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MAGNETIC MOMENTS OF COMPOSITE FERMIONS

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There have been a considerable number of papers proposing composite models for leptons and quarks. Recently, Gluck and Lipkin have stated that reproducing the observed magnetic moments of these fermions presents a *serious* difficulty for these composite models. We show for a renormalizable theory that, in contrast to Gluck's and Lipkin's nonrelativistic arguments, a deeply bound system (with heavy constituent particle masses m_c) of (total) spin $1/2$, charge e and mass m has the magnetic moment $(e/2m)$ [1 + "usual" (QED + QCD + weak) corrections + $O(m/m_c)$ "new" binding corrections]. Although there remains the *considerable* dynamical problem of obtaining "light" bound fermions from heavy constituents, there is no separate, additional magnetic moment difficulty.

The large number of "fundamental" fermions is a problem of substantial difficulty and has not as yet been understood. The six quark flavors (including the expected t quark) together with the e , μ and τ leptons and their associated neutrinos now fill three generations of SU_5 families [1], $(\bar{5} + 10)$, of fermions. Several approaches have been suggested: (1) It is possible that there is a larger group G containing SU_5 in which all the fermion families are contained in a single low lying representation [2,3], e.g., SO_{4n+6} where n is the number (now three) of fermion families. (2) Alternatively, all the fermions might be assigned to a large irreducible representation of a smaller unifying group, SO_{10} or E_6 [3]. A third approach considered by many authors [4] is to treat the quarks and leptons as composites of new elementary constituents. (4) It is possible that only one fermion family is elementary and the other quarks and leptons are composite, as in our recent E_6 model [5].

In recent papers [6], Gluck and Lipkin have stated

that reproducing the observed magnetic moments of the leptons and quarks present a serious difficulty for composite models [such as in (3) and (4) above]. Lipkin notes that obtaining the moments of the electron and muon are particularly difficult "because the mass scale of the moment must be precisely the mass of the composite system rather than some function of basic building block masses". A composite model for the leptons would appear to be an absolute disaster. The *exact* agreement of QED with the astonishingly precise experimental results for $g - 2$ for the electron and muon stands today as one of the finest achievements in physics. The CERN muon storage ring group has recently reported [7] the final results (3) in an 18 year series of three extraordinarily accurate experiments:

$$a_\mu = \frac{1}{2}(g - 2) = (1\,162\,000 \pm 5000) \times 10^{-9}, \quad (1)$$

$$= (1\,166\,160 \pm 310) \times 10^{-9}, \quad (2)$$

$$= (1\,165\,924 \pm 8.5) \times 10^{-9}, \quad (3)$$

resulting in a measurement good to a part in 10^8 . The comparison of eq. (3) with the theoretical sixth order diagrams result of

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$$a_\mu^{\text{th}} = (1\,165\,921 \pm 8.3) \times 10^{-9}, \quad (4)$$

is impressive! An hadronic vacuum polarization contribution of $(66.7 \pm 8.1) \times 10^{-9}$ is absolutely necessary for the precise agreement between theory (4) and experiment (3). (There is a weak contribution of $(2.1 \pm 0.2) \times 10^{-9}$ to eq. (4) predicted by the electroweak theory which is below the sensitivity of eq. (3).) Thus any composite model would seem impossible. However, we show for a renormalizable theory that, contrary to the nonrelativistic arguments, a deeply bound system (with heavy constituent particle masses m_c) of spin $1/2$, charge e and mass m has the magnetic moment $(e/2m) [1 + \text{"usual"} (\text{QED} + \text{weak} + \text{QCD}) \text{ corrections} + O(m/m_c) \text{"new"} \text{ binding corrections}]$. (5)

In fact, if the composite fermion is made of a constituent boson of mass m_B and a constituent fermion of mass m_F , then for $m_B \gg m_F$, the "new" binding corrections to $g - 2$ are

$$O(mm_F/m_B^2). \quad (6)$$

There is indeed the *considerable* dynamical problem of obtaining light bound fermions from heavy constituents. However, there is no separate, *additional* magnetic moment difficulty.

