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Dynamical Properties and Flux Tubes of the Friedberg-Lee Model*

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Abstract

A new nonpolynomial parametrization for the dielectric function of the Friedberg-Lee model is suggested to enforce the confinement of dynamical gluons. We investigated flux tube solutions of this model and show how divergences of the colour magnetic quark interaction can be avoided. As effective models for confinement, we contrast the Friedberg-Lee model and Abelian Higgs models.

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1 Introduction

In this paper we study the gluon sector of the Friedberg-Lee model [1,2,3]. In that model the nonperturbative dynamics responsible for confinement is simulated in terms of a nonlinear coupling to a scalar condensate field σ. An effective colour dielectric function $\kappa(\sigma)$ is introduced with the property that $\kappa(\sigma_V) = 0$ in the nonperturbative vacuum as characterized by $\sigma = \sigma_V$. In the interior of hadrons or in a quark gluon plasma the perturbative vacuum, $\sigma = 0$, is recovered resulting in an MIT bag model like description. However, unlike the bag model there is a finite surface interface between the perturbative and nonperturbative regions in this model. As shown by Wilets and collaborators [2,3] static properties of hadrons can be well described in terms of this model. The problem we address here is related to *non-static*, dynamic properties of the model. Interest in dynamical applications of effective models of confinement has arisen in connection with nuclear collisions [4,5]. In such collisions the high energy density may lead to the "melting" of the nonperturbative vacuum and to the formation of a quark-gluon plasma. All attempts thusfar to formulate QCD transport theories [6] needed to address the dynamical evolution of nuclear collisions have assumed that the vacuum is perturbative. However, without a dynamical treatment of the nonperturbative vacuum itself, such transport theories are not able to describe the transition from the initial confined phase to the deconfined phase and back again. Unfortunately, it is not known yet what are the most important degrees of freedom that control the vacuum dynamics. Many candidates ranging from gluon condensates, quark condensates to magnetic monopole condensates have been proposed [7].

The attractive feature of the Friedberg-Lee model is the simple postulate that one scalar field σ coupling to quarks and via an effective dielectric function, $\kappa(\sigma)$, to gluons may provide insight into the dynamics of confining theories. However, as we point out below, present formulations of the model become problematic for the treatment of dynamical problems. The crux of the problem is that a dielectric constant approaching zero does not automatically confine high frequency gluons.

To overcome the above problems we propose a new nonpolynomial form of the dielectric function that enforces confinement of even high energy gluons. Therefore, this new version of the model may be better suited for constructing transport theories in which the confinement mechanism is treated as a dynamical variable. The present paper is however more limited in scope as we study only a few aspects of the new model. In particular, we investigate string like flux tube solutions and compare them with vortex solutions in an alternate confining model, the Abelian Higgs model [8]. As we emphasize below the advantage of our model over the Abelian Higgs is that the latter also suffers from the problem of non-confinement of dynamical vector bosons. However, in contrast to the Abelian Higgs model only vortices corresponding to type I superconductors are possible in Friedberg-Lee models and thus only attractive vortex-vortex interactions can be modelled. The dynamics of flux tubes is of interest because of the successful phenomenology of multiparticle production in terms of string models such as Lund and dual parton models [9]. Thus, in the confining phase, Nambu string degrees of freedom may be relevant for dynamical problems [10]. The finite thickness of flux tubes of the Friedberg-Lee model may in turn provide a proper generalization of the Nambu strings which allows for a transition to the deconfinement domain in which the string degrees of freedom evaporate [11].

Finally, we study the problem of divergences of the colour magnetic interaction of quarks with our proposed $\kappa(\sigma)$. We show that those problems can be solved by postulating an effective $(\kappa(\sigma)^{-1} - 1)\overline{\psi}\psi$ coupling to the quarks modeling the quark self interaction [12].

The outline of this paper is as follows. In section 2 we review how the Friedberg Lee model implements confinement via a medium with zero dielectric constant κ. We examine the physical implications of such a medium. In particular we show that the electromagnetic fields $F_{\mu\nu}$ have no physical significance for $\kappa = 0$, which is crucial to avoid long distance interactions of flux tubes. However, we point out in section 3 that the confinement mechanism of the Friedberg-Lee mechanism does not work for Abelian colour fields since they carry no colour charge. We discuss the issue of gluon confinement and show how the model can be improved by choosing a non-polynomial dependence of the dielectric constant on the condensate field at $\sigma = \sigma_V$. We show that such a behaviour is also necessary in order to avoid any influence of $F_{\mu\nu}$ on the field σ at σ_V, thus ensuring that $F_{\mu\nu}$ has no physical significance in the nonperturbative vacuum. In section 4 we study flux tube solutions of the modified model. In section 5 we compare the

Friedberg-Lee model with the Abelian Higgs model [8]. In section 6 we discuss divergences of the colour magnetic interaction of the quarks that arise with our modified $\kappa(\sigma)$. We show that in order to overcome this problem we have to take the quark self-energy properly into account. Summary and outlook are presented in section 7.

2 The Friedberg Lee Model

The Lagrangian of the Friedberg Lee model is given by [1,2,3]

$$\begin{aligned} \mathcal{L} &= -\frac{\kappa(\sigma)}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \sum_f \overline{\psi}_f (i\partial\!\!\!/ - g_s A\!\!\!/_a T_a - m_f - g\sigma)\psi_f \\ &\quad + \frac{1}{2}(\partial_\mu \sigma)^2 - U(\sigma) \quad , \end{aligned} \tag{1}$$

where a are the colour components, T_a the generators for the colour group and f denotes the flavour index. The field tensor $F^a_{\mu\nu}$ is given by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f_{abc} A^b_\mu A^c_\nu \tag{2}$$

with the structure constants f_{abc} of SU(3). The field σ is supposed to describe a condensate, being zero for the perturbative vacuum and taking on some value σ_V in the QCD ground state, which we shall also refer to as the nonperturbative vacuum. The two phase structure is ensured by choosing a functional form for $U(\sigma)$ as depicted in fig. 1 where U vanishes for $\sigma = \sigma_V$ and has a second minimum for $\sigma = 0$, corresponding to the perturbative vacuum. The energy difference between the two phases, $B \equiv U(0)$, is the volume energy energy of a bubble of perturbative vacuum. This corresponds to the bag constant in the MIT bag model. However, at the surface of a bag one has to make a transition between the two phases, leading to a surface energy due to the corresponding kinetic energy and to the fact that there is a potential barrier in $U(\sigma)$. This is not included in MIT bag model fits, so that one has to use a renormalized bag constant B here which is somewhat larger in order to include the surface energy [13,14].

A soliton bag model of the Friedberg Lee type works in the following way. The coupling term $g\overline{\psi}\sigma\psi$ provides a large mass term for the quarks in the nonperturbative vacuum $\sigma = \sigma_V$, whereas the in the perturbative vacuum the masses m_f are rather small. In a more general model one can also choose a nonlinear coupling between σ and ψ [12,15]. A nucleon is modelled as a bubble of perturbative vacuum containing quarks. As in the MIT model the vacuum pressure B balances the fermi pressure of the quarks. However, coupling σ linearly to the quarks neither gives rise to absolute quark confinement, as the mass of a quark in the nonperturbative vacuum is still finite, nor does it explain why the quarks have to be coupled to colour singlets. Confinement is achieved by coupling the gluon field to the σ-field via a dielectric constant $\kappa(\sigma)$ [1,2]. The functional form of κ is chosen so that it is 1 for $\sigma = 0$ and approaches zero for $\sigma \rightarrow \sigma_V$, as shown in fig. 2. This leads to charge confinement because an electric charge creates an electric displacement $\vec{D}_a$ with the energy $\int d^3r\, \vec{D}_a^2/2\kappa$ which is infinite for nonzero total charge if κ falls off faster then $1/\sqrt{r}$ for large r.

In this paper we are mainly interested in the purely gluonic sector of the model, which is obtained by omitting all terms containing quark fields. Furthermore, we consider only the Abelian approximation of the model neglecting all self-coupling terms of the gluons, treating all eight components as independent Maxwell fields[1]. The resulting effective Lagrangian reads

$$\mathcal{L} = -\frac{\kappa(\sigma)}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2}(\partial_\mu \sigma)^2 - U(\sigma) - j^\mu A_\mu \tag{3}$$

where j^μ are the quark currents $\overline{\psi}\gamma^\mu\psi$. The Abelian charge Q of the quarks is adjusted in such a way that the relation

$$Q^2 = g_s^2 {<}T_a^2{>} \tag{4}$$

holds. For the doublet representation, ${<}T_a^2{>} = 4/3$. Thus the quark charge Q is related to the strong coupling constant by

$$\alpha_s = \frac{g_s^2}{4\pi} = \frac{3Q^2}{16\pi} \quad . \tag{5}$$

The classical field equations resulting from (3) are

$$\begin{aligned} \partial_\nu(\kappa F^{\mu\nu}) &= j^\mu & (6)\\ \partial_\alpha F_{\mu\nu} + \partial_\mu F_{\nu\alpha} + \partial_\nu F_{\alpha\mu} &= 0 & (7)\\ \Box\sigma + \frac{dU}{d\sigma} + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \frac{d\kappa}{d\sigma} &= 0 \quad . & (8) \end{aligned}$$

Given two opposite charges, e. g. a quark and an antiquark, the electric displacement between them will go along a flux tube which we can assume

[1] We can think of that as considering only fields which point into only one direction in colour space. For this case all nonlinear terms vanish due to the antisymmetry of f_{abc}.

