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BEAM DYNAMICS IN HEAVY ION INDUCTION LINACS*

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Summary

Interest in the use of an induction linac to accelerate heavy ions for the purpose of providing the energy required to initiate an inertially confined fusion reaction has stimulated a theoretical effort to investigate various beam dynamical effects associated with high intensity heavy ion beams. This paper presents a summary of the work that has been done so far; transverse, longitudinal and coupled longitudinal transverse effects are discussed.

Introduction

An inertial fusion power plant would require the delivery of some megajoules of energy to a target a few millimeters in diameter in a time measured in tens of nano-seconds. These numbers imply particle energies of 5-10 GeV and currents of ten kilo-amperes for atomic weights greater than 200. While it might be possible to achieve such performance with conventional r.f. linacs and a system of storage rings, an induction linac is also attractive because it provides good electrical efficiency at high current and would avoid the complex beam handling necessitated by the use of a large number of accumulator rings. However, in order to capitalize on the potential efficiency the current must be kept as high as possible throughout the accelerator without degradation in either transverse or longitudinal beam quality as determined by the need to focus the beam, or beams, on a small target several meters in from a reactor wall. The central question in beam dynamics is thus the effect of high intensity on momentum spread and emittance.

Induction linacs have been used to accelerate electrons for some time; recently, with currents up to 10 kiloamperes. That application is simpler to analyze than for heavy ions since the electrons are soon ultra-relativistic; space charge effects are relatively small and differential longitudinal motion negligible. A better comparison is with an r.f. ion linac; the tune depressions contemplated are similar to those encountered at the front end of an r.f. linac but must be maintained during the entire length of the machine, the instantaneous current being increased by suitably ramping the voltage on the accelerating modules. There is nothing analogous to r.f. defocussing and a good match is more easily achieved at the front end, but neither is there an automatic phase stability, so that small positive and negative fields would be provided at the ends of the bunch to counteract longitudinal self-fields and thermal drifting.

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Scaling Laws

It can be shown from the structure of Vlasov's equation that the electric current transported in a magnetic quadrupole channel is related to the r.m.s. emittance by an expression of the form[1]:

$$I = K_1 \left(\frac{A}{q}\right)^{1/3} B^{2/3} (\beta\gamma)^{1/3} \varepsilon^{2/3} \qquad (1)$$

and to the beam radius, a, by another expression

$$I = K_2 (\beta\gamma)^2 Ba. \qquad (2)$$

where A and Z are atomic weight and ionization state and B is the quadrupole field strength at the beam edge. The constants K_1 and K_2 depend on details of the lattice, the phase space distribution function and the tune depression. As the tune goes to zero, K_1 approaches infinity but K_2 is finite; in that limit, equation(2) is the quadrupole analog of the Briliuin flow formula for solenoidal focussing.

Equation(1) was first given by Maschke[2)] with a value of K_1, corresponding to a tune depression of about a factor of two, a formula which became known as the Maschke limit. Because the two expressions are so different in functional form, there was early confusion concerning scaling, particularly since atomic weight and charge state are additional free parameters for this application. The confusion was largely laid to rest by Reiser[3)] in a paper which spelled out the reasons for the differences and presented a smooth approximation version which exhibited emittance and channel acceptance simultaneously:

$$I = \frac{1}{2} I_0 (\beta\gamma)^3 \sigma_0 \frac{\alpha}{L} \left[1 - \left(\frac{\varepsilon}{\alpha}\right)^2\right] = \left(\frac{\sigma}{\sigma_0}\right)^2 = 1-\mu \quad (3)$$

where $I_0 = 3.1 \times 10^7$ A/Z amperes, α is the channel acceptance and L is the lattice period. Even so, care is required in using these formulas because important practical parameters do not appear explicitly.

Transverse Stability

In 1970 Gluckstern[4] investigated the possible modes of a perturbed K-V beam, infinite in longitudinal extent with constant external restoring force. In connection with the study of induction linacs, a generalization to the case of quadrupole focusing was carried out[5]. Since the bunch in an induction linac is typically 10-20 meters long and 10 cm in radius, an infinite beam is a reasonable first approximation. In both models, one finds a great number of modes that become unstable and grow rapidly at sufficiently high intensity (or tune depression). The quadrupole analysis shows in addition the occurrence of "structure resonances"; that is, instabilities which occur at lower intensity when a mode frequency approaches a low order rational relation to the quadrupole period. A striking fact that has not been explained is that, as a function of tune depression, the thresholds and growth rates beyond threshold appear to be identical in the two models, to a precision beyond what one might expect from a smooth approximation to quadrupole focusing. Concurrent with the analytic quadrupole work, simulation programs were developed to attack the problem[6]. Early runs were in qualitative agreement with theory and further indicated that non-KV distributions were also unstable and that emittances grow by factors of two of three before saturating. In one run, a low-order structure resonance appeared in isolation from other unstable modes; a quantative comparison with theory showed excellent agreement and lent credence to both theory and simulation.

