

# A Synchronized Lissajous-based Approach to Achieve Situational Awareness Using Synchronized Waveform Measurements

Milad Izadi, *Student Member, IEEE* and Hamed Mohsenian-Rad, *Fellow, IEEE*

**Abstract**—This paper proposes a new method to achieve situational awareness in power distribution systems. The proposed method uses synchronized data from *waveform measurement units* (WMUs); which are an emerging class of smart grid sensors. WMUs provide GPS-synchronized voltage waveform and current waveform measurements. In the center of the proposed method is to construct synchronized Lissajous curve; where we plot the voltage waveform versus the current waveform measurements. Under normal operating conditions, the synchronized Lissajous curve shows an ellipse; however, once an event occurs in the network, the shape of the Lissajous curve may change. The changes depend on the type of the event, the location of the event, and other characteristics of the event. Therefore, an event can be characterized based on the changes in the synchronized Lissajous curve. These changes, such as in the area of the synchronized Lissajous curve, can be used also to detect an event, even if the duration of the event is very short. Importantly, the proposed method does not require any prior information of the network as it only requires data from as few as only two WMUs. We examine the performance of the proposed method on the IEEE 33-bus test system for the cases of different events including sustained events, such as equipment switching, and temporary events, such as incipient faults.

**Keywords:** Synchronous waveforms, synchronized Lissajous curves, situational awareness, substitution theorem, equivalent two-port network, power distribution, waveform measurement unit, WMU, data-driven analysis, event-based analysis.

## I. INTRODUCTION

The deployment of smart meters over the past decade has provided a wide range of benefits for data-driven monitoring and situational awareness in power distribution systems [1]. However, there are major limitations to smart meters; such as due to their slow reporting rates; at once every 15 minutes.

The reading interval, resolution, and time-precision of measurements have drastically improved in recent years by the use of distribution-level phasor measurement units (DPMUs). D-PMUs provide GPS-synchronized voltage phasor and current phasor measurements in power distribution circuits [2], [3]. Applications of D-PMUs are discussed in several paper, e.g., in [4], [5]. However, PMUs are not suitable to study power quality events in distribution systems. Examples of such events are incipient faults and high impedance faults.

Motivated by the above facts, a new class of smart grid sensors has been introduced recently, called *waveform measurement units* (WMUs). WMUs provide precise GPS-synchronized voltage waveform and current waveform measurements in time domain at very high resolutions [6], [7]. So far, WMUs have had only limited applications, e.g., to study

harmonics in transformers [8], sub-synchronous resonance [9], and to identify the location of power quality events [6].

The very high reporting rate of WMUs and the fact that they provide *synchronized* voltage waveform and current waveform measurements, can significantly improve situational awareness and operational intelligence in distribution systems. This is a breakthrough when compared with the prior studies in the literature that look into the waveform measurements *at only one location*, e.g., see [10], [11]. Having access to multiple synchronized waveform measurements can enhance accuracy and robustness in situational awareness.

Here, situational awareness is broadly defined as gaining various information about the operational status of the grid and its components with focus on the analysis of events that occur across the power distribution system. This definition has been used in the literature, such as in [5], [10].

In this paper, we propose a fundamentally new approach to the analysis of synchronized voltage and current waveform measurements with the ultimate goal of achieving situational awareness in power distribution systems. The contributions in this paper can be summarized as follows:

- 1) On one hand, this paper introduces a new way to work with the measurements from WMUs, based on a novel concept, called *synchronized Lissajous curve*; which is constructed based on the substitution theorem. On the other hand, this paper addresses the important problem of achieving situational awareness in distribution systems, in particular, with respect to power quality events.
- 2) The proposed method is capable of studying both *transient* events, such as incipient faults, and *sustained* events, such as high impedance faults and equipment switching. The events are detected and their types and locations are characterized based on the shape, area, and other parameters of the synchronized Lissajous curve.
- 3) Unlike most existing situational awareness methods that are done in *phasor domain*, the analysis in this paper is done in *time domain*. Note that, even in our recent studies in [6] that we use waveform measurements, the waveform measurements are first converted into phasors before they are analyzed. Our ability to conduct the analysis in time domain is the direct result of using the proposed concept of synchronized Lissajous curve.
- 4) The proposed method does *not* require any prior knowledge about the network model. Instead, the proposed method is *data-driven* and it *only* requires the synchronized data from as few as two WMUs.

