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Efficiency of Pulsed-Jet Propulsion via Thrust-Drag Decomposition

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By using an axisymmetric immersed-boundary model, fluid dynamics of a cephalopod-inspired propeller undergoing periodic inflation-deflation deformation in background flow is numerically studied in low Reynolds number regime. A thrust-drag decoupling method based on physical analysis is proposed, in which the jet-related thrust is obtained as the summation of three parts: the jet momentum flux, the normal stress at the exit plane, and the flow acceleration inside the body. This method enables the calculation of the propulsive efficiency, especially the efficiency at the steady-swimming state. Systematic simulations are then conducted to study the effects of Reynolds number and stroke ratio on force generation and efficiency. Two Reynolds numbers, the incoming-flow Reynolds number Re_∞ and the jet-flow Reynolds number Re_j , are involved. When Re_∞ is fixed, the thrust generation is found to depend mostly on jet-flow velocity at high Re_j , while the effect of incoming-flow velocity is pronounced at relatively low Re_j , mostly through its influence on the excessive pressure at the nozzle. Within the range of incoming-flow Reynolds number considered in this study (40-150), our results show that the whole-cycle propulsive efficiency of the propeller lies in the range of 11-30%.

I. INTRODUCTION

Since it is relatively easy to balance self weight in water, aquatic locomotion methods display a higher level of diversity than flying mechanisms in air. Examples include body undulation (anguiliform), caudal fin activation (carangiform/thunniform), pectoral fin activation (labriform), mollusk shell flapping, and jetting^{1,2}. Among these swimming methods the jetting mechanism involves coordinated deformation of a large portion of the body. It is an effective method for soft-bodied creatures to achieve significant swimming speed since the subsequent hydrodynamic pressure is distributed over the body rather than concentrated on a small part of it (e.g. the caudal or pectoral fin). Besides, during jetting the mechanical load is sustained by the tension inside the body so that the requirement on the bending stiffness of the body is relatively low. Indeed, through this locomotion method squids are among the fastest swimmers in the aquatic world³.

The morphology and jet propulsion process of squids are sketched in Fig. 1. Inside the body of a squid there exists an empty chamber called the mantle cavity. Driven by an actuation system including the mantle wall as well as two groups of muscles, radial and circular, the body deforms so that the volume of the mantle cavity changes with time. When the volume shrinks water is expelled through a funnel tube, creating tremendous thrust. After that, the animal expands the pressure chamber to suck water into the body and get ready for next jetting. The body size of squid and squid larva ranges from $\mathcal{O}(10^{-3})\text{m}$ to $\mathcal{O}(1)\text{m}$ so that this propulsion mechanism works over a huge range of Reynolds number $(\mathcal{O}(1 - 10^6))^4$.

The swimming mechanisms of live squids have been observed and measured. The kinematics of the fin and the near-body flow fields in both the flapping and the jetting modes of squids have been recorded experimentally⁵⁻⁷. These studies illustrate the complicated nature of the underlying physics.

For example, *Digital Particle Image Velocimetry* (DPIV) measurements demonstrated various vorticity patterns in both fin-activated and jet-propelled motions. The fin-activated mode is utilized on both head-first and tail-first swimming activities with multiple sub-modes distinguishable by vorticity patterns in the wake. The jetting mode, on the other hand, consists of two sub-modes, corresponding to the subcritical and supercritical behaviors of vortex ring development illustrated in theoretical and experimental fluid dynamics⁸⁻¹¹. These studies indicate that dynamics of vortex rings generated by a cylindrical piston are mostly determined by the stroke ratio of the piston, which coincides with the length-to-diameter ratio of the jet plug. When the stroke ratio is lower than a critical value, the wake vorticity is concentrated into a single dominant vortex ring (the subcritical case). Otherwise the vorticity is distributed along the wake following a leading vortex ring (the supercritical case). The exact value of the critical stroke ratio (i.e. the formation number) depends on factors such as the time history of the jet speed¹².

Based on this locomotion mechanism, bio-inspired swimmers with various mechanical designs have been proposed.¹³ developed a cephalopod-inspired underwater vehicle and tested its performance in escaping maneuver. Their device includes a rigid internal frame with an elastic outer skin, which is initially pressurized by water. By releasing this vehicle it reaches a speed comparable to those of its natural counterparts. Without the ability to refill the pressure chamber, this design can only perform one burst so that any long-distance swimming is out of the question. More sophisticated robotic systems were also reported^{14,15}. By using a tendon-pulley system this design undergoes repeated inflation and deflation of the pressure chamber so that long-distance swimming is achieved through intermittent jetting. More recently, a soft-bodied robot that is capable of pulsed-jet swimming through cyclic shape deformations has been developed by Christianson *et al*¹⁶. By adjusting the orientation of the nozzle, this device is not only able to swim forward but also capable of performing turning maneuver via thrust vectoring.

There have also been numerical simulations of squid-

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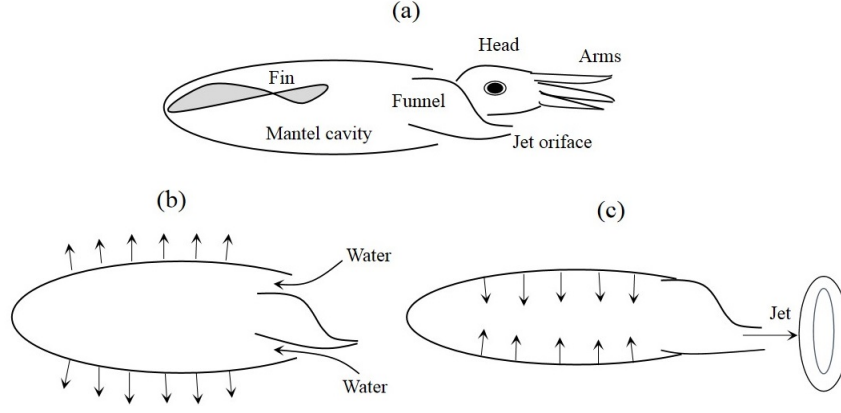


FIG. 1. (a) Sketch of a squid. (b) Inflation phase, during which water is sucked into the cavity by expanding the mantle. (c) Deflation phase, during which water is ejected out of the body through the orifice, forming a jet flow behind.

inspired swimming. Based on idealized fluid dynamics theory, the thrust generation by the repulsive mechanism of a jet has been examined by Giorgio-Serchi *et al*¹⁵. We recently applied immersed-boundary methods and other numerical tools to solve dynamics of fluids inside and outside of the pressure chamber. In two-dimensional studies the propulsive performance is found to be related to the relative strength between the body-generated and jet-generated vortices^{17–19}. There is, however, significant difference in terms of underlying physical mechanisms between two-dimensional and three-dimensional scenarios. Specifically, three-dimensional cases involve generation and development of vortex rings, whose dynamics (especially the stability properties) cannot be accurately duplicated in two dimensions. To overcome this issue we extended the two-dimensional immersed-boundary model to axisymmetric cases²⁰. In that work we examined the thrust generation by a single discharge from the pressure chamber of a deformable body without incoming flow, a scenario roughly corresponding to the escape jetting. It does not include the dynamics of such a system in repeated inflation and deflation motions in an incoming flow, a scenario associated with long-distance swimming. There was also no consideration about the efficiency of this locomotion method.

The efficiency of locomotion via pulsed jetting has been the topic in some existing studies^{4,21–24}. Among these studies, Staaf *et al*⁴ measured the dynamics of squid larvae around 1 mm in body length and found that they could reach swimming efficiency around 20%. Moslemi & Krueger^{23,24} experimentally measured the propulsive efficiency of a squid-inspired pulsed-jet propulsion device with a rigid-body design for Reynolds number in the range of 37 – 60 and 1300 – 2700, respectively. They found that the efficiency of the jetting process reached 15% – 32%. Hereby the efficiency is defined the conventional way, i.e. the useful power (thrust multiplied by forward speed) divided by the total power expenditure (excess kinetic energy shed into the wake plus the useful power). However, in the calculation of thrust these studies rely on simplified models based on the jet flux momentum or the momentum change in the wake (which includes not only the jet but also the body wake; the latter is actually related to

drag rather than thrust). In fact, the efficiency of jet propulsion is often calculated as the Froude efficiency, which has been extended to study pulsed-jet propulsion including both the contraction and the refilling phases²⁵. Nevertheless, this is a quasi-steady method using the average forward speed and average jet speed. The detailed time history of the jetting and refilling processes is not considered.

In a broader context of the dynamics of bio-inspired swimmers, for a reasonable estimation of efficiency it is necessary to decompose the overall force on the body into thrust and drag. Otherwise the efficiency based on overall force in steady-swimming scenario, in which thrust and drag are balanced, will be zero. Traditionally, it was proposed to use the drag on a rigid swimmer as a rough approximation for the drag on the deformable body²⁶. The accuracy of this approach is limited since body deformation may significantly affect the drag force. In an alternative approach, the longitudinal force is separated into two parts, pressure force and viscous force. At any instant the contribution from each part is counted as either thrust if it is in the swimming direction or drag if it is in the opposite direction²⁷. In terms of physical interpretation, this method is very different from the traditional view, in which the thrust force can be either positive (towards the swimming direction) or negative (against the swimming direction). Moreover, with this method there will be nonzero drag force even if the forward speed is zero.

We hereby propose a novel thrust-drag decoupling strategy for pulsed-jet propulsion by utilizing its special characteristics in terms of thrust and drag generation. We then apply the aforementioned axisymmetric immersed-boundary model to study dynamics of a repeatedly deforming object that generates thrust in a constant-speed incoming flow through pulsed jetting. This setup, together with the thrust-drag decoupling method, allows us to define the propulsive efficiency in the traditional manner.

The rest of the paper is organized as follows. We will first describe the physical problem, including the geometry of the squid-inspired propulsive system and its prescribed deformation over time. Afterwards a brief description of the mathematical formulation and the numerical model is provided.

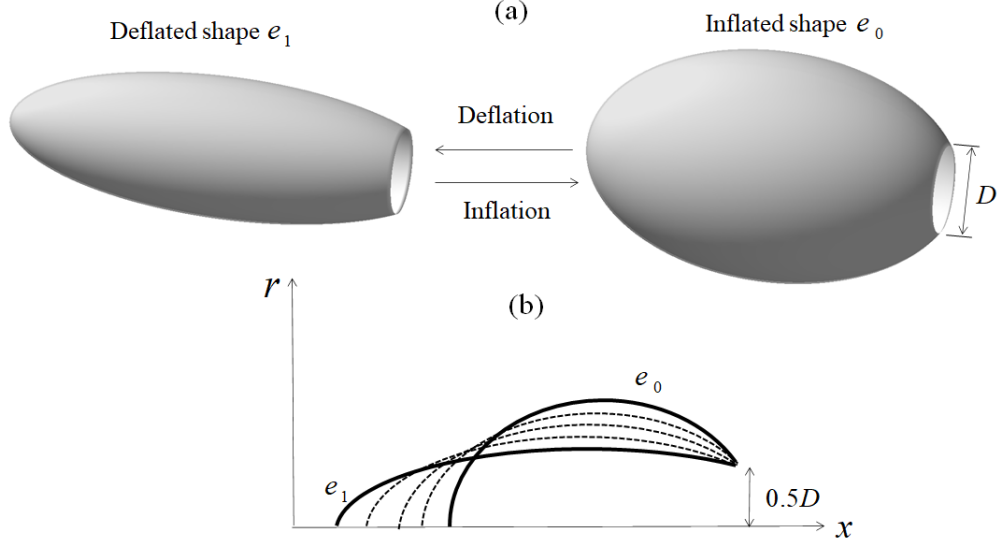


FIG. 2. (a) Rendition of the axisymmetric squid-inspired system, (b) the simulation model and body deformation.