Our result is readily obtained from the use of the relativistic sidewise dispersion relation [8] for the magnetic moment. All "new" contributions to the magnetic moment due to the bound state nature of our lepton of mass m start at the large constituent threshold m_c and are of order $O(m/m_c)$. Roughly, let us consider the range of size R of the deeply bound system to be

$$R \approx 1/m_{\text{ex}} + 1/m_c, \quad (7)$$

where m_{ex} is the vector boson mass associated with the (broken symmetry) exchange binding force. We expect to see no "new" contributions to the anomalous moment when

$$(mR) \ll 1. \quad (8)$$

Let us contrast this with the usual non-relativistic system with a size R . Now *all* our experience has been with composite systems having $mR \gg 1$: Atoms have $mR \approx 10^6$, nuclei $\approx 10^2$, and nucleons ≈ 5 . In all these *loosely* bound systems we measure the expected classical, nonrelativistic result that the magnetic mo-

ment is the vector sum of the moments of the constituents (electrons, nucleons and quarks, respectively) and the contribution from their orbital motion (plus "small" corrections). However, in the case of the light mass, deeply bound relativistic system, each of the heavy constituents (if nonrelativistic) would separately have a moment $\lesssim e/2m_c \ll e/2m$. We have no simple intuitive picture for the Dirac moment of the light, composite fermion where the constituents are relativistic, far off the mass shell and in the presence of many pairs. Instead, we start with our deeply bound composite system and consider relativistic *corrections* from its point-like structure.

The electromagnetic vertex, fig. 1a, of a spin $1/2$ fermion f with charge $+e$ and mass m has the usual description

$$M = ie \bar{u} (\gamma_\mu F_1 + (i\sigma_{\mu\nu}/2m) q^\nu F_2) u, \quad (9)$$

where $F_2(m)$ is the anomalous magnetic moment in units of $e/2m$. A clear dimensional argument concerning the magnitude of $F_2(m)$ is obtained from its side-wise dispersion relation [8]:

$$F_2(m) = \frac{1}{\pi} \int_{m^2}^{\infty} \frac{dW^2}{W^2 - m^2} \text{Im } F_2^*(W^2). \quad (10)$$

The absorptive part of the amplitude $f(W) \rightarrow f + \gamma$ is obtained by cutting this process as in fig. 1b and considering all possible on-shell intermediate states $\{n\}$. Now consider f to be a *deeply bound* composite system involving a very short range interaction [satisfying eqs. (7) and (8)] so that a "new" set of states $\{n_c\}$, where c denotes the real breakup of f involving its constituents, must be included at a large energy $W = m_c$ in addition to the "old" states $\{n_o\}$ not explicitly involving the constituents of f : $\{n\} = \{n_o\} + \{n_c\}$. Thus we can simply break up $\text{Im } F_2$ as $\text{Im } F_2 = \text{Im } F_2^0 + \text{Im } F_2^c$ where $\text{Im } F_2^0$ would include all the usual QED and QCD corrections to the fermion's magnetic moment involving states $\{n_o\}$ and $\text{Im } F_2^c$ gives the contribution from f 's composite nature^{†1}. For example, we readily calculate, using eq. (10), the usual $\alpha/2\pi$ contribution to F_2^0 of the one photon intermediate state in fig. 1c in which [8]

^{†1} We assume that there is only one bound state for a given channel.

Fig. 1. (a) The electromagnetic vertex of a spin 1/2 fermion f with charge e . (b) The absorptive part of the $J = 1/2$ partial wave amplitude $f(W) \rightarrow f + \gamma$ with an n particle intermediate state as used in the sidewise dispersion relation (10). (c) The one photon intermediate state which gives the usual $\alpha/2\pi$ contribution to the anomalous magnetic moment. (d) and (e) are the lowest order diagrams in which the constituent heavy fermion F (with charge q_F) and heavy boson B (with charge q_B) contribute to the anomalous magnetic moment of the composite f , g denotes the fFb coupling.

$$\text{Im } F_2^{0+}(W^2) = (\alpha/2W^4) m^2(W^2 - m^2). \quad (11)$$

The contribution F^c , from the composite nature of f

$$F_2^c(m) = \frac{1}{\pi} \int_{m_c^2}^{\infty} \frac{dW^2}{W^2 - m^2} \text{Im } F_2^{c+}(W^2), \quad (12)$$

is readily estimated since the asymptotic behavior of $\text{Im } F_2^{c+}(W^2)$ for any intermediate state n , is (mm_c/W^2) where m_c arises from the helicity flip nature of the anomalous moment, m from the definition (9) of F_2 , and W^{-2} from dimensional arguments in a renormalizable field theory. (The bound state structure of a composite f will introduce additional form factor powers of m_c^2/W^2 .) Then from eq. (12) we obtain the conclusion

