Computing Private Information Equilibria: Moral Hazard in an Indian Village*

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1. Introduction

Simulata is the name I've given to an imaginary village, modelled very loosely after the ICRISAT villages in India. The life of the inhabitants of Simulata is pretty simple; all they ever do is grow grain and eat it. They do this every period, forever.[1] The sole inputs to grain production are land and labor, and output is uncertain.

Since the inhabitants of Simulata don't like risk, they have an incentive to try to reduce the risk they face by pooling their output. For simplicity, we'll suppose that the (risk neutral) village chief acts as an intermediary. In each period, the denizens of Simulata first go work in their fields, applying some level of effort a, producing some random output q which they give to the village chief. The chief in turn piles up the entire output of the village, and gives some grain back to each

*This work is highly preliminary, and its author does not yet take it very seriously as a model of an actual village economy. Neither should the reader.

[1] The infinite horizon assumed here is not in any way essential to the results. Assuming an infinite horizon does reduce the dimension of the problem, however.

agent, depending on what the chief knows about how hard the agent worked and and much grain each agent produces in total.

What I'm principally interested in is how different sorts of information problems might affect life in Simulata. For now, I'll discuss only two different cases: the benchmark case of full information, in which everybody knows everything there is to know about Simulata; and the hidden action case, in which the production effort made by each agent is private information.

Because the case with full information is well understood, I'll rely principally on a brief discussion of some analytical results for this case. The hidden action problem is more interesting, as well as more difficult; although a variety of analytical results are available for this case, I'll chiefly present some numerical simulations of behavior with a hidden action.

Because the numerical simulations require a complete description of the economy of Simulata, I'll be forced to take a stand on exactly what the utility functions of the Simulatans is, as well as taking a stand on what their grain production function looks like. In order to pin down just what these functions are, I'll present the results of some empirical work I've done using data from the three principal ICRISAT villages: Aurepalle, Shirapur, and Kanzara. The data from these three villages provides the information we need to make some educated guesses regarding production and preferences, which we'll then apply to Simulata.

2. Primitives

Preferences

As remarked above, the chief is risk neutral. Agents are assumed to have separable momentary utility of the CES variety:

$$U(c) + V(a) = \frac{c^{1-\gamma}}{1-\gamma} + \frac{(1-a)^{1-\alpha}}{1-\alpha}$$

where γ and α are the coefficients of relative risk aversion in consumption and action, respectively. Agents and principal share a common constant discount factor β.

Technology

Crops either succeed or fail; the value of the crop under each of these circumstances is given respectively by q_H and q_L. The probability of agent i producing high output in period t is given by

$$\Pr(q_H|a_{it}) = \frac{A_i a_{it}^{\eta}\theta_t}{1 + A_i a_{it}^{\eta}\theta_t}$$

where A_i denotes an agent-specific fixed effect, a_{it} gives the agent i's time t action, and θ_t is some aggregate productivity shock publicly observed at the beginning of the period, with distribution $\lambda(\theta)$, and support Θ.

In addition to grain production, the chief has access to a linear storage technology that transforms one unit of grain today into $1/\beta$ units of grain tomorrow.

3. Full Information

Each Simulatan is born at time zero, and is endowed with a certain amount of land, L. The value of this land is determined by the discounted value of the produce that can be grown using this land. Immediately after a Simulatan is born, the village chief offers him a take-it-or-leave-it deal; if he turns his land over to the chief, the chief will enroll him in a communal insurance arrangement. The chief will tell him each day how hard to work, and will give him some grain at the end of the day. What makes this deal attractive is that the chief promises that his instructions regarding work, and the consumption he awards, will be such that the agent will receive $w(L)$ expected utils. Since the chief wants to enroll every Simulatan in the communal scheme, he will pick $w(L)$ larger than the expected utils the agent would receive if he decided to farm his land on his own. Let $\mu(w)$ be the distribution of $w(L)$ in the population.