This is followed by the description of the thrust-drag decoupling method, including the derivation and validation. After that, the decoupling strategy will be used to determine the jet-related thrust and steady locomotion efficiency for the squid-inspired swimmer. Finally, conclusions are drawn.

II. PROBLEM DESCRIPTION

As shown in Fig. 2a, the squid-inspired propulsion system is idealized as an ellipsoidal shell of zero thickness with a circular exit whose diameter is D . The axisymmetric numerical model used in this study is presented in Fig. 2b. The contour length of the body, measured from the top point of the nozzle to the bottom point along the body via its front end, remains a constant L . In this study we choose $L = 10D$. The propulsion process includes two phases, deflation and inflation, both accomplished by prescribed body shape change. For example, during the deflation phase, the eccentricity e of the body, whose initial value is e_0 , is increased to shrink the body so that fluid inside the chamber is expelled out through the exit to form a jet flow. After the fully deflated state e_1 is reached, the inflation phase is initialized by decreasing e back to e_0 , with external fluid being sucked back into the chamber. A detailed description of how to create a truncated ellipsoid with given L , D and e can be found in our previous publication²⁸. The system is tethered at the exit plane during the body deformation.

The deformation trajectory is depicted by the time history of the eccentricity $e(t)$, which in its turn is determined by the jet speed profile $V_j(t)$ we prescribe, where V_j (the spatially averaged axial flow speed at the exit) is defined as

$$V_j(t) = -\frac{1}{A} \frac{d\forall}{dt}, \quad (1)$$

in which $\forall(e)$ is the volume of the internal chamber depend-

ing on the eccentricity e , $A = \pi D^2/4$ denotes the area of the exit. In the current study, sinusoidal jet speed profiles are employed for both the inflation and deflation phases. The prescribed jet profile is displayed in Fig. 3, where T is the period of a full inflation-deflation cycle. $\tilde{V}_j$ is the upper bound of the jet speed. Equation (1) enables us to determine the time evolution of the internal body volume $\forall$ with the prescribed $V_j(t)$, then map onto $e(t)$, which describes the body deformation trajectory.

The volume of fluid ejected during the jetting process can be measured by the instantaneous stroke ratio Γ defined as

$$\Gamma = \frac{\forall(e_0) - \forall(e)}{AD}. \quad (2)$$

The maximum stroke ratio Γ_m is reached at the end of the deflation when $e = e_1$. Note the definition of Γ is equivalent to the stroke ratio defined in previous experiments where the jet flow was generated by a cylindrical piston⁸. Both correspond to the length-to-diameter ratio of the discharged jet plug.

Besides the stroke ratio, there are three important dimensionless parameters, the incoming-flow Reynolds number Re_∞ , the jet-flow Reynolds number Re_j , and the Strouhal number St , whose definitions are

$$Re_\infty \equiv \rho u_0 D / \mu, \quad Re_j \equiv \rho \tilde{V}_j D / \mu, \quad St \equiv D / (u_0 T), \quad (3)$$

where ρ is the fluid density, μ represents the dynamic viscosity, and u_0 is the incoming flow speed. The correlation among those quantities can be expressed as

$$Re_j = 4\Gamma_m St Re_\infty. \quad (4)$$

To ensure that the problem remains axisymmetric, we only consider relatively low Reynolds number ($Re_\infty \leq 150$). Unless otherwise specified, we choose $\Gamma_m = 10.4$, achieved by setting $e_0 = 0.8$ and $e_1 = 0.95$.

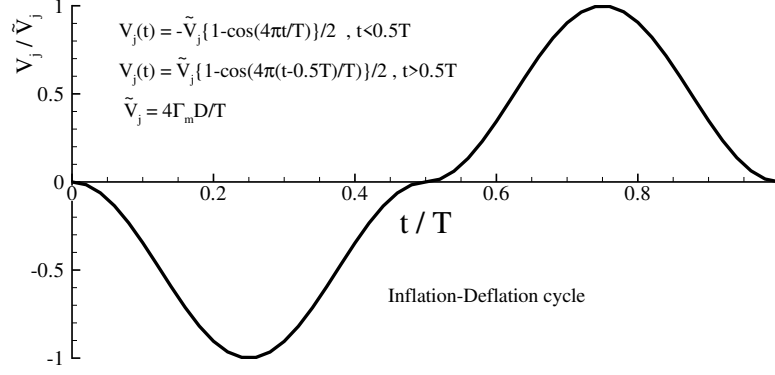


FIG. 3. The prescribed jet speed profile of a complete deflation-inflation cycle. T is the period of deformation.

III. MATHEMATICAL FORMULATION AND NUMERICAL APPROACH

Within the immersed-boundary framework^{29–31}, the incompressible axisymmetric Navier-Stokes (N-S) equations with an additional forcing term to impose solid boundary conditions are employed. Defined in a cylindrical coordinate system $\mathbf{x}(x, r)$ (where x lies on the rotating axis pointing towards the direction of the incoming flow and r is the radial coordinate), $\mathbf{u} \equiv (u_x, u_r)$ denotes the fluid velocity vector, $p(x, r)$ is the pressure, and $\mathbf{f} \equiv (f_x, f_r)$ stands for the force density exerted by the immersed boundary. The equations of fluid motion in the fluidic domain Ω_f are

$$\begin{aligned} \rho \left(\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_r \frac{\partial u_x}{\partial r} \right) &= -\frac{\partial p}{\partial x} + \\ &\quad \mu \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial r^2} + \frac{\partial u_x}{r \partial r} \right) + f_x \\ \rho \left(\frac{\partial u_r}{\partial t} + u_x \frac{\partial u_r}{\partial x} + u_r \frac{\partial u_r}{\partial r} \right) &= -\frac{\partial p}{\partial r} + \\ &\quad \mu \left(\frac{\partial^2 u_r}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_r}{\partial r} \right) - \frac{u_r}{r^2} \right) + f_r \\ \frac{\partial u_x}{\partial x} + \frac{1}{r} \frac{\partial (r u_r)}{\partial r} &= 0, \end{aligned}$$

and the boundary conditions are

- Inflow: $u_x = u_0$, $u_r = 0$, $\frac{\partial p}{\partial x} = 0$;
- Outflow: $\frac{\partial (u_x, u_r)}{\partial t} + u_0 \frac{\partial (u_x, u_r)}{\partial x} = 0$, $\frac{\partial p}{\partial x} = 0$;
- Rotating axis: $\frac{\partial u_x}{\partial r} = 0$, $u_r = 0$, $\frac{\partial p}{\partial r} = 0$;
- Far field: $u_x = u_0$, $u_r = 0$, $\frac{\partial p}{\partial r} = 0$.

The structural kinematics are defined in a Lagrangian coordinate s , which is the arclength along the body between the nozzle and the head so that $0 \leq s \leq L/2$. The Lagrangian variables include body position $\mathbf{X}(s, t)$, body velocity $\mathbf{V}(s, t)$

and the fluid force acting on the body $\mathbf{F}(s, t)$. Here $\mathbf{X}(s, t)$ and $\mathbf{V}(s, t)$ are prescribed quantities according to §II. $\mathbf{F}(s, t)$ is calculated with a feedback scheme³²

$$\mathbf{F}(s, t) = \alpha \int_0^t [\mathbf{U}(s, \tau) - \mathbf{V}(s, \tau)] d\tau + \beta [\mathbf{U}(s, t) - \mathbf{V}(s, t)], \quad (5)$$

$$\mathbf{U}(s, t) = \int_{\Omega_f} \mathbf{u}(\mathbf{x}, t) \delta(\mathbf{x} - \mathbf{X}(s, t)) d\mathbf{x}, \quad (6)$$

where $\mathbf{U}(s, t)$ represents the local fluid velocity at the immersed boundary point s , which is obtained via interpolation using the Dirac delta function. α and β are negative parameters applied to penalize the discrepancy between fluid velocity $\mathbf{U}$ and body velocity $\mathbf{V}$ at the fluid-body interface. The penalty force is based on both the current state and the histories of fluid and structure motions. Physically, this feedback equation implies that the adjacent fluid particle and structural point are linked via a linear spring with stiffness α and damping coefficient β . The exact values of α and β do not affect the simulation results as long as their absolute values are sufficiently large. Herein we choose $\alpha = -10^3 \rho \tilde{V}_j^2 / D$ and $\beta = -150 \rho \tilde{V}_j$. By using the same delta function, the interaction force in the Lagrangian domain can be distributed to the Eulerian domain to update the flow field, i.e.

$$\mathbf{f}(\mathbf{x}, t) = \int_{\Gamma_{ib}} \mathbf{F}(s, t) \delta(\mathbf{X}(s, t) - \mathbf{x}) ds, \quad (7)$$

where Γ_{ib} is the contour along the body between the nozzle and the head. With the feedback strategy, the converged solution for the dynamic equations and fluid-structure interacting force $\mathbf{F}(s, t)$ can be obtained after iterations.

Integrating the distributed interaction force in the x direction $F_x(s, t)$ along the body Γ_{ib} , we have the net axial force acting on the body, which is given as

$$F_n = - \int_{\Gamma_{ib}} 2\pi r(s) F_x(s, t) ds. \quad (8)$$

A conventional fractional-step method is employed to solve the fluid dynamics equations. The numerical strategy has

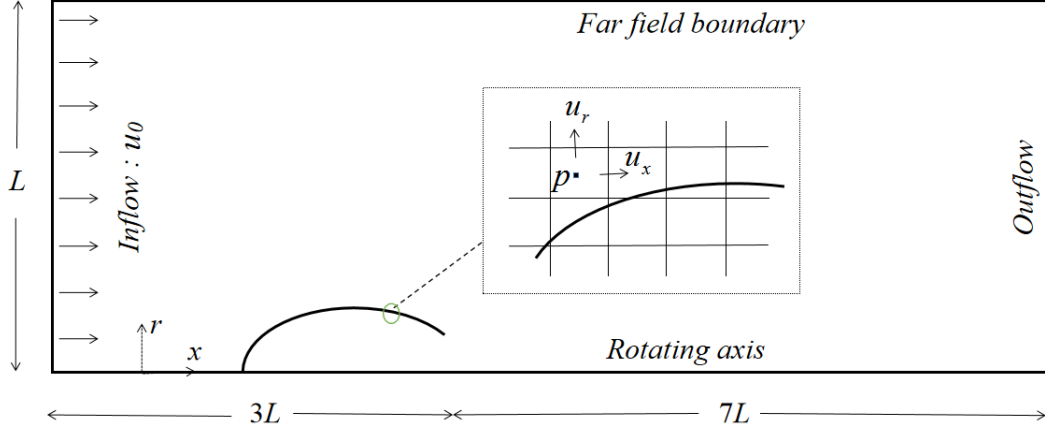


FIG. 4. Sketch of the computational domain (not to scale).

been well documented in our previous studies and other publications^{20,33}. A brief summary is presented herein.

