the a_0 and a_1 at low quark masses.

In summary, gauge field configurations created from the centre vortices identified in the original gauge field configurations in Maximal Centre Gauge (MCG) are able to capture the essence of the QCD vacuum structure. We observe the spin splittings between the nucleon and Delta baryons to be preserved and the pion maintains its pseudo-Goldstone boson nature in the light quark-mass regime. While some reduction in the mass of the low-lying hadron spectrum is seen, it can be attributed to the small amount of cooling applied that is necessary to evolve the thin P-vortices (identified from the plaquette values in MCG) towards the physical *thick* vortices that describe the topologically nontrivial QCD vacuum. Simultaneously, the cooling ensures that the smoothness condition required for the overlap operator is satisfied.

4. Chiral Symmetry Restoration

Before presenting results for the vortex-removed ensemble, it is worth carefully considering our expectations for the ground-state hadron spectrum upon removal of dynamical chiral symmetry breaking.

Under the complete restoration of chiral symmetry, we expect baryon currents

related by chiral transformations to become degenerate. The massless QCD Lagrangian has an $SU(2)_L \times SU(2)_R \times U(1)_A$ symmetry. The $U(1)_A$ symmetry is, however, explicitly broken by the axial anomaly. We must therefore admit the possibility that the $U(1)_A$ and $SU(2)_L \times SU(2)_R$ symmetries are restored separately. The complete restoration of chiral symmetry would imply the following degeneracies [40],

$$
\begin{aligned}
\pi &\leftrightarrow a_0 \quad [U(1)_A] \\
\rho &\leftrightarrow a_1 \quad [SU(2)_L \times SU(2)_R] \\
N &\leftrightarrow \Delta \quad [SU(2)_L \times SU(2)_R] \,.
\end{aligned}
\tag{2}
$$

In lattice simulations a non-zero bare quark mass is used. Chiral symmetry is thus explicitly broken even in the absence of dynamical chiral symmetry breaking. At small bare quark masses, the explicit breaking of chiral symmetry is negligible and so we expect these degeneracies to be manifest.

At larger masses, chiral symmetry no longer holds even approximately. We thus expect to see something close to a non-interacting constituent-quark like model, where the mass of each state is simply the sum of the dressed quark masses composing it, possibly with some momentum. This is the result seen in Ref. [25]; degenerate π- and ρ-meson masses were observed, even though the two are not related by a chiral transformation. The mesons had a mass of approximately 2/3 of the mass of the baryons.

As we are considering pure gauge theory, one must also consider multi-particle states contributing to the a_0 and a_1 correlators. In the pure gauge sector, quark flows such as the "hairpin", illustrated in Fig. 3, can carry the quantum numbers of the a_0 or a_1 through π-η' or ρ-η' intermediate states respectively. Because the sea-quark loops vital to generating the mass of the singlet η' meson are absent, $m_{\eta'} = m_\pi$. The associated mass thresholds of these multi-particle states carrying the quantum numbers of the a_0 and a_1 are thus $2\,m_\pi$ and $m_\pi + m_\rho$ respectively. We will refer to these multi-particle states as π-η' and ρ-η' states.

One might also be concerned about the "double-hairpin" graph illustrated in Fig. 4 that can provide a negative-metric contribution to the correlators. However, all of our correlators on the vortex-removed configurations remain positive. There is some evidence of a nontrivial contribution in the a_0 correlator at the lightest quark mass considered, as its effective mass function rises sharply from below at the earliest Euclidean times. This is highlighted in the discussion below.
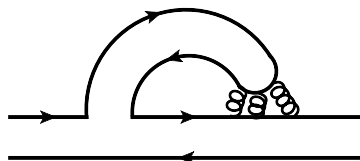

Figure 3. The "hairpin" diagram, showing a π-η' or ρ-η' intermediate state with the quantum numbers of the a_0 or a_1 mesons respectively.
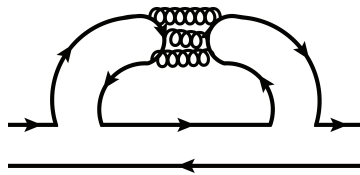
Figure 4. The "double-hairpin" diagram, associated with a negative-metric contribution to the a_0 and a_1 correlators.

In summary, if vortex removal does indeed result in the loss of dynamical chiral symmetry breaking, we can make a number of predictions. At light quark masses, we expect a chiral regime to hold where the hadron spectrum has the following qualities:

- In the absence of dynamical chiral symmetry breaking, the pion is no longer a pseudo-Goldstone boson, and so there is no *a priori* reason for it to have a much lower mass than the other mesons.

- The restoration of the $U(1)_A$ symmetry will be shown by the degeneracy of the π and ground state a_0 at low quark masses.

- The restoration of the $SU(2)_L \times SU(2)_R$ symmetry will be shown by the degeneracy of the ρ and ground state a_1 at low quark masses.

- The N and Δ should also be degenerate via $SU(2)_L \times SU(2)_R$ symmetry.

- There is no chiral transformation relating the π and ρ mesons, and so at light quark masses we expect the two to differ in mass.

At heavy quark masses, we expect a constituent regime to hold where the light hadron masses should simply be estimated by counting quarks. However, it should be noted that, due to their positive parity, there is no way to make the quantum numbers of the a_0 or a_1 with two constituent quarks at rest. To create overlap with an $l = 1$ orbital angular momentum state needed for positive parity, we must excite at least one of the constituent quarks with the lowest non-trivial momentum available on the lattice. Hence, for the hadron spectrum in the constituent regime we predict the following:

- The π should be degenerate with the ρ at high quark masses. Likewise, the N and Δ should be degenerate, each with a mass $3/2$ times that of the mesons.

- The a_0 mass will be the lower of two possibilities: a π-η' state with mass $2\,m_\pi$, or a two quark state excited with the lowest non-trivial momentum

- Similarly, the a_1 mass should be the lower of two possibilities: a ρ-η' state, or a two quark state excited with the lowest non-trivial momentum.

Note that the most interesting predictions are within the meson spectrum. The baryon spectrum is simple, as the nucleon and Delta are expected be degenerate at all quark masses. In the constituent regime they are both composed of three dressed quarks, while in the chiral regime they are related through symmetry restoration.