to be axially symmetric. In the limit that their mutual distance goes to infinity one will also get translational invariance along it. Assuming that the charges lie on the z-axis we can make the ansatz that the magnetic field vanishes and that the electric field goes only in z-direction, i. e. $\vec{E} = E\vec{e}_z$. In this case eq. (7) reduces to

$$\vec{\nabla} \times \vec{E} = 0 \quad \Longrightarrow \quad E = const. \tag{9}$$

At first sight this result looks quite surprising, because it means that the flux tube leads to an electromagnetic field in the whole universe. However, in the nonperturbative vacuum this field is neither related to an electric displacement nor to an electrostatic energy, since $\vec{D} = \kappa\vec{E} = 0$ and $\vec{E} \cdot \vec{D}/2 = \kappa\vec{E}^2/2$ in this phase. Furthermore, looking at the Maxwell equations

$$\begin{aligned} \vec{\nabla} \cdot (\kappa\vec{E}) &= \rho \\ \frac{\partial}{\partial t}(\kappa\vec{E}) - \vec{\nabla} \times (\kappa\vec{B}) &= \vec{j} \\ \vec{\nabla} \cdot \vec{B} &= 0 \\ \dot{\vec{B}} + \vec{\nabla} \times \vec{E} &= 0 \end{aligned} \tag{10}$$

one finds that for $\kappa = 0$, the first two equations do not depend on $\vec{E}$ or $\vec{B}$ any more, they only state that ρ and $\vec{j}$ vanish in the nonperturbative vacuum. Thus any fields $\vec{E} = -\vec{\nabla}A_0$ and $\vec{B} = \nabla \times \vec{A}$ with arbitrary A_0 and $\vec{A}$ fulfil the Maxwell equations for $\kappa = 0$. However, these fields cannot intrude into regions with $\kappa \neq 0$. To see this consider the limit that the dielectric constant is ε in the nonperturbative vacuum and that we send ε to zero. We can look at that limit in a different way if we divide κ by ε. This changes the energy by an overall factor, but not the equations of electrodynamics. The limit $\varepsilon \to 0$ then implies that the dielectric constant κ/ε goes to infinity in regions with $\kappa \neq 0$, i. e. these regions behave like a metal. As we know from electrostatics, electric fields cannot intrude into metals because the corresponding energy would be infinitely large. We conclude that electromagnetic fields at $\kappa = 0$ do not influence any colour charges, because any colour charge density expels the nonperturbative vacuum around it and prevents the external fields from intruding. Thus the constant E-field associated to a flux tube does not lead to any long range interaction. It also does not create pairs. One can think of pair creation as a two step process. First one creates a pair locally at the expense of mass and kinetic energy, and afterwards one seperates the pair gaining potential energy. However, this mechanism does not work any more for $\kappa = 0$, because once the pair is created locally the corresponding charge density expels the nonperturbative vacuum so that the external field cannot reach the quarks any more to pull them apart. We therefore conclude that E and B have no physical meaning at all in the nonperturbative vacuum. Therefore we have to construct the model carefully in order to ensure this. As we shall see in the next section this imposes severe constraints on the functional form of $\kappa(\sigma)$, which have not been taken into account in previous approaches [1,2,3].

Assuming that the constant E-field of the flux tube goes into the z-direction and furthermore assuming axial symmetry the equation for σ becomes

$$\sigma'' + \frac{1}{r}\sigma' = \frac{dU}{d\sigma} - \frac{1}{2}E^2\frac{d\kappa}{d\sigma} \quad , \tag{11}$$

where the prime denotes the derivative with respect to the radial coordinate $r = \sqrt{x^2 + y^2}$. For an electrostatic field eq. (6) reduces to Gauss' law:

$$\vec{\nabla} \cdot \vec{D} = \vec{\nabla} \cdot (\kappa \vec{E}) = 0 \quad . \tag{12}$$

For the flux tube solution we have to take this into account as follows. All the flux that originates from the quark has to go the antiquark with opposite charge. Therefore the total flux through a cross section of the flux tube this has to be equal to the quark charge. Thus we get the condition

$$\int d^2r \, \kappa E = Q \quad , \tag{13}$$

where Q is related to α_s by eq. (5). The string tension t is the energy per unit length of the flux tube, which is

$$t = \int d^2r \left\{ \frac{\kappa}{2}E^2 + \frac{1}{2}\sigma'^2 + U(\sigma) \right\} \quad . \tag{14}$$

Equation (11) can be obtained by minimizing t with the constraint (13), i. e. by minimizing

$$-l \equiv t - \lambda Q \quad . \tag{15}$$

Minimizing this with respect to E one finds $\lambda = E$, which implies that l is equal to $\int d^2r\,\mathcal{L}$. Eq. (11) can also be obtained by minimizing t after eliminating E via (13). As a measure for the radius of the flux tube we define

$$r_{ms}^2 = \frac{1}{t}\int d^2r \cdot r^2 \left\{\frac{\kappa}{2}E^2 + \frac{1}{2}\sigma'^2 + U(\sigma)\right\} \quad . \tag{16}$$

In order to make a calculation we have to specify the functional forms of U and κ. As in previous references [1,2,3] we use a quartic polynomial for U:

$$U(\sigma) = \frac{c}{24} \cdot \left(\sigma^4 - \frac{4}{3}(\sigma_M + \sigma_V)\sigma^3 + 2\sigma_M\sigma_V\sigma^2 + \frac{1}{3}\sigma_V^3(\sigma_V - 2\sigma_M)\right) \quad . \tag{17}$$

For $c > 0$ and $0 < \sigma_M < \sigma_V/2$ this potential has a minimum at 0 and σ_V and a maximum at σ_M. The ratio σ_M/σ_V determines the shape of the potential, whereas c and σ_V simply rescale it[2]. Most previous approaches made the ansatz that $\kappa(\sigma)$ is a polynomial of the form $|1-(\sigma/\sigma_V)^n|^m$ [1,2,3]. For reasons we discuss in the next section we use a different functional form for κ, namely

$$\kappa(\sigma) = \exp\left(-\frac{1}{2}\lambda\left[\ln\left(1 - \frac{\sigma}{\sigma_V}\right)\right]^2\right)\theta(\sigma_V - \sigma) \quad . \tag{18}$$

This is depicted in fig. 2 for $\lambda = 12$.

Finally we note some interesting aspects of the theory defined by the Lagrangian (3). Independent of the functional form of U and κ there is a scaling property relating different parametrizations of this Lagrangian. Suppose we have found a solution $\sigma(x)$ for the field equations corresponding to a given $U(\sigma)$ and $\kappa(\sigma)$. Now let us consider a theory with a different parametrization of U and κ by replacing $U(\sigma)$ by $U(s\sigma)$ and $\kappa(\sigma)$ by $\kappa(s\sigma)$. For the parametrizations (17) and (18) this means

$$c \longrightarrow s^4 c \quad , \quad \sigma_M, \sigma_V \longrightarrow \frac{1}{s}\sigma_M, \frac{1}{s}\sigma_V \quad , \quad B \longrightarrow B \quad . \tag{19}$$

[2] The parameter c is the same as in reference [3]. The parameter f in this reference is given by $f = 2(1+\xi)^2/3\xi$ with $\xi = \sigma_M/\sigma_V$.

Then we get a solution for the new parametrization by the replacement

$$\sigma(x) \longrightarrow \frac{1}{s}\sigma(sx) \quad , \quad F_{\mu\nu}(x) \longrightarrow F_{\mu\nu}(sx) \quad . \tag{20}$$

Under this transformation the Lagrangian density remains invariant[3], and so does the energy density, because inserting $\sigma(sr)/s$ into $U(s\sigma)$ gives the same result as inserting $\sigma(r)$ into $U(\sigma)$ and analogously for $\kappa(\sigma)$, whereas one gets an additional factor of s which cancels the factor $1/s$ when taking the derivative of σ. The size of the solution scales like $1/s$. Since both κE and the the energy density remain constant this means that the total flux as well as the string tension go like $1/s^2$, so that we get

$$t \longrightarrow \frac{1}{s^2}t \quad , \quad Q \longrightarrow \frac{1}{s^2}Q \quad , \quad \alpha_s \longrightarrow \frac{1}{s^4}\alpha_s \quad , \tag{21}$$

whereas the r_{ms} goes like $1/s$.

In addition to the scaling property described above we also find the usual scaling property by rescaling each parameter according to its dimension [16]. In natural units with $\hbar = c = 1$, where an energy corresponds to an inverse length, this transformation implies

$$\begin{aligned} \sigma_M, \sigma_V &\longrightarrow s'\sigma_M, s'\sigma_V \\ c &\longrightarrow c \\ B &\longrightarrow s'^4 B \\ \sigma(x) &\longrightarrow s'\sigma(s'x) \\ F_{\mu\nu}(x) &\longrightarrow s'^2 F_{\mu\nu}(s'x) \\ Q &\longrightarrow Q \\ \alpha_s &\longrightarrow \alpha_s \\ t &\longrightarrow s'^2 t \\ r_{ms} &\longrightarrow \frac{1}{s'} r_{ms} \quad . \end{aligned} \tag{22}$$

[3]more precisely: The transformed Lagrangian density at the transformed position is equal to the original Lagrangian density at the original position.

We can use the two scaling properties to adjust any parametrization to fit two given parameters. For example we can use s to rescale α_s and subsequently s' to adjust the string tension t.