A fully implicit time-advancement method, the Crank-Nicolson scheme, is employed to perform the temporal discretization. To calculate the spatial derivatives, second-order central finite difference is applied on staggered grids, in which the fluidic velocity is defined on the cell boundaries, whereas the pressure is defined at the cell center (see the inset in Fig. 4) so that it is not necessary to enforce *ad hoc* pressure boundary conditions³⁴. Since the implicit treatment of the convective term makes the equations nonlinear, we employ the procedure of Beam & Warming³⁵ to linearize the N-S equations with second-order temporal accuracy. Meanwhile, the N-S equations and the continuity equation are converted into block LU decomposition to decouple the velocity and pressure and avoid large sparse matrices. Subsequently, we obtain a system of tri-diagonal matrices, whose computation requires far less memory and cost. In our axisymmetric problem, a more general elliptic equation rather than a Poisson equation is solved to update the pressure field using the *biconjugate gradient stabilized method* (BiCGSTAB)³⁶.

The computational domain is a $10L \times L$ rectangle as displayed in Fig. 4. The finest fluidic grid $\Delta x = \Delta r$ is used in the inner area around the body, while coarser grids are applied in the far field region. The optimal grid size and time step are determined through grid and time step convergence studies, which have been clearly depicted in the previous study²⁰. Uniform distributed Lagrangian grid ($\Delta s = 0.0167D$) is applied to discretize the body. Fluidic grid size and time step are set to be $\Delta x = 0.02D$ and $\Delta t = 5.0 \times 10^{-4}D/\tilde{V}_j$, respectively.

Details of this numerical method can be found in our previous study, where it was applied to simulate pulsed-jet device with the same geometry and similar kinematics as the one in the current work²⁰. In that study the validity and accuracy of this numerical method were also delineated through comparisons with benchmark results of canonical problems. For example, uniform flow past a fixed sphere was simulated using our fluid solver, and our results match well with experimental data. Moreover, in the pulsed jetting simulations of this propeller, the vortex ring evolution (i.e. vortex ring circulation

as function of stroke ratio Γ) is in good agreement with previous experiments, in which the vortex ring was created in a piston-cylinder arrangement under the same Re_j .

IV. THRUST-DRAG DECOUPLING AND THRUST DECOMPOSITION

A. Control volume analysis

As shown in Figure 5, a uniform flow with constant speed u_0 passes around the deformable ellipsoidal propeller. We consider a cylindrical control volume, which is bounded by the lateral surface $\delta\Omega_{side}$, the flow inlet $\delta\Omega_{in}$, and the exit plane $\delta\Omega_{out}$ (which includes the nozzle exit A). The control volume is separated into two parts (Ω_I and Ω_{II}) by the body, whose inner surface and outer surface are Γ_{in} and Γ_{out} , respectively. The conservation of momentum equation in the axial direction is

$$\rho \frac{\partial}{\partial t} \iiint_{\Omega_I + \Omega_{II}} u_x dV - \rho \iint_{\delta\Omega_{in}} u_0^2 dS + \rho \iint_{\delta\Omega_{out}} u_x^2 dS - \iint_{\delta\Omega_{in}} p_\infty dS + \iint_{\delta\Omega_{out}} (-\sigma) dS = F_n, \quad (9)$$

where F_n represents the axial hydrodynamic force (net force) acting on the body, which includes jet-related thrust F_T , drag F_d , and the added-mass-related force F_a ³⁷. p_∞ denotes the background pressure. σ is the normal fluid stress, i.e. $-p + \tau$, in which τ is the viscous normal stress. Equation (9) can be rewritten as

$$\begin{aligned} \rho \frac{\partial}{\partial t} \iiint_{\Omega_I} u_x dV + \rho \frac{\partial}{\partial t} \iiint_{\Omega_{II}} u_x dV - \rho \iint_{\delta\Omega_{in}} u_0^2 dS + \\ \rho \iint_{\delta\Omega_{out} - A} u_x^2 dS + \rho \iint_A u_x^2 dS + \iint_A (-\sigma - p_\infty) dS + \\ \iint_{\delta\Omega_{out} - A} (-\sigma - p_\infty) dS = F_T + F_d + F_a. \end{aligned} \quad (10)$$

In these expressions the positive directions of the forces (F_n , F_T , F_d and F_a) are the $-x$ direction.

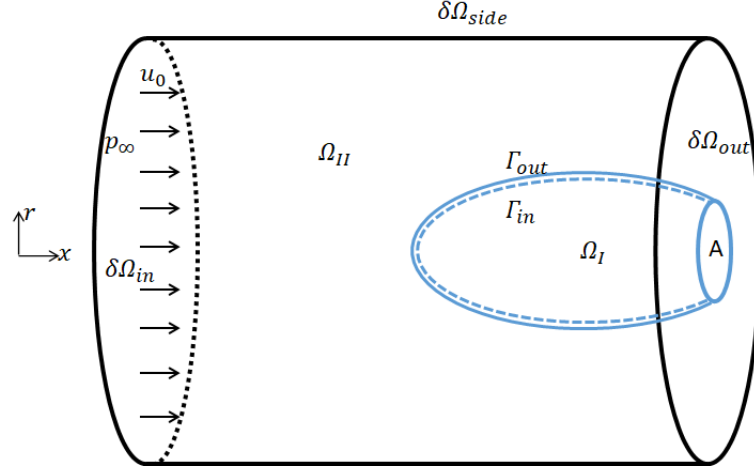


FIG. 5. Definition of the control volume and its boundaries.

On the left-hand side of Eq. (10), the first, fifth, and sixth terms are associated with the internal flow and the generation of the jet flow, whereas the rest of the terms are associated with the outside flow attributed mostly to the incoming flow u_0 . Based on the physical insight that the drag force F_d and added-mass-related force F_a are related to the incoming flow and the thrust F_T is related to the jet, we categorize these terms into F_T or $F_d + F_a$ according to their physical origins. Thus we have

$$F_T = \rho \frac{\partial}{\partial t} \iiint_{\Omega_I} u_x dV + \rho \iint_A u_x^2 dS + \iint_A (-\sigma - p_\infty) dS, \quad (11)$$

and

$$F_d + F_a = \rho \frac{\partial}{\partial t} \iiint_{\Omega_{II}} u_x dV - \rho \iint_{\delta\Omega_{in}} u_0^2 dS + \rho \iint_{\delta\Omega_{out}-A} u_x^2 dS + \iint_{\delta\Omega_{out}-A} (-\sigma - p_\infty) dS. \quad (12)$$

By doing so, we are able to extract the jet-generated thrust F_T from the net force F_n . Meanwhile, F_T is further decomposed into three components with different physical meaning. The first term on the right-hand side of Eq. (11) represents the change rate of the fluid momentum inside the body (hereafter referred to as F_m), the second term is the jet flux (referred to as F_j), and the third term is the excessive normal stress at the exit plane (F_σ).

Indeed, the jet-generated thrust defined in Eq. (11) can be alternatively interpreted as the axial hydrodynamic force acting on the inner surface of the body. Towards this end, we perform control volume analysis on the deformable control volume Ω_I . According to the conservation of momentum in axial direction³⁸, we have

$$\rho \frac{\partial}{\partial t} \iiint_{\Omega_I} u_x dV + \rho \iint_{\Gamma_{in}} (\mathbf{V}_r \cdot \mathbf{n}) u_x dS + \rho \iint_A u_x^2 dS + \iint_A -\sigma dS = F_{in}, \quad (13)$$

where $\mathbf{n}$ is the unit normal vector on the control volume boundaries pointing outwards. $\mathbf{V}_r$ is the relative velocity of fluid crossing the control surface Γ_{in} , whose value is zero in this case because Γ_{in} is a non-penetrable boundary. F_{in} denotes the axial hydrodynamic force on the inner surface with $-x$ as its positive direction. A comparison between Eq. (11) and Eq. (13) leads to

$$F_{in} = F_T + \iint_A p_\infty dS. \quad (14)$$

It is clear that F_{in} and F_T are the same except for a hydrostatic term related to the background pressure p_∞ . Similar analysis on the deformable control volume Ω_{II} can also be conducted. It indicates that $F_d + F_a$ is the hydrodynamic force acting on the outside of the body. This is physically reasonable since the jet-generated force acts primarily on the inside surface, and the outer surface is directly in contact with the incoming flow.

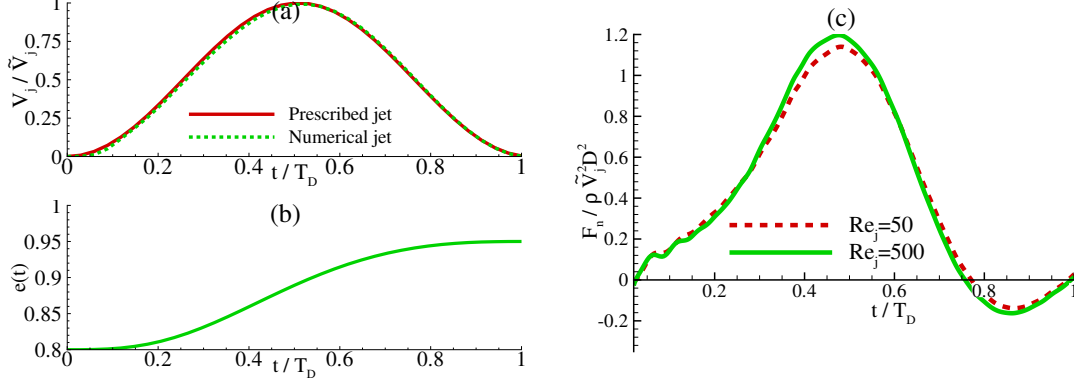
The physical assumption behind our thrust-drag decoupling method is that the drag force is associated with incoming flow or forward speed, whereas the thrust is associated with jet flow caused by body deformation. Subsequently, to demonstrate the accuracy of our approach we need to show the following: (a) without body deformation the thrust force F_T we compute through Eq. (11) is small in comparison with the drag force on the body; (b) without incoming flow the thrust force F_T is close to the overall hydrodynamic force on the body so that the drag force is negligibly small. These tests are carried out in the following.

B. Tests with simple scenarios

To test the performance of our thrust-drag decoupling method, we conduct simulations of three simple scenarios. The first scenario is a rigid body tethered in background flow ($Re_j = 0$), in which the jet-generated thrust in our definition is expected to be small so that $F_d \approx F_n$ (in this scenario $F_a = 0$

TABLE I. Thrust, drag comparison when $Re_j = 0$ and $e = 0.88$.

