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### Publication Date

1974-07-01

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July, 1974

Prepared for the U. S. Atomic Energy Commission  
under Contract W-7405-ENG-48

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A MICROSCOPIC STUDY OF THE VARIABLE-MOMENT-OF-INERTIA

MODEL FOR RARE EARTH NUCLEI\*

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ABSTRACT

Based on the microscopic cranking model, the moment-of-inertia parameter  $J_0$  and the force constant  $C_{VMI}$  associated with the variable-moment-of-inertia model are calculated microscopically for rare-earth nuclei. Higher-order effects representing quadrupole and hexadecapole centrifugal stretching, proton and neutron Coriolis-anti-pairing effects and fourth-order cranking correction are included. The present calculations are able to reproduce the trend and the magnitude of both  $J_0$  and  $C_{VMI}$  fairly well with discrepancies ranging from 10 to 40 percent.

NUCLEAR STRUCTURE rare earth even-even nuclei, calculated moment-of-inertia and force constant. Variable-moment-of-inertia model, cranking model. Coriolis-anti-pairing effect, fourth-order cranking, centrifugal stretching.

## I. INTRODUCTION

It is now well established that the quasi-rotational spectrum plays a central role in the excitations of even-even deformed nuclei.<sup>1,2</sup> The general features of the quasi-rotational states are as follows: 1) their spins and parities follow the sequence of  $0^+$ ,  $2^+$ ,  $4^+$ ,  $6^+$ , . . . , and 2) their energies deviate from the  $I(I+1)$  rule as the spins increase. Recently, it was discovered that at very high angular momenta, the rotational energies of some nuclei may exhibit anomalous behavior, the so called back-bending.<sup>1,2,3</sup> We shall not discuss the back-bending phenomenon in this paper, but shall limit our calculations only to those states with moderate high spins. There exist many two-parameter formulas which fit very well the energy levels up to spin  $I=12$ . Among them we may mention the centrifugal stretching model of Diamond, Stephens and Swiatecki<sup>4</sup> (which was later extended by Sood<sup>5</sup>), the fourth order cranking model of Harris<sup>6</sup>, the variable moment-of-inertia model (VMI model) of Mariscotti, Scharff-Goldhaber and Buck<sup>7</sup>, and the EXP model of Draper.<sup>8</sup> Recently the VMI model has also been extended to high spins by several authors to deal with the back-bending phenomenon.<sup>9,10</sup> Compared to the phenomenological fits, the microscopical calculations of the nuclear rotational energies,<sup>11-17</sup> on the other hand, have only moderate success in reproducing the experimental data. For example, the authors of Ref. 11 to Ref. 15 (Udagawa and Sheline; Chan and Valatin; Sano and Wakai; Bes, Landowne and Mariscotti; Krumlande) took into consideration the centrifugal stretching and the Coriolis-anti-pairing effect<sup>18</sup> (CAP effect) and obtained fairly good agreement with the experiment. However, other calculations<sup>16,17</sup> have shown that the fourth-order cranking contribution is as important as the CAP effect and the inclusion of the former makes the theoretical results much worse. Indeed, Marshalek's calculations<sup>16</sup> showed that in general the calculated

values of the B coefficient associated with the  $I^2(I+1)^2$  correction term in an expansion of the rotational energies is about a factor of 1.5 to 3 too large compared with the experimental data in the rare-earth region. The calculations by Ma and Rasmussen<sup>17</sup> were likewise only capable to produce results of the right order of magnitude; however, the quantitative significance of their results is subject to some uncertainty due to the use of a simple basis where the single particle angular momentum is kept as a good quantum number. More recently, several authors<sup>19-21</sup> have done Hartree-Fock-Bogoliubov variational calculations to study the back-bending phenomenon at high spin states; which, however, will not be discussed here. In summary, the above situations indicate that the microscopic calculation of the rotational energy deserves much further study.

The present calculations are based on the cranking model of Inglis.<sup>22</sup> We follow closely the formulations of Ma and Rasmussen<sup>17</sup> (hereafter referred to as (I)), and make use of the single particle wave functions of Nilsson et al.<sup>23</sup> with the inclusion of both quadrupole ( $\epsilon_2$ ) and hexadecapole ( $\epsilon_4$ ) deformation. Since it has been shown that most of the two-parameter formulas are related to each other,<sup>7,17,24</sup> we shall calculate specifically the parameters associated with the VMI model and the B coefficient connected with the  $I^2(I+1)^2$  term.

The following section will briefly review the formulations developed in (I), the detailed calculations and formulas are given in Sec. III, and the last two sections will give the results and discussions.

## II. REVIEW OF MICROSCOPIC THEORY

The VMI model<sup>7</sup> can be expressed as follows:

$$\begin{cases} E_I = \frac{1}{2} C_{\text{VMI}} (J_I - J_0)^2 + \frac{I(I+1)}{2J_I} \\ \frac{\partial E_I}{\partial J_I} = 0 \end{cases} \quad (1)$$

where  $E_I$  and  $J_I$  are respectively the energy and moment-of-inertia of the excited state with spin  $I$ . The force constant  $C_{\text{VMI}}$  and the ground state moment-of-inertia  $J_0$  are the two parameters which can be determined by a least squared fit to the experimental energy levels. The VMI model is able to give very good fit for states up to spin  $I \sim 12$ . Recently Saethre et al.<sup>25</sup> have improved the fitting by using a three-parameter and a four-parameter cranking model formulas. The two-parameter VMI model has been shown<sup>7</sup> to be mathematically identical to the Harris fourth order cranking model; in addition, Klein et al.<sup>24</sup> have proved that the VMI model and cranking model are equivalent to all orders.

The microscopic derivation of the VMI model has been given in (I) and will be briefly outlined below. One first expresses the total energy of a rotating system as

$$E = \sum_i \frac{1}{2} C_i (x - x_i)^2 + \frac{I(I+1)}{2J(x_i)} \quad (2)$$

where the potential energy is expressed approximately as a sum of harmonic terms each of which represents contributions from various collective degrees of freedom denoted here by  $x_i$ .  $C_i$  is the spring constant associated with

the  $i$ -th degree of freedom. The second term is the kinetic energy. The rotational solutions are obtained by minimizing Eq. (2) with respect to various  $x_i$  at a given value of spin  $I$ . In the present calculation we introduce as collective degrees of freedom the quadrupole and hexadecapole shape deformations  $\epsilon_2$  and  $\epsilon_4$  involved in the centrifugal stretching effect; the proton and neutron pairing correlation parameters  $v_p$  and  $v_n$  involved in the Coriolis-anti-pairing effect; and a new collective variable  $\eta \equiv \omega^2$  involved in the fourth order cranking correction where  $\omega$  is the angular velocity. Thus, we define (see (I) for details)

$$\{x_1, x_2, x_3, x_4, x_5\} \equiv \{\epsilon_2, \epsilon_4, v_p, v_n, \eta\} \quad (3a)$$

$$\{c_1, c_2, c_3, c_4, c_5\} \equiv \{c_{22}, c_{44}, c_{vp}, c_{vn}, c_\eta\} \quad (3b)$$

We have not included the asymmetric degree of freedom (gamma shape vibration), since its contribution is rather insignificant as was shown by the calculations of Marshalek.<sup>16</sup>

In the first order approximation, Eq. (2) can be reduced to Eq. (1) through a normal coordinate transformation and one obtains<sup>17</sup>

$$c_{VMI}^{-1} = \sum_i \frac{1}{c_i} \left( \frac{\partial J(x_i)}{\partial x_i} \right)_{\{x_{i0}\}}^2 \quad (4)$$

where  $x_{i0}$  is the value of  $x_i$  at the ground state, thus  $\eta_0 \equiv 0$ . The moment-of-inertia  $J(x_i)$  can be expressed as

$$J(x_i) = J_0(x_1, x_2, x_3, x_4) + 2 c_\eta (x_1, x_2, x_3, x_4) \cdot \eta \quad (5)$$



The first term is the ground state moment-of-inertia which can be calculated by the well known cranking formula of Inglis<sup>22</sup> and Belyaev,<sup>26</sup>

$$J_0 = 2 \sum_{m_\alpha} \frac{|\langle \alpha' | j_x | \alpha \rangle|^2}{E_{\alpha'} + E_\alpha} (U_{\alpha'} V_\alpha - V_{\alpha'} U_\alpha)^2 \quad (6)$$

where  $|\alpha\rangle$  is the deformed single particle state with  $\alpha$  denoting the appropriate quantum numbers,  $m_\alpha$  is the magnetic quantum number along the symmetry axis,  $U_\alpha$  and  $V_\alpha$  are the probability amplitudes in the presence of pairing interaction and  $E_\alpha$  is the quasi-particle energy.

The Inglis and Belyaev cranking formula (6) is based on the independent quasi-particle approximation. However, a recent calculation by Meyer, Speth and Vogeler<sup>27</sup> showed that the two correction terms arising from the particle-particle and particle-hole interactions nearly cancel each other. It has also been shown by Rich<sup>28</sup> that correction due to particle-number conservation is also small. Thus it seems that the use of the cranking formula (6) is rather well justified numerically.

The second term of Eq. (5) represents the fourth order cranking correction which was first studied by Harris<sup>6</sup> and the fourth order cranking constant  $C_\eta$  can be expressed as<sup>6</sup>

$$C_\eta = 2 \sum_{m,n,p} \frac{\langle \psi_0 | j_x | \psi_m \rangle \langle \psi_m | j_x | \psi_p \rangle \langle \psi_p | j_x | \psi_n \rangle \langle \psi_n | j_x | \psi_0 \rangle}{(\epsilon_m - \epsilon_0) (\epsilon_n - \epsilon_0) (\epsilon_p - \epsilon_0)} - 2 \sum_{m,n} \frac{\langle \psi_m | j_x | \psi_0 \rangle^2 \langle \psi_n | j_x | \psi_0 \rangle^2}{(\epsilon_m - \epsilon_0)^2 (\epsilon_n - \epsilon_0)} \quad (7)$$

where the ground state  $\psi_0$  is the quasiparticle vacuum state,  $\psi_m$  and  $\psi_n$  are two-quasiparticle states and the intermediate state  $\psi_p$  can be either two-quasiparticle or four-quasiparticle excitations. The corresponding energies are denoted by  $\epsilon_0$ ,  $\epsilon_m$ ,  $\epsilon_n$  and  $\epsilon_p$ . The "prime" on the summation indicates that the ground state is excluded from the summation.

It is obvious from Eq. (5) that

$$\frac{\partial J}{\partial \eta} = 2C_\eta$$

Thus, the contribution of the fourth order cranking in Eq. (4) is simply  $4C_\eta$  while the contributions of the other degrees of freedom are given by

$$\frac{1}{C_i} \left( \frac{\partial J_0}{\partial x_i} \right)_{\{x_{i0}\}}^2$$

The B coefficient associated with the  $I^2(I+1)^2$  term in the angular momentum expansion of the rotational energy

$$E_I = \frac{I(I+1)}{2J_0} + B I^2(I+1)^2 + C I^3(I+1)^3 + \dots \quad (8)$$

can be expressed as

$$B = -\sum_i \frac{1}{8C_i J_0^4} \left( \frac{\partial J}{\partial x_i} \right)_{\{x_{i0}\}}^2 \quad (9)$$

The value of the force constant  $C_{VMI}^{-1}$  or the B coefficient indicates the degree to which the spectrum deviates from the  $I(I+1)$  rule. Both Eqs. (4) and (9) show that the contributions from various degrees of freedom are all positively added.

A simple relation between  $C_{\text{VMI}}$  and B can be obtained by combining Eq. (4) with Eq. (9), which yields

$$8 B C_{\text{VMI}} J_0^4 = -1 \quad . \quad (10)$$

### III. DETAILED CALCULATION AND FORMULAS

#### A. Single Particle Basis and the Pairing Problem

The deformed single particle basis used in the present calculations is chosen to be identical to that of Nilsson et al.<sup>23</sup> The diagonalization is carried out over the space of 11 shells for proton and 12 shells for neutron. The values of  $\epsilon_2$  and  $\epsilon_4$  of each nucleus are taken from the work of Nilsson et al. and are listed in Table I.

