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Modelling exchange rate volatility

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Two types of statistical models are empirically applied to test the pattern of volatility in the exchange rate markets. One considers the autoregressive models and tests the random walk hypothesis. The other considers the conditional variance process and tests the hypothesis of chaotic dynamics. Empirical results mostly support the random walk hypothesis and also the existence of Lorenz-type chaos.

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1. Introduction

The foreign exchange market is by far the largest financial market in the world. Daily turnover in these markets has exceeded \$1000 billion in 1995, up from only \$200 billion in 1986. This impressive increase is expected to continue in the future, as financial markets continue to integrate all over the world and the world trade continues to rise with cross-border investments continuing to mount. The foreign exchange market is the only major 24-hour financial market, which is a continuous one with no opening or closing hours. It opens in Tokyo, then moves to Hong Kong, on to London and then to the United States. The role of central banks and hence the government is not a passive one in the foreign exchange markets. They continue to intervene to maintain an “orderly market”, through trading in the exchange market. This trading in the foreign exchange markets generally involves the US dollar. This is because of the depth and importance of the dollar currency market and the associated lower transactions costs. A British importer for instance wishing to buy Japanese yen to pay a Japanese exporter would sell pounds for dollars and then buy yen with the proceeds. Increasingly however the German mark is used as a reference currency for cross-transactions in Europe.

Exchange rate volatility has acquired a special importance in this global framework of international trade due to two main reasons. One is that the national governments have increasingly felt the impact of this volatility on their own monetary policies and this has been more so for those countries where export growth provides a large stimulus to their domestic economy's growth. Secondly, the investors today are increasingly participating in international portfolios and the asset market approach has become the dominant model for such investors. Our object here is two-fold. One is to model the adjustment process in the exchange rate market

through a representative government's policy aspiration, where an orderly market is considered desirable. This type of approach has been followed by Hall and Henry (1988) and more recently by Fisher (1992) in connection with the NIESR (National Institute of Economic and Social Research) econometric model for the UK over the period 1973 to 1984 and later. A second object is to model the stochastic process underlying the exchange rate volatility through a logistic process to test if there is any chaotic instability in such markets. This type of modelling the nonlinear stochastic process and its empirical testing provide some answers to the question whether the exchange rate process is nonstationary over time. Recently, the empirical studies by Meese and Rogoff (1983) and others have shown the existence of such nonstationarity. They show that the existing structural models of exchange rates, e.g., the sluggish price adjustment models of Dornbusch (1976) and the portfolio balance models of Branson *et al.* (1979) failed to significantly outperform a random walk model in predicting the behavior of exchange rates out of sample.

2. Dynamic models of volatility

Consider a simple adjustment cost function $C(E_t)$ representing government aspirations

$$C(E_t) = \sum_{t=0}^T (1/2)[a_1(E_t - E_t^0)^2 + a_2(E_t - E_{t-1})^2] \quad (1)$$

where E_t is the real exchange rate and E_t^0 is the rate which the market would set in the absence of government intervention. We further suppose an inverse intervention function

$$E_t = g(I_t)$$

representing government intervention either through direct intervention or through official financing flows on the foreign exchange market. If we minimise this function (1) with respect to the control variable I_t we obtain the first-order condition

$$a_1 g (g(I_t) - E_t^0) + a_2 g (g(I_t) - g(I_{t-1})) - a_2 g (g(I_{t+1}) - g(I_t)) = 0$$

$$t=0,1,\dots,T-1$$

where g denotes the partial derivative of g with respect to I_t . On dividing through by g this yields the optimising condition

$$E_t = (a_1 + 2a_2)^{-1} \{a_1 E_t^0 + a_2(E_{t-1} + E_{t+1})\} . \quad (2)$$

To estimate this model one needs to specify the determinants of E_t^0 . A common hypothesis frequently used in the finance literature is to assume