The problem faced by the chief, then, is to choose for each w, given the current shock θ the action taken by the agent, as well as the consumption and future expected utility under both high and low realizations of output. This problem has a natural recursive formulation, given by

$$\begin{aligned} S(w,\theta) = \max_{\{a,(c_L,c_H),(w'_L,w'_H)\}} \quad & \Pr(q_H|a)[q_H - c_H + \beta \int S(w'_H,\theta')\mathrm{d}\lambda(\theta')] \\ + \quad & \Pr(q_L|a)[q_L - c_L + \beta \int S(w'_L,\theta')\mathrm{d}\lambda(\theta')] \end{aligned} \tag{1}$$

subject to keeping promises regarding expected utility:

$$V(a) + \Pr(q_H|a)[U(c_H) + \beta w'_H] + \Pr(q_L|a)[U(c_L) + \beta w'_L] = w. \tag{2}$$

Because the chief is risk neutral, agents will face no uncertainty in consumption. The constant level of consumption for each agent is determined by the agent's initial endowment, $w(L)$. Agents may face uncertainty in labor, since the random productivity shock θ will give rise to different recommended actions.

4. Private Information

If the principal is unable to observe effort, then the remuneration provided to agents, in addition to satisfying the promise-keeping constraint (2), must also be incentive compatible:

$$w \geq V(\hat{a}) + \Pr(q_H|\hat{a})[U(c_H) + \beta w'_H] + \Pr(q_L|\hat{a})[U(c_L) + \beta w'_L] \tag{3}$$

for all recommended and deviation actions $(a, \hat{a})$. Note that by substituting for w from (2) and manipulating the result, we have

$$V(\hat{a}) - V(a) \geq [\Pr(q_H|a) - \Pr(q_L|\hat{a})][U(c_H) - U(c_L) + \beta(w'_H - w'_L)] \tag{4}$$

The principal must seek to prevent shirking on the part of the agent. Since V is decreasing in a, if the agent were to consider a level of effort of $\hat{a} < a$, then the left hand side of this expression is positive. Since the likelihood of high output is increasing in a, (4) implies that the agent's utility in the high output state must be strictly greater than that in the low output state: the hidden action problem does not exhibit full insurance. Furthermore, since agents seek to smooth their consumption over time, the efficient means of satisfying (4) requires the principal to set both $c_H \geq c_L$ and $w_H \geq w_L$. This latter fact, of course, implies that the expected utility, or wealth, of agents is shifting about over time.

5. Estimation and Calibration

Ligon (1994), using data from three ICRISAT VLS villages (Aurepalle, Shirapur, and Kanzara) and assuming a model of private information, estimates the coefficient of relative risk aversion to be 0.83. Pender and Walker (1990), using experimental methods, estimate $\beta \approx 0.8$.

Using maximum likelihood techniques, estimating the productivity parameter η gives values of around 0.3.

Assuming on the preference side that $\alpha = \gamma = 0.83$, and neglecting time and household effects in production gives us an almost fully specified economic model.

Let $C = [\underline{c}, \overline{c}] \subset \mathbb{R}$ be the set of feasible consumptions, where $\underline{c}$ and $\overline{c}$ are determined by the minimum and maximum levels of observed consumption in the VLS villages between 1976-83.

Let W be the set of feasible expected utilities, with upper and lower bounds dictated by perpetual consumption at the upper and lower bounds of feasible consumption, and with no action taken[2].

Over the period observed in the ICRISAT data, there were no large aggregate shocks (droughts). Though this was fortunate for the inhabitants of these villages, it was in some sense unfortunate for us. However, we'll revel in the villager's good fortune by setting the aggregate shock, θ, equal to one in every period.

6. Computation

Using the parameters suggested in the last section, I computed the efficient solution to the private information economy described above. The techniques used are essentially those of Phelan and Townsend (1991). Figures 1 through 7 summarize some of the relevant findings of this exercise.

Figure 1 describes the utility possibility frontier for principal and agent. As described above, the upper bound $\overline{w}$ on W can be achieved only if the agent receives the upper bound on c and the lower bound on effort. These are the idle rich. The lower bound $\underline{w}$ on W can be attained only the unemployed; those whose effort is minimal, and whose consumption is also minimal. Alternative assumptions regarding the upper and lower bounds on W are critical, in the sense that the optimal contract for *every* w depends on the bounds of this compact set. Note that it would be extremely costly for the principal to maintain an agent at $\overline{w}$; if the principal is unable to commit to do so, then the upper bound on W would have to be redefined to be the point in $\mathbb{R}$ where the principal's surplus is non-negative. Since any change in the bounds of w will in general lead to a change in the position of this "crossing point," some sort of recursive calculation of this upper bound would be required.