We test and verify the performance of the proposed method by analyzing different cases of sustained events and temporary events, on the IEEE 33-bus distribution test system.

The authors are with the Department of Electrical and Computer Engineering, University of California, Riverside, CA, USA; e-mails: {mizadi, hamed}@ece.ucr.edu. This work was supported in part by UCOP grant LFR-18-548175. The corresponding author is H. Mohsenian-Rad.

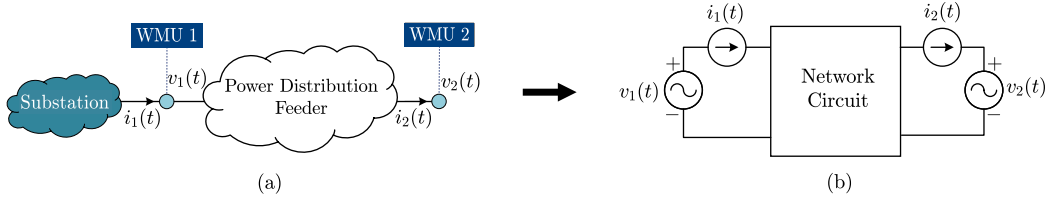


Fig. 1. Application of the substitution theorem: (a) a distribution feeder that is equipped with two WMUs; (b) the equivalent two-port network.

## II. PROPOSED METHODOLOGY

In this section, we present the method for our analysis; which is built upon two fundamental concepts: the substitution theorem and the voltage-current Lissajous curve.

### A. Substitution Theorem

According to the *substitution theorem* in Circuit Theory, an element in a circuit can be replaced by a voltage source or a current source that represents the characteristics of the element, c.f. [12]. Hence, we propose to model the network that is seen by each WMU with a voltage source based on the WMU's voltage waveform measurements and a current source based on the WMU's current waveform measurements.

As an example, consider the power distribution feeder in Fig. 1(a). Suppose it is equipped with two WMUs, where WMU 1 is installed at the beginning of the feeder and WMU 2 is installed at the end of the feeder. Let  $v_1(t)$  denote the voltage waveform measurements and  $i_1(t)$  denote the current waveform measurements that are obtained by WMU 1. Also, let  $v_2(t)$  denote the voltage waveform measurements and  $i_2(t)$  denote the current waveform measurements that are obtained by WMU 2. According to the *substitution theorem*, we can replace the network at WMU 1 with voltage source  $v_1(t)$  and current source  $i_1(t)$ , which are connected in *series*, as shown in Fig. 1(b). Similarly, we can replace the network at WMU 2 with voltage source  $v_2(t)$  and current source  $i_2(t)$ , which are connected in *series*, as shown in Fig. 1(b). It should be noted that, the two-port network model is a *virtual* model, whose voltage and current sources are *ideal* sources.

The above procedure transforms the power distribution feeder in Fig. 1(a) to the *two-port network* in Fig. 1(b).

### B. Synchronized Lissajous Curve

The Lissajous curve is a graph that is obtained by plotting the voltage waveform measurements versus the current waveform measurements. It has applications in signal and image processing; such as in electrocardiogram analysis and dielectric discharge analysis [13]. It also been used for detecting different types of internal faults in power transformers [14].

In this paper, we propose to construct a new type of Lissajous curve, that we refer to as the *synchronized* Lissajous curve. It is constructed based on the *difference* between the voltage waveforms at WMUs 1 and 2 versus the *difference* between the current waveforms at WMUs 1 and 2. In other words, it is the Lissajous curve that is obtained by plotting

$$\Delta v(t) = v_1(t) - v_2(t) \quad (1)$$

versus

$$\Delta i(t) = i_1(t) - i_2(t). \quad (2)$$

As we will explain in Section II-C, the above novel concept is well-suited to help us understand different types of events that occur at power distribution networks and captured by WMUs. However, before we discuss the synchronized Lissajous curve during an event; let us first understand its characteristics during *normal* operating conditions.