$$E_t^0 = b_0 + b_1 E_{t-1} . \quad (3)$$

This finally yields the estimable equation

$$E_t = \alpha_0 + \alpha_1 E_{t+1} + \alpha_2 E_{t-1} \quad (4)$$

where

$$\alpha_0 = (a_1 + 2a_2)^{-1} (b_0 a_1)$$

$$\alpha_1 = (a_1 + 2a_2)^{-1} b_1 (a_1 + a_2)$$

$$\alpha_2 = (a_1 + 2a_2)^{-1} a_2 .$$

So long as the coefficients $\alpha_0, \alpha_1, \alpha_2$ are constants, this equation may be transformed to a causal form, suitable for ordinary least squares (OLS) estimation as follows:

$$E_t = \alpha_0 + \alpha_1 E_{t-1} + \alpha_2 E_{t-2} \quad (5)$$

where

$$\alpha_0 = -\alpha_0 / \alpha_1, \quad \alpha_1 = 1 / \alpha_1, \quad \alpha_2 = \alpha_2 / \alpha_1 .$$

This is a backward looking view, since the current level of exchange rate is determined by the lagged exchange rates. By comparison the equation (4) provides a forward looking view of exchange rate determination. Note that by adding an error term on the right hand side of (5), this equation may be estimated by the OLS method and if the error term is a white noise process, the estimated coefficients $\alpha = (\alpha_0, \alpha_1, \alpha_2)$ would have optimum properties of unbiasedness and minimum variance.

The characteristic equation for the second order process (5) is

$$\lambda^2 - \alpha_1 \lambda + \alpha_2 = 0 . \quad (6)$$

If the natural exchange rate E_t^0 is assumed to be a constant in the short run (i.e., $b_1 = 0$), then the exchange rate equation (5) becomes simpler

$$E_t = -\alpha_0/\alpha_1 + (1/\alpha_1) E_{t-1} - E_{t-2} . \quad (7)$$

The characteristic equation now takes the form

$$\lambda^2 - (1/\alpha_1)\lambda + 1 = 0 . \quad (8)$$

Since the constant term here is unity, the two characteristic roots here are reciprocal. Hence if one is positive but less than one, the other must be positive and greater than one. The latter root may contribute an explosive component to the time series of exchange rates in the sense that as $T \rightarrow \infty$, E_T may tend to be unbounded. It is clear that a similar phenomenon may occur with the second order process (6) also, if the coefficient α_2 is positive and one of the two roots exceeds unity.

A second way of modelling exchange rate volatility is to postulate a model of time-varying variances of exchange rates. The random shocks to the mean exchange rates, which may cause volatility are represented here by what is known as the Arch (autoregressive conditional heteroscedastic) model and its various generalizations in the modern financial literature. The simplest version of the Arch model uses the following specification for the conditional variance

$$y_t = E_{t-1}(y_t) + e_t; \quad \sigma_t^2 = E_{t-1}(e_t^2) \quad (9)$$

$$\sigma_t^2 = w + e_{t-1}^2 + \sigma_{t-1}^2$$

where y_t is the real exchange rate and $E_{t-1}(y_t)$ is its conditional mean as of time $t-1$. The variance term σ_t^2 is conditional on information up to time point $t-1$, i.e., $E_{t-1}(e_t^2)$ is the conditional expectation of the error term e_t^2 . Note that the random shock to mean exchange rate is e_t and the shock to variance is $[E_t(e_t^2) - E_{t-1}(e_t^2)] = \eta_t$, where η_t is a random term. A more general model for variance is

$$\sigma_t^2 = (B) \sigma_t^2 + w$$

where (B) is a lag polynomial in B , the backshift operator with $(0) = 0$. Clearly this conditional variance model (9) can be estimated by the maximum likelihood method (or by least squares if η_t is approximately normally distributed), since an estimated series $\{\hat{\sigma}_t^2\}$ can be constructed from the estimated residuals $\hat{e}_t$ obtained from (1). If the sum of the estimated coefficients and in (9) equals one or more in a statistically significant sense, then the variance shocks are persistent or permanent.