[2]An alternative lower bound on w could be given by the autarkic level of utility. If the upper bound on W were also modified to insure that the principal received non-negative profit at this upper bound, then this would yield the model described by Dutta, Ray, and Sengupta (1989), in which the lower bound on w is described by the utility received by the agent if the principal reneges.

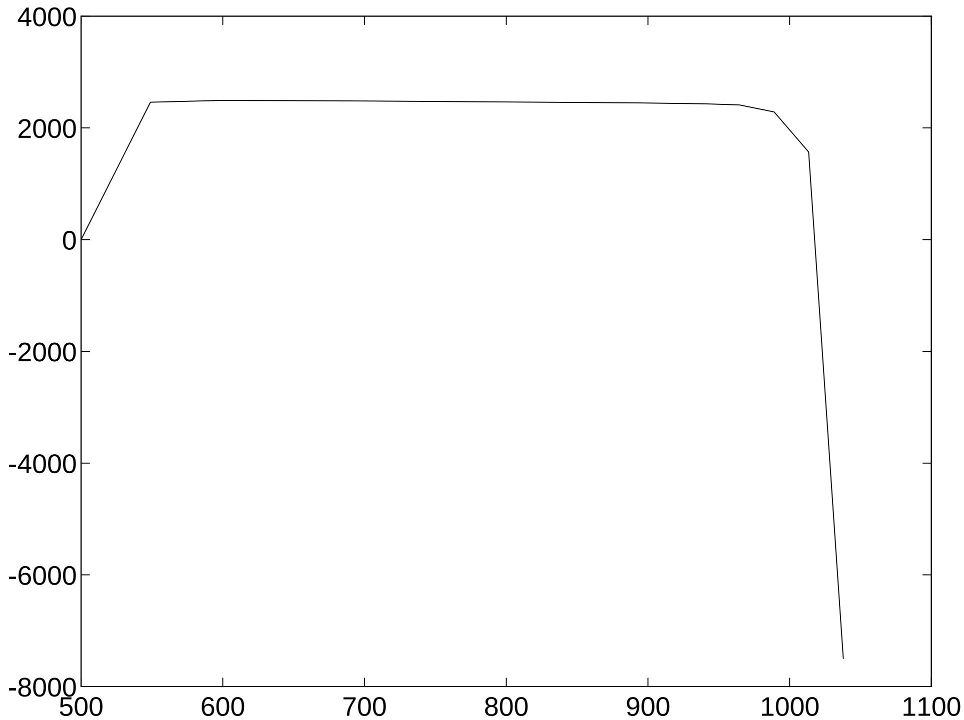

Figure 1: Utility Possibility Frontier

The principal faces a different problem of commitment with agents who are near the lower bound of w. In the region of Figure 1 in which the utility possibility curve is upward sloping, both parties would be better off if the contract were renegotiated, and the agent assigned a higher value of w. This would induce a higher level of effort (and hence expected profit) from the agent, and a higher level of expected utility for the agent. It should be possible to recursively determine the set of w over which the efficient contract is renegotiation proof in this sense, much as described above for determining the commitment-free upper bound on W.

The technology is shown in Figure 2. This conditional likelihoold function was estimated using maximum likelihood techniques on a sample of land plots from the three VLS villages. The ICRISAT data includes information on the labor used with each plot, separated by the labor of the plot owner, along with hired labor. One hypothesis which the model itself suggests is that the labor effort of hired labor might be misreported, so I experimented with estimation restricting the sample to plots on which only the owner (or members of his household) worked. The difference between the two estimates is significant but small, so I have neglected it here.

The measure of labor input is given as the proportion of the total time endowment spent working. Following earlier authors who have worked with the labor dataset (Rosenzweig and Wolpin 1985), I rather arbitrarily take the total amount of time available for work to be sixteen hours per day, six days per week. The actual labor input for the households in the VLS sample is much less that this; the largest village average is around four hours per day, rather than the approximately 14 hours per day implied for some agents by Figure 3.

Figure 3 indicates the level of effort the principal is able to induce the agent into taking. As noted, the predicted proportion (up to 93%) seems extremely high; however, the level of effort is quite sensitive to the choice of bounds on the set of feasible utilities W, as well as to the specification of the time endowment. All of these were chosen rather arbitrarily; a more careful effort would choose these quantities to match, say, the observed labor effort, and then test to see whether other predicted quantities could also be approximately matched.