5. Vortex-Removed Hadron Spectrum

Results for the four light quark masses on the vortex removed ensemble are plotted in Fig. 5. We first turn our attention to the meson spectrum, starting with the lightest mass ($m_q = 13$ MeV). The pion on the vortex-removed ensemble is below 100 MeV, compared to a ground state mass of over 200 MeV for the untouched case. The change for the rho is much more drastic, having a mass of around 170 MeV as compared to around 1000 MeV in the untouched ensemble. Both the pion and the rho now have masses smaller than their physical values. On the vortex-removed ensemble, the majority of dynamical mass generation is gone, with only a small remnant reflected by both the pion and the rho having masses larger than twice the bare quark mass. This is consistent with our results for the vortex-removed quark propagator [26,27]. We note that while the rho is greatly reduced in mass, it is not degenerate with the pion, providing the first indication that we are within the chiral regime where physics beyond simple quark counting can contribute.

In the vortex-removed case, the a_0 effective mass is observed to behave differently from the untouched case. This time the correlator remains positive. However, evidence of a negative-metric contribution to the source-sink-symmetric correlator is manifest as the effective mass rises from below at the earliest Euclidean times. Thus, short-distance quenched artifacts survive the process of vortex removal as anticipated, noting that the static quark potential indicates that Coulombic interactions associated with one-gluon exchange also remain present [24]. Thereafter, the effective mass stabilises to an approximate plateau for time slices 15 to 20. Here the mass in the a_0 channel is higher than the ρ meson. However, this meta-stable plateau eventually gives way to a low-lying effective-mass plateau associated with a relatively small coupling to an eigenstate degenerate with the pion. This ultimate degeneracy provides evidence of the effective restoration of the $U(1)_A$ symmetry. The excited state seen at earlier Euclidean times has a mass consistent with the two-particle π-η' state which can contribute in the a_0 channel.

The a_1 behaves similarly to the a_0, showing an excited state consistent with a ρ-η' state, then a ground state consistent with the ρ meson. The a_1, however, has much larger error bars and is not shown for $t > 22$.

Turning now to the remaining three light quark masses ($m_q = 25,\ 38,\ 50$ MeV) we see similar trends continue for the π and ρ mesons; both have lower masses than in the untouched case, increasing with increasing bare quark mass. The pion continues to be lighter than the rho, although the gap is reduced at higher values of m_q. As the bare quark mass is increased, so is the explicit chiral symmetry breaking, and so the results move towards to the predictions for the constituent quark regime. While the non-degeneracy of the π and ρ mesons at these masses reveals that chiral physics remains manifest, by $m_q = 50$ MeV, the π and ρ mesons have become almost degenerate, signaling the start of the transition to the constituent quark regime.

At $m_q = 25$ MeV, both the a_0 and a_1 are too noisy to extract a clean signal, while
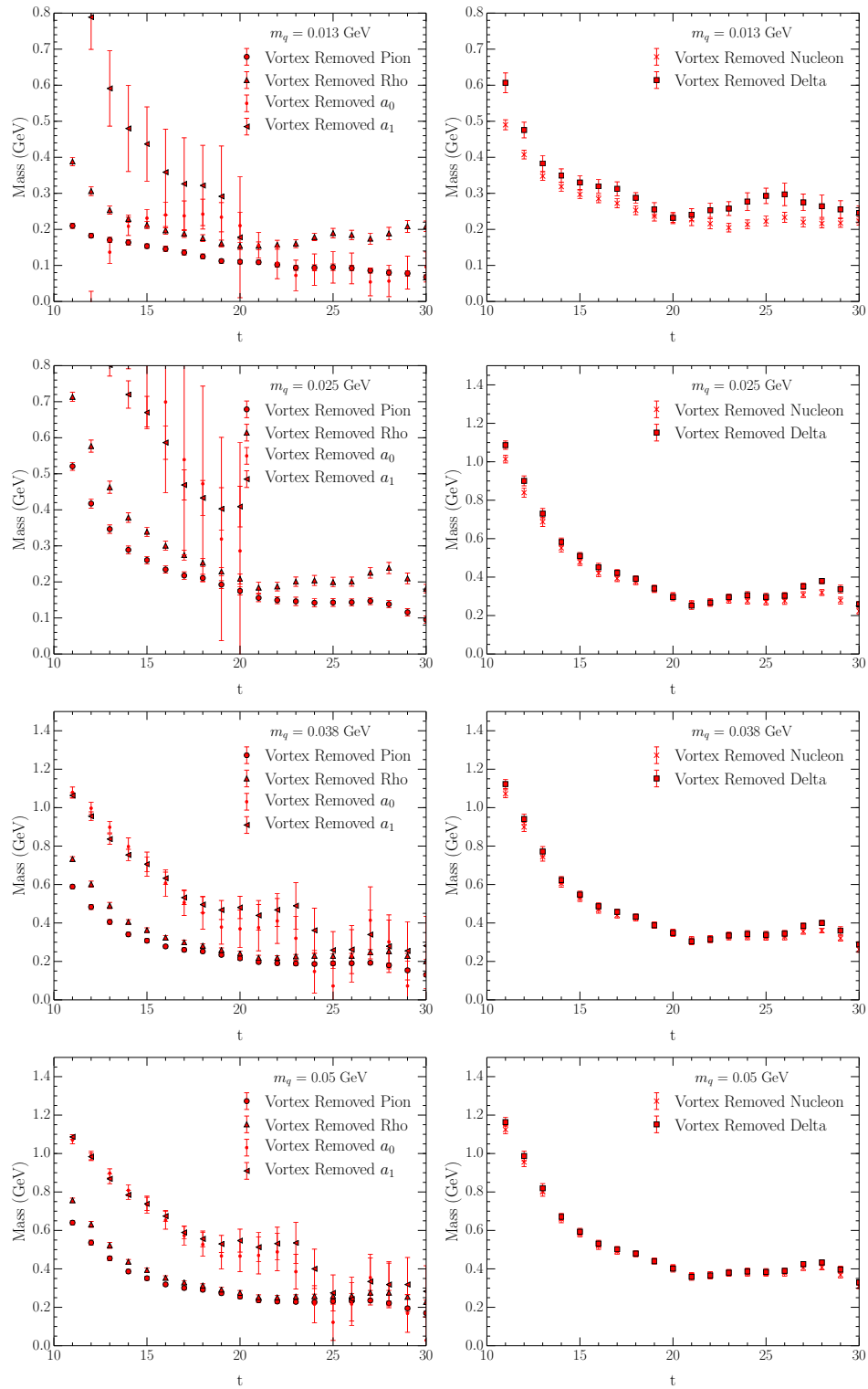
Figure 5. The effective masses for the low-lying mesons (left) and baryons (right) on the vortex-removed ensemble. Results are shown for light bare quark masses, with values of $m_q = 13, 25, 38, 50$ MeV from top to bottom respectively. Note the smaller scale on the vertical axis in plots on the top row, and for the meson plot in the second row.

at the other two quark masses ($m_q = 38$, 50 MeV), a similar result is seen to that at the lightest mass. There is an excited state with a mass higher than the other two mesons, followed by a ground state plateau similar to the π and ρ mesons. The degeneracy of the a_0 with the pion, and the a_1 with the ρ, is a signal of the restoration of both the $U(1)_A$ and $SU(2)_L \times SU(2)_R$ symmetries. This, combined with the non-degeneracy of the π and ρ mesons, suggests that at $m_q = 50$ MeV, explicit chiral symmetry breaking is still small enough that the predictions of chiral symmetry restoration hold.