An alternative version of the variance model (9) is the second order process

$$\sigma_t^2 = w + \alpha_1 \sigma_{t-1}^2 + \alpha_2 \sigma_{t-2}^2 + \text{error} \quad (10)$$

which yields the characteristic equation

$$\lambda^2 - \alpha_1 \lambda - \alpha_2 = 0$$

with two roots. The character of the two roots determines the stability or oscillatory property of temporal variance σ_t^2 .

One way to generalize the variance model is to test the presence of nonlinearity in the process. One way to introduce nonlinearity is by a second order process defined for the conditional skewness of the process as $s_t = E_{t-1}(e_t^3)$, e.g.,

$$s_t = \alpha + \beta_1 s_{t-1} + \beta_2 s_{t-2} + \text{error} . \quad (11)$$

Clearly by this method one could test if there is any asymmetry in the variance process or not. A second type of nonlinear model is to use a logistic model for variance. Denoting the estimated conditional variance $\hat{\sigma}_t^2$ by v_t the logistic model may be written as

$$dv_t/dt = v_t(a - bv_t), \quad b > 0.$$

In discrete time it can be transformed as

$$z_t = \alpha + \beta z_{t-1}, z_t = 1/v_t \quad (12)$$

where $\alpha = b$, $\beta = 1 - a$. In canonical form suitable for testing the existence of chaotic behavior the logistic model may be written as

$$v_t/v_{t-1} = (1 - v_{t-1}) + \gamma_t \quad (13)$$

where γ_t is the random error component and γ the critical parameter generating the nonlinear behavior of volatility. This canonical logistic equation defines a classic chaos model due to Lorenz (1963), that has been discussed widely in modern physics. Economic applications to financial economics have been recently discussed by Sengupta and Sfeir (1994).

The stochastic process underlying the logistic model can be characterized in two different ways. One is the specification of the conditional variance model (9) in a nonlinear form

$$v_t = \alpha_0 v_{t-1} - v_{t-1}^2 + \text{error}$$

where v_t is the conditional variance. On using the estimates of α_0 and γ this equation may be written in a logistic form

$$v_t/v_{t-1} = (1 - v_{t-1}) + a_0, a_0 = 0 - \quad (14)$$

which yields the canonical equation (13) on setting the intercept a_0 to zero on a change of scale.

On rewriting (14) in a functional form: $v_t = f(v_{t-1})$ the stability of the dynamic trajectory in chaos theory is determined by the map $f(\cdot)$ and an initial starting point v_0 . The sequence of points $v_0, f(v_0), f^2(v_0), \dots$ is called the orbit of the map where the iterates $f^n(v_0)$ are defined by $f^n(v_0) = f[f^{n-1}(v_0)]$ with $f^0(v_0) = v_0$. A periodic orbit is one which repeats itself after a finite number of steps. The following classification of the map $f(\cdot)$ is often used in chaos theory:

$$\begin{aligned} |f'(\cdot)| > 1 &: \text{linearly unstable} \\ |f'(\cdot)| < 1 &: \text{linearly stable} \\ |f'(\cdot)| < 1 &: \text{strongly stable} \\ |f'(\cdot)| = 1 &: \text{marginally stable} \\ f'(\cdot) = 0 &: \text{super stable} \end{aligned}$$

where $f'(\cdot)$ is the slope of the map at a fixed point with $|f'(\cdot)|$ denoting its absolute value. Here the critical parameter r acts as a bifurcation parameter in the sense that the qualitative behavior of v_t suddenly changes for different value of r . For instance with $0 < r < 2$ one obtains a monotonic growth in v_t converging to the steady state, but for $r > 3$ the steady state becomes unstable and a two-period cycle emerges. In his simulation studies Lorenz (1963) showed that as r increases beyond $(1 + \sqrt{6}) = 3.45$, higher and higher even-order cycles appear. These diminish in order until at about a level 3.8, when a cycle of period 3 emerges.