Accepting these shortcomings for a moment, Figure 3 indicates, as specified above, that both the very rich and the very poor work not at all. There is a wide range of possible expected utilities for which effort varies relatively little; this is due in part to the nature of the technology, which exhibits large returns to labor for small effort levels, and sharply diminishing returns for higher levels.

These sharply diminishing returns are reflected also in Figure 4, which gives the expected level of output by wealth level, given the relation between wealth and effort exhibited in Figure 3. Note that, since expected output is monotonically increasing in effort, Figure 4 goes up and down in all the same places as does Figure 3; however, for w between about 550 and 975, there is much less variation in output than there is in effort.

The principal has three tools he can use to induce high levels of effort, and it is the efficacy with which he can wield these tools with agents of varying wealths that determines the variation in effort by wealth. The first such tool is the expected level of consumption. The principal is extremely stingy in his use of this incentive device, since it has an immediate and one-for-one effect on his own utility. This stinginess is reflected in Figure 5; agents must be moderately wealthy before they

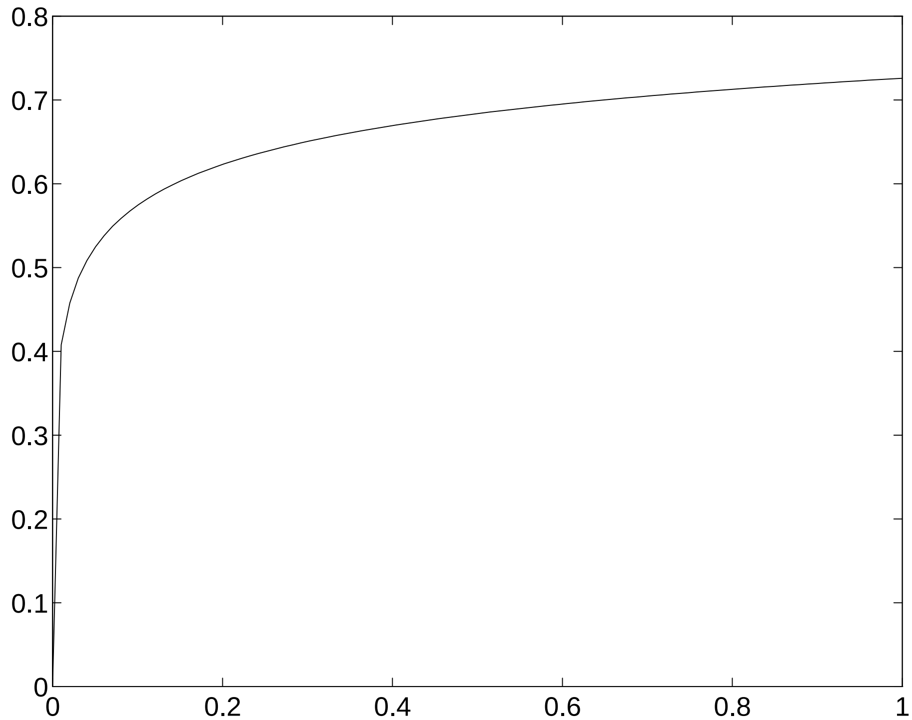

Figure 2: Production Function

receive *any* of the consumption good. Similarly, the flat area in the curve reflects the principal's preference for rewarding higher w agents with a cheaper coin. The principal cannot avoid giving the upper bound of consumption to agents at the upper bound of W, so expected consumption skyrockets as wealths approach this limit. This rapid increase is due to the success of the principal at satisfying lower w agents with instruments other than consumption.

The second tool that the principal has at his disposal is consumption insurance. Since the principal is risk-neutral, it costs him nothing to provide this sort of incentive, however, he must guard against providing too much insurance for fear of violating the incentive compatibility constraints given by equation (3). Figure 6 gives a measure of the insurance provided; the difference in consumption in the high and low output states. If there is no difference (the curve coincides with