Re_∞	7	20	60	100	150
$F_d / \rho u_0^2 D^2$	-8.15	-3.97	-2.03	-1.52	-1.22
$F_T / \rho u_0^2 D^2$	-0.165	-0.063	-0.034	-0.030	-0.028
F_T / F_d	2.0%	1.6%	1.66%	1.96%	2.29%

FIG. 6. (a) Time histories of jet speed V_j (both prescribed and numerically calculated values) in a single deflation, (b) time histories of body eccentricity e corresponding to the jet profile, (c) time histories of net force F_n when $Re_j = 50$ and 500.

since there is no body deformation). The second and third scenarios involve single deflation and inflation motions without incoming flow (i.e. $Re_\infty = 0$), in which the incoming-flow-related drag force F_d and added mass force F_a are expected to be negligible so that $F_T \approx F_n$. In these two scenarios the underlying physics of force generation are also examined by using the thrust decomposition technique developed in §IV A.

a) Rigid body within incoming flow

Physically there should be no thrust generated in this scenario when $Re_j = 0$. Based on our definition, however, there might be a nonzero F_T due to the disturbance in the wake, the exit normal stress F_σ is not exactly zero. To examine how much error this brings to the result, we simulate the steady flow around a rigid ellipsoidal swimmer with $e = 0.88$ at various values of Re_∞ . Table I lists the measured thrust F_T via Eq. (11) and the drag force F_d calculated as $F_n - F_T$. It shows that even though F_T is non-zero, the values are extremely small (less than 3% in all the cases) in comparison with F_d .

b) Single deflation without incoming flow

In this part we simulate the jetting process through a single deflation when the eccentricity changes from e_0 to e_1 without any incoming flow. Two different values of jet-flow Reynolds number Re_j are considered. The jet speed profile is prescribed as $V_j = 0.5\tilde{V}_j[1 - \cos(2\pi t/T_D)]$, where T_D is the duration of deflation and $\tilde{V}_j = 2\Gamma_m D/T_D$. Figure 6a shows time histories of the prescribed jet speed and the numerically obtained jet speed, which is calculated as $4\iint_A u_x(r)dS/\pi D^2$. The consistency between these two quantities verifies the robustness of our immersed-boundary strategy in imposing the no-flux boundary conditions. Figure 6b displays the time evolution of eccentricity $e(t)$ applied to accomplish the desired jet speed. Figure 6c presents time histories of the normalized axial net

force ($F_n/\rho\tilde{V}_j^2 D^2$, where F_n is calculated by Eq. (8)). It indicates that the propeller generates positive net force during most of the process except in a relatively short period near the end.

We then calculate the jet-related thrust F_T defined in Eq. (11), i.e. $F_T = F_m + F_j + F_\sigma$, where $F_m = d(\iint_{\Omega_j} \rho u_x dV)/dt$, $F_j = \iint_A \rho u_x^2 dS$, and $F_\sigma = \iint_A (-\sigma - p_\infty) dS = \iint_A (p - p_\infty - \tau) dS$. Hereby we present the obtained F_T in Figure 7a, where the time history of the net force F_n is also plotted. It confirms that F_T stay close to F_n over the whole jetting process, verifying the accuracy of our thrust extraction strategy.

Moreover, the decomposition of thrust into three distinctive components helps us understand the underlying physics of force generation. For this purpose we present the three thrust components in Figs. 7b, c, and d. Based on their definitions, F_j depends on V_j^2 , which is always positive. On the other hand, F_m is related to jet acceleration dV_j/dt so that negative F_m is generated when $t > 0.5T_D$. Meanwhile, F_σ is found to be roughly proportional to the jet speed V_j . The small disturbance in F_σ during the initial stage of the deflation, particularly when $Re_j = 50$, is mainly attributed to the initial effect associated with the entrainment of ambient fluid into the vortex ring and the accumulation of added mass surrounding the vortex ring bubble¹². It is seen that the jet-related thrust F_T could be negative near the end of the deformation when the negative contribution from F_m outweighs the positive ones from F_j and F_σ .

In terms of the effect of Re_j , we found that lower Re_j tends to create larger normalized flux momentum F_j . For further insight, we present the axial velocity distributions along the nozzle radius of these two cases at $t = 0.5T_D$ in Fig. 8. The case of $Re_j = 500$ displays a narrower boundary layer near

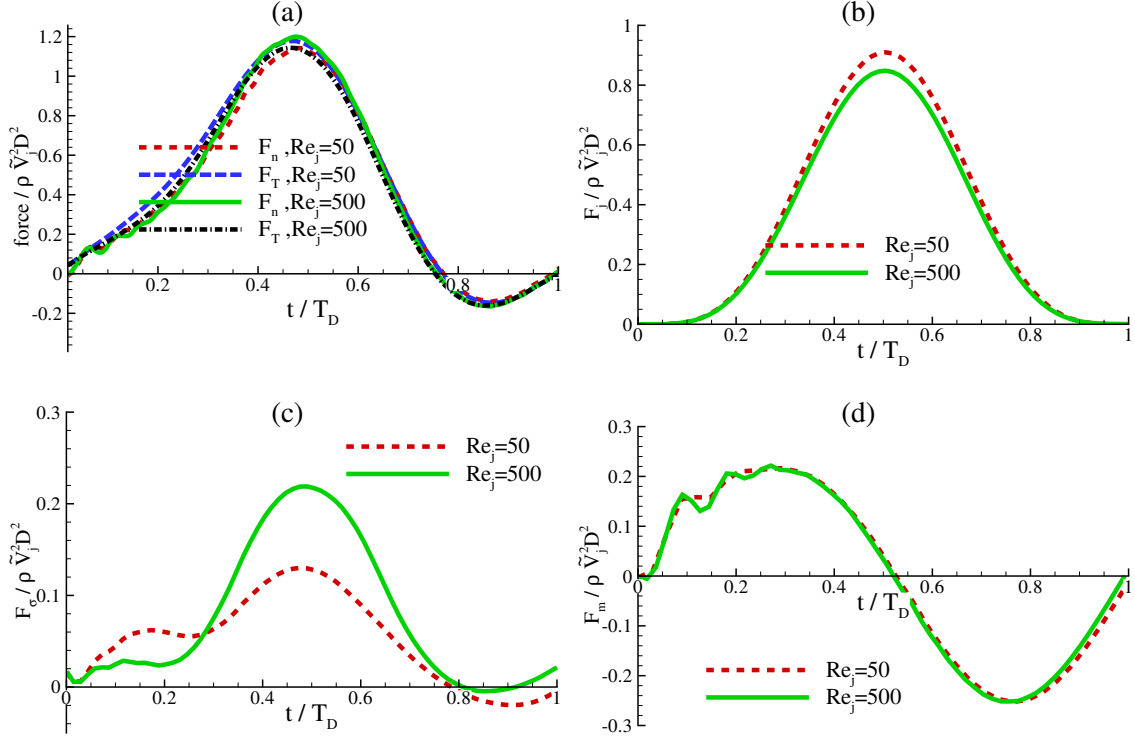


FIG. 7. (a) Comparison between the net force F_n and jet-generated thrust F_T in a single deflation, (b,c,d) thrust contributions F_j , F_σ and F_m .

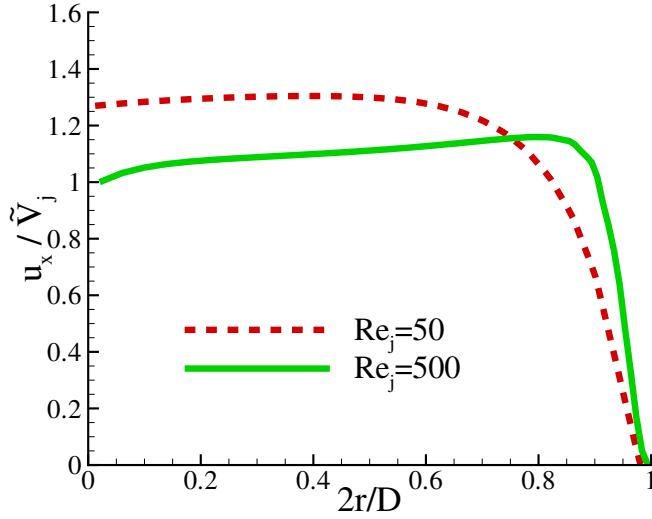


FIG. 8. Comparison of axial velocity distribution $u_x(r)$ at the nozzle plane between $Re_j = 50$ and 500. Both cases are shown at $t = 0.5T_D$.

the outer surface of the nozzle ($r = D/2$) owing to reduced viscosity, so that the axial flow speed at the nozzle core area, which is the main contributor to F_j , is slightly reduced to keep V_j consistent. Therefore $Re_j = 500$ creates slightly smaller jet flux momentum than $Re_j = 50$ (Fig. 7b). It is also seen that higher Re_j leads to larger F_σ due to the reduced viscous stress

(Fig. 7c). For F_m , Re_j does not play any role since it only depends on jet acceleration.

Figure 9 presents the evolution of the vorticity field during the deflation deformation. Most of the discharged fluid is entrained into a vortex ring at the initial stage. Afterwards a clear trailing shear layer is formed behind the ring when the jetting process fails to deliver vorticity and energy to the vortex ring at a sufficient rate²⁰. However, the vortex ring does not separate from the trailing flow, nor is there any secondary ring generated behind the leading vortex ring. Similar phenomenon has been reported by Moslemi & Krueger²⁴ and Zenit³⁹. It is also observed that the vortex ring tends to be thicker at lower Reynolds number ($Re_j = 50$), which is consistent with previous experimental results³⁹.

c) Single inflation without incoming flow

Numerical simulations are also conducted for the dynamics of the system during a single inflation. Figure 10 presents the time histories of the desired jet speed (defined as $V_j = -0.5\tilde{V}_j[1 - \cos(2\pi t/T_I)]$, where T_I is the duration of inflation and $\tilde{V}_j = 2\Gamma_m D/T_I$), numerically calculated jet speed, body eccentricity and net force F_n . Similar to the deflation case, F_T matches well with F_n due to the negligibly small effects of F_d and F_a , as shown in Fig. 11a. Previously it was believed that the inflation phase generates mostly drag force⁴, however we find that although negative thrust is indeed created at the initial stage, it is followed by positive thrust in the remaining time (note that this result is obtained without any incoming flow). To understand the physical mechanism, Figs. 11b,c