A second way of modelling the logistic process is to use a generalization of the linear birth and death process, where the birth and death rate parameters themselves change as a linear function of the level of population. For example Tier and Hanson (1981) have analysed environmental stochasticity resulting from the random fluctuations of the environment affecting the population as a whole. They have shown that if a Gaussian white noise is introduced into the parameters of the logistic equation, a diffusion process $x(t)$ is obtained with mean $M(x)$ and variance $V(x)$

$$M(x) = (1 - x/k), \quad k > 0$$

$$V(x) = x^2 \tilde{\epsilon}(x), \quad \tilde{\epsilon}(x) \geq 0$$

where $X(0) = x$ is the initial population size and the term $\tilde{\epsilon}(x)$ is related to the variance of noise.

This diffusion process $X(t)$ satisfies the differential equation

$$dM(t)/dt = (ak_2 + bk_1) M(t) - (a + b)[V(t) + M^2(t)]$$

where the birth rate λ_x and death rate (μ_x) parameters are defined as $\lambda_x = a(k_2 - x) x$ and

$\mu_x = bx(x - k_1)$. Clearly this equation can be rewritten in discrete time form and estimated by the

conditional means and variances of the real exchange rate. On using variance v_t as the dependent

variable and μ_t as the conditional mean the estimating equation of the logistic process model may

be specified as

$$y_t = \mu_0 + \mu_1 z_t + \text{error} \quad (15)$$

where $y_t = (v_t/\mu_t) + \mu_t$ and $z_t = \mu_t/\mu_t$.

This formulation is useful in two ways. First, if the estimated coefficient μ_1 is negative in some time period, it indicates asymmetry, i.e., a decrease in mean levels is likely to be followed by an increase in volatility and vice versa. Second, the mean process may be viewed as a function of variance, e.g.,

$$\mu_t = (1/\mu_1)[(v_t + \mu_t^2) - \mu_0\mu_t]$$

which shows that the effect of the variance term v_t is to inflate or reduce the change in mean μ_t according as μ_1 is positive or negative. Furthermore, the mean solution of the stochastic logistic model is greater or less than the solution of the deterministic logistic model according as μ_1 is positive or negative.

3. Estimates of the exchange rate models

The real exchange rate (Rx_t) on a monthly basis is here calculated as (SP^*/P) , where S is the nominal exchange rate expressed in domestic currency per unit of the foreign currency, P and P^* are the consumer price indices of the domestic and foreign currency. Thus for example, the real exchange for Japan is computed as yens per dollar multiplied by the ratio of US price index

to the price index in Japan. Similarly for France, UK and West Germany the monthly data on nominal exchange rates are obtained from *International Financial Statistics* published by IMF, while the consumer price indices are from the DRI data bank. With US as the foreign country, the following four countries: France (FR), Japan (JP), United Kingdom (UK) and West Germany (WG) are used here with three sample periods: Sample I: February 1985 to January 1988, Sample II: February 1988 to January 1991 and Sample III: February 1991 to August 1995. These sample periods are selected from the scatter diagrams of fluctuations indicating turning points of the trend of the time series.