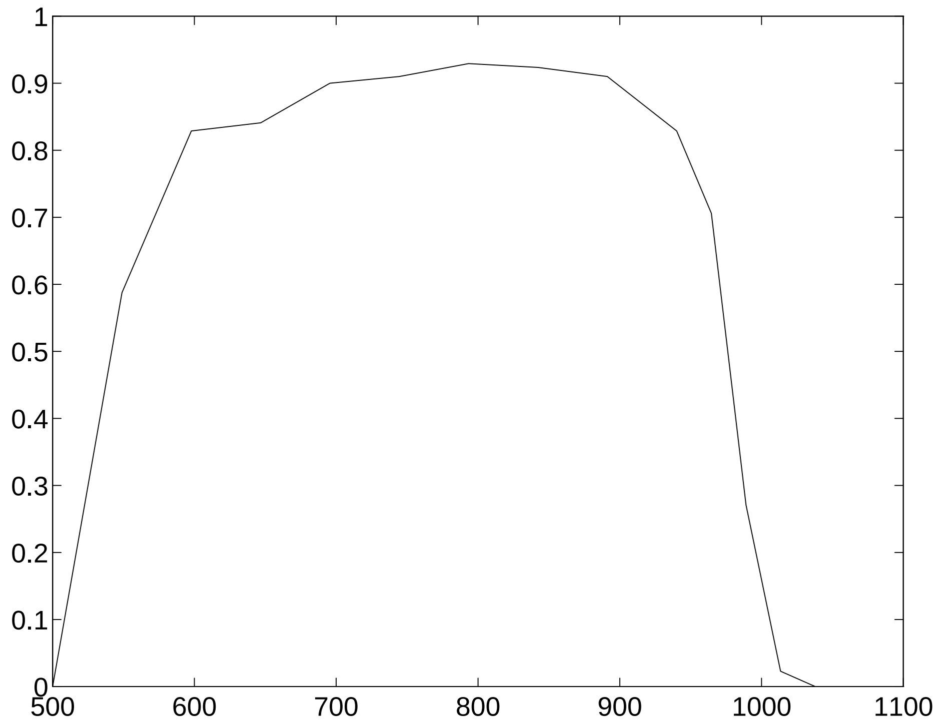

Figure 3: Effort Level by Wealth

zero) then there is perfect insurance; the higher the curve, the less the insurance. Note that the peak of curve in Figure 6 coincides with the beginning of the flat segment in Figure 5 depicting expected consumption; the decrease in uncertainty is the principal tool used by the principal over this range of wealths to reward higher w agents. The second, smaller peak in the figure between about 970 and 1040 is interesting, and coincides with the rapid increase in expected consumption seen in Figure 5. The principal is evidently constrained to increase expected consumption, but also increases consumption uncertainty over a certain range in order to reduce agents' expected utility, and induce them to take a higher action than they might otherwise.

The third and final tool available to the principal is the one I find the most intriguing. The principal can seek to induce high effort today in exchange for high