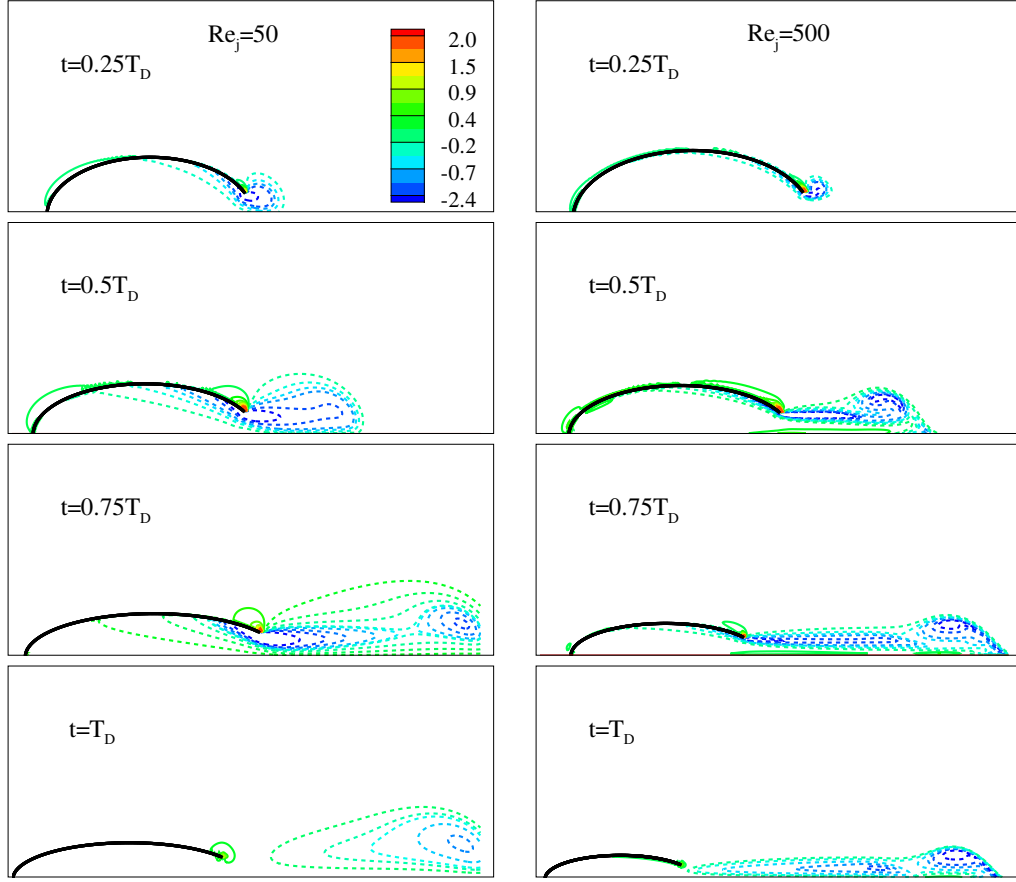


FIG. 9. Snapshots of azimuthal vorticity (normalized by $\tilde{V}_j/D$) contour during the single deflation. The dashed lines correspond to negative values of vorticity (counterclockwise vortices) and the solid lines correspond to positive values of vorticity (clockwise vortices).

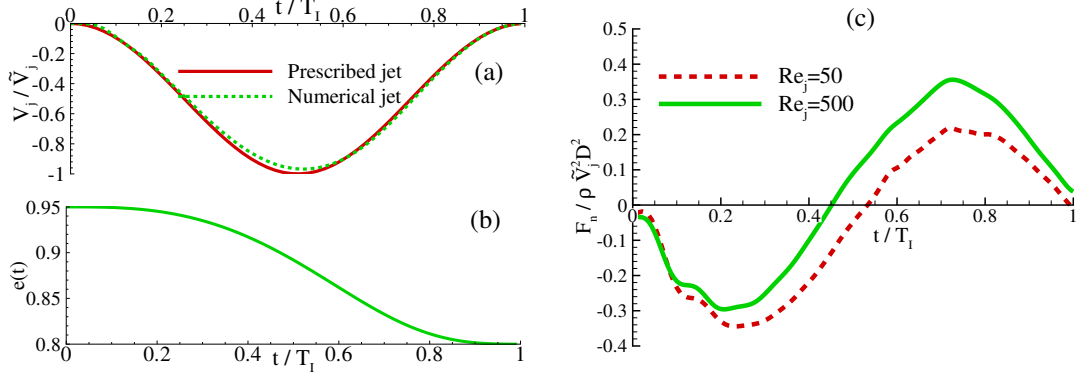


FIG. 10. Same as Fig. 6 except that the deformation is inflation.

and d present the thrust contributions from F_j , F_σ and F_m , which are roughly proportional to V_j^2 , V_j and dV_j/dt , respectively. It indicates that F_j is largely balanced by F_σ so that F_T undergoes similar tendency as F_m . The case with $Re_j = 500$ generates greater thrust than the case of $Re_j = 50$. The thrust enhancement mainly results from the decrease of F_σ at the nozzle exit while the other two components, F_j and F_σ , vary little as Re_j changes.

It is interesting that the momentum flux term F_j remains positive during deflation and inflation, even though the directions of the jet flow are opposite in these cases. This is explained by the fact that in terms of the total momentum of the body (including the water inside), sucking water in has the same effect as push water out. If we define the forward direction as the positive direction, in the former case there is an increase in the positive momentum inside the body, while

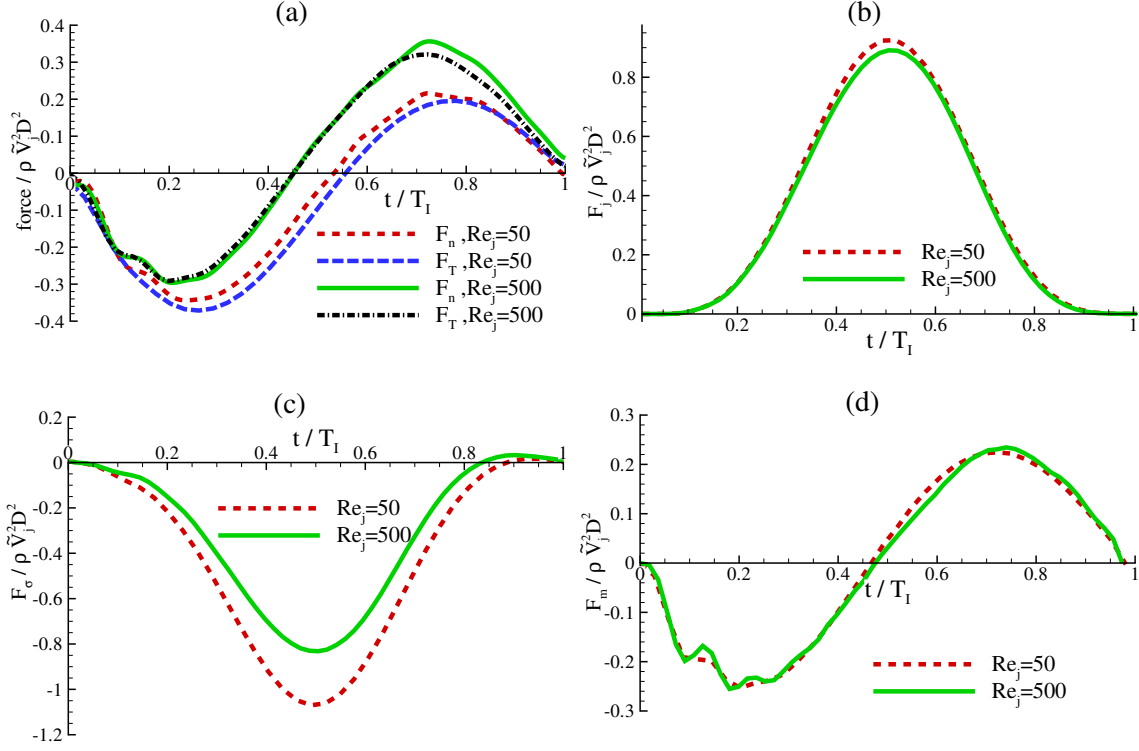


FIG. 11. Same as Fig. 7 except that the deformation is inflation.

in the latter case there is a reduction in the negative momentum. These two cases are thus physically equivalent in their contributions to the momentum change.

The flow fields visualized through azimuthal vorticity are displayed in Fig. 12. It is shown that internal vortices are formed as fluid is sucked into the pressure chamber, yet the vorticity evolution differs significantly between the two cases with different Reynolds numbers. When $Re_j = 500$, the internal flow consists of a leading vortex and a trailing flow behind. The leading vortex is stopped when it approaches the internal surface of the body, inducing a counter-rotating vortex as the body expands. At $t = 0.75T_I$, this new vortex splits the jet flow into two parts. The leading vortex, the secondary vortex, and the trailing flow then interact with each other till the end of the inflation, when there still exist fairly strong vortices in the chamber. When $Re_j = 50$, however, the jet flow evolution takes a different path. In the chamber no leading vortex is formed due to large viscous effect. Indeed, the jet flow even fails to reach the front wall. At the end of inflation, there is no residual vorticity in the chamber due to dissipation.

These tests have demonstrated that the thrust-drag decoupling method is a feasible method to extract the jet-related thrust F_T , based on which the propulsive efficiency can be obtained when an incoming flow is added.

V. CYCLIC DEFORMATION WITH INCOMING FLOW

We now consider complete deflation-inflation cycles (see Fig. 3 for the jet speed profile) and study thrust generation and hydrodynamic efficiency in the tethered mode with incoming flow u_0 . The thrust-drag decoupling and thrust decomposition methods in §IV A will be utilized to obtain the jet-generated thrust and the hydrodynamic efficiency. We will examine the effect of Reynolds number and stroke ratio on the whole-cycle propulsive performance. Comparison between the propulsive efficiency obtained using our thrust-drag decoupling method and those from traditional methods will also be conducted.

According to our numerical tests, periodic responses are established after the first deformation cycle. Therefore all results presented in this section will be extracted from the third deformation cycle.

A. Time histories over a full cycle

In Figs. 13a and b we plot the time evolutions of jet speed and the corresponding body eccentricity over an full deformation cycle for a representative case with $S_r = 0.06$ and $Re_\infty = 150$ (in this case $u_0 = 0.4\tilde{V}_j$). Time histories of F_n , F_T (calculated as $F_j + F_\sigma + F_m$) and $F_r = F_d + F_a$ (calculated as $F_n - F_T$) are also displayed in the same figure (see Fig. 13c). Since cyclic propulsion is performed in an incoming flow, the drag force F_d and the added-mass-related force F_a can not be ignored. The added-mass-related force is given as

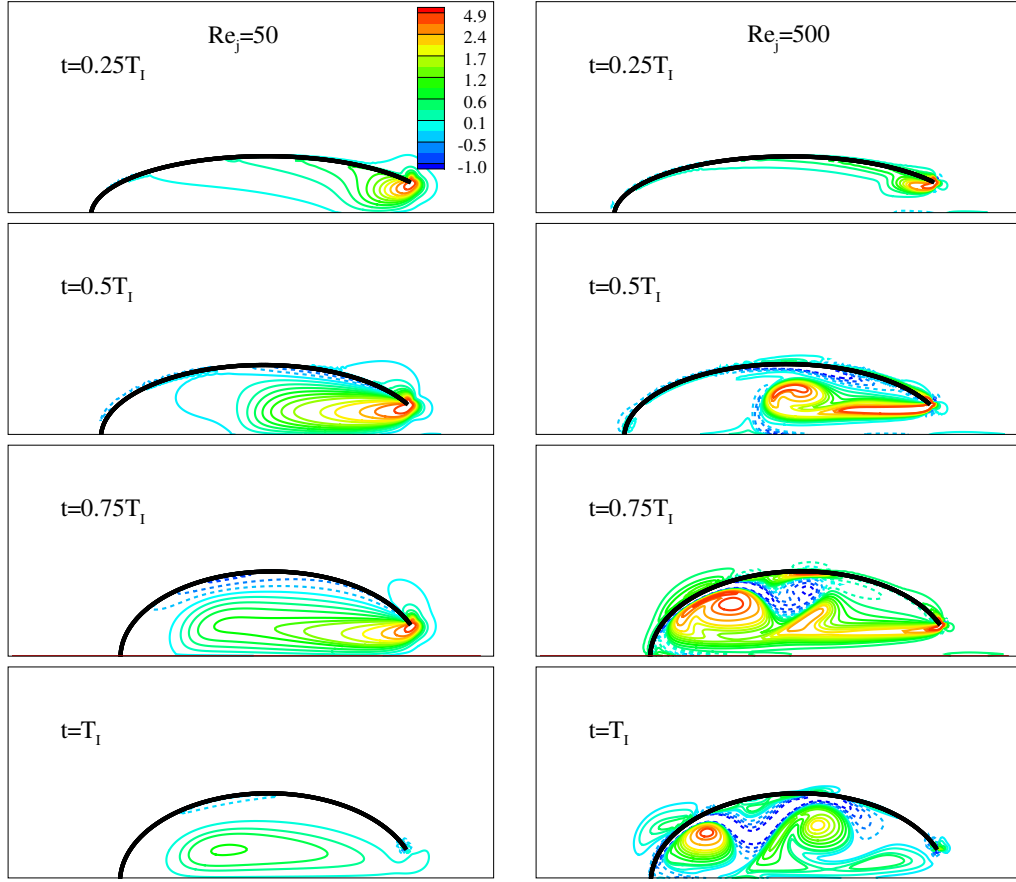


FIG. 12. Same as Fig. 9 except that the deformation is inflation.