Another interesting relation is found in the following way. Defining l as the integral over the Lagrangian density over a cross section of the flux tube,

$$l = \int d^2r \, \mathcal{L} = \int d^2r \left\{ \frac{1}{2}\kappa E^2 - \frac{1}{2}\sigma'^2 - U(\sigma) \right\} \tag{23}$$

we find

$$\frac{dl}{dE} = \int d^2r \, \kappa E = Q \quad . \tag{24}$$

Changing E the solution $\sigma(r)$ also changes, but this gives no contribution in (24) since $\sigma(r)$ is an extremum of l, i. e. because of eq. (11). t can be regarded as the Legendre transform of $-l$:

$$t = -l + EQ \tag{25}$$

leading to

$$\frac{dt}{dQ} = E \quad . \tag{26}$$

3 Dynamical Problems in the Friedberg Lee Model

We discuss next some of the problems which arise in the Friedberg Lee model due to the fact that $\kappa \to 0$ alone does not guarantee the confinement of dynamical gluons for a static external κ. These problems are also related to the question whether the $\vec{E}$-field has a physical reality in the nonperturbative vacuum phase. As noted in the last section at least the electric displacement and the energy density associated to a given field $\vec{E}$ vanish. Furthermore, for $\kappa = 0$ any longitudinal $\vec{E}$ and any transverse $\vec{B}$ fulfil the Maxwell equations (10), since the inhomogeneous equations do not impose any restrictions on these fields. Given this arbitrariness of $\vec{E}$ and $\vec{B}$ in the nonperturbative vacuum, we must make the stronger postulate that those fields do not influence the condensate degrees of freedom either. This imposes severe restrictions on the functional form of $\kappa(\sigma)$.

First of all we observe that the dielectric constant approaching zero does not necessarily imply that *dynamical* gluons are confined. It is sufficient to confine colour charges. But note that the classical colour charge associated to an Abelian wave vanishes. Thinking of an Abelian configuration as one in which all fields point into the same direction in colour space the colour charge density given by $g_s f_{abc} \vec{A}_b \cdot \vec{E}_c$ vanishes due to the antisymmetry of the structure constants f_{abc}. Therefore the mechanism of colour charge confinement does not work for classical Abelian colour fields. On the other hand, even for an arbitrarily small constant value of κ dynamical gluons do exist. This obvious since starting from the Lagrangian $-\kappa F_{\mu\nu}F^{\mu\nu}/4$ one can simply rescale the electromagnetic field, defining $A'_\mu = \sqrt{\kappa} A_\mu$, so that the Lagrangian becomes $-F'_{\mu\nu}F'^{\mu\nu}/4$. In this representation the theory is equivalent to normal electrodynamics. For small κ this implies that the amplitude of a plane wave in the original representation becomes large. It is instructive to study the following problem. Suppose that κ varies in space, with $\kappa = 1$ in one region and $\kappa = 0$ in another, being constant in time. One may think of this as inside and outside of a bag. We first want to study the free propagation of a gluon wave in this medium, i. e. we first neglect the feedback to κ via its coupling to the field σ. What will happen to a gluon wave moving from one region into the other? For simplicity we study the one dimensional case, i. e. $\kappa = \kappa(z)$ with $\kappa = 1$ for $z \to -\infty$ and $\kappa \to 0$ for $z \to \infty$, assuming a plane wave going into the z-direction. If $\vec{E}$ and $\vec{B}$ depend only on z and t and are polarized in the xy-plane then the first and the third equation of (10), are automatically fulfilled. For the case $\dot{\kappa} = 0$ we can eliminate $\vec{B}$ by taking the time derivative of the second equation and inserting the fourth, yielding

$$\frac{\partial^2}{\partial t^2}(\kappa \vec{E}) + \vec{\nabla} \times (\kappa \vec{\nabla} \times \vec{E}) = 0 \quad . \tag{27}$$

With the ansatz

$$\vec{E}(z,t) = \vec{e}_\perp \frac{1}{\sqrt{\kappa(z)}} \psi(z)\, e^{-i\omega t} \tag{28}$$

eq. (27) becomes

$$-\psi'' + \frac{\sqrt{\kappa}''}{\sqrt{\kappa}} \psi = \omega^2 \psi \quad , \tag{29}$$

where the prime denotes the derivative with respect to z. This equation has the same structure as a Schrödinger equation for a particle with mass $1/2$, where $\sqrt{\kappa}''/\sqrt{\kappa}$ represents a potential and ω^2 corresponds to its energy. Whether or not the plane wave can intrude into the region with $\kappa = 0$ depends on this potential. If κ approaches zero exponentially for large z then this potential will go to a finite value. This means that for ω^2 smaller than this value $\psi(z)$ decreases exponentially and the wave cannot intrude, whereas for larger frequencies part of the wave is reflected and part of it transmitted. With decreasing κ the amplitude of the outgoing wave grows like $1/\sqrt{\kappa}$. This cancels the factor κ in the electric energy $\kappa \vec{E}^2/2$, so that the wave carries away energy. Thus $\kappa \to 0$ does not necessarily lead to gluon confinement for a static external field κ.

On the other hand, even in the limit $F \to 0$ one cannot neglect the feedback to the field σ. Rewriting the coupling term in (8) as

$$\frac{1}{4}\kappa F^2 \frac{d\ln\kappa}{d\sigma} = -\frac{1}{2}\kappa(\vec{E}^2 - \vec{B}^2)\frac{d\ln\kappa}{d\sigma} \tag{30}$$

one finds that it becomes arbitrarily large for large z, no matter how small E and B was originally. This is so because $\kappa(\vec{E}^2 - \vec{B}^2)$ remains of the same order of magnitude for a wave which propagates to large z, whereas

$d\ln\kappa/d\sigma$ blows up. Thus no small field limit exists in that case. This mechanism damps the original wave dissipating the energy to the field σ. In this way the divergent feedback may somehow confine even high energy gluons. However, the details of this mechanism depend on whether we treat the problem classically or quantum mechanically. Classically an outgoing wave of freuquency ω leads to a driving term for σ. Being proportional to F^2 it has the frequency 2ω (with possible admixtures of frequency zero). Thus the feedback to the field σ corresponds to the absorption of 2 identical gluons with frequency ω, so that we cannot think of the classical gluon wave as representing one single outgoing gluon. In a quantum treatment we get quite a different picture. In order to treat the fluctuations ξ around the external field $\sigma_0(\vec{r})$ quantum mechanically we expand the Lagrangian:

$$
\begin{aligned}
\mathcal{L} &= -\frac{\kappa(\sigma_0)}{4}F^2 - \frac{1}{4\kappa}\frac{d\kappa}{d\sigma}\xi\kappa F^2 - \frac{1}{8\kappa}\frac{d^2\kappa}{d\sigma^2}\xi^2\kappa F^2 - \dots \\
&\quad +\frac{1}{2}(\partial_\mu\xi)^2 - \frac{1}{2}\frac{d^2U}{d\sigma}\xi^2 - \dots
\end{aligned}
\tag{31}
$$

We have omitted all terms that do not depend on the dynamical fields F and ξ, furthermore all terms linear in ξ except the one proportional to F^2, which ensures that σ_0 is a stable configuration in the absence of an electromagnetic field. If U and κ are polynomials in σ the expansions terminate after a finite number of terms. Having one gluon in the original state this gluon is propagated according to the term $\kappa(\sigma_0)F^2$. This propagation does not leave the subspace of one-gluon states. Thus the terms $\xi^n F^2$ does not lead to the absorption of two identical gluons as suggested by the classical treatment, but rather to a scattering of the gluon emitting ξ excitations as bremsstrahlung. As in the classical limit the couplings to ξ will become very large in the limit $\kappa \to 0$, so that it is no longer possible to describe them perturbatively. As we shall see below the whole issue is quite different for our new functional form of κ since the gluons cannot intrude into regions with large $d\ln\kappa/d\sigma$, thus avoiding all the problems mentioned above.

The second problem we encounter with the usual parametrization of κ is the following. We can regard the quantity $U - \kappa(\vec{E}^2 - \vec{B}^2)/2$ as an effective potential for the σ-field. For the flux tube solution this becomes $U_{eff} = U - \kappa E^2/2$. This means that the effective potential outside the flux tube is influenced by the arbitrary electric field E. Solving the field equations for σ under the assumption that σ is constant one finds that one has to minimize the effective potential. If $d\kappa/d\sigma$ at σ_V were not zero this would imply that for $E \neq 0$ the value of σ would be shifted since σ_V would no longer be a minimum of the effective potential, and for the flux tube solution σ does not go to σ_V in the outside region. This would be a strong contradiction to the requirement that E has no effect in the nonperturbative vacuum! Therefore, as it has been noted earlier [2,3], $d\kappa/d\sigma$ must vanish at σ_V.