The pairing strength G is chosen to be a smooth function of A as suggested by Nilsson et al.

$$G \cdot A = g_0 \pm g_1 \frac{N-Z}{A}$$

$$g_0 = 19.2 \text{ MeV}$$

$$g_1 = 7.4 \text{ MeV} \quad (11)$$

with plus sign for protons and minus sign for neutrons. They also put in a linear surface dependence of G, which may be important for large deformation. The BCS equation is then solved by including  $(15Z)^{1/2}$  or  $(15N)^{1/2}$  states above and below the proton or neutron Fermi level. The pairing gap parameters  $\Delta_p$  and  $\Delta_n$  thus obtained are given in Table I.

The energy of a quasiparticle can be expressed as

$$E_k = (\epsilon_k - \lambda) (U_k^2 - V_k^2) + 2U_k V_k G \sum_{\ell > 0} U_\ell V_\ell \quad (12)$$

where  $\epsilon_k$  is the single-particle energy and  $\lambda$  is the chemical potential.

Following (I) we parametrize the probability amplitudes  $\{U_k, V_k\}$  by introducing a pairing correlation parameter  $v$

$$\frac{U_k^2}{V_k^2} = \frac{1}{2} \left[ 1 \pm \frac{\epsilon_k - \lambda}{\sqrt{(\epsilon_k - \lambda)^2 + v^2}} \right] \quad (13)$$

If  $v = \Delta$  (the energy gap  $\Delta$  is the equilibrium value of  $v$  at ground state),

Eq. (12) reduces to the familiar BCS result

$$E_k = \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2} \quad (14)$$

In what follows we shall vary  $v$  to calculate the corresponding derivatives of moment-of-inertia as well as the pairing spring constant for a fixed pairing strength  $G$  as given by Eq. (11). Since for  $v \neq \Delta$ , the BCS gap equation no longer holds and Eq. (14) is not valid. Thus, it is important to use Eq. (12) rather than Eq. (14) as the expression for the quasiparticle energy.

### B. Derivatives of the Moment-of-Inertia

We shall calculate the derivative of the moment-of-inertia  $J_0$  with respect to the pairing correlation parameter  $v$  while the average particle number  $n$  and the pairing strength  $G$  are held fixed. One obtains

$$\left(\frac{\partial J_0}{\partial v}\right)_{n,G} = \left(\frac{\partial J_0}{\partial v}\right)_{\lambda,G} + \left(\frac{\partial J_0}{\partial \lambda}\right)_{v,G} \left(\frac{\partial \lambda}{\partial v}\right)_{n,G} \quad (15)$$

where  $J_0$  is given by Eq. (6) and the average particle number  $n$  is given by

$$\sum_{k > 0} 2v_k^2 = n \quad (16)$$

It then follows

$$\left(\frac{\partial J_0}{\partial v}\right)_{\lambda,G|v=\Delta} = 2 \sum_{k,l} \frac{|\langle k | j_x | -l \rangle|^2}{E_k + E_l} (U_k v_l - U_l v_k) \cdot \left\{ \begin{aligned} & (U_l^2 - v_l^2) (U_k U_l + v_k v_l) / E_l \\ & - \frac{2U_l v_l (U_k v_l - v_k U_l)}{E_k + E_l} \cdot \frac{G}{\Delta} \sum_{m > 0} U_m v_m (U_m^2 - v_m^2)^2 \end{aligned} \right\} \quad (17a)$$

$$\left(\frac{\partial J_0}{\partial \lambda}\right)_{v,G|v=\Delta} = 2 \sum_{k,l} \frac{|\langle k | j_x | -l \rangle|^2}{E_k + E_l} (U_k v_l - U_l v_k) \cdot \left\{ \begin{aligned} & 2U_l v_l (U_k U_l + v_k v_l) / E_l \\ & + \frac{(U_k v_l - U_l v_k)}{E_k + E_l} \left[ (U_l^2 - v_l^2) - 4U_l v_l \frac{G}{\Delta} \sum_{m > 0} U_m^2 v_m^2 (U_m^2 - v_m^2) \right] \end{aligned} \right\} \quad (17b)$$

$$\left(\frac{\partial \lambda}{\partial v}\right)_{n,G} = - \sum_{k > 0} U_k^2 v_k^2 (U_k^2 - v_k^2) / 2 \sum_{k > 0} U_k^3 v_k^3 \quad (17c)$$

Note that in taking the derivative, the quasiparticle energy is given by Eq. (12). After the derivative is taken, its value at  $\nu = \Delta$  is then evaluated.

The derivatives of  $J_0$  with respect to the deformation parameters  $\epsilon_2$  and  $\epsilon_4$  are calculated by finite differences. The mesh point interval is taken to be 0.02 for both  $\epsilon_2$  and  $\epsilon_4$ .

The various derivatives of the moment-of-inertia are listed in Table II. The derivative with respect to pairing are quite stable over the rare-earth region of nuclei. For example, with respect to the neutron pairing, the derivatives fluctuate around  $-(36 \pm 10) \text{ MeV}^{-2}$ , while the derivatives with respect to proton pairing are roughly equal to  $-(19 \pm 3) \text{ MeV}^{-2}$  for nuclei in the region of  $A \sim 165$  and  $-(11 \pm 2) \text{ MeV}^{-2}$  for those in the region of  $A \sim 187$ . The derivatives with respect to the deformation, on the other hand, are quite different as one goes from one nucleus to another. In the case of quadrupole deformation ( $\epsilon_2$ ), the derivatives are largest at the beginning of the rare-earth region and generally decrease towards the end of the region; while in the case of hexadecapole deformation ( $\epsilon_4$ ), the derivatives are strongly negative at the beginning of the rare-earth region and change to positive values near the end. A negative value for the inertia derivative with respect to hexadecapole deformation ( $\epsilon_4$ ) has some interesting consequences. The equilibrium value of  $\epsilon_4$  at a given spin  $I$  in first order approximation is given in (I) as

$$\epsilon_4 = \epsilon_{40} + \frac{I(I+1)}{2C_{44}J^2} \frac{\partial J}{\partial \epsilon_4} \quad (18)$$

where  $\epsilon_{40}$  is the hexadecapole deformation at ground state. In the beginning

of the rare-earth region, the values of both  $\epsilon_{40}$  and the inertia derivative with respect to  $\epsilon_4$  are negative, thus, as the spin goes up, the hexadecapole deformation will increase in magnitude which is just the familiar stretching effect. However, for nuclei in the middle of the rare-earth region, the values of  $\epsilon_{40}$  become positive while in most cases the inertia derivatives with respect to  $\epsilon_4$  still remain negative. Thus, as the spin goes up, the hexadecapole deformation will actually decrease.

Equilibrium values of quadrupole deformation  $\epsilon_2$  and of pairing parameters  $v_p$  and  $v_n$  at a given spin  $I$  can also be determined by equations similar to Eq. (18) which then yield the quadrupole stretching and the Coriolis-anti-pairing effect.

### C. Pairing Spring Constant

The ground state energy can be expressed as

$$\begin{aligned} \epsilon_0 = & \sum_{k > 0} \epsilon_k 2v_k^2 - G \left( \sum_{k > 0} U_k v_k \right)^2 - G \sum_{k > 0} v_k^4 \\ & - G \sum_{k \neq l > 0} U_k v_k^3 U_l^3 v_l^3 / \sum_{k > 0} U_k^2 v_k^2 \end{aligned} \quad (19)$$

where the first three terms are the BCS ground state energy, and the last term approximately accounts for the fixed-particle-number correction. <sup>29,30</sup>

The pairing spring constant  $C_v$  can be obtained by taking the second derivative of  $\epsilon_0$  with respect to the pairing correlation parameter  $v$

$$C_v = \left( \frac{\partial^2 \epsilon_0}{\partial v^2} \right)_{n,G}$$

$$\begin{aligned}
&= \left( \frac{\partial^2 \epsilon_0}{\partial v^2} \right)_\lambda + 2 \left( \frac{\partial^2 \epsilon_0}{\partial \lambda \partial v} \right) \left( \frac{\partial \lambda}{\partial v} \right)_n + \left( \frac{\partial^2 \epsilon_0}{\partial \lambda^2} \right)_v \left( \frac{\partial \lambda}{\partial v} \right)_n^2 \\
&\quad + \left( \frac{\partial \epsilon_0}{\partial \lambda} \right)_v \left( \frac{\partial^2 \lambda}{\partial v^2} \right)_n
\end{aligned} \tag{20}$$

Evaluation of the derivatives of  $\epsilon_0$  with respect to  $\lambda$  and  $v$  are straightforward by using Eq. (19). The derivative of  $\lambda$  with respect to  $v$  can also be easily obtained in terms of Eq. (16). The proton and neutron pairing spring constants  $C_{vp}$  and  $C_{vn}$  as calculated by Eq. (20) are given in Table III.

It is interesting to note that inclusion of the fixed-particle-number correction in the ground state energy will in general increase the pairing spring constant by about 20 percent, hence, reduces the CAP effect. Some of the examples are given in Table IV.

A simple formula for  $C_v$  based on the continuous model is given in (I) which reads

$$C_v = \left( 2\rho + \frac{2}{\pi\Delta} \right) (1 - \rho G) \tag{21}$$

where  $\rho$  is the average nucleon orbital level density. The spring constant given by Eq. (21) (see Table I of (I)) are somewhat larger than those given by the present calculations by about 5 to 15 percent in the case of proton and 10 to 25 percent in the case of neutron. In view of the tremendous numerical work involved in Eq. (20), the simple formula Eq. (21) is indeed a very useful approximation.



D. Shape Spring Constant

The shape spring constants  $C_{22}$  and  $C_{44}$  associated with the quadrupole  $\epsilon_2$  and hexadecapole  $\epsilon_4$  deformation degrees of freedom can be obtained similarly by taking the second derivative of the ground state energy  $\epsilon_0$  with respect to  $\epsilon_2$  and  $\epsilon_4$

$$C_{22} = \frac{\partial^2 \epsilon_0}{\partial \epsilon_2^2}, \quad C_{44} = \frac{\partial^2 \epsilon_0}{\partial \epsilon_4^2} \quad (22)$$

In applying Eq. (22), the ground state energy  $\epsilon_0$  is calculated according to the Strutinsky average prescription as described in Ref. 23 by Nilsson et al. The  $C_{22}$  and  $C_{44}$  are then obtained by finite differences with the mesh point interval taken to be 0.02 both for  $\epsilon_2$  and  $\epsilon_4$ ; the results are listed in Table III.

The curvatures  $C_{22}$  and  $C_{44}$  at the ground state deformation are due to contributions from the liquid drop energy part, the shell correction part and the pairing energy part, which make up the potential energy surface. The shell correction part gives the largest positive contribution and in fact determines, to the larger extent, the deformation of the ground state nucleus. The pairing effect tends to smooth out the level density and thus acts against the shell effect. It provides a negative contribution to the curvature. The liquid drop energy part in general gives a small positive contribution.

The Strutinsky normalization replaces the smooth part of the eigenenergy summation by the liquid drop energy. Due to the inadequacy of volume normalization of the single particle potential well, the former has a much stronger curvature than that of the liquid drop part, as is obvious from the fact that its value would be infinite at  $\epsilon = 1.5$  (which is of course quite far

away from the ground state deformation of  $\epsilon \approx 0.25$ ), whereas the liquid drop energy would be finite. Hence, one would expect the value of curvature calculated in a scheme with the employment of Strutinsky normalization to be smaller than the value calculated without it. This is indeed borne out by our detailed calculation which show that the Strutinsky normalization generally reduces the values by about 20 percent.

On comparison with Table I of (I), our present results for  $C_{22}$  are about 40 to 100 percent larger. The first reason is that we are currently using a finer set of grids,  $\Delta\epsilon = 0.02$ , as compared with  $\Delta\epsilon = 0.05$  used in the older calculation. Thus, the new calculations are less likely to suffer from the problem of anharmonic effects in the potential energy surface which, in the present case, will tend to reduce the effective value of the curvature. The second and probably the main reason behind the discrepancy is that the older calculation used a surface-independent pairing force; whereas the new calculation has a pairing force dependent on the surface area. For most properties of the nucleus near the ground state, this difference does not present significant discrepancy. But for such higher-order effect as the curvature, we find it does make a difference. When we calculated the pairing energy contribution, we found that the new calculation with a surface-dependent pairing force gives a smaller negative value than the old calculation, and the change is sufficient to account for the discrepancy between the two results. A detailed discussion was made by other authors<sup>23,34</sup> on the choice of these two versions of the pairing force. We have not pursued the question regarding which is the more appropriate form of pairing force to be used. However, as will be seen later, the contribution of centrifugal stretching effect to the VMI

force constant is very small when compared with the contributions from Coriolis-anti-pairing and fourth-order cranking effect, so that either choice of the pairing force will have little effect on our final results and conclusions.