$$F_2^c(m) \approx O(m/m_c). \quad (13)$$

Again, we stress that all the usual composite systems that we know, atoms, nuclei and hadrons, have an $mR > 1$ so that the expansion (13) in terms of mass parameters cannot be used. It is only for the assumed deeply bound system that this sidewise dispersion relation result (13) holds. From the amazing agreement

between theory (4) and experiment (3) for a_μ we must have

$$F_2^c(m) \lesssim 10^{-9}. \quad (14)$$

Thus on quite general grounds, a composite renormalizable theory for the muon is compatible with experiment if it has constituent mass

$$m_c \gtrsim 10^8 \text{ GeV}. \quad (15)$$

Clearly, much looser restrictions would be necessary for models of composite quarks.

From lowest order perturbation theory we find that the anomalous moments are proportional to the mass of the constituent fermion since the fermion mass violates chirality and allows the chiral or helicity flip nature of the anomalous moment. Consider the light fermion f to be composed of a massive fermion F and boson B (with masses m_F and m_B) as in figs. 1d and 1e. Then for $m_B \gg m_F$ we find

$$F_2^c \approx O(mm_F/m_B^2) \quad (16)$$

and the extra reduction by the ratio m_F/m_B allows constituent masses to be smaller than in (15). In particular, for B having spin 0 (scalar or pseudoscalar) we have to lowest order in m from figs. 1d and 1e, respectively, with $e < 0$ and $r = (m_F^2/m_B^2)$,

$$F_2^c = -\frac{q_F}{e} \frac{1}{4\pi} \left(\frac{g^2}{4\pi} \right) \left(\frac{mm_F}{m_B^2} \right) \frac{1}{(1-r)^3} \quad (17)$$

$$\times [(3-r)(1-r) + 2 \ln r],$$

$$F_2^c = -\frac{q_B}{e} \frac{1}{4\pi} \left(\frac{g^2}{4\pi} \right) \left(\frac{mm_F}{m_B^2} \right) \frac{1}{(1-r)^3} \quad (18)$$

$$\times [(1-r^2) + 2r \ln r].$$

For B a massive spin one gauge boson we have calculated the result of fig. 1d in the unitary gauge to eliminate unphysical Higgs boson graphs and obtain for vector coupling (axial vector coupling changes the sign);

$$F_2^c = -\frac{q_F}{e} \frac{1}{4\pi} \left(\frac{g^2}{4\pi} \right) \left(\frac{mm_F}{m_B^2} \right) \frac{1}{(1-r)^3} \quad (19)$$

$$\times [(1-r)(4-7r+r^2) + 10r \ln r].$$

The diagram of fig. 1e for a massive vector gauge boson B has been evaluated by Fujikawa et al. [9] [eq. (3.12)]

and the result may be given for vector coupling (axial vector coupling changes the sign)

$$F_2^c = -\frac{q_B}{e} \frac{1}{4\pi} \left(\frac{g^2}{4\pi} \right) \left(\frac{mm_F}{m_B^2} \right) \frac{1}{(1-r)^3} \times [(1-r)(4-11r+r^2) - 6r^2 \ln r] . \quad (20)$$

All of these cases show the result (16) for $m_F < m_B$ or $r \rightarrow 0$. (We *again* stress that for the *usual* cases of composite systems, i.e., atoms, nuclei, and hadrons, $mR \gtrsim 1$ and eqs. (17)–(20) become of order 1 in the mass dependence.) The QCD corrections to fig. 1d and 1e due to gluons coupling to quarks, F and B should (in an asymptotically free theory) give only additional powers of $\ln(m_B^2/m^2)$ or $\ln(m_F^2/m^2)$ to eqs (17)–(20), as in the QCD corrections to the penguin diagrams for non-leptonic weak decays [10].

Clearly there is a considerable dynamical problem in obtaining very low mass bound fermions from heavy constituents. However, we have shown ^{‡2} that there is no additional magnetic moment difficulty as suggested by nonrelativistic arguments [6].

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^{‡2} Similar conclusions have been reached using methods complementary to ours by Brodsky and Drell [11]. Eq. (5) was suggested by Brodsky [12] and more recently mentioned by Visnjic-Triantafillou [13] and Derman [13].

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