However, this condition is not strong enough. Suppose the second derivative of κ with respect to σ is different from zero. Since the mass of a σ-excitation in the nonperturbative vacuum is given by

$$m_\sigma^2 = \frac{d^2 U_{eff}}{d\sigma^2} = \frac{d^2 U}{d\sigma^2} - \frac{1}{2}E^2 \frac{d^2\kappa}{d\sigma^2} \tag{32}$$

this would imply that this mass is modified if $E \neq 0$. In general, the effective potential U_{eff} determines the self-coupling of the σ-field. The n-th derivative of U_{eff} corresponds to a Feynman graph with n external σ-legs. If physics should remain unaltered in the nonperturbative vacuum all self-coupling must remain the same for $E \neq 0$, which in turn implies that all derivatives of κ vanish at $\sigma = \sigma_V$. An example for a possible ansatz with this behaviour is

$$\kappa(\sigma) = \exp\left(-\frac{1}{2}\lambda\left[\ln\left(1 - \frac{\sigma}{\sigma_V}\right)\right]^2\right)\theta(\sigma_V - \sigma) \quad , \tag{33}$$

which vanishes faster than any power of $(1-\sigma/\sigma_V)$ for $\sigma \to \sigma_V$. We shall use this functional form to calculate flux tubes in the next section. Note that in this approach all derivatives of κ are smooth, whereas for a polynomial ansatz some derivative would be discontinuous at $\sigma = \sigma_V$ because of the θ-function.

The ansatz (33) has also an interesting consequence for the expression $\sqrt{\kappa}''/\sqrt{\kappa}$ which we found to represent a potential for outgoing gluons. Let us again assume that σ approaches σ_V exponentially[4]. Since $\kappa(\sigma)$ given

[4] At least approximately. For the flux tube solution for example σ goes like $\exp(-mr)/\sqrt{r} = \exp(-mr - \ln r/2)$ for large r. We are only interested in the dominating term which is mr in this case.

by (33) becomes very small in this case the behaviour of σ should be dominated by the potential $U(\sigma)$, especially by its curvature at σ_V which corresponds to a mass. Thus it is consistent to assume that $\sigma_V - \sigma$ decreases exponentially. Because of (33) κ then goes like $\exp(-\gamma z^2)$ with some constant γ for large z, and we find that $\sqrt{\kappa}''/\sqrt{\kappa}$ grows like z^2, leading to gluon confinement[5]. As can be seen e. g. from a WKB approach ψ falls off like $\exp(-\varepsilon z^2)$ for large z with some constant ε. On the other hand, up to power corrections $d \ln \kappa / d\sigma$ grows only exponentially for large z, so that the coupling of F^2 to σ in (8) goes to zero for $z \to \infty$ in contrast to the usual parametrization of κ where it diverges. Thus we avoid the problems mentioned above.

[5] A linear rise can be obtained by choosing κ so that it goes like $\exp(-\gamma' z^{3/2})$ for large z

4 Flux Tube Solutions

The structure of the flux tubes with our version of the Friedberg-Lee model is obtained by solving the field equation (11) for σ with the additional condition (13) for the total electric flux, where Q and α_s are related by (5). To get an idea what values of λ are reasonable in (33) we relate λ to the value $\sigma_{1/2}$ of the σ-field for which κ becomes 1/2:

$$\lambda = \frac{2\ln 2}{\left[\ln\left(1-\frac{\sigma_{1/2}}{\sigma_V}\right)\right]^2} \quad , \quad \frac{\sigma_{1/2}}{\sigma_V} = 1 - \exp\left(-\sqrt{\frac{2\ln 2}{\lambda}}\right) \quad . \tag{34}$$

In the calculation we shall present below we have chosen $\lambda = 12$ corresponding to $\sigma_{1/2} = 0.288\sigma_V$ and $\lambda = 4$ corresponding to 0.445.

We proceeded in the following way. Defining

$$\xi = \sigma_M/\sigma_V \quad , \quad m_\sigma = \left.\frac{d^2U}{d\sigma^2}\right|_{\sigma=\sigma_V} \quad , \quad \sigma_0 = \sigma(r=0) \tag{35}$$

we fixed the parameters ξ, λ, $B^{1/4}$, m_σ, and σ_0/σ_V. The values of σ_V and c in (17) followed from m_σ and $B^{1/4}$. Given these parameters we adjusted E so that the boundary condition $\sigma \to \sigma_V$ is fulfilled for $r \to \infty$. For every solution we calculated the string tension t, r_{ms} and Q, from which we got α_s by using (5). Then we used the transformation laws discussed in section 2 to transform the string tension to $t' = 0.2\,(\mathrm{GeV})^{-1} \approx 1\,\mathrm{GeV/fm}$ and the coupling constant to either $\alpha'_s = 0.4$ or $\alpha'_s = 2$ using the relations

$$\begin{aligned} B'^{1/4} &= \left(\frac{t'}{t}\right)^{1/2}\left(\frac{\alpha_s}{\alpha'_s}\right)^{1/4} B^{1/4} \\ m'_\sigma &= \left(\frac{t'\alpha_s}{t\alpha'_s}\right)^{1/2} m_\sigma \\ E' &= \frac{t'}{t}\left(\frac{\alpha_s}{\alpha'_s}\right)^{1/2} E \\ r'_{ms} &= \left(\frac{t\alpha'_s}{t'\alpha_s}\right)^{1/2} r_{ms} \end{aligned} \tag{36}$$

with ξ, λ, and σ_0/σ_V invariant. $\alpha_s = 0.4$ is close to the parametrization of Hasenfratz et. al. [14], whereas $\alpha_s = 2$ is close to the original MIT fit [13].

We have compiled some untransformed results for $\xi = 0.3$ and $\lambda = 12$ in table 1 (All quantities are in units of GeV). As one can see for decreasing σ_0 the electric field changes rather slowly, whereas the radius increases. This is so because for smaller σ_0, the field σ stays longer close to zero before it reaches a value $\sigma \not\ll \sigma_V$ where one gets a relatively sharp surface. Since t and $\alpha_s^{1/2}$ are given by volume integrals they become larger with increasing radius, but they can be renormalized by using (36). On the other hand the surface energy becomes less important, so that $\sigma_0 \rightarrow 0$ corresponds to the MIT-bag model limit. Table 2 shows the corresponding rescaled quantities when we demand $t = 0.2\,\mathrm{GeV}^2$ (corresponding to $1\,\mathrm{GeV/fm}$) and $\alpha_s = 0.4$, and table 3 the corresponding values for $\alpha_s = 2$, which is closer to the original MIT fit [13] with $\alpha_s = 2.2$. For decreasing σ_0/σ_V the electric field very slowly approaches the MIT-limit $\sqrt{2B}$, which we have included as $\sigma_0 = 0$. Note that in this case $r_{ms} = R/\sqrt{2}$ with the bag radius R. We have plotted the solution for $\sigma_0/\sigma_V = 0.001$ in fig. 3. The solid line represents the σ-field, the dashed line corresponds to the electric displacement and the dotted line to the energy density. There is a relatively large region where all quantities remain constant and a relatively distinct surface where σ rises to σ_V. For comparison we show in fig. 4 the solution for the parameters $\lambda = 4$ and $\xi = 0.2$. For these parameters the potential barrier in U becomes smaller and the electric displacement reaches farther out reducing the region where there is no flux anymore but still kinetic and potential energy of the σ-field. Both effects tend to decrease the surface energy, so that the model comes closer to the MIT-bag limit.

For given α_s and $B^{1/4}$ the string tension obtained in the Friedberg Lee model comes out larger than in the MIT-bag model. There are two reasons for that. In the MIT-model inside the flux tube $\vec{D}$ is equal to $\vec{E}$, whereas in the Friedberg Lee model $\vec{D}$ is smaller because κ becomes smaller than 1. Secondly, in the MIT-model inside the flux tube the volume energy is equal to B, whereas in the Friedberg Lee model $U(\sigma)$ becomes larger than B for nonzero σ. Only in the outside region it becomes smaller again, but on the other hand in this region the flux is much smaller. In addition one has a kinetic energy term $(\vec{\nabla}\sigma)^2/2$ in the Friedberg Lee model which is not present in the MIT-model. In turn this means that in the Friedberg Lee model one has to chose a smaller value of B in order to fit given values for α_s and t. In table 2 we have to use smaller values for $B^{1/4}$ than in the MIT fit of Hasenfratz et. al. [14] with $\alpha_s = 0.385$ and $B^{1/4} = 0.235\,\text{GeV}$, and in table 3 smaller values than in the original MIT fit with $\alpha_s = 2.2$ and $B^{1/4} = 0.145\,\text{GeV}$.

The values for r_{ms} obtained in table 3 are much larger than those in table 2. This is so because on one hand the electromagnetic energy is larger, tending to increase the radius of the flux tube, and on the other hand the counteracting bag pressure is smaller. This is also reflected in the transformation law (36) for r_{ms}.

An interesting effect occurs for $\lambda \to 0$, i. e. when κ falls off only slowly as σ approaches σ_V. One does not find a solution any more for which σ is close to zero at the origin. This can be understood by neglecting the surface energy, studying some kind of bag model in which $\sigma = \sigma_V$ outside and a value σ_0 inside which is not necessarily zero. In this case the string tension is given by

$$t = \Big(\frac{\kappa(\sigma_0)}{2}E^2 + U\Big)\pi R^2 = \frac{Q^2}{2\kappa(\sigma_0)\pi R^2} + U\pi R^2 = \sqrt{\frac{2U}{\kappa(\sigma_0)}}Q \quad , \tag{37}$$

where we have minimized the energy with respect to the bag radius R in the lase step. $\sigma = 0$ is always an extremum of the energy because in our ansatz $dU/d\sigma = d\kappa/d\sigma = 0$ for $\sigma = 0$. However, for small λ a second minimum appears near $\sigma = \sigma_V$, and for even smaller λ this becomes the absolute minimum. This is due to the fact that one can have a relatively large value of κ already at σ close to σ_V with only a small expense of volume energy. Thus it is not necessary to have σ close to zero in order to accomodate an electric flux. For $\sigma_M/\sigma_V = 0.3$ the second minimum appears at about $\lambda = 1.2$, and for values somewhat smaller than one it becomes the absolute minimum.

