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ABSTRACT

The resonant response of a uniform plasma to an external plane-wave field is formulated in terms of the mismatch between the driving frequency and the complex nonlinear normal-mode frequency at the driving wave number. Simulation exhibits good agreement with theory.

While much study has been devoted to the topic of nonlinear plasma waves, propagating freely,[1] relatively little attention has been paid to the problem of the driven nonlinear plasma wave.[2] Such a problem arises in two situations of current interest: (a) The nonlinear interaction of two high-frequency electromagnetic waves $(\omega_1,\vec{k}_1)$, $(\omega_2,\vec{k}_2)$ occurs through an effective beat potential[3] at $(\omega_0 = \omega_1 - \omega_2, \vec{k}_0 = \vec{k}_1 - \vec{k}_2)$, which can excite a nonlinear Langmuir wave if the beat potential is strong and nearly resonant, i.e., if $\varepsilon(\omega_0,\vec{k}_0) << 1$. (b) A single nonlinear Langmuir wave $(\omega_1,\vec{k}_1)$ may be amplitude-modulated at $(\omega_0,\vec{k}_0)$, and thereby drive a nonlinear ion wave[2a] if $\omega_0 \approx k_0 c_s$.

In the present paper, we study the nonlinear excitation in detail, taking the driving potential as given (i.e., we ignore the reaction back on it) and treating the excitation of a one-dimensional Langmuir wave for definiteness, with the nonlinearity due to electron trapping. We develop an analytic theory for the response in terms of the nonlinear eigenfrequency, and test the theory by computer simulation.

To illustrate the phenomenon, we show in Fig. 1: (a) the driving field $E_0(x)$ and the total plasma response $E(x)$ at a given time; (b) the electron phase space (x,v) at that time; and (c) the electron velocity distribution at that time. (The parameters are given in the caption.) Figure 2a shows the magnitude and phase of the response relative to the driving field.

The basic idea of our paper is that the amplitude of the plasma response is inversely proportional to the frequency mismatch, i.e., to the difference between the driving frequency ω_0 and the normal mode (complex) frequency $\omega(\vec{k}_0)$ of a freely propagating wave. The latter, however, is in turn a nonlinear function of wave amplitude, as is known from several theories.[1,4] The evolution of the response (Fig. 2a) is thus tied to that of the complex nonlinear eigenfrequency (Fig. 2b). Finally, the asymptotic value of the nonlinear frequency shift, as deduced from the response in our simulations for various nonlinear amplitudes, can be compared (Fig. 3) with the theory of Morales and O'Neil,[4a] based on trapped-particle orbits.

We consider then a given external driving potential $\Phi_0(x,t) \equiv \Phi_0(t)\exp(i\,k_0\,x) + c.c.$, and the self-consistent plasma potential $\Phi_{sc}(t)$ at the same wave number k_0 (suppressing harmonics of k_0; see Fig. 1a). The latter is determined by the Poisson equation $\Phi_{sc}(t) = (4\pi/k_0^2)\,\rho(t)$, while the charge density $\rho(t)$ is determined by the relevant kinetic equation. We postulate a (weakly) nonlinear susceptibility kernel $\bar{\chi}(\tau)$ relating the charge density at t to the total potential $\Phi \equiv \Phi_0 + \Phi_{sc}$ at time $t - \tau$: $(4\pi/k_0^{\,2})\,\rho(t) = -\int_0^\infty d\tau\ \bar{\chi}_{NL}(\tau)\ \Phi(t-\tau)$. (The nonlinearity is implicit in $\bar{\chi}$.) We now factor out the dominant (ω_0) time-dependence for each function of time, e.g., $\rho(t) \equiv \tilde{\rho}(t)\exp(-i\,\omega_0\,t) + \text{c.c.}$, and obtain

$$(4\pi/k_0^{\,2})\,\tilde{\rho}(t) = -\int d\tau\ \bar{\chi}_{NL}(\tau)\exp(i\omega_0\tau)\,\tilde{\Phi}(t-\tau)$$
$$= -\int d\tau\ \bar{\chi}_{NL}(\tau)\exp(i\omega_0\tau - \tau d/dt)\,\tilde{\Phi}(t)\ .$$