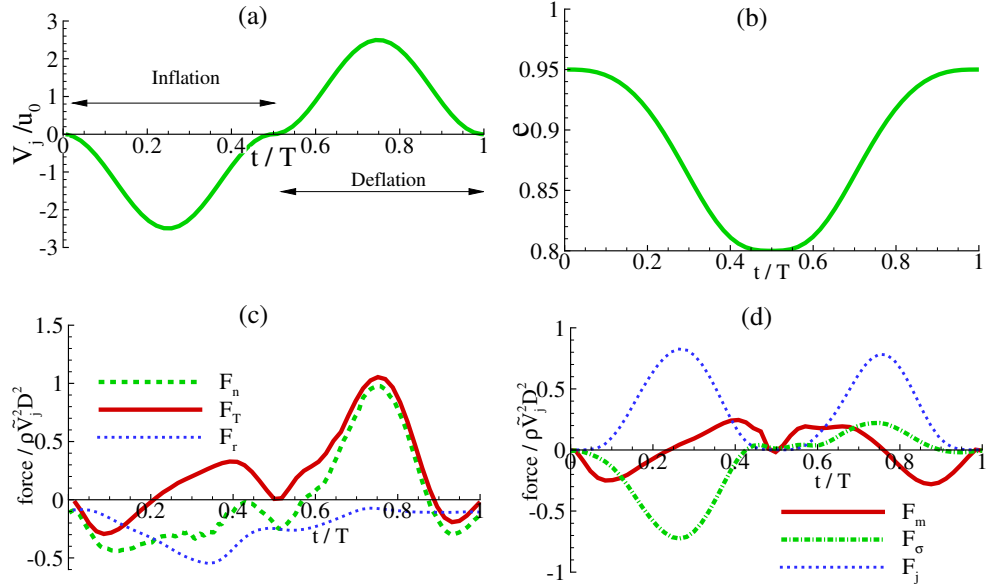


FIG. 13. Time histories of (a) the jet speed V_j , (b) the body eccentricity e , (c) the net force F_n , the thrust force F_T and the drag force F_r , and (d) the components of the thrust force F_T during a full deflation-inflation cycle. $S_l = 0.06$, $Re_\infty = 150$.

$F_a = -u_0 \frac{dm_a}{dt}$, where m_a is the added mass^{18,37}. Clearly, it is positive during the deflation phase (herein it can be considered as an additional thrust contribution besides the jet-generated thrust) and negative during the inflation phase (thus it can be regarded as additional drag force besides the free stream-generated drag). This explains why the magnitude of F_r is much higher during the inflation phase than during the deflation phase, as shown in Fig. 13c. Even though the instantaneous force includes the added mass contribution, the average value of F_a over a full cycle is zero when u_0 is a constant¹⁸. In this sense, the jet-generated thrust F_T can be regarded as the only source of thrust over a full cycle, and F_d is the only source of resistance.

The components of F_T over a cycle are displayed in Fig. 13d. The time-averaged values of F_j , F_σ and F_m are found to be $\overline{F_j} = 0.312\rho\tilde{V}_j^2 D^2$, $\overline{F_\sigma} = -0.105\rho\tilde{V}_j^2 D^2$, and $\overline{F_m} = 0$, respectively. It indicates that only F_j and F_σ contribute to time-averaged thrust generation, and the effect of F_m is negligible with the particular jet profile we use.

B. Parametric studies

We then perform parametric investigations about the effects of Re_∞ and Re_j upon the locomotion dynamics.

a) Thrust generation

Figure 14 summarizes the dependence of the time-averaged jet-related thrust $\overline{F_T}$ upon Re_∞ under various Re_j . Hereby Γ_m is fixed as 10.4 so that Re_j is determined by the Strouhal number S_t . It shows that when Re_j is relatively small (e.g. $Re_j = 20$), the average thrust varies significantly with respect to Re_∞ . Specifically, increasing Re_∞ tends to cause reduced $\overline{F_T}$. Negative thrust is observed when $Re_\infty > 36$. However, the dependence of $\overline{F_T}$ upon Re_∞ becomes less pronounced as Re_j increases. Indeed, after normalization $\overline{F_T}$ remains almost a constant within the range of Re_∞ when $Re_j > 100$ ($\overline{F_T} \approx 0.21\rho\tilde{V}_j^2 D^2$).

To further understand the effect of Re_∞ upon thrust generation, we present the two thrust components F_σ and F_j for the cases of $Re_j = 20$ in Fig. 15 (as mentioned earlier the other component F_m does not contribute to time-averaged thrust). According to its definition, the jet flux F_j is determined by the jet velocity, so that Re_∞ has negligible effect on it. Therefore the impact of Re_∞ on thrust generation rests solely on F_σ (see Fig. 15b). It shows that increasing Re_∞ tends to generate reduced F_σ . For further insight, Fig. 16 displays the pressure and streamline distributions of the flow field at different time instants. It is demonstrated that for low Re_∞ cases (e.g. $Re_\infty = 10$), the free stream starts to separate from the surface of the outer shell, forming a closed recirculating vortex ring in the wake at $t = T/2$ (the end of inflation phase). The body-generated vortex ring is quickly pushed downstream and dissipated away in the deflation phase under the effect of the high-speed jet flow (note that in this case the jet flow is relatively strong compared with the incoming flow). As Re_∞ grows (e.g. $Re_\infty = 100$), a much stronger body-generated vortex ring exists after the inflation phase, accompanying the presence of

a negative pressure region on the rear side of the body (see Figs. 16f,g). This can be the reason why higher Re_∞ leads to lower F_σ , as displayed in Fig. 15b. Meanwhile, we find that this vortex ring is sufficiently strong to survive well into the deflation phase. Indeed the reverse flow within the vortex ring forms a ‘virtual wall’ that blocks the jet flow. However, the influence of this virtual ‘ground effect’ on thrust generation is not pronounced in our simulations.

For relatively high Re_j (e.g. $Re_j = 100$), The jetting dynamics will be dominated by the high-speed jet and insensitive to the free stream. Figure 17 displays the time histories of F_j and F_σ when $Re_j = 100$. As expected, Re_∞ makes little difference on both of them.

b) Propulsive Efficiency

As noted earlier, the thrust-drag decoupling method enables us to define the propulsive efficiency based on jet-created thrust F_T . We have

$$\eta = \frac{\overline{F_T} u_0}{\overline{P_{in}}}, \quad (15)$$

where $\overline{P_{in}}$ represents the average power expenditure per cycle, i.e.

$$\overline{P_{in}} = \frac{\int_T \int_{\Gamma_{ib}} 2\pi r(s) \mathbf{F} \cdot \mathbf{U} ds dt}{T}. \quad (16)$$

In Fig. 18a we present the dependencies of efficiency η , average net force $\overline{F_n}$ and average drag $\overline{F_r}$ over a complete cycle upon Re_∞ for the case of $Re_j = 200$ (since the time-averaged value of the added-mass-related force F_a is zero, in the cyclic motion case $\overline{F_r}$ coincides with the time-averaged drag force). As expected, both efficiency and drag force rise with increasing Re_∞ . This efficiency definition is physically relevant in the scenario when the propeller is applied in an underwater locomotion system with constant swimming speed u_0 . However, it is inappropriate for efficiency measurements if the propeller works as a stand-alone swimmer since the prescribed u_0 does not necessarily match the free-swimming speed. To address this issue, we approximate the average free-swimming speed as the free-stream speed u_0 (or, alternatively, the incoming flow Reynolds number Re_∞) when the average net force on the system $\overline{F_n}$ disappears. For example, this ‘steady-swimming’ (or ‘free-swimming’) state of $Re_j = 200$ is reached at $Re_\infty = 60$, where the efficiency η equals 0.21. Through this approach, the steady-swimming efficiency at other values of Re_j and the corresponding steady-swimming Re can be obtained. As shown in Fig. 18b, increasing the jet speed (Re_j) requires higher Re_∞ to achieve steady swimming. The efficiency η is also increased as Re_j rises.

Meanwhile, we have also evaluated the steady-swimming efficiencies using existing approaches discussed in the introduction. These definitions of efficiency are physically equivalent to the one used in the present study. Specifically, η_R is defined by using the traditional thrust-drag decoupling method in which the drag on the deforming body is approximated by the drag on a rigid body²⁶, i.e.

$$\eta_R = \frac{(\overline{F_n} - F_{D_R}) u_0}{\overline{P_{in}}}, \quad (17)$$

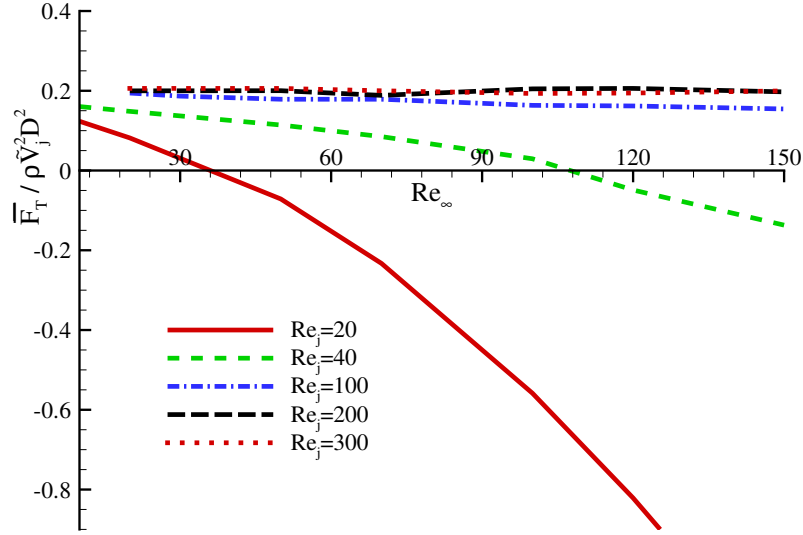


FIG. 14. Dependence of the time-averaged thrust $\bar{F}_T$ upon Re_∞ with different values of Re_j .