Three types of statistical tests are conducted on the monthly time series data of real exchange rates for the four countries: France, Japan, UK and Germany. The first group of tests reported in Tables 1 and 2 is intended to detect nonstationarity in the monthly time series of real exchange rates. Autoregressive models of first, second and third orders are empirically fitted to see if there is nonstationarity in a statistically significant sense. The second group of tests reported in Tables 3-5 considers the conditional variance process and tests if there is a unit root indicating variance persistence. Assuming the conditional variance to be a measure of market volatility, the existence of unit roots implies volatility persistence or clustering and this aspect has been extensively analyzed in the context of fluctuations of stock market returns. To test if volatility measured by the conditional variance is 'excessive' or not, a Lorenz-type model of chaos is empirically estimated here. Finally, the third group of tests reported in Tables 6 and 7 is applied to detect asymmetry in the temporal variance process. For stock market volatility this aspect of asymmetry has played important roles, which has been analyzed by Sengupta and Sfeir (1994). Since the exchange rate market may be viewed in terms of the investors' return on the

world financial markets, where the internationalization of capital markets is reflected today in the large flows of capital across borders and in the cross-listing of securities, the investors and corporate managers are increasingly aware of the various currency risks associated with the exchange rate volatility. Over the years the asset market or the stock portfolio approach has become the dominant model for international investors and this approach argues that exchange rates are not determined by the flows of buying and selling during a particular period, but rather by the market's assessment of value measured in term of the outstanding amount of the two currencies. By applying the mean variance frontier analysis of portfolio theory, this approach argues that the movements in exchange rates are driven by changes in expectations due to 'news' and that relative currency demands are driven by expectations. This aspect has been empirically investigated by Sengupta and Sfeir (1996) for the exchange rate markets. On extending this line of reasoning one could test if the asymmetry in the exchange rate process is affected by the skewness parameter. The so-called phenomenon of skewness preference by the investors in a bullish market tends to disturb the normal mean variance relationship. This phenomenon says that if the average investor is optimistic about the future in the sense that tomorrow's returns are likely to be higher than today's, then this 'good news' effect tends to depress the conditional variance estimated from the past data in a backward looking sense. Thus the mean variance relationship is negatively correlated in a bullish market and vice versa in a bearish market.

The autoregressive estimates in Table 1 of the real exchange rate process show very clearly the predominance of the case where the coefficient of Rx_{t-1} equals or exceeds one. This predominance of the unit root is 100% for Japan, 67% for UK and France for all the samples taken together. A more comprehensive way to test for the existence of the random walk

hypothesis (i.e., which amounts to unit roots) is to run the regression equation in the third order autoregressive form

$$Rx_t = (\alpha_1 - 1) Rx_{t-1} + \alpha_2 Rx_{t-2} + \alpha_3 Rx_{t-3} + \text{error}$$

and then test if the estimated coefficient $(\alpha_1 - 1)$ is significantly different from zero in a statistical sense. The mostly commonly used statistic in this case is the ADF (augmented Dicky-Fuller) statistic which was then used in Table 2 and in all cases for all samples the four time series are found to be nonstationary, i.e., the unit root exists which implies a random walk. The most important implication of the random walk hypothesis is that the conditional variance process may itself be time varying.

4. Variance process of market volatility

The estimates of the second order variance process equation (10) in Table 3 show that the two characteristic roots calculated from these estimates are complex for each case of samples II and III except for Japan in the third sample. For example the two roots for UK are: $0.601 \pm 0.612i$ (sample II) and $1.444 \pm 0.771i$ (sample III) with amplitudes $R = 0.857$ and 0.669 respectively. Even for Japan, which is a special case due to its persistent trade surplus with US for a long time, a complex root of $0.631 \pm 0.149i$ is exhibited by the estimates of the second order variance process equation. Thus the variance process exhibits a persistent feature of

oscillations. Even the log-linear variance equation estimated in Table 5 shows the persistent tendency of the regression coefficient of $\log \left(\frac{2}{t-1} \right)$ to exceed unity.

To test for chaotic instability in the nonlinear variance process, Table 4 estimates the critical chaos parameter in the Lorenz model for variance. Although the multiple correlation coefficient is very low, the estimate of the critical parameter in the logistic map (13) is significantly higher than the critical threshold 3.57 in all cases except Japan. This suggests that chaotic instability cannot be ruled out in the international exchange rate market. Since the real and nominal exchange rates are highly correlated, such chaotic behavior is most likely for the nominal rate also. The case of Japan is an exception for reasons indicated before.