Substituting this into the Poisson equation $\tilde{\Phi} - \tilde{\Phi}_0 = (4\pi/k_0^{\,2})\tilde{\rho}$, and using the usual definitions $\varepsilon(\omega) \equiv 1 + \chi(\omega)$ and $\chi(\omega) \equiv \int_0^\infty d\tau\ \bar{\chi}(\tau)\exp(i\omega\tau)$, we obtain the formal equation for the total response amplitude $\tilde{\Phi}(t)$:

$$\varepsilon_{NL}(\omega_0 + i\ d/dt)\ \tilde{\Phi}(t) = \tilde{\Phi}_0(t)\ . \qquad (1)$$

We note that ε_{NL} depends implicitly on Φ, and that if Φ_0 acts only on electrons, but not on ions, the right side of (1) should be replaced by $\left[1 + \chi_{NL}^{\,i}(\omega_0 + i\ d/dt)\right]\Phi_0(t)$.

To introduce the nonlinear normal mode frequency ω_{NL} (at wavenumber k_0), we let the driver Φ_0 vanish, whence Eq. (1) reduces to $\varepsilon_{NL}(\omega) = 0$ (with ω replacing $\omega_0 + i\ d/dt$). We define ω_{NL} as the complex root of the nonlinear dielectric function $\varepsilon_{NL}(\omega)$. Defining the nonlinear increments $\delta\varepsilon \equiv \varepsilon_{NL} - \varepsilon_L$ and $\delta\omega \equiv \omega_{NL} - \omega_L$, where ω_L is the linear normal mode frequency (i.e., the complex root of the linear dielectric function $\varepsilon_L(\omega)$), we have the relation $\delta\omega = -\,\delta\varepsilon/\bar{\varepsilon}_L$ to lowest order, where $\bar{\varepsilon} \equiv \partial\varepsilon/\partial\omega$.

Returning to Eq. (1), we expand $\varepsilon_{NL}(\omega_0 + i\ d/dt)$ in a Taylor series about ω_{NL} (where ε_{NL} vanishes), and obtain to first order

$$(\omega_0 - \omega_{NL} + i\ d/dt)\ \tilde{\Phi}(t) = \tilde{\Phi}_0(t)/\bar{\varepsilon}_{NL}\ , \qquad (2)$$

where $\bar{\varepsilon}_{NL} \equiv \partial\varepsilon_{NL}/\partial\omega$ at ω_{NL}. We note that ω_{NL} is implicitly a function of $\tilde{\Phi}$, so that this equation of evolution for $\tilde{\Phi}$ is nonlinear. Writing Eq. (2) in the form $\tilde{\Phi}(t) = (\omega_0 - \omega_{NL} + i\ d/dt)^{-1}\ \tilde{\Phi}_0(t)/\bar{\varepsilon}_{NL}$, we see that, in the limit of slow variation, the amplitude of the response varies reciprocally as the complex nonlinear frequency mismatch $\omega_0 - \omega_{NL}$. Thus for small mismatch, the response is large, thereby modifying ω_{NL} and the mismatch, and either enhancing or depressing the response (depending on the signs of the mismatch and of $\delta\omega$).

Since ω_{NL} depends on the history of $\tilde{\Phi}$, not just on its instantaneous value, and since no theory yet exists for this relationship (except qualitatively,[4a] or asymptotically in time[4]), we use Eq. (2) to determine the evolution of $\omega_{NL}(t)$ (Fig. 2b) in terms of the complex response ratio $R(t) \equiv \tilde{\Phi}(t)/\tilde{\Phi}_0(t)$ (Fig. 2a), where the latter is obtained from computer simulation. Specifically, we have $\omega_{NL}(t) = \omega_0 - \left[R(t)\bar{\varepsilon}\right]^{-1} + i\ \dot{R}/R$, when $\tilde{\Phi}_0$ is a step function in time.