5 Comparison with the Abelian Higgs Model

It is interesting to compare the Friedberg Lee model with the Abelian Higgs model, defined by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left|(i\partial_\mu - qA_\mu)\phi\right|^2 - \frac{h}{4}\left(|\phi|^2 - \frac{\mu^2}{h}\right)^2 \quad , \qquad (38)$$

where q is the charge of the field ϕ. A nonrelativistic version of this model can be used to describe superconductors. The field ϕ corresponds to an order parameter, which is to be identified with a Cooper pair condensate in the nonrelativistic case with $q = 2e$ where e is the electron charge. This model confines magnetic monopoles because of the Meissner Ochsenfeld effect. This mechanism works in the following way. Due to the last term in (38) the ground state of this theory is characterized by a nonvanishing field ϕ with $|\phi|^2 = \mu^2/h$, ϕ being constant. In a nonrelativistic model this corresponds to the superconducting phase. The coupling of this vacuum value to the gauge field given by $(2q^2\mu^4/h^2)A_\mu A^\mu/2$ yields a mass term for A_μ with

$$m_A = \frac{\sqrt{2}q\mu^2}{h} \quad . \qquad (39)$$

This mass term prevents the intrusion of a magnetic field into the superconducting medium. In order to accomodate a magnetic flux one has to break the condensate ϕ, forcing the magnetic field to go along a flux tube. However, it does not correspond to absolute confinement of the vector bosons, because they can still intrude into the nonperturbative vacuum if their energy exceeds the mass they have in that phase. This is exactly the problem that we can avoid in the Friedberg-Lee model by introducing our nonpolynomial parametrization of κ.

In order to compare this to the Friedberg-Lee model let us study the equations for a static, axially symmetric flux tube along the z-direction, resulting from the ansatz

$$\begin{aligned} A_\varphi &= A_\varphi(\rho) \quad , \quad A_0 = A_z = A_\rho = 0 \\ \phi &= G(\rho)e^{in\varphi} \quad , \end{aligned} \qquad (40)$$

where ρ and φ are polar coordinates in the xy-plane. n has to be an integer number so that ϕ is unique. Using this ansatz the energy density becomes

$$\mathcal{H} = \frac{1}{2}\Big(A'_\varphi + \frac{1}{\rho}A_\varphi\Big)^2 + G'^2 + \Big(qA_\varphi - \frac{n}{\rho}\Big)^2 G^2 + \frac{h}{4}\Big(G^2 - \frac{\mu^2}{h}\Big)^2 \quad . \tag{41}$$

The total energy remains finite only if the boundary conditions

$$G \longrightarrow \pm\sqrt{\frac{\mu^2}{h}} \quad , \quad A_\varphi \longrightarrow \frac{n}{q\rho} \qquad \text{for} \quad \rho \longrightarrow \infty \tag{42}$$

are fulfilled. The total magnetic flux is given by

$$\Phi = \oint d^2r\, B_z = \lim_{\rho\to\infty} \rho \int_0^{2\pi} A_\varphi d\varphi = \frac{2\pi n}{q} \quad , \tag{43}$$

i. e. the magnetic flux is quantized. One can also look at this argument in the opposite way. Any magnetic flux leads to a vector potential which goes like $1/\rho$ for large ρ, yielding an infinite contribution to the energy due to the third term in $\mathcal{H}$ unless it is cancelled by a change of phase in the field ϕ, giving rise to a nonzero winding number of the mapping $\varphi \to \arg(\phi)$. This argument does not dependent on the assumption of axial symmetry.

We can also derive a set of boundary conditions for $r \to 0$. If the magnetic field is finite at the origin we can deduce

$$2\pi\rho A_\varphi \longrightarrow \pi\rho^2 B_z \qquad \Longrightarrow \qquad A_\varphi \longrightarrow 0 \quad . \tag{44}$$

Because of this the third term in $\mathcal{H}$ gives an infinite contribution to the energy unless the boundary condition

$$G \longrightarrow 0 \qquad \text{for} \qquad = \rho \longrightarrow 0 \tag{45}$$

is fulfilled forcing φ to be zero at the origin for axially symmetric solutions. This result can be generalized in the following way. As we have shown above any magnetic flux would lead to a nonzero winding number of the mapping $\varphi \to \arg(\phi)$. However, one can show that for any field which is nonzero everywhere this winding number must be zero[6]. In turn this implies that any mapping with nonzero winding number with φ being nonzero

[6]One can regard $\arg(\phi(\rho,\varphi))$ as a homotopy map connecting $\arg(\phi(\rho_0,\varphi))$ for a given ρ_0 with the constant function $\arg(\phi(0,\varphi))$.

everywhere cannot be smoothly continued to the whole plane, so that φ has to be zero somewhere. In the ansatz (40) for the field ϕ for example with winding number $n \neq 0$ ϕ is singular at the origin unless it vanishes there. In this respect the Abelian Higgs model differs from the Friedberg Lee model, where σ can be nonzero everywhere. In fact, it is even necessary that $\sigma(0) \neq 0$ since otherwise the differential equation (11) would imply $\sigma = 0$ everywhere except in the MIT-limit.

Minimizing the energy $\int d^2r\, \mathcal{H}$ yields the field equations

$$\begin{aligned} G'' + \frac{1}{\rho}G &= \Big(qA_\varphi - \frac{n}{\rho}\Big)^2 G + \frac{h}{2}\Big(G^2 - \frac{\mu^2}{h}\Big)G \\ A''_\varphi + \frac{1}{\rho}A'_\varphi - \frac{1}{\rho^2}A_\varphi &= 2q\Big(qA_\varphi - \frac{n}{\rho}\Big)G^2 \quad . \end{aligned} \tag{46}$$

which can be solved numerically. An important parameter in the Abelian Higgs model is the ratio between the mass of the gauge field in the superconducting phase and the mass parameter corresponding to an excitation of the Higgs field in this phase. The square of the latter is given by $m_H^2 = -d^2\mathcal{L}/d\phi^2|_{\phi_{vac}} = \mu^2$. The case $m_A \gg m_H$ corresponds to a type I superconductor where the magnetic flux falls off rapidly in the superconducting phase. In a type II superconductor on the other hand characterized by $m_A \ll m_H$ it falls off only slowly after ϕ has nearly reached its vacuum value. Such a behaviour cannot be reproduced in the Friedberg-Lee model for the following reason. Taking into account that $\sigma_V - \sigma$ goes like $\exp(-m_\sigma\rho)$ for large ρ we cannot get an electric flux falling off like $\exp(-M\rho)$ with $M < m_\sigma$, since this would only be possible if κ went like $(1-\sigma/\sigma_V)^{M/m_\sigma}$ which contradicts the requirement $d\kappa/d\sigma|_{\sigma_V} = 0$. For κ behaving like $(1-\sigma/\sigma_V)^n$ near σ_V (so that $d\kappa/d\sigma \to 0$) one gets $M = nm_\sigma$ being the analogue of a type I superconductor. In the functional form we have introduced above κ and thus $\vec{D}$ falls off faster than exponential for large ρ, which definitely corresponds to the behaviour of a type I superconductor.

This is also reflected in the behaviour of the interaction of two flux tubes with the flux going into the same direction. In the Friedberg-Lee model there is a volume energy as well as a positive surface energy. By merging the flux tubes one can reduce the surface, resulting in an attractive interaction between them. The same argument applies for type I superconductors.

For a type II superconductor on the other hand one has a large region of condensed vacuum where the magnetic fields fall of exponentially. As the flux tubes approach each other one gets a superposition of these magnetic fields, which has a higher energy than the isolated magnetic fields if both have the same signs, leading to a repulsive interaction between vortices.

6 Quark Magnetic Interaction

With our new parametrization we have studied only string-like solutions, i. e. only the purely gluonic sector of the Friedberg-Lee model. We should also be able to include quarks within that model and describe a soliton bag with our new parametrization. However, doing so we encounter problems concerning the magnetic interaction of the quarks if we couple σ linearly to ψ. Suppose that we first neglect the colour interaction solving for σ and ψ self-consistently and subsequently calculate the magnetic interaction in first order perturbation theory by regarding the quarks as colour currents [3]. This approach fails if we use our functional form of κ, leading to a divergent expression for the magnetic energy for the following reason. Due to the coupling $g\overline{\psi}\sigma\psi$ the quark fields die out exponentially for large r in the region where σ is equal to its vacuum expectation value, since $g\sigma$ provides an effective mass term for the quarks. The divergence of the magnetic energy can be understood as follows. To first order in g one finds

$$\vec{\nabla} \times \vec{H}_a = \vec{j}_a \quad , \tag{47}$$

where $\vec{H}_a = \kappa \vec{B}_a$. This equation results from minimizing the magnetostatic energy

$$E_{magn} = \int d^3r \left\{ \frac{1}{2}\kappa(\vec{B}_a)^2 - \vec{j}_a \cdot \vec{A}_a \right\} = -\frac{1}{2} \int d^3r \, \frac{1}{\kappa} (\vec{H}_a)^2 \quad , \tag{48}$$

where the last expression is obtained by using (47) and performing a partial integration. Each quark contributes a term