In the baryon spectrum, at the lightest mass the nucleon and Delta both have masses around 220 - 260 MeV, dramatically lower than in the untouched cases. Notably, they are also approximately degenerate. At the three remaining light masses the nucleon and Δ effective masses show remarkably similar behaviour, and are degenerate within error bars. This is consistent with the restoration of the $SU(2)_L \times SU(2)_R$ symmetry and our predictions for the chiral regime. Similar to the case for the π and ρ mesons, the baryons show a loss of almost all dynamical mass generation, with a much lower plateau reached than in the untouched case (but larger than three times the bare quark mass). Unlike in the meson channel, as the nucleon and Δ baryons are predicted to be degenerate in both the chiral and constituent regime, there is no signal of a transition as the bare quark mass is increased.

We also note that all four of the light hadrons (π, ρ, N, Δ) show a slow approach to the mass plateau, indicating a dense tower of excited states. This echoes results seen using a Wilson fermion action in Ref. [25]. Also of note is the stability of the ground state seen in both the mesons and baryons, a reflection of the near-empty gauge field background in the vortex removed case.

Results at the four heavy quark masses ($m_q = 101$, 126, 151, 177 MeV) are presented in Fig. 6. At these masses, the π and ρ mesons have become approximately degenerate, indicating that explicit chiral symmetry breaking is now large enough that both hadrons behave as though composed of two weakly-interacting constituent quarks. The nucleon and Delta baryons remain degenerate in the constituent regime.

At these higher quark masses, the a_0 and a_1 no longer reach a plateau degenerate with the π and ρ mesons respectively, as chiral symmetry is no longer approximately restored. Instead, in the constituent regime the a_0 and a_1 are degenerate with each other, and the lightest state in these channels is now a two quark state excited with the lowest non-trivial momentum.

Qualitatively, the results seen suggest agreement with the predictions of chiral symmetry restoration below a bare quark mass of 50 MeV, and above that, agreement with the predictions of a constituent-quark like model. We now turn to quantitative measures of these predictions. Fits of the ground state masses of the pion, rho, nucleon, and Delta on the vortex-removed ensemble across all eight quark masses are summarised in Table 3.

We first consider the validity of the constituent-quark like model of the hadron spectrum in the heavy quark mass region. In Fig. 7, we have plotted the hadron masses divided by the number of valence quarks as a function of the bare quark mass.
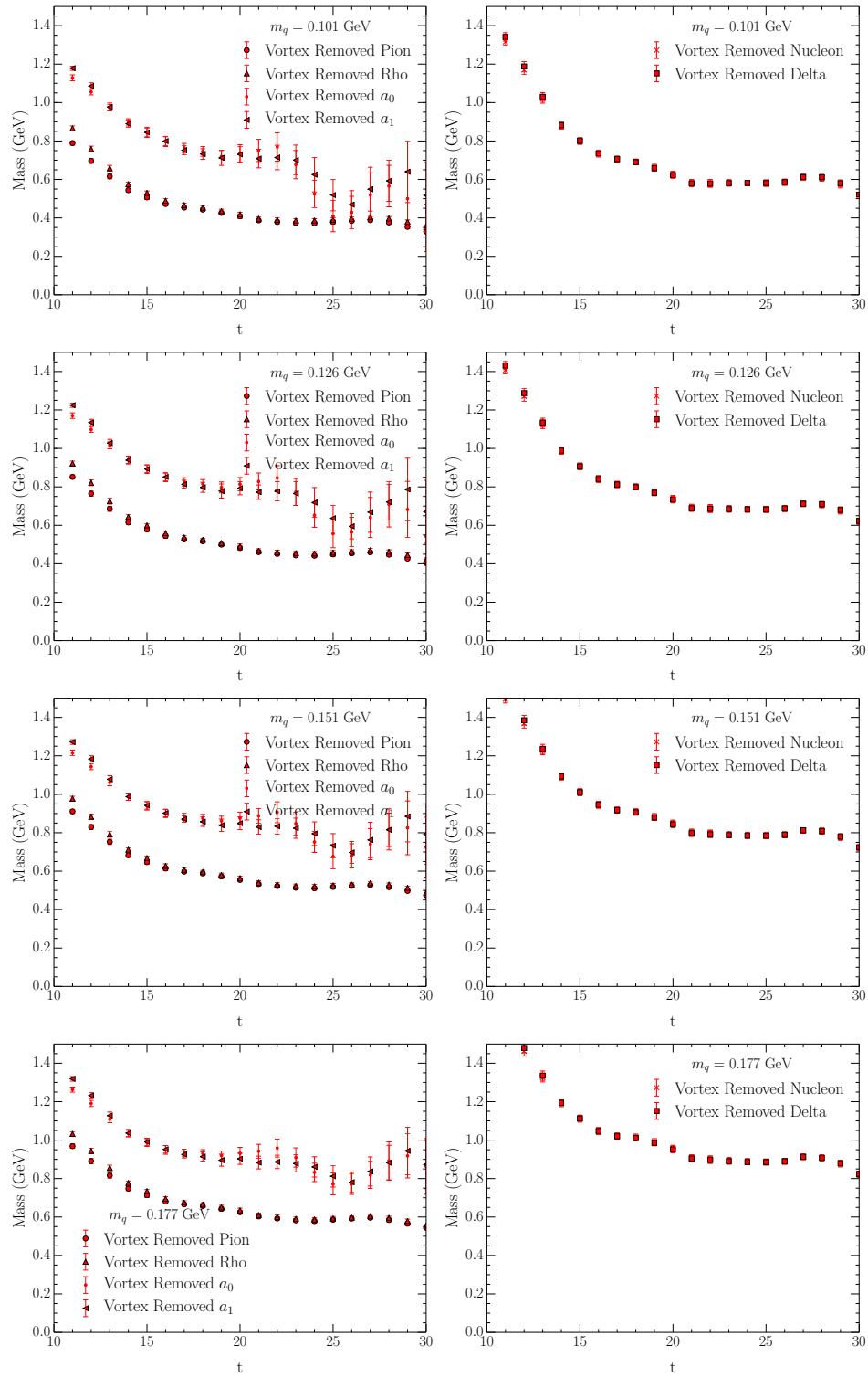
Figure 6. The effective masses for the low-lying mesons (left) and baryons (right) on the vortex-removed ensemble. Results are shown for heavy bare quark masses, with values of $m_q = 101$, 126, 151, 177 MeV from top to bottom respectively.