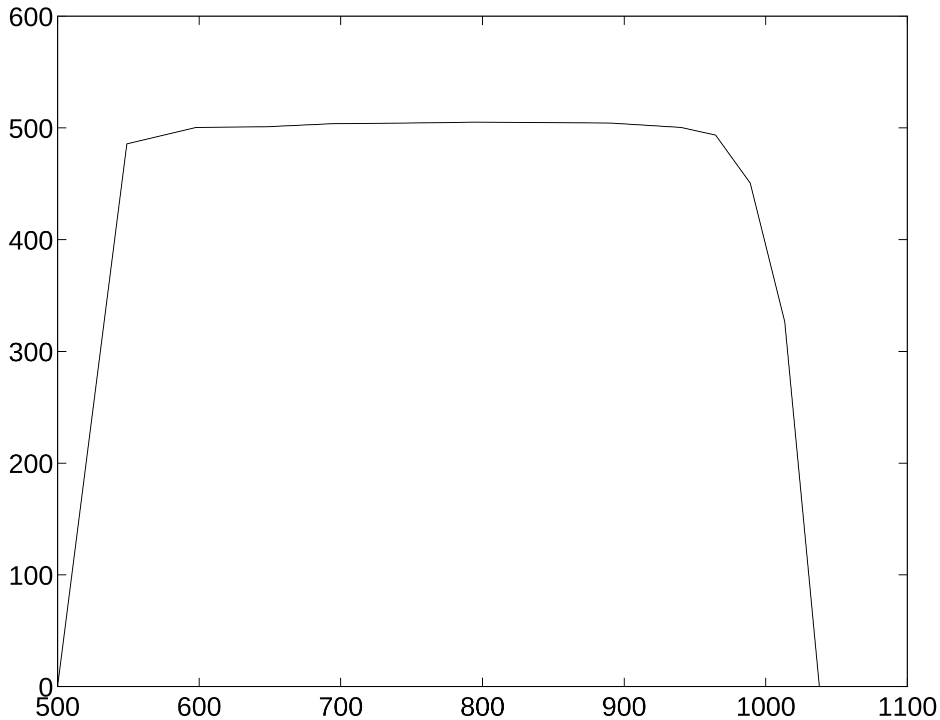

Figure 4: Expected Output by Wealth

expected utility tomorrow, or similarly, can punish low output by offering lower expected utility. The transition diagram in Figure 7 summarizes the principal's use of this device. The upper line labelled 'w_H' gives the future expected utility awarded to an agent who produces high output today. Similarly, the lower line labelled 'w_L' gives the expected future utility which punishes an agent who produces the low output. The two lines coincide only for the idle rich and the unemployed; no effort is expected of either of these two classes, and so none is rewarded. These two extrema comprise the set of stationary w's.

The middle line of this figure denotes today's expectation regarding tomorrow's expected utility, before output is observed. This line crosses the 45 degree line at about 730 utils; for w's below this point, the principal offers agents a carrot; above this he threatens them with a stick. Note the tendency of such a scheme to

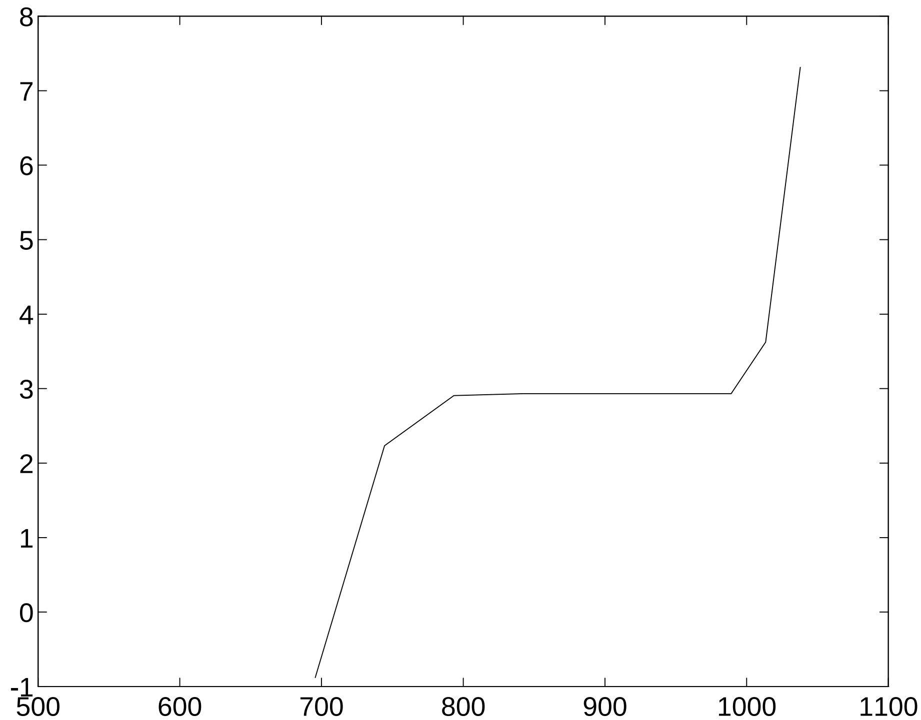

Figure 5: Expected Consumption by Wealth

push agents toward the center of the wealth distribution. The principal chooses to push agents toward the center of the wealth distribution because that is the region in which he is most easily able to induce them to high levels of exertion. Interestingly, this egalitarian feature of the evolution of the wealth distribution is in some sense a short-run phenomenon; $\{\underline{w}, \overline{w}\}$ is the ergodic set to which all wealths must eventually belong.

The fact that the limiting wealth distribution has this degenerate character was first observed by Thomas and Worrall (1990); Phelan (1990) seeks to ameliorate this somewhat unpleasant feature by observing that in the long run we are all dead; he models an overlapping generations world, in which agents begin life at a fixed (interior) w, and then die before their wealth goes to an extreme. Though this one example falls far short of a general demonstration, the short-run egalitarianism of