5. Asymmetry in exchange rate volatility

Two types of tests are performed in order to quantify the impact of skewness. One is to estimate a second order autoregressive process for the conditional skewness model (11). The other is to test the impact of changes in the mean on the variance. Table 6 reports the estimates of the second order skewness process. The characteristic roots computed from the slope coefficients are in each case less than unity in absolute value. This implies that there is no skewness persistence in the observed data. However there is some oscillatory behavior due to complex roots. For example, UK displays characteristic roots of $0.159 \pm 0.517i$ and $0.346 \pm 0.330i$ for the two sample periods II and III. Likewise for Japan in sample period III the characteristic root turns out to be $0.322 \pm 0.021i$. Clearly the oscillatory behavior due to external shocks to the market cannot be ruled out.

The mean variance hypothesis of portfolio theory implies a positive correlation between variance and the required change in mean return. In terms of the nonlinear stochastic process model estimated in Table 7, this implies a positive value for the slope coefficient β_1 . Except for Japan in the first two sample periods and for France and Germany in the first period, this is clearly supported by the estimates. This suggests a globalization of the investor behavior, where his asset diversification is increasingly on an international scale. The fact that the estimate $\hat{\alpha}_0$ of the intercept term is always positive and significant statistically at 1% level in all the cases implies a strong positive correlation of mean and variance, though this relationship is greatly influenced by the nonlinear term (μ_t/μ_t) . The cases of UK and Germany in particular provide significant evidence of this nonlinear incidence.

6. Conclusions

Several broad results emerge from the empirical analysis of four exchange markets for the three sample periods from February 1985 to August 1995. One is the nonstationarity of the real exchange rate process, implying support for the random walk hypothesis. Since the real exchange rate is highly correlated with the nominal exchange rate, this implies a random walk behavior for the nominal exchange rate also. Secondly, the market volatility measured by conditional variances follows a persistent nonlinear behavior, where the Lorenz-type chaotic behavior cannot be ruled out. Finally, the skewness process shows in some cases an oscillatory pattern of behavior, along with an asymmetry where the mean variance correlation is not always positive. This suggests some scope for policy intervention through international coordination.

Table 1. Real exchange rate processes
(Dependent variable: Rx_t)

Country and Sample		Intercept	Rx_{t-1}	Rx_{t-2}	$\bar{R}^2$	n	D
FR	I	0.504*	0.906	-	0.96	35	yes
		0.412	0.589**	0.326*	0.96	34	yes
	II	0.321	0.942**	-	0.82	35	no
		0.826	0.995**	-0.066	0.81	34	no
	III	0.997*	0.819**	-	0.67	54	yes
		0.870*	0.949**	-0.109	0.74	53	no
JP	I	6.328	0.947**	-	0.97	35	no
		6.490	0.796**	0.147	0.97	34	no
	II	17.84	0.889**	-	0.82	35	no
		20.92	0.941**	-0.069	0.81	34	no
	III	5.21	0.959**	-	0.93	54	no
		8.81	1.225**	-0.292*	0.94	53	no
UK	I	0.121**	0.798**	-	0.87	35	yes
		0.058	0.874**	0.024	0.88	34	no
	II	0.031	0.936**	-	0.79	35	no
		0.032	0.968**	-0.031	0.79	34	no
	III	0.063*	0.886**	-	0.83	54	yes
		0.054	1.149**	-0.249	0.85	53	no
WG	I	0.140	0.922**	-	0.96	35	yes
		0.111	0.583**	0.345*	0.96	34	yes
	II	0.159	0.919**	-	0.79	35	no
		0.200	0.986**	-0.087	0.78	34	no
	III	0.218	0.881**	-	0.76	54	no
		0.241*	1.048**	-0.181	0.81	53	no

Notes: 1. One and two asterisks denote significance at 5 and 1% levels of t-statistics respectively.