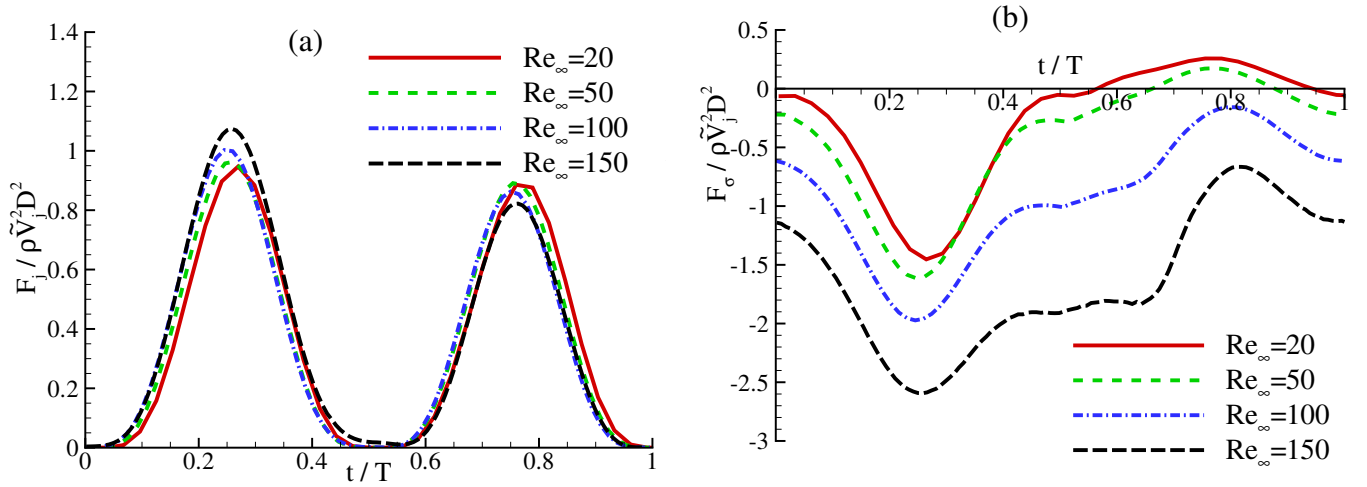


FIG. 15. Time histories of F_j and F_σ for various Re_∞ . $Re_j = 20$.

where F_{D_R} is the drag force on the body whose eccentricity $e = (e_0 + e_1)/2$.

Another propulsive efficiency, η_F , is calculated as

$$\eta_F = \frac{2u_0\bar{V}_j}{2u_0\bar{V}_j + 3u_0^2 + \bar{V}_j^2}, \quad (18)$$

where $\bar{V}_j$ is the average jet speed. Originated from the Froude efficiency, η_F was proposed by Anderson & Demont²⁵ to calculate the whole-cycle propulsive efficiency of jet-propelled organisms such as squid and scallops. Note that Eq. (18) is valid when the average jet speed in deflation equals the average flow intake speed in inflation, which is consistent with the jet profile we consider herein.

As shown in Fig. 18b, η_R is only slightly below η and they share the same trend as Re_j increases. η_F , on the other hand, is significantly higher than η or η_R , especially when Re_j is small. The implication is that this simplified definition may significantly overestimate the propulsive efficiency, at least in the low Reynolds number regime.

It is necessary to point out that the ‘steady-swimming’ or ‘free-swimming’ state in this analysis is defined in the time-averaged sense. In a real free-swimming case the forward speed is unlikely to be a constant. In fact, a more likely scenario is that the forward speed is larger in the deflation phase and smaller in the inflation phase. The net contribution of the added mass related thrust F_a averaged over a cycle will be positive¹⁷. Due to this effect the analysis based on the teth-

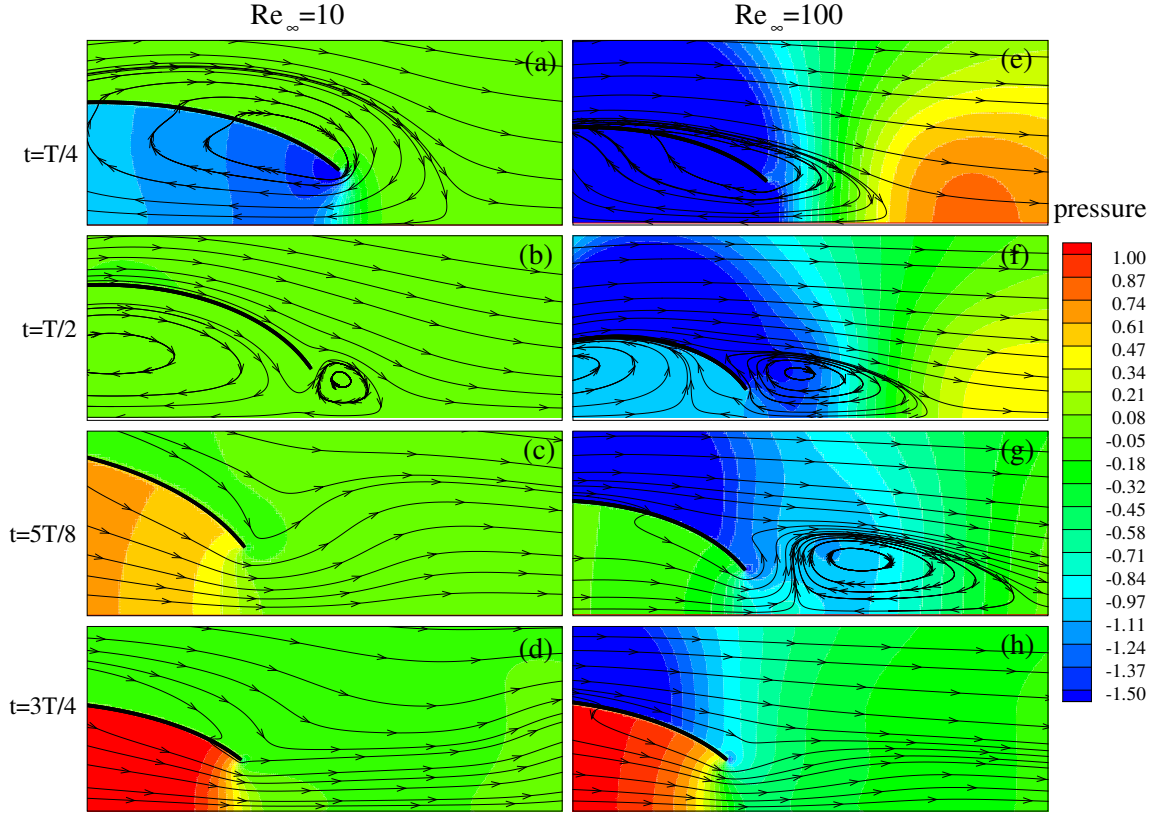


FIG. 16. Snapshots of the flow fields for the case of $Re_\infty = 10$ and 100 with $Re_j = 20$. The flow fields are visualized through streamlines and normalized pressure distribution ($p/\rho\tilde{V}_j^2$).

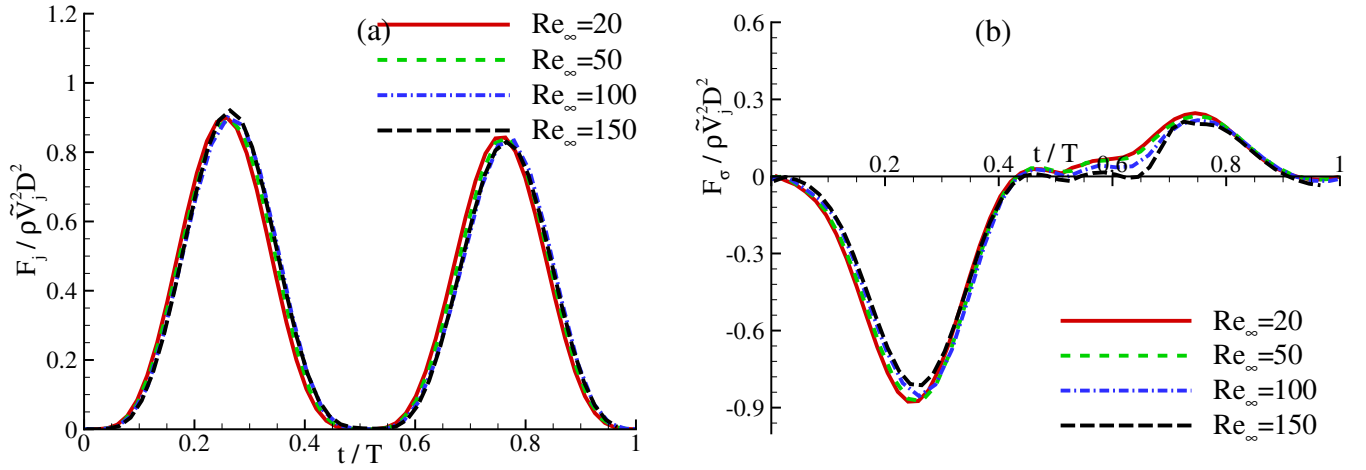


FIG. 17. Same as Figure 15 except that $Re_j = 100$.

ered scenario may underestimate the propulsive performance of the system. The predicted steady-swimming efficiency is thus a lower bound of the actual efficiency in free swimming.

c) Effect of stroke ratio

As mentioned in the introduction, previous studies suggest that there is a universal scaling law that governs the vortex

ring formation of pulsed jetting at intermediate and high Re_j (≈ 2000)⁸. They found that characteristics of the vortex ring were determined by the stroke ratio. Increasing this parameter will enhance the vortex ring until a threshold called *formation number* is reached. Beyond this threshold, the fluid discharged thereafter forms minor vortex rings behind the lead-

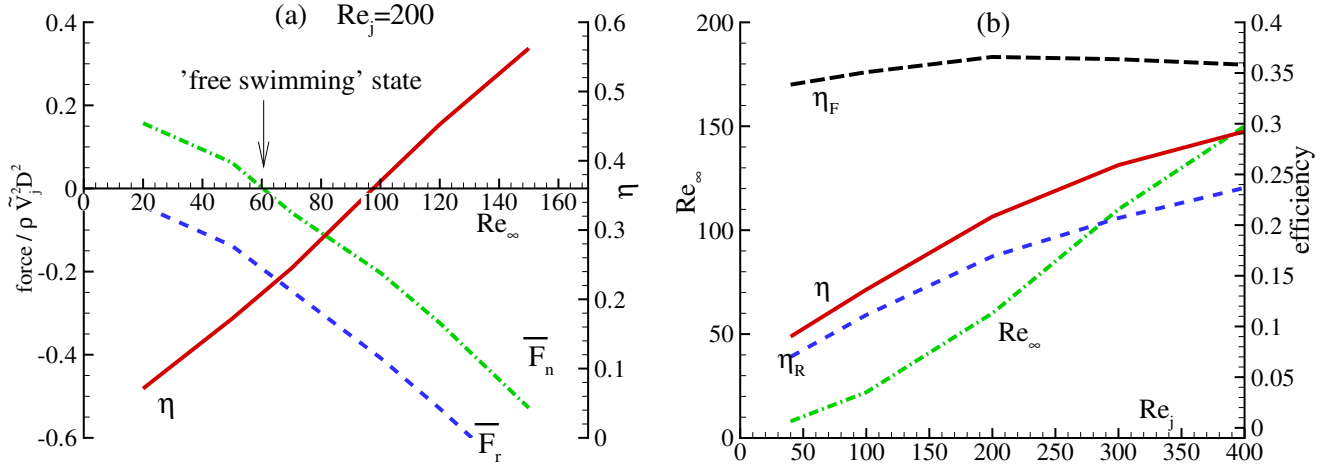


FIG. 18. (a) Dependencies of the average net force $\overline{F}_n$, average drag $\overline{F}_r$ and efficiency η upon Re_∞ when $Re_j = 200$, (b) steady-swimming efficiency and Re_∞ as functions of Re_j .

ing ring, rather than being entrained into it, i.e. the leading vortex ring reaches saturation state at the formation number. Incidentally, the optimal propulsive performance is found to coincide with the formation number⁹. Nevertheless, our earlier publication revealed that at low Re_j ($\simeq 150$) the pulsed jetting is a very smooth process, in which the above pinch-off scenario does not happen²⁰. With a sinusoidal jet profile (similar to the one used in the current study) there does not exist a universal formation number featuring the saturated state of the vortex ring. Instead, the circulation of the vortex ring grows gradually and reaches a peak, followed by a decrease because of viscous dissipation (see Fig. 12b in the reference²⁰).