Consider the two-port network in Fig. 1(b). Suppose the network is in *normal* operating condition, i.e., there is no fault, harmonic, equipment mis-operation, or switching event. Therefore, each voltage or current waveform comprises only the fundamental component and can be expressed as

$$\begin{aligned} v_1(t) &= V_1 \cos(\omega_o t + \theta_1), \\ v_2(t) &= V_2 \cos(\omega_o t + \theta_2), \\ i_1(t) &= I_1 \cos(\omega_o t + \gamma_1), \\ i_2(t) &= I_2 \cos(\omega_o t + \gamma_2), \end{aligned} \quad (3)$$

where  $\omega_o$  denotes the fundamental rotational frequency;  $V_1$  and  $V_2$  denote the magnitudes of the voltage waveforms;  $I_1$  and  $I_2$  denote the magnitudes of the current waveforms;  $\theta_1$  and  $\theta_2$  denote the phase angles of the voltage waveforms; and  $\gamma_1$  and  $\gamma_2$  denote the phase angles of the current waveforms.

From (1) and (3), the voltage difference under the normal operating condition can be expressed as:

$$\Delta v(t) = V \cos(\omega_o t + \theta), \quad (4)$$

where

$$\begin{aligned} V &= \sqrt{V_1^2 + V_2^2 - 2V_1V_2 \cos(\theta_1 - \theta_2)}, \\ \theta &= \tan^{-1} \left\{ \frac{V_1 \sin \theta_1 - V_2 \sin \theta_2}{V_1 \cos \theta_1 - V_2 \cos \theta_2} \right\}. \end{aligned} \quad (5)$$

Similarly, from (2) and (3), the current difference under normal operating condition can be expressed as:

$$\Delta i(t) = I \cos(\omega_o t + \gamma), \quad (6)$$

where

$$\begin{aligned} I &= \sqrt{I_1^2 + I_2^2 - 2I_1I_2 \cos(\gamma_1 - \gamma_2)}, \\ \gamma &= \tan^{-1} \left\{ \frac{I_1 \sin \gamma_1 - I_2 \sin \gamma_2}{I_1 \cos \gamma_1 - I_2 \cos \gamma_2} \right\}. \end{aligned} \quad (7)$$

Next, we obtain the relationship between waveform  $\Delta v(t)$  and waveform  $\Delta i(t)$  by eliminating  $\omega_o t$  from equations (4) and (6). Accordingly, from (4), we have:

$$\omega_o t = \cos^{-1} \left\{ \frac{\Delta v(t)}{V} \right\} - \theta. \quad (8)$$

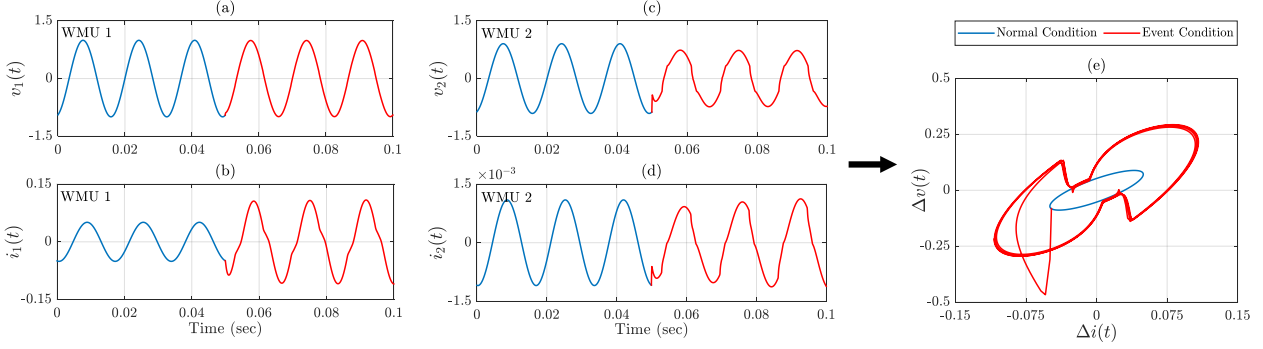


Fig. 2. An example of the synchronized Lissajous curve for a high impedance fault on a distribution feeder that is seen by two WMUs: (a)-(b) voltage and current waveform measurements from WMU 1 that is installed at the beginning of the feeder; (c)-(d) voltage and current waveform measurements from WMU 2 that is installed at the end of the feeder; (e) the corresponding synchronized Lissajous curve.