Table 3. Fitted masses of the pion, rho, nucleon, and Delta on the vortex-removed ensemble as a function of the bare quark mass, m_q.

m_q (MeV)	m_π (MeV)	m_ρ (MeV)	m_N (MeV)	m_Δ (MeV)
13	85(3)	171(7)	219(6)	260(10)
25	132(4)	203(5)	272(7)	295(7)
38	173(4)	228(5)	316(7)	334(6)
50	213(4)	257(4)	365(5)	378(5)
100	366(3)	386(3)	572(5)	575(5)
126	439(3)	453(3)	676(4)	676(4)
151	510(3)	521(3)	780(4)	779(4)
177	578(3)	588(3)	881(4)	880(5)
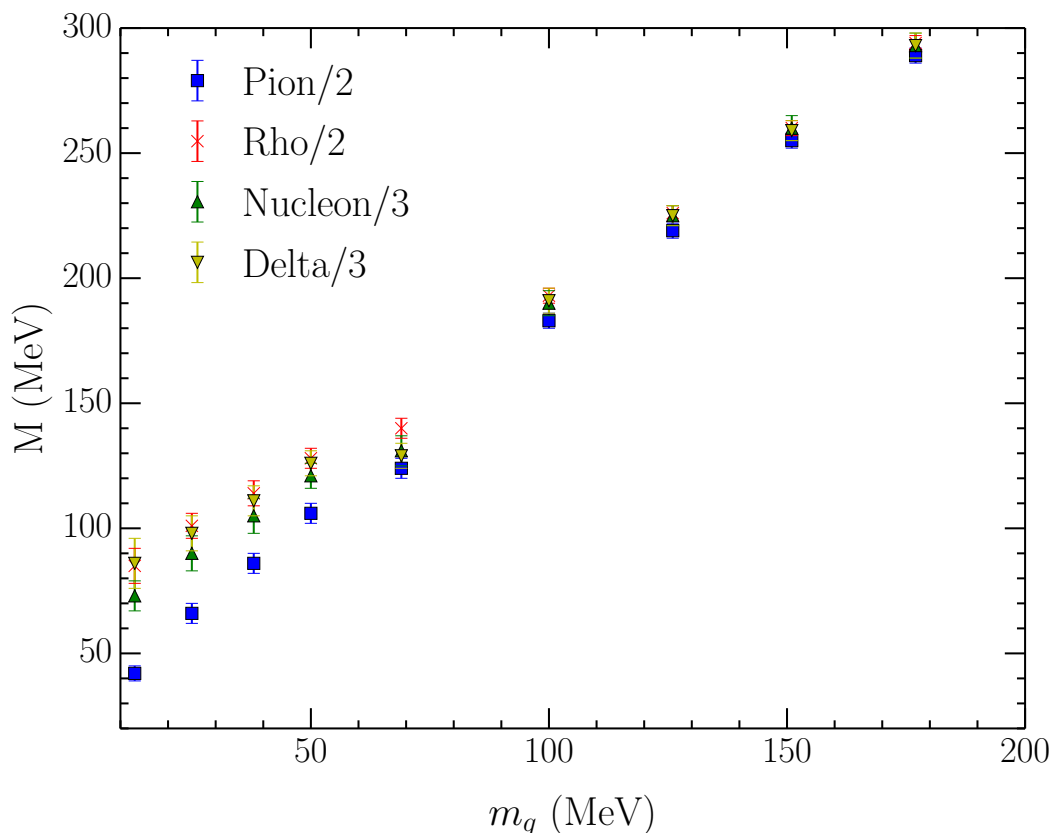
Figure 7. The implied constituent quark mass from each of the hadrons considered as a function of the input bare quark mass.

At masses of $m_q = 101\,\text{MeV}$ and beyond, the constituent-quark like model is highly successful in describing the behaviour of the spectrum, with all hadrons approximately degenerate after division by the number of constituent quarks. At these quark masses, all four hadrons can be accurately modelled as weakly interacting dressed quarks. Below this value, while the rho, nucleon and Delta are still in agreement within statistical uncertainties, the pion is lighter. It is in this region, therefore, that we expect the
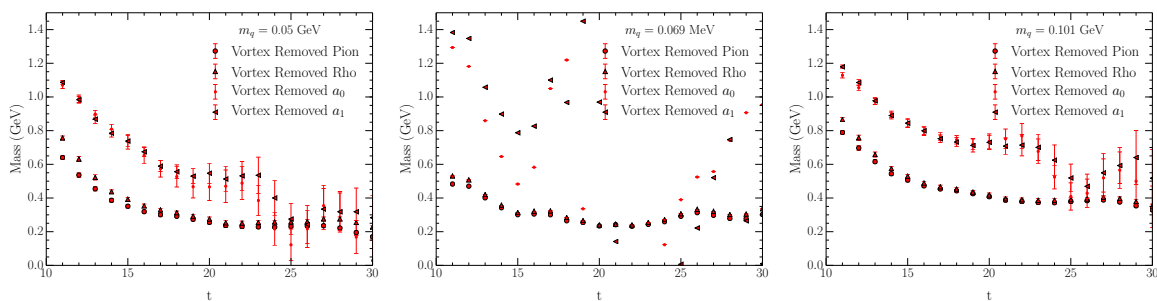
Figure 8. The effective masses for the a_0 and a_1 mesons at bare quark masses of 50 (left), 69 (middle), and 100 (right) MeV. The sequence of plots shows the transition from the chiral regime (left), passing through an intermediate region (middle) before reaching the constituent regime (right).

predictions of the chirally restored theory to be valid.

In Figure 7 we also include points at $m_q = 69$ MeV, in the transition region between the chiral and constituent regimes. While we are able to obtain fits for the π, ρ, N, and Δ at this intermediate quark mass in the intermediate region, in Fig. 8 we see the a_0 and a_1 correlators fluctuate wildly, perhaps indicating that the nature of these two states is ill-defined in the transition region. Indeed, due to their distinct properties in each regime it is on the a_0 and a_1 mesons that we now focus.

6. Mass ratios for the a_0 and a_1 mesons

The nature of the a_0 and a_1 mesons strongly differs between the chiral and constituent regimes. While the large error bars on the a_0 and a_1 correlators make fitting a mass value difficult, we can test the predictions for these two mesons in both regimes using ratios of masses.