We interpret the oscillations (Fig. 2b) of $\delta\Omega$ and $\delta\gamma$, the real and imaginary parts of the frequency shift $\delta\omega(t)$, by generalizing the momentum and energy balance considerations of Morales and O'Neil[4a] to the driven case. The total particle momentum density $P(t)$ evolves as $dP/dt = i\ k_0\ \tilde{\rho}(t)\ \tilde{\Phi}^*(t) + c.c.$ As in the derivation of Eq. (1), we have $\tilde{\rho}(t) = (k_0^2/4\pi)\left[1 - \varepsilon_{NL}(\omega_0 + i\ d/dt)\right]\ \tilde{\Phi}(t)$. Substituting $\tilde{\rho}(t)$ above, and expanding ε_{NL} about ω_{NL}, we obtain to lowest order, $(d/dt)\left[P(t) - (k_0\bar{\varepsilon}_L)(k_0^2/4\pi)|\Phi(t)|^2\right] = -2(\gamma_L + \delta\gamma)(k_0\bar{\varepsilon}_L)(k_0^2/4\pi)|\Phi(t)|^2$. The left side of this equation is the evolution of the difference between the total particle momentum and the wave momentum, and thus of the momentum of the resonant particles. Since this oscillates at the trapping frequency, so does the normal-mode nonlinear damping rate $-(\gamma_L + \delta\gamma)$.

The total particle kinetic energy density $K(t)$ evolves as $dK/dt = -\Phi^*(t)\, d\rho(t)/dt + c.c.$ After more extensive algebraic manipulation, we obtain analogously[5]

$$(d/dt)\left[\!\left[K(t) - \left\{\left[1 + 2(\omega_0 - \Omega_L)/\omega_0\right]\Omega_L\bar{\varepsilon}_L - 1\right\}(k_0^2/4\pi)|\Phi(t)|^2\right]\!\right]$$

$$= \left\{-2(\gamma_L + \delta\gamma) - 2d(\delta\Omega/\omega_0)/dt + \left[\beta - (\delta\Omega/\omega_0)\right]d/dt\right\}$$

$$\times\ \Omega_L\bar{\varepsilon}_L(k_0^2/4\pi)|\Phi(t)|^2, \quad \text{where} \quad \beta \equiv \mathrm{Re}(\delta\bar{\varepsilon}/\bar{\varepsilon}_L).$$

(We note that the Morales-O'Neil version of this equation is considerably simpler, since they use the linear wave frame, where $\Omega_L = 0$.) The left side now represents the evolution of the kinetic energy of the resonant particles: the total nonlinear kinetic energy appears on the left side with the linear wave kinetic energy subtracted away (obtained by taking the difference of the total linear wave energy, corrected by the linear mismatch $\omega_0 - \Omega_L$, and the field energy). In the wave frame only the terms in $\delta\Omega$ survive on the right, from which Morales and O'Neil deduce oscillations in $\delta\Omega$ also at twice the trapping frequency. (This is less evident in our simulations, where variations at the trapping frequency are observed but variations at the harmonic cannot be resolved. We are reminded of the similar time dependence of the nonlinear frequency shift of a single large wave driven by the beam-plasma instability (see Fig. 2 of Ref. 6)).

Asymptotically in time, a steady state is reached $(d/dt,\ \gamma_L + \delta\gamma \to 0)$, and the frequency shift $\delta\Omega$ can be compared with the theory of Morales and O'Neil (Fig. 3), which predicts a dependence as $|\tilde{\Phi}|^{\frac{1}{2}}$. Their theory requires that the trapping velocity be small compared to (thermal speed squared/phase velocity): $|e\tilde{\Phi}/m|^{\frac{1}{2}} \ll k_0 v_e^{\,2}/\omega_0$, and that only the tail of the electron velocity distribution is involved: $\omega_0/k_0 > 4v_e$, which guarantees that linear Landau damping will be weak. Both inequalities are violated in our simulations, where $|e\tilde{\Phi}/m|^{\frac{1}{2}} \sim k_0 v_e^{\,2}/\omega_0$ and $\omega_0/k_0 = 3v_e$; thus the qualitative agreement is quite satisfactory. Although the electron distribution function suffers considerable modification due to the interaction of many resonant particles with the wave, its main body remains Maxwellian (Fig. 1c).