$$\vec{j}_a = g_s \overline{\psi} \vec{\alpha} T_a \psi \tag{49}$$

to the colour current. For large r the quark fields fall off roughly exponentially and so will $\vec{j}_a$ and also $\vec{H}_a$, the solution of (47). But since κ falls off much faster than exponentially in our parametrization this means that the expression for the magnetostatic energy density (48) blows up for large r. If the magnetic energy density were positive this problem could be avoided in a self-consistent nonperturbative approach. Trying to minimize the total energy the quarks would certainly avoid a configuration with a positive energy density blowing up for large r. Unfortunately, in classical magnetostatics the total energy of an external current is always negative, as can be seen from the last expression in (48). In a quantum mechanical calculation the situation is somewhat different. Following De Grand et al. [17] one usually omits the magnetic self-interaction within all kind of bag models. Since the self-interaction is negative the total energy is increased and becomes positive for the Δ-resonance. Alas, for the proton it is still negative, so that the divergence of the magnetic energy implies that the energy of the system negative infinite. This presents a severe problem of the model for the case that κ falls off faster than exponentially. However, even for other parametrizations of κ which do not lead to a divergence one gets a large contribution from regions with small κ, even though the total quark density there may be very small. Furthermore the magnetic energy depends sensitively on the behaviour of κ in these regions. But even worse, as long as κ is allowed to go to zero the energy is not bounded from below. To see this look at the proton solution of the soliton bag model obtained by omitting the magnetic interaction first. The resulting quark currents create a magnetic field which gives rise to the magnetic interaction. Now suppose we change the configuration by letting σ approach σ_V, so that κ approaches 0, without changing the quark states. Due to the coupling term $g\overline{\psi}\sigma\psi$ this will increase the energy, but only by a finite amount. On the other hand the magnetic field $\vec{B}_a$ will become arbitrarily large due to (47) and so will the magnetic energy. On can also look at this problem in the following way: Due to the linear term $\vec{j}_a \cdot \vec{A}_a$ one can decrease the energy by turning on a magnetic field. Increasing its amplitude this term can be made arbitrarily small, but for large amplitudes the quadratic term will be dominant preventing a collapse of the system. On the other hand, for $\kappa = 0$ this term is no longer present, so that the system becomes unstable! In the first place one might think that the quarks would avoid regions with small κ due to confinement, because the electric energy will become very large there. But how is this built into the model? The term $g\overline{\psi}\sigma\psi$ does not lead to absolute confinement because it is finite, so that it cannot prevent the collapse due the magnetic interaction. We have convinced ourselves that classical charges are confined within this model because their electric energy is very large for small κ. One might suspect that this is also the case if one treats the quark self-energies properly. It is not sufficient to simply calculate the electric energy corresponding to the classical quark charge density resulting from the quark orbitals. When the quark orbitals differ only in spin the total electric field vanishes since the quarks are coupled to colour singlets and so does the colour electric energy in this approach, no matter how small κ is. However, calculating the colour-electric interaction one has to include both graphs shown in fig. 5, corresponding to mutual interaction and self-interaction of the quarks. They only cancel each other when the quark in the self-energy graph is restricted to the original quark orbital in the intermediate state. For a proper treatment one has to take into account all the other quark states as well. The so calculated self-energy will depend on $\kappa(\vec{r})$ in a complicated way through the dependence of the gluon propagator. For κ = const the gluon propagator is proportional to $1/\kappa$. As in QED this yields a renormalization of the mass term which also goes like $1/\kappa$ in lowest order, i. e. for the one gluon exchange. This motivates a simplified way to incorporate the self-energy in an approximate manner by including a term proportional to

$$\left(\frac{1}{\kappa} - 1\right) \overline{\psi}\psi \tag{50}$$

in the Lagrangian which vanishes for $\kappa = 1$ and gives rise to a mass term growing like $1/\kappa$ for small κ [12]. Introducing such a term solves the problems described above because it leads to absolute confinement and prevents the quarks from intruding into regions with small κ. It also makes the coupling term $g\overline{\psi}\sigma\psi$ unnecessary.

7 Summary and Conclusions

In this article we studied properties of confinement in the Friedberg-Lee model. We found that the electromagnetic fields $\vec{E}$ and $\vec{B}$ have no physical significance in the nonperturbative vacuum with zero dielectric constant, because the associated energy density vanishes and the electromagnetic fields cannot intrude into regions with a nonvashing dielectric constant. Thus they have no influence on colour charges, because due to confinement any colour charge density has to be surroundend by a region with $\kappa \neq 0$. On the other hand, in order to prevent any influence of $\vec{E}$ and $\vec{B}$ on the field σ in the nonperturbative vacuum one has to postulate that all derivative of κ vanish at σ_V. Such a behaviour of κ also solves the problem of gluon confinement in a satisfactory way, which is especially important if one applies this model to dynamical problems.

We found that the coupling between σ and the quarks has to be chosen so that the quarks cannot intrude into regions with small κ, in order to make sure that the magnetic interaction of the quarks is not dominated by the quark currents in these regions or even diverges. This is probably guaranteed if this potential is an effective description of the electric self-energy of the quarks as proposed in reference [12].

Choosing a parametrization of $\kappa(\sigma)$ which satisfies the constraints mentioned above we have looked for flux tube solutions. We could always reproduce a given string tension and a given radius of the flux tube by choosing the parameters appropriately. This will not be so easy if one wants to use the same set of parameters to describe hadrons as soliton bags, since the coupling constant α_s is fixed by the $N - \Delta$ splitting resulting from the magnetic interaction of the quarks. On the other hand, that interaction has to be reconsidered anyway since it may be changed considerably by the introduction of a quark-σ coupling which prevents quarks from intruding into regions with small κ, since without that term one gets large contributions from quark currents in those regions.

We have also compared the Friedberg-Lee model to the Abelian Higgs model, which confines magnetic monopoles. Even though the Friedberg-Lee model has a much larger degree of arbitrariness due to the different couplings and possible functional forms the possible physical scenarios are more restricted since they are more analogous to type I superconductors independent of the detailed form of the model, as we have shown in section 5. This has a direct consequence for the interaction between two flux-tubes. For a type I superconductor as well as for the Friedberg-Lee model this interaction is attractive, whereas it is repulsive for a type II superconductor. We also noted that there is no absolute confinement of dynamical vector bosons in the Abelian Higgs model, since they aquire only a finite mass in the outside region. That mass is smaller for superconductors of type II than for those of type I. Thus in a dynamical calculation one encounters the problem that vector bosons with sufficiently high energy can propagate freely. We have shown that this can be avoided in the Friedberg-Lee model. If one tries to circumvent this in the Abelian Higgs model by choosing a very large vector boson mass one is lead to type I superconductors.

Finally, as we note interest in dynamical applications of effective confinement models has arisen in connection with modeling nuclear collisions [6]. The problem of calculating the transport phenomena in such collisions including constraints of confinement remains an important open problem. Effective confinement models such as the Friedberg-Lee model discussed here may help to provide new insight into that problem, and thus desevers further study.

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Figure Captions

Figure 1: The potential energy $U(\sigma)$ for $\sigma_M/\sigma_V = 0.3$. The local minimum at $\sigma = 0$ corresponds to the perturbative vacuum and the global minimum at $\sigma = \sigma_V$ to the nonperturbative QCD ground state with condensation energy B.

Figure 2: Our nonpolynomial form for of dielectric function $\kappa(\sigma)$ for $\lambda = 12$ with an ultrasmooth approach to zero for $\sigma \to \sigma_V$ that avoids problems of previous parametrizations.

Figure 3: The flux tube solution for $\sigma_M/\sigma_V = 0.3$ and $\lambda = 12$, $\sigma_0/\sigma_V = 0.001$ and $t = 0.2$, with a) $\alpha_s = 0.4$, $B^{1/4} = 0.187$ and b) $\alpha_s = 2$, $B^{1/4} = 0.125$ (in units derived from GeV). The solid line represents the condensate field σ, the dashed line the electric flux density κE, and the dashed-dotted line the energy density ε, all fields in arbitrary units.

Figure 4: The resulting flux tube solution for $\sigma_M/\sigma_V = 0.2$ and $\lambda = 4$ with a) $\alpha_s = 0.4$, $B^{1/4} = 0.212$ and b) $\alpha_s = 2$, $B^{1/4} = 0.142$ is closer to the MIT-bag model limit.

Figure 5: Graphs contributing to the electric energy in a bag,
a) Mutual Interaction b) Quark Self-Energy
that cancel only if the intermediate quark states in b) are restricted to the lowest orbital.

Table 1: Flux tube characteristics for fixed $B^{1/4} = 0.235$ and $m_\sigma = 1$ for different $\sigma = \sigma_0$ at the origin. Listed are the electric field E, the corresponding coupling constant α_s, the string tension t and the mean square radius r_{ms}. All results are given in units derived from GeV. One

can rescale these solutions so that they correspond to given α_s and t (cf. tables 2 and 3).

Table 2: The rescaled flux tube solution for $\alpha_s = 0.4$ and $t = 0.2$ (units derived from GeV). In the MIT bag model limit this corresponds approximately to the parametrization of Hasenfratz et. al. with $B^{1/4} = 0.235$ and $\alpha_s = 0.385$.

Table 3: The rescaled flux tube solution for $\alpha_s = 2$ and $t = 0.2$ (units derived from GeV). This corresponds more to the original MIT bag model fit with $B^{1/4} = 0.145$ and $\alpha_s = 2.2$.