For the a_0, the U(1)$_A$ symmetry predicts degeneracy with the pion in the chiral regime. As the two-particle π-η' state has the same quantum numbers as the a_0, it can appear as an excited state, or possibly as the ground state in the constituent quark regime. In the weakly-interacting constituent-quark regime, we can model the lowest energy two-quark energy eigenstate having overlap with the $l = 1$ orbital angular momentum required to form the desired quantum numbers by

$$|\psi_{q\bar{q}}\rangle = \frac{1}{2}\Big(|0, \vec{p}\rangle - |0, -\vec{p}\rangle + |\vec{p}, 0\rangle - |-\vec{p}, 0\rangle\Big), \tag{3}$$

where $|\vec{p}| = 2\pi/L$ is the minimum non-trivial momentum available to a free constituent quark on a lattice with spatial length L. The energy E associated with this quark model state is given by

$$E^2 = \left(M^2 + \left(\frac{2\pi}{L}\right)^2\right), \tag{4}$$

where $M = 2\,m_{q\,\text{cons.}}$ is twice the constituent quark mass.

Table 4. The two mass ratios considered, together with their expected values in the chirally-restored and weakly-interacting constituent-quark regimes.

Ratio	Definition	Chiral regime value	Constituent quark regime value
R_0	$\frac{m_{a_0}}{2\,m_\pi}$	$\frac{1}{2}$	Smaller of 1 (π-η' state) or $E/2m_\pi$ (2 quark state)
R_1	$\frac{m_{a_1}}{2\,m_\rho}$	$\frac{1}{2}$	Smaller of 1 (ρ-η' state) or $E/2m_\rho$ (2 quark state)

In consideration of all of the above, we define the a_0 mass ratio as

$$R_0 = \frac{m_{a_0}}{2\,m_\pi}. \tag{5}$$

In the chiral regime, we expect this to have a value of $1/2$, as $m_{a_0} = m_\pi$. If the a_0 is described by a π-η' state, it will have a value of approximately 1. For our two quark state in the constituent-quark regime, this ratio will be given by $E/2m_\pi$.

For the a_1, in the chiral regime the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry predicts degeneracy with the ρ meson. The quantum numbers of the a_1 can be produced by a ρ-η' state, and so we expect this to appear as an excited state in the chiral regime, and as either an excited state or the ground state in the constituent regime. Again, we can construct a model two quark state that describes the a_1 in the constituent regime. This model state has the same expected energy E given by Eq. (4), such that the a_1 is degenerate with the a_0 at heavy quark masses.

We therefore define the a_1 mass ratio as

$$R_1 = \frac{m_{a_1}}{2\,m_\rho}. \tag{6}$$

Again, in the chiral regime we expect this to have a value of $\frac{1}{2}$, as $m_{a_1} = m_\rho$. In the constituent quark regime, we expect a value of $E/2m_\rho$ for a two-quark state. We note that while a ρ-η' state can create the quantum numbers of the a_1, in the constituent regime the pion and the rho become approximately degenerate, and so this state will have mass $m_\rho + m_\pi = 2\,m_\rho$ and once again produce a value of $R_1 \simeq 1$. In the chiral regime, the pion is lighter than the rho, and so R_1 for this state will be less than 1, varying from 0.75 at $m_q = 13$ MeV to 0.91 at $m_q = 50$ MeV. This still allows a clean separation from the prediction of restored chiral symmetry, where $R_1 = 1/2$.

We have summarised these ratios and their expected values in Table 4. Based on the results in Fig. 7, we have defined the constituent quark mass $m_{q\,\mathrm{cons.}}$ to be half the fitted mass of the rho meson. These values are listed in Table 5, together with the corresponding energy E of a two quark state and the values of the mass ratios $E/2m_\pi$, $E/2m_\rho$, and $(m_\pi + m_\rho)/2m_\rho$.. Interestingly, a comparison of the different mass ratios reveals that at all four heavy quark masses, the two quark state is lighter than the corresponding π-η' or ρ-η' multi-particle state (while at the four light quark masses the reverse is true). Hence, we predict that in the constituent regime the value of R_0 and R_1 should approach $E/2m_\pi$ or $E/2m_\rho$ respectively.

Table 5. For each bare quark mass m_q, the constituent quark masses $m_{q\,\text{cons.}}$ inferred from the fitted ground-state rho meson masses and the corresponding energy E of a two quark state with the smallest non-trivial lattice momentum are indicated. Also shown are the values of the ratios $E/2m_\pi$, $E/2m_\rho$, and $(m_\pi+m_\rho)/2m_\rho$.

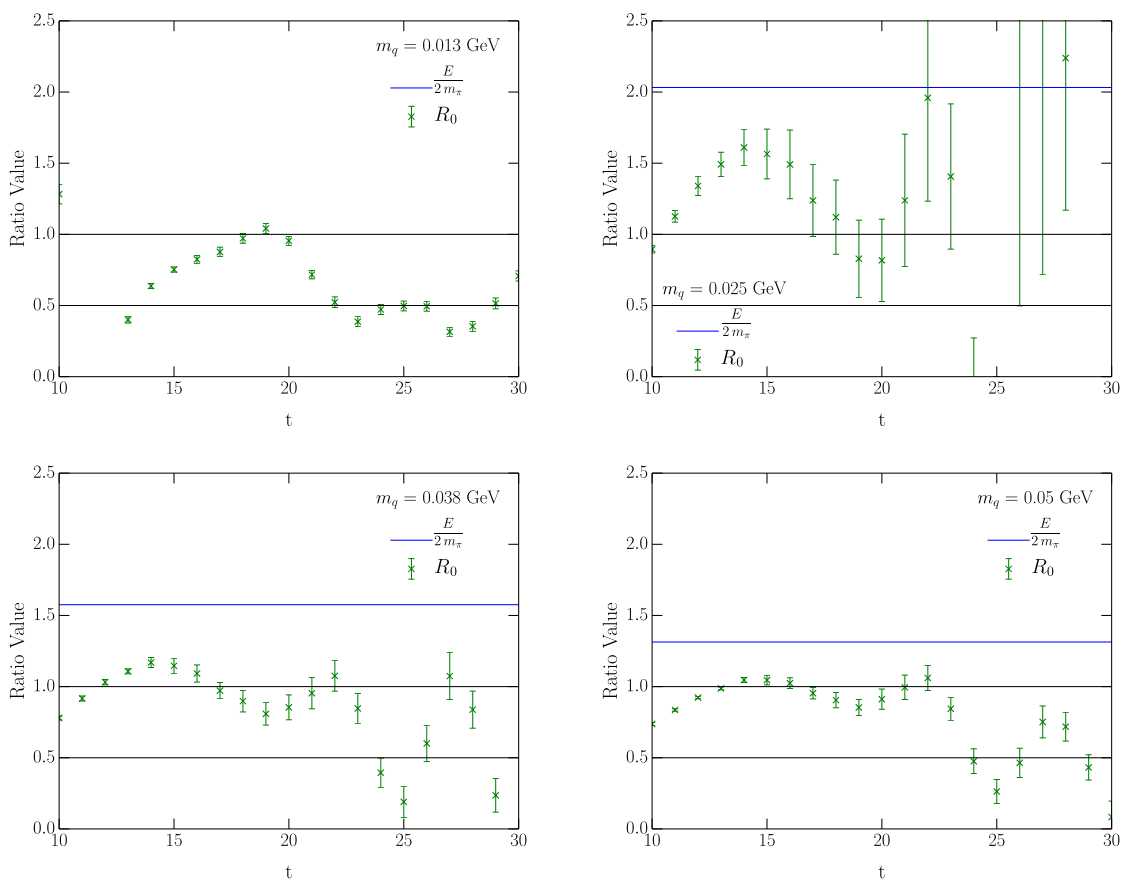| m_q (MeV)	$m_{q\,\text{cons.}}$ (MeV)	E (MeV)	$E/2m_\pi$	$E/2m_\rho$	$\frac{m_\pi+m_\rho}{2m_\rho}$
13	85	525	3.09	1.53	0.75
25	101	536	2.03	1.32	0.83
38	114	546	1.58	1.20	0.88
50	128	559	1.31	1.09	0.91
100	193	628	0.86	0.81	0.97
126	226	672	0.76	0.74	0.98
151	260	719	0.71	0.69	0.99
177	294	769	0.67	0.65	0.99