The algebraic dependence of the frequency shift on the wave amplitude $\delta\Omega \propto |\tilde{\Phi}|^{\frac{1}{2}}$ admits the possibility of the existence of multiple equilibrium solutions to Eq. (2). In fact under certain circumstances there can be three equilibrium wave amplitudes of which one is always unstable against small perturbations.[7] For the parameters of our simulations, Eq. (2) admits only one equilibrium solution, which is stable. (The equilibrium and stability analysis will be considered elsewhere.[8])

In conclusion, we have formulated the resonant response of a plasma wave to excitation, in terms of the mismatch between the driving frequency and the time-dependent, complex, nonlinear eigenfrequency of a normal mode. Simulation shows good agreement with existing theory.

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5. We assume the following ordering in deriving the energy and momentum laws: $\omega_0, \Omega_L \sim O(\omega_p)$; $\delta\Omega, \omega_0 - \Omega_L, \omega_0\beta, (d/dt) \sim O(\eta\, \omega_p)$; and $\gamma_L, \delta\gamma \sim O(\eta^2\, \omega_p)$ where $\eta \ll 1$.

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FIGURE CAPTIONS

Fig. 1. Simulation of resonant response of a Maxwellian (thermal speed v_e) electron plasma (uniform positive background) to a plane wave driving field, of frequency ω_0 (chosen to equal ω_p, the plasma frequency) and phase velocity ω_0/k_0 (chosen to equal $3.0\ v_e$). The linear normal mode frequency Ω_L is $1.17\ \omega_p$, and the linear Landau damping $-\gamma_L$ is $0.03\ \omega_p$. For a typical simulation, we exhibit at $\omega_p t = 300$:

(a) the driving field E_0 and the total field E, as functions of x, in natural units;

(b) the electron phase space;

(c) the velocity distribution, in arbitrary units.

Fig. 2. For the same simulation as in Fig. 1, we show, as functions of time,

(a) the magnitude r and phase θ of the relative response function $r \exp i\theta \equiv \tilde{\Phi}/\tilde{\Phi}_0$;

(b) the deduced frequency shift $\delta\Omega$ and nonlinear damping γ_{NL}.

Fig. 3. Asymptotic $(t \to \infty)$ frequency shift $\delta\Omega$ as a function of asymptotic wave amplitude $|\tilde{\Phi}|$, in natural units, as deduced from simulations, for sudden driver switch-on (●) and "adiabatic" switch-on over $\omega_p t = 50\ \pi$ (○). The solid line represents the Morales-O'Neil theoretical prediction.