σ_0/σ_V	E	α_s	t	r_{ms}
0.01	0.11042	1.9257	0.8339	3.9600
0.001	0.09914	10.5701	1.6239	5.7780
0.0001	0.09384	33.6192	2.6261	7.5379
0.00001	0.09078	81.2148	3.8377	9.2662

Table 1

σ_0/σ_V	$B^{1/4}$	m_σ	E	r_{ms}
0.01	0.17048	1.07456	0.058105	3.6852
0.001	0.18698	1.80431	0.062765	3.2023
0.0001	0.19636	2.53003	0.065522	2.9794
0.00001	0.20251	3.25287	0.067401	2.8486
0	0.23372	∞	0.077255	2.3094

Table 2

σ_0/σ_V	$B^{1/4}$	m_σ	E	r_{ms}
0.01	0.11400	0.48055	0.025986	8.2404
0.001	0.12504	0.80691	0.028070	7.1606
0.0001	0.13132	1.13146	0.029302	6.6621
0.00001	0.13542	1.45473	0.030146	6.3697
0	0.15630	∞	0.034549	5.1640

Table 3

Figure 1

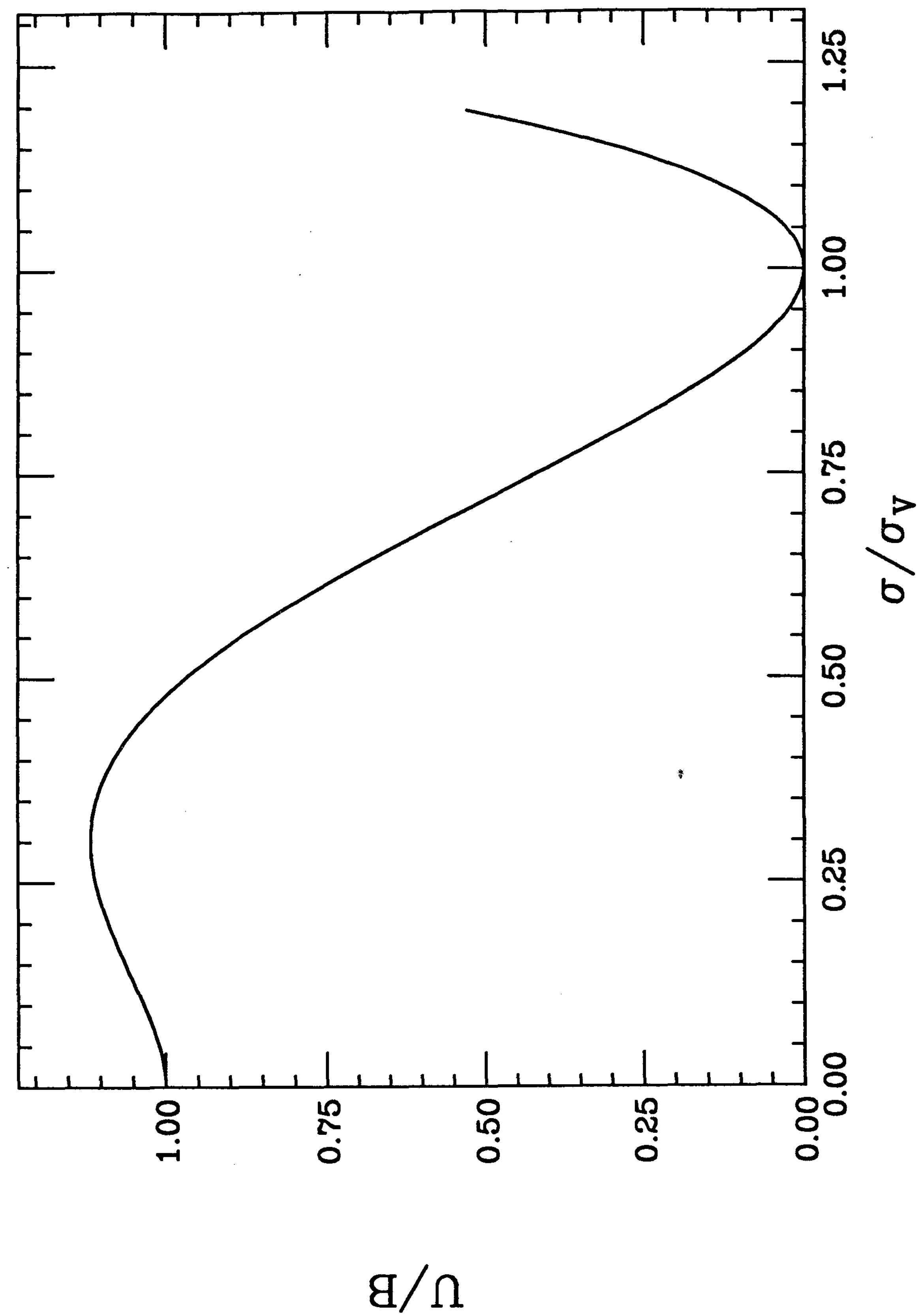

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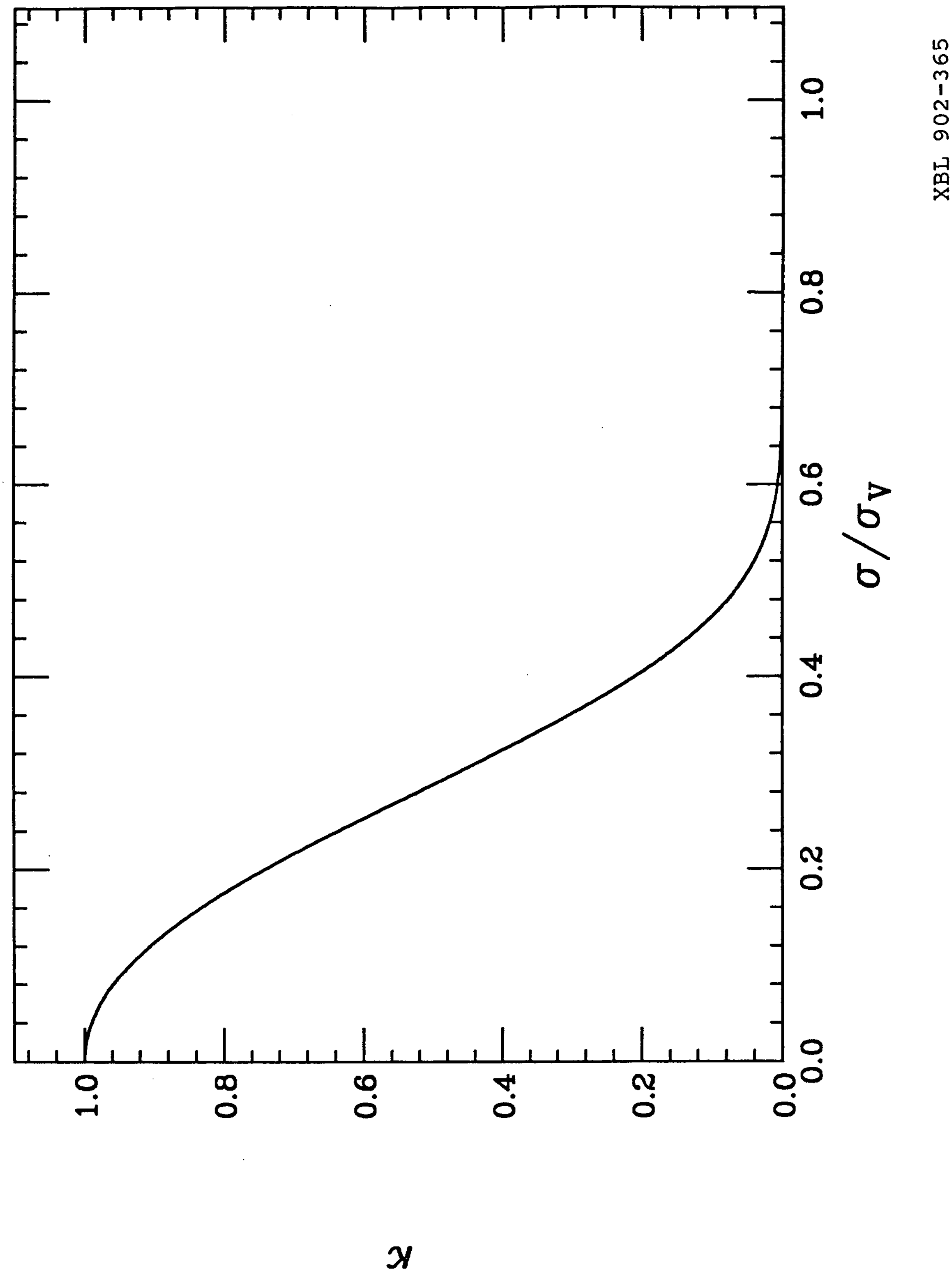
Figure 2