Figure 9. The ratio R_0 for the a_0 meson on the vortex-removed ensemble in the chiral regime, at light bare quark masses with values of $m_q = 13$, 25, 38, 50 MeV increasing from left to right then top to bottom. Horizontal lines are drawn at $\frac{1}{2}$ (U(1)$_A$ symmetry), 1 (π-η' state), and $\frac{E}{2m_\pi}$ (two-quark state) to guide the eye. Note that at the lightest mass the value of $E/2m_\pi$ is above the range of the vertical axis.
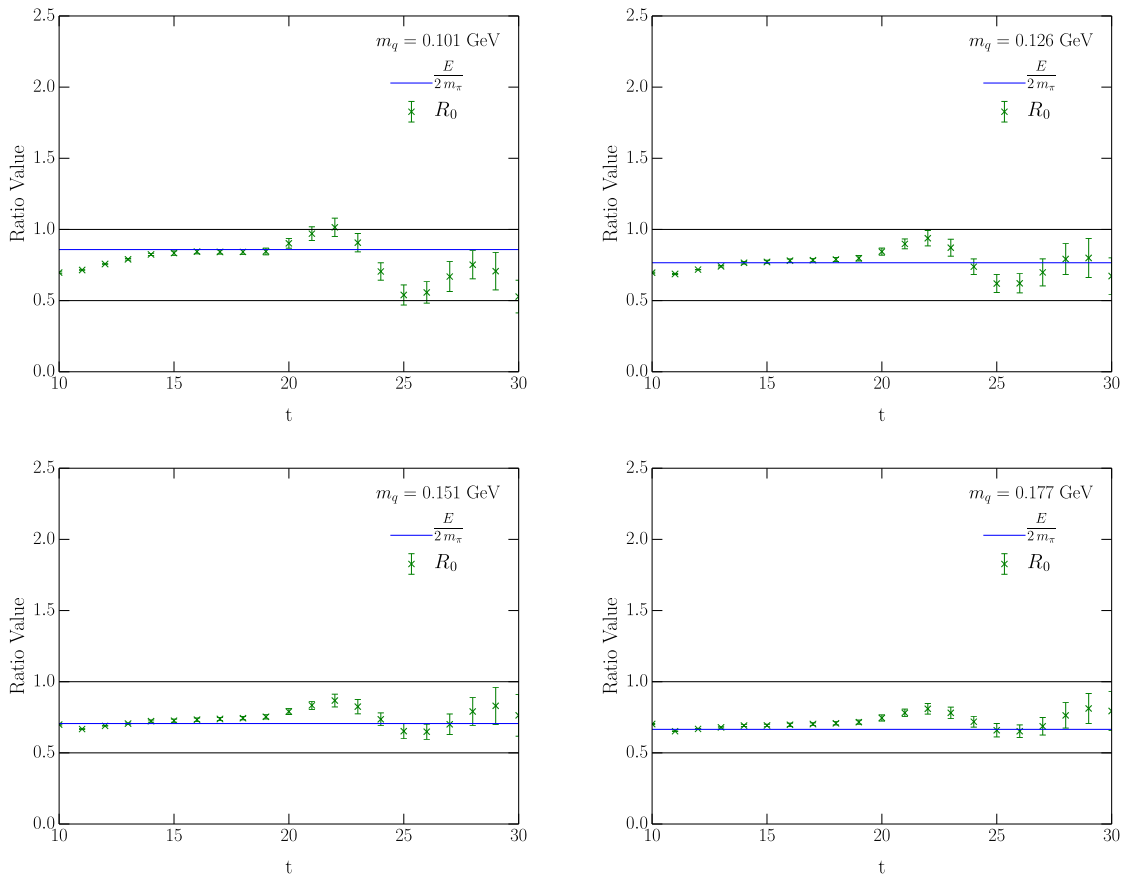
Figure 10. The ratio R_0 for the a_0 meson on the vortex-removed ensemble in the the constituent regime, at heavy bare quark masses with $m_q = 101, 126, 151, 177$ MeV increasing from left to right then top to bottom. Horizontal lines are drawn at $\frac{1}{2}$ (U$(1)_A$ symmetry), 1 (π-η' state), and $\frac{E}{2m_\pi}$ (two-quark state) to guide the eye.

In Fig. 9 we present the ratio R_0 for the a_0 meson at the four light bare quark masses. At the lightest mass ($m_q = 13$ MeV), R_0 touches 1, before dropping down to a stable value at $\frac{1}{2}$. The plateau at $\frac{1}{2}$ shows a restoration of the U$(1)_A$ symmetry; degeneracy of the a_0 and pion. There is also evidence of a π-η' state in the same channel, reflected by the value around 1 at earlier time slices.