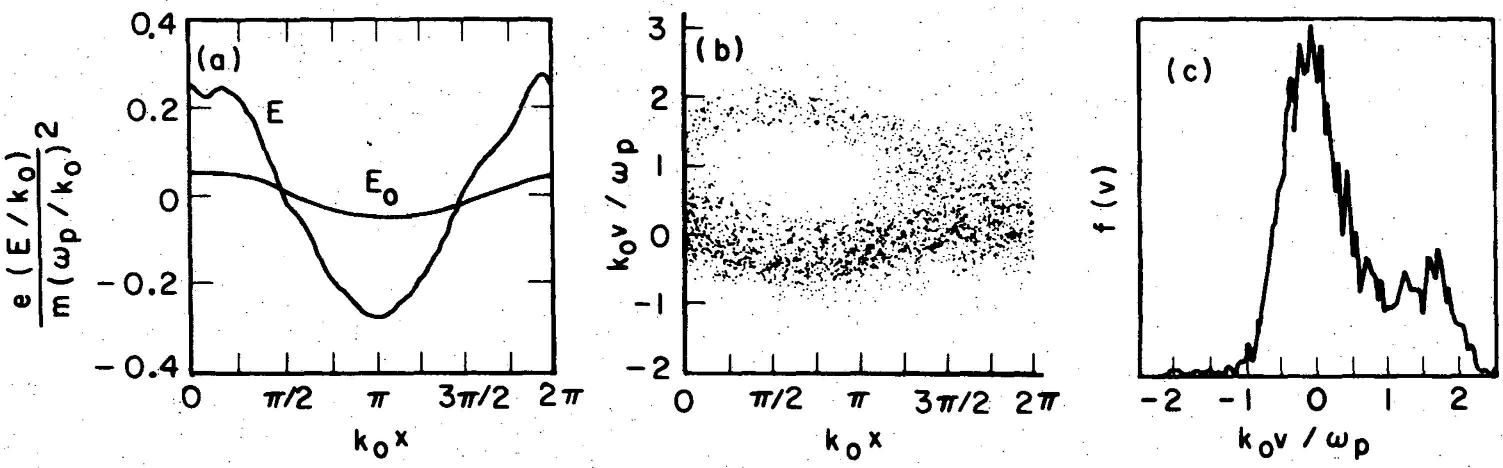

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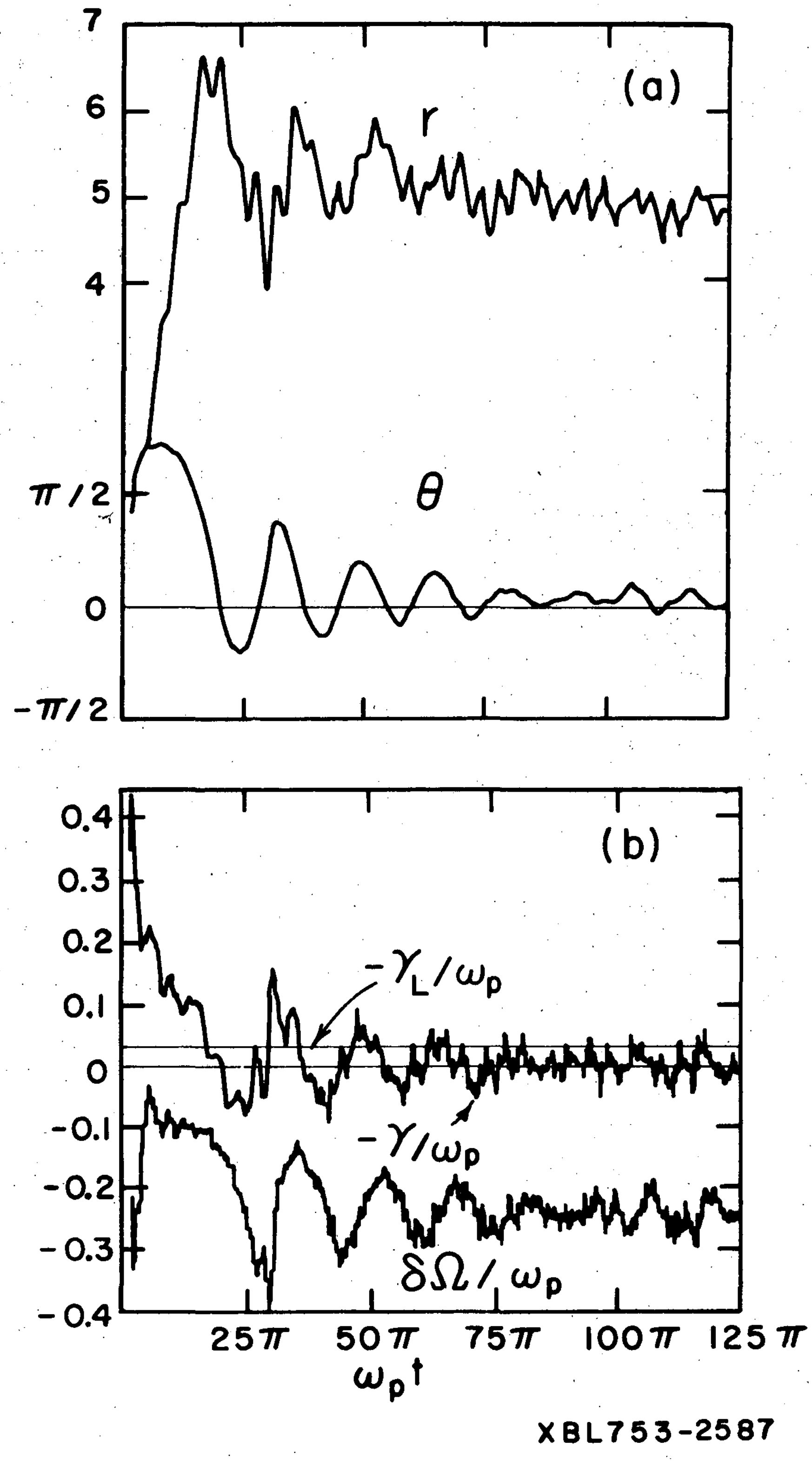
Fig. 1