2. $\bar{R}^2$ is squared multiple correlation coefficient adjusted for degrees of freedom.

3. D denotes if the slope coefficient of Rx_{t-1} is significantly less than one.

Table 2. Augmented Dickey-Fuller unit root tests

$$RX_t = (\alpha - 1)RX_{t-1} + \beta_1 RX_{t-2} + \beta_2 RX_{t-3}$$

Sample		$\hat{\alpha} - 1$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\bar{R}^2$	n
FR	I	-0.097 (-2.64)	-0.280 (-1.58)	-0.048 (-0.29)	0.16	33
	II	-0.081 (-0.91)	0.072 (0.38)	0.078 (0.40)	-0.07	33
	III	-0.157 (-1.94)	0.149 (1.01)	-0.018 (-0.14)	0.03	52
JP	I	-0.063 (-1.98)	0.124 (-0.71)	0.147 (0.82)	0.08	33
	II	-0.143 (-1.78)	0.062 (.034)	0.025 (0.14)	0.006	33
	III	-0.054 (-1.54)	0.385 (2.53)	-0.048 (-0.32)	0.12	52
UK	I	-0.113 (-1.71)	0.023 (0.12)	-0.029 (-0.20)	-0.001	33
	II	-0.073 (-0.72)	0.025 (0.12)	0.044 (0.23)	-0.08	33
	III	-0.099 (-1.76)	0.313 (2.24)	-0.107 (-0.81)	0.08	52
WG	I	-0.079 (-2.2)	-0.324 (-1.79)	-0.035 (-0.20)	0.13	33
	II	-0.125 (-1.31)	0.098 (0.52)	0.156 (0.82)	-0.03	33
	III	-0.128 (-1.85)	0.183 (1.23)	-0.024 (-0.18)	0.03	52

Note: t-statistics in parentheses; McKinnon critical values are -3.56 at 1% and -2.92 at 5%.

Table 3. Estimates of the linear second-order variance process

$$\frac{2}{t} = w + \frac{1}{t-1} + \frac{2}{t-2}$$

	Sample	w	1	2	$\bar{R}^2$	n
FR	I	0.008	0.903**	-0.041	0.72	29
	II	0.016**	1.036**	-0.489**	0.66	29
	III	0.012**	0.991**	-0.329**	0.63	48
JP	I	16.72	1.263**	-0.421*	0.81	29
	II	18.0**	1.265**	-0.717**	0.75	29
	III	6.962**	1.219**	-0.510**	0.74	48
UK	I	1.49E-4	0.809**	-0.083	0.64	29
	II	0.0002**	1.202**	-0.676**	0.72	29
	III	0.0001*	1.444**	-0.670**	0.86	48
WG	I	0.002	0.974**	-0.143	0.72	29
	II	0.002**	1.054**	-0.470**	0.69	29
	III	0.002**	0.992**	-0.321**	0.63	48

Note: One and two asterisks denote significance at 5 and 1% of t statistics.

Table 4. Estimate of the Lorenz logistic process for variance

$$\frac{\sigma_t^2}{\sigma_{t-1}^2} = \alpha + (1 - \alpha) \frac{\sigma_{t-2}^2}{\sigma_{t-1}^2}$$

	Sample	$\hat{\alpha}$	$\hat{\alpha}$	$\bar{R}^2$	n
FR	I	-1.74 (-0.84)	3.30 (1.44)	0.04	30
	II	-20.1 (-1.69)	22.4 (1.81)	0.07	30
	III	-5.96 (-2.16)	7.36* (2.56)	0.12	49
JP	I	1.303** (7.21)	0.002 (1.21)	0.02	30
	II	1.46** (8.04)	0.01 (2.11)	0.11	30
	III	1.69** (5.42)	0.017 (1.51)	0.14	48
UK	I	-261.6 (-1.77)	262.9 (1.77)	0.07	30
	II	-383.5 (-1.13)	384.7 (1.13)	0.01	30
	III	-164.7 (-0.76)	166.0 (0.77)	-0.01	49
WG	I	-27.1 (-1.71)	28.7 (1.79)	0.07	30
	II	-205.3 (-1.58)	207.8 (1.59)	0.05	30
	III	-63.3** (-2.79)	64.8** (2.84)	0.08	48

Note: t-statistics are in parentheses with one and two asterisks for significance at 5 and 1% respectively.