At $m_q = 25$ MeV the signal for R_0 is poor, with large fluctuations in the central value. By contrast, at the next two masses ($m_q = 38, 50$ MeV), the ratio hovers around 1 at early time slices, providing evidence of the formation of a multi-particle π-η' state. At later time slices, however, while there is some evidence of the value decreasing, the signal becomes too noisy to see a clear plateau at $\frac{1}{2}$. It may be that due to the additional symmetry breaking from the axial anomaly, the restoration of the U$(1)_A$ symmetry is particularly sensitive to explicit symmetry breaking from the bare quark mass.

The plots of R_0 for the a_0 meson at the four heavy quark masses are shown in Fig. 10. We have seen previously that at these masses the hadrons behave like weakly-interacting constituent quarks, and this is quantified here. Up to Euclidean times of
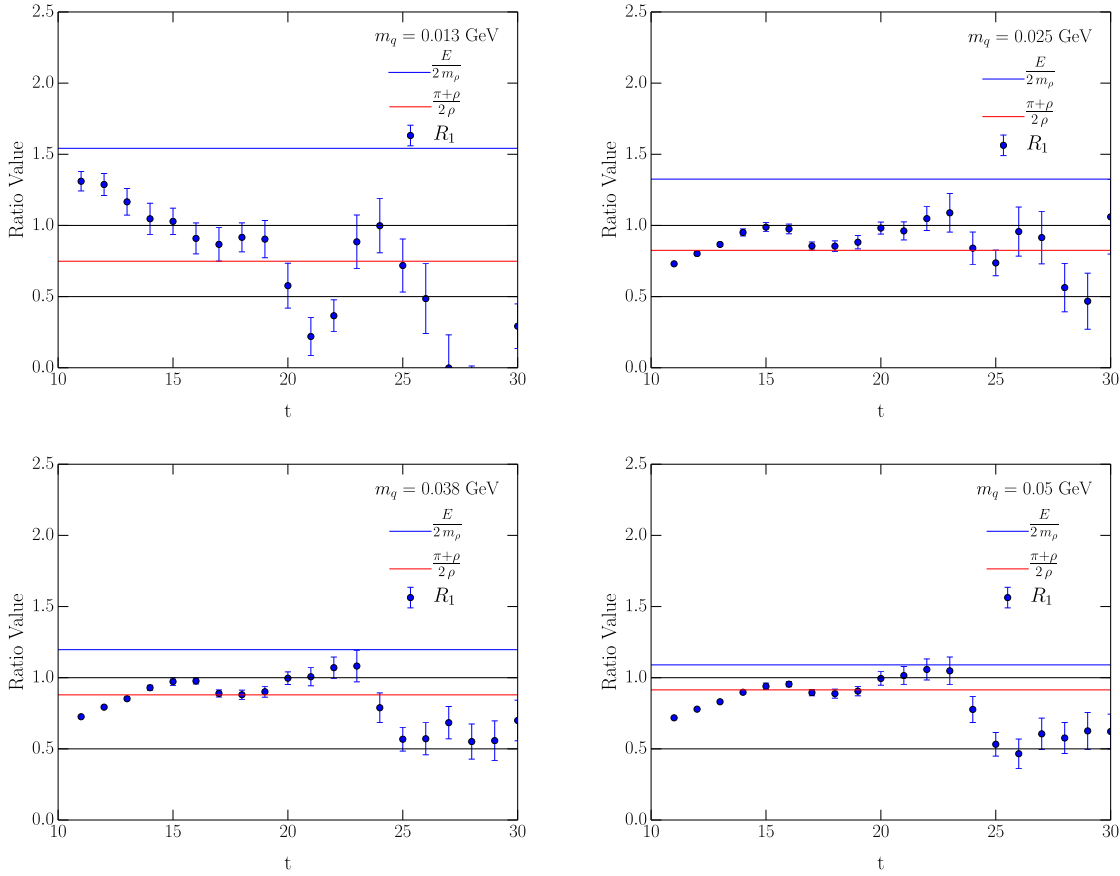
Figure 11. The ratio R_1 for the a_1 meson on the vortex-removed ensemble in the chiral regime, at light bare quark masses with $m_q = 13$, 25, 38, 50 MeV increasing from left to right then top to bottom. Horizontal lines are drawn at $\frac{1}{2}$ ($\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry), $\frac{(m_\pi + m_\rho)}{2\,m_\rho}$, 1, and $\frac{E}{2m_\rho}$ (two-quark state) to guide the eye.

$t \simeq 20$, R_0 lies almost exactly on the line drawn at $E/2m_\pi$, indicating the a_0 is best described by a two quark state excited with the minimum lattice quantum of momenta. The ratio $E/2m_\pi$ is less than one for all four heavy masses, implying that the two quark state is lighter than the π-η' state. Hence, the observations are consistent with our predictions for R_0 in the constituent regime. At later times, the signal for the ratio oscillates, with no clear evidence of any other states in this region.