-9-

0 0 0 0 4 2 0 2 7 5 5



7
6
5
4
π/2
0
-π/2
(a)
r
θ
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
(b)
$-\gamma_L/\omega_p$
$-\gamma/\omega_p$
$\delta\Omega/\omega_p$
25π
50π
75π
100π
125π
$\omega_p t$
XBL753-2587

Fig. 2

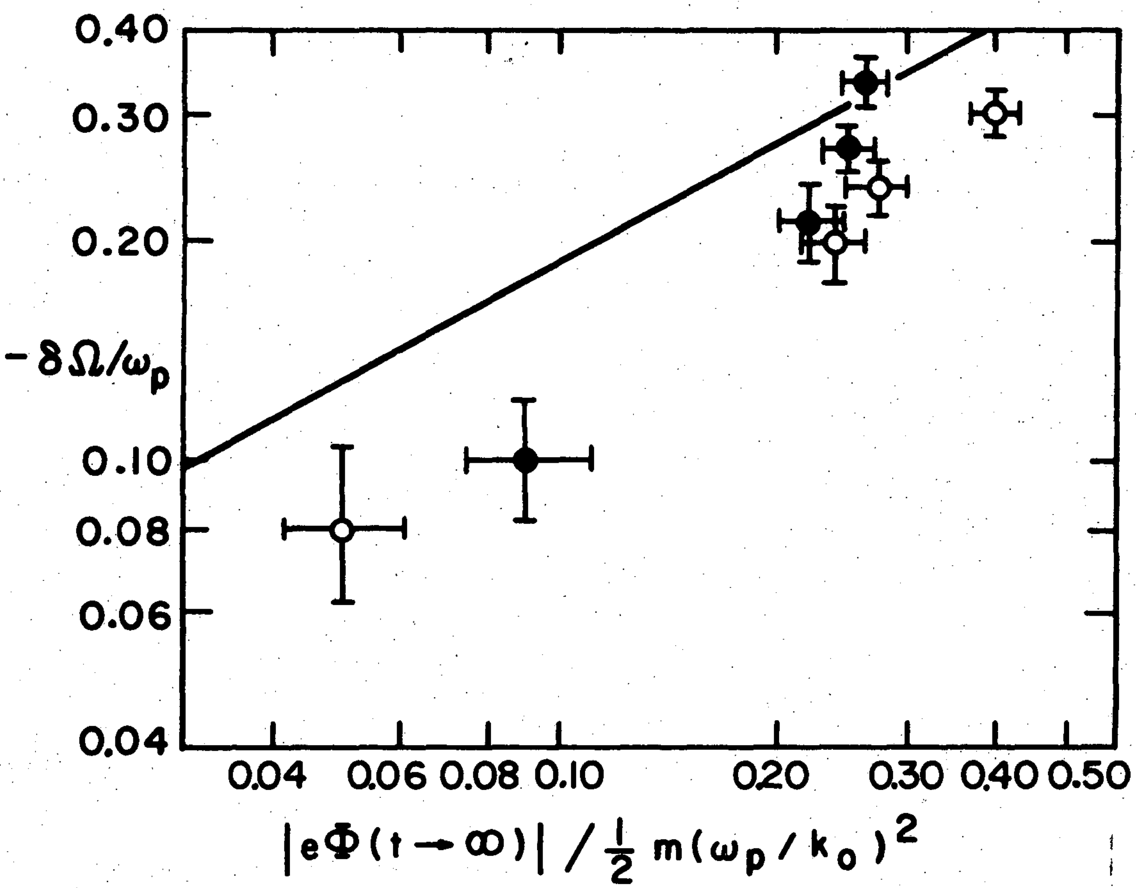


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Fig. 3

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