On the basis of this work a criterion was established that maximum current could be transported by a quadrupole lattice with zero intensity phase advance less than 60° (to avoid the structure resonance) and a tune depressed to 24° (to avoid the intrinsic resonances). There is however, a later development. According to linear theory, there is no growth in r.m.s. emittance and it was assumed that the observed growth in simulation work was a non-linear effect. Haber then discovered for a continuous solenoid, and Hofmann[7] for a quadrupole array of 60° zero intensity phase advance, that a great rearrangement occurs in phase space (instability) but that the r.m.s. emittance does not change even for tune depressions as low as the simulation technique permits reliable results. There now appears to be no limit on current if r.m.s. emittance is the sole criterrion for hitting a target since K_1, in equation(1), becomes infinite. However, equation(2) indicates that the aperture must then increase also so that, for practical reasons, there is not much room for improvement over the 60°-24° criterion.

Hofmann[8] has extended Gluckstern's work to the case of unequal restoring forces and emittances in the two planes, leading to a further proliferation of modes and possible instabilities. These results, however, are of more interest to the question of equipartition and the ultimate stable distribution[9] than to the performance of induction linacs.

Finally in the category of transverse stability, a calculation was made[10] for the beam break-up mode, the coherent transverse instability which has been so bothersome to electron linac performance. The accelerating modules were represented for this purpose as a cavity with a single, low Q, mode. Because of the low Q and strong transverse focusing for coherent motion, the beam break-up mode does not present a problem for a heavy ion induction linac.

Longitudinal Stability

The single bunch in an induction linac differs from a bunch in an rf linac in several ways. It is very long compared to its diameter, rather than almost spherical. In order to exploit the maximum current criterion obtained from the considerations of the previous section, the instantaneous current should be constant along the bunch and in order to achieve maximum acceleration the voltage on the modules should be constant during the passage of the bunch except for a slight increase during passage to compress the bunch in time. The confining potential is then a square well defined by the auxiliary modules which prevent the ends of the bunch from deteriorating. To first approximation, the ions see a d.c. field all the way from source to final energy — what, then, is the source and magnitude of momentum spread in the bunch? We believe it will be due to errors in timing and wave shape of the module voltage pulses — estimates indicate an average value of 10^{-4}-10^{-3}. At that level, individual particles would move from front to rear and back at most one or two times in several kilometers of accelerator. At the same time, the characteristic velocity of space charge waves for a perturbation in charge density is at least an order of magnitude larger than the thermal spread, so that a good approximation for stability analysis is to neglect energy spread altogether — a cold beam in plasma physics jargon.

The principal worry concerning longitudinal stability arises from the fact that if the module parameters are selected to give good electrical efficiency in transferring energy to the beam, then the beam sees a module as an L-C circuit with a resistive component of several hundred ohms,[11] though there appear to be ways to modify the module circuitry to reduce the impedance. In the CERN PS, primarily, there has been observed a micro-wave instability; i.e. one at wavelengths short compared to bunch length which leads to a damaging increase in momentum spread but for which no satisfactory theoretical explanation existed. According to a semiempirical criterion used by CERN, an induction linac bunch would be highly unstable and so an urgent need arose to understand the phenomenon, at least in the parameter range of interest. Kim[12] has made a perturbation analysis for a bunch in a square well potential and concludes that the system is stable for a monotonically decreasing momentum distribution. Channell et al.[13] considered a parabolic density distribution with no momentum spread and find stability, provided that the resistive component is sufficiently small. An estimate of the convergence of their expansion procedure by Bisognano[14] suggests that the e-folding distance for a density perturbation moving along the bunch must be small compared to the bunch length.