We show the ratio R_1 for the a_1 meson at the light quark masses in Fig. 11. At all four masses, R_1 hovers around the line $\frac{m_\pi + m_\rho}{2\,m_\rho}$ at early Euclidean times; this corresponds to the a_1 correlator being dominated by a ρ-η' excited state. At the lightest two masses, the signal is too poor to provide evidence of any other state. However, at $m_q = 38$ MeV and $m_q = 50$ MeV, after time slice 25 another stable plateau is seen at $R_1 = \frac{1}{2}$. This indicates the degeneracy of the a_1 with the rho meson, evidence of the restoration of the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry. This reveals that at $m_q = 50$ MeV chiral symmetry breaking from the quark mass is still sufficiently small that the symmetry holds. This concurs with the results for the π and ρ mesons, which are not yet degenerate with
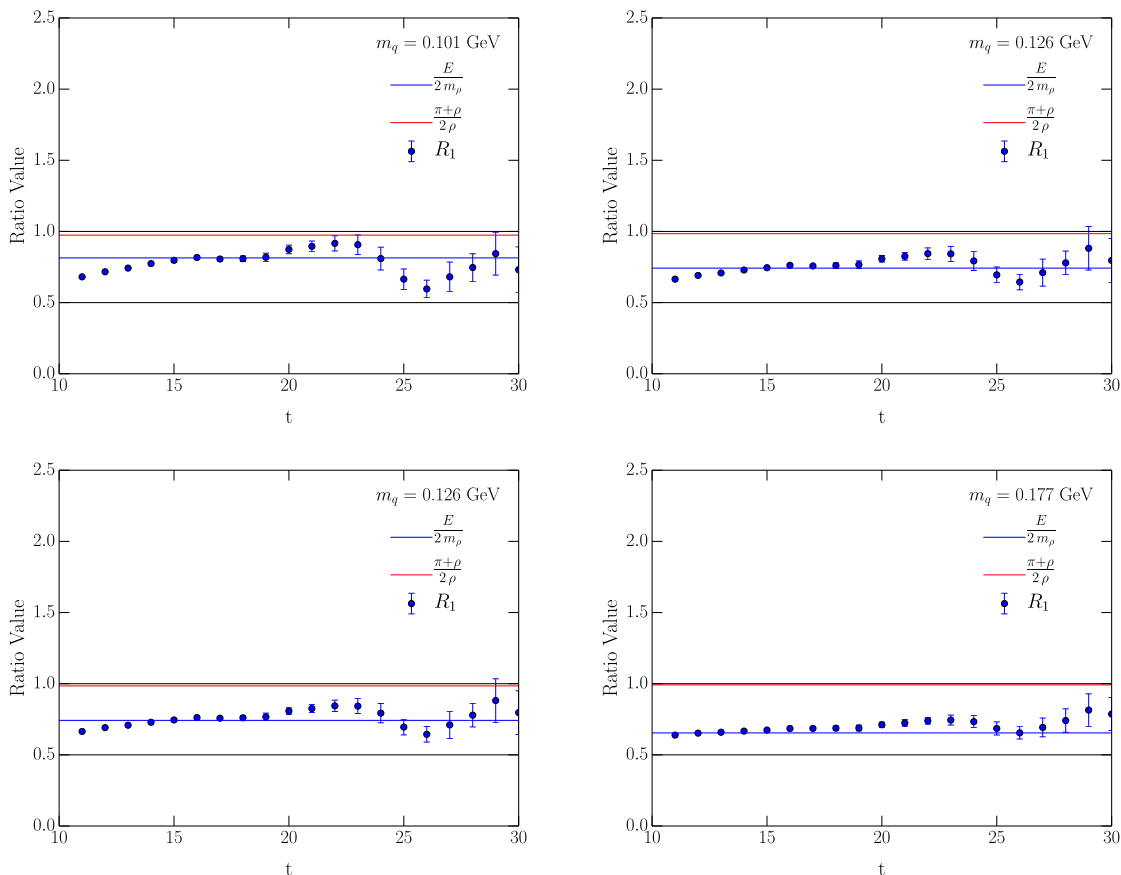
Figure 12. The ratio R_1 for the a_1 meson on the vortex-removed ensemble in the constituent regime, at heavy bare quark masses with $m_q = 101$, 126, 151, 177 MeV increasing from left to right then top to bottom. Horizontal lines are drawn at $\frac{1}{2}$ $(\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry), $\frac{(m_\pi + m_\rho)}{2\,m_\rho}$, 1, and $\frac{E}{2m_\rho}$ (two-quark state) to guide the eye.

$\frac{m_\pi + m_\rho}{2\,m_\rho} \simeq 0.91$ indicating a small but significant splitting at this quark mass.

The ratio R_1 for the a_1 meson at the four heavy quark masses is shown in Fig. 12. The π and ρ are approximately degenerate here, such that the value of $E/2m_\rho$ is less than the value of $\frac{m_\pi + m_\rho}{2\,m_\rho}$, *i.e.* the two quark state is lighter than the multi-particle ρ-η' state at all four heavy masses. Indeed, we see that at high quark masses the ratio R_1 lies along the line at $E/2m_\rho$, showing almost perfect agreement up to $t \simeq 20$, with some small fluctuations at later times as the signal degrades. The two quark state behaviour mirrors that of R_0, indicating an onset of degeneracy between the a_0 and a_1 at high quark masses. While this is a feature seen also in the untouched ensemble, remarkably the masses of both are given within error bars by the value E predicted by our simple model in Eq. (4). At the four heavy quark masses, the weakly-interacting constituent-quark like model is remarkably successful; all six hadrons considered are in agreement with the predictions at all of these masses.

7. Summary

We have presented a novel examination of the influence of centre vortices on the low-lying hadron spectrum over a wide range of bare quark masses using the chirally sensitive overlap operator. This has allowed us to use the behaviour of the low-lying hadron spectrum as a probe of the role of centre vortices in dynamical chiral symmetry breaking.

After a small amount of cooling, the vortex-only backgrounds are capable of recreating all the salient features of the low-lying hadron spectrum. While the ground state masses are slightly lower due to the use of smoothing [38], the qualitative features of the spectrum are intact, in agreement with results seen for the quark propagator [26,27]. In particular, the pion remains much lighter than the other mesons. Its behaviour as a pseudo-Goldstone boson is a clear signal of the presence of dynamical chiral symmetry breaking on the vortex-only ensemble. Furthermore, there is a significant splitting in the masses of chiral partners.

On the vortex-removed ensemble, we have observed the loss of dynamical chiral symmetry breaking. At low quark masses, there is strong evidence of the restoration of $SU(2)_L \times SU(2)_R$ chiral symmetry. The nucleon and Δ baryons become degenerate, as do the a_1 and ρ mesons. The evidence for the restoration of the $U(1)_A$ symmetry at our lowest quark mass is clear; at this mass the a_0 shows a degeneracy with the pion. We have also observed that the $U(1)_A$ symmetry is more sensitive to explicit chiral symmetry breaking from the bare quark mass.

At high quark masses, the vortex-removed hadron spectrum is consistent with the behaviour of weakly-interacting dressed constituent quarks in an otherwise near-trivial background, as seen in previous studies using a Wilson action [25]. In accord with the quark-propagator results [26,27], there is some residual dynamical mass generation on the vortex-removed ensemble. The π and the ρ are approximately degenerate, as are the nucleon and Delta, such that the implied constituent quark mass from all four hadrons are in agreement. A constituent quark mass higher than the bare quark mass indicates a nontrivial dressing of the quarks from the gauge field fluctuations surviving vortex removal. Using the constituent quark mass extracted, the a_0 and a_1 mesons can be described successfully as a two quark state excited with the minimal non-trivial lattice momentum.

Here, for the first time, using a vortex-removed gauge field ensemble we have been able to produce the hadronic degeneracies associated with the restoration of chiral symmetry. The use of the overlap fermion action, which respects chiral symmetry, is vital to revealing this property. Remarkably, we are able to reproduce all the salient features of QCD in the low-lying hadron spectrum from smoothed vortex-only backgrounds, while upon vortex removal we see a loss of dynamical chiral symmetry breaking. These results provide a further contribution to the already significant body of evidence [28] that centre vortices are the fundamental mechanism underlying dynamical chiral symmetry breaking in SU(3) gauge theory.

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