E. Fourth Order Cranking Constant  $C_\eta$

The evaluation of the fourth order cranking constant  $C_\eta$  given by Eq. (7) is rather tedious, since now one has to calculate the contributions from the four-quasiparticle intermediate states as well as those from the two-quasiparticle states. Fortunately, we are able to reduce Eq. (7) to a sum of quadratic terms; as a result, the numerical work is considerably simplified. We shall quote the final result below while the derivation will be given in the appendix.

$$\begin{aligned}
 C_\eta = & -4 \sum_{\substack{m_p > 0 \\ m_q = m_p - 1 \\ m_{q'} = m_p + 1}} \sum_{t_q, t_{q'}} (E_q + E_{q'}) \cdot \\
 & \cdot \left[ \sum_{t_p} \langle p | j_x | q \rangle \langle p | j_x | q' \rangle \frac{(U_{p,q} V_q - V_{p,q} U_q) (U_{p,q'} V_{q'} - U_{q',p} V_p)}{(E_p + E_q) (E_p + E_{q'})} \right]^2 \\
 & - 2 \sum_{\substack{m_p = m_{p'} > 0 \\ m_q = m_p + 1}} \sum_{t_p, t_{p'}} (E_p + E_{p'}) \cdot \\
 & \cdot \left[ \sum_{t_q} \frac{\langle p | j_x | q \rangle \langle p' | j_x | q \rangle (U_{p,q} V_q - V_{p,q} U_q) (U_{p',q} V_q - V_{p',q} U_q)}{(E_p + E_q) (E_{p'} + E_q)} \right]^2 +
 \end{aligned}$$

$$+4 \sum_{\substack{m_p \neq 0 \\ m_q = m_p - 1 \\ m_{q'} = m_p + 1}} \sum_{t_q, t_{q'}} \frac{1}{E_q + E_{q'}} \left\{ \sum_{t_p} \langle p | j_x | \varphi \rangle \langle p | j_x | \varphi' \rangle \right.$$

$$\left. \left[ \frac{(U_{p q} V_{p q} - V_{p q} U_{p q}) (U_{p q'} U_{p q'} + V_{p q'} V_{p q'})}{E_p + E_q} \right]^2 + \left[ \frac{(U_{p q'} V_{p q'} - V_{p q'} U_{p q'}) (U_{p q} U_{p q} + V_{p q} V_{p q})}{E_p + E_{q'}} \right]^2 \right\}$$

$$+2 \sum_{m_p = m_{p'}} \sum_{t_p, t_{p'}} \frac{1}{E_p + E_{p'}}$$

$$\left\{ \sum_{\substack{t_q \\ m_q = m_p \pm 1}} \langle p | j_x | \varphi \rangle \langle p' | j_x | \varphi' \rangle \right.$$

$$\left. \left[ \frac{(U_{p q} V_{p q} - V_{p q} U_{p q}) (U_{p' q} U_{p' q} + V_{p' q} V_{p' q})}{E_p + E_q} \right]^2 + \left[ \frac{(U_{p' q} V_{p' q} - V_{p' q} U_{p' q}) (U_{p q} U_{p q} + V_{p q} V_{p q})}{E_{p'} + E_q} \right]^2 \right\}$$

(23)

where  $m_p$  is the magnetic quantum number of the particle in state  $|p\rangle$ ,  $t_p$  denotes the quantum numbers other than  $m_p$  and  $E_p$  is the corresponding quasi-particle energy. The first two terms represent the contributions from four-quasiparticle intermediate states while the last two terms represent those from two-quasiparticle intermediate states, as can readily be seen from the form of the products of the  $U, V$  coefficients. The fourth order cranking constants

$C_{\eta}$  calculated in terms of Eq. (23) are listed in Table III. Our calculations show that the two-quasiparticle contribution is always positive while the four-quasiparticle contribution is always negative. Furthermore, the former is generally about three to four times larger than the latter in magnitude. Some of the examples are given in Table IV.

#### IV. RESULTS AND DISCUSSIONS

##### A. Moment-of-Inertia

The ground state moment-of-inertia  $J_0$  is calculated according to Eq. (6). In addition, we have followed Nilsson and Prior<sup>31</sup> and increased the calculated values by 5 percent which represents approximately the effects of the coupling between the shells  $N$  and  $N + 2$ , due to the  $j_x$  operator. This is because the Nilsson wave functions of Ref. 23 are expressed in the stretched coordinates and the  $j_x$  operator in these stretched coordinates will give rise to a term which will couple the shells  $N$  and  $N + 2$ . The results are listed in Table II and plotted in Fig. 1. From Fig. 1, it is seen that the trends of the experimental moment-of-inertia are well reproduced by the calculations. The calculated magnitudes, however, are generally too small by 10 to 40 percent, the average discrepancy being 25 percent. This disagreement in magnitude seems to be somewhat too large compared to a similar calculation by Nilsson and Prior<sup>31</sup> where the calculated  $J_0$  are generally 10 to 30 percent too small, the average discrepancy being 20 percent. But it should be pointed out that in the present calculation, the single particle states and the parameters  $G$ ,  $\epsilon_2$ ,  $\epsilon_4$  are all taken directly from the works of Nilsson et al.<sup>23</sup> without any readjustment. One may, for example, obtain very good agreement with the experimental data by choosing  $g_0 = 18.0$  MeV instead of 19.2 MeV in Eq. (11). We shall return to this question at the end of this section.

##### B. Force Constant $C_{VMI}$

The force constant  $C_{VMI}^{-1}$  associated with the VMI model of Eq. (1) is calculated using Eq. (4) and the results are listed in Table V and plotted in Fig. 2. The contributions to  $C_{VMI}^{-1}$  from various sources are also given

separately in Table V. One notices first that except for nuclei with neutron number  $N = 90$  and  $92$ , both quadrupole and hexadecapole centrifugal stretching contribute very little to the energy deviation from the  $I(I+1)$  rule. Typically they amount to only a few percent of the total contribution and hence in most cases can be entirely neglected. This result is consistent with other microscopic calculations and also with experimental observations.<sup>32</sup> It is important to note that the contributions of hexadecapole stretching are comparable with those of quadrupole stretching. Hence they should both be taken into account in other relevant analyses, such as the study<sup>16</sup> of change of nuclear mean-square radius  $\Delta \langle r^2 \rangle$  or the study of the deviation of the transition probability from the rigid rotor formula.

It is shown in Table V that the neutron Coriolis-anti-pairing and the fourth-order cranking corrections are the two largest contributions and are comparable with each other. The proton Coriolis-anti-pairing term is relatively smaller and amounts to about 10 to 20 percent of the total contribution. In general, the present results are very different quantitatively from those of (I). However, many qualitative discussions of (I) are still valid.

We observe in Fig. 2 that except for nuclei with neutron number  $N = 90$ , 104 and 108, both the experimental trend and magnitude of the force constant  $C_{VMI}^{-1}$  are fairly well reproduced in general by the present calculation. In most cases the discrepancy ranges from 10 to 40 percent; the average discrepancy for all nuclei (excluding those with  $N = 90$ ) is about 34 percent.

The large discrepancies of the calculated force constants which occurred at neutron number  $N = 90, 104$  and  $108$  deserve more careful study. Perhaps the  $90$ -neutron nuclei are so close to being shape unstable that the present model of stable deformation may be somewhat questionable. This argument, however, cannot be applied to nuclei with  $N = 104$  and  $108$ , all of which appear to be good rotors. We then compare the calculated Nilsson single neutron levels around  $N = 104$  and  $108$  with those deduced semi-empirically by Ogle *et al.*<sup>33</sup> and are not surprised to find some discrepancies between them. Consequently, we make the following preliminary neutron level shifts

$$\text{Calculation B: } \begin{cases} [512, 5/2^-]_n + 0.05 \hbar\omega \\ [510, 1/2^-]_n - 0.05 \hbar\omega \\ [512, 3/2^-]_n - 0.05 \hbar\omega \end{cases} \quad \text{for } A \geq 170 \quad (24)$$

With the above neutron level shifts and assuming further that the wave function and the quadrupole and hexadecapole stretching are the same, we repeat the calculation on the moment-of-inertia and the force constant which will be called calculation B while the previous calculation without level shifts will be called calculation A. The results of calculation B are listed in Table VI. In general, the results of calculation B are similar to those of calculation A except for nuclei around neutron number  $N = 104$  and  $108$ . Note in Table VI that the moments-of-inertias from calculation B change only slightly over the results of calculation A. On the other hand, the force constants of calculation B are considerably improved over the results of calculation A around  $N = 104$  and  $108$ , as can be seen in Fig. 3. The serious discrepancies of calculation A which occurred at  $N = 104$  and  $108$  are now much reduced; in most cases, both the trend and magnitude of the



experimental force constants are now fairly well reproduced. However, in addition to the 90-neutron nuclei there still remain three nuclei ( $^{170}_{102}\text{Er}$ ,  $^{174}_{104}\text{Yb}$  and  $^{180}_{108}\text{Hf}$ ) whose calculated force constants are a factor of 2 too large compared to the experiment. The average discrepancy for all nuclei (excluding  $N = 90$ ) is now 29 percent. Thus the agreement between the theoretical and experimental values of the force constant is comparable to that of the moment-of-inertia calculations.

The above results indicate that the force constant  $C_{\text{VMI}}$  which represents the higher-order effects in the rotational energy calculation is much more sensitive to the single particle levels than the moment-of-inertia. We have no intention here to do a detailed searching for better single neutron levels. Instead, the emphasis is to indicate that by removing the discrepancy of the Nilsson neutron levels around  $N = 104$  and  $108$  (although only in a preliminary way), the calculated results of the force constant  $C_{\text{VMI}}^{-1}$  can be considerably improved.

### C. B Coefficients

The B coefficient associated with the  $I^2(I+1)^2$  term in an expansion of the rotational energy can be evaluated either by Eq. (9) or in a more straightforward way, since we already know  $J_0$  and  $C_{\text{VMI}}^{-1}$ , by Eq. (10). The results of calculation B are given in Table VI and plotted in Fig. 4. The experimental B coefficients are obtained by a least-squared-fit to the first three excited states by using the first three terms in Eq. (8) with  $J_0$  taken from Ref. 7. It is seen in Fig. 4 that the trend of the B coefficient is fairly well reproduced; the calculated magnitudes, however, are generally too large by a factor of 2 to 5. Thus, the agreements are much worse than those of the force constant  $C_{\text{VMI}}^{-1}$  although both of them represent the higher-order effects. The reason is easy to understand, because the B coefficient depends on the

inverse fourth power of the moment-of-inertia  $J_0$  according to Eq. (10). Our calculated  $J_0$  are roughly 10 to 40 percent smaller while our calculated  $C_{VMI}^{-1}$  are roughly 10 to 40 percent larger; so combined they yield the B values by a factor 2 to 5 too large.

Because of this  $J_0^{-4}$  dependence, it seems that in order to get reasonable agreement for B coefficient one probably should first fit the moment-of-inertia as accurately as possible. We have mentioned in the beginning of this section that very good agreements of the moment-of-inertia could be achieved provided one uses a reduced pairing strength  $g_0 = 18.0$  MeV instead of 19.2 MeV in Eq. (11). The values of  $J_0$ , B and  $C_{VMI}^{-1}$  calculated with  $g_0 = 18.0$  MeV and without neutron levels shifts are listed in Table VII. In addition, the second set of results of  $J_0$  and B of Marshalek<sup>16</sup> which are obtained by adjusting the pairing strength so as to exactly reproduce the experimental moment-of-inertia are also included for comparison. Our results are roughly similar to those of Marshalek at the middle of the rare-earth region, though discrepancies occur at both ends of this region. Note also in Table VII that our B values are now improved over those obtained previously with  $g_0 = 19.2$  MeV, although they are still too large by a factor of 1.5 to 3 in general. On the other hand, however, the force constants  $C_{VMI}^{-1}$  in Table VII are much worse than those obtained previously with  $g_0 = 19.2$  MeV.

We seem to be in a very interesting situation. On the one hand, our calculation with pairing strength  $g_0 = 19.2$  MeV is able to reproduce fairly well  $J_0$  and  $C_{VMI}^{-1}$ ; however, it yields very poor B coefficients. On the other hand, very good  $J_0$  and improved values of B (but still too large by a factor of 1.5 to 3) could be obtained from calculation with the reduced pairing

strength  $g_0 = 18.0$  MeV, but now the  $C_{VMI}$  becomes very poor. We feel that an accurate microscopic theory should be able to reproduce both force constant and the B coefficient correctly. For calculations involving as many approximations as these, however, we suggest that the force constant  $C_{VMI}$  is a more meaningful quantity to be compared with. The reasons are as follow: .1) Because of the  $J_0^{-4}$  dependence, the large discrepancy of the calculated B coefficient may be misleading since it may essentially be a result of small to moderate deviation in  $J_0$ . It is also probably misleading for one to obtain better B coefficient by adjusting the pairing strength alone in order to reduce the error arising from the  $J_0^{-4}$  dependence, because in doing so, the force constant  $C_{VMI}$  will become unduly worse. 2) It is well known that the expansion of the rotational energy in terms of the angular velocity  $\omega^2$  is much better than the expansion in terms of the angular momentum  $I(I+1)$ . Thus, the force constant  $C_{VMI}$  also appears to be a more physically significant parameter than the B coefficient.

### V. SUMMARY

Based on the microscopic cranking model, the present calculations are able to reproduce fairly well the moment-of-inertia  $J_0$  and the force constant  $C_{\text{VMI}}$  associated with the VMI model with discrepancy ranging from 10 to 40 percent in general. On the other hand, the calculated B coefficients are quantitatively poor, which resemble the calculations of Marshalek.<sup>16</sup> However, as we have mentioned, one must use care in interpreting the large discrepancy of the B coefficient because of its  $J_0^{-4}$  dependence.

We have taken into account the fixed-particle-number correction for the potential energy; obviously it will also affect the moment-of-inertia and the fourth order cranking calculations. According to the calculation of Rich,<sup>28</sup> the fixed-particle-number correction will reduce the inertia derivative with respect to pairing by 10 to 20 percent. Since the force constant  $C_{\text{VMI}}^{-1}$  is proportional to the square of the inertia derivative, this will cause 20 to 50 percent reduction of the Coriolis-anti-pairing effect, which is in the right direction of improvement. We feel that the present approach is not accurate enough to study nuclear rotation at very high spin; to do that, the perturbation treatment on  $\omega J_x$  probably will have to be avoided altogether.

### ACKNOWLEDGMENTS

We are grateful to S. Y. Chu for helpful discussions and for assistance in numerical calculations, and to J. O. Rasmussen for many valuable suggestions. We also wish to thank J. O. Rasmussen and J. Meyer ter Vehn for critical reading of the manuscript.

APPENDIX

We first express the  $J_x$  operator in the quasi-particle representation as

$$J_x = J_{11} + J_{20} \tag{A.1}$$

with

$$\langle p | J_{11} | q \rangle = \langle p | j_x | q \rangle (U_p U_q + V_p V_q) \tag{A.2a}$$

$$\langle 0 | J_{20} | pq \rangle = (-1)^{m_q + \frac{1}{2}} \langle p | j_x | -q \rangle (U_p V_q - V_p U_q) \tag{A.2b}$$

where  $|0\rangle$  is the quasi-particle vacuum state. Note that  $J_x$  only operates between states with  $J_z$  components differed by  $\pm 1$ .

Consider now the contribution of the four-quasiparticle excitations to the first term in Eq. (7)

$$c'_\eta(4QP) = 1/8 \cdot 2 \sum_{\substack{(pq)\pm 1 \\ (p'q')\pm 1 \\ (p_1q_1)\pm 1 \\ p \neq q \neq p' \neq q'}} \frac{\langle 0 | J_{20} | pq \rangle}{(E_p + E_q)} \times$$

$$\times \frac{\langle pq | J_{20} | pq, p'q' \rangle \langle pq, p'q' | J_{20} | p_1q_1 \rangle \langle p_1q_1 | J_{20} | 0 \rangle}{(E_p + E_q + E_{p'} + E_{q'}) (E_{p_1} + E_{q_1})}$$

where  $(pq)_{\pm 1}$  denotes a complete set of two-quasiparticle states  $|pq\rangle$  with  $J_z$  component  $m_p + m_q = \pm 1$ . Since for each  $|pq\rangle$  there is a corresponding state  $|qp\rangle$ , the factor  $1/8$  thus accounts for the double counting of  $pq$ ,  $p'q'$  and  $p_1q_1$ . Writing the non-zero terms of the above expression explicitly one obtains

$$c'_\eta(4QP) = \frac{1}{2} \sum_{\substack{(pq)_{\pm 1} \\ (p'q')_{\pm 1} \\ p \neq q \neq p' \neq q'}} \left\{ \frac{\langle 0 | J_{20} | pq \rangle^2 \langle 0 | J_{20} | p'q' \rangle}{(E_p + E_q)^2 (E_{p'} + E_{q'})} \right.$$

$$- \frac{\langle 0 | J_{20} | pq \rangle \langle 0 | J_{20} | p'q' \rangle \langle 0 | J_{20} | pq' \rangle \langle 0 | J_{20} | p'q \rangle}{(E_p + E_q)(E_p + E_{q'})(E_{p'} + E_q)}$$

$$\left. = \frac{\langle 0 | J_{20} | pq \rangle \langle 0 | J_{20} | p'q' \rangle \langle 0 | J_{20} | pp' \rangle \langle 0 | J_{20} | qq' \rangle}{(E_p + E_q)(E_p + E_{p'})(E_q + E_{q'})} \right\}$$

We notice first that the constraint  $p \neq q \neq p' \neq q'$  can be dropped because the additional terms thus created will cancel each other. Secondly, the third term may be made equal to the second term by exchanging  $p'$  with  $q'$ . It then follows

$$c'_\eta(4QP) = \frac{1}{2} \sum_{\substack{(pq)_{\pm 1} \\ (p'q')_{\pm 1}}} \frac{\langle 0 | J_{20} | pq \rangle^2 \langle 0 | J_{20} | p'q' \rangle^2}{(E_p + E_q)^2 (E_{p'} + E_{q'})}$$

$$- \frac{1}{2} \sum_{\substack{(pq)_{\pm 1} \\ (p'q')_{\pm 1}}} \frac{\langle 0 | J_{20} | pq \rangle \langle 0 | J_{20} | p'q' \rangle \langle 0 | J_{20} | pq' \rangle \langle 0 | J_{20} | p'q \rangle}{(E_p + E_q)(E_{p'} + E_{q'})(E_p + E_{q'})(E_{p'} + E_q)} \quad \times \quad (A.3)$$

$$\times (E_p + E_{p'} + E_q + E_{q'})$$

The first term in this equation obviously cancels the second term in Eq. (7); we thus defined the full contributions of the four-quasiparticle excitations as

$$C_{\eta}(4QP) = C'_{\eta}(4QP) + \text{second term of Eq. (7)}$$

$$= \text{second term of (A.3)}$$

The summation of the second term in Eq. (A.3) can be separated into four terms

$$\sum_{\substack{(pq)\pm 1 \\ (p'q')\pm 1}} = \sum_{\substack{(pq)_1 \\ (p'q')_1}} + \sum_{\substack{(pq)_1 \\ (p'q')_{-1}}} + \sum_{\substack{(pq)_{-1} \\ (p'q')_1}} + \sum_{\substack{(pq)_{-1} \\ (p'q')_{-1}}}$$

The first term yields

$$\sum_{\substack{(pq)_1 \\ (p'q')_1}} = -\frac{1}{2} \sum_{\substack{m_p = \pm 1/2, \pm 3/2, \dots \\ m_{p'} = m_p \\ m_q = m_{q'} = -m_p + 1}} \sum_{\substack{t_p, t_{p'} \\ t_q, t_{q'}}} \frac{\langle 0 | J_{20} | pq \rangle \langle 0 | J_{20} | p'q' \rangle \langle 0 | J_{20} | pq' \rangle \langle 0 | J_{20} | p'q \rangle}{(E_p + E_q)(E_{p'} + E_{q'})(E_p + E_{q'})(E_{p'} + E_q)} \times (E_p + E_{p'} + E_q + E_{q'})$$

$$= -\sum_{\substack{m_p = \pm 1/2, \pm 3/2, \dots \\ m_{p'} = m_p \\ m_q = -m_p + 1}} \sum_{t_p, t_{p'}}^{(E_p + E_{p'})} \left[ \sum_{t_q} \frac{\langle 0 | J_{20} | pq \rangle \langle 0 | J_{20} | p'q' \rangle}{(E_p + E_q)(E_{p'} + E_q)} \right]^2$$

where we have applied the conditions

$$m_p + m_{q'} = \pm 1, \quad m_{p'} + m_q = \pm 1$$

The other three terms can be evaluated in a similar way, and we obtain finally

$$\begin{aligned}
 C_{\eta}(4QP) = & -4 \sum_{\substack{m_p > 0 \\ m_q = -m_p + 1 \\ m_{q'} = -m_p - 1}} \sum_{t_q, t_{q'}} (E_q + E_{q'}) \left[ \sum_{t_p} \frac{\langle 0 | J_{20} | pq \rangle \langle 0 | J_{20} | pq' \rangle}{(E_p + E_q)(E_p + E_{q'})} \right]^2 \\
 & - 2 \sum_{m_{p'} = m_p > 0} \sum_{t_p, t_{p'}} (E_p + E_{p'}) \left[ \sum_{t_q} \frac{\langle 0 | J_{20} | pq \rangle \langle 0 | J_{20} | p'q \rangle}{(E_p + E_q)(E_{p'} + E_q)} \right]^2.
 \end{aligned} \tag{A.4}$$

Substitution of Eq. (A.2) into Eq. (A.4) then yields the first two terms in Eq. (23).

The contribution of the two-quasiparticle excitations to the first term in Eq. (7) is

$$\begin{aligned}
 C_{\eta}(2QP) = & 1/8 \cdot 2 \sum_{\substack{(pq) \pm 1 \\ (p'q') 0, \pm 2 \\ (p_1 q_1) \pm 1}} \frac{\langle 0 | J_{20} | pq \rangle \langle pq | J_{11} | p'q' \rangle \langle p'q' | J_{11} | p_1 q_1 \rangle}{(E_p + E_q)(E_{p'} + E_{q'})} \times \\
 & \times \frac{\langle p_1 q_1 | J_{20} | 0 \rangle}{(E_{p_1} + E_{q_1})}
 \end{aligned}$$

Since

$$\begin{aligned}
 \langle pq | J_{11} | p'q' \rangle = & \delta_{pp'} \langle q | J_{11} | q' \rangle + \delta_{qq'} \langle p | J_{11} | p' \rangle - \delta_{pq} \langle q | J_{11} | p' \rangle \\
 & - \delta_{qp'} \langle p | J_{11} | q' \rangle
 \end{aligned}$$



one obtains

$$\begin{aligned}
 c_{\eta}(2QP) = & 2 \sum_{\substack{(pq)\pm 1 \\ (pq')_0, \pm 2 \\ (pq_1)\pm 1}} \frac{\langle 0 | J_{20} | pq \rangle \langle q | J_{11} | q' \rangle \langle q' | J_{11} | q_1 \rangle \langle pq_1 | J_{20} | 0 \rangle}{(E_p + E_q)(E_p + E_{q'})(E_p + E_{q_1})} \\
 & + 2 \sum_{\substack{(pq)\pm 1 \\ (pq')_0, \pm 2 \\ (p'q')\pm 1}} \frac{\langle 0 | J_{20} | pq \rangle \langle q | J_{11} | q' \rangle \langle p | J_{11} | p' \rangle \langle p' q' | J_{20} | 0 \rangle}{(E_p + E_q)(E_p + E_{q'})(E_{p'} + E_{q'})}
 \end{aligned}$$

Recalling that  $J_x$  only operates between states with  $J_z$  components differed by  $\pm 1$ , we get

$$\begin{aligned}
 c_{\eta}(2QP) = & \left\{ \begin{array}{ccc} 4 \sum_{(pq)_1} & + & 4 \sum_{(pq)_1} & + & 4 \sum_{(pq)_1} \end{array} \right\} \times \\
 & \begin{array}{ccc} (pq')_0 & (pq')_0 & (pq')_2 \\ (pq_1)_1 & (pq_1)_{-1} & (pq_1)_1 \end{array} \\
 & \times \left( \frac{\langle 0 | J_{20} | pq \rangle \langle q | J_{11} | q' \rangle \langle q' | J_{11} | q_1 \rangle \langle pq_1 | J_{20} | 0 \rangle}{(E_p + E_q)(E_p + E_{q'})(E_p + E_{q_1})} \right) \\
 & + \left\{ \begin{array}{ccc} 4 \sum_{(pq)_1} & + & 4 \sum_{(pq)_1} & + & \sum_{(pq)_1} \end{array} \right\} \times \\
 & \begin{array}{ccc} (pq')_0 & (pq')_0 & (pq')_2 \\ (p'q')_1 & (p'q')_{-1} & (p'q')_1 \end{array} \\
 & \times \left( \frac{\langle 0 | J_{20} | pq \rangle \langle q | J_{11} | q' \rangle \langle p | J_{11} | p' \rangle \langle p' q' | J_{20} | 0 \rangle}{(E_p + E_q)(E_q + E_{q'}) (E_{p'} + E_{q'})} \right) \tag{A.5}
 \end{aligned}$$

where we have used the concise notation

$$\{\Sigma_1 + \Sigma_2 + \Sigma_3\} (A) = \Sigma_1 A + \Sigma_2 A + \Sigma_3 A$$

We regroup the third and the sixth term in Eq. (A.5) where the  $J_z$  components of the two-quasiparticle intermediate states are  $m_p + m_{q'} = 2$ , and obtain

$$C'_\eta (2QP) = 4 \sum_{\substack{m_p = \pm 1/2, \pm 3/2, \dots \\ m_{q'} = -m_p + 2}} \sum_{t_p, t_{q'}} \frac{1}{E_p + E_{q'}} \left[ \sum_{m_q = -m_p + 1} t_q \frac{\langle 0 | J_{20} | pq \rangle \langle q | J_{11} | q' \rangle}{E_p + E_q} \right]^2$$

$$+ 4 \sum_{\substack{m_p = \pm 1/2, \pm 3/2, \dots \\ m_{q'} = -m_p + 2}} \sum_{t_p, t_{q'}} \frac{1}{E_p + E_{q'}} \left[ \sum_{m_q = -m_p + 1} t_q \frac{\langle 0 | J_{20} | pq \rangle \langle q | J_{11} | q' \rangle}{E_p + E_q} \right] \chi$$

$$\chi \left[ \sum_{m_q = m_p - 1} t_q \frac{\langle 0 | J_{20} | qq' \rangle \langle p | J_{11} | q \rangle}{E_{q'} + E_q} \right]$$

The above expression can be rewritten finally as

$$C'_\eta (2QP) = 4 \sum_{\substack{m_p \geq 3/2 \\ m_{q'} = -m_p + 2}} \sum_{t_p, t_{q'}} \frac{1}{E_p + E_{q'}} \left[ \sum_{m_q = -m_p + 1} t_q \frac{\langle 0 | J_{20} | pq \rangle \langle q | J_{11} | q' \rangle}{E_p + E_q} + \sum_{m_q = m_p - 1} t_q \frac{\langle 0 | J_{20} | qq' \rangle \langle p | J_{11} | q \rangle}{E_{q'} + E_q} \right]^2 \quad (A.6)$$

We then regroup the remaining four terms in Eq. (A.5) where the  $J_z$  components of the two-quasiparticle intermediate states are  $m_p + m_q = 0$ , and obtain

$$\begin{aligned}
 C_{\eta}''(2QP) = & 4 \sum_{\substack{m_p = \pm 1/2, \pm 3/2, \dots \\ m_q = -m_p}} \sum_{t_p, t_q} \frac{1}{E_p + E_q} \left\{ \left[ \sum_{\substack{t_q \\ m_q = -m_p + 1}} \frac{\langle 0 | J_{20} | pq \rangle \langle q | J_{11} | q' \rangle}{E_p + E_q} \right]^2 \right. \\
 & + \left[ \sum_{\substack{t_q \\ m_q = -m_p + 1}} \frac{\langle 0 | J_{20} | pq \rangle \langle q | J_{11} | q' \rangle}{E_p + E_q} \right] \left[ \sum_{\substack{t_q \\ m_q = -m_p - 1}} \frac{\langle 0 | J_{20} | pq \rangle \langle q | J_{11} | q' \rangle}{E_p + E_q} \right] \\
 & + \left[ \sum_{\substack{t_q \\ m_q = -m_p + 1}} \frac{\langle 0 | J_{20} | pq \rangle \langle q | J_{11} | q' \rangle}{E_p + E_q} \right] \left[ \sum_{\substack{t_q \\ m_q = m_p + 1}} \frac{\langle 0 | J_{20} | qq' \rangle \langle p | J_{11} | q \rangle}{E_q + E_{q'}} \right] \\
 & \left. + \left[ \sum_{\substack{t_q \\ m_q = -m_p + 1}} \frac{\langle 0 | J_{20} | pq \rangle \langle q | J_{11} | q' \rangle}{E_p + E_q} \right] \left[ \sum_{\substack{t_q \\ m_q = m_p - 1}} \frac{\langle 0 | J_{20} | qq' \rangle \langle p | J_{11} | q \rangle}{E_q + E_{q'}} \right] \right\}
 \end{aligned}$$

which can be rewritten finally as

$$\begin{aligned}
 C_{\eta}''(2QP) = & 2 \sum_{\substack{m_p > 0 \\ m_q = -m_p}} \sum_{t_p, t_q} \frac{1}{E_p + E_q} \left[ \sum_{\substack{t_q \\ m_q = -m_p \pm 1}} \frac{\langle 0 | J_{20} | pq \rangle \langle q | J_{11} | q' \rangle}{E_p + E_q} \right. \\
 & \left. + \sum_{\substack{t_q \\ m_q = m_p \pm 1}} \frac{\langle 0 | J_{20} | qq' \rangle \langle q | J_{11} | p \rangle}{E_q + E_{q'}} \right]^2 \quad (A.7)
 \end{aligned}$$

Substitution of Eq. (A.2) into Eqs. (A.6) and (A.7) then yields the last two terms in Eq. (23).

FOOTNOTES AND REFERENCES

- \* Work performed under the auspices of the U. S. Atomic Energy Commission.
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Table I. The quadrupole and hexadecapole deformation parameters  $\epsilon_2$  and  $\epsilon_4$  are taken from Ref. 23. The energy gap  $\Delta_p$  and  $\Delta_n$  are calculated with pairing strength  $G$  as given in Eq. (11).

| Nucleus | $\epsilon_2$ | $\epsilon_4$ | $\Delta_p$<br>(MeV) | $\Delta_n$<br>(MeV) |
|---------|--------------|--------------|---------------------|---------------------|
| Sm 152  | 0.202        | -0.036       | 1.114               | 0.975               |
| 154     | 0.227        | -0.039       | 1.024               | 0.888               |
| Gd 154  | 0.206        | -0.029       | 1.101               | 1.001               |
| 156     | 0.233        | -0.030       | 1.020               | 0.935               |
| 158     | 0.245        | -0.024       | 0.980               | 0.895               |
| 160     | 0.255        | -0.015       | 0.948               | 0.849               |
| Dy 160  | 0.245        | -0.015       | 0.988               | 0.934               |
| 162     | 0.256        | -0.006       | 0.945               | 0.880               |
| 164     | 0.264        | 0.003        | 0.910               | 0.836               |
| Er 162  | 0.242        | -0.007       | 0.989               | 0.969               |
| 164     | 0.254        | 0.001        | 0.941               | 0.906               |
| 166     | 0.261        | 0.010        | 0.898               | 0.861               |
| 168     | 0.272        | 0.020        | 0.847               | 0.815               |
| 170     | 0.273        | 0.031        | 0.807               | 0.786               |
| Yb 166  | 0.246        | 0.004        | 1.002               | 0.926               |
| 168     | 0.255        | 0.014        | 0.956               | 0.883               |
| 170     | 0.265        | 0.025        | 0.902               | 0.835               |
| 172     | 0.270        | 0.037        | 0.845               | 0.799               |
| 174     | 0.266        | 0.048        | 0.799               | 0.739               |
| 176     | 0.258        | 0.053        | 0.785               | 0.661               |
| Hf 174  | 0.258        | 0.034        | 0.915               | 0.822               |
| 176     | 0.256        | 0.043        | 0.879               | 0.734               |
| 178     | 0.250        | 0.052        | 0.844               | 0.672               |
| 180     | 0.243        | 0.063        | 0.808               | 0.561               |
| W 180   | 0.236        | 0.050        | 0.870               | 0.699               |
| 182     | 0.232        | 0.060        | 0.828               | 0.602               |
| 184     | 0.216        | 0.061        | 0.793               | 0.735               |
| 186     | 0.197        | 0.060        | 0.777               | 0.790               |
| Os 184  | 0.213        | 0.053        | 0.750               | 0.690               |
| 186     | 0.198        | 0.055        | 0.665               | 0.780               |
| 188     | 0.178        | 0.055        | 0.592               | 0.819               |

Table II. The moment-of-inertia and the inertia derivatives, where  $D_{xi} = \left( \frac{\partial}{\partial x_i} 2J_0 \right)_{x_{io}}$

| Nucleus | $D_{\epsilon_2^{-1}}$<br>(MeV <sup>-1</sup> ) | $D_{\epsilon_4^{-1}}$<br>(MeV <sup>-1</sup> ) | $D_{vp}^{-2}$<br>(MeV <sup>-2</sup> ) | $D_{vn}^{-2}$<br>(MeV <sup>-2</sup> ) | $2J_p^{-1}$<br>(MeV <sup>-1</sup> ) | $2J_n^{-1}$<br>(MeV <sup>-1</sup> ) | $2J_o^{-1}$<br>(MeV <sup>-1</sup> ) | $2J_{exp}^{-1}$<br>(MeV <sup>-1</sup> ) |
|---------|---|---|---------------------------------------|---------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|---|
| Sm 152  | 196   | -266  | -16.68                                | -32.35                                | 13.62                               | 24.84                               | 40.38                               | 46.8                                    |
| 154     | 163   | -266  | -20.17                                | -46.17                                | 17.16                               | 35.19                               | 54.97                               | 73.0                                    |
| Gd 154  | 195   | -236  | -15.51                                | -29.85                                | 13.26                               | 23.14                               | 38.22                               | 46.6                                    |
| 156     | 168   | -235  | -18.49                                | -41.16                                | 16.33                               | 32.00                               | 50.75                               | 66.6                                    |
| 158     | 119   | -176  | -19.88                                | -37.59                                | 17.45                               | 33.27                               | 53.26                               | 74.8                                    |
| 160     | 126   | -150  | -20.98                                | -40.19                                | 18.20                               | 35.40                               | 56.28                               | 79.4                                    |
| Dy 160  | 131   | -169  | -19.48                                | -35.24                                | 16.55                               | 30.63                               | 49.53                               | 68.6                                    |
| 162     | 126   | -137  | -20.55                                | -38.17                                | 17.60                               | 33.33                               | 53.48                               | 73.8                                    |
| 164     | 94  | -116  | -21.25                                | -43.05                                | 18.33                               | 35.84                               | 56.88                               | 81.2                                    |
| Er 162  | 148   | -147  | -16.15                                | -32.72                                | 15.07                               | 27.99                               | 45.21                               | 58.6                                    |
| 164     | 134   | -109  | -17.25                                | -36.35                                | 16.38                               | 31.41                               | 50.17                               | 65.4                                    |
| 166     | 102   | -87   | -18.22                                | -41.23                                | 17.41                               | 34.00                               | 53.98                               | 73.8                                    |
| 168     | 85  | -35   | -19.04                                | -36.48                                | 18.74                               | 35.11                               | 56.54                               | 75.0                                    |
| 170     | 108   | 4   | -19.93                                | -40.27                                | 19.59                               | 36.77                               | 59.19                               | 75.6                                    |
| Yb 166  | 154   | -96   | -14.94                                | -35.08                                | 14.16                               | 29.59                               | 45.94                               | 57.8                                    |
| 168     | 135   | -68   | -16.52                                | -39.72                                | 15.54                               | 32.27                               | 50.20                               | 68.4                                    |
| 170     | 118   | -19   | -18.14                                | -36.34                                | 17.20                               | 33.76                               | 53.51                               | 70.8                                    |
| 172     | 124   | 7   | -19.70                                | -39.98                                | 18.75                               | 35.96                               | 57.45                               | 75.8                                    |
| 174     | 89  | -4  | -21.11                                | -41.15                                | 19.78                               | 36.83                               | 59.44                               | 78.4                                    |
| 176     | 49  | 11  | -21.87                                | -31.59                                | 19.90                               | 35.45                               | 58.12                               | 72.8                                    |
| Hf 174  | 141   | -11   | -13.18                                | -37.80                                | 14.90                               | 34.40                               | 51.77                               | 65.4                                    |
| 176     | 108   | -17   | -13.38                                | -45.53                                | 15.38                               | 38.19                               | 56.25                               | 67.6                                    |
| 178     | 53  | -7  | -13.68                                | -35.14                                | 15.74                               | 36.42                               | 54.77                               | 64.0                                    |
| 180     | 15  | 35  | -13.88                                | -33.54                                | 16.18                               | 39.02                               | 57.96                               | 64.2                                    |
| W 180   | 63  | 3   | -8.69                                 | -36.56                                | 12.34                               | 35.26                               | 49.98                               | 57.6                                    |
| 182     | 21  | 52  | -8.94                                 | -35.55                                | 12.90                               | 37.84                               | 53.28                               | 59.6                                    |
| 184     | 69  | 65  | -9.91                                 | -28.92                                | 12.88                               | 31.00                               | 46.07                               | 53.6                                    |
| 186     | 89  | 74  | -10.87                                | -22.25                                | 12.55                               | 24.76                               | 39.18                               | 48.6                                    |
| Os 184  | 49  | 62  | -9.25                                 | -36.92                                | 11.92                               | 34.35                               | 48.58                               | 49.4                                    |
| 186     | 79  | 68  | -10.75                                | -29.99                                | 12.62                               | 28.53                               | 43.20                               | 43.0                                    |
| 188     | 86  | 68  | -12.97                                | -22.67                                | 13.29                               | 22.64                               | 37.72                               | 37.4                                    |

Table III. The spring constants associated with various degrees of freedom.

| Nucleus | $C_{22}$<br>(MeV) | $C_{44}$<br>(MeV) | $C_{vp}$<br>(MeV <sup>-1</sup> ) | $C_{vn}$<br>(MeV <sup>-1</sup> ) | $C_{\eta}$<br>(MeV <sup>-3</sup> ) |
|---------|-------------------|-------------------|----------------------------------|----------------------------------|------------------------------------|
| Sm 152  | 760               | 1205              | 3.67                             | 3.88                             | 27.32                              |
| 154     | 966               | 1519              | 3.74                             | 3.27                             | 37.68                              |
| Gd 154  | 725               | 1222              | 3.72                             | 3.93                             | 23.61                              |
| 156     | 899               | 1453              | 3.75                             | 3.50                             | 31.48                              |
| 158     | 1019              | 1556              | 3.78                             | 4.18                             | 24.81                              |
| 160     | 1092              | 1620              | 3.81                             | 4.13                             | 28.44                              |
| Dy 160  | 989               | 1450              | 3.36                             | 4.18                             | 23.79                              |
| 162     | 1108              | 1592              | 3.36                             | 4.19                             | 27.36                              |
| 164     | 1215              | 1711              | 3.40                             | 3.50                             | 33.25                              |
| Er 162  | 926               | 1318              | 3.73                             | 4.22                             | 19.27                              |
| 164     | 1043              | 1503              | 3.66                             | 4.24                             | 22.38                              |
| 166     | 1175              | 1663              | 3.62                             | 3.59                             | 27.44                              |
| 168     | 1205              | 1780              | 3.58                             | 4.40                             | 17.75                              |
| 170     | 1181              | 2005              | 3.57                             | 4.18                             | 23.34                              |
| Yb 166  | 956               | 1390              | 3.93                             | 4.27                             | 21.21                              |
| 168     | 1064              | 1517              | 3.83                             | 3.69                             | 26.56                              |
| 170     | 1097              | 1645              | 3.66                             | 4.40                             | 19.49                              |
| 172     | 1113              | 1714              | 3.46                             | 4.25                             | 25.66                              |
| 174     | 1220              | 1710              | 3.34                             | 4.03                             | 30.56                              |
| 176     | 1275              | 1709              | 3.39                             | 5.20                             | 19.89                              |
| Hf 174  | 1032              | 1637              | 4.10                             | 4.56                             | 20.90                              |
| 176     | 1178              | 1643              | 4.20                             | 3.71                             | 31.15                              |
| 178     | 1243              | 1603              | 4.29                             | 4.92                             | 21.16                              |
| 180     | 1300              | 1600              | 4.39                             | 2.19                             | 14.27                              |
| W 180   | 1238              | 1600              | 4.15                             | 4.76                             | 18.34                              |
| 182     | 1280              | 1595              | 4.18                             | 2.49                             | 12.16                              |
| 184     | 1255              | 1450              | 4.37                             | 5.25                             | 15.29                              |
| 186     | 1175              | 1225              | 4.60                             | 5.37                             | 12.46                              |
| Os 184  | 1280              | 1663              | 2.45                             | 3.06                             | 13.53                              |
| 186     | 1225              | 1488              | 2.06                             | 5.14                             | 16.28                              |
| 188     | 1188              | 1325              | 1.77                             | 5.36                             | 13.12                              |



TABLE IV. The first four columns list pairing spring constants with and without fixed-particle-number correction (PBCS and BCS). The fifth and sixth columns give separately the two and four quasiparticle contributions to the fourth-order cranking constant.

| Nucleus           | $C_{vp}$ ( $\text{MeV}^{-1}$ ) |      | $C_{vn}$ ( $\text{MeV}^{-1}$ ) |      | $C_{\eta}$ ( $\text{MeV}^{-3}$ ) |       |
|-------------------|--------------------------------|------|--------------------------------|------|----------------------------------|-------|
|                   | BCS                            | PBCS | BCS                            | PBCS | 2Q.P.                            | 4Q.P. |
| $^{154}\text{Sm}$ | 3.17                           | 3.74 | 3.07                           | 3.27 | 49.8                             | -12.2 |
| $^{158}\text{Gd}$ | 3.21                           | 3.78 | 3.43                           | 4.18 | 35.2                             | -10.4 |
| $^{162}\text{Dy}$ | 3.07                           | 3.36 | 3.50                           | 4.19 | 37.7                             | -10.4 |
| $^{166}\text{Er}$ | 3.09                           | 3.62 | 3.36                           | 3.59 | 37.7                             | -10.3 |
| $^{170}\text{Yb}$ | 3.18                           | 3.66 | 3.60                           | 4.40 | 28.8                             | -9.35 |
| $^{174}\text{Hf}$ | 3.44                           | 4.10 | 3.70                           | 4.56 | 30.0                             | -9.06 |
| $^{180}\text{W}$  | 3.32                           | 4.15 | 3.40                           | 4.76 | 27.3                             | -8.97 |
| $^{184}\text{Os}$ | 2.45                           | 2.45 | 2.89                           | 3.06 | 22.3                             | -8.76 |

TABLE V. The force constant  $C_{VMI}^{-1}$  and the separate contributions from various higher order effects. The experimental values of  $C_{VMI}^{-1}$  are taken from Ref. 7.

We define  $K_{xi} \equiv \left( \frac{\partial}{\partial x_i} J_0 \right)^2 / C_i$ . All units are in  $\text{MeV}^{-3}$ .

| Nucleus | $K_{22}$ | $K_{44}$ | $K_{vp}$ | $K_{vn}$ | $4C_{\eta}$ | $C_{VMI}^{-1}$<br>(Calc.) | $C_{VMI}^{-1}$<br>(exp.) |
|---------|----------|----------|----------|----------|-------------|---------------------------|--------------------------|
| Sm152   | 13       | 15       | 18.98    | 67.45    | 109.53      | 224.0                     | 595                      |
| 154     | 7        | 12       | 27.20    | 163.11   | 150.73      | 360.0                     | 229                      |
| Gd154   | 13       | 11       | 16.16    | 56.68    | 94.46       | 191.3                     | 544                      |
| 156     | 8        | 10       | 22.78    | 121.15   | 125.94      | 287.9                     | 338                      |
| 158     | 3        | 5        | 26.13    | 84.55    | 99.24       | 217.9                     | 245                      |
| 160     | 4        | 4        | 28.85    | 97.89    | 113.76      | 248.5                     | 215                      |
| Dyl60   | 4        | 5        | 28.25    | 74.28    | 95.15       | 206.7                     | 219                      |
| 162     | 4        | 3        | 31.43    | 86.97    | 109.44      | 234.8                     | 195                      |
| 164     | 2        | 2        | 33.23    | 132.55   | 132.99      | 302.8                     | 238                      |
| Erl62   | 6        | 4        | 17.48    | 63.43    | 77.07       | 168.0                     | 255                      |
| 164     | 4        | 2        | 20.29    | 77.95    | 89.50       | 193.7                     | 197                      |
| 166     | 2        | 1        | 22.96    | 118.26   | 109.76      | 254.0                     | 240                      |
| 168     | 2        | 0        | 25.35    | 75.57    | 70.98       | 173.9                     | 110                      |
| 170     | 2        | 0        | 27.80    | 96.92    | 93.36       | 220.1                     | 132                      |
| Ybl66   | 6        | 2        | 14.19    | 71.98    | 84.83       | 179.0                     | 255                      |
| 168     | 4        | 1        | 17.79    | 106.91   | 106.23      | 235.9                     | 258                      |
| 170     | 3        | 0        | 22.48    | 75.03    | 77.96       | 178.5                     | 160                      |
| 172     | 3        | 0        | 28.05    | 93.90    | 102.64      | 227.6                     | 213                      |
| 174     | 2        | 0        | 33.33    | 105.01   | 122.26      | 262.6                     | 108                      |
| 176     | 0        | 0        | 35.23    | 48.01    | 79.57       | 162.8                     | 128                      |
| Hf174   | 5        | 0        | 10.60    | 78.40    | 83.61       | 177.6                     | 215                      |
| 176     | 2        | 0        | 10.66    | 139.57   | 124.61      | 276.8                     | 170                      |
| 178     | 1        | 0        | 10.90    | 62.80    | 84.66       | 159.4                     | 135                      |
| 180     | 0        | 0        | 10.97    | 128.56   | 57.06       | 196.6                     | 73                       |
| Wl80    | 1        | 0        | 4.55     | 70.13    | 73.35       | 149.1                     | 188                      |
| 182     | 0        | 0        | 4.79     | 126.76   | 48.65       | 180.2                     | 98                       |
| 184     | 1        | 1        | 5.61     | 39.81    | 61.16       | 108.6                     | 102                      |
| 186     | 2        | 1        | 6.42     | 23.04    | 49.84       | 82.3                      | 93                       |
| Os184   | 1        | 1        | 8.74     | 111.23   | 54.14       | 176.1                     | 180                      |
| 186     | 1        | 1        | 14.04    | 43.71    | 65.11       | 124.9                     | 162                      |
| 188     | 2        | 1        | 23.82    | 23.96    | 52.49       | 103.3                     | 196                      |

Table VI. The results of calculation B where neutron levels have been shifted according to Eq. (24).

| Nucleus | $\Delta n$<br>(MeV) | $\frac{\partial}{\partial V_n} 2J_n$<br>(MeV <sup>-2</sup> ) | $C_{vn}$<br>(MeV <sup>-1</sup> ) | $C_n$<br>(MeV <sup>-3</sup> ) | $2J_n$<br>(MeV <sup>-1</sup> ) | $2J_o$<br>(MeV <sup>-1</sup> ) | $C^{-1}$<br>VMI<br>(MeV <sup>-3</sup> ) | $-B_{theo}$<br>(eV) | $-B_{exp}$<br>(eV) |
|---------|---------------------|--|----------------------------------|-------------------------------|--------------------------------|--------------------------------|---|---------------------|--------------------|
| Sm152   | 0.975               | -32.35   | 3.88                             | 27.32                         | 24.84                          | 40.38                          | 224.0                                   | 169                 | 195                |
| 154     | 0.888               | -46.17   | 3.27                             | 37.68                         | 35.19                          | 54.97                          | 353.0                                   | 77.3                | 14.9               |
| Gd154   | 1.001               | -29.85   | 3.93                             | 23.61                         | 23.14                          | 38.22                          | 191.3                                   | 179                 | 180                |
| 156     | 0.935               | -41.16   | 3.50                             | 31.48                         | 32.00                          | 50.75                          | 287.9                                   | 86.8                | 33.8               |
| 158     | 0.895               | -37.59   | 4.18                             | 24.81                         | 33.27                          | 53.26                          | 217.9                                   | 54.2                | 17.5               |
| 160     | 0.849               | -40.19   | 4.13                             | 28.44                         | 35.40                          | 56.28                          | 248.5                                   | 49.5                | 11.8               |
| Dy160   | 0.934               | -35.24   | 4.18                             | 23.79                         | 30.63                          | 49.53                          | 206.7                                   | 68.7                | 20.7               |
| 162     | 0.880               | -38.17   | 4.19                             | 27.36                         | 33.33                          | 53.48                          | 234.8                                   | 57.4                | 12.0               |
| 164     | 0.836               | -43.05   | 3.50                             | 33.25                         | 35.84                          | 56.88                          | 302.8                                   | 57.9                | 12.0               |
| Er162   | 0.969               | -32.72   | 4.22                             | 19.27                         | 27.99                          | 45.21                          | 168.0                                   | 80.4                | 40.0               |
| 164     | 0.906               | -36.35   | 4.24                             | 22.38                         | 31.41                          | 50.17                          | 193.7                                   | 61.1                | 19.9               |
| 166     | 0.861               | -41.23   | 3.59                             | 27.44                         | 34.00                          | 53.98                          | 254.0                                   | 59.8                | 17.4               |
| 168     | 0.815               | -36.48   | 4.40                             | 17.75                         | 35.11                          | 56.54                          | 173.9                                   | 34.0                | 6.74               |
| 170     | 0.771               | -44.06   | 3.48                             | 25.57                         | 38.84                          | 61.35                          | 271.7                                   | 38.4                | 10.1               |
| Yb166   | 0.926               | -35.08   | 4.27                             | 21.21                         | 29.59                          | 45.94                          | 179.0                                   | 80.4                | 49.5               |
| 168     | 0.883               | -39.72   | 3.69                             | 26.56                         | 32.27                          | 50.20                          | 235.9                                   | 74.3                | 22.9               |
| 170     | 0.813               | -37.41   | 4.38                             | 19.58                         | 35.34                          | 55.17                          | 183.8                                   | 39.7                | 12.7               |
| 172     | 0.786               | -43.59   | 3.61                             | 27.80                         | 37.86                          | 59.44                          | 273.9                                   | 43.9                | 10.4               |
| 174     | 0.789               | -36.41   | 4.70                             | 26.46                         | 34.99                          | 57.52                          | 211.6                                   | 38.7                | 8.31               |
| 176     | 0.746               | -29.64   | 4.95                             | 17.55                         | 33.14                          | 55.70                          | 149.8                                   | 31.1                | 14.6               |

(continued)

Table VI. continued

| Nucleus | $\Delta_n$<br>(MeV) | $\frac{\partial}{\partial \Delta_n} 2J_0$<br>(MeV <sup>-2</sup> ) | $C_{vn}$<br>(MeV <sup>-1</sup> ) | $C_n$<br>(MeV <sup>-3</sup> ) | $2J_n$<br>(MeV <sup>-1</sup> ) | $2J_0$<br>(MeV <sup>-1</sup> ) | $C_{VWI}^{-1}$<br>(MeV <sup>-3</sup> ) | $-B_{theo}$<br>(eV) | $-B_{exp}$<br>(eV) |
|---------|---------------------|---|----------------------------------|-------------------------------|--------------------------------|--------------------------------|--|---------------------|--------------------|
| Hf174   | 0.825               | -41.11  | 3.98                             | 22.85                         | 35.61                          | 53.04                          | 213.2                                  | 53.9                | 21.0               |
| 176     | 0.803               | -39.93  | 4.42                             | 26.53                         | 35.30                          | 53.21                          | 208.9                                  | 52.1                | 14.7               |
| 178     | 0.770               | -32.51  | 4.62                             | 18.10                         | 33.19                          | 51.38                          | 141.5                                  | 40.6                | 15.1               |
| 180     | 0.721               | -30.89  | 3.68                             | 13.82                         | 33.48                          | 52.13                          | 131.0                                  | 35.5                | 6.36               |
| W 180   | 0.795               | -33.53  | 4.55                             | 15.19                         | 31.88                          | 46.44                          | 128.1                                  | 55.1                | 39.9               |
| 182     | 0.747               | -31.94  | 3.84                             | 11.13                         | 32.48                          | 47.65                          | 115.7                                  | 44.9                | 16.8               |
| 184     | 0.749               | -31.75  | 4.75                             | 14.65                         | 31.09                          | 46.18                          | 119.2                                  | 52.4                | 24.4               |
| 186     | 0.759               | -27.90  | 4.78                             | 14.91                         | 26.98                          | 41.52                          | 109.8                                  | 73.9                | 34.2               |
| Os184   | 0.807               | -32.37  | 4.08                             | 11.75                         | 29.71                          | 43.71                          | 121.9                                  | 66.8                | 59.2               |
| 186     | 0.809               | -31.47  | 4.76                             | 15.16                         | 27.93                          | 42.58                          | 128.7                                  | 78.3                | 88.2               |
| 188     | 0.814               | -26.63  | 5.09                             | 14.92                         | 23.84                          | 38.99                          | 121.3                                  | 105                 | 174                |

TABLE VII. The moment-of-inertia, B coefficient and the force constant calculated with  $g_o = 18.0$  MeV in Eq. (11), the results of Marshalek<sup>16</sup> are also listed for comparison.

| Nucleus | This Calculation               |            |  | Marshalek                      |            |
|---------|--------------------------------|------------|--|--------------------------------|------------|
|         | $2J_0$<br>(MeV <sup>-1</sup> ) | -B<br>(ev) | $C_{VMI}^{-1}$<br>(MeV <sup>-3</sup> ) | $2J_0$<br>(MeV <sup>-1</sup> ) | -B<br>(ev) |
| Sm152   | 55.57                          | 99.8       | 476                                    | 46.77                          | 221        |
| 154     | 78.02                          | 50.4       | 934                                    | 72.31                          | 37.5       |
| Gd154   | 52.17                          | 109        | 403                                    | 46.30                          | 160        |
| 156     | 70.69                          | 56.9       | 711                                    | 66.53                          | 43.0       |
| 158     | 69.66                          | 32.4       | 382                                    | 74.96                          | 26.7       |
| 160     | 73.42                          | 29.4       | 427                                    | 79.05                          | 25.4       |
| Dy160   | 66.06                          | 42.7       | 407                                    | 68.45                          | 37.4       |
| 162     | 70.74                          | 34.6       | 433                                    | 74.02                          | 33.4       |
| 164     | 78.00                          | 39.6       | 733                                    | 81.37                          | 26.0       |
| Er164   | 65.54                          | 36.9       | 340                                    | 65.19                          | 43.7       |
| 166     | 73.17                          | 43.5       | 623                                    | 73.96                          | 33.7       |
| 168     | 71.40                          | 19.3       | 251                                    | 74.96                          | 25.0       |
| 170     | 75.56                          | 21.2       | 346                                    | 75.13                          | 30.0       |
| Yb170   | 68.33                          | 25.5       | 278                                    | 70.82                          | 31.8       |
| 172     | 73.90                          | 25.0       | 373                                    | 75.93                          | 30.0       |
| 174     | 76.61                          | 28.1       | 485                                    | 77.82                          | 27.8       |
| 176     | 71.03                          | 20.8       | 265                                    | 72.46                          | 27.7       |
| Hf176   | 73.30                          | 39.5       | 570                                    | 67.43                          | 42.6       |
| 178     | 66.44                          | 26.1       | 254                                    | 64.10                          | 44.0       |
| W 184   | 54.67                          | 29.8       | 133                                    | 53.48                          | 68.5       |
| 186     | 46.41                          | 50.4       | 117                                    | 48.40                          | 99.1       |

FIGURE CAPTIONS

Fig. 1. The moment-of-inertia  $J_0$ . The theoretical values are calculated with the single particle states and pairing strength (Eq. (11)) as given by Nilsson et al.<sup>23</sup> The experimental values are taken from Mariscotti et al.<sup>7</sup>

Fig. 2. The force constant  $C_{VMI}^{-1}$ . The theoretical values are calculated with the single particle states and pairing strength (Eq. (11)) as given in Nilsson et al.<sup>23</sup> The experimental values are taken from Mariscotti et al.<sup>7</sup> Note the large discrepancies at neutron number  $N = 90, 104$  and  $108$ .

Fig. 3. Same as Fig. 2, except in these calculations the neutron levels have been shifted according to Eq. (24).

Fig. 4. The B-coefficient. The theoretical values are calculated with the pairing strength as given in Eq. (11) and with the neutron levels shifted according to Eq. (24).

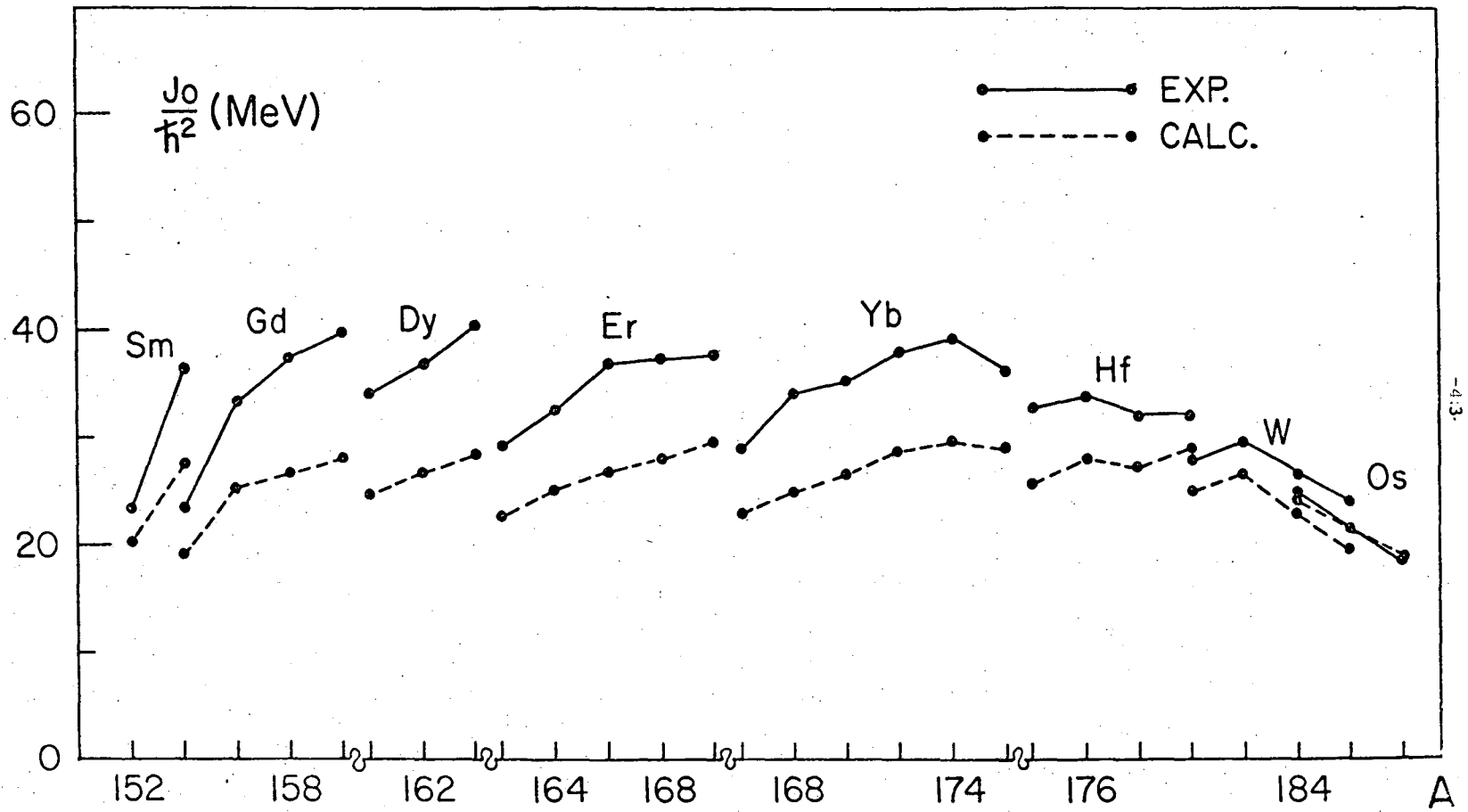


Fig. 1

-413-

LBL-2347

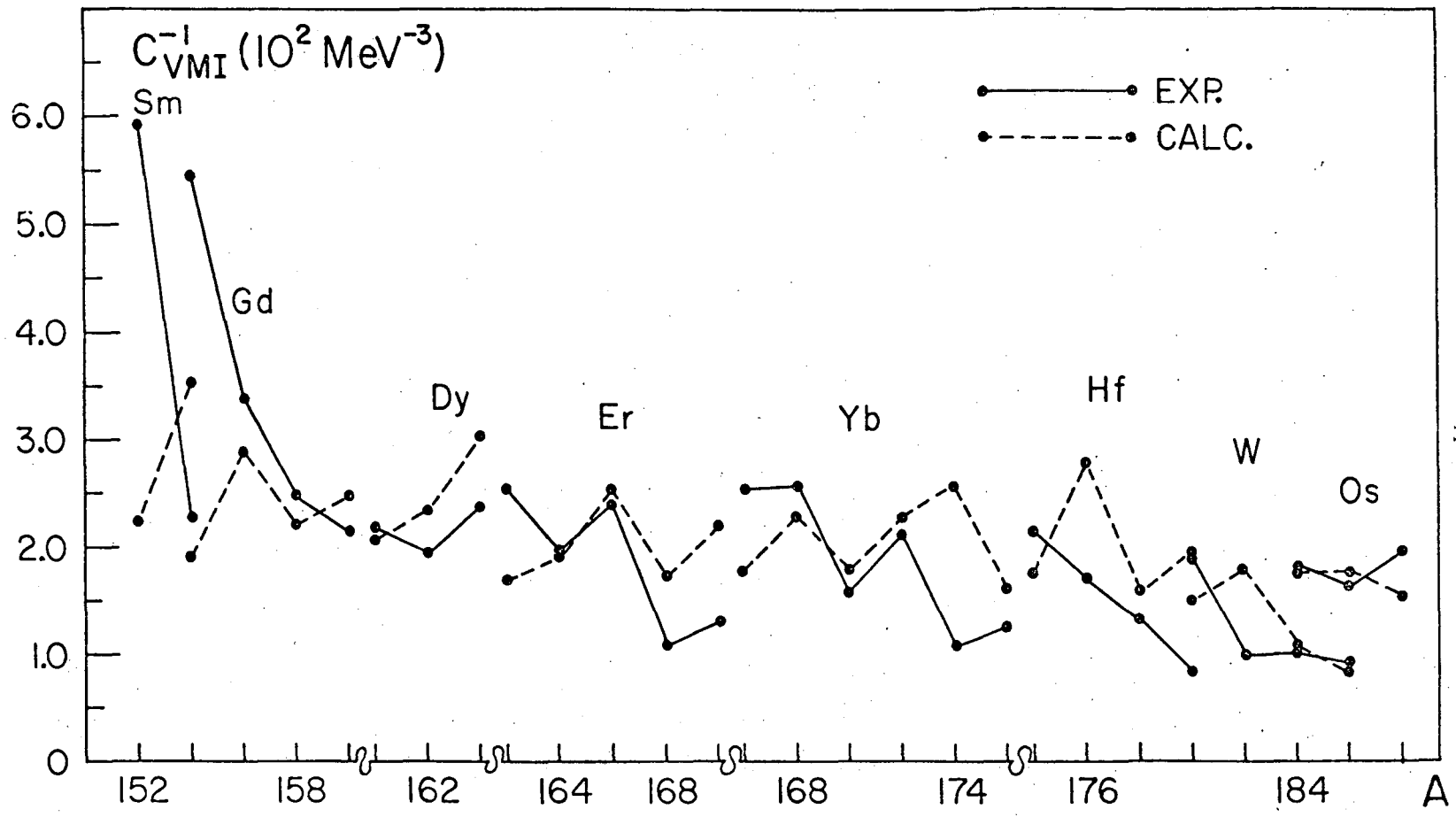


Fig. 2

-44-

LBL-2347



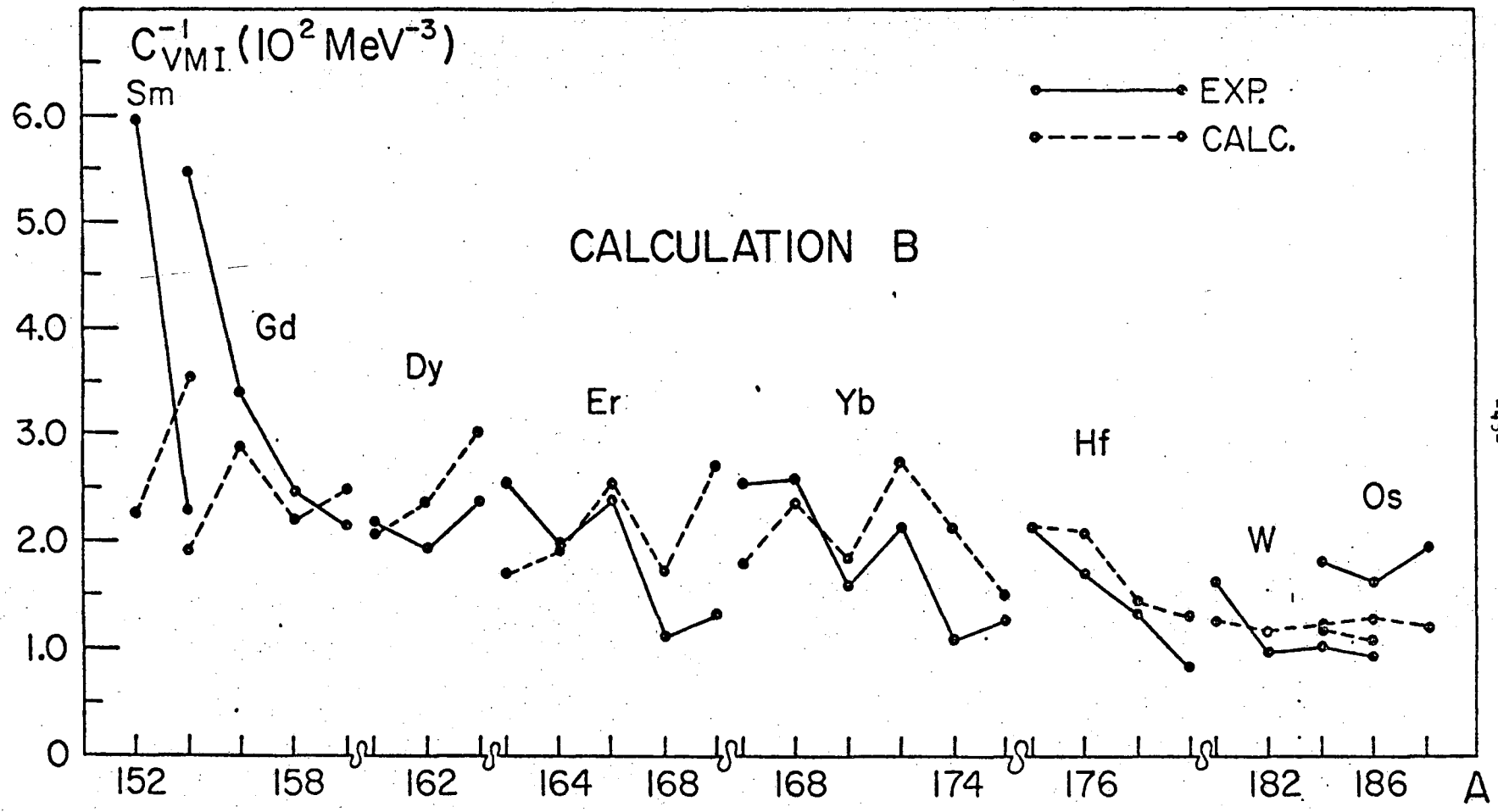


Fig. 3

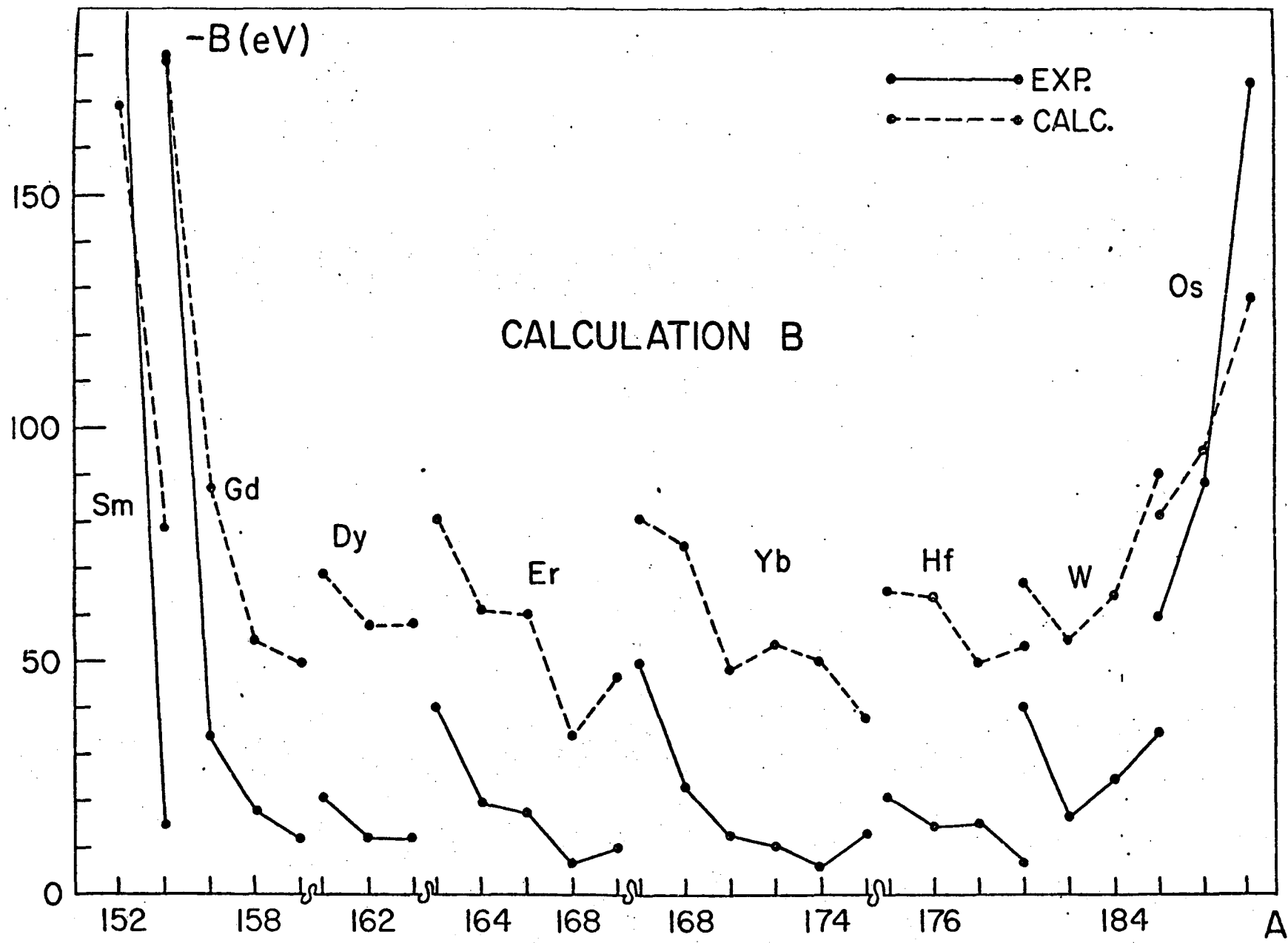


Fig. 4

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