Computational work has been done by Neuffer[15] using a code developed by Neil et al.[16] and by Haber[17] and Sternlieb[18] using a modified NRL simulation code. These codes have been applied only to parabolic charge distributions, the more realistic model analyzed by Kim being more difficult to deal with computationally. The results appear to corroborate Bisognano's speculation; a ten percent density bump propagating from the center and growing more than one e-folding chews the bunch apart, starting from the disturbed end, while one with less growth causes some disturbance but then reflects and dies out. In a square bunch, the bump would reflect more quickly and, we hope, with less disruptive effect — clearly more work must be done.

In Haber's simulation work,[17] various fascinating phenomena appeared, such as soliton formation — the Vlasov equation for a cold beam with dispersion at short space charge wavelengths in fact closely resembles the Korteweg deVries equation — but such effects probably do not occur in the parameter range of interest to the induction linac. Runs were made using the best impedance functions we could construct[11] applied to two full scale Fusion driver designs that had been developed for cost and systems studies. The result was catastrophic for the earlier of the designs, but the later one easily passed the test[19].

Longitudinal-Transverse Coupling

There are several three dimensional effects which must be explored. The beam is visualized as occupying a large fraction of the available aperture; as a result there will be a significant variation with radius of longitudinal electric field, which must drop to zero at the conducting walls. At the ends of the bunch, the self-field pattern is complicated and it is not clear that the trimming voltages mentioned earlier are sufficient to control beam behavior at the ends. Finally, the matching lenses at the entrance of the accelerator would be adjusted to accommodate space charge repulsion in the body of the bunch, leaving the lower density ends mis-matched. Loss of the leading and trailing ions could be tolerated but a continuous erosion of the bunch from the ends inward could not.

Questions such as these present a formidable analytic problem. The tactic we have adopted is to develop simulation codes, first in r and z only and eventually, we hope, fully three dimensional. Hofman[20] has an r-z code in operation and Haber[21] expects to be in the same position in the near future.

There is another aspect of three-dimensional behavior which is amenable to an analytic treatment. Since the bunch is long compared to its diameter, the problem of stability resembles more closely the problem of stability of a coasting beam in a high energy storage ring, which has been studied exhaustively over the years, than it resembles the problem of stability of an r.f. linac bunch. However, because of the strong tune depression, both transverse and longitudinal modes are of the order of the plasma frequency and the assumption used in storage ring theory that longitudinal and transverse effects can be treated separately is suspect.

In order to investigate the interaction of longitudinal and transverse modes, we have considered a beam infinite and uniform longitudinally, subject to a constant linear transverse focusing force and with a longitudinal velocity spread. There are two choices of distribution function in transverse phase space amenable to analytic treatment — circular counter-rotating orbits or a K-V distribution.

The circular orbit model is even less realistic than the K-V distribution but has the mathematical advantage of leading to a simple differential equation and a dispersion equation in closed form for fully three-dimensional modes.[22] The principal result of this work was to show that for an arbitrary wall impedance the longitudinal unstable modes are suppressed more easily by a velocity spread than in a purely longitudinal treatment[23] while transverse modes are little affected.

Analysis of the K-V case was restricted to axially symmetric perturbations but even the the mathematical treatment is exceedingly complicated, leading to a dispersion relation in the form of an infinite determinant. An approximate solution leads to the result that the familiar longitudinal mode, as treated in one dimension, is not significantly affected by the coupling nor are Gluckstern's[4] transverse modes, with one perhaps significant exception; at a tune depression of about a factor of three one of his low order modes couples with a low order longitudinal mode, the coupled system being unstable for all greater tune depressions. The effects of wall impedance and velocity spread are currently being investigated.

The tentative conclusion of this work is that, at least for a beam of infinite length, it is still a good approximation to regard transverse and longitudinal effects as independent.

Experiments

The Heavy Ion Fusion program has chronically suffered from meager financial support and consequently we have no experimental information regarding beam dynamics. Although much work has been done with electron beams in the parameter range of interest, beam quality is of minor interest at best in the development of klystron tubes and other electronic devices and is scarcely mentioned in the available literature. This situation is due to change in the near future. An electron beam transport system using solenoids has been set up at the University of Maryland[24] to investigate transverse phenomena and an electrostatic quadrupole array forty periods long to propagate a 20kV cesium beam is being set up at Lawrence Berkeley Laboratory for the same purpose. Both experiments are designed to cover a wide range of parameters, including extreme tune depressions and should provide valuable information not only for theory but, equally important, for the practical problem of maintaining a nearly Brillouin flow pattern for a long distance, a feat which has never been demonstrated.

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