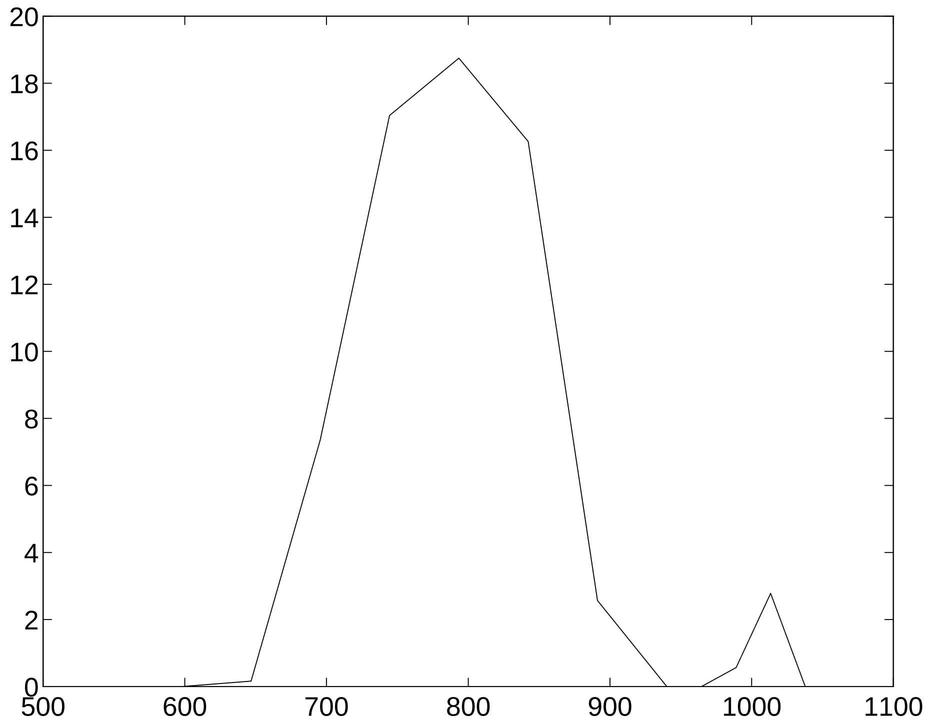

Figure 6: Difference Between High Consumption to Low Consumption by Wealth

the wealth distribution seems to be an underappreciated feature of this model.

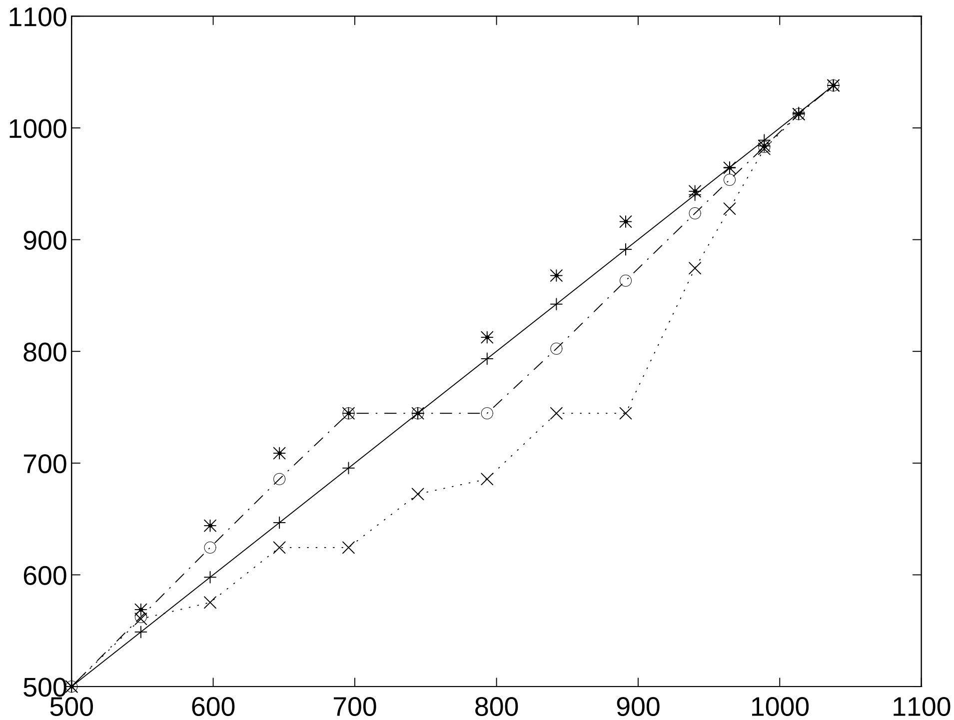

Figure 7: Wealth Transition

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