Table 5. Estimate of the log-linear variance process

$$\log\left(\frac{\sigma_t^2}{\sigma_t^2}\right) = \alpha + \beta_1 \log\left(\frac{\sigma_{t-1}^2}{\sigma_{t-1}^2}\right) + \beta_2 \log\left(\frac{\sigma_{t-2}^2}{\sigma_{t-2}^2}\right)$$

	Sample	$\hat{\alpha}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\bar{R}^2$	n
FR	I	-0.623	0.995**	-0.192	0.72	29
	II	-1.778**	1.033**	-0.522**	0.57	29
	III	-1.162**	1.034**	-0.358**	0.64	48
JP	I	0.651	1.234**	-0.389	0.81	29
	II	1.298**	1.188**	-0.559**	0.69	29
	III	0.858**	1.105**	-0.412**	0.68	48
UK	I	-3.025**	1.033**	-0.425*	0.59	29
	II	-3.643**	1.332**	-0.791**	0.74	29
	III	-1.067	0.909**	-0.031	0.76	48
WG	I	-0.993	1.073**	-0.274	0.72	29
	II	-2.498**	1.073**	-0.507**	0.61	29
	III	-1.921	1.071**	-0.491*	0.64	48

Note: One and two asterisks denote significance at 5 and 1% levels respectively in terms of t-statistics.

Table 6. Estimate of the second order skewness process

$$s_t = \hat{\alpha} + \hat{\alpha}_1 s_{t-1} + \hat{\alpha}_2 s_{t-2}$$

	Sample	$\hat{\alpha}$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\bar{R}^2$	n
FR	I	-0.089	0.720**	-0.208	0.37	22
	II	0.197	0.608*	-0.299	0.21	22
	III	-0.087	0.416**	-0.189	0.28	41
JP	I	-0.534**	0.125	-0.194	0.01	22
	II	0.129	0.297	0.275	0.12	22
	III	-0.108	0.644**	-0.281	0.28	41
UK	I	-0.144	0.169	0.472*	0.22	22
	II	0.212	0.318	-0.293	0.05	22
	III	-0.175	0.693**	-0.229	0.34	41
WG	I	-0.017	0.491*	0.004	0.21	22
	II	0.157	0.328	-0.372	0.03	22
	III	-0.100	0.551**	0.040	0.32	41

Note: One and two asterisks denote significance at 5 and 1% levels respectively in terms of t-statistics.

Table 7. Impact of mean on variance of the exchange rate process

$$\frac{\sigma_t^2}{\mu_t + \mu_t} = \hat{\alpha}_0 + \hat{\alpha}_1 \mu_t / \mu_t$$

	Sample	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\bar{R}^2$	n
FR	I	5.80**	-31.19**	0.29	24
	II	5.90**	16.58**	0.26	24
	III	5.51**	6.21	0.06	43
JP	I	157.21**	-884.44**	0.52	24
	II	163.30**	-88.62	-0.03	24
	III	139.08**	342.62	0.05	43
UK	I	0.637**	1.276*	0.13	24
	II	0.545**	1.442**	0.40	24
	III	0.546**	0.137	-0.02	43
WG	I	1.988**	-12.27**	0.34	24
	II	2.049**	4.646*	0.22	24
	III	1.854**	3.365**	0.17	43

Note: One and two asterisks denote significant t-values at 5 and 1% levels respectively.

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