If we substitute (8) into (6), we can rewrite (6) as follows:

$$\begin{aligned} \Delta i(t) &= I \cos \left( \cos^{-1} \left\{ \frac{\Delta v(t)}{V} \right\} - \theta + \gamma \right) \\ &= I \cos(\theta - \gamma) \frac{\Delta v(t)}{V} \\ &\quad + I \sin(\theta - \gamma) \sqrt{1 - \frac{\Delta v(t)^2}{V^2}}. \end{aligned} \quad (9)$$

We square both sides and rearrange the terms to obtain:

$$A \Delta v(t)^2 + B \Delta v(t) \Delta i(t) + C \Delta i(t)^2 + D = 0, \quad (10)$$

where

$$\begin{aligned} A &= 1/V^2, & B &= -2 \cos(\theta - \gamma)/VI, \\ C &= 1/I^2, & D &= -\sin^2(\theta - \gamma). \end{aligned} \quad (11)$$

Equation (10) always represents an *ellipse* because

$$B^2 - 4AC < 0. \quad (12)$$

Therefore, the synchronized Lissajous curve that is obtained based on the difference  $\Delta v(t)$  versus the difference  $\Delta i(t)$  is always an ellipse during *normal* operating conditions.

### C. Application in Situational Awareness

Recall from Section II-B that under normal operating conditions, the synchronized Lissajous curve is an *ellipse*. However, once an “event” occurs somewhere on the distribution feeder, the voltage waveform and the current waveform may be distorted. Here, the event can be any disturbance, fault, mis-operation, load switching, or equipment switching.

As an example, consider the two-port network that we discussed in Fig. 1(b). Suppose a high impedance fault occurs in the network; with a small fault current and overall nonlinear characteristics. Fig. 2 shows the voltage waveform measurements and the current waveform measurements as well as the corresponding synchronized Lissajous curve during both the normal operating condition (blue) and also the event condition (red). As we see in Fig. 2(e), under normal operating conditions, the synchronized Lissajous curve is an ellipse. However, once the event occurs, the synchronized Lissajous curve takes a *very different shape*, making the impact of the high impedance fault clearly visible.

Importantly, the changes in the shape of the synchronized Lissajous curve carries information about the *type* of the

event, the *location* of the event, and other *dynamics characteristics* of the event, which will be discussed for different types of events through multiple case studies in Section III.

A key step in event-based situational awareness is to *detect* the events that occur in the system. In this regard, we propose an event detection method based on the synchronized Lissajous curve and the measurements that come from WMUs. The proposed method relies on the changes in the *area* of the synchronized Lissajous curve during two successive cycles.

Let us define the area of the synchronized Lissajous curve at time  $t$  over period  $T$  of the past cycle as follows:

$$Area(t) = \left| \int_{\Delta i(\tau=t-T)}^{\Delta i(\tau=t)} \Delta v(\tau) d(\Delta i(\tau)) \right|, \quad (13)$$

where  $|\cdot|$  is the operator to take the absolute value.

During normal operating conditions, there is little to no difference between two successive calculations of the areas in (13). However, once an event occurs, such difference suddenly becomes significant. This can help us detect the event. In this regard, let us define the *similarity index* at time  $t$  as:

$$S(t) = 1 - \left| \frac{Area(t) - Area(t - \Delta t)}{Area(t - \Delta t)} \right|, \quad (14)$$

where  $\Delta t$  is the reporting interval. Note that,  $S(t)$  is 1 if the two successive areas are equal; and less than one otherwise.

Next, we propose a time-adaptive threshold for event detection by considering the past similarity indices so as to lessen the number of false alarms. In this regard, consider the past areas right before time  $t$  from  $t - W$  to  $t - \Delta t$ , where  $W$  is the window duration for the threshold calculation and  $W > \Delta t$ . We define the median of their similarity indices as:

$$\mathcal{M}(t) = \text{median} \left\{ S(t - W), \dots, S(t - \Delta t) \right\}, \quad (15)$$

where  $\text{median}(\cdot)$  returns the median. Accordingly, we may detect an event at time  $t$  if the following inequality holds:

$$S(t) < \alpha \mathcal{M}(t), \quad (16)$$

where  $\alpha$  is the sensitivity factor to control the event detector. Note that, once an event is detected, the next step is to update the threshold. As the similarity index is very small at the event time, the area corresponding to the event should be discarded from the next calculation of the adaptive threshold.

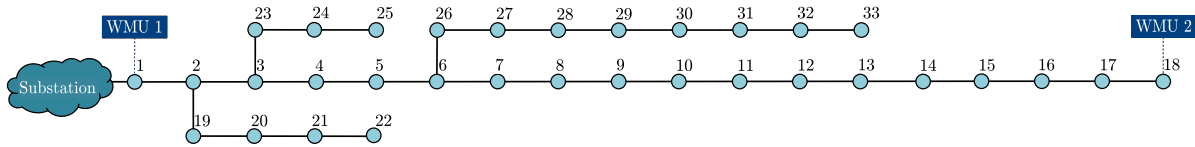


Fig. 3. The IEEE 33-bus distribution system with two WMUs. The proposed method is designed to capture events in form of synchronized Lissajous curves. An event can occur at any bus across the feeder; in form of a device or load switching, device mis-operation, incipient fault, etc.

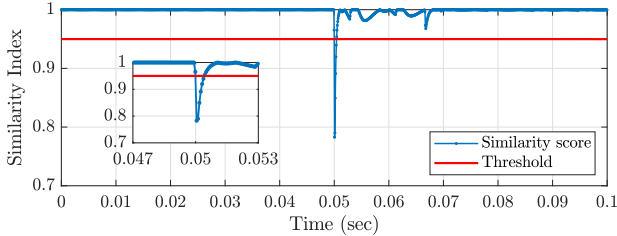


Fig. 4. The similarity index and the adaptive threshold for the synchronized Lissajous curve during the event condition in Fig. 2(e).

Fig. 4 shows the profile of the calculated similarity index and also the profile of the adaptive event detection threshold for the synchronized Lissajous curves that we saw in Fig. 2(e). The sensitivity factor is 0.95 and the window size is 65 msec. The similarity index drops from 1 to nearly 0.78 at  $t = 0.05$  sec, indicating that an event occurs at  $t = 0.05$  sec, which is the correct start time of the event. The similarity index returns to 1 at  $t = 0.068$  sec, which is the end time of the event. We can conclude that the event is a sustained event. Furthermore, the profile of the similarity index can also help us identify the *start time* and the *end time* of the event.

### III. CASE STUDIES

In this section, we consider the IEEE 33-bus power distribution test system to assess the performance of the proposed situational awareness method; see Fig. 3. We assume that two WMUs are available. WMU 1 is installed at bus 1 and WMU 2 is installed at bus 18. We use PSCAD [15] to generate the voltage waveform and current waveform measurements that are captured by WMU 1 at bus 1 and WMU 2 at bus 18.

#### A. Impact of the Type of Events

We study *sustained events*; such as high impedance and low impedance faults and capacitor switching; and *temporary events*, such as incipient faults and voltage sag. The synchronized Lissajous curve for high impedance fault was already shown in Fig. 2(e). The synchronized Lissajous curves for the other events are shown in Figs. 5(a)-(d). The normal condition is shown in *blue* and the event condition is shown in *red*.

As shown in Fig. 5(a), if a capacitor bank switches on, then the synchronized Lissajous curve oscillates for a very short period of time and then converges to a new different ellipse shape. This happens because of the new transient mode of oscillation originated from the capacitor bank [6]. As shown in Fig. 5(b), if a low impedance fault occurs, then the synchronized Lissajous curve becomes a larger ellipse during the event condition than during the normal condition. This originates from the fact that the low impedance fault only changes the characteristics of the fundamental mode of the waveform measurements and it does not create any new mode. This is in contrast to the case of the high impedance

fault that we saw in Fig. 2(e). As shown in Fig. 5(c), if an incipient fault occurs, then the shape of the synchronized Lissajous curve changes for the short period of the incipient fault, and then it turns back to the ellipse shape during the normal condition. This happens because incipient faults are *self-clearing*. As shown in Fig. 5(d), if a voltage sag occurs, then the synchronized Lissajous curve has a small spark once the voltage sag occurs, and then it creates a larger ellipse than the one during the normal condition. It finally returns back to the initial ellipse shape during the normal condition. This occurs because of the decrease in voltage for a short duration.

The above cases confirm the effectiveness of the proposed situational awareness method for a wide range of events.

#### B. Impact of the Location of Events

Next, we study the impact of the location of a high impedance fault on the synchronized Lissajous curve. The fault may happen at bus 4, 9, or 14. Fig. 6 shows the synchronized Lissajous curves during the event conditions. The impact of the location of the events is clearly visible in the synchronized Lissajous curves. When the high impedance fault occurs at bus 4, the synchronized Lissajous curve rotates *clockwise* with a significant decrease in its area. However, when the high impedance fault occurs at bus 14, the synchronized Lissajous curve rotates *counter-clockwise* with a significant increase in its area. We can conclude that the location of the event can drastically change the shape of the synchronized Lissajous curve. This is very informative in identifying the location of events in power distribution systems; *without* using any prior knowledge about the network.

It bears mentioning that if the event occurs on a lateral, then the two WMUs can still observe its impact [6]. Thus, the synchronized Lissajous curve can still provide information about the event on the lateral. We can better study such event if a WMU is available at the end of the lateral.

#### C. Impact of the Characteristics of Events

We also study the impact of the characteristics of a certain event on the shape of the synchronized Lissajous curve. The event that is studied here is capacitor bank switching. The size of the capacitor is 300 kVAR, 400 kVAR, or 500 kVAR. The location is the same for all these events. Fig. 7 shows the corresponding synchronized Lissajous curves. When the size of the capacitor changes, the shape of the synchronized Lissajous curve changes as well. When a 300 kVAR capacitor is switched on, the area of the synchronized Lissajous curve decreases. However, when a 500 kVAR capacitor is switched on, the area increases. We can conclude that the characteristics of the event significantly affects the shape of the synchronized Lissajous curve. This is very useful in understanding the characteristics of events in power distribution systems without prior information about the network.

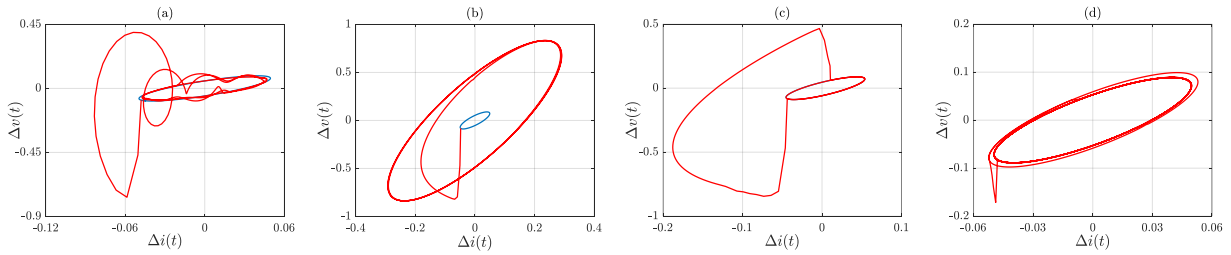


Fig. 5. The synchronized Lissajous curves for different events: (a) capacitor bank switching; (b) low impedance fault; (c) incipient fault; (d) voltage sag.

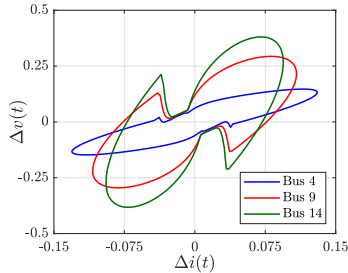


Fig. 6. The synchronized Lissajous curves when the high impedance fault occurs at three different locations: bus 4; bus 9; and bus 14.

#### D. Impact of Event on Phases

If the waveform data is available from all *three* phases, then we can construct *three* synchronized Lissajous curves, one for each phase. During the normal operating conditions, each of the three synchronized Lissajous curves would be ellipses. The three ellipses may or may not be identical depending on whether the network is balanced or unbalanced during normal operating conditions. In either case, once an event occurs, the synchronized Lissajous curve for each phase would change depending on how each phase is affected by the event. In particular, if the event is unbalanced, then changes in the shapes of the three synchronized Lissajous curves can be drastically different, making the phase of the event clearly distinguishable. Hence, the Lissajous curves based on three-phase synchronized waveform measurements can be very helpful for investigating unbalanced events.

## IV. CONCLUSIONS

A new method is proposed to achieve situational awareness using the data from synchronized WMUs in power distribution systems. The first step in the proposed method is to generate the equivalent two-port network of the distribution feeder via the substitution theorem. The second step is to obtain the voltage difference waveform and current difference waveform from WMUs. The final step is to construct the synchronized Lissajous curve based on the synchronized waveforms. Once an event occurs, the shape of the synchronized Lissajous curve changes. These changes can be used to detect the event. The results reveal that the shape of the synchronized Lissajous curves carries information about the type, location, and other characteristics of the event, which can be used in important applications, such as event detection and event location. Furthermore, the results confirm the effectiveness of the proposed method in identifying a wide range of events, including sustained events and temporary events. Importantly, the proposed method does not require prior knowledge of the network information; because it only

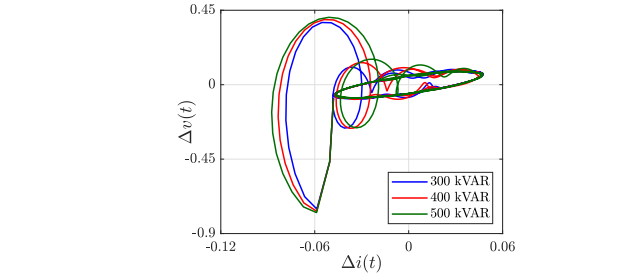


Fig. 7. The synchronized Lissajous curves when the capacitor switching occurs with three different sizes: 300 kVAR; 400 kVAR; and 500 kVAR.

requires data from as few as only two WMUs; one at the beginning of the feeder and one at the end of the feeder.

## REFERENCES

- [1] E. Boardman, "The role of integrated distribution management systems in smart grid implementations," in *Proc. IEEE PES GM*, Providence, RI, USA, 2010, pp. 1–6.
- [2] A. von Meier, E. Stewart, A. McEachern, M. Andersen, and L. Mehrmanesh, "Precision micro-synchrophasors for distribution systems: a summary of applications," *IEEE Trans. Smart Grid*, vol. 8, no. 6, pp. 2926–2936, Nov. 2017.
- [3] H. Mohsenian-Rad, E. Stewart, and E. Cortez, "Distribution synchrophasors: pairing big data with analytics to create actionable information," *IEEE Power Energy Mag.*, vol. 16, pp. 26–34, May 2018.
- [4] O. Ardakanian, Y. Yuan, R. Dobbe, A. von Meier, S. Low, and C. Tomlin, "Event detection and localization in distribution grids with phasor measurement units," in *Proc. IEEE PES GM*, Chicago, IL, 2017.
- [5] A. Shahsavari, M. Farajollahi, E. M. Stewart, E. Cortez, and H. Mohsenian-Rad, "Situational awareness in distribution grid using micro-PMU data: A machine learning approach," *IEEE Trans. Smart Grid*, vol. 10, no. 6, pp. 6167–6177, Nov. 2019.
- [6] M. Izadi and H. Mohsenian-Rad, "Event location identification in distribution networks using waveform measurement units," in *Proc. IEEE PES ISGT Europe*, The Hague, Netherlands, 2020, pp. 924–928.
- [7] iPSR sensors. [Online]. Available: <https://www.candura.com>
- [8] A. F. Bastos, S. Santoso, W. Freitas, and W. Xu, "Synchrowaveform measurement units and applications," in *Proc. IEEE PES GM*, Atlanta, GA, USA, 2019, pp. 1–6.
- [9] B. Gao, R. Torquato, W. Xu, and W. Freitas, "Waveform-based method for fast and accurate identification of subsynchronous resonance events," *IEEE Trans. Power Syst.*, vol. 34, pp. 3626–3636, Sep. 2019.
- [10] J. A. Wischkaemper, C. L. Benner, B. D. Russell, and K. Manivannan, "Application of waveform analytics for improved situational awareness of electric distribution feeders," *IEEE Trans. Smart Grid*, vol. 6, no. 4, pp. 2041–2049, Jul. 2015.
- [11] A. F. Bastos and S. Santoso, "Universal waveshape-based disturbance detection in power quality data using similarity metrics," *IEEE Trans. Power Deliv.*, vol. 35, no. 4, pp. 1779–1787, Aug. 2020.
- [12] C. A. Desoer and E. S. Kuh, *Basic Circuit Theory*. New Delhi, India: Tata McGraw-Hill, 2009.
- [13] D. Karacor, S. Nazlibilek, M. H. Sazli, and E. S. Akarsu, "Discrete Lissajous figures and applications," *IEEE Trans. Instrum. Meas.*, vol. 63, no. 12, pp. 2963–2972, Dec. 2014.
- [14] A. Abu-Siada and S. Islam, "A novel online technique to detect power transformer winding faults," *IEEE Trans. Power Deliv.*, vol. 27, no. 2, pp. 849–857, Apr. 2012.
- [15] Manitoba HVDC Research Centre. ver. 4.2 PSCAD/EMTDC (Software Package), Winnipeg, MB, Canada.