For this reason we anticipate that the maximum stroke ratio Γ_m plays a less important role in the performance of the system we consider. Instead, it is indirectly involved since it determines the value of Re_j in the steady-swimming case (see Eq. (4)). To illustrate its effect we conduct simulations by using two additional values of Γ_m , 2.9 and 4.6, in which the fully inflated states correspond to $e_0 = 0.92$ and 0.9, and the fully deflated state e_1 is fixed at 0.95.

As shown in Fig. 19a, the variation of Γ_m does not lead to much difference in the thrust with Re_j unchanged. However Γ_m is found to play certain role in drag generation. As displayed in Fig. 19b, less drag force is generated in $\Gamma_m = 4.6$ than in $\Gamma_m = 10.4$, particularly within the large Re_∞ regime. This is because smaller Γ_m corresponds to less inflated body. Therefore, at $\Gamma_m = 4.6$ higher Re_∞ is required to compensate the drag loss to maintain the steady-swimming state. As shown in Fig. 19b, at $Re_j = 200$ the steady-swimming Re_∞ for $\Gamma_m = 4.6$ rises to 70 (instead of 60 for $\Gamma_m = 10.4$). We note that the inflated state at $e_0 = 0.92$ is geometrically close to the one at $e_0 = 0.9$ so that the difference between $\Gamma_m = 2.9$ and 4.6 is far less pronounced. We then present the steady-swimming Re_∞ and efficiency as functions of Re_j for these three stroke ratios in Fig. 20. As expected, decreasing Γ_m results in higher steady-swimming Re_∞ and propulsive efficiency.

VI. CONCLUSIONS

Fluid dynamics of a cephalopod-inspired jet propulsion system in low Reynolds number have been numerically investigated by using the immersed-boundary method. The idealized axisymmetric system includes a pressure chamber enclosed within a shell with zero thickness and a circular nozzle. The body deformations (inflation and deflation) are prescribed to achieve sinusoidal jet speed profile with the nozzle size fixed. The present study concentrates on the calculation of jet-related thrust and whole-cycle propulsive efficiency.

A thrust-drag decoupling strategy designed specifically for this problem is proposed based on control volume analysis. The jet-related thrust in our definition can be further decomposed into three distinct components, the jet flux force F_j , the exit excessive stress F_σ and the flow momentum force inside the body F_m . F_j and F_σ are closely associated with jet speed, whereas F_m is proportional to jet acceleration. The feasibility and accuracy of the thrust-drag decoupling method are demonstrated through three special scenarios, rigid-body dynamics and single deflation or inflation dynamics without incoming flow. The thrust-drag decoupling method enables the calculation of jet-related thrust, which is then used to define the propulsive efficiency of repeated inflation-deflation cycles in a uniform free stream.

Through systematic simulations, we found that the whole-cycle propulsive performance depended mostly on the incoming flow Reynolds number Re_∞ and the jet flow Reynolds number Re_j . For relatively low Re_j (e.g. 20), the time-averaged thrust varies considerably within the range of Re_∞ we consider. This variation mainly results from the exit stress term F_σ , which is sensitive to wake characteristics. It is shown that at low Re_j when the jet is relatively weak, F_σ can be easily affected by the negative pressure region induced by the vorticity shed from the outer side of the shell. However, for relatively high Re_j in which high-speed jet is discharged, the body-shed vortex ring becomes too weak to affect the jet flow

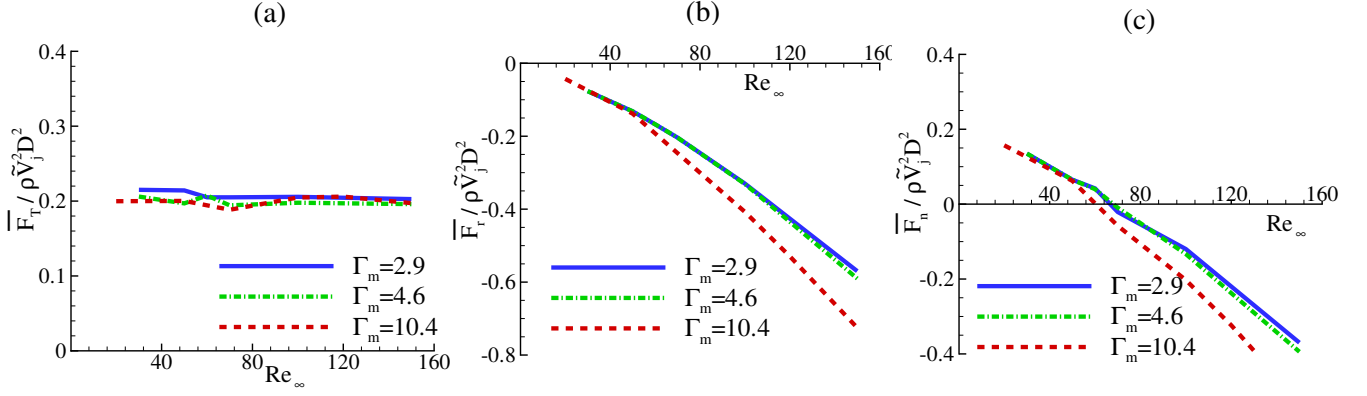


FIG. 19. Dependencies of (a) the average thrust $\overline{F}_T$, (b) the average drag $\overline{F}_r$ and (c) the average net force $\overline{F}_n$ on Re_∞ at various values of Γ_m . $Re_j = 200$.

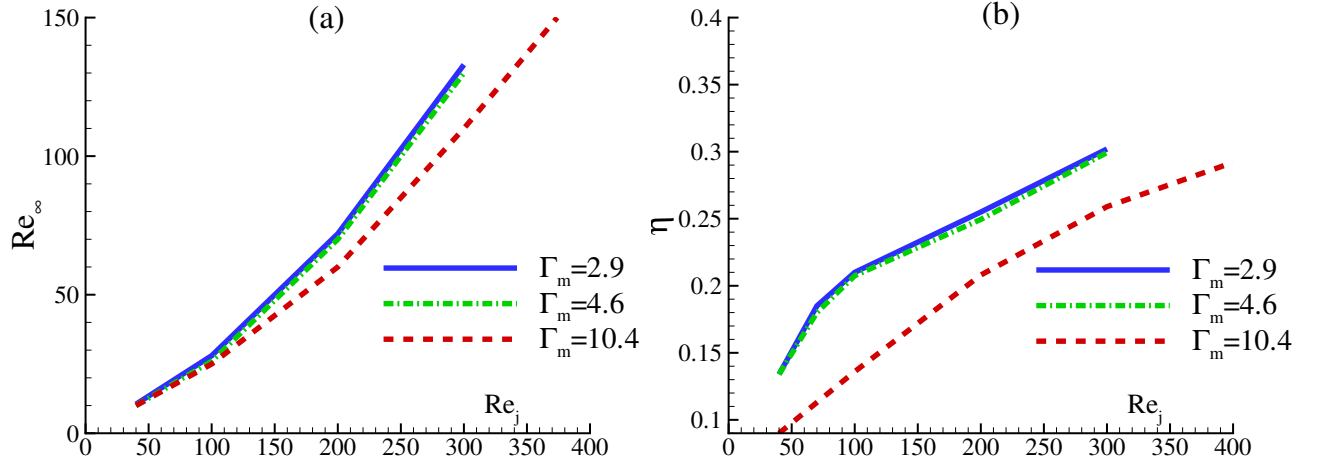


FIG. 20. (a) Steady-swimming Reynolds number Re_∞ and (b) steady-swimming efficiency as functions of Re_j at various values of Γ_m .

or the exit stress component. In these cases the effect of Re_∞ on thrust generation is negligible. Our results suggest that the time-averaged thrust over a full cycle is linearly dependent on the jet speed when $Re_j > 100$.

The propulsive efficiency at the steady-swimming state (i.e. the state when the time-averaged net force on the body is zero) is then calculated at various values of Re_j . The results show that the efficiency increases as Re_j goes up. Finally, the effect of the maximum stroke ratio has also been examined. It is demonstrated that within the current range of Re_j , and with the prescribed sinusoidal jet speed profile, the maximum stroke ratio does not directly affect thrust generation.

Unlike previous studies^{12,20}, for better accuracy in the present study we consider effects of not only the pressure but also the normal viscous stress at the nozzle. In our formulation F_σ is a linear combination of a pressure term F_p and normal viscous stress term F_τ . At relatively high Reynolds number, F_τ becomes negligibly small so that F_σ is mostly equal to F_p . Our results show that even in the low Reynolds number regime we consider in this study, F_τ is small in comparison with F_p .

For example, when $Re_j = 100$ and $Re_\infty = 0$, F_τ accounts for less than 8% of F_σ . Its effect is further reduced when Re_∞ is none zero because F_p is amplified at the presence of incoming flow.

The current study concentrates on a simple sinusoidal jet profile, yet we are aware that performance of the system can be highly sensitive to the detailed time history of the jet²⁰, an effect beyond the reach of methods stemming from the concept of Froude efficiency since they are based on time-averaged values. On the other hand, the method proposed herein is fully capable of simulating cases with different jet profiles and studying the dynamics of untethered swimming. Future studies will be conducted to understand how these factors will affect the efficiency of the system.

The thrust-drag decoupling strategy proposed here is a novel method to compute thrust force and the corresponding propulsive efficiency of squid-like underwater locomotion. It can be easily extended to other locomotion problems involving pulsed-jet propulsion, e.g. locomotion of other cephalopods such as octopuses and cuttlefish, salps, or jelly-

fish. This method, however, depends on the existence of clear boundaries between thrust generating and drag generating regions. It is thus not applicable to cases where these regions are not easily distinguishable (e.g. flapping or undulating swimming modes).

Moreover, our axisymmetric numerical model is only applicable within the low Reynolds number regime. Several high Reynolds number mechanisms are not considered, including, e.g. the non-axisymmetric vortex shedding from the body, the pinch-off of leading vortex ring, and evolution of secondary vortex rings in the wake. To address these issues, three-dimensional modeling with turbulence rendition is necessary.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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