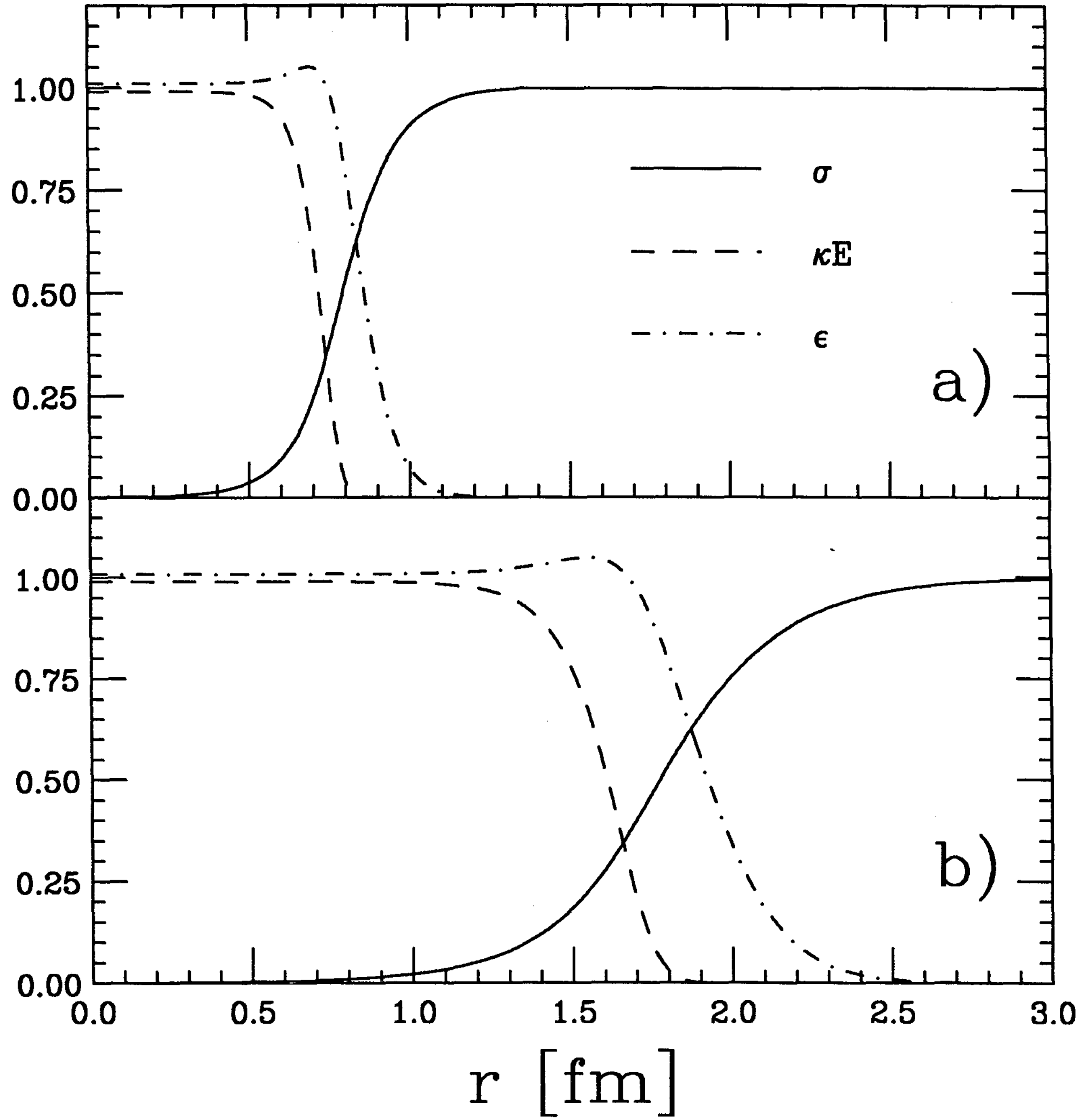


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Figure 3

Figure 4

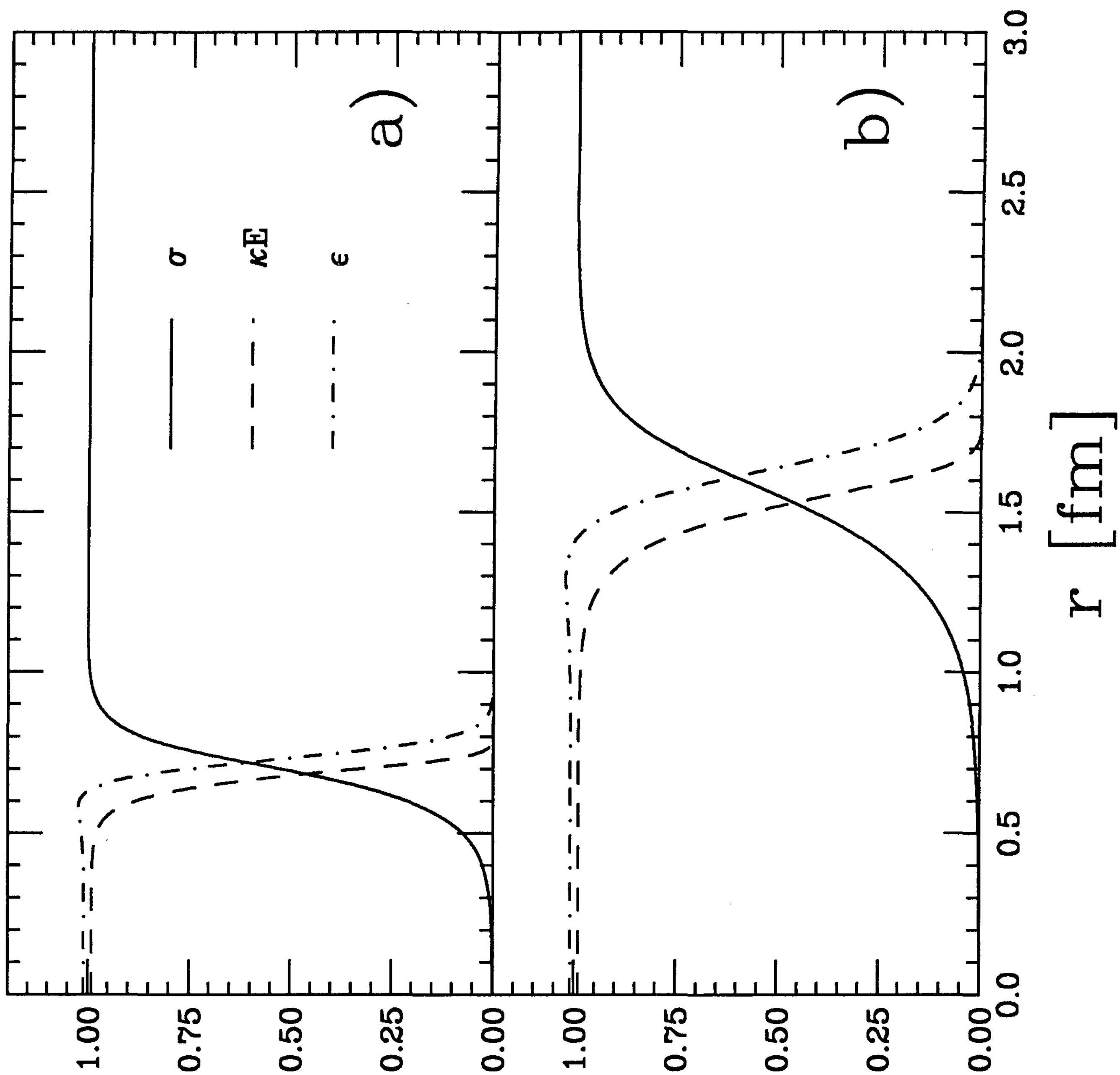

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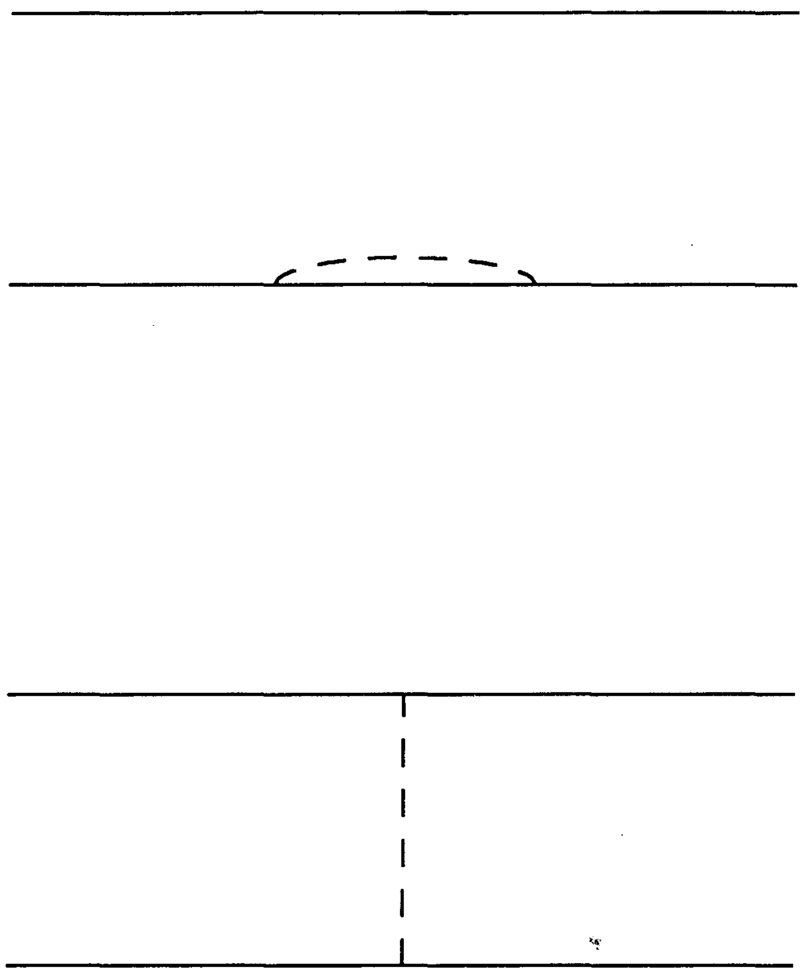
Figure 